4.5.1

$$\hat{g}(k) = \int_{-\infty}^{0} -e^{ax}e^{-ikx}dx + \int_{0}^{\infty} e^{-ax}e^{-ikx}dx = \frac{-e^{x(a-ik)}}{a-ik} + \frac{e^{x(a+ik)}}{a+ik}$$

The decay rate of $\hat{g}(k)$ is $\frac{1}{k}$. There is a discontinuity in $g(x)$.

4.5.8

We only illustrate with $f(x) = e^{-|x|}$.
$\hat{f}(k) = \frac{2}{1+k^2}$
$\hat{g}(k) = \frac{2a}{a^2+k^2}$

Starting with $g(x) = f(ax)$, take the derivative of each side to get $\frac{dg}{dx} = \frac{1}{a}\frac{df}{dx}$.
Now take the transform of each side:

$$ik\hat{g}(k) = \frac{1}{a}ik\hat{f}(k)$$

Then $\hat{g}(k) = \frac{1}{a}\hat{f}(k)$

4.5.11

$$\frac{du}{dx}+au=\delta(x-d)\rightarrow ik\hat{u}(k)+a\hat{u}(k)=e^{-ikd}\rightarrow\hat{u}(k)=\frac{e^{-ikd}}{a+ik}$$

$$e^{ikd}\hat{u}(k)=\frac{1}{a+ik}=\hat{v}(k)\rightarrow v(x)=\begin{cases}e^{-ax} & \text{for } x\geq 0\\ 0 & \text{for } x<0\end{cases}\rightarrow u(x)=\begin{cases}e^{-a(x-d)} & \text{for } x\geq d\\ 0 & \text{for } x<d\end{cases}$$

4.5.12

$$\left(\text{integral of u}\right) - \left(\text{derivative of u}\right) = \delta\left(x\right)$$

$$\rightarrow \frac{\hat{u}\left(k\right)}{ik} - ik\hat{u}\left(k\right) = 1 \rightarrow \hat{u}\left(k\right) = \frac{ik}{1+k^2} \xrightarrow{\text{Example 6}} u\left(x\right) = \begin{cases} -\dfrac{e^{-x}}{2} & \text{for } x > 0 \\ \dfrac{e^{x}}{2} & \text{for } x < 0 \end{cases}$$

4.5.22

(22) $\left((ik)^4 - 2(ik)^2 + 1\right)\hat{G}(k) = 1$

$$\hat{G}(k) = \frac{1}{k^4 + 2k^2 + 1} = \frac{1}{(k^2+1)^2}$$

Since $f(x) = \frac{1}{2}e^{-|x|}$ = 2-way pulse with $a = 1$ has $\hat{f}(k) = \frac{1}{k^2+1}$

we can compute $G(x)$ by convolution $f(x) * f(x)$