MATLAB PDE Solving

Boundary Value Problems

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CONTENTS

EXAMPLE 27.10

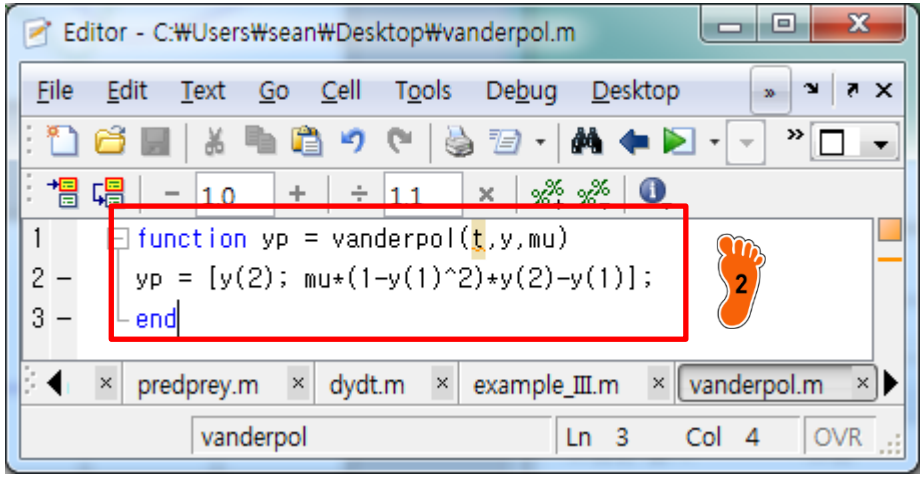
van der Pol equation

$$\frac{d^2 y_1}{dt^2} - \mu\left(1 - y_1^2\right)\frac{dy_1}{dt} + y_1 = 0$$

initial condition

$$t = 0, y_1 = 1, \frac{dy_1}{dt} = 1$$

$$\xrightarrow{\text{convert process}} \begin{cases} \dfrac{dy_1}{dt} = y_2 \\ \dfrac{dy_2}{dt} = \mu\left(1 - y_1^2\right) y_2 + y_1 \end{cases}$$



1 stiff 한 정도가 mu 값에 따라서 변하는 van der Pol equation.

In 1920 the Dutch physicist Balthasar van der Pol studied a differential equation that describes the circuit of a vacuum tube.
It has been used to model other phenomenon such as the human heartbeat by Johannes van der Mark.

2 매틀랩 함수

EXAMPLE 27.10: MAIN & RESULT

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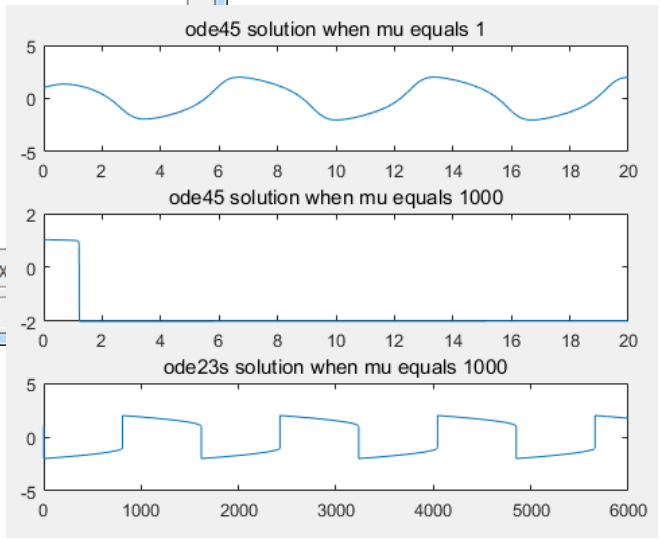
```
1 -  clc; clear all; close all;
2
3 -  %% ode45 solution when mu equals 1
4 -  [t_45a,y_45a] = ode45(@vanderpol,[0 20],[1,1],[],1);
5 -  %% ode45 solution when mu equals 1000
6 -  [t_45b,y_45b] = ode45(@vanderpol,[0 20],[1,1],[],1000);
7 -  %% ode23s solution when mu equals 1000
8 -  [t_23s,y_23s] = ode23s(@vanderpol,[0 6000],[1,1],[],1000);
9 -  %% plot solution
10 - subplot(3,1,1)
11 - plot(t_45a,y_45a)
12 - title('ode45 solution when mu equals 1')
13 - subplot(3,1,2)
14 - plot(t_45b,y_45b(:,1))
15 - title('ode45 solution when mu equals 1000')
16 - subplot(3,1,3)
17 - plot(t_23s,y_23s(:,1))
18 - title('ode23s solution when mu equals 1000')
```

Heun.m | example II.m | predprev.m | dvdt.m | ex

Ln 1

t_23s	1827x1 double
t_45a	213x1 double
t_45b	67705x1 double
y_23s	1827x2 double
y_45a	213x2 double
y_45b	67705x2 double



1. mu = 1 일 때 실행 코드

2. mu = 1000 일 때 ode45 로 푸는 코드
결과는 그래프 2 번째 그림이며 20초 계산하는데 67705 pnt. 필요

3. mu = 1000 일 때 ode23s 로 푸는 코드
결과는 그래프 3번째 그림이며 6000초 계산하는데 1827 pnt. 필요(ode45로 20초 푸는 pnt개수의 약 1/37)

따라서 ode45로는 해당 미분방정식을 푸는 것은 매우 비 효율적(또한 결과의 정확도 보장하기 어려움)

CASE STUDY I

Background. Electric circuits where the current is time-variable rather than constant are common. A transient current is established in the right-hand loop of the circuit shown in Fig. 28.11 when the switch is suddenly closed.

Equations that describe the transient behavior of the circuit in Fig. 28.11 are based on Kirchhoff's law, which states that the algebraic sum of the voltage drops around a closed loop is zero (recall Sec. 8.3). Thus,

$$L\frac{di}{dt} + Ri + \frac{q}{C} - E(t) = 0 \tag{28.9}$$

where $L(di/dt)$ = voltage drop across the inductor, L = inductance (H), R = resistance (Ω), q = charge on the capacitor (C), C = capacitance (F), $E(t)$ = time-variable voltage source (V), and

$$i = \frac{dq}{dt} \tag{28.10}$$

$$E = E_0 \sin(\omega t)$$

$$L = 1\ \text{H}$$

$$E_0 = 1\ \text{V}$$

$$C = 0.25\ \text{C}$$

$$\omega^2 = 3.5\ \text{s}^2$$

$$R = 0$$

RLC circuit equation

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} - E(t) = 0$$

$$\frac{d^2q}{dt^2} = \frac{1}{L}\left(E(t) - R\frac{dq}{dt} - \frac{q}{C}\right) \xrightarrow{\text{convert process}} \begin{cases} q = y_1 \\ i = \dfrac{dq}{dt} = y_2 \\ \dfrac{di}{dt} = \dfrac{d^2q}{dt^2} = \dfrac{1}{L}\left(E(t) - Ry_2 - \dfrac{y_1}{C}\right) \end{cases}$$

initial condition

$t = 0,\ q = 0,\ i = 0$

CASE STUDY I

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```
function yp = circuit(t,y,L,E0,omega,R,C)
yp = [y(2); 1/L*E0*sin(omega*t)-R*y(2)-y(1)/C];
end
```

q i

case_I.m × circuit.m ×

circuit Ln 3 Col 4 OVR

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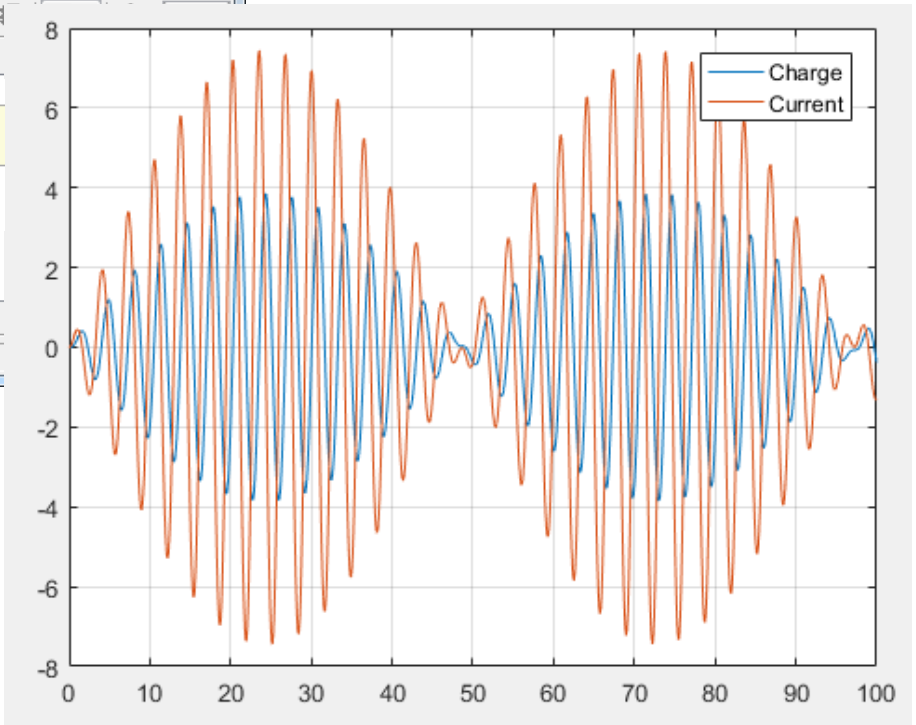
```
clc; clear all; close all;
%% ode45 solution
[t,y] = ode45(@circuit,[0 100],[0,0],[],1,1,sqrt(3.5),0,0.25);
%% plot solution
plot(t,y)
```

case_I.m × circuit.m ×

script Ln



CASE STUDY II

Therefore, in polar coordinates ($\alpha = d^2\theta/dt^2$),

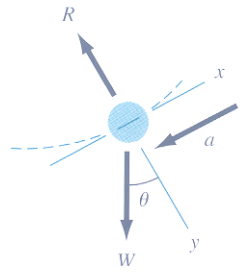
$$-W \sin\theta = \frac{Wl}{g}\alpha = \frac{Wl}{g}\frac{d^2\theta}{dt^2}$$

or

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0 \qquad (28.15)$$

This apparently simple equation is a second-order nonlinear differential equation. In general, such equations are difficult or impossible to solve analytically. You have two choices regarding further progress. First, the differential equation might be reduced to a form that can be solved analytically (recall Sec. PT7.1.1), or second, a numerical approximation technique can be used to solve the differential equation directly. We will examine both of these alternatives in this example.

FIGURE 28.16
A free-body diagram of the swinging pendulum showing the forces on the particle and the acceleration.



pendulum equation

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta$$

initial condition

$$t = 0,\ \theta = 0, \frac{d\theta}{dt} = 0$$

convert process

$$\begin{cases} \theta = y_1 \\ \dfrac{d\theta}{dt} = y_2 \\ \dfrac{d^2\theta}{dt^2} = -\dfrac{g}{l}\sin\theta \end{cases}$$

$$\sin\theta \approx \theta$$

$$g = 9.81\ \text{m/s}^2$$

$$\theta_0 = \pi / 4$$

$$l = 0.6096\ \text{m}$$

CASE II: RESULT

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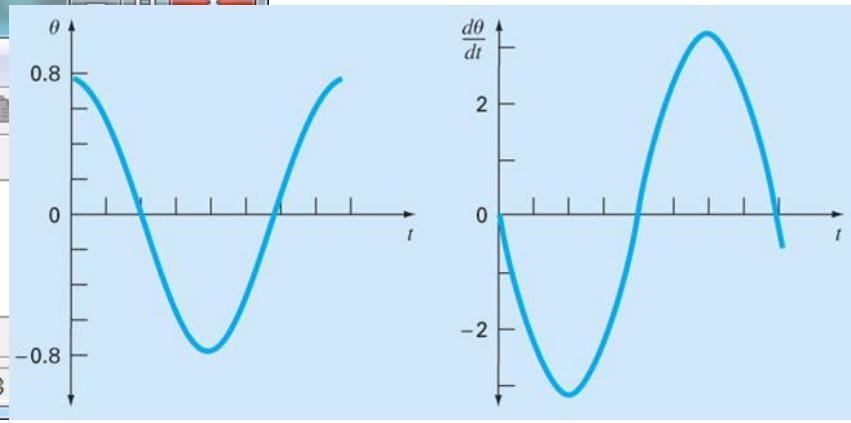
```
1  function yp = pendulum(t,y,g,l)
2  yp = [y(2); -g/l*y(1)];
3  end
```

Θ dΘ/dt

case_I.m* | circuit.m | assignment6.m | pendulum.m | case_II.m

pendulum Ln 3



편집기 - E:₩CAE실습₩CAE_자료₩3rd week m-file₩case_II.m

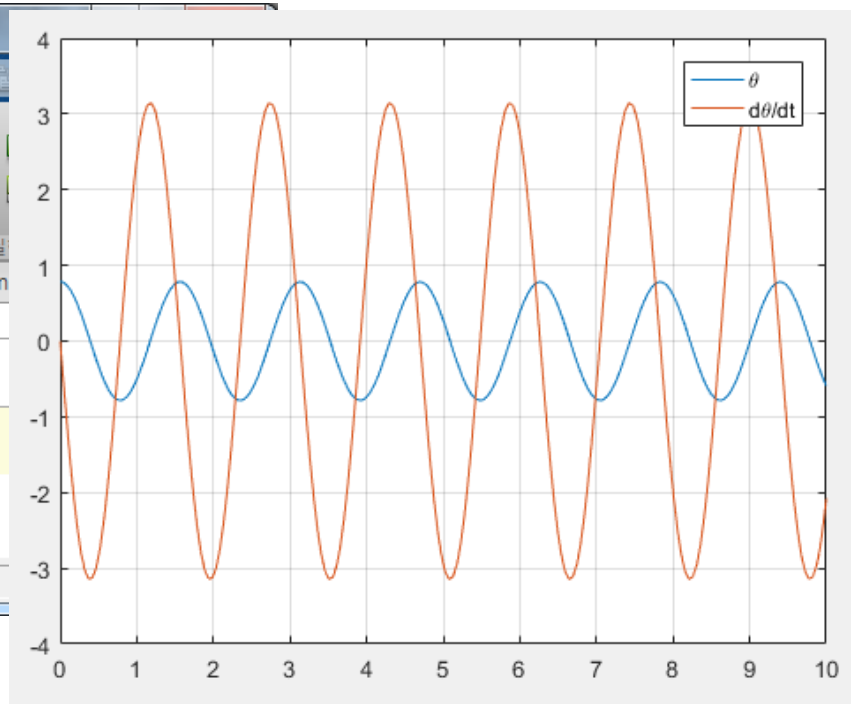
편집기 퍼블리시 보기

새로 만들기 열기 저장 파일 찾기 비교 인쇄 파일 이동 찾기 탐색 편집 중단점 실행 실행 및 진행

+23 getAscendingRank.m | assignment1_2_21.m | sec_RK.m | case_I.m

```
1  clc; clear all; close all;
2  %% ode45 solution
3  [t,y] = ode45(@pendulum,[0 10],[pi/4,0],[],9.81,0.6096);
4  %% plot solution
5  plot(t,y)
```

스크립트



- **Boundary Value Problems for ODE**

  - ✓ **Shooting method for linear ODE**
  - ✓ **Shooting method for nonlinear ODE**
  - ✓ **Finite difference method**

EXAMPLE

- Heat balance for a long, thin rod
  - Not insulated along its length
  - Steady state

$$\frac{d^2T}{dx^2} + h'(T_a - T) = 0$$

$$\left.\begin{aligned} T(0) &= T_1 = 40°C \\ T(L) &= T_2 = 200°C \end{aligned}\right\} \text{boundary conditions}$$

$$T_a = 20°C \left(\text{temperature of the surrounding air}\right)$$

$$L = 10\,m$$

$$h' = 0.01\,m^{-2} \left(\text{heat transfer coefficient}\right)$$

rate of heat dissipation to the surrounding air

analytic solution: $T = 73.4523e^{0.1x} - 53.4523e^{-0.1x} + 20$

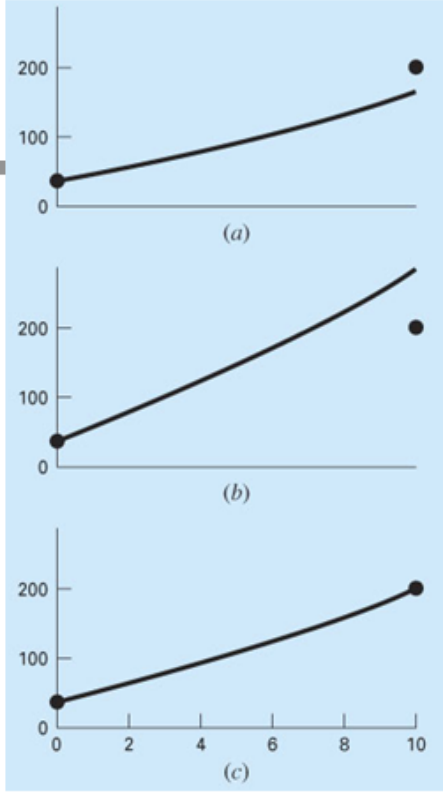
EXAMPLE

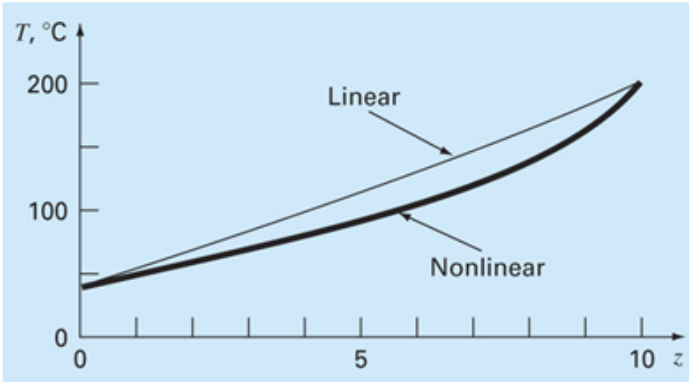
Shooting Method

$$\begin{cases} \dfrac{dT}{dx} = z \\ \dfrac{dz}{dx} = h'(T - T_a)\,[\text{linear}] \quad \dfrac{dz}{dx} = h''(T - T_a)^4\,[\text{nonlinear}] \end{cases}$$

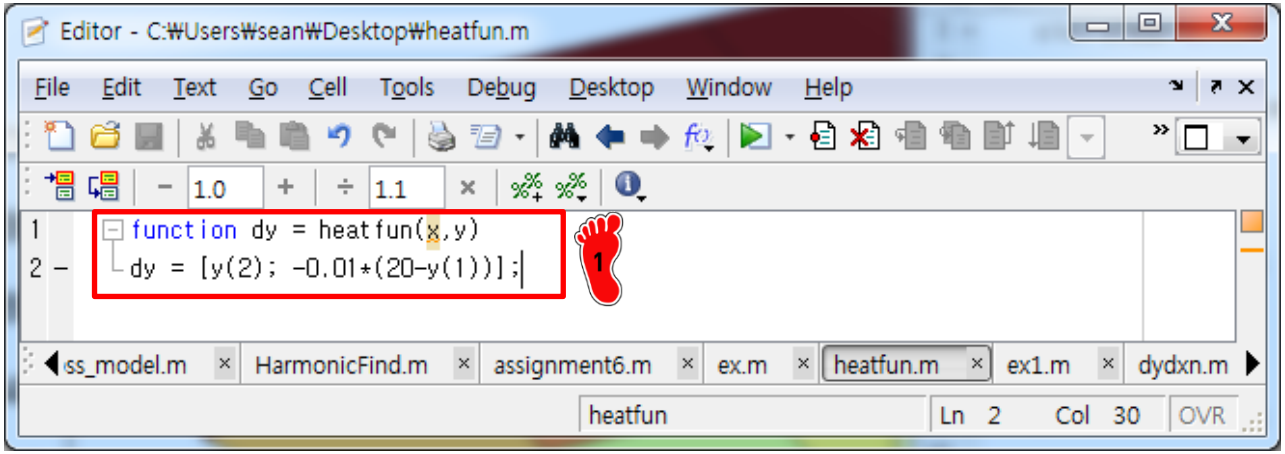
$4th$ RK, $h = 2, T(10) = 200$

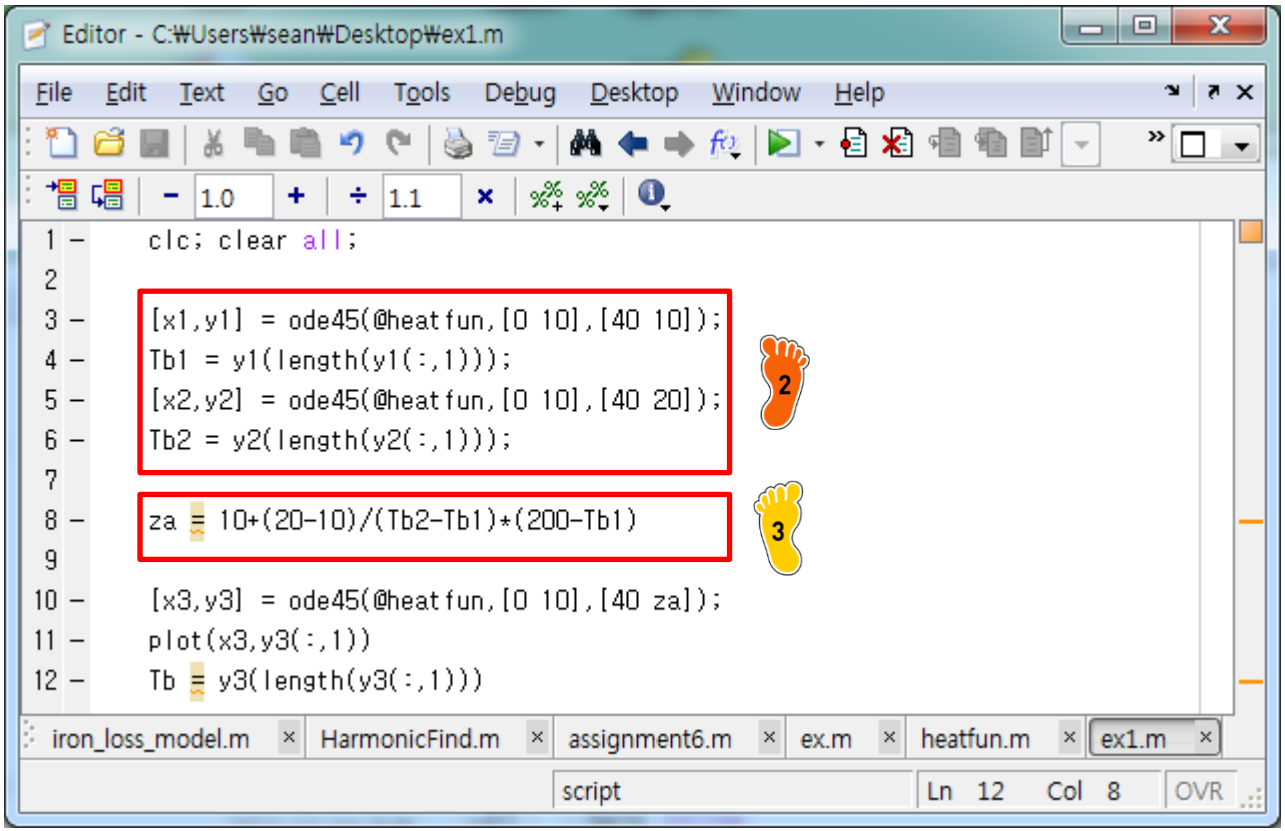
$$\rightarrow \begin{cases} \text{guess / linear: } \begin{cases} z(0) = 10 \rightarrow T(10) = 168.3797 \\ z(0) = 20 \rightarrow T(10) = 285.8980 \end{cases} \\ \xrightarrow[\text{interpolation}]{\text{linear}} z(0) = 12.6907 \\ \text{non-linear: } T(10) = f(z(0)) \\ \rightarrow g(z(0)) = f(z(0)) - 200 \end{cases}$$





SHOOTING METHOD: LINEAR





1 아래에 표기된 시스템 ODE 함수를 heatfun 이름으로 저장

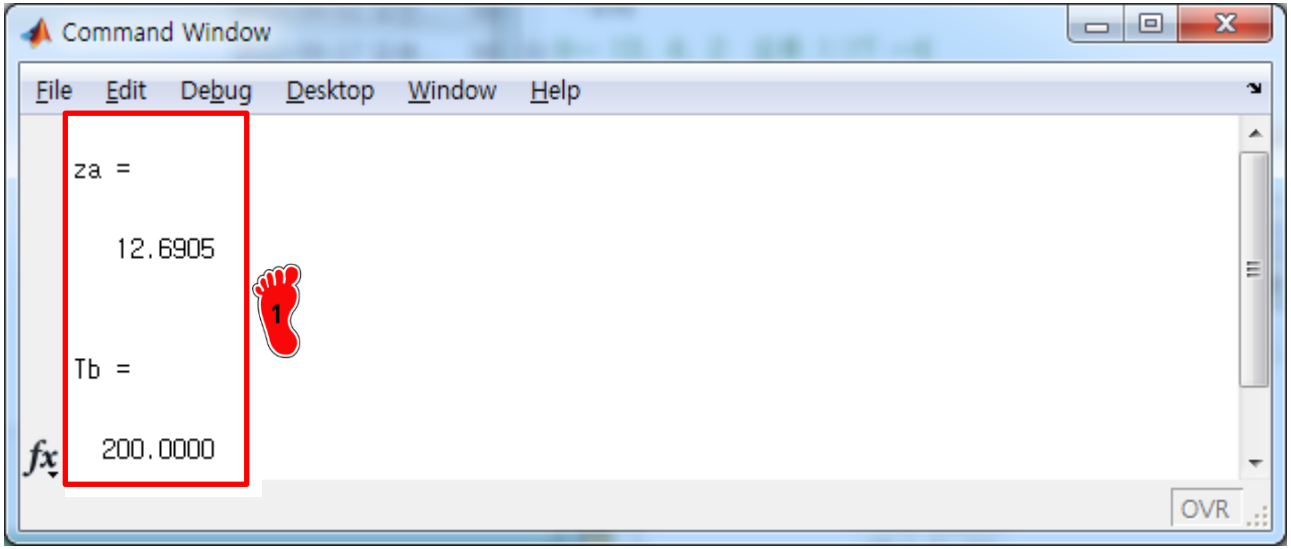
$$\left.\begin{aligned}\frac{dT}{dx} &= z \\ \frac{dz}{dx} &= -0.01(20-T)\end{aligned}\right\} \xrightarrow{\substack{y(1)=T \\ y(2)=z}} \begin{cases}\dfrac{dy(1)}{dx} = y(2) \\ \dfrac{dy(2)}{dx} = -0.01(20-y(1))\end{cases}$$

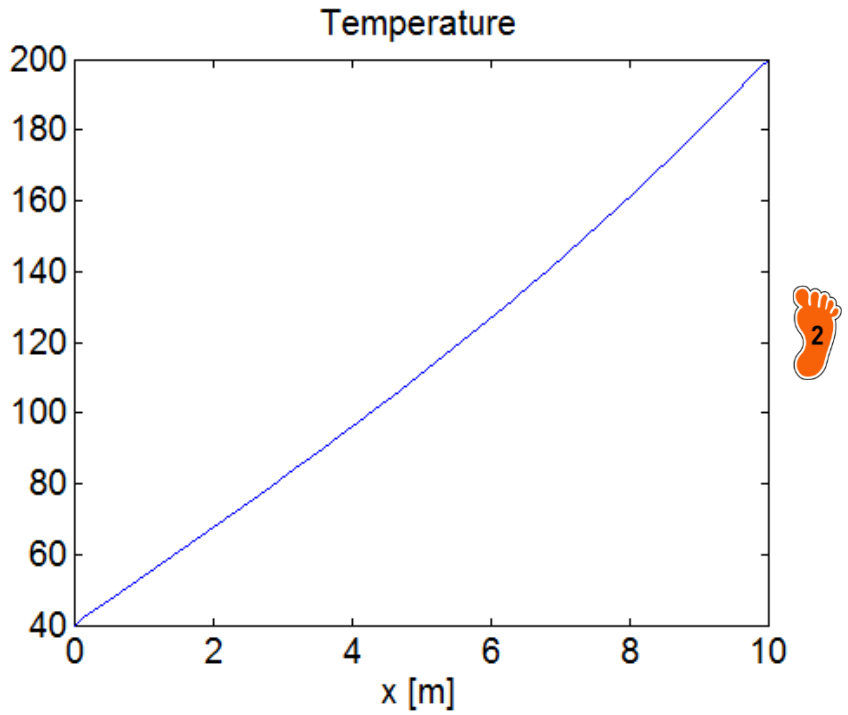
2 z_{a1} 이 10 일 때와 z_{a2} 가 20 일 때 값을 T_{b1} 과 T_{b2} 에 저장

3 아래 수식을 이용하여 z_a 를 선형 보간

$$z_a = z_{a1} + \frac{z_{a2} - z_{a1}}{T_{b2} - T_{b1}}\left(T_b - T_{b1}\right)$$

RESULT



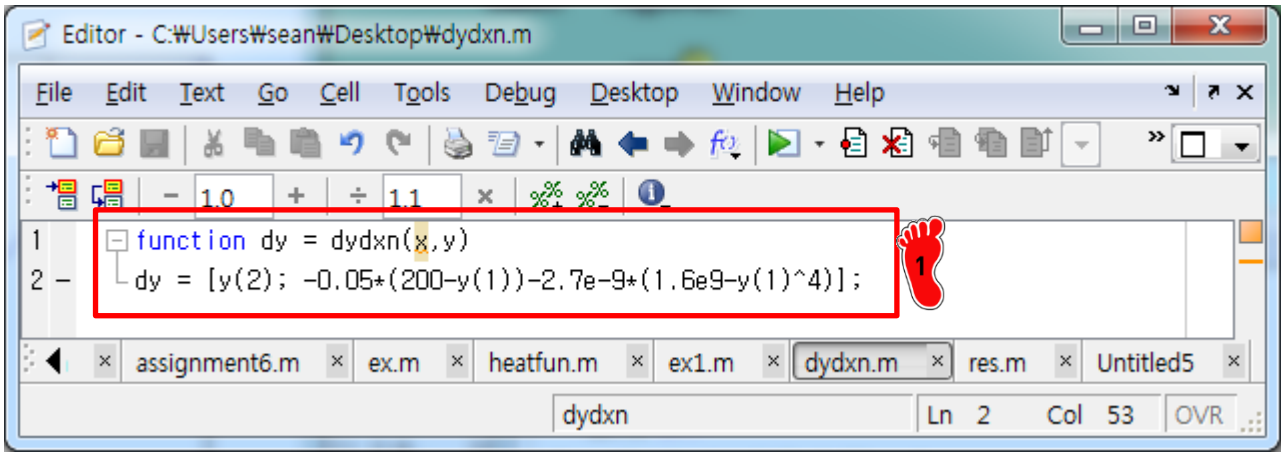


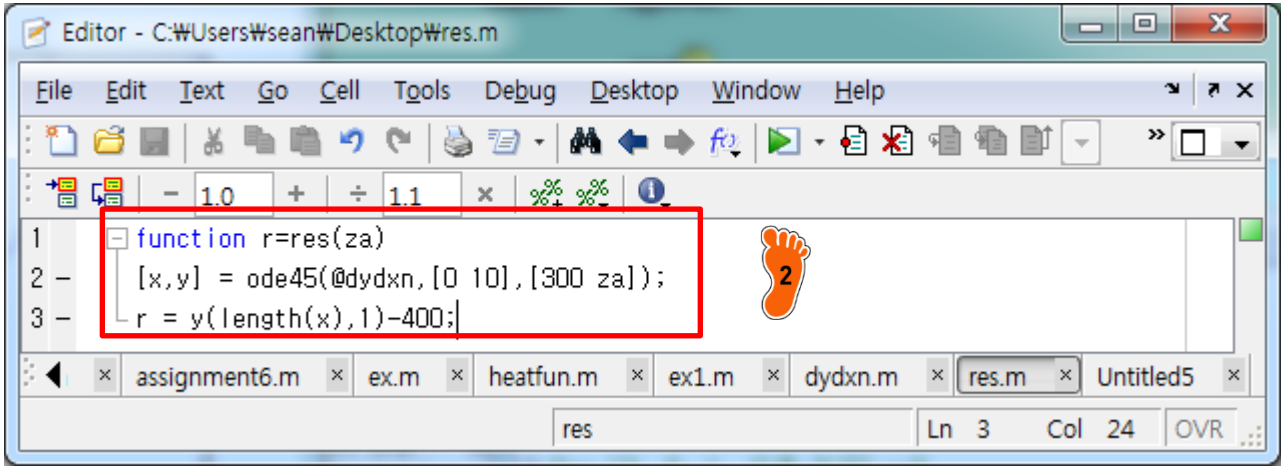
1 z_a 가 12.6905 일 때 B.C. 이 일치

2 결과 그래프

SHOOTING METHOD: NONLINEAR

$$\begin{cases} \dfrac{dT}{dx} = z \\ \dfrac{dz}{dx} = -h'\left(T_\infty - T\right) - h''\left(T_\infty^4 - T^4\right) \end{cases}$$





1 아래에 표기된 상수대로 왼쪽의 ODE 함수를 입력

$$h' = 0.05\ \mathrm{m}^{-2}$$

$$h'' = 2.7\times10^{-9}\ \mathrm{K}^{-3}\mathrm{m}^{-2}$$

$$T_\infty = 200\ \mathrm{K}$$

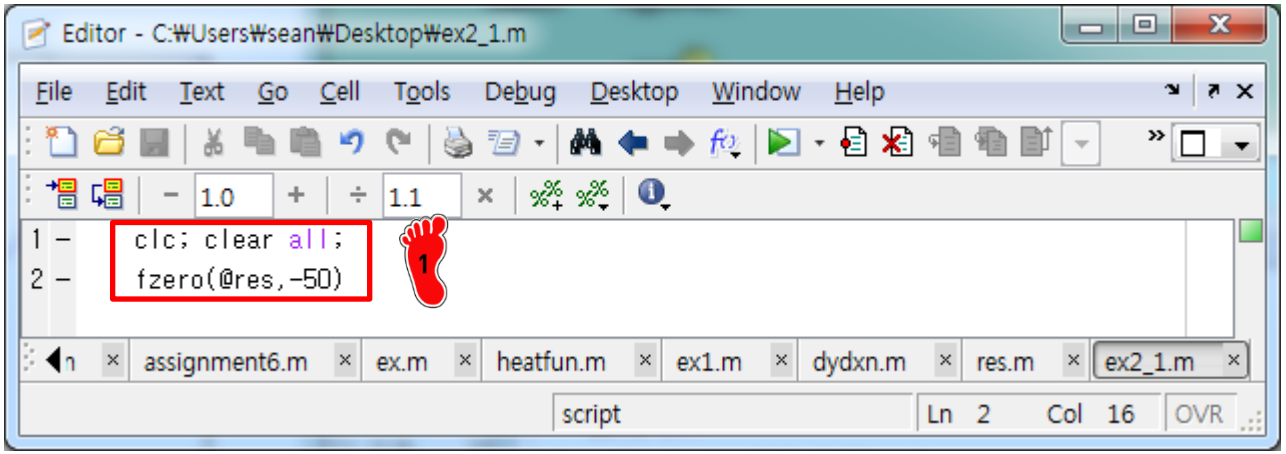
$$T(0) = 300\ \mathrm{K}$$

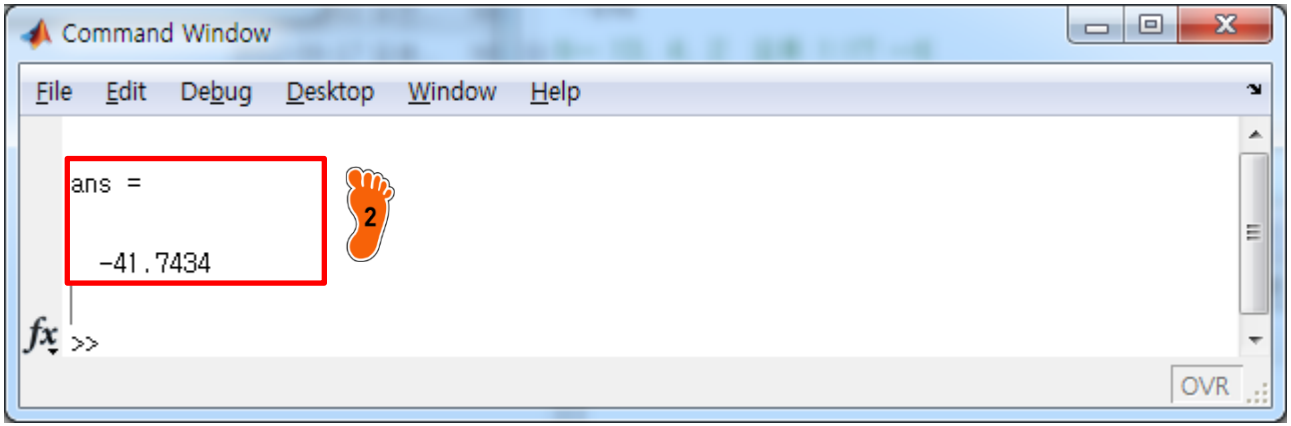
$$T_\infty(10) = 400\ \mathrm{K}$$

2 근을 찾기 위한 함수 설정

r 값(residual: 해석해와 수치해의 차이)이 0 일 경우 정확한 z_a 값

SHOOTING METHOD: NONLINEAR





1 fzero(@funname, IV)

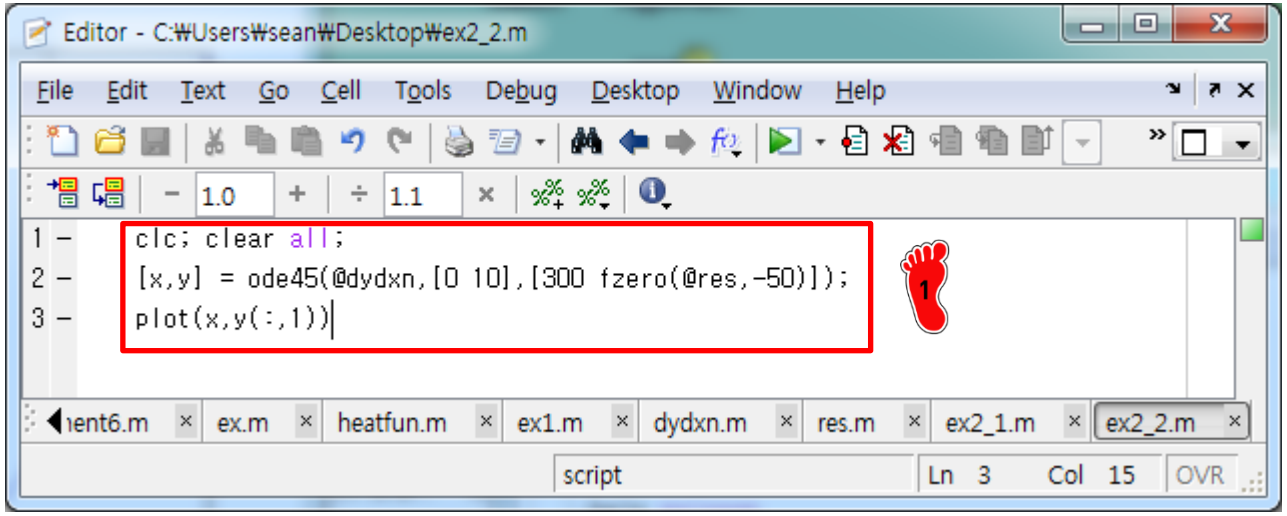
funname의 근을 찾는 매틀랩 함수
IV 는 initial value 를 뜻함

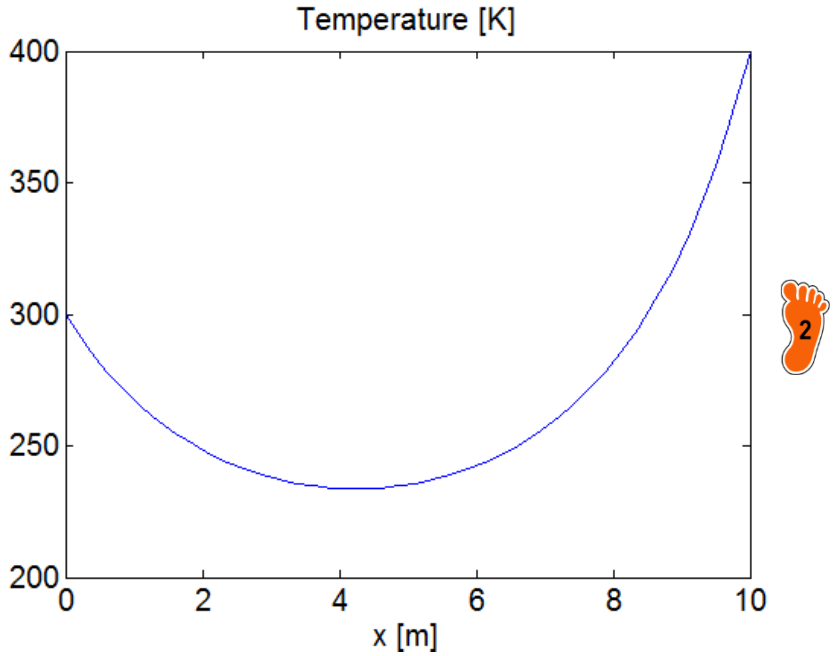
[주의] 비선형함수이므로 초기값에 따른 영향 존재

2 결과 z_a = -41.7434

즉, 이 값을 이용하여 ode 함수를 풀면 nonlinear BC 를 만족하는 해

RESULT





1 혹은 initial 값을 입력하는 곳에 fzero 함수를 직접 입력 가능

2 결과 그래프

FINITE DIFFERENCE METHOD

Forward FDM	$f'(x_i)=\frac{f(x_{i+1})-f(x_i)}{h}$	$O(h)$
	$f'(x_i)=\frac{-f(x_{i+2})+4f(x_{i+1})-3f(x_i)}{2h}$	$O(h^2)$
	$f''(x_i)=\frac{f(x_{i+2})-2f(x_{i+1})+f(x_i)}{h^2}$	$O(h)$
	$f''(x_i)=\frac{-f(x_{i+3})+4f(x_{i+2})-5f(x_{i+1})+2f(x_i)}{h^2}$	$O(h^2)$
Backward FDM	$f'(x_i)=\frac{f(x_i)-f(x_{i-1})}{h}$	$O(h)$
	$f'(x_i)=\frac{3f(x_i)-4f(x_{i-1})+f(x_{i-2})}{2h}$	$O(h^2)$
	$f''(x_i)=\frac{f(x_i)-2f(x_{i-1})+f(x_{i-2})}{h^2}$	$O(h)$
	$f''(x_i)=\frac{2f(x_i)-5f(x_{i-1})+4f(x_{i-2})-f(x_{i-3})}{h^2}$	$O(h^2)$
Centered FDM	$f'(x_i)=\frac{f(x_{i+1})-f(x_{i-1})}{2h}$	$O(h^2)$
	$f'(x_i)=\frac{-f(x_{i+2})+8f(x_{i+1})-8f(x_{i-1})+f(x_{i-2})}{12h}$	$O(h^4)$
	$f''(x_i)=\frac{f(x_{i+1})-2f(x_i)+f(x_{i-1})}{h^2}$	$O(h^2)$
	$f''(x_i)=\frac{-f(x_{i+2})+16f(x_{i+1})-30f(x_i)+16f(x_{i-1})-f(x_{i-2})}{12h^2}$	$O(h^4)$

FINITE DIFFERENCE METHOD

$$\frac{d^2T}{dx^2} = -h'(T_\infty - T) \rightarrow \frac{T_{i-1} - 2T_i + T_{i+1}}{\Delta x^2} = -h'(T_\infty - T_i)$$

$$\rightarrow -T_{i-1} + (2 + h'\Delta x^2)T_i - T_{i+1} = h'\Delta x^2 T_\infty$$

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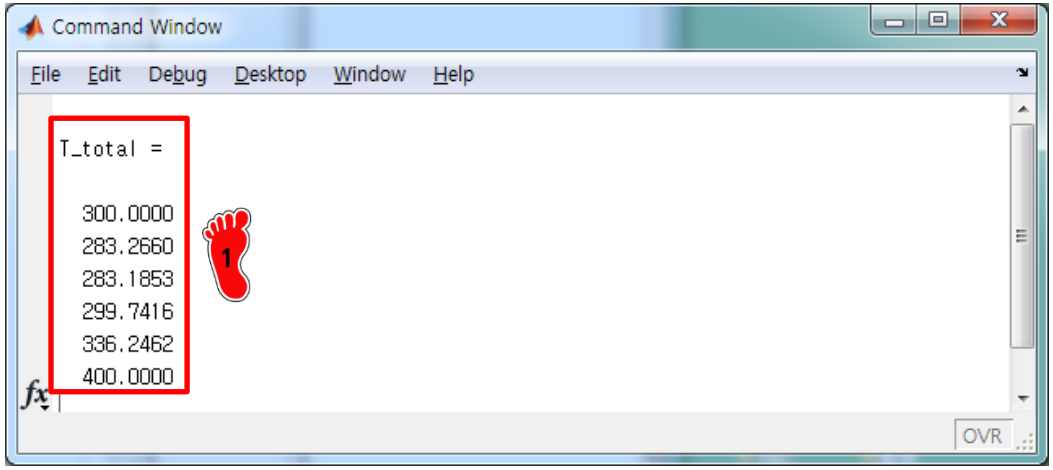
```
1 -  clc; clear all
2
3 -  step_size = 2; h = 0.05; T_inf = 200;
4 -  length = 10; T_a = 300; T_b = 400;
5
6 -  number = length/step_size;
7
8 -  K = zeros((number+1),(number+1));
9 -  f = ones((number-1),1)*h*step_size^2*T_inf;
10
11 - f(1,1) = f(1,1) + T_a;
12 - f((number-1),1) = f((number-1),1) + T_b;
13
14 - i = 1:3;
15 - a = [-1, 2+h*step_size^2,-1];
16
17 - for j = 1:(number+1)
18 -     K(j,i+j-1) = a;
19 - end
20
21 - K = K(1:(number-1),2:number);
22
23 - T_sol = K₩f;
24 - T_total = [T_a;T_sol;T_b]
25 - plot([0:step_size:length],T_total)
```

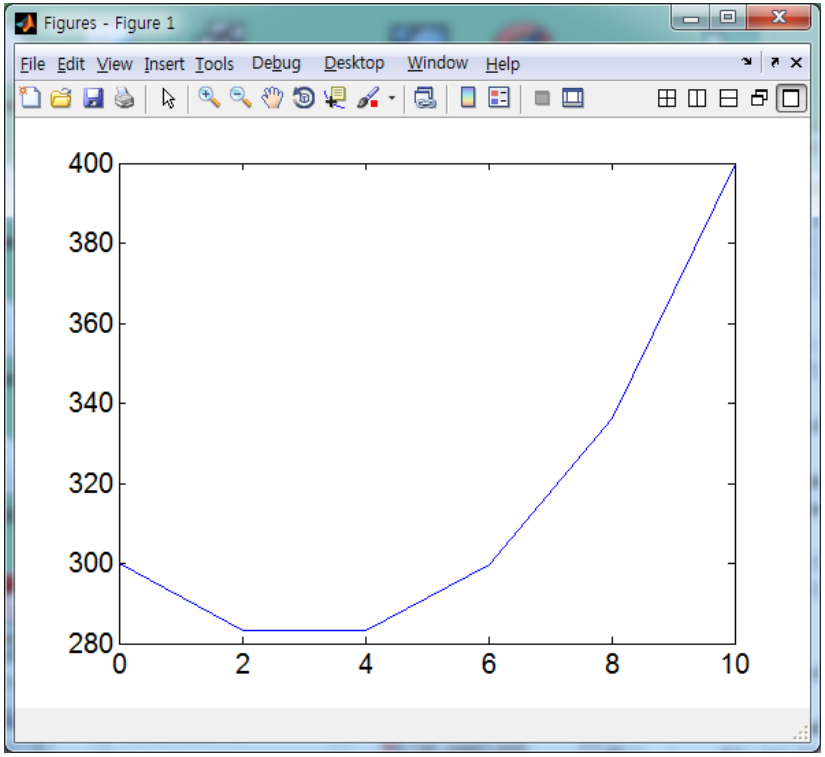
heatfun.m | ex1.m | dydxn.m | res.m | ex2_1.m | ex2_2.m | FDM_ex1.m

script Ln 9 Col 44 OVR

1. step size 및 계수 선언
2. 매트릭스 및 온도 행렬 선언
3. 매트릭스 구성
4. 솔루션 계산 및 결과 출력

RESULT





1 결과 온도 벡터

2 결과 그래프

- **Boundary Value Problems for PDE**

  ✓ **Finite Difference Method: Elliptic equation**

LAPLACE EQUATION

– PDE → algebraic difference equation

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0 \leftarrow \begin{cases} \dfrac{\partial^2 T}{\partial x^2}=\dfrac{T_{i+1,j}-2T_{i,j}+T_{i-1,j}}{\Delta x^2} \\ \dfrac{\partial^2 T}{\partial y^2}=\dfrac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{\Delta y^2} \end{cases}$$

$$\rightarrow \frac{T_{i+1,j}-2T_{i,j}+T_{i-1,j}}{\Delta x^2}+\frac{T_{i,j+1}-2T_{i,j}+T_{i,j-1}}{\Delta y^2}=0$$

$$\xrightarrow{\Delta x=\Delta y} T_{i+1,j}+T_{i-1,j}+T_{i,j+1}+T_{i,j-1}-4T_{i,j}=0$$

apply boundary conditions (fixed/Dirihlet)

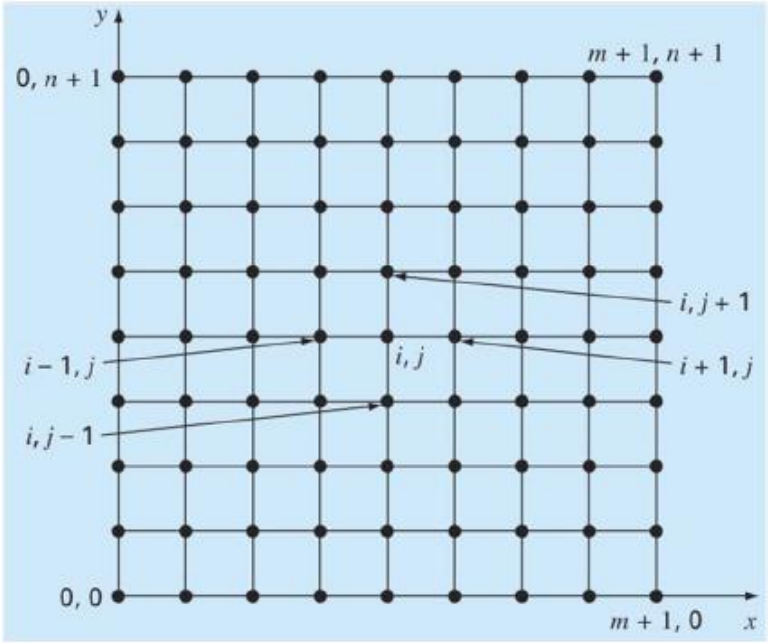
$$@(1,1): T_{21}+\underbrace{T_{01}}_{75°C}+T_{12}+\underbrace{T_{10}}_{0°C}-4T_{11}=0 \rightarrow 4T_{11}-T_{21}-T_{12}=75$$

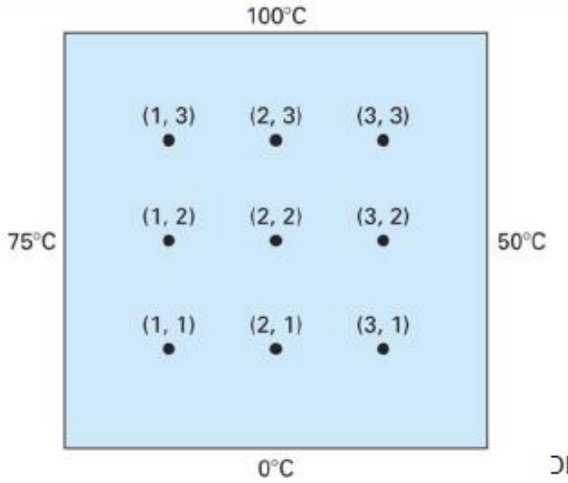
$$@(2,1): T_{31}+T_{11}+T_{22}+\underbrace{T_{20}}_{0°C}-4T_{21}=0 \rightarrow -T_{11}+4T_{21}-T_{31}-T_{22}=0$$

$$@(3,1): \underbrace{T_{41}}_{50°C}+T_{21}+T_{32}+\underbrace{T_{30}}_{0°C}-4T_{31}=0 \rightarrow -T_{21}+4T_{31}-T_{32}=50$$

$$\vdots$$

9 equations





$$[K_{ij}]\{T_i\}=\{f_i\}$$

$[K_{ij}]$: coefficient matrix, $\{T_i\}$: solution vector, $\{f_i\}$: force vector

MATLAB CODE

```
clc; clear all;
T_left = 75; T_right = 50; T_bottom = 0; T_upper = 100;
nx = 3; ny = 3;
T_numbering = [];
iter = 0;
i = [1:nx]';
j = ones(nx,1);
for k = 1:ny
    T_numbering = [T_numbering;i,j*k];
end
f = zeros(nx*ny,1);
for j = 1:ny
    for i = 1:nx
        T_numbering(:,3) = zeros(nx*ny,1);
        a = find(T_numbering(:,1) == i+1 & T_numbering(:,2) == j);
        b = find(T_numbering(:,1) == i-1 & T_numbering(:,2) == j);
        c = find(T_numbering(:,1) == i & T_numbering(:,2) == j+1);
        d = find(T_numbering(:,1) == i & T_numbering(:,2) == j-1);
        e = find(T_numbering(:,1) == i & T_numbering(:,2) == j);
        T_numbering(a,3) = -1;
        T_numbering(b,3) = -1;
        T_numbering(c,3) = -1;
        T_numbering(d,3) = -1;
        T_numbering(e,3) = 4;
        iter = iter + 1;
        T(iter,:) = T_numbering(:,3)';
        a = [a 1];
        b = [b 1];
        c = [c 1];
        d = [d 1];
        if length(a) == 1
            f(iter) = f(iter) + T_right;
        end
        if length(b) == 1
            f(iter) = f(iter) + T_left;
        end
        if length(c) == 1
            f(iter) = f(iter) + T_upper;
        end
        if length(d) == 1
            f(iter) = f(iter) + T_bottom;
        end
    end
end
Temp_temp1 = T\f;
iter = 0;
for i = ny:-1:1
    for j = 1:nx
        iter = iter + 1;
        Temp_temp2(i,j) = Temp_temp1(iter,1);
    end
end
Temp = zeros(ny+2,nx+2);
Temp(:,1) = T_left;
Temp(:,nx+2) = T_right;
Temp(1,:) = T_upper;
Temp(ny+2,:) = T_bottom;
Temp(2:ny+1,2:nx+1) = Temp_temp2;
x=[0:1:nx+1]; y=[ny+1:-1:0];
surf(x,y,Temp)
xlabel('x'); ylabel('y');zlabel('Temp'); colorbar
```

INITIALIZATION

clc; clear all;

```
T_left = 75; T_right = 50; T_bottom = 0; T_upper = 100;
```

```
nx = 3; ny = 3;
```

```
T_numbering = [];
iter = 0;
```

```
i = [1:nx]';
j = ones(nx,1);

for k = 1:ny
    T_numbering = [T_numbering;i,j*k];
end

f = zeros(nx*ny,1);
```

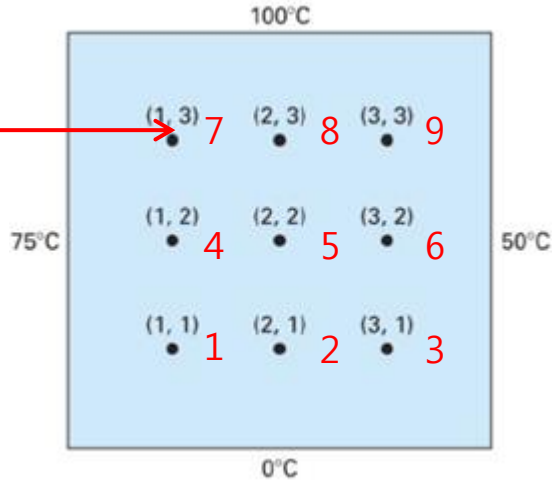
1 경계조건 선언

2 경계를 제외한 node 개수 선언

3 매트릭스를 구성하기 위한 정보를 저장

numbering

	1	2
1	1	1
2	2	1
3	3	1
4	1	2
5	2	2
6	3	2
7	1	3
8	2	3
9	3	3



COEFFICIENT MATRIX

1 매트릭스 생성

2 nx = 3, ny = 3 일 때의 매트릭스 요소 정보

```
for j = 1:ny
    for i = 1:nx
        T_numbering(:,3) = zeros(nx*ny,1);
        a = find(T_numbering(:,1) == i+1 & T_numbering(:,2) == j);
        b = find(T_numbering(:,1) == i-1 & T_numbering(:,2) == j);
        c = find(T_numbering(:,1) == i & T_numbering(:,2) == j+1);
        d = find(T_numbering(:,1) == i & T_numbering(:,2) == j-1);
        e = find(T_numbering(:,1) == i & T_numbering(:,2) == j);
        T_numbering(a,3) = -1;
        T_numbering(b,3) = -1;
        T_numbering(c,3) = -1;
        T_numbering(d,3) = -1;
        T_numbering(e,3) = 4;
        iter = iter + 1;

        T(iter,:) = T_numbering(:,3)';
```

2

T <9x9 double>

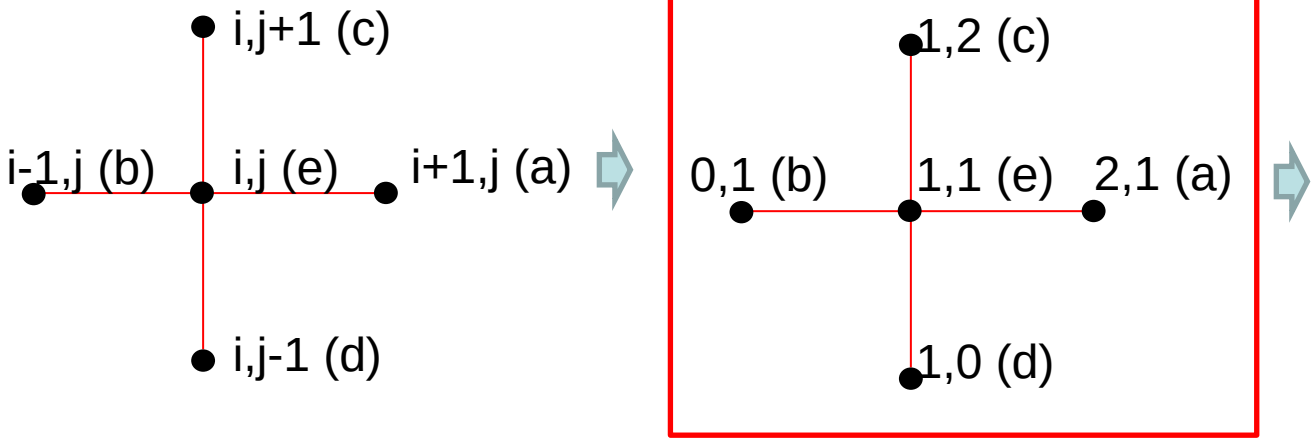
	1	2	3	4	5	6	7	8	9
1	4	-1	0	-1	0	0	0	0	0
2	-1	4	-1	0	-1	0	0	0	0
3	0	-1	4	0	0	-1	0	0	0
4	-1	0	0	4	-1	0	-1	0	0
5	0	-1	0	-1	4	-1	0	-1	0
6	0	0	-1	0	-1	4	0	0	-1
7	0	0	0	-1	0	0	4	-1	0
8	0	0	0	0	-1	0	-1	4	-1
9	0	0	0	0	0	-1	0	-1	4

COEFFICIENT MATRIX

```
a = find(T_numbering(:,1) == i+1 & T_numbering(:,2) == j);
b = find(T_numbering(:,1) == i-1 & T_numbering(:,2) == j);
c = find(T_numbering(:,1) == i & T_numbering(:,2) == j+1);
d = find(T_numbering(:,1) == i & T_numbering(:,2) == j-1);
e = find(T_numbering(:,1) == i & T_numbering(:,2) == j);
```

find 는 벡터 혹은 매트릭스 내 요소 중 같은 값을 갖는 요소의 위치정보를 저장하는 명령어

i = 1, j = 1 일 때 T_numbering 벡터의 위치 정보를 저장



	1	2
1	1	1
2	2	1
3	3	1
4	1	2
5	2	2
6	3	2
7	1	3
8	2	3
9	3	3

a=1
b=[]
c=4
d=[]
e=1

COEFFICIENT MATRIX

```
T_numbering(a,3) = -1;
T_numbering(b,3) = -1;
T_numbering(c,3) = -1;
T_numbering(d,3) = -1;
T_numbering(e,3) = 4;
```

	1	2	3
1	1	1	4
2	2	1	-1
3	3	1	0
4	1	2	-1
5	2	2	0
6	3	2	0
7	1	3	0
8	2	3	0
9	3	3	0

1 위치 정보를 이용하여 계수 값을 T_numbering 내에 입력

2 입력된 3번째 벡터를 coefficient matrix 의 첫 번째 열로 저장

```
T(iter,:) = T_numbering(:,3)';
```

	1	2	3	4	5	6	7	8	9
1	4	-1	0	-1	0	0	0	0	0

FORCE VECTOR

```
a = [a 1];
b = [b 1];
c = [c 1];
d = [d 1];
```

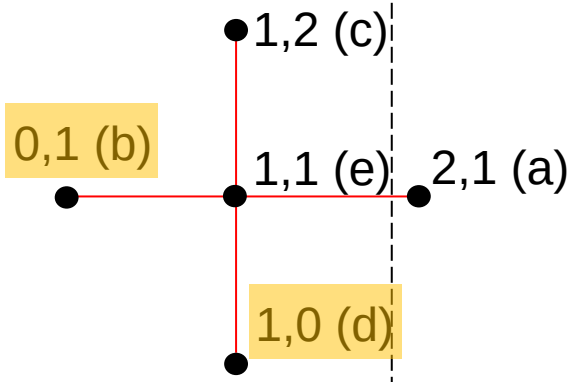
→

```
a = [2,1]
b = 1
c = [4,1]
d = 1
```

경계조건을 입력할 번호를 구분, length 명령어를 이용하여 값이 1 일 경우 경계에 존재하는 번호

a 가 경계일 경우 항상 오른쪽 경계
마찬가지 논리로 입력할 경계 위치를 결정

```
if length(a) == 1
    f(iter) = f(iter) + T_right;
end
if length(b) == 1
    f(iter) = f(iter) + T_left;
end
if length(c) == 1
    f(iter) = f(iter) + T_upper;
end
if length(d) == 1
    f(iter) = f(iter) + T_bottom;
end
    end
end
```



경계조건이 입력될 부분

POST-PROCESSING

```
Temp_temp1 = T\f;
```

1 solve

2 솔루션 매트릭스 재배열

재배열을 하는 것은 온도분포가 1열 벡터로 출력되기 때문

surf 명령어로 후처리를 할 예정이므로 매트릭스로 재배열이 필요

```
iter = 0;
for i = ny:-1:1
    for j = 1:nx
        iter = iter + 1;
        Temp_temp2(i,j) = Temp_temp1(iter,1);
    end
end
```

2

Temp

5x5 double

	1	2	3	4	5
1	100	100	100	100	100
2	75	78.5714	76.1161	69.6429	50
3	75	63.1696	56.2500	52.4554	50
4	75	42.8571	33.2589	33.9286	50
5	0	0	0	0	0

3 후처리

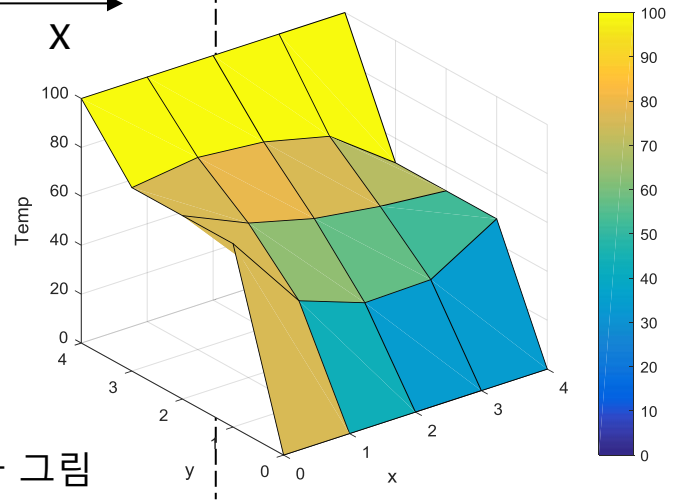
```
Temp = zeros(ny+2,nx+2);
Temp(:,1) = T_left;
Temp(:,nx+2) = T_right;
Temp(1,:) = T_upper;
Temp(ny+2,:) = T_bottom;
Temp(2:ny+1,2:nx+1) = Temp_temp2;
x=[0:1:nx+1]; y=[ny+1:-1:0];
surf(x,y,Temp)
xlabel('x'); ylabel('y');zlabel('Temp'); colorbar
```

y

0

x

3



결과 그림

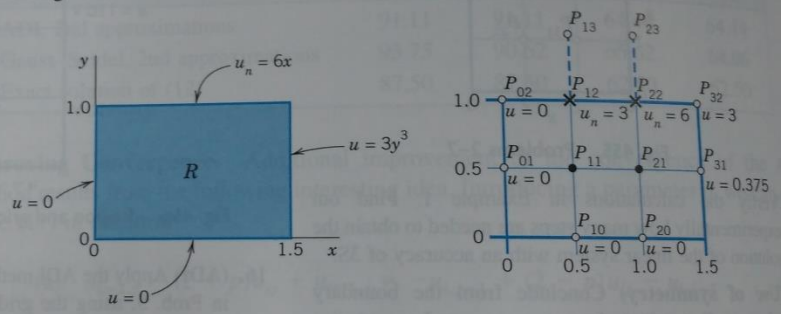
ASSIGNMENT

EXAMPLE 1 Mixed Boundary Value Problem for a Poisson Equation

Solve the mixed boundary value problem for the Poisson equation

$$\nabla^2 u = u_{xx} + u_{yy} = f(x, y) = 12xy$$

shown in Fig. 457a.



(a) Region R and boundary values (b) Grid ($h = 0.5$)

Fig. 457. Mixed boundary value problem in Example 1

(3)
$$\begin{bmatrix} -4 & 1 & 1 & 0 \\ 1 & -4 & 0 & 1 \\ 2 & 0 & -4 & 1 \\ 0 & 2 & 1 & -4 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{21} \\ u_{12} \\ u_{22} \end{bmatrix} = \begin{bmatrix} 0.75 \\ 1.125 \\ 1.5 - 3 \\ 0 - 6 \end{bmatrix} = \begin{bmatrix} 0.75 \\ 1.125 \\ -1.5 \\ -6 \end{bmatrix}.$$

(The entries 2 come from u_{13} and u_{23}, and so do -3 and -6 on the right). The solution of (3) (obtained by Gauss elimination) is as follows; the exact values of the problem are given in parentheses.

$u_{12} = 0.866$ (exact 1) $u_{22} = 1.812$ (exact 2)

$u_{11} = 0.077$ (exact 0.125) $u_{21} = 0.191$ (exact 0.25). ■

$$u_{xx} + u_{yy} = f(x, y) = 12xy$$

Dirichet B.C

$$u(0, y) = u(y, 0) = 0,\ u(1.5, y) = 3y^3$$

Neumann B.C

$$u_y(x, 1) = 6x$$

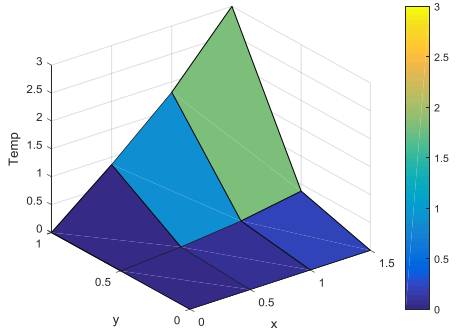
1) $h = 0.5$

```
u12 =          u22 =

    0.8665         1.8121

u11 =          u21 =

    0.0769         0.1910
```



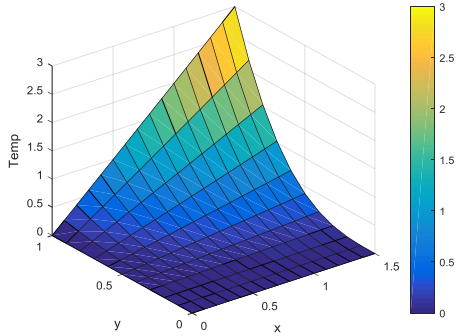
2) $h = 0.1$

```
u12 =          u22 =

    0.9941         1.9910

u11 =          u21 =

    0.1229         0.2474
```



Ref. : Advanced Engineering Mathematics, 9th edition, Chap. 21, pp918-919

APPENDIX

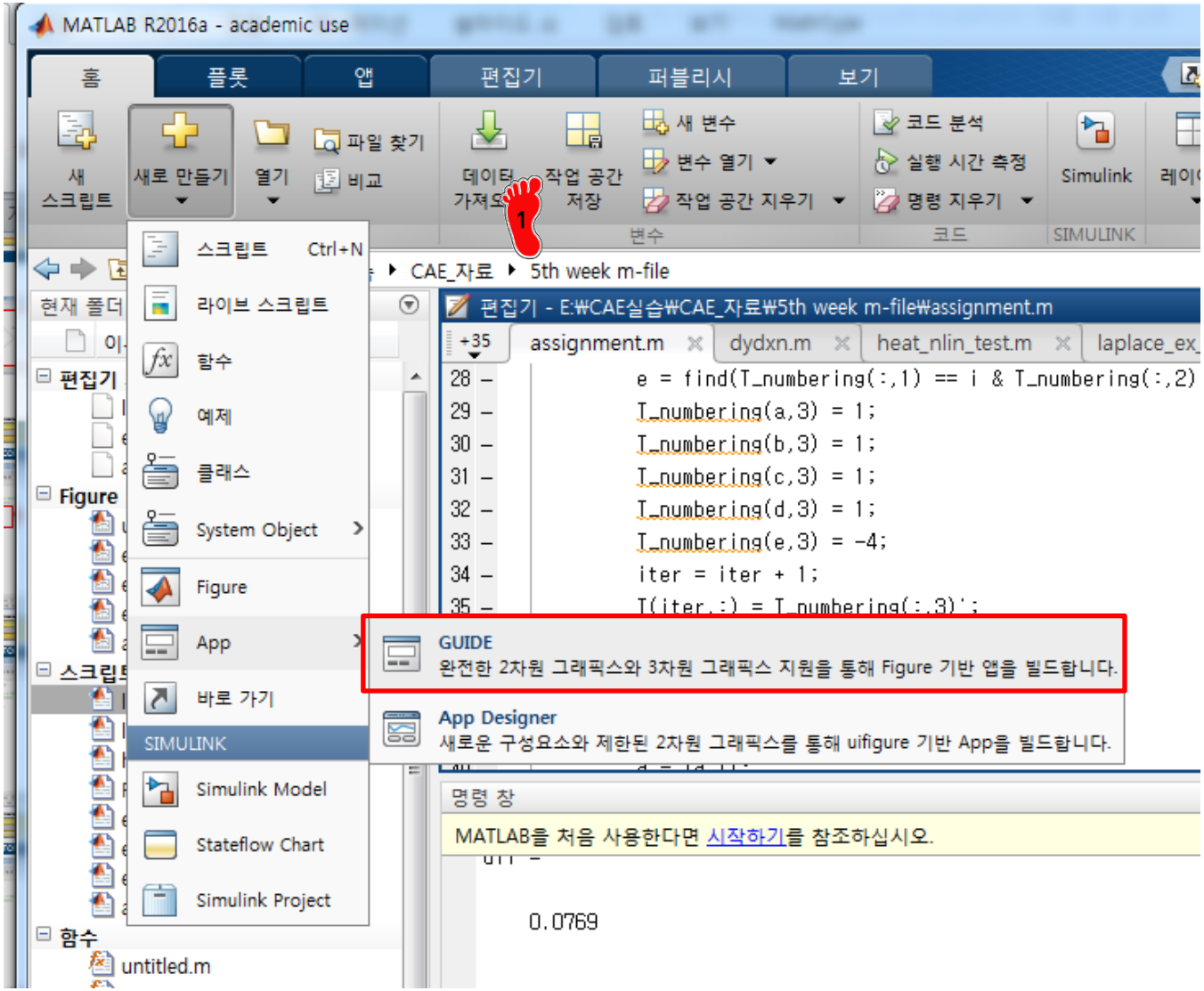
- **MATLAB GUI implementation**

  - ✓ **GUI**

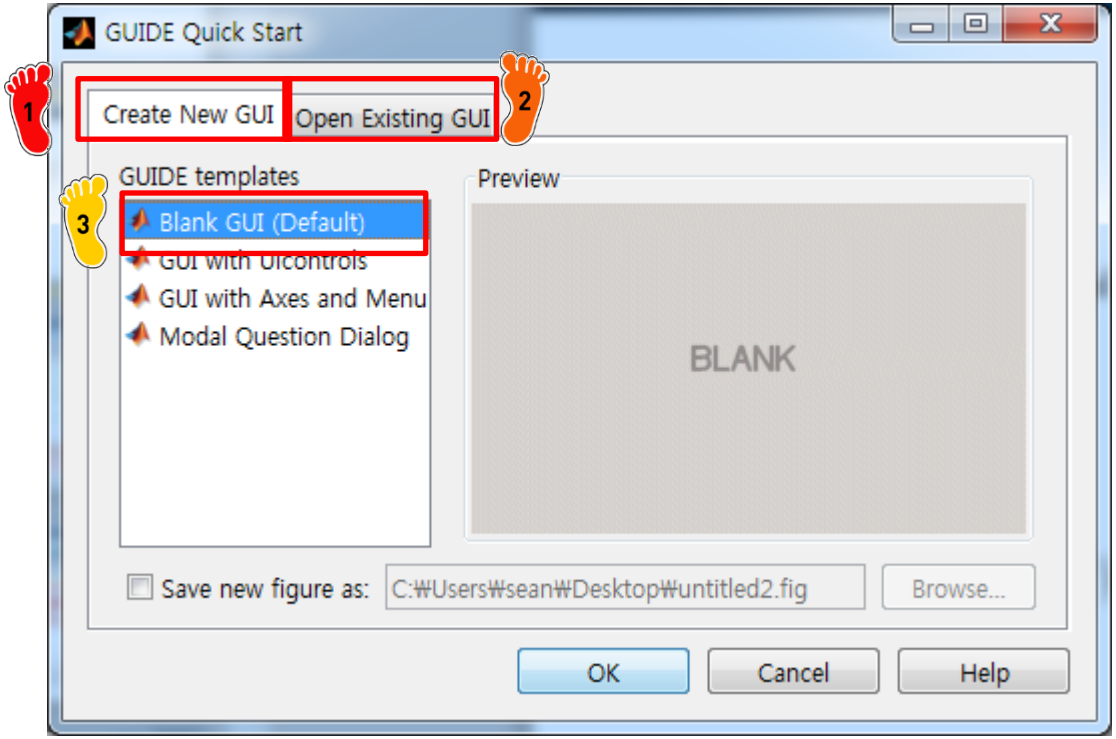
  - ✓ **Example**

START

1 home – New – APP - GUIDE 클릭



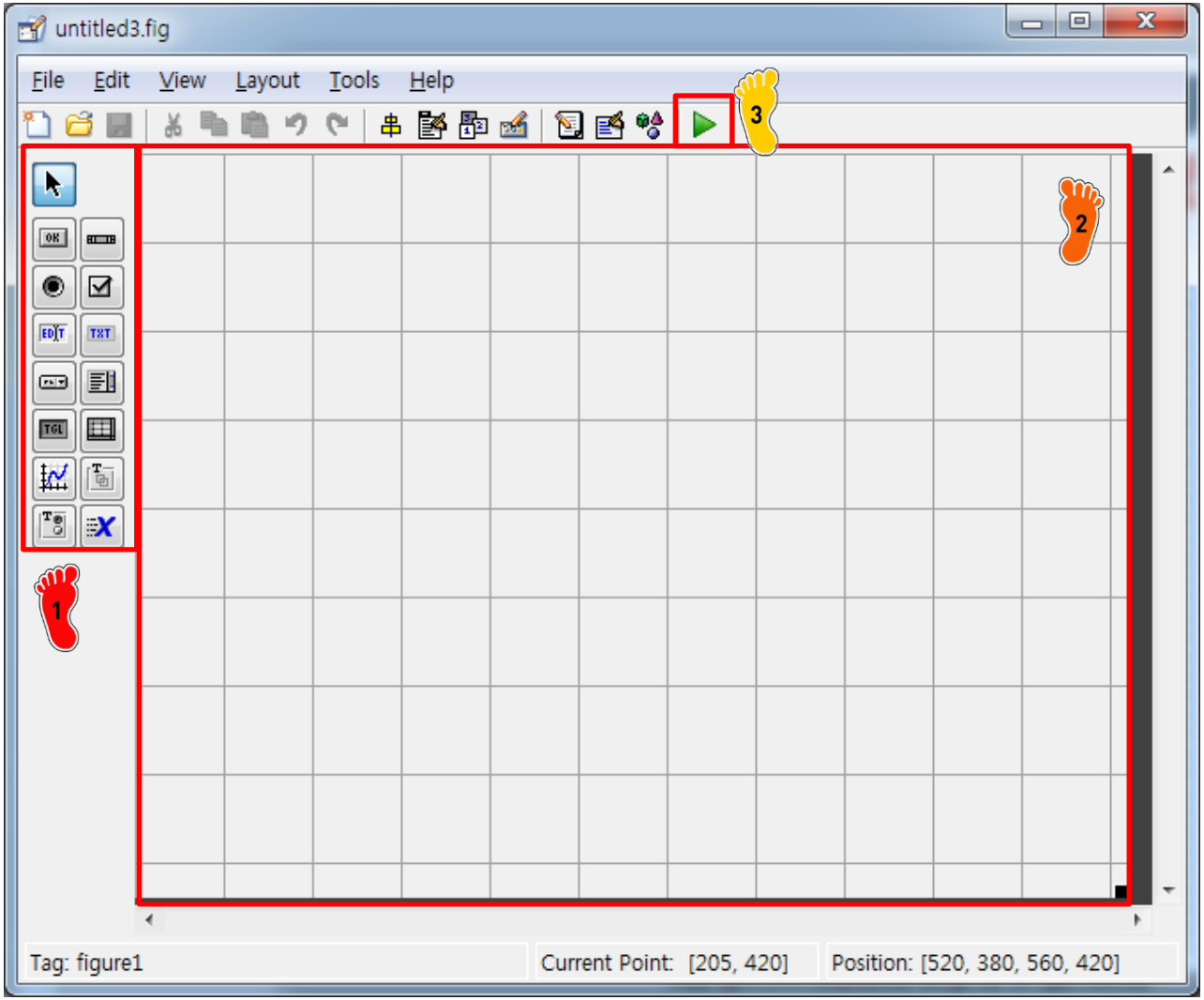
GUI SETTING WINDOW



1. 새로운 GUI 를 만드는 탭 메뉴
2. 기존 GUI 파일을 불러오는 탭 메뉴
3. 빈 GUI 창 생성(기본)

그 아래 메뉴는 예제

GUI WINDOW

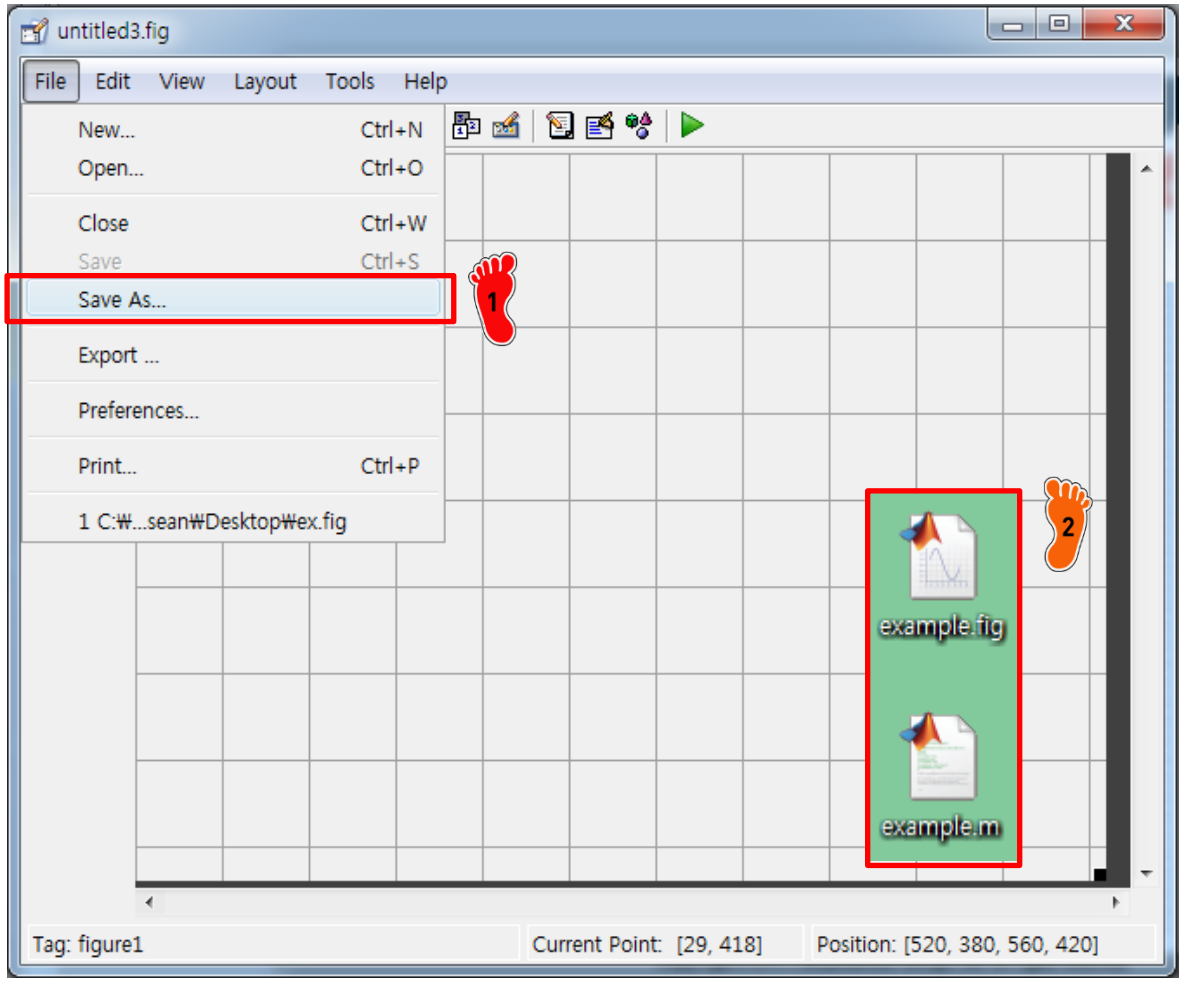


1 GUI 아이콘

2 GUI 아이콘을 배치하는 창

3 GUI 실행 아이콘

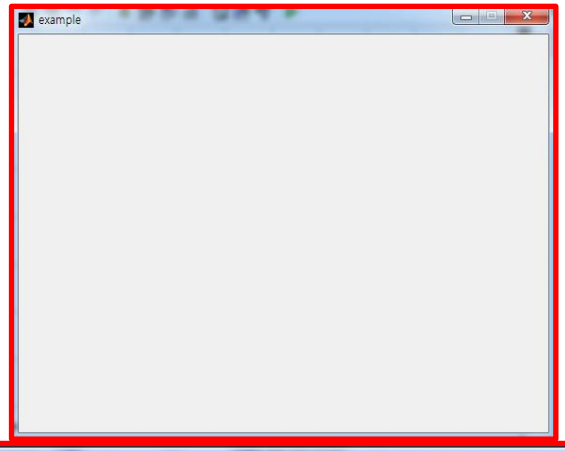
GUI FILES



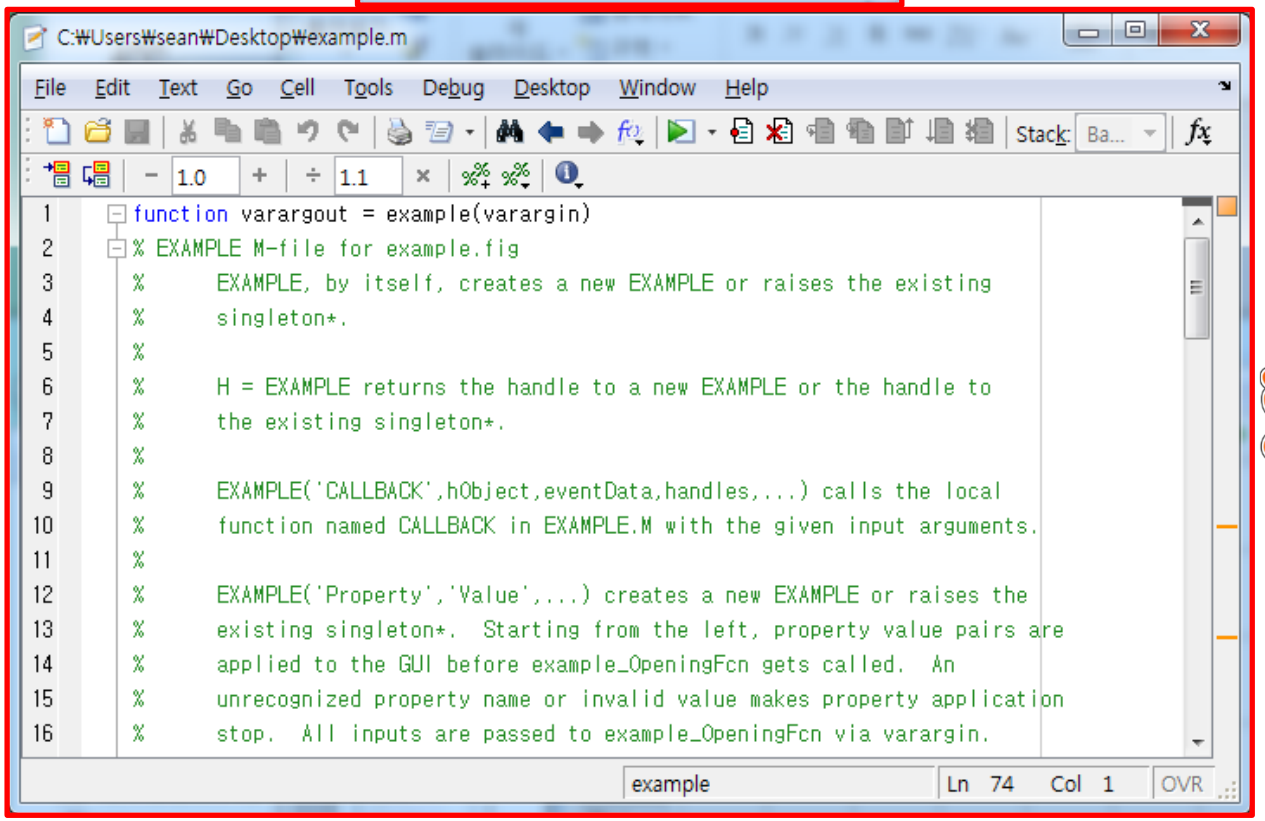
1 현재 GUI 창을 저장

2 저장하면 *.fig 파일과 *.m 파일이 동시에 생성됨

GUI M-FILE



1



2

1. GUI 를 실행하면 설정한 아이콘이 없기 때문에 빈 창이 pop up 됨

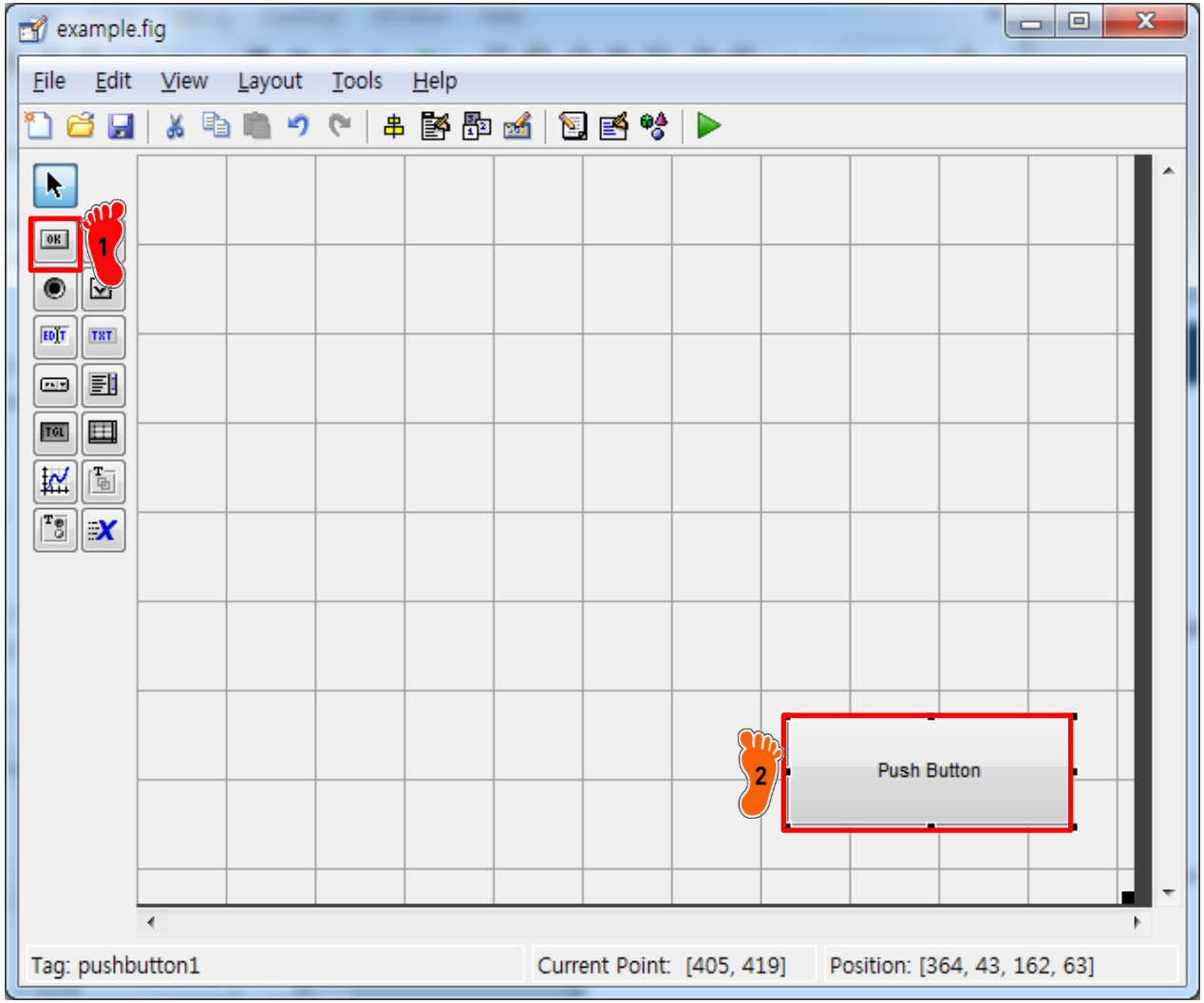
2. m-file 내용은 GUI 를 이용하는 방법 설명과 현재 GUI 창을 pop up 시키는 명령어로 이루어짐

APPENDIX

- **MATLAB GUI implementation**

  - ✓ **GUI**
  - ✓ **Example (Prob.27-27에 참고)**
    - ✓ **ODE 문제를 입력하여 다양한 방법으로 수치해를 구하고 비교할 수 있는 GUI 구축**

PUSH BUTTON

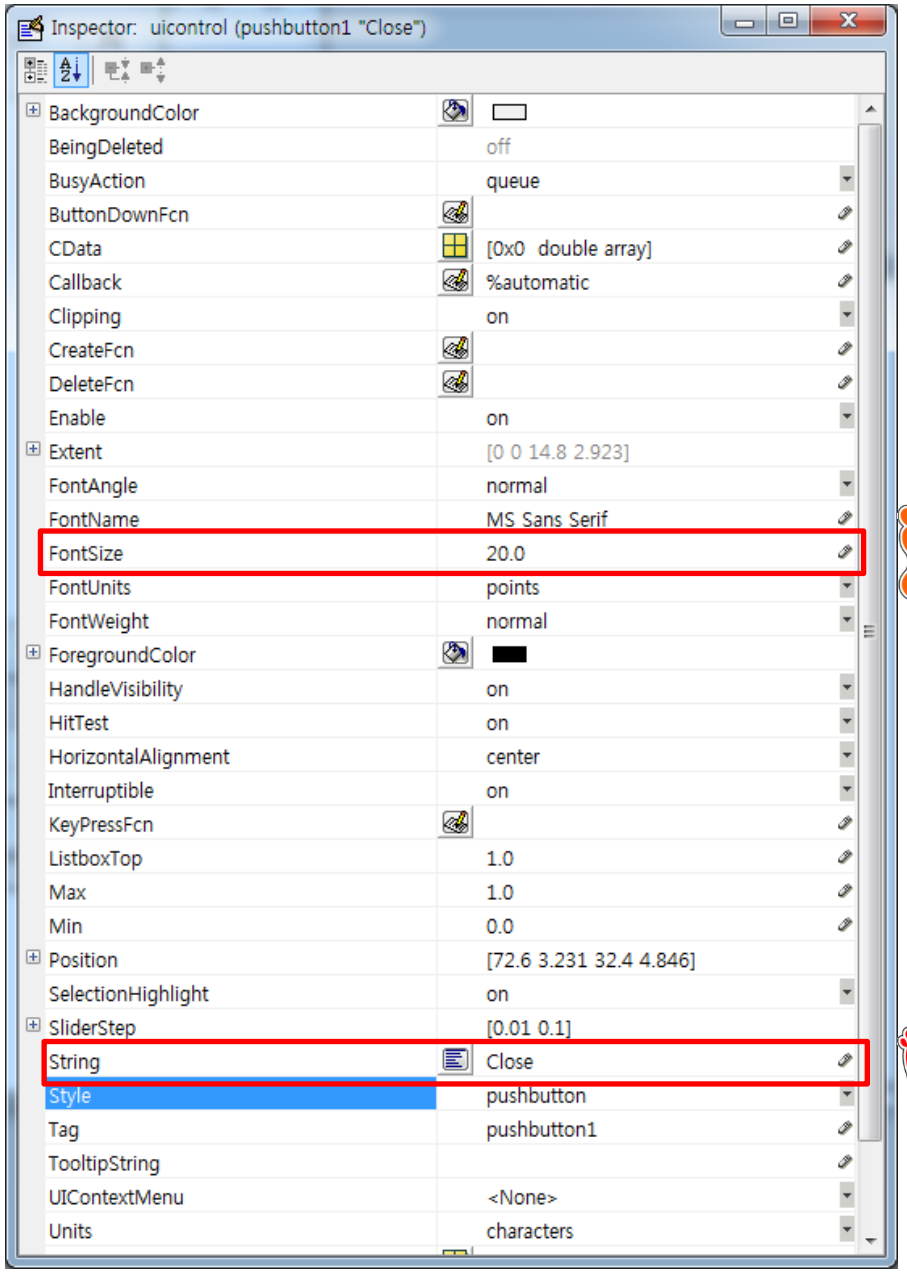


1 Push Button 아이콘 클릭

2 마우스를 드래그해서 Push Button 아이콘 생성

그 후 아이콘 더블 클릭

PUSH BUTTON: INSPECTOR



1 String 을 Close 로 변경

2 FontSize 20 으로 변경

PUSH BUTTON: M-FILE

C:₩Users₩sean₩Desktop₩example.m

File Edit Text Go Cell Tools Debug Desktop Window Help

```
60
61    % UIWAIT makes example wait for user response (see UIRESUME)
62    % uiwait(handles.figure1);
63
64
65    % --- Outputs from this function are returned to the command line.
66    function varargout = example_OutputFcn(hObject, eventdata, handles)
67    % varargout  cell array for returning output args (see VARARGOUT);
68    % hObject    handle to figure
69    % eventdata  reserved - to be defined in a future version of MATLAB
70    % handles    structure with handles and user data (see GUIDATA)
71
72    % Get default command line output from handles structure
73    varargout{1} = handles.output;
74
75
76    % --- Executes on button press in pushbutton1.
77    function pushbutton1_Callback(hObject, eventdata, handles)
78    % hObject    handle to pushbutton1 (see GCBO)
79    % eventdata  reserved - to be defined in a future version of MATLAB
80    % handles    structure with handles and user data (see GUIDATA)
81
```

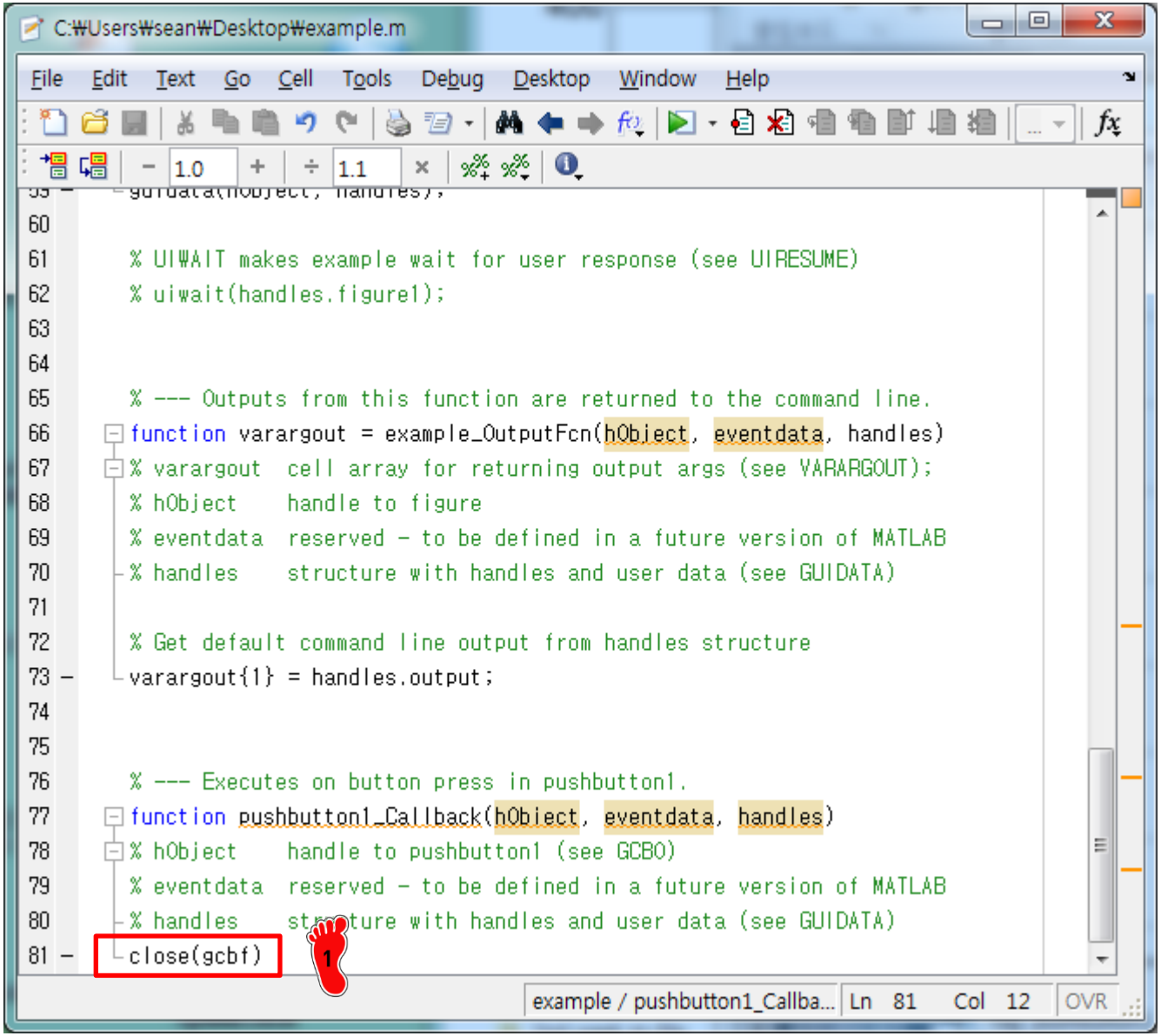
example / example_OutputFcn Ln 73 Col 31 OVR

1

1 M-file 에 pushbutton1_Callback 함수가 생성됨

이 함수 밑에 m-file 명령어를 입력하고 저장하면 pushbutton 을 클릭했을 때 실행됨

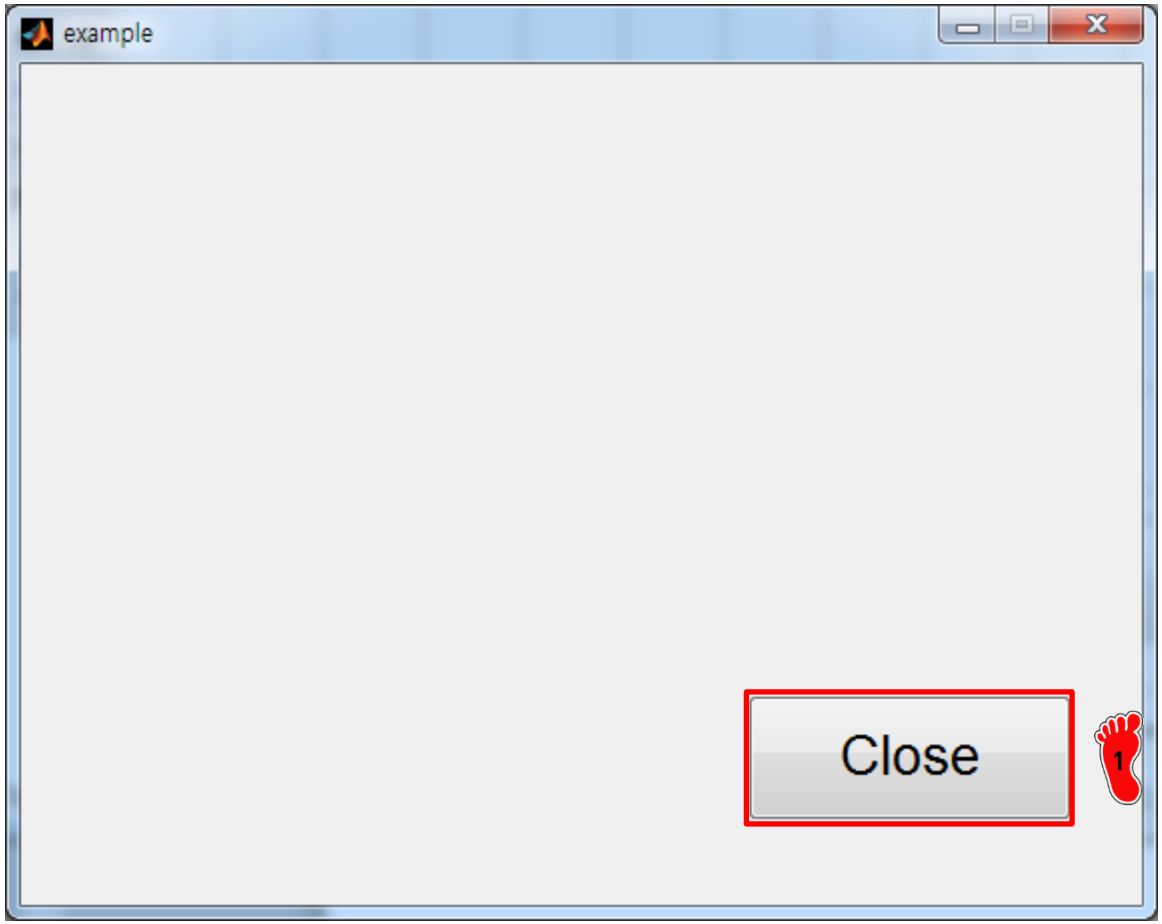
PUSH BUTTON: M-FILE



1 close(gcbf) 입력

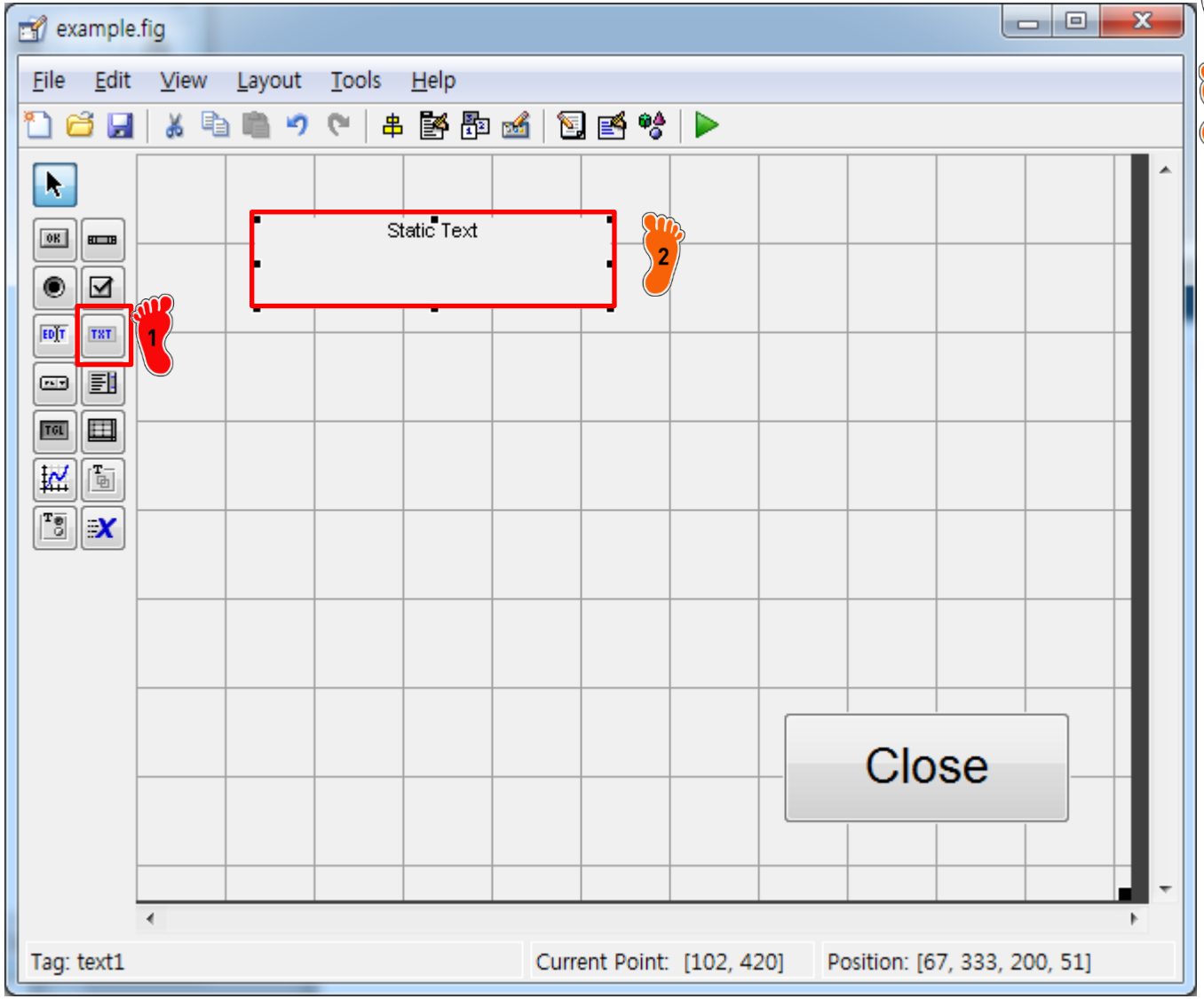
이 명령어는 현재 GUI 창을 닫는 명령어

PUSH BUTTON: GUI



1 GUI 창을 실행시켜 Close pushbutton 을 클릭하면 창이 닫힘

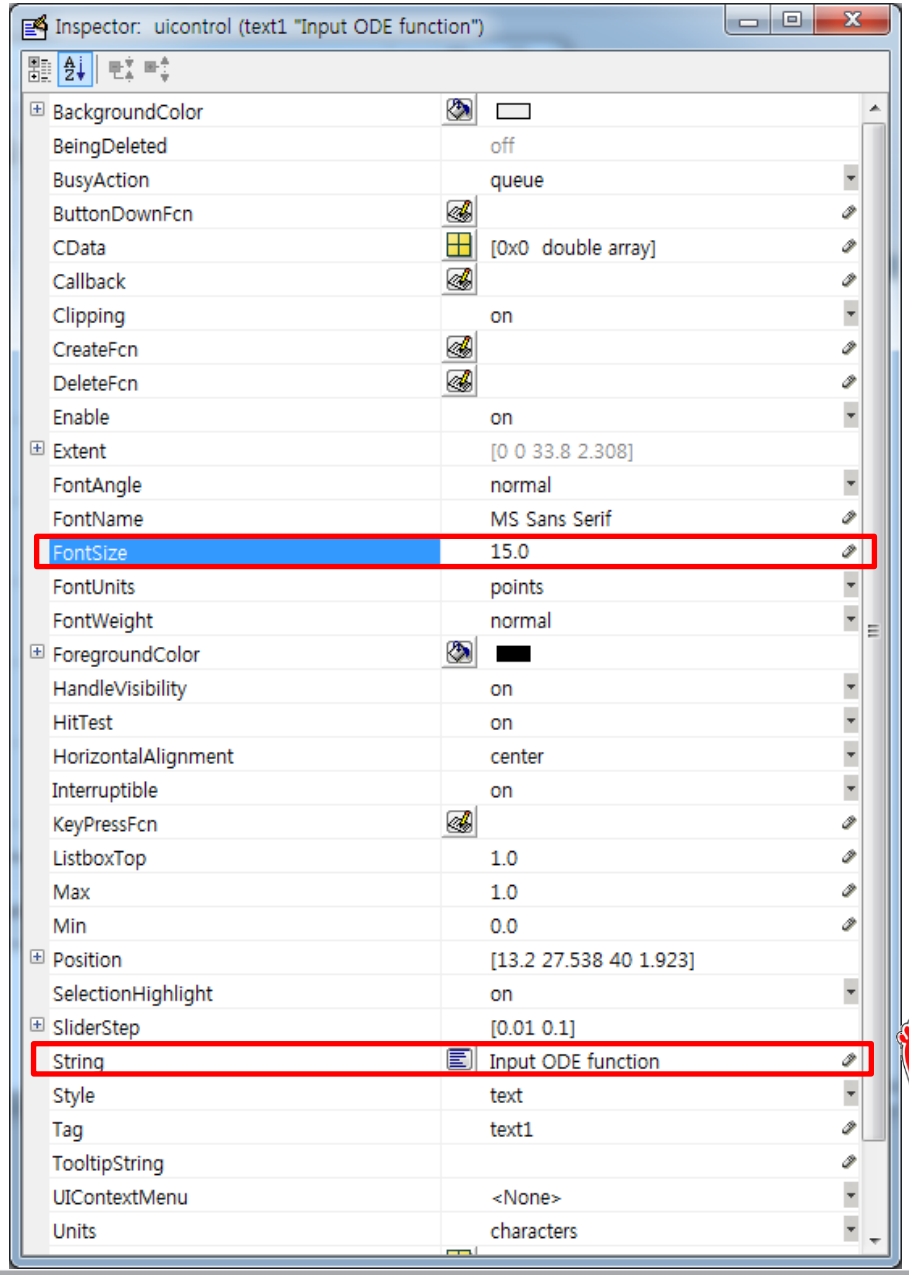
STATIC TEXT BOX



1 static text 아이콘 클릭

2 static text box 생성
더블 클릭

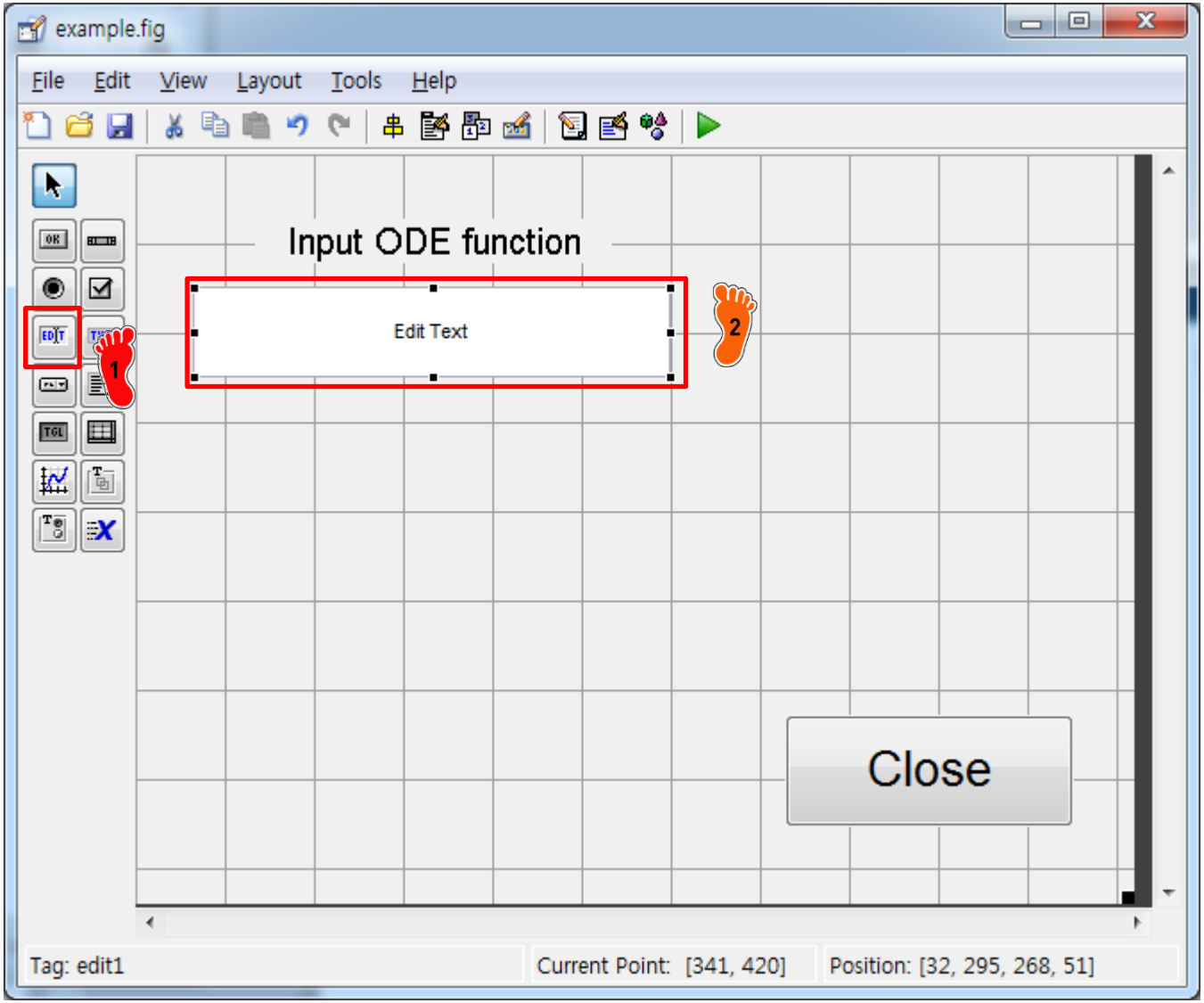
STATIC TEXT BOX: INSPECTOR



1 String 을 Input ODE function 으로 변경

2 FontSize 15로 변경

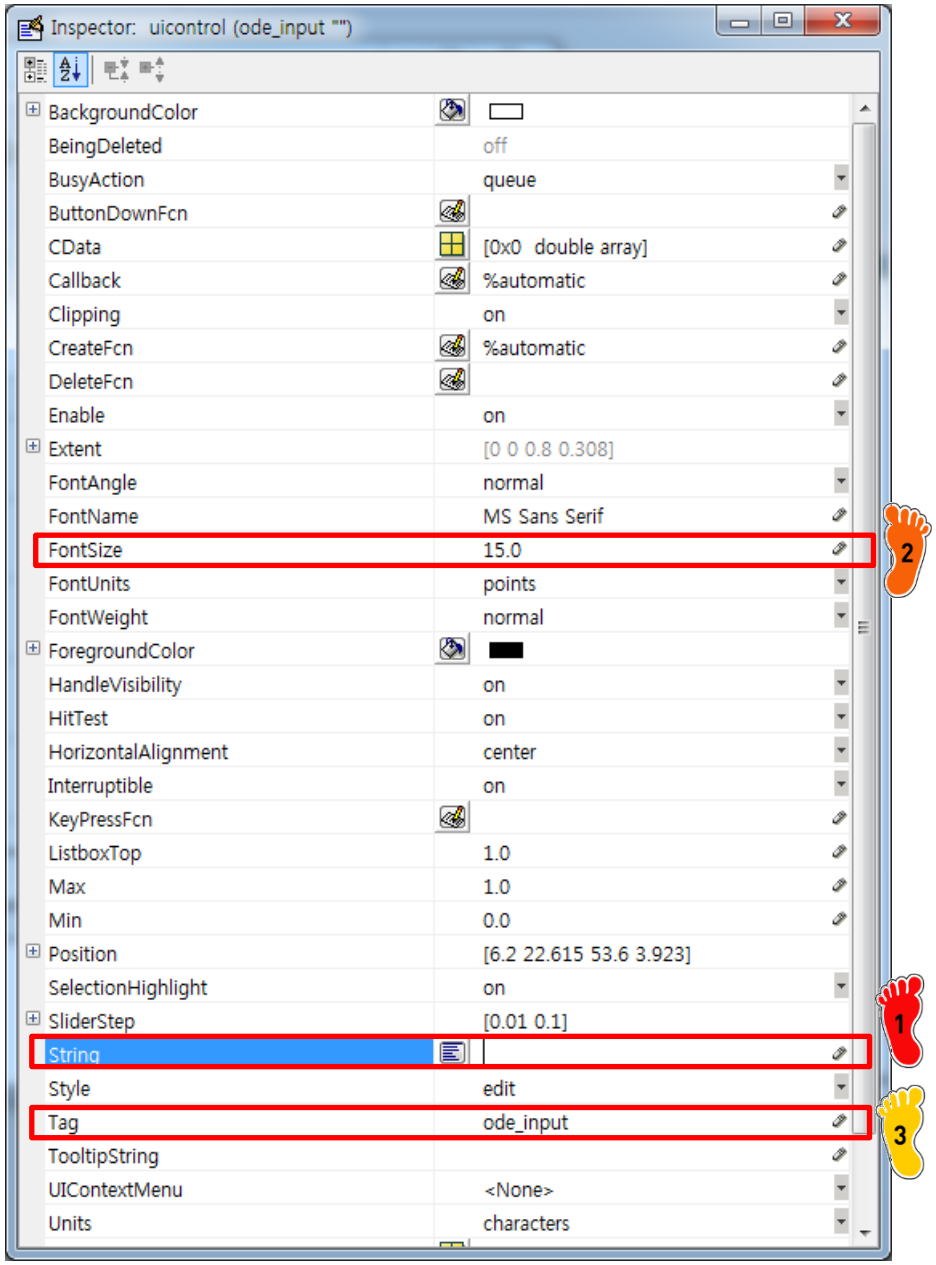
EDIT TEXT BOX



1 edit text 아이콘 클릭

2 edit text box 생성
더블 클릭

EDIT TEXT BOX: INSPECTOR

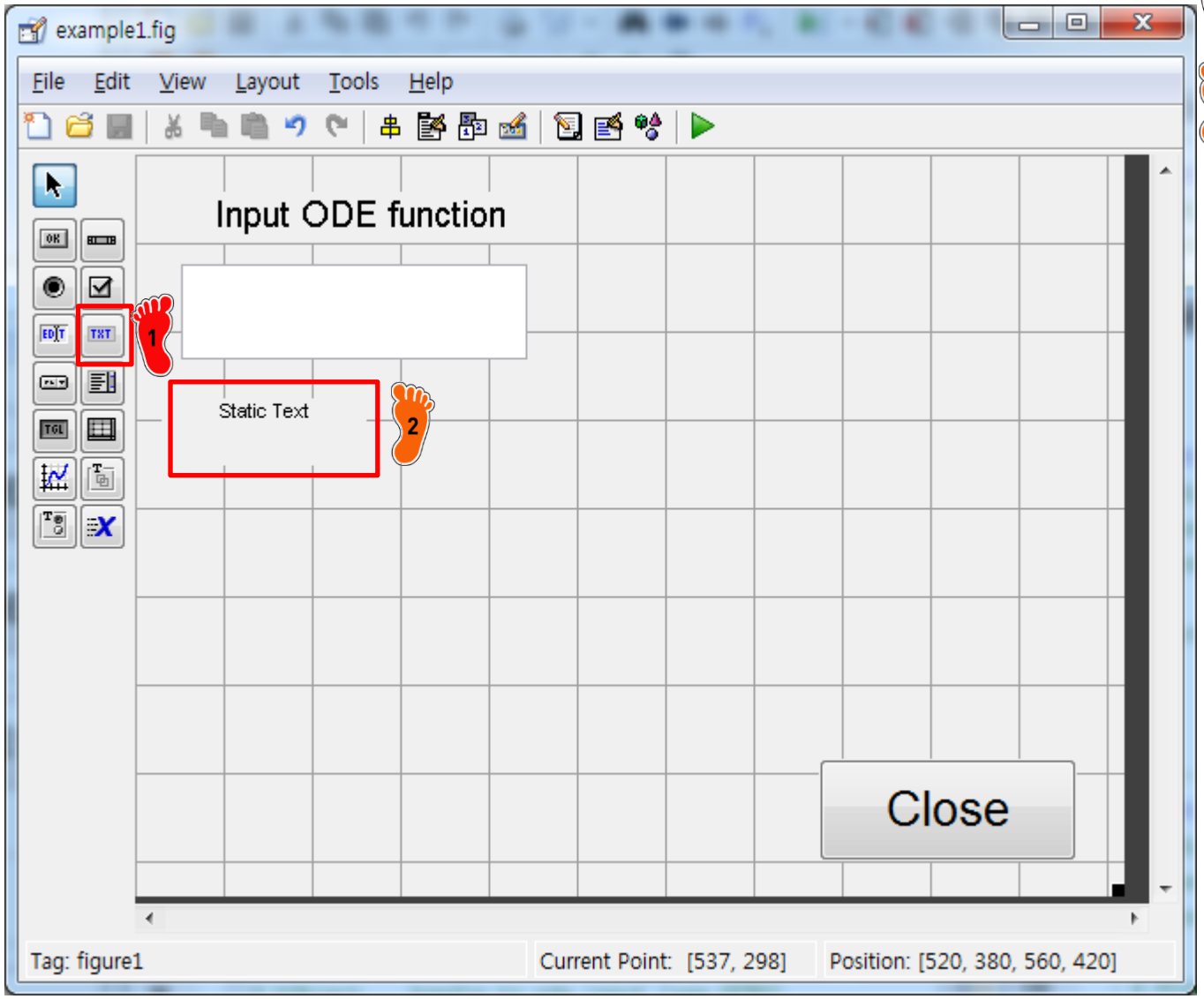


1 String 빈칸으로 변경

2 FontSize 15로 변경

3 Tag 를 ode_input 으로 변경

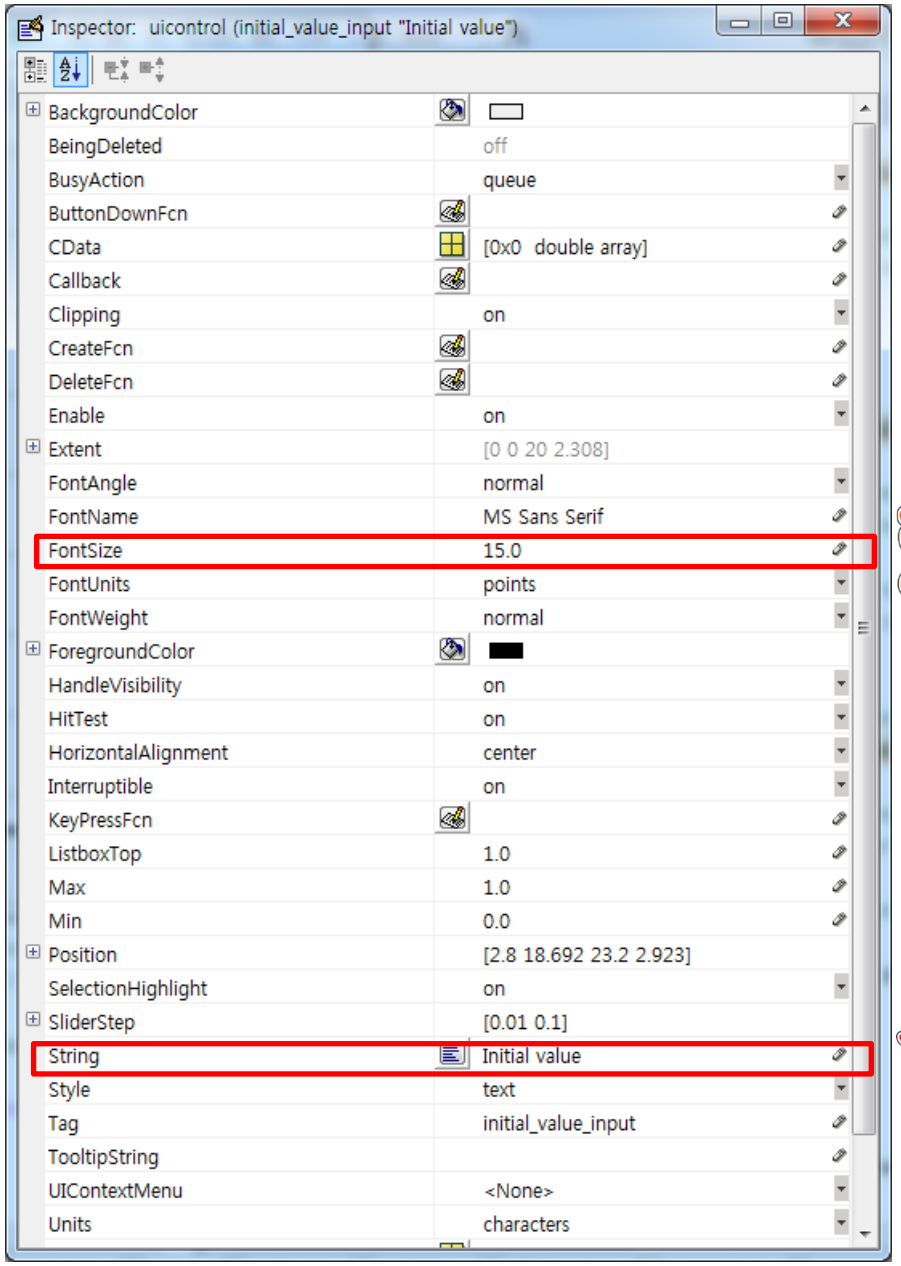
STATIC TEXT BOX



1 static text 아이콘 클릭

2 static text box 생성
더블 클릭

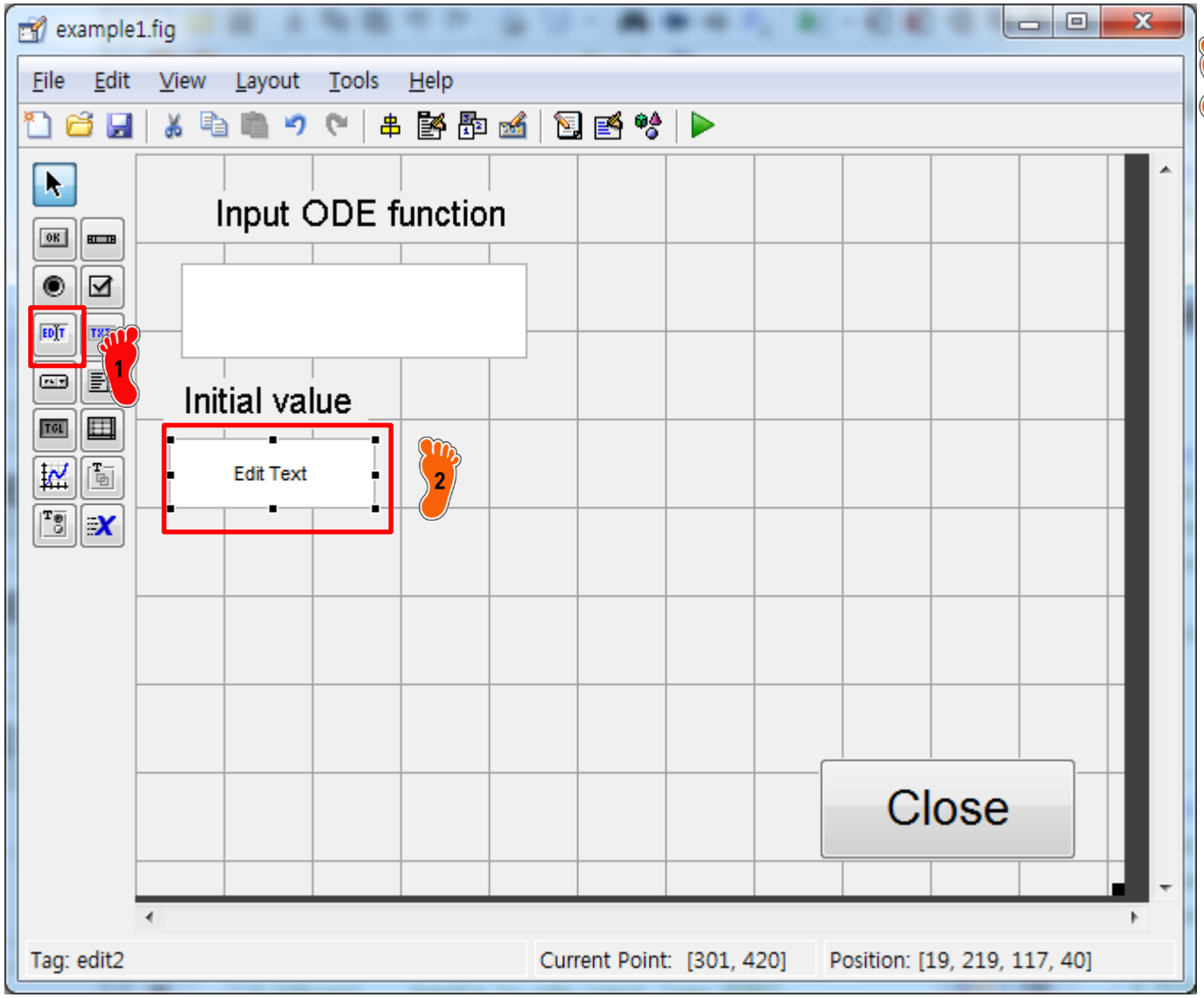
STATIC TEXT BOX: INSPECTOR



1 String 을 Initial value 로 변경

2 FontSize 15로 변경

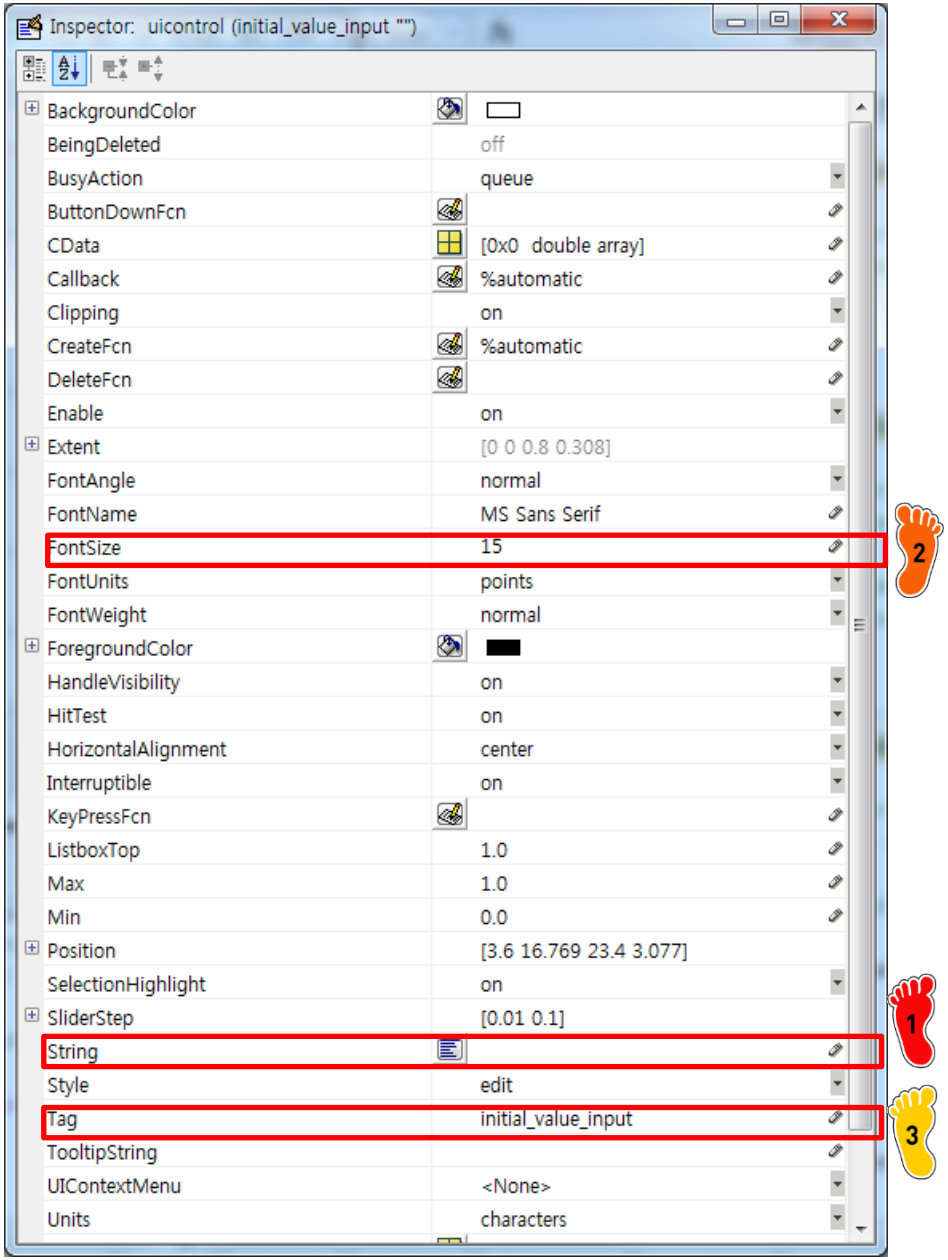
EDIT TEXT BOX



1 edit text 아이콘 클릭

2 edit text box 생성
더블 클릭

EDIT TEXT BOX: INSPECTOR

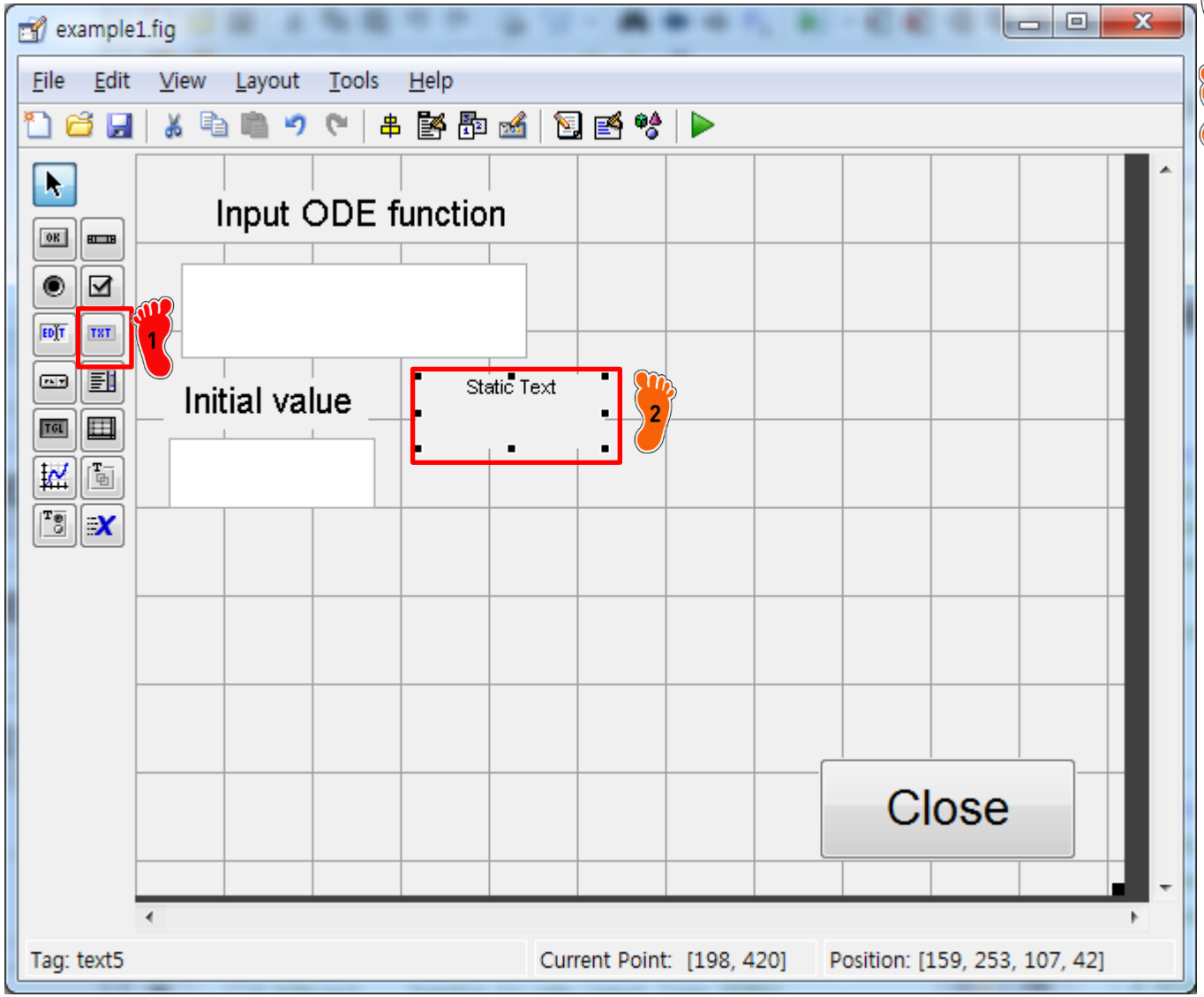


1 String 빈칸으로 변경

2 FontSize 15로 변경

3 Tag 를 initial_value_input 으로 변경

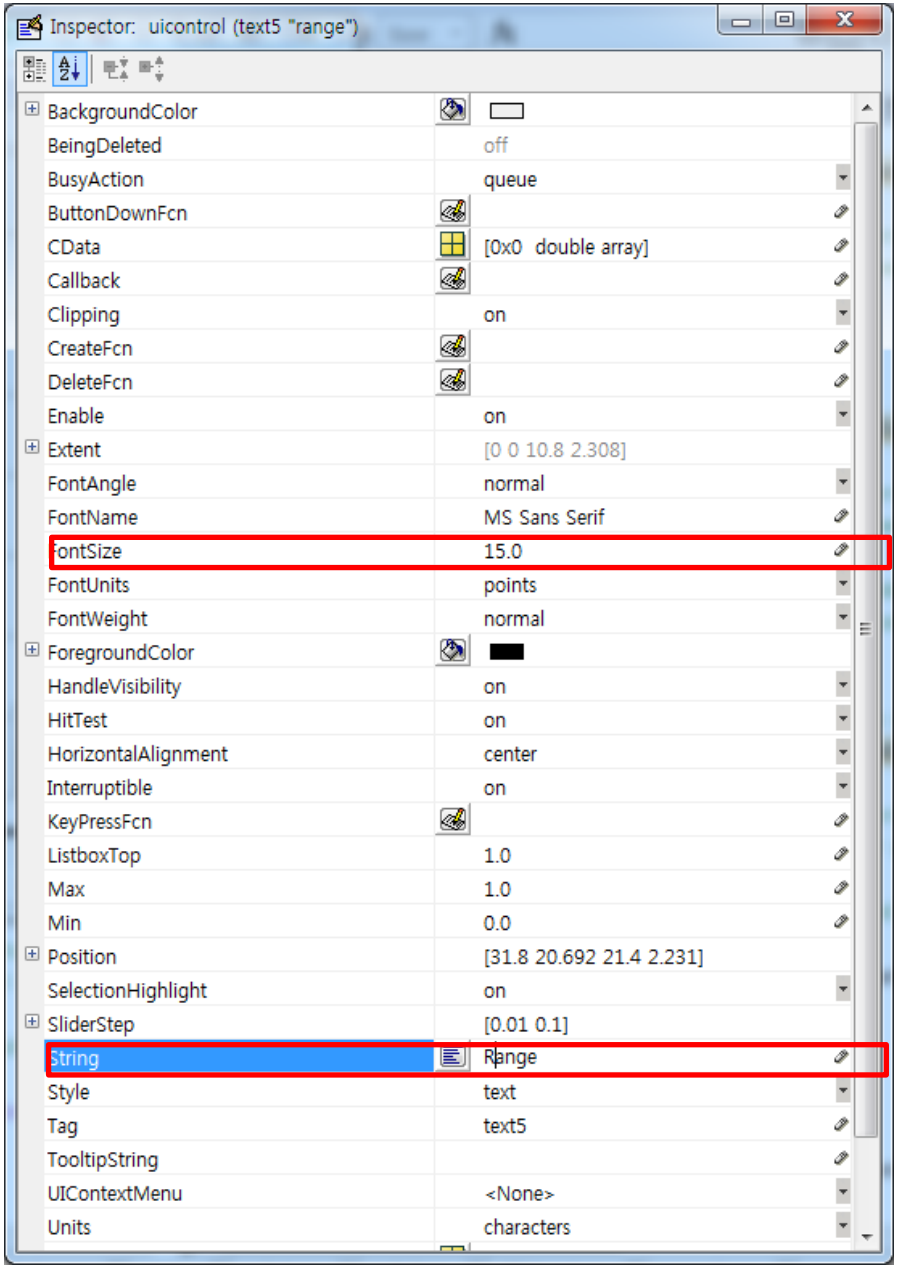
STATIC TEXT BOX



1 static text 아이콘 클릭

2 static text box 생성
더블 클릭

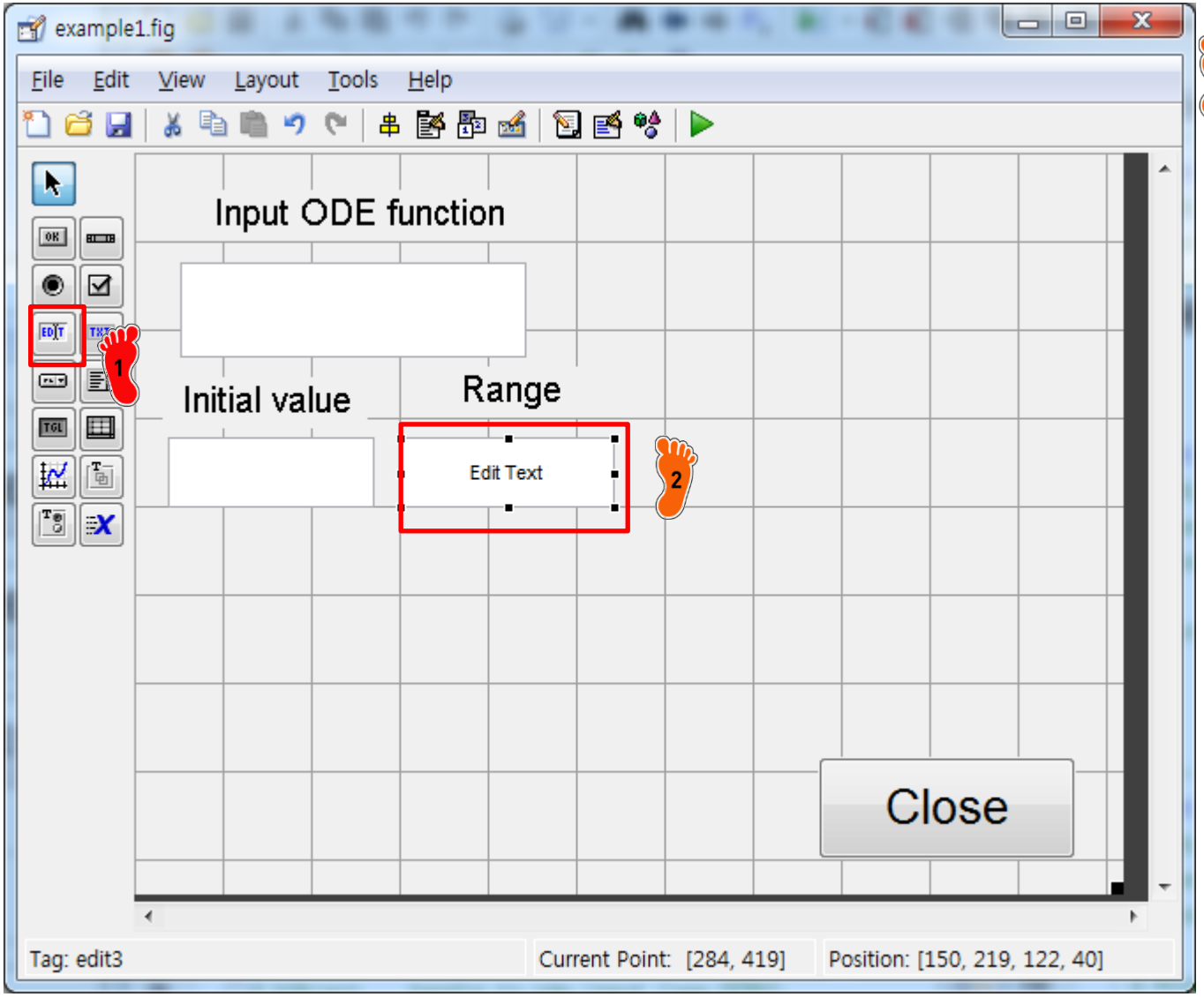
STATIC TEXT BOX: INSPECTOR



1 String 을 Range 로 변경

2 FontSize 15로 변경

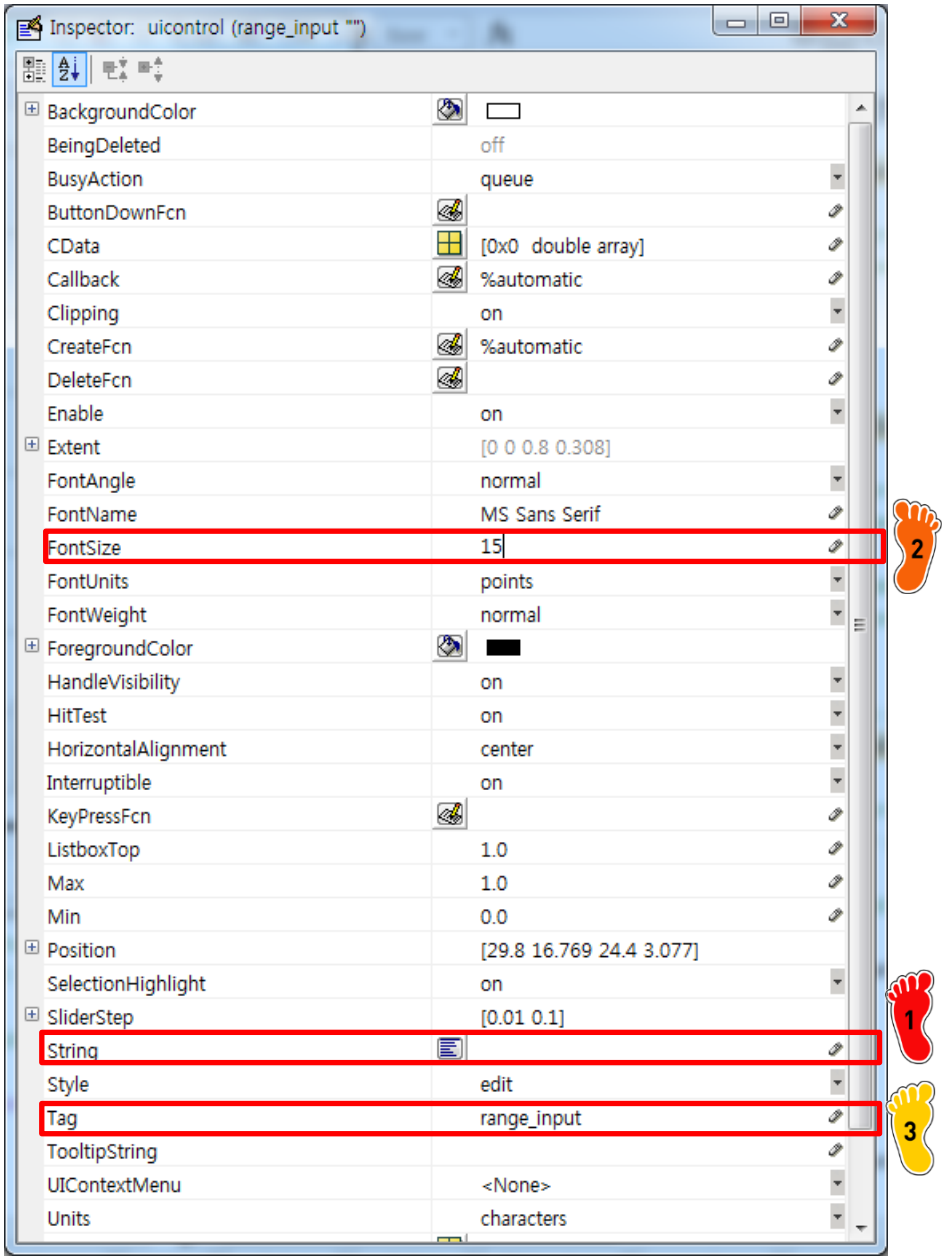
EDIT TEXT BOX



1 edit text 아이콘 클릭

2 edit text box 생성
더블 클릭

EDIT TEXT BOX: INSPECTOR



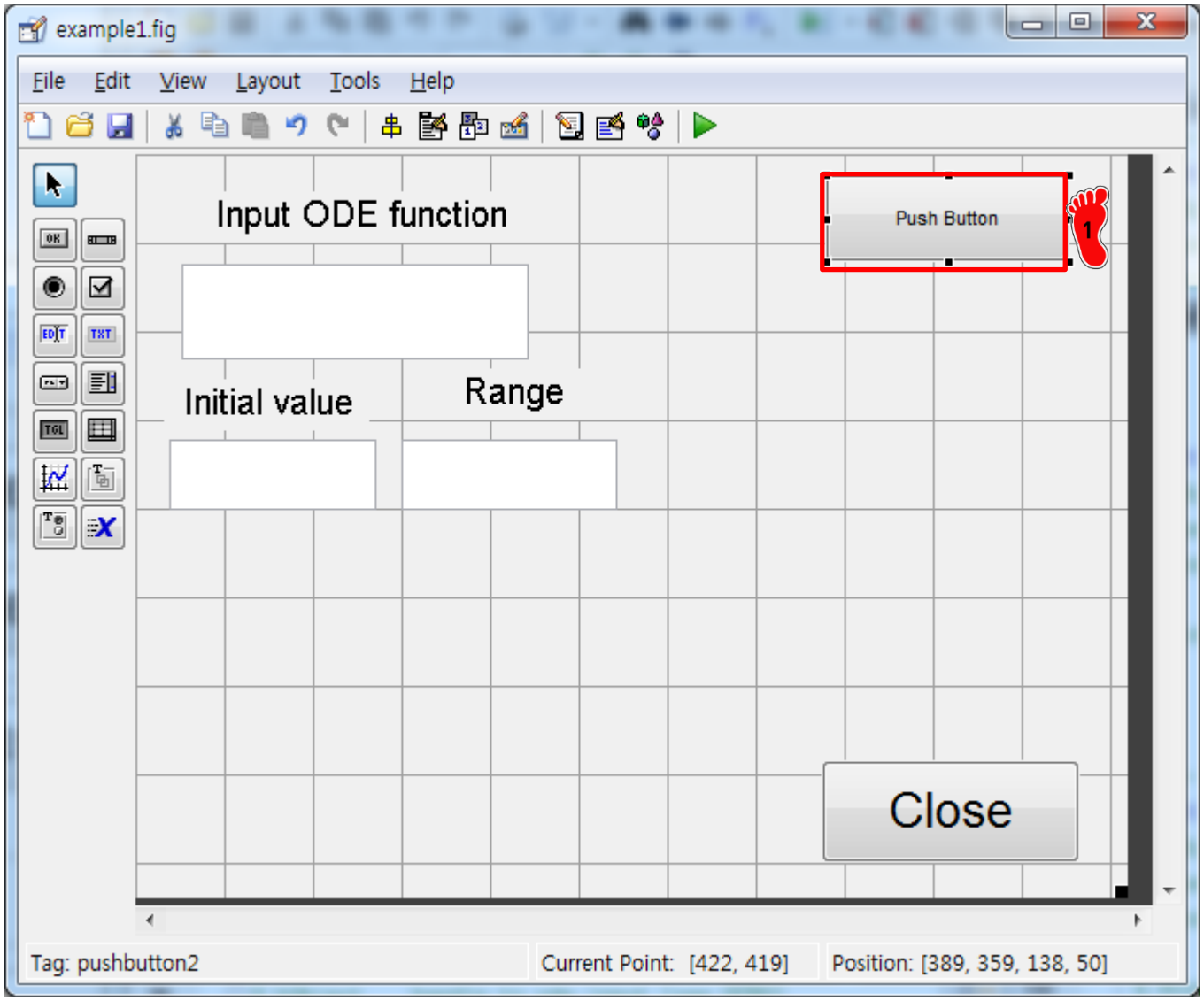
1 String 빈칸으로 변경

2 FontSize 15로 변경

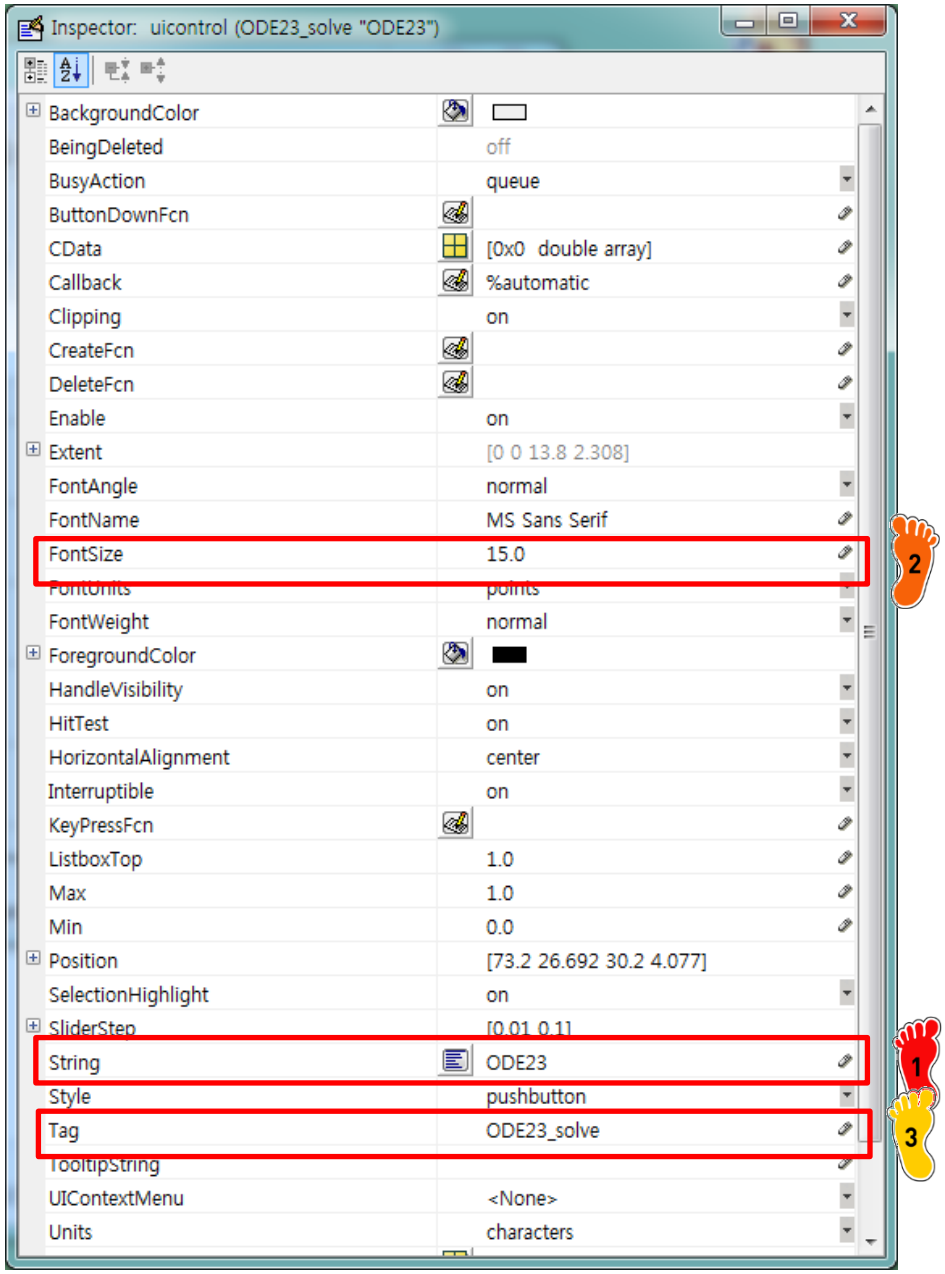
3 Tag 를 range_input 으로 변경

ODE23 PUSH BUTTON

1 pushbutton 생성



ODE23 PUSH BUTTON: INSPECTOR



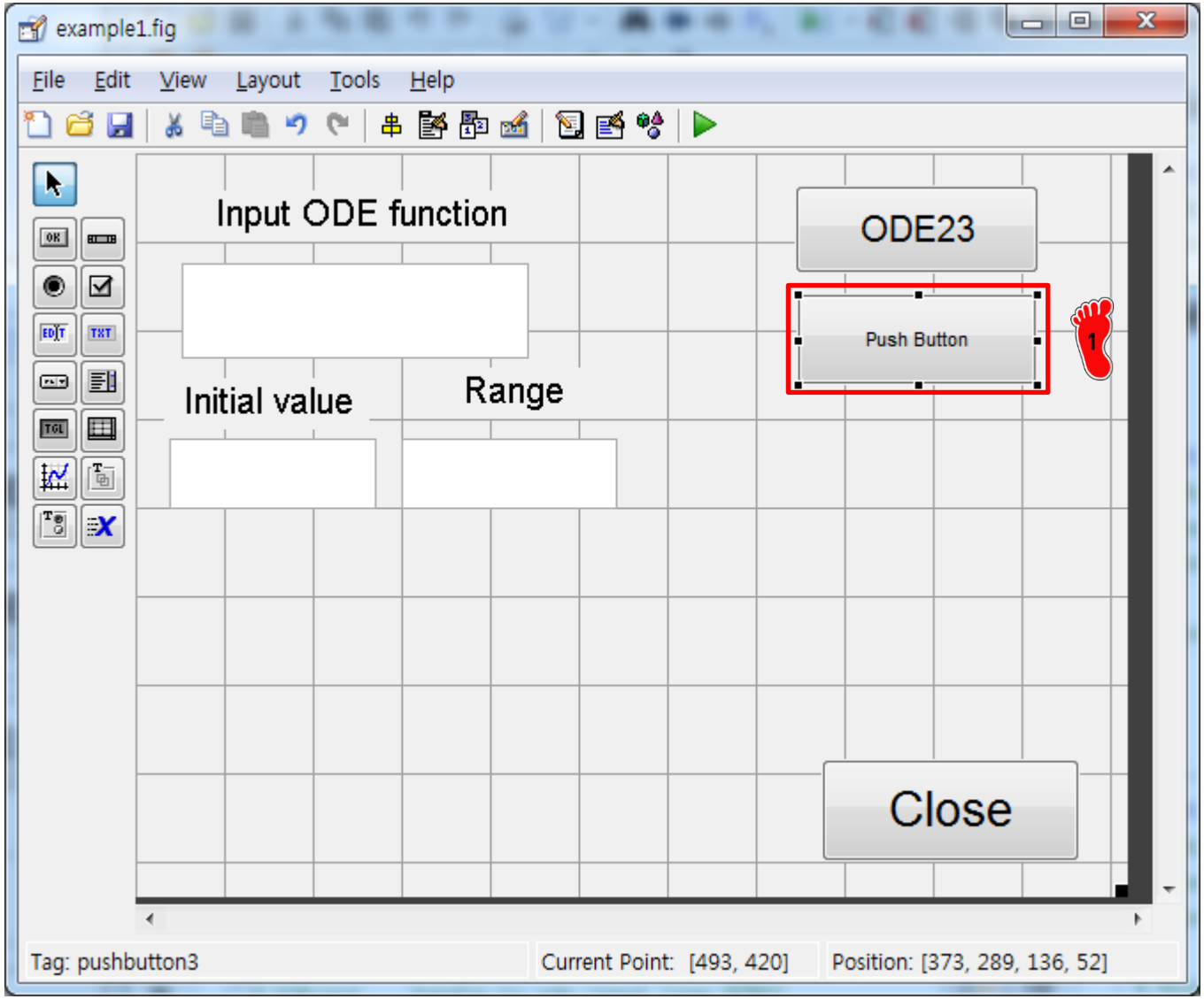
1 String ODE23으로 변경

2 FontSize 15로 변경

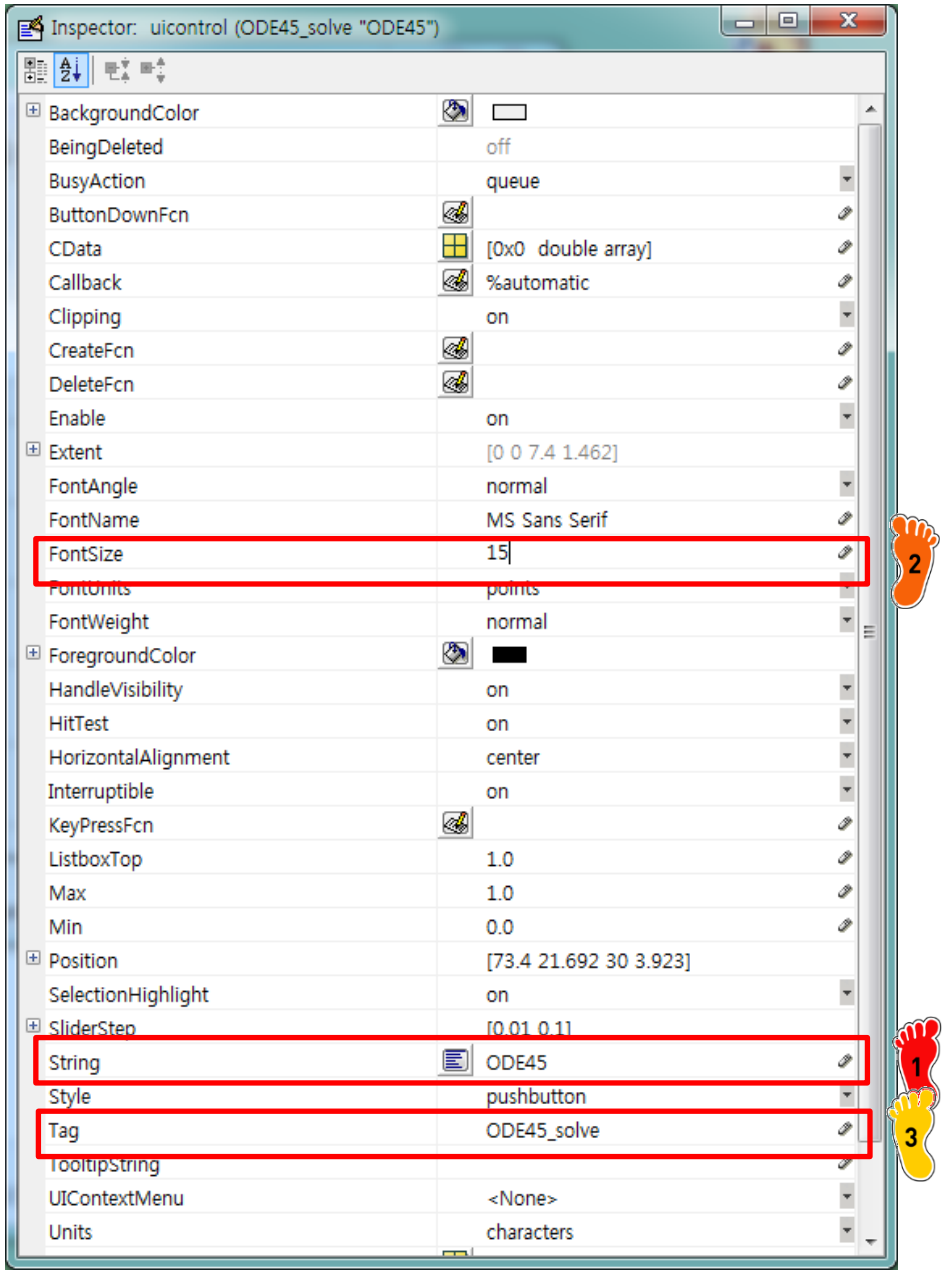
3 Tag 를 ODE23_solve 로 변경

ODE45 PUSH BUTTON

1 pushbutton 생성



ODE45 PUSH BUTTON: INSPECTOR



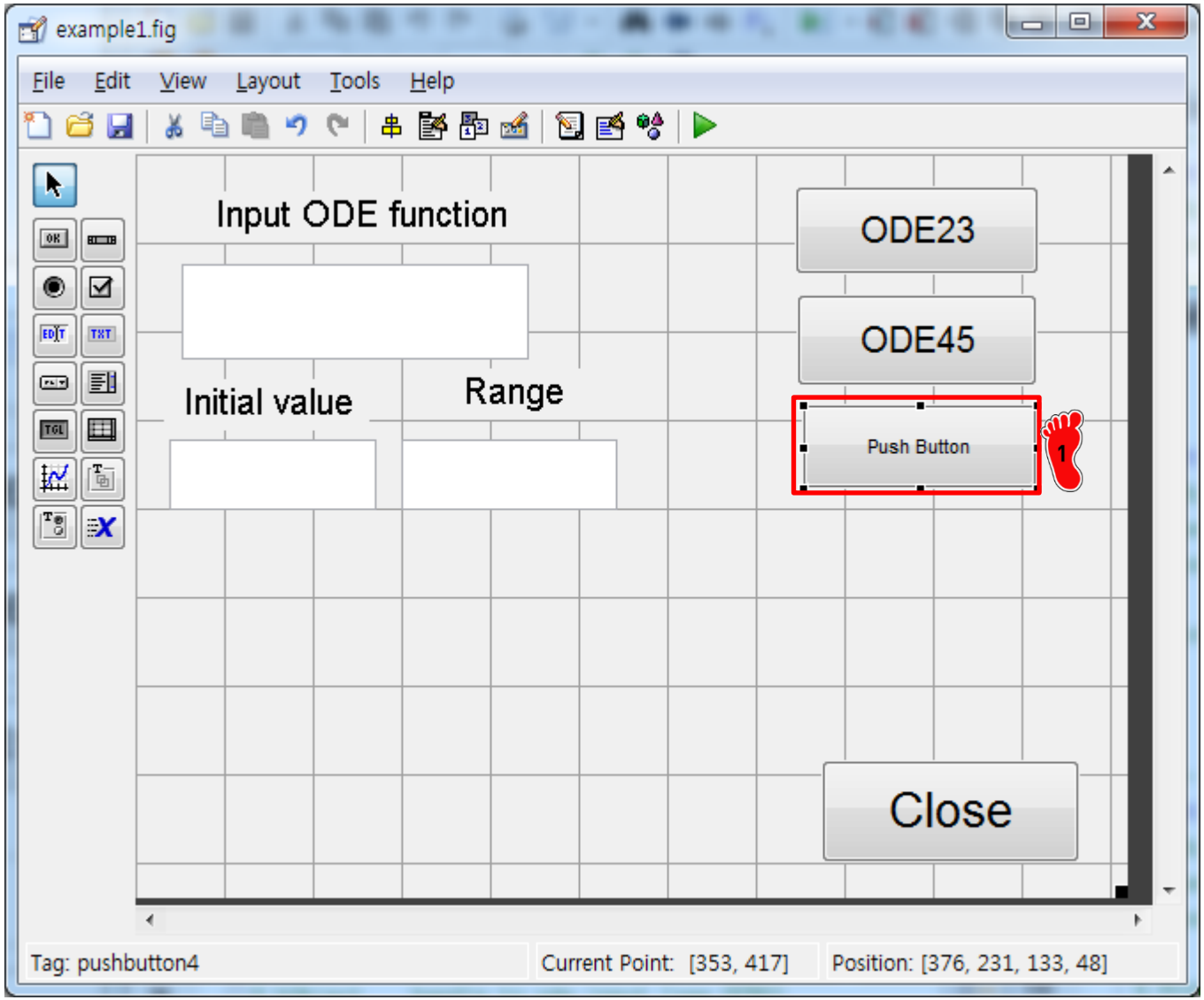
1 String ODE45 로 변경

2 FontSize 15 로 변경

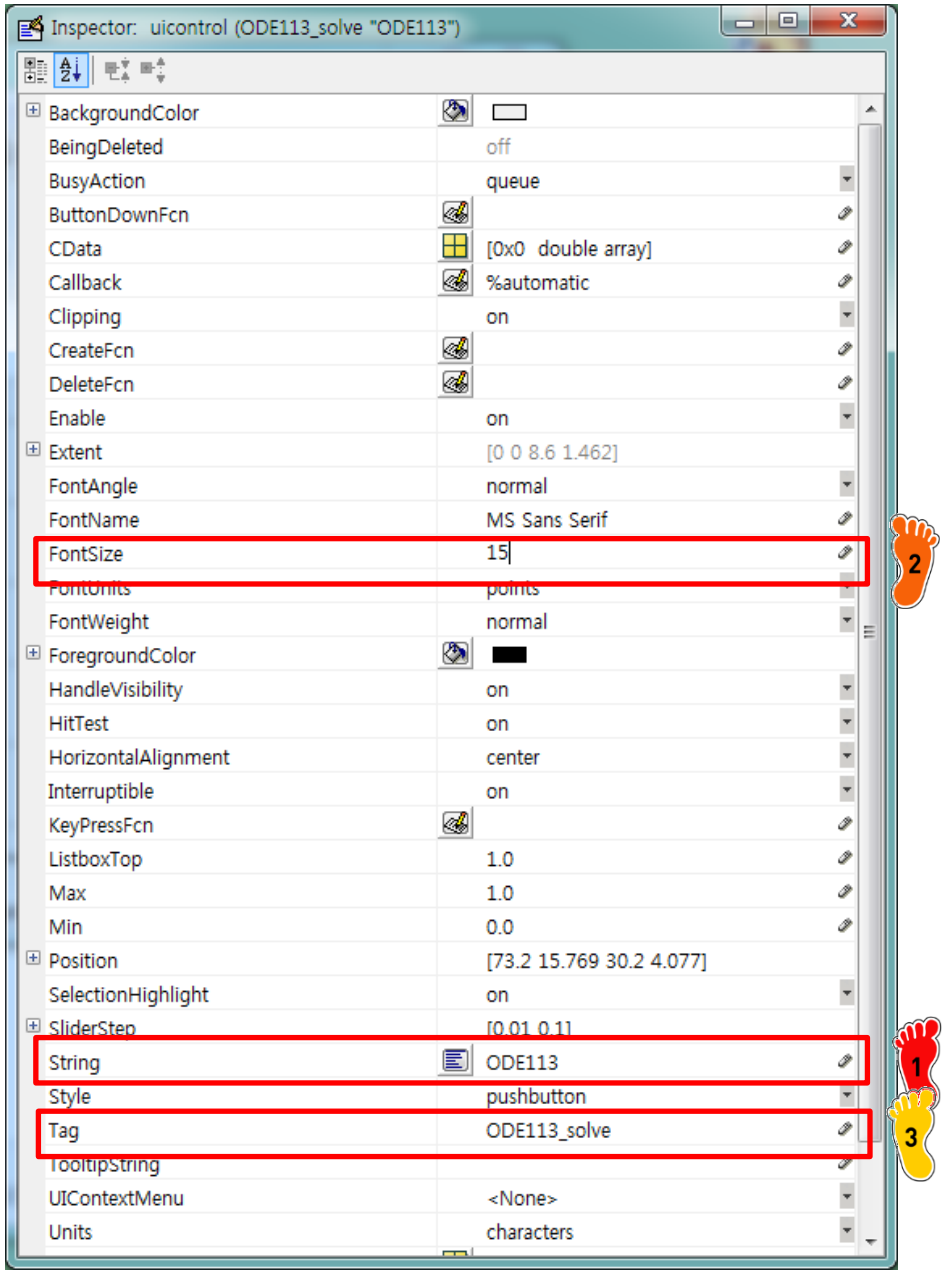
3 Tag 를 ODE45_solve 로 변경

ODE113 PUSH BUTTON

1 pushbutton 생성



ODE113 PUSH BUTTON: INSPECTOR



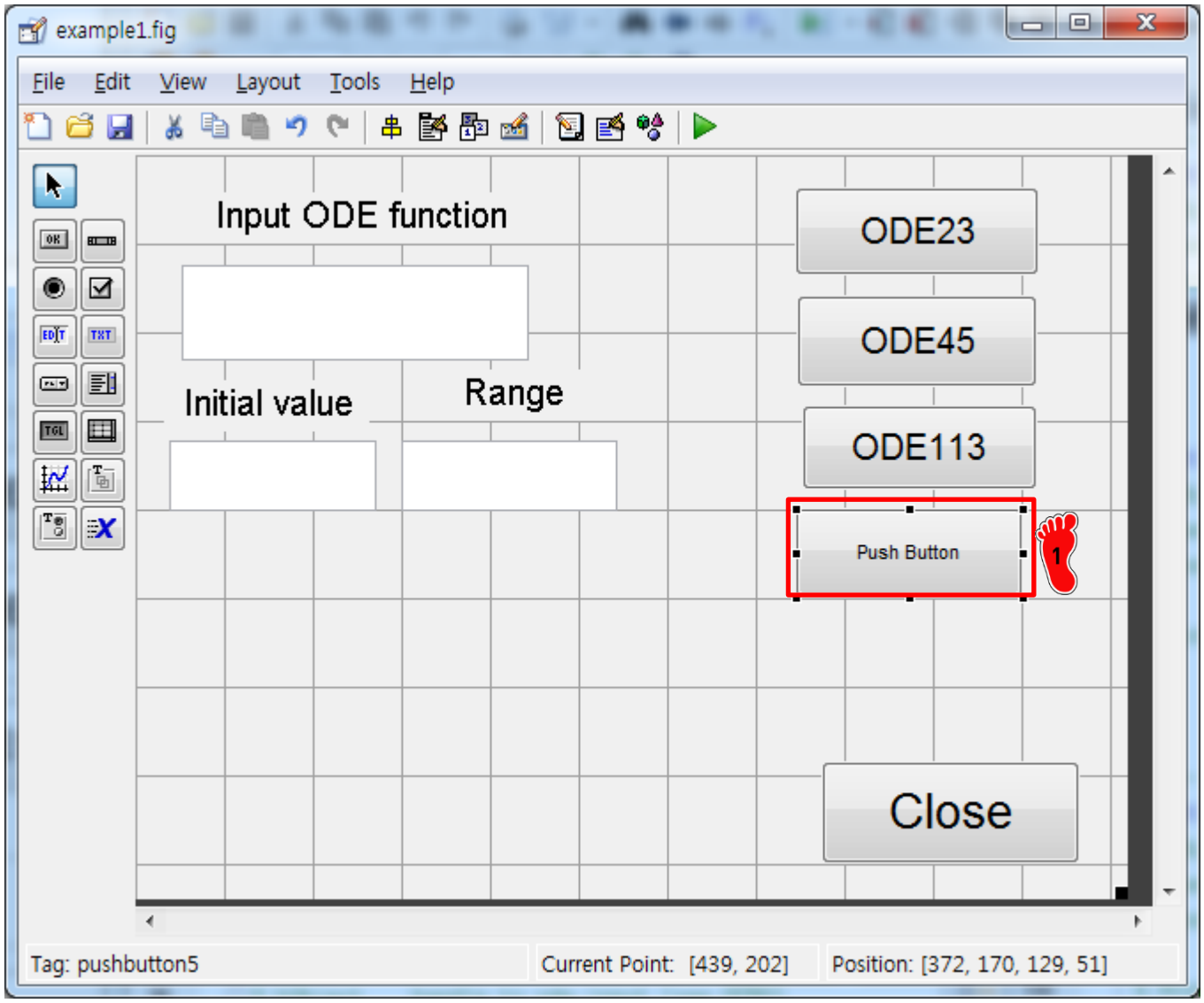
1 String ODE113 으로 변경

2 FontSize 15 로 변경

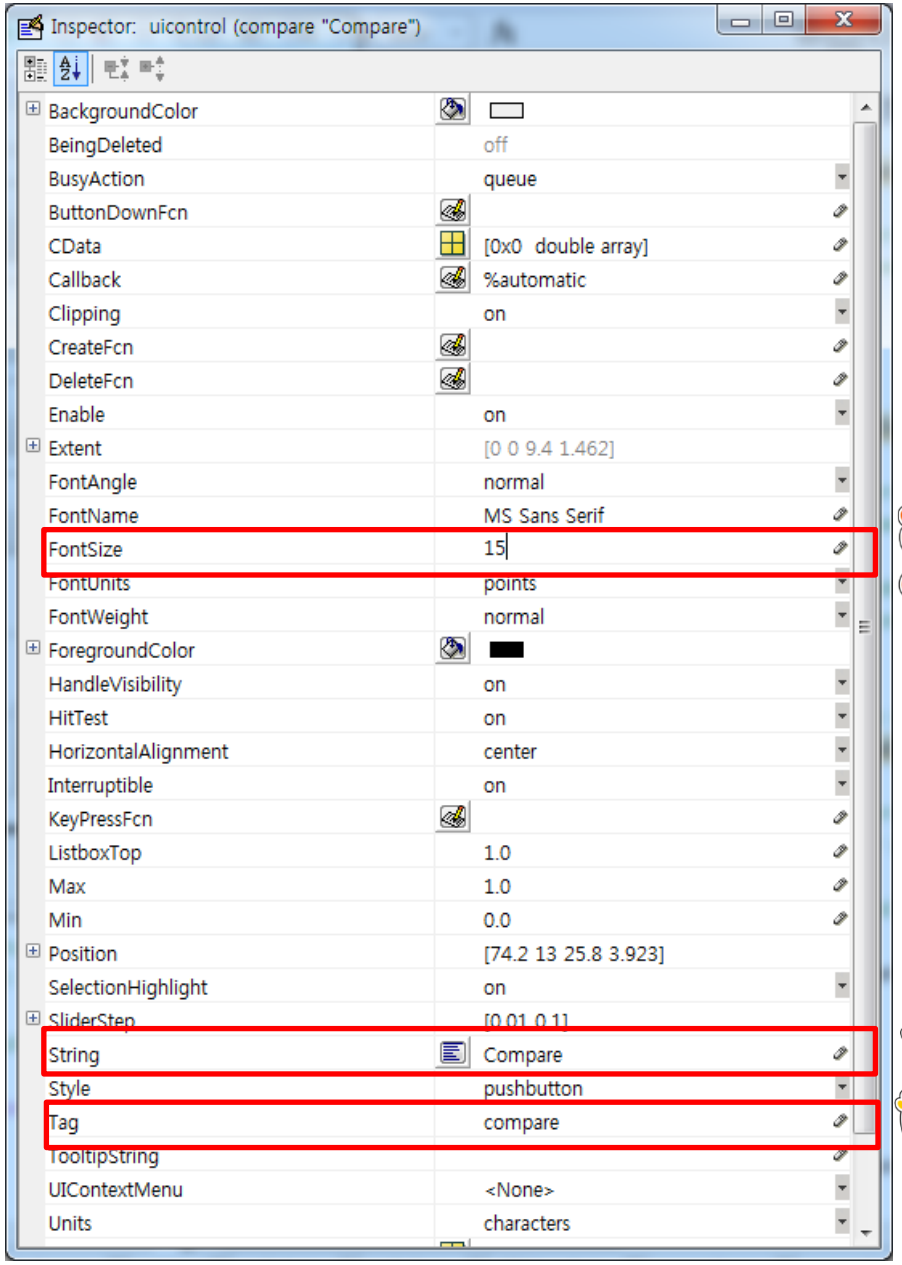
3 Tag 를 ODE113_solve 로 변경

COMPARE PUSH BUTTON

1 pushbutton 생성



COMPARE PUSH BUTTON: INSPECTO

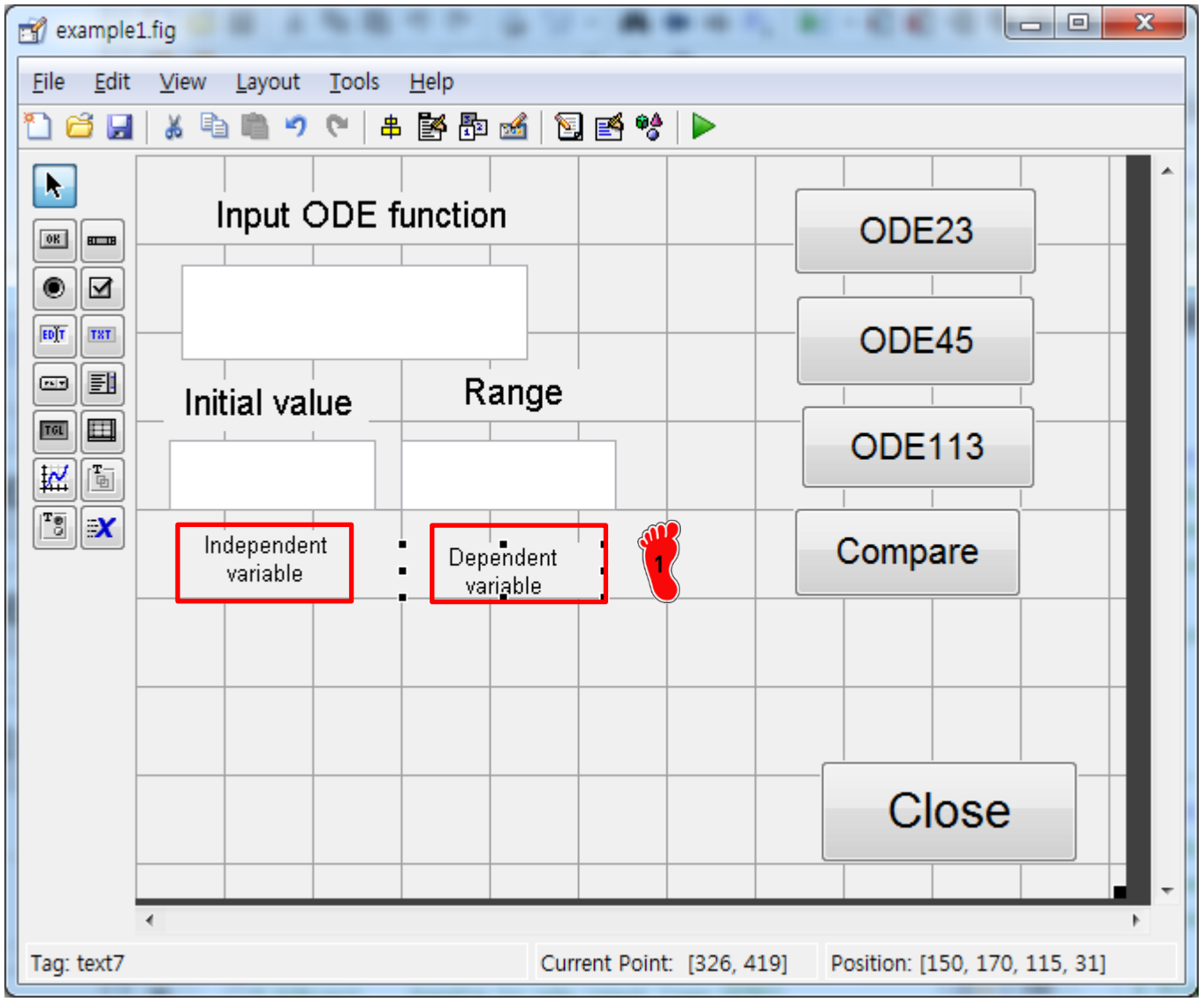


1 String Compare 으로 변경

2 FontSize 15 로 변경

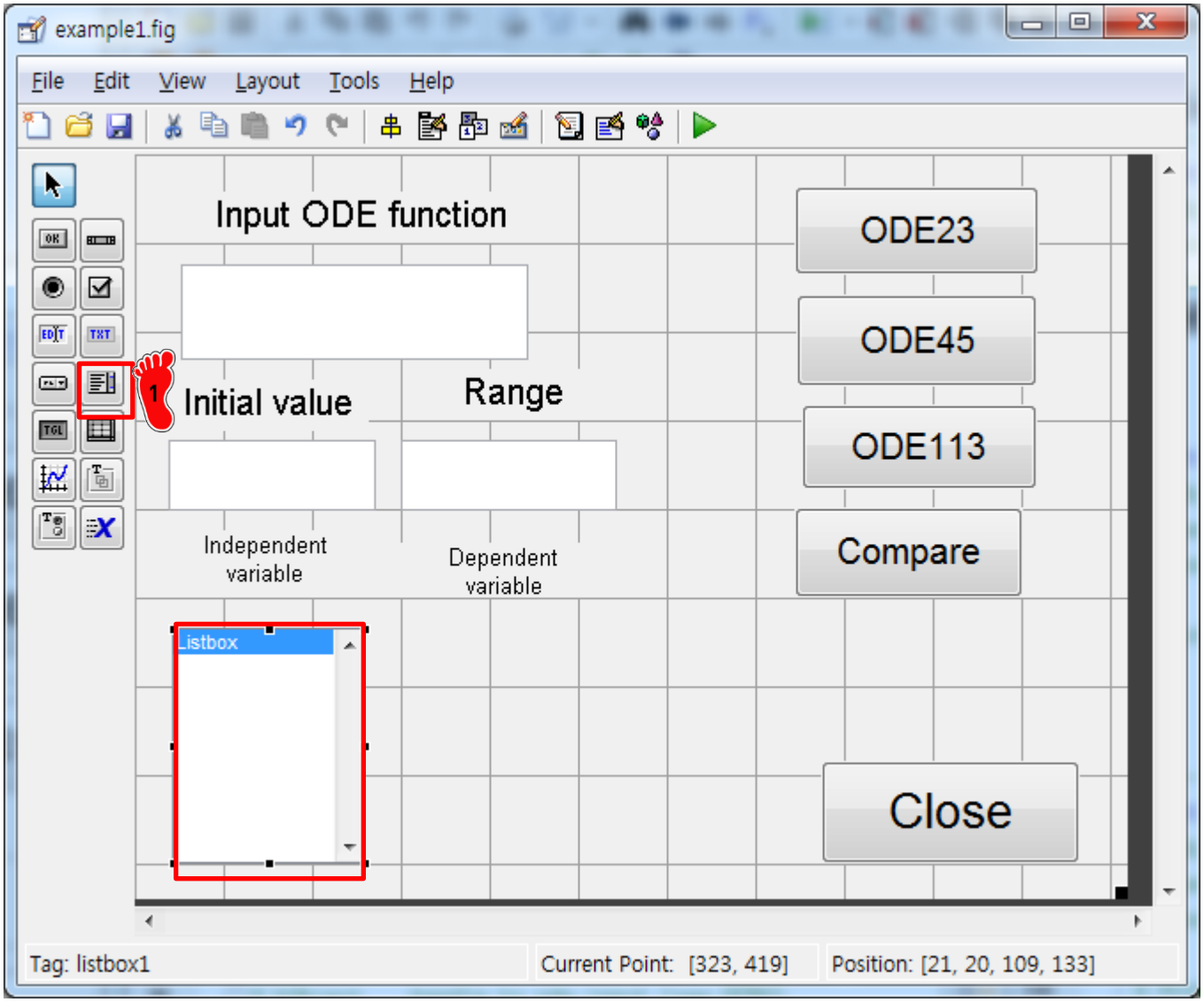
3 Tag 를 compare 로 변경

STATIC TEXT BOX



1 Independent variable 과 Dependent variable 이름을 갖는 static text box 2개 생성

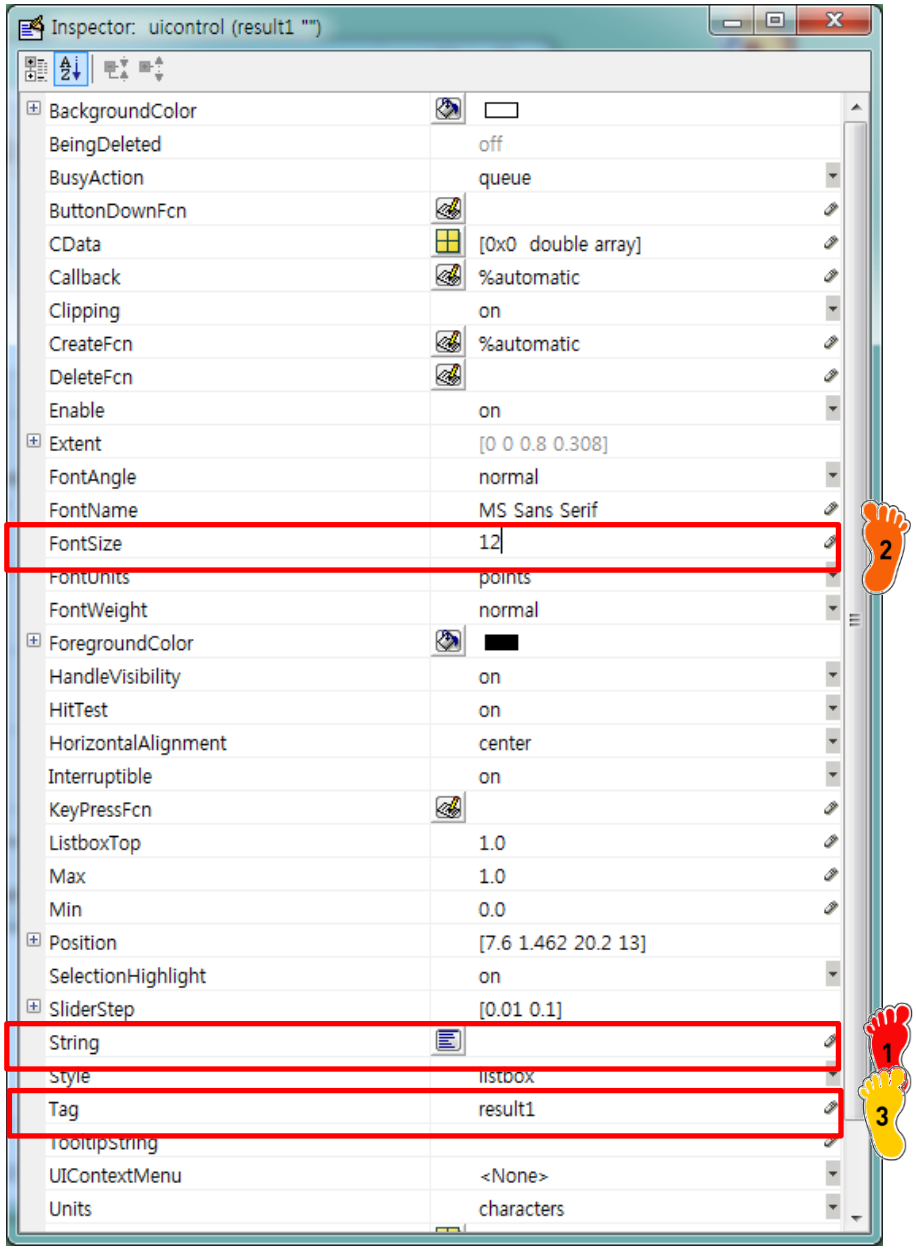
LIST BOX



1 List box icon 클릭

2 List box 생성

LIST BOX: INSPECTOR

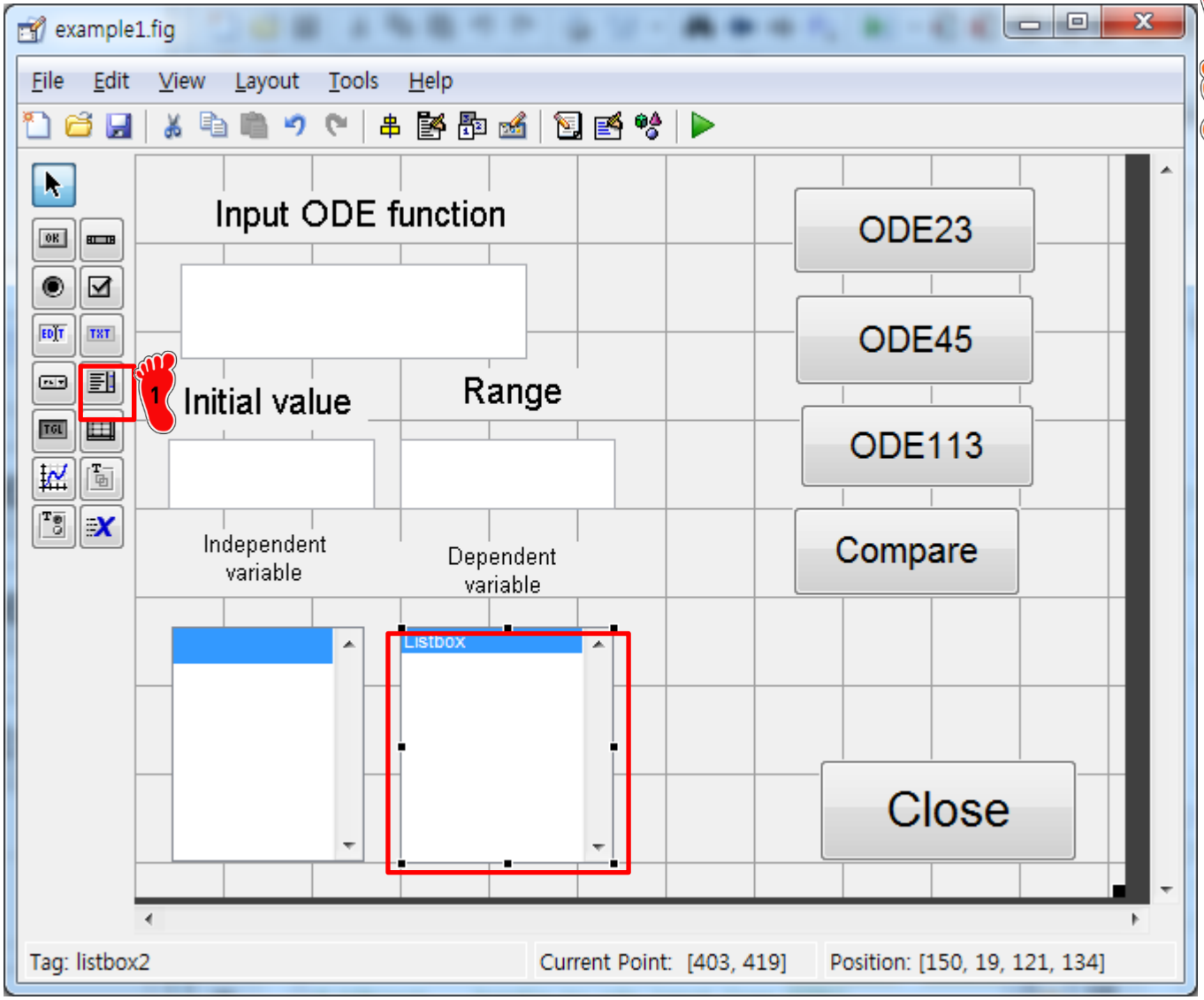


1 String 빈칸으로 변경

2 FontSize 12 로 변경

3 Tag 를 result1 으로 변경

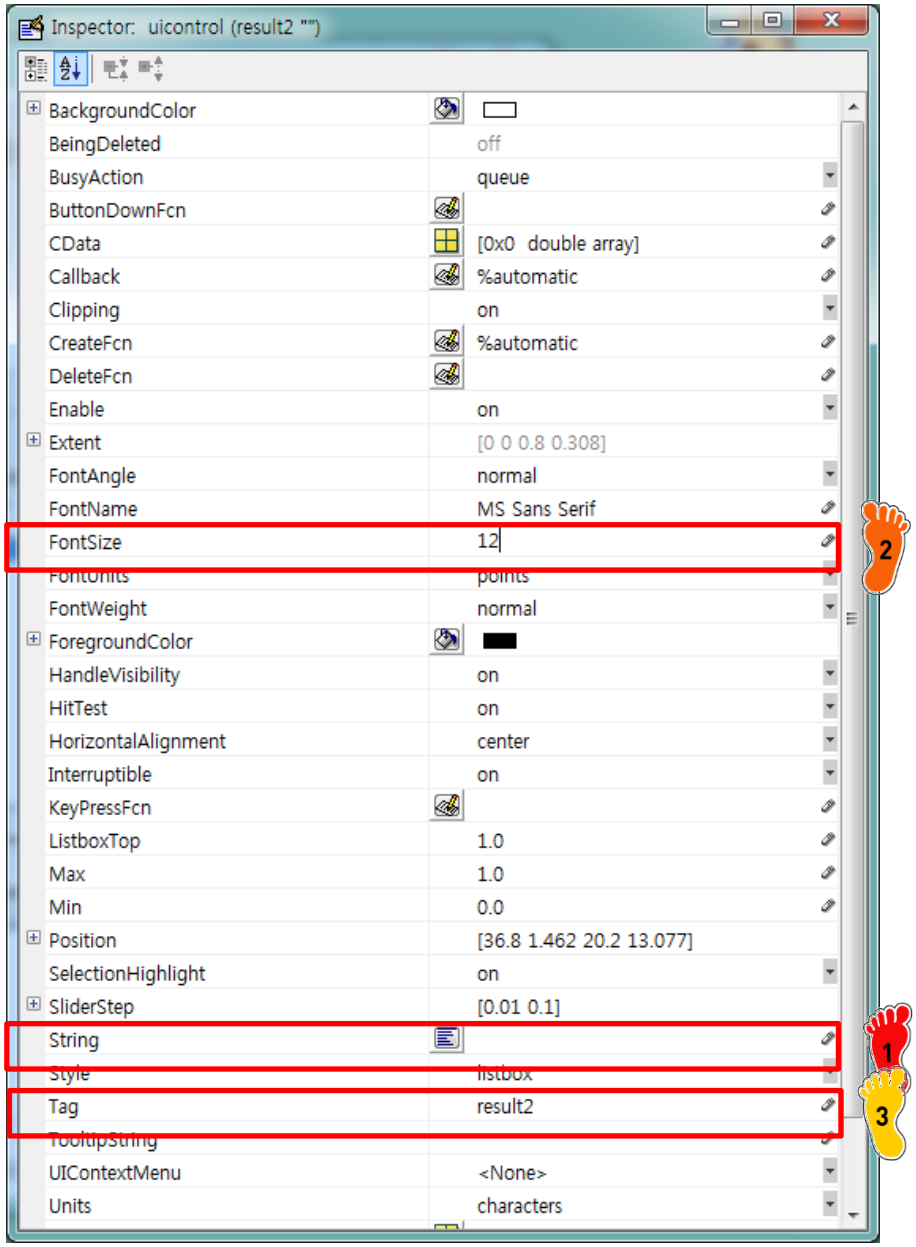
LIST BOX



1 List box icon 클릭

2 List box 생성

LIST BOX: INSPECTOR



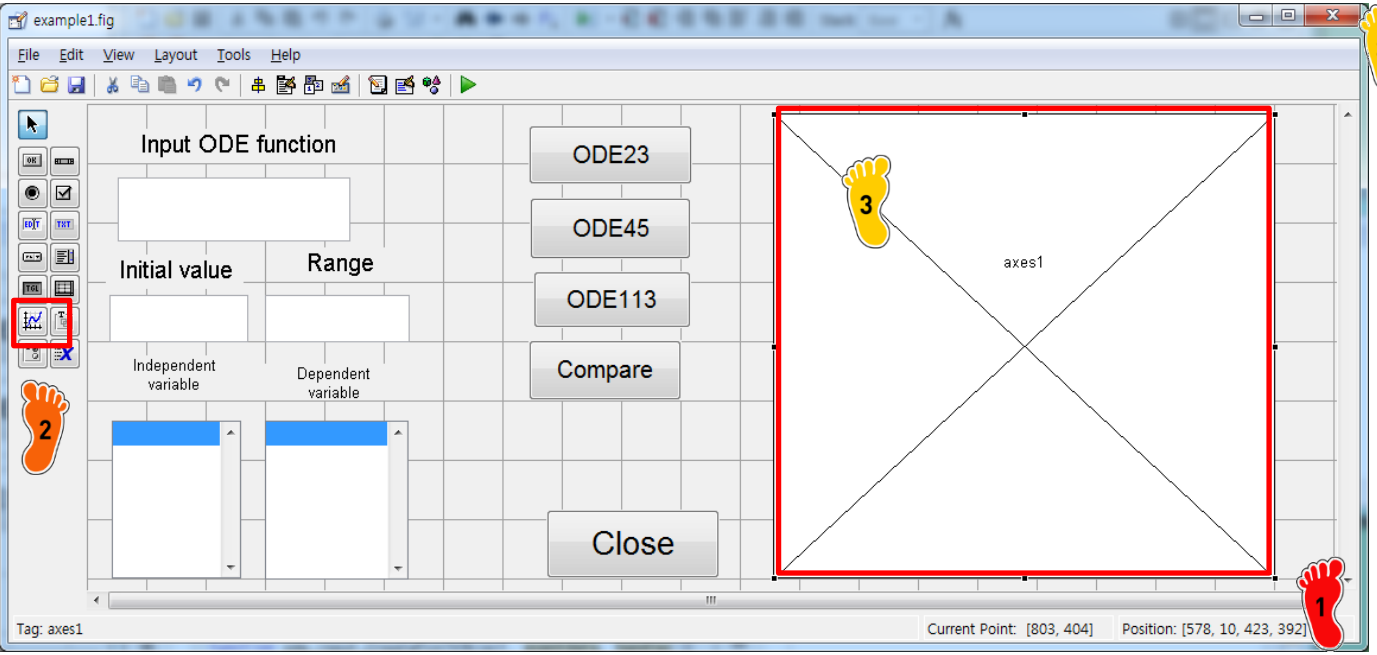
1. String 빈칸으로 변경
2. FontSize 12 로 변경
3. Tag 를 result2 로 변경

GRAPH WINDOW

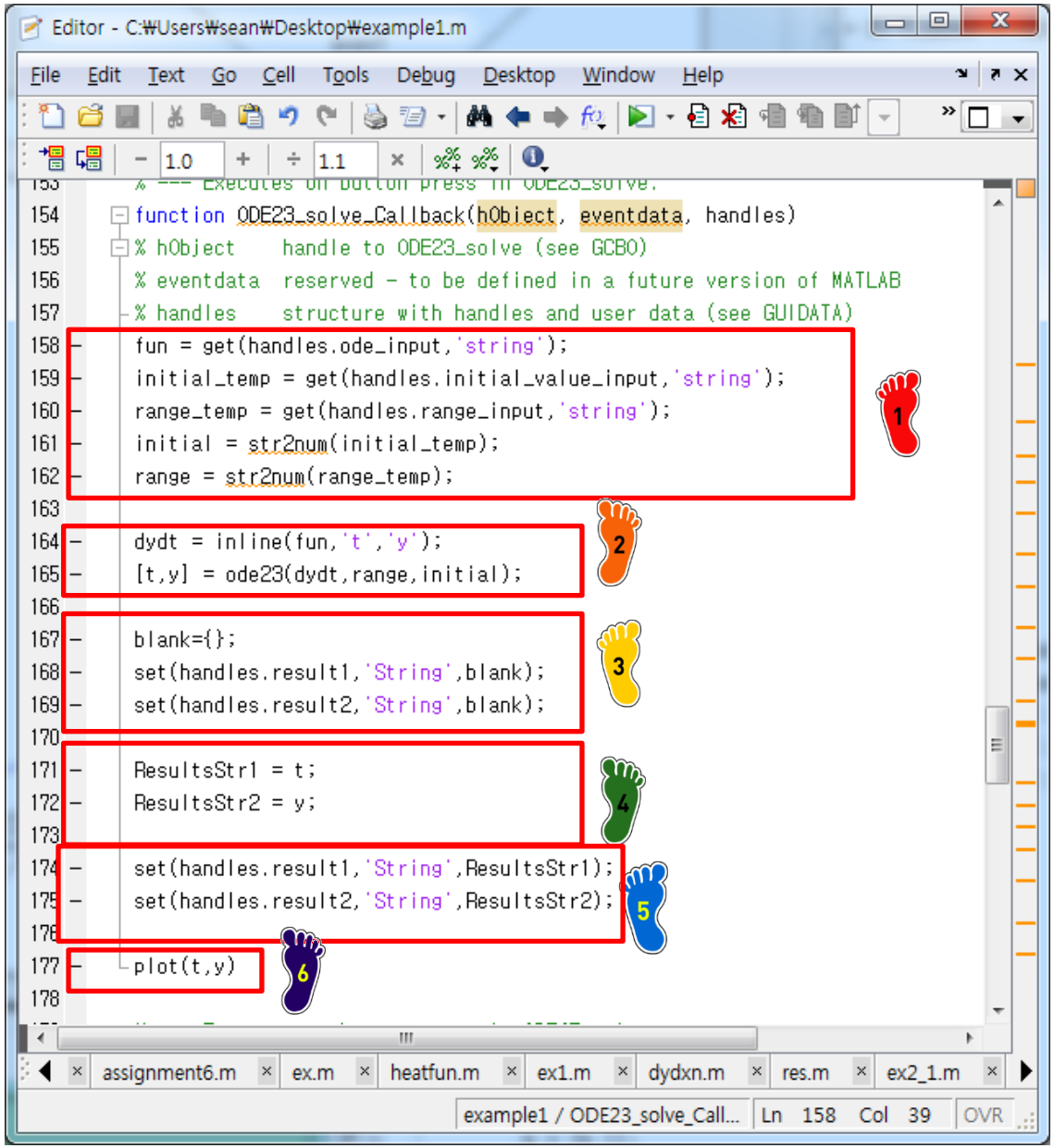
1 GUI window 창을 마우스 드래그 하여 늘림

2 axes 아이콘 클릭

3 graph 를 표시해주는 창 생성



ODE23 PUSHBUTTON CODING



1 ODE23_solve 태그로 이동

ode_input 태그로 지정된 edit box 입력 값을 get 함수로 호출

initial 과 range 값은 숫자로 입력 해야 하기 때문에 str2num 명령어를 이용하여 숫자로 변경 후 저장

2 호출한 함수를 inline 명령어를 이용하여 함수로 지정

ode23 함수로 실행

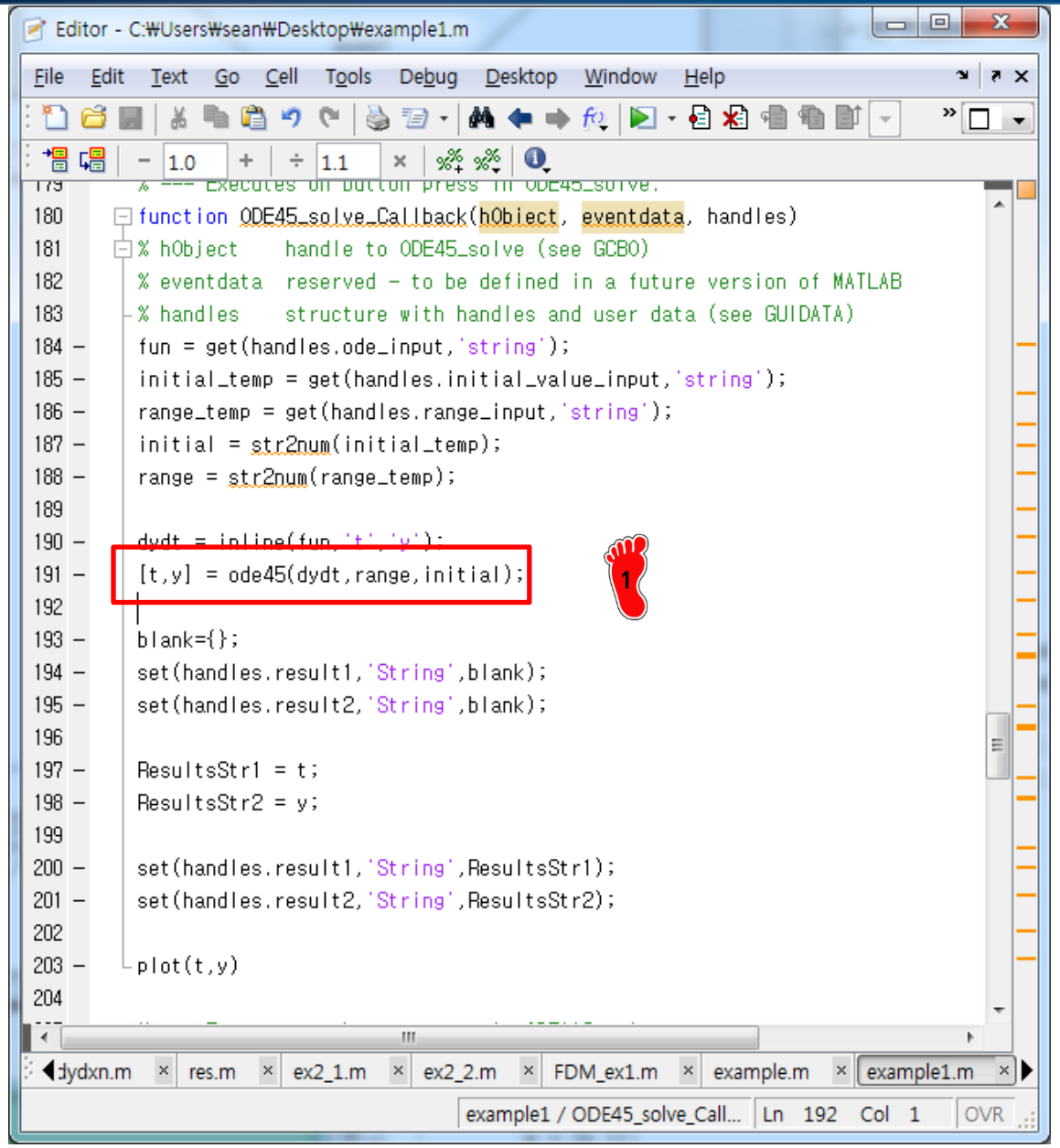
3 result1 과 result2 의 태그로 지정된 listbox 내용을 clear

4 결과 저장

5 결과를 listbox에 출력

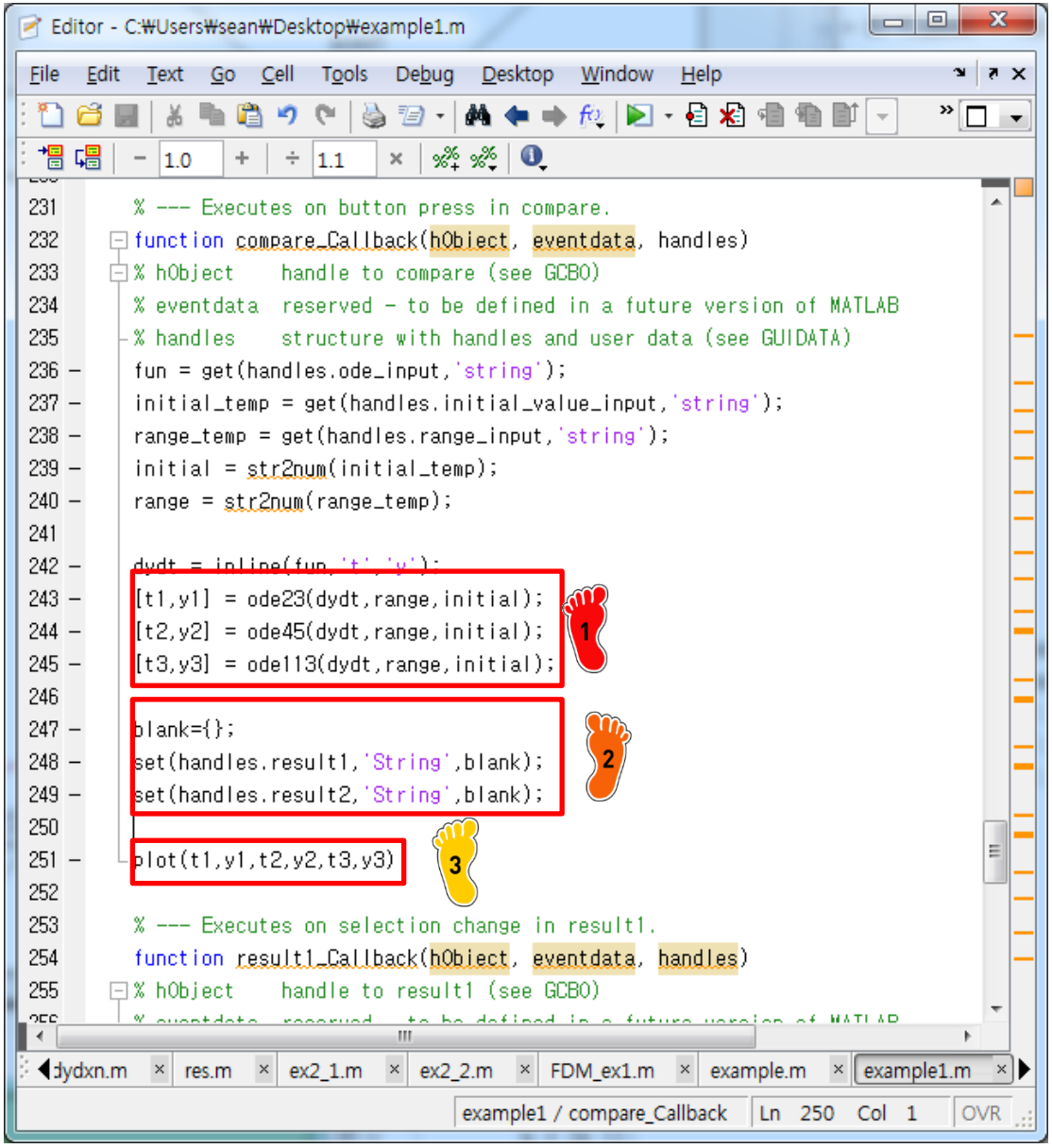
6 그래프 출력

ODE45&113 PUSHBUTTON CODING



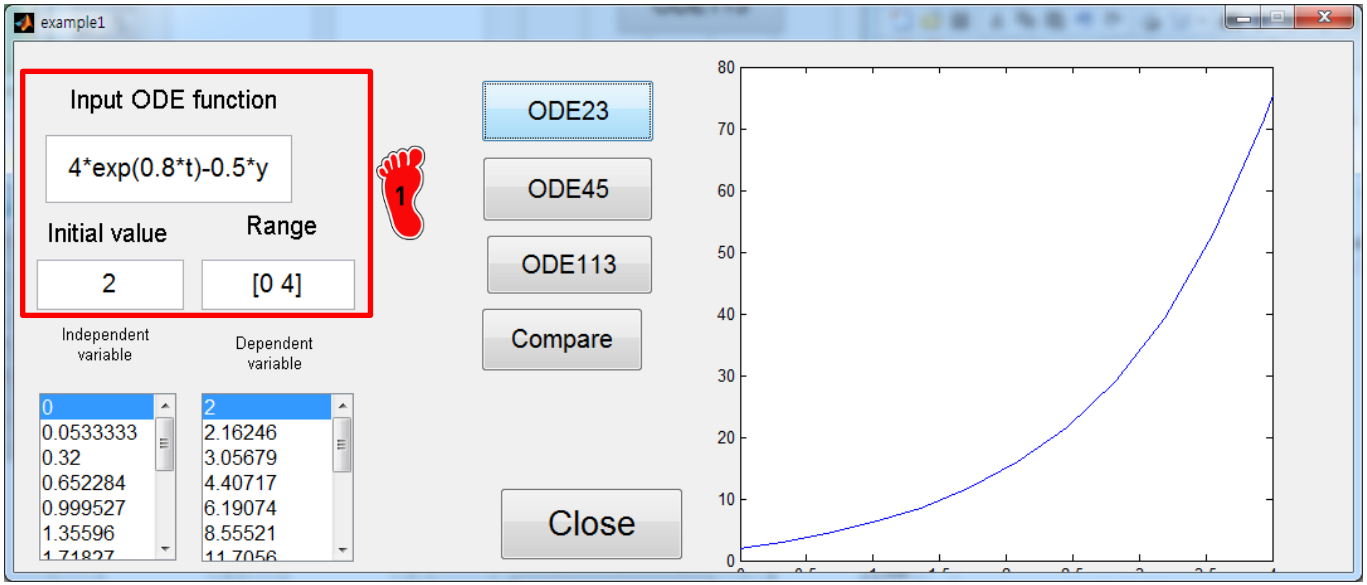
1 ode함수 명령어만 변경 후 나머지는 동일한 코드로 입력

ODE45&113 PUSHBUTTON CODING



1. 세 가지 ode 함수를 비교하기 위해 결과를 따로 저장
2. listbox 내용 출력코드는 삭제
3. 세 가지 결과를 동시에 plot

RESULT



1 함수 및 초기값과 range 입력 후 실행 버튼을 클릭하여 결과 확인

$$\frac{dy}{dx} = f(x,y) = 4e^{0.8x} - 0.5y$$

with $y(0) = 2$ from $x = 0 \sim 4$