Example

x	Exact	Least Square	Collocation	Moment	Galerkin
0.25	11.2963	11.2803 (0.14%)	11.3131 (0.15%)	11.2788 (0.15%)	11.2979 (0.01%)
0.50	13.3023	13.2713 (0.23%)	13.3333 (0.23%)	13.2692 (0.25%)	13.2955 (0.05%)
0.75	16.1440	16.1266 (0.11%)	16.1869 (0.27%)	16.1251 (0.12%)	16.1453 (0.01%)

least square method:

$$r\left(U\left(x\right)\right)=\left[2-x\left(x-1\right)\right]\alpha_1+\left[6x-x\left(x^2-1\right)\right]\alpha_2+\left(1-x\right)10+20x$$

$$\left\{\begin{aligned}&i=1:\ \frac{\partial r}{\partial\alpha_1}=2-x\left(x-1\right)\Rightarrow\int_0^1\left[2-x\left(x-1\right)\right]r\left(U\right)dx=0\\&i=2:\ \frac{\partial r}{\partial\alpha_2}=6x-x\left(x^2-1\right)\Rightarrow\int_0^1\left[6x-x\left(x^2-1\right)\right]r\left(U\right)dx=0\end{aligned}\right.\rightarrow\begin{bmatrix}\frac{47}{10}&\frac{141}{20}\\\frac{141}{20}&\frac{1436}{105}\end{bmatrix}\begin{Bmatrix}\alpha_1\\\alpha_2\end{Bmatrix}=\begin{Bmatrix}\frac{65}{2}\\\frac{323}{6}\end{Bmatrix}\Rightarrow\left\{\begin{aligned}\alpha_1&=\frac{109118}{24487}\\\alpha_2&=\frac{854}{521}\end{aligned}\right.$$

collocation method:

$$\left\{\begin{aligned}&i=1,\ \chi_1=\delta\left(x-\frac{1}{2}\right)\Rightarrow r\left(U\right)\Big|_{x=1/2}=0\\&i=2,\ \chi_2=\delta\left(x-\frac{1}{4}\right)\Rightarrow r\left(U\right)\Big|_{x=1/4}=0\end{aligned}\right.\rightarrow\begin{bmatrix}\frac{9}{4}&\frac{27}{8}\\\frac{35}{16}&\frac{111}{64}\end{bmatrix}\begin{Bmatrix}\alpha_1\\\alpha_2\end{Bmatrix}=\begin{Bmatrix}15\\\frac{50}{4}\end{Bmatrix}\Rightarrow\left\{\begin{aligned}\alpha_1&=\frac{460}{99}\\\alpha_2&=\frac{400}{297}\end{aligned}\right.$$

method of moment:

$$\int_0^1 x^{i-1}r\left(U\right)dx=0\rightarrow\left\{\begin{aligned}&i=1,\ \chi_1=1\Rightarrow\int_0^1 r\left(U\right)dx=0\\&i=2,\ \chi_2=x\Rightarrow\int_0^1 xr\left(U\right)dx=0\end{aligned}\right.\rightarrow\begin{bmatrix}\frac{13}{6}&\frac{13}{4}\\\frac{13}{12}&\frac{32}{15}\end{bmatrix}\begin{Bmatrix}\alpha_1\\\alpha_2\end{Bmatrix}=\begin{Bmatrix}15\\\frac{25}{3}\end{Bmatrix}\Rightarrow\left\{\begin{aligned}\alpha_1&=\frac{3540}{793}\\\alpha_2&=\frac{100}{61}\end{aligned}\right.$$

Galerkin method:

$$\int_0^1 \Phi_i\left(x\right) r\left(U\right) dx = 0 \rightarrow \begin{cases} i=1,\ \chi_1 = \Phi_1 = x\left(x-1\right) \Rightarrow \int_0^1 x\left(x-1\right) r\left(U\right) dx = 0 \\ i=2,\ \chi_2 = \Phi_2 = x\left(x^2-1\right) \Rightarrow \int_0^1 x\left(x^2-1\right) r\left(U\right) dx = 0 \end{cases}$$

$$\begin{bmatrix} \frac{11}{30} & \frac{11}{20} \\ \frac{11}{20} & \frac{92}{105} \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \end{Bmatrix} = \begin{Bmatrix} \frac{5}{2} \\ \frac{23}{6} \end{Bmatrix} \Rightarrow \begin{cases} \alpha_1 = \frac{2070}{473} \\ \alpha_2 = \frac{70}{43} \end{cases}$$

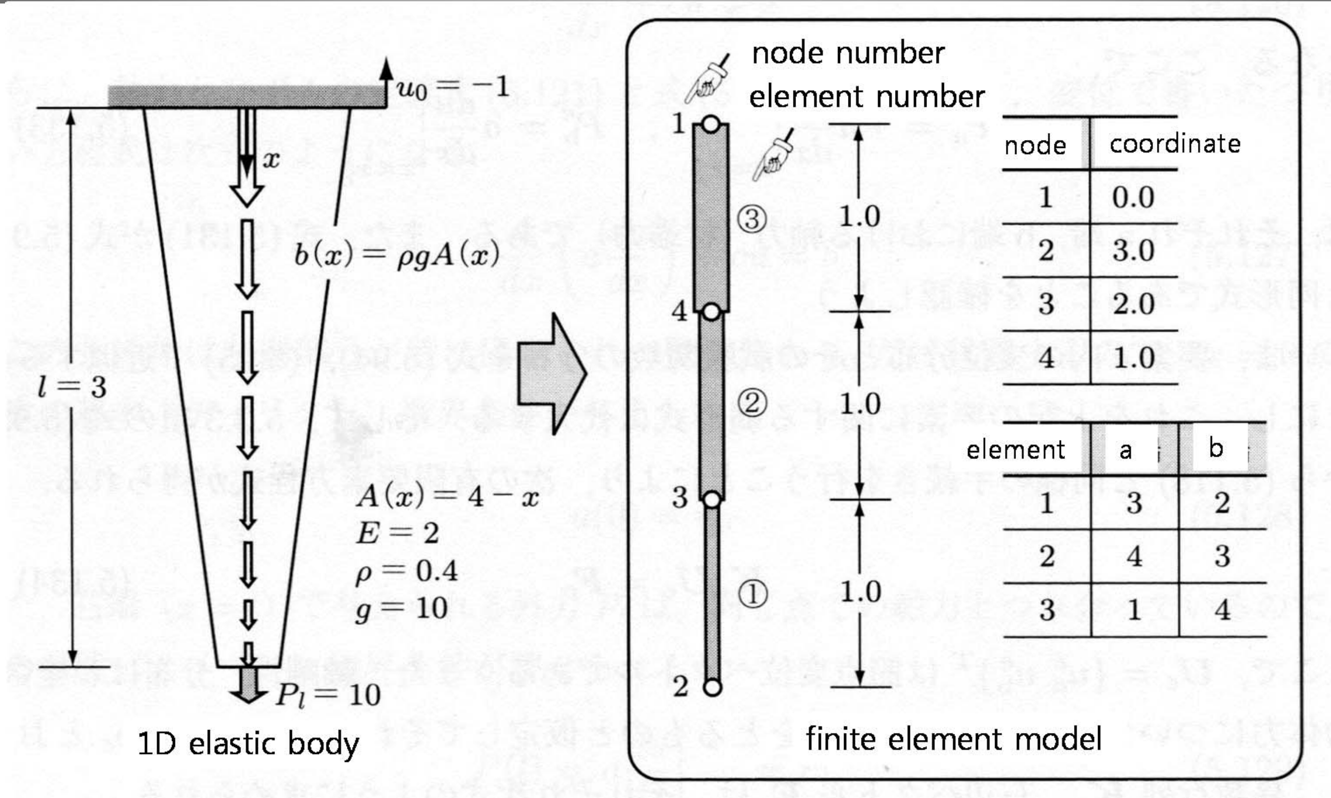
Example (2)

$$M=3,\ m=1,\ u_0=10,\ l=1,\ Q_l=0$$

$$U(x)=10-\frac{35175}{4658}x+\frac{10725}{2329}x^2-\frac{10675}{18632}x^3 \rightarrow \left.\frac{dU}{dx}\right|_{x=1}=-\frac{1125}{18632}\neq 0\left(=Q_l\right)$$

x	u(x)	U(x)		
		M=2	M=3	M=4
0.25	8.3903	8.4186 (0.34%)	8.3910 (0.9×10^{-2}%)	8.3902 (7.8×10^{-4}%)
0.50	7.3076	7.3055 (0.03%)	7.3039 (5.1×10^{-2}%)	7.3076 (1.0×10^{-4}%)
0.75	6.6841	6.6607 (0.35%)	6.6850 (1.2×10^{-2}%)	6.6842 (7.8×10^{-4}%)

1D Elasticity: Example



element	x_a	x_b	h	A	E	a = EA	b = ρgA
①	2.0	3.0	1.0	3/2	2.0	3.0	6.0
②	1.0	2.0	1.0	5/2	2.0	5.0	10.0
③	0.0	1.0	1.0	7/2	2.0	7.0	14.0

$$\boldsymbol{K}_{①} = a_{①}\begin{bmatrix} \dfrac{1}{h_{①}} & -\dfrac{1}{h_{①}} \\ -\dfrac{1}{h_{①}} & \dfrac{1}{h_{①}} \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ -3 & 3 \end{bmatrix},\ \boldsymbol{F}_{①} = \frac{b_{①}h_{①}}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \begin{Bmatrix} P_a^{①} \\ P_b^{①} \end{Bmatrix} = \begin{Bmatrix} 3 \\ 3 \end{Bmatrix} + \begin{Bmatrix} P_a^{①} \\ P_b^{①} \end{Bmatrix}$$

$$\boldsymbol{K}_{②} = a_{②}\begin{bmatrix} \dfrac{1}{h_{②}} & -\dfrac{1}{h_{②}} \\ -\dfrac{1}{h_{②}} & \dfrac{1}{h_{②}} \end{bmatrix} = \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix},\ \boldsymbol{F}_{②} = \frac{b_{②}h_{②}}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \begin{Bmatrix} P_a^{②} \\ P_b^{②} \end{Bmatrix} = \begin{Bmatrix} 5 \\ 5 \end{Bmatrix} + \begin{Bmatrix} P_a^{②} \\ P_b^{②} \end{Bmatrix}$$

$$\boldsymbol{K}_{③} = a_{③}\begin{bmatrix} \dfrac{1}{h_{③}} & -\dfrac{1}{h_{③}} \\ -\dfrac{1}{h_{③}} & \dfrac{1}{h_{③}} \end{bmatrix} = \begin{bmatrix} 7 & -7 \\ -7 & 7 \end{bmatrix},\ \boldsymbol{F}_{③} = \frac{b_{③}h_{③}}{2}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + \begin{Bmatrix} P_a^{③} \\ P_b^{③} \end{Bmatrix} = \begin{Bmatrix} 7 \\ 7 \end{Bmatrix} + \begin{Bmatrix} P_a^{③} \\ P_b^{③} \end{Bmatrix}$$

$$①:\begin{Bmatrix} U_a^{①} \\ U_b^{①} \end{Bmatrix} \equiv \begin{Bmatrix} U_3 \\ U_2 \end{Bmatrix},\ ②:\begin{Bmatrix} U_a^{②} \\ U_b^{②} \end{Bmatrix} \equiv \begin{Bmatrix} U_4 \\ U_3 \end{Bmatrix},\ ③:\begin{Bmatrix} U_a^{③} \\ U_b^{③} \end{Bmatrix} \equiv \begin{Bmatrix} U_1 \\ U_4 \end{Bmatrix}$$

$$\begin{bmatrix} (0)+(0)+7 & (0)+(0)+(0) & (0)+(0)+(0) & (0)+(0)-7 \\ (0)+(0)+(0) & 3+(0)+(0) & -3+(0)+(0) & (0)+(0)+(0) \\ (0)+(0)+(0) & -3+(0)+(0) & 3+5+(0) & (0)-5+(0) \\ (0)+(0)-7 & (0)+(0)+(0) & (0)-5+(0) & (0)+5+7 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{Bmatrix} = \begin{Bmatrix} (0)+(0)+7 \\ 3+(0)+(0) \\ 3+5+(0) \\ (0)+5+7 \end{Bmatrix} + \begin{Bmatrix} (0)+(0)+P_a^{③} \\ P_b^{①}+(0)+(0) \\ P_a^{①}+P_b^{②}+(0) \\ (0)+P_a^{②}+P_b^{③} \end{Bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 0 & 0 & 7 \\ 0 & 3 & -3 & 0 \\ 0 & -3 & 8 & -5 \\ -7 & 0 & -5 & 12 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 3 \\ 8 \\ 12 \end{Bmatrix} + \begin{Bmatrix} P_a^{③} \\ P_b^{①} \\ P_a^{①}+P_b^{②} \\ P_a^{②}+P_b^{③} \end{Bmatrix}$$

$$\begin{Bmatrix} P_a^{③} \\ P_b^{①} \\ P_a^{①}+P_b^{②} \\ P_a^{②}+P_b^{③} \end{Bmatrix} = \begin{Bmatrix} R_1 \\ P_l \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ 10 \\ 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{bmatrix} 7 & 0 & 0 & 7 \\ 0 & 3 & -3 & 0 \\ 0 & -3 & 8 & -5 \\ -7 & 0 & -5 & 12 \end{bmatrix} \begin{Bmatrix} -1 \\ U_2 \\ U_3 \\ U_4 \end{Bmatrix} = \begin{Bmatrix} 7 \\ 3 \\ 8 \\ 12 \end{Bmatrix} + \begin{Bmatrix} R_1 \\ 10 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{bmatrix} 3 & -3 & 0 \\ -3 & 8 & -5 \\ 0 & -5 & 12 \end{bmatrix} \begin{Bmatrix} U_2 \\ U_3 \\ U_4 \end{Bmatrix} = \begin{Bmatrix} 13 \\ 8 \\ 12 \end{Bmatrix} + \begin{Bmatrix} 10 \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 0 \\ -7\times(-1) \end{Bmatrix} = \begin{Bmatrix} 13 \\ 8 \\ 5 \end{Bmatrix} \Rightarrow \begin{Bmatrix} U_2 \\ U_3 \\ U_4 \end{Bmatrix} = \begin{Bmatrix} \dfrac{1286}{105} \\ \dfrac{277}{35} \\ \dfrac{26}{7} \end{Bmatrix}$$

$$R_1 = \begin{Bmatrix} 7 & 0 & 0 & -7 \end{Bmatrix} \begin{Bmatrix} -1 \\ \dfrac{1286}{105} \\ \dfrac{277}{35} \\ \dfrac{26}{7} \end{Bmatrix} - 7 = -40$$

$$\varepsilon = \frac{du}{dx} \approx \frac{d\boldsymbol{N}_{③}}{dx}\boldsymbol{U}_{③} = \boldsymbol{B}_{③}\boldsymbol{U}_{③} = \begin{Bmatrix} -\dfrac{1}{h_{③}} & \dfrac{1}{h_{③}} \end{Bmatrix} \begin{Bmatrix} U_a^{③} \\ U_b^{③} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{1}{h_{③}} & \dfrac{1}{h_{③}} \end{Bmatrix} \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} = \begin{Bmatrix} -1 & 1 \end{Bmatrix} \begin{Bmatrix} -1 \\ \dfrac{26}{7} \end{Bmatrix} = \frac{33}{7}$$

$$\sigma = E\varepsilon \approx \frac{66}{7}$$