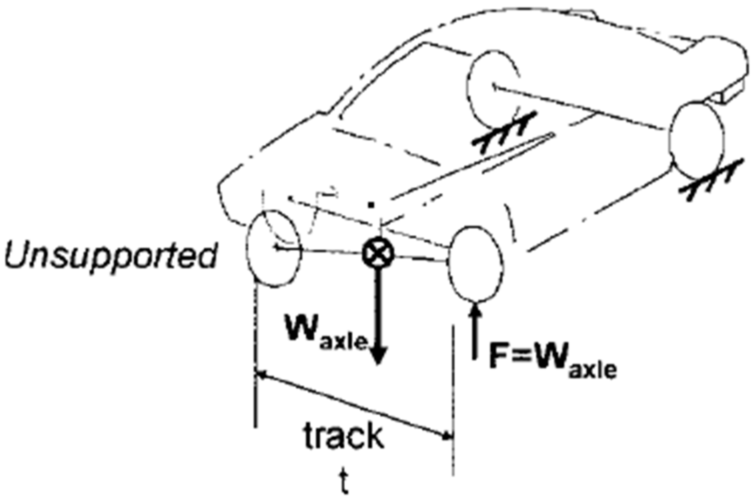
Design for Body Torsion

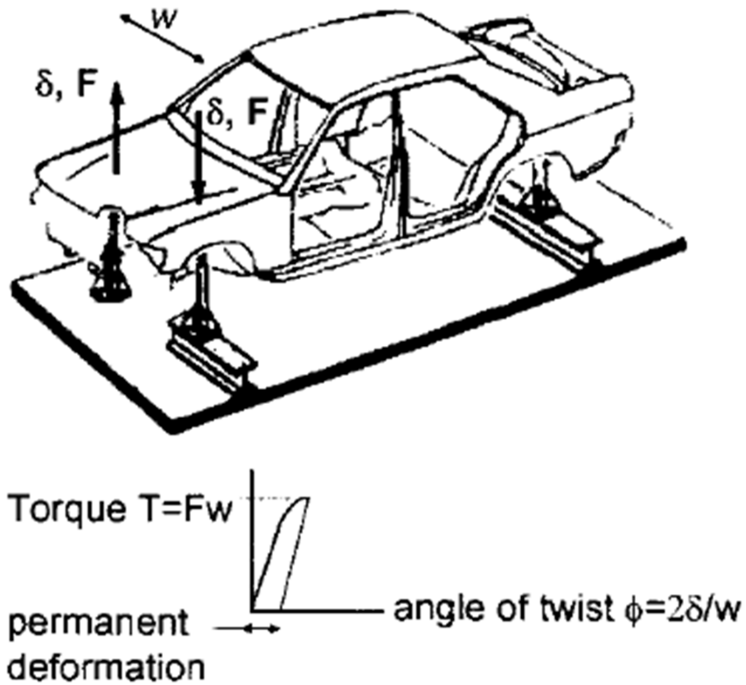
- Body torsion requirements
  - Body torsion strength
  - Body torsion stiffness
  - For midsize vehicle: K = 12000 Nm/°, T = 6250 Nm
- Internal loads during global torsion: load path analysis
- Analysis of body torsional stiffness
  - Shear strain energy
  - Effective shear stiffness
- Torsional stiffness of convertibles and framed vehicles

5.1 Body Torsion Strength Requirement

- Maximum torque to recover its shape with little deformation upon removal
- Vehicle-use condition: twist ditch maneuver
  - Input: δ, output: F (load cell)

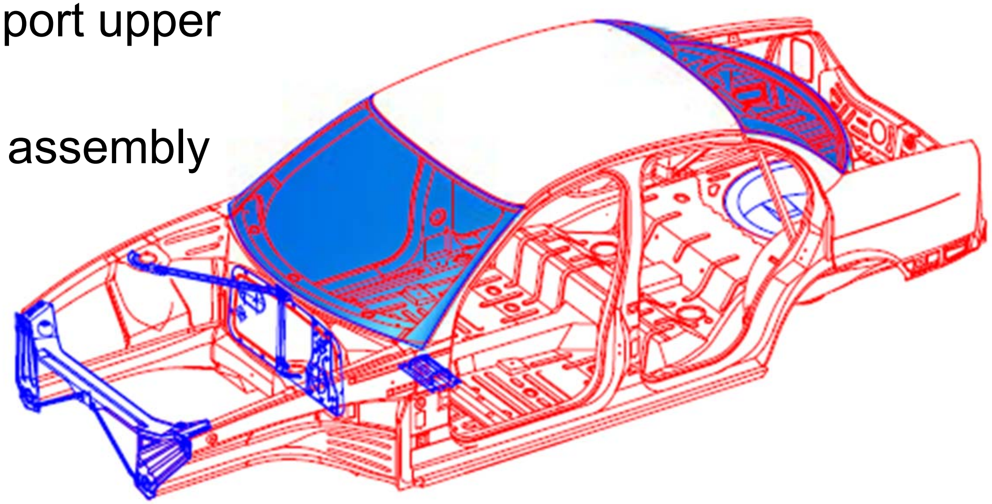


$$T_{\max} = W_{axle} \frac{t}{2}$$



Test Configuration

- Welded body structure
- Bonded windshield and back light (aluminum panels)
- Bonded and bolted panel dash insert
- Bonded panel spare tire tub
- Bolted reinforcement panel dash brake booster
- Bolted braces radiator
- Bolted reinforcement radiator rail closeout RH/LH
- Bolted reinforcement radiator support upper
- Bolted tunnel bridge lower/upper
- Bolted brace cowl to shock tower assembly

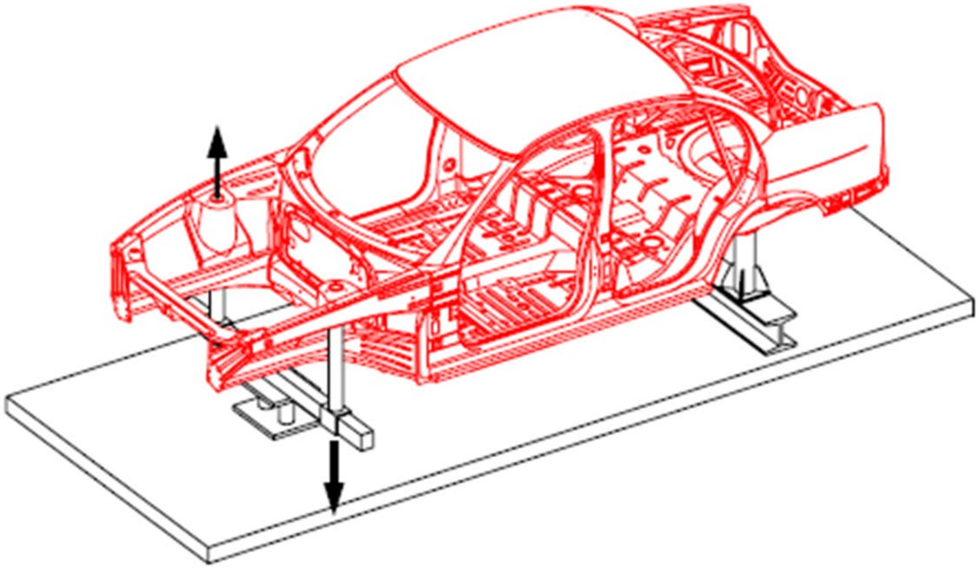


– Holding
  - Front: at panel skirt RH/LH
  - Rear: at plate rear spring upper

– Measurement
  - 12 stadia rods along the front rails, rockers, rear rails

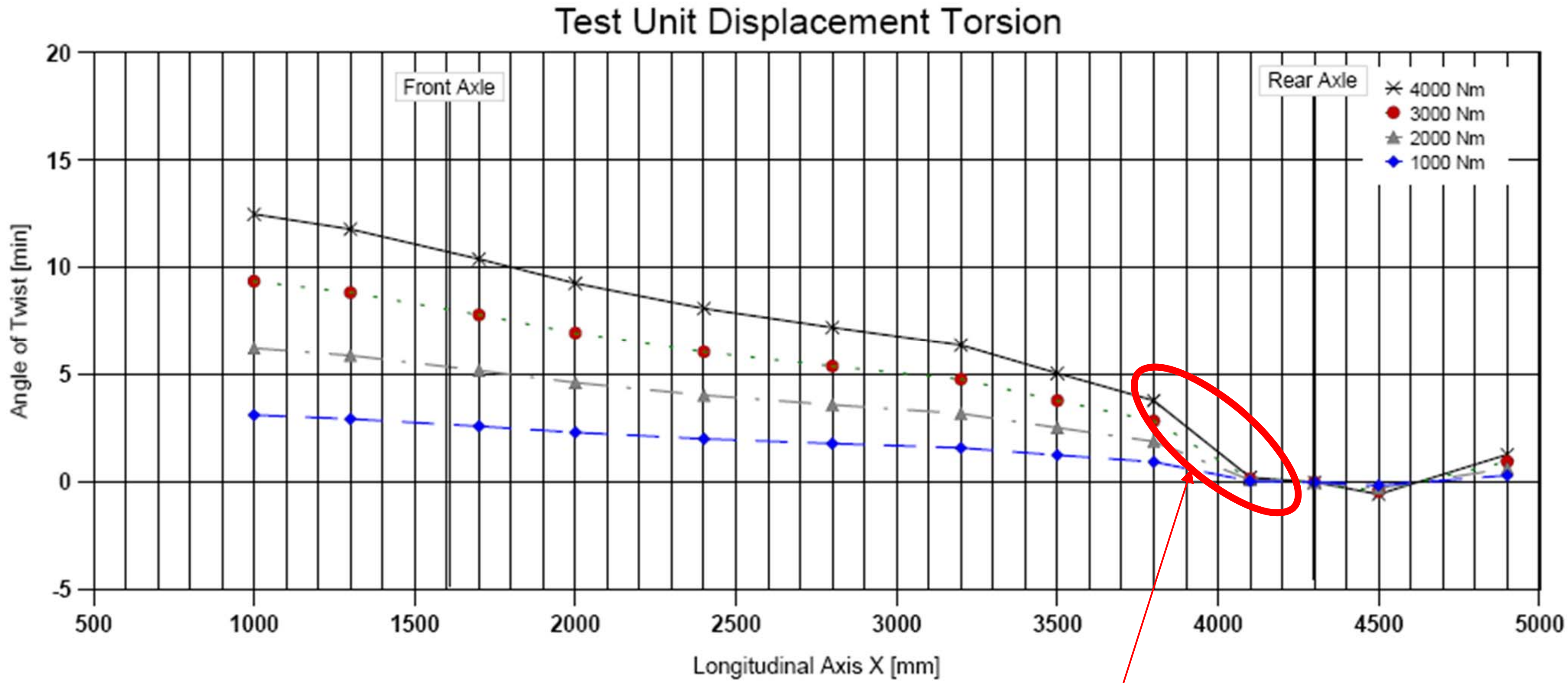
Static Torsion

– Constraint: two locations at the plate spring rear upper
– Load: panel skirt RH/LH by a scale beam, from M = 1000Nm to 4000Nm



Static Torsion: Test Results

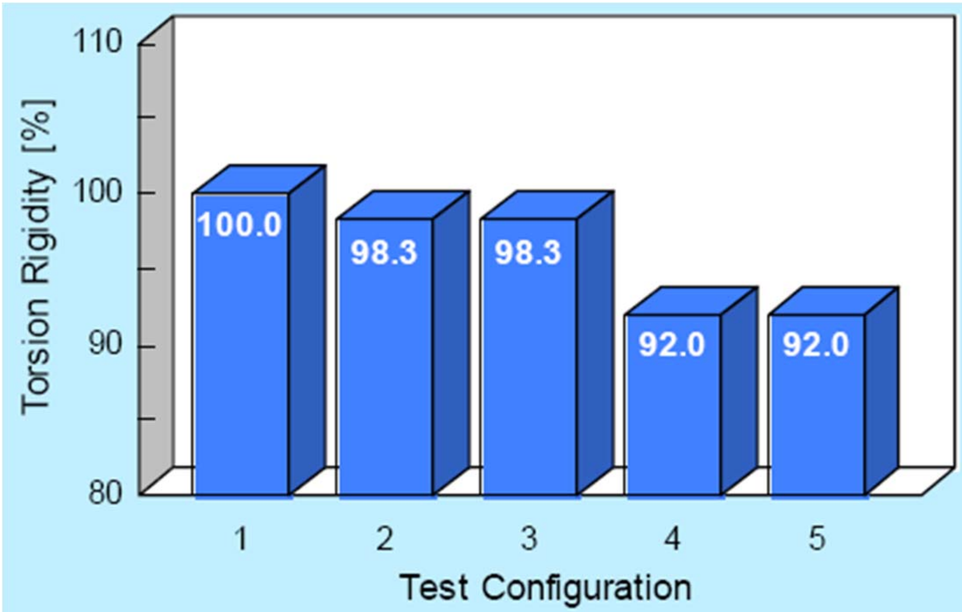
– With glass: 21,620 Nm/deg
– Without glass: 15,790 Nm/deg



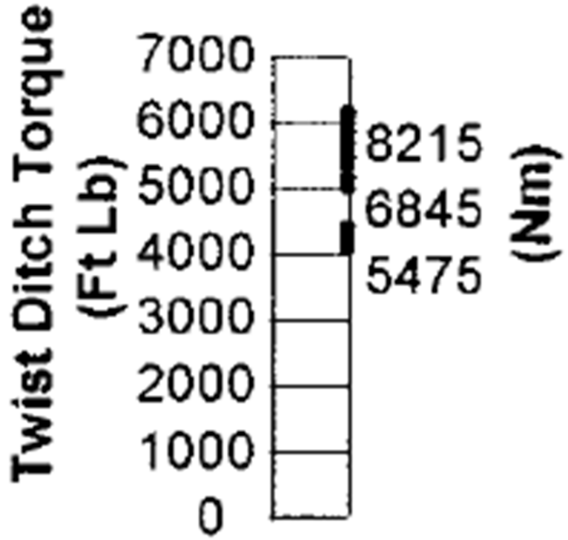
positive impact of the "member pass through" (Part No.090)

Static Torsion: impact of bonded and/or bolted parts

- 1. full configuration
- 2. as 1, but without braces radiator
- 3. as 2, but without radiator support upper
- 4. as 3, but without bolted brace cowl to shock tower assembly
- 5. as 4, but without tunnel bridge



Torsion Strength Benchmark Data



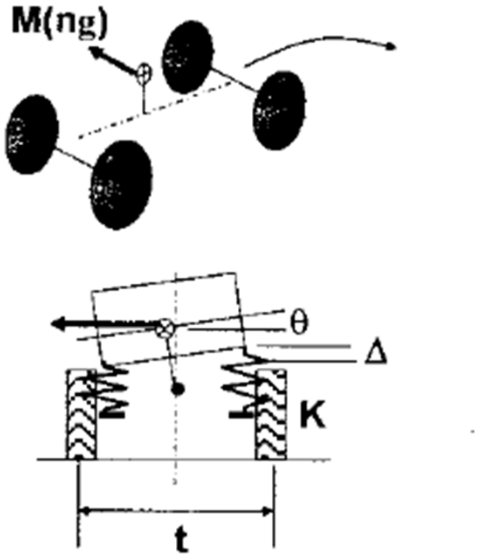
Range for 20 Cars from Small to Luxury Segments

5.2 Body Torsion Stiffness Requirement

- Body torsion test
  - Slope in the linear region of the applied couple vs. angular rotation
- Required functions for high torsional stiffness
  - Good handling property: torsionally stiff body relative to the suspension stiffness
    - Torsional stiffness: 10,000 Nm/°
  - Solid structural feel: minimize relative deformations which cause squeaks and rattles
    - Feel of solidness over road irregularities
    - Related to fundamental natural frequency of the body twisting mode: the higher, the more desirable solid feel
    - Desirable vehicle torsional frequency range: 22~25 Hz
    - Torsional stiffness (benchmarking): 12,000 Nm/°

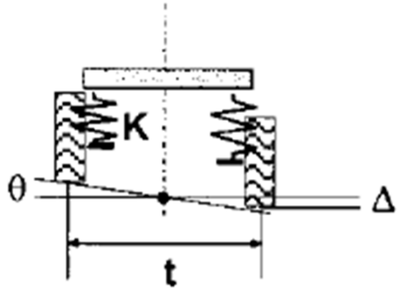
Good Handling Properties (1)

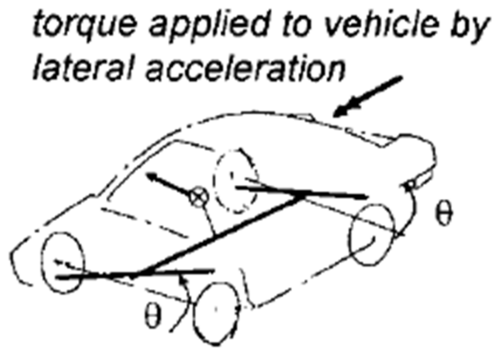
- Corner turn: roll on the suspension ride spring
- Weight transfer from the inside wheels to the outside wheels
- Affect the steering characteristics of the vehicle
- Suspension design: rigid body assumption → high torsional stiffness (much stiffer than the roll stiffness: 1000 Nm/°)



Roll Gain: Degrees of Vehicle Roll per g of Lateral Acceleration: θ/n

First order estimate of vehicle roll stiffness



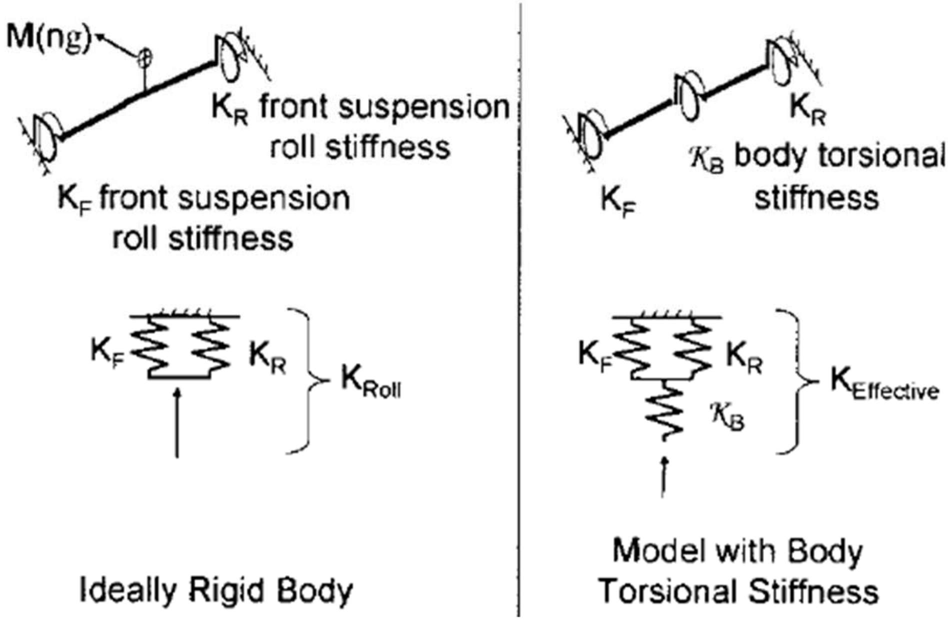


$$K_{RollVehicle} = K_{RollFront} + K_{RollRear} = \frac{t^2 K_{RideFront}}{2} + \frac{t^2 K_{RideRear}}{2}$$

$$\left.\begin{array}{l} t = 1560mm \\ K_{Ride} = 23.4N/mm \end{array}\right\} \rightarrow K_{Roll} = 57,000Nm/rad = 1000Nm/\deg$$

Good Handling Properties (2)

- For typical passenger cars, K_{body} = 10,000 Nm/°



K_{eff} : stiffness with a torsionally flexible body

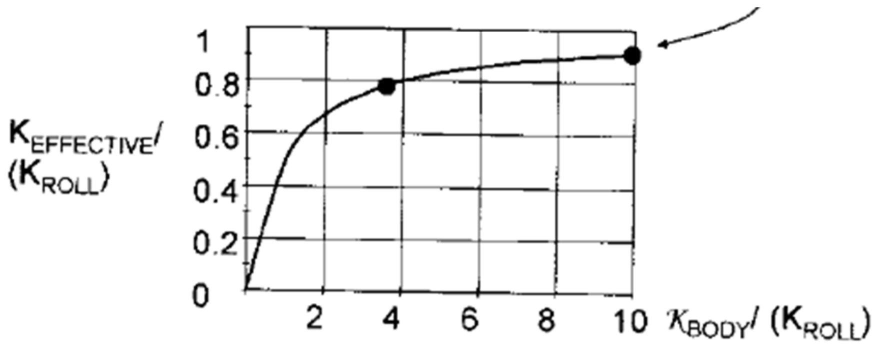
K_{roll} : suspension stiffness with a rigid body

K_B : body torsional stiffness

$$K_{eff} = \frac{K_{roll} K_B}{K_{roll} + K_B} \rightarrow \frac{K_{eff}}{K_{roll}} = \frac{1}{\frac{K_{roll}}{K_B} + 1} = \frac{1}{\frac{1}{\frac{K_B}{K_{roll}}} + 1}$$

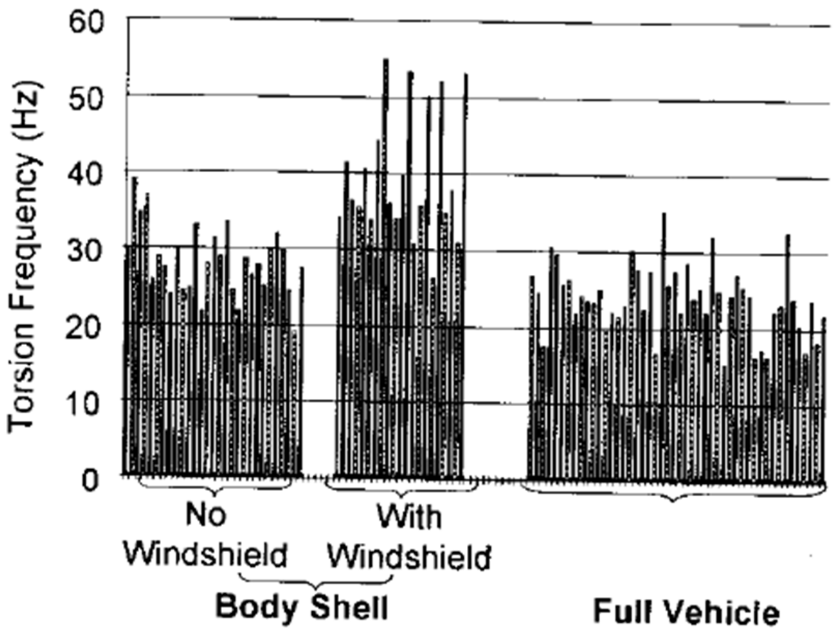
$$\frac{K_{eff}}{K_{roll}} = 0.9 \left(\text{wish to approach } 1 \right) \rightarrow K_B = 10 K_{roll}$$

<u>Only suspension roll rate</u>

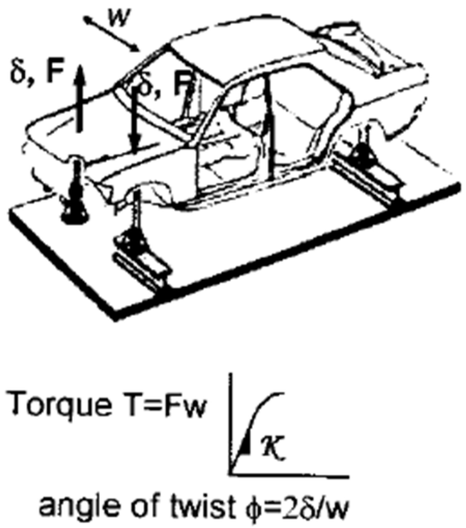


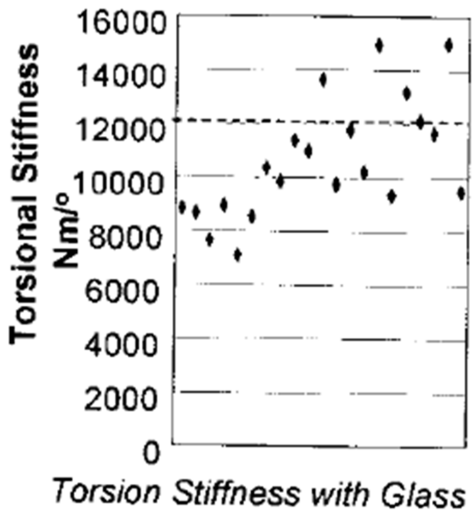
Benchmark Data

- Torisional frequency



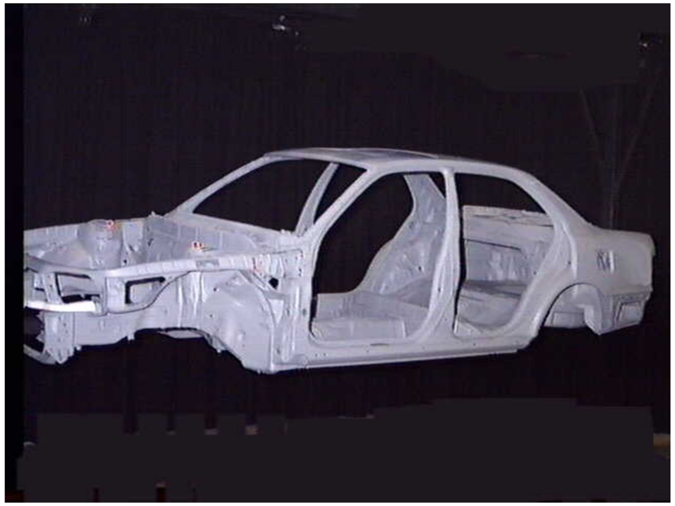
- Torsional stiffness

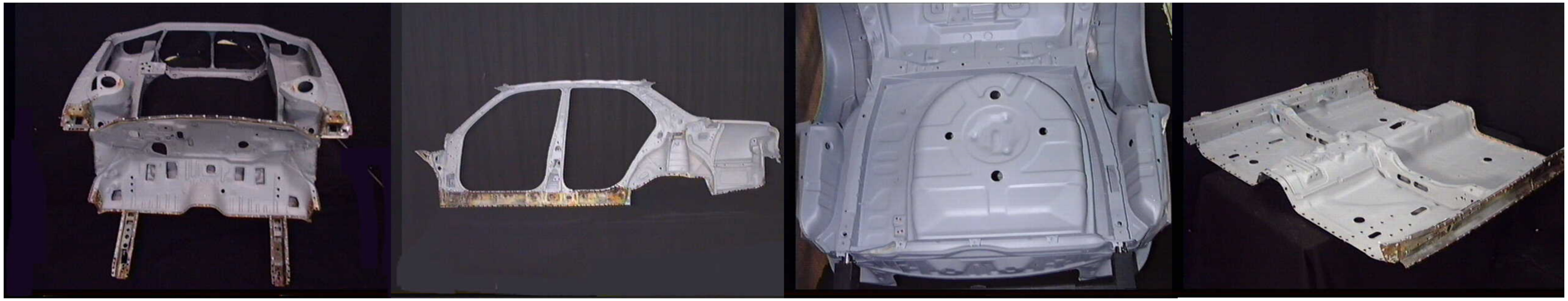




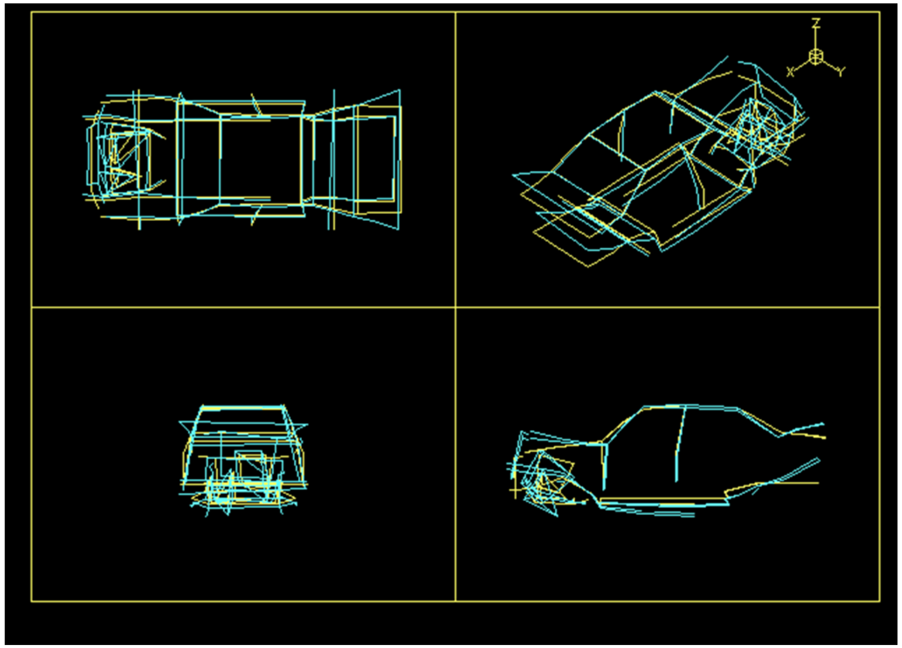
Body-In-White: TOYOTA-CAMRY

- Nameplate CAMRY - Model XLE -Year 1992
  Style 4 DR , Vehicle class MID-SIZE
  Overall length (m) 4.77 , Overall width (m) 1.77 ,
  Overall height (m) 1.4 , Wheelbase (m) 2.62
  Base price ($) 20,508 , Seating capacity 5 , Curb weight (Kg) 1493
  Body/frame UNIBODY , Body material STEEL
  Fuel economy (MPG) 18/24 , Engine 3.0L, V6, TRANS DOHC , Chassis layout FRONT ,
  Transmission 4 SP AUTO
  Suspension front MacPHERSON STRUT ,
  Suspension rear INDEPENDENT DUAL-LINK

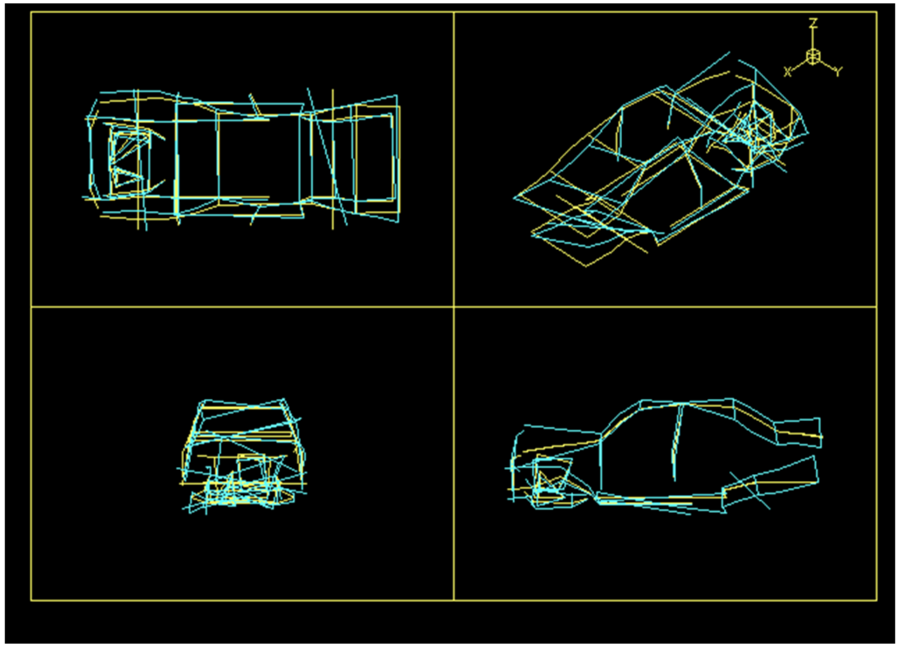




Vibration: TOYOTA-CAMRY



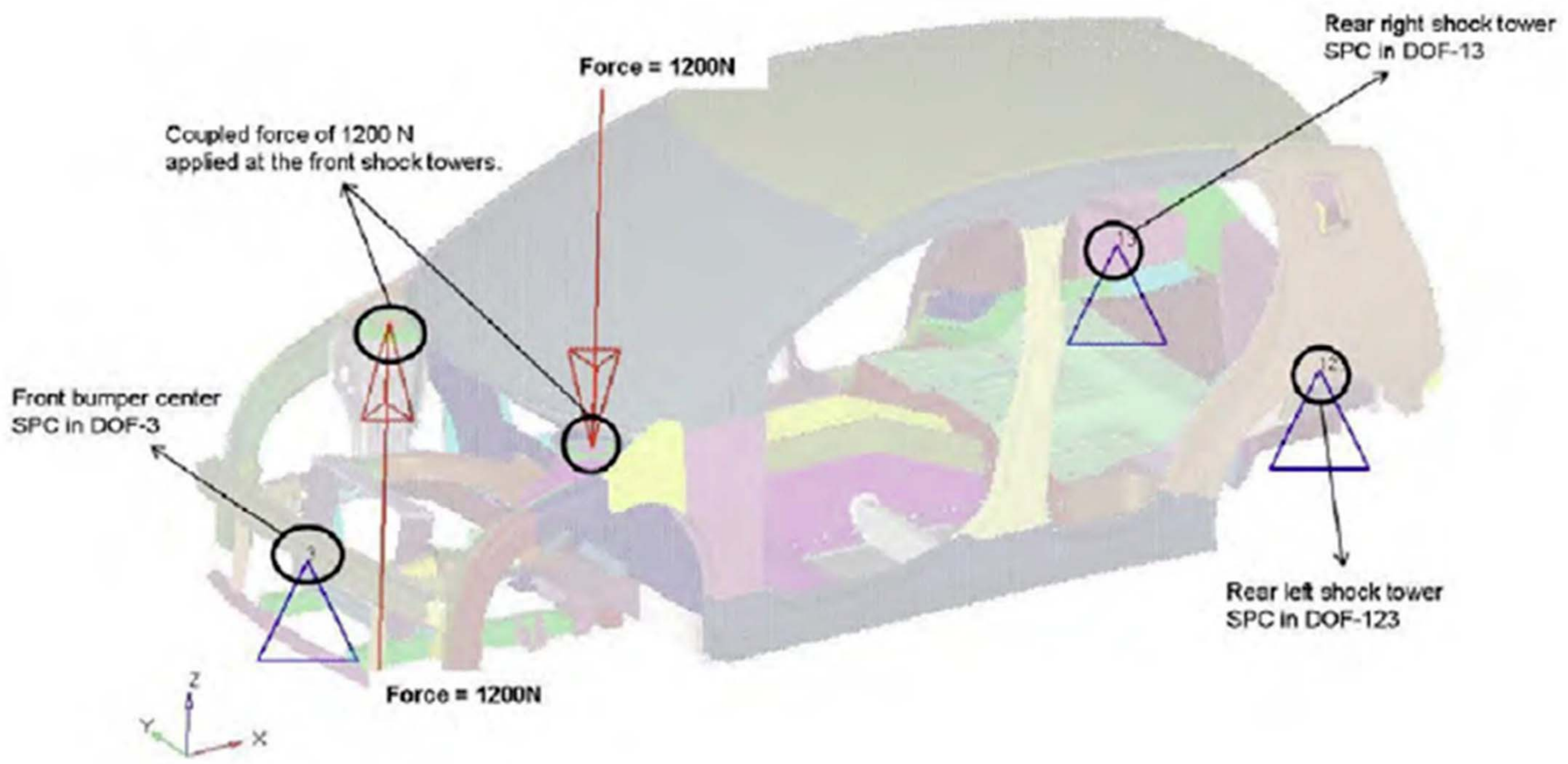
Bending



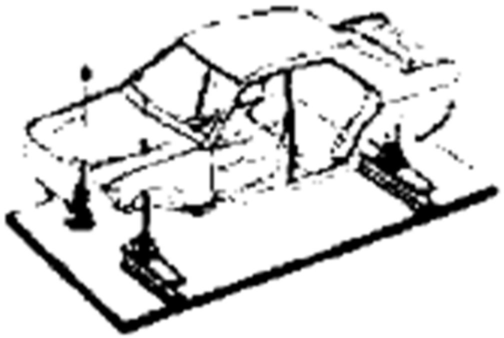
Torsion

Torsion Stiffness

- Constraints and Loading in FSV Report



Typical Torsional Requirements: Midsize Vehicle



Restraints at Suspension Attachments

Torsion Stiffness

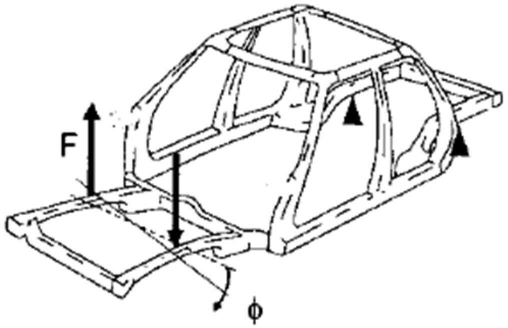
Nominal Value
Stiffness = 12000 Nm/o

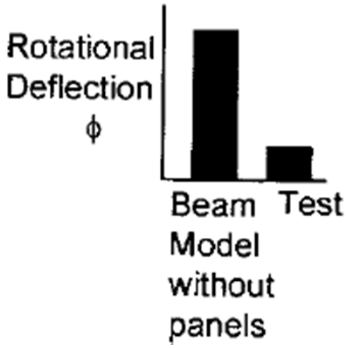
Torsion Strength

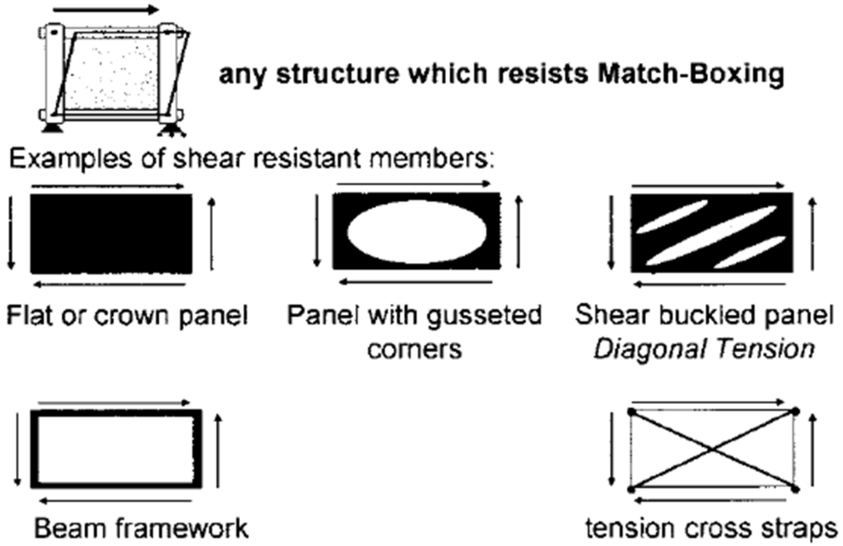
Nominal Value
T= 6250Nm
no permanent deformation

5.3 Load Path Analysis: Global Torsion

- Understand how global body requirements flow down to loads on structural elements
- Idealized structure as a framework of beams
  - Torsional stiffness: 10~30% of experimental values
- Dominant structure in reaction torsion loading
  - Surface: shear resistant members





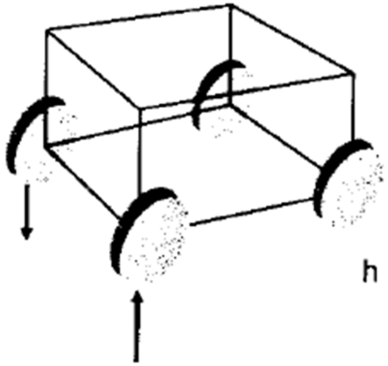


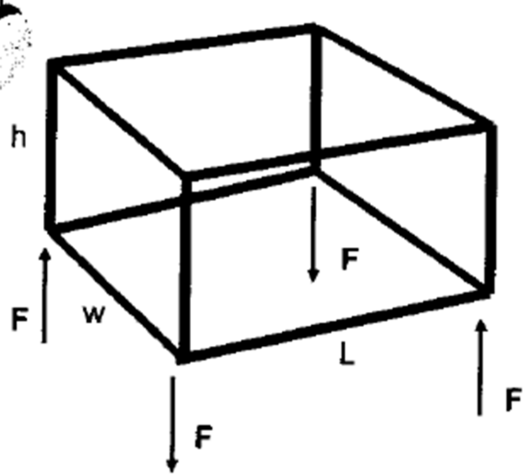
sideframe, rear hatch opening, windshield ring

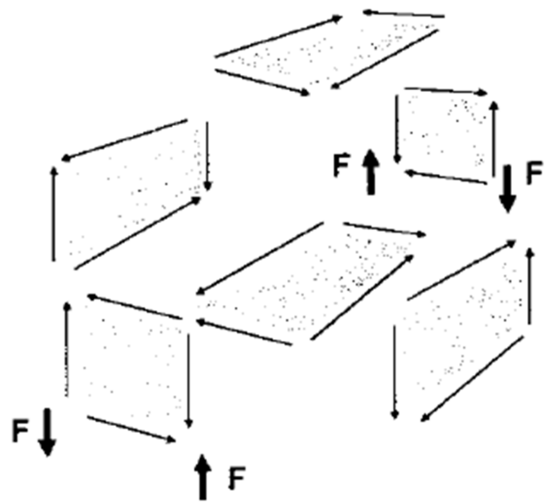
Simple Box Model

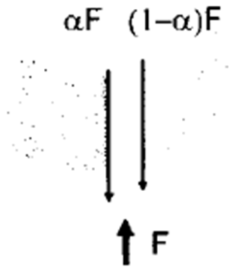
– All surfaces are loaded
– Internal loads are independent of material properties
– Each surface is necessary to react the applied torsional couple: removal of any single surface will not allow the required equilibrium and the box will collapse
– Shear flow is equal for all edges

α: load taken by front surface
1-α: load taken by side surface

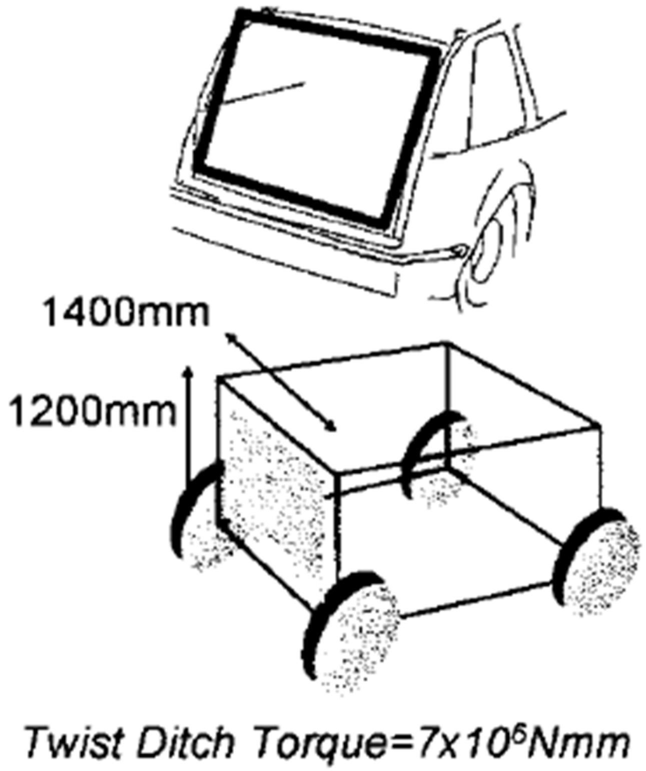






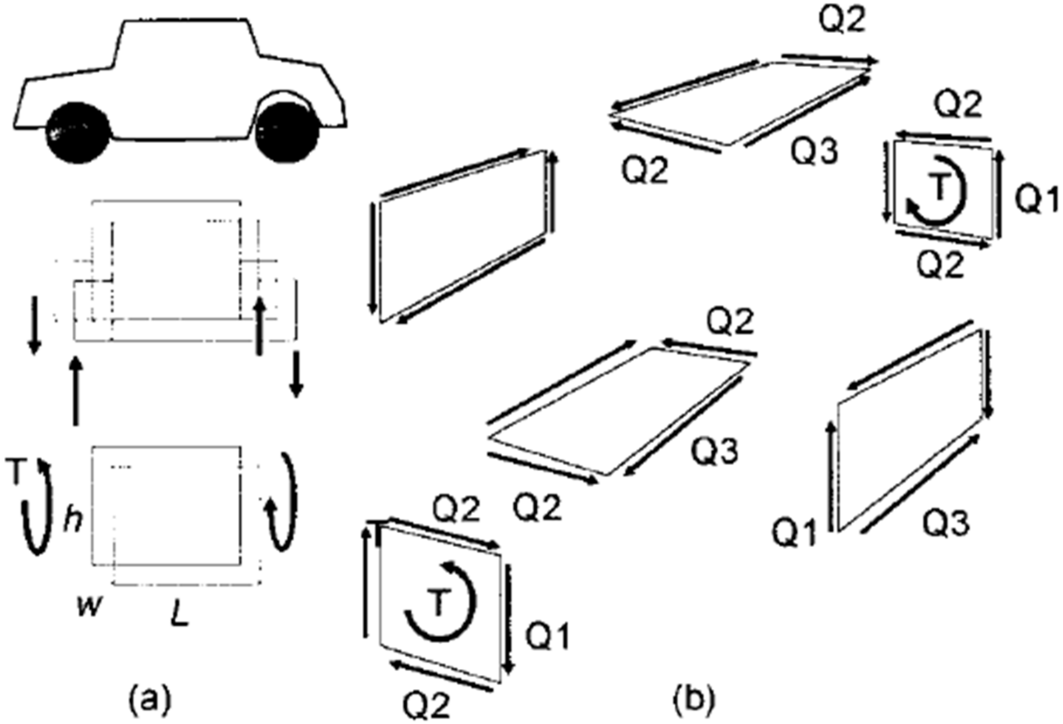


Example: Van Rear Hatch Opening

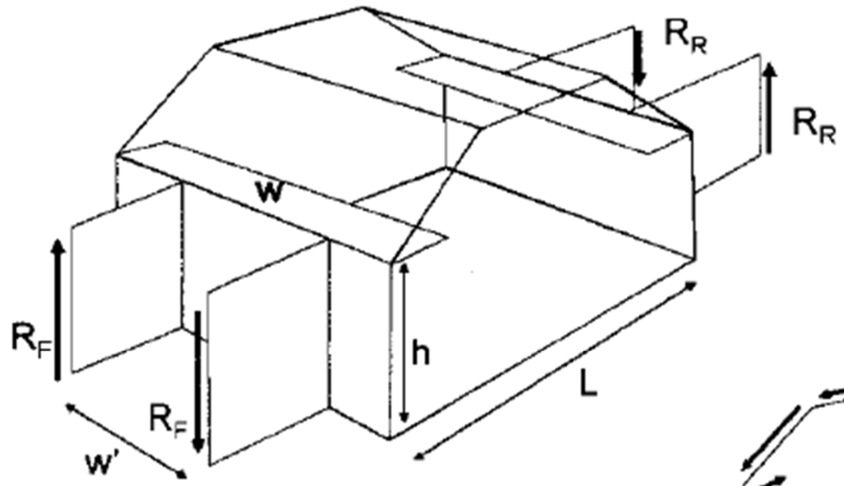


- Determine shearing loads which need to be reacted by the rear hatch structure.

Passenger Cabin Internal Loads

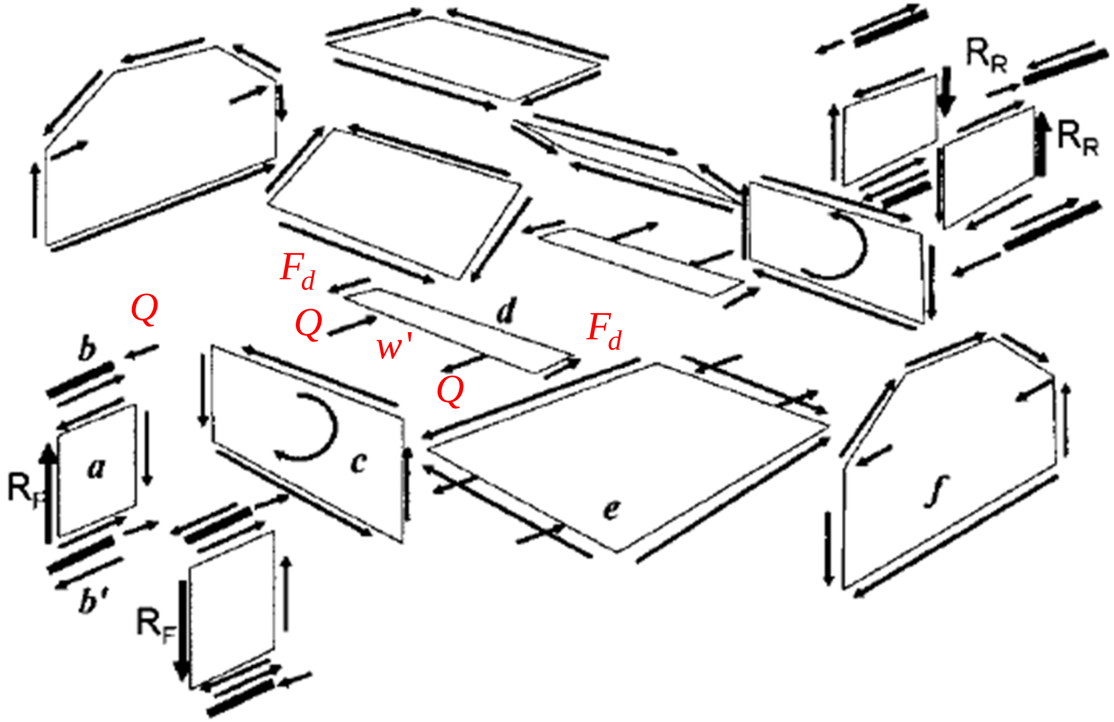


Structural Surface and Bar Model

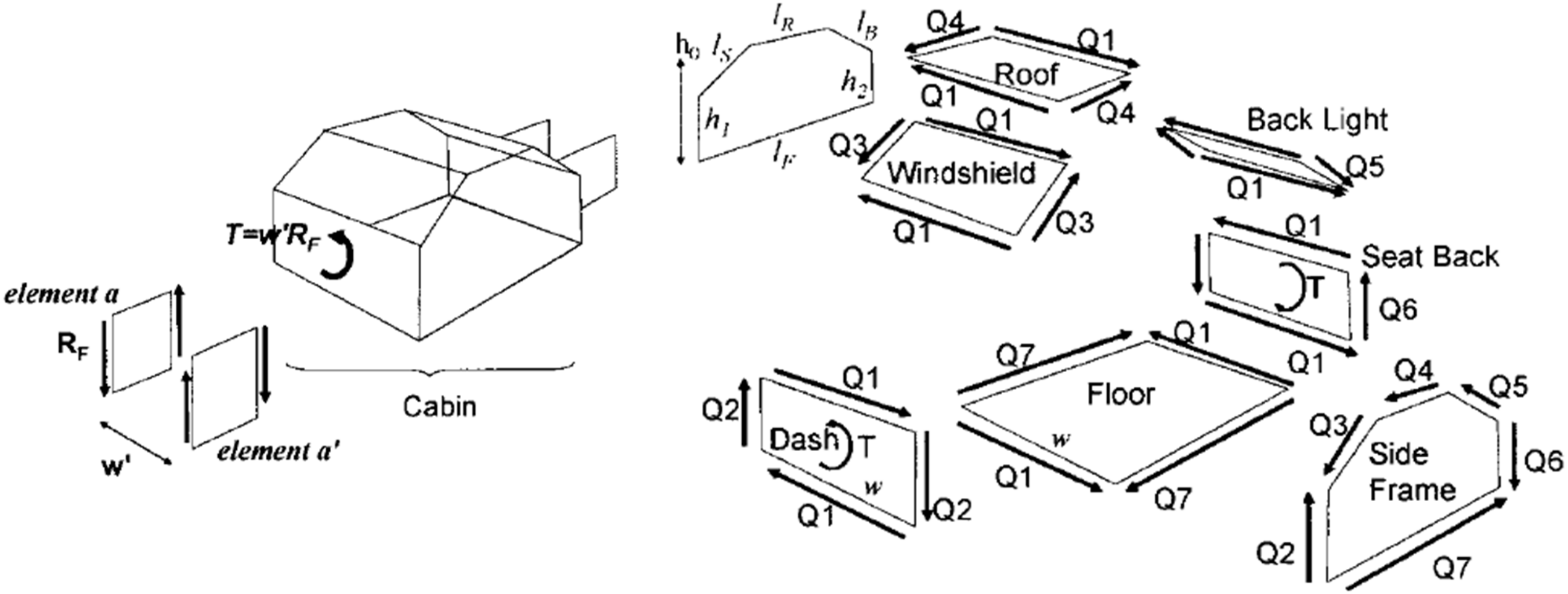


more realistic model

$$R_F = \frac{T_{max}}{w'}$$



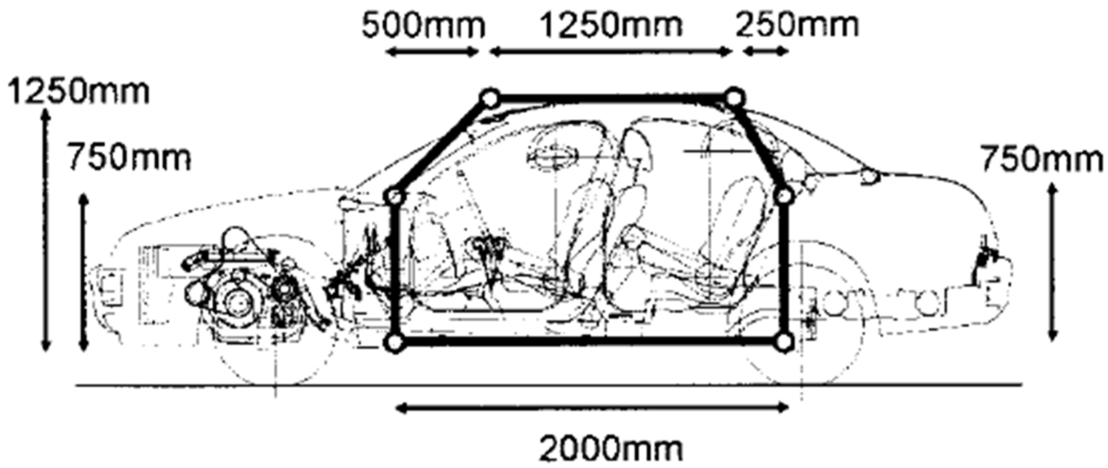
Shear Loads on Cabin Panels



$$\begin{bmatrix} h_1 & w & 0 & 0 & 0 & 0 & 0 \\ l_S & 0 & -w & 0 & 0 & 0 & 0 \\ l_R & 0 & 0 & -w & 0 & 0 & 0 \\ l_B & 0 & 0 & 0 & -w & 0 & 0 \\ h_2 & 0 & 0 & 0 & 0 & w & 0 \\ l_F & 0 & 0 & 0 & 0 & 0 & -w \\ 0 & 0 & 0 & (h_0 - h_1) & \left[l_F (h_0 - h_2) + l_2 (h_2 - h_1) \right] / l_B & -l_F & h_1 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \end{bmatrix} = \begin{bmatrix} T \\ 0 \\ 0 \\ 0 \\ T \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \text{dash} \\ \text{windshield} \\ \text{roof} \\ \text{backlight} \\ \text{rear seat panel} \\ \text{floor} \\ \text{side frame} \end{matrix}$$

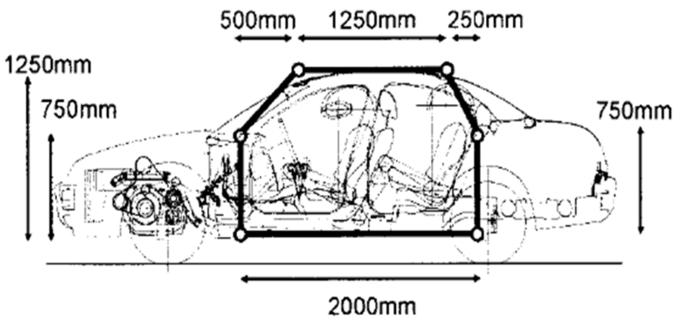
Example: Midsize Sedan Data

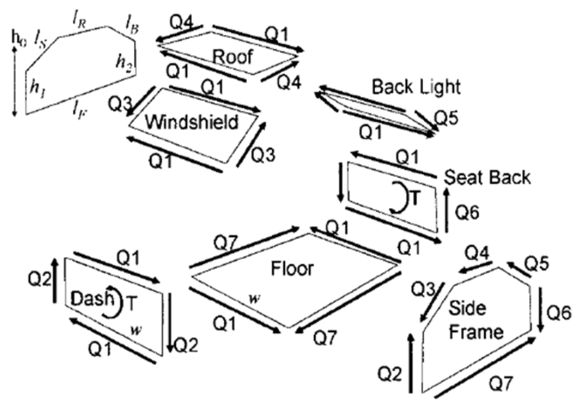
- Determine internal shear loads
  - Track = 1560 mm
  - Twist ditch torque = 7,730 Nm



Example: Shear Loads

Torque	7730000
track	1560
h0	1250
h1	750
h2	750
Ls	707
Lr	1250
Lb	559
Lf	2000





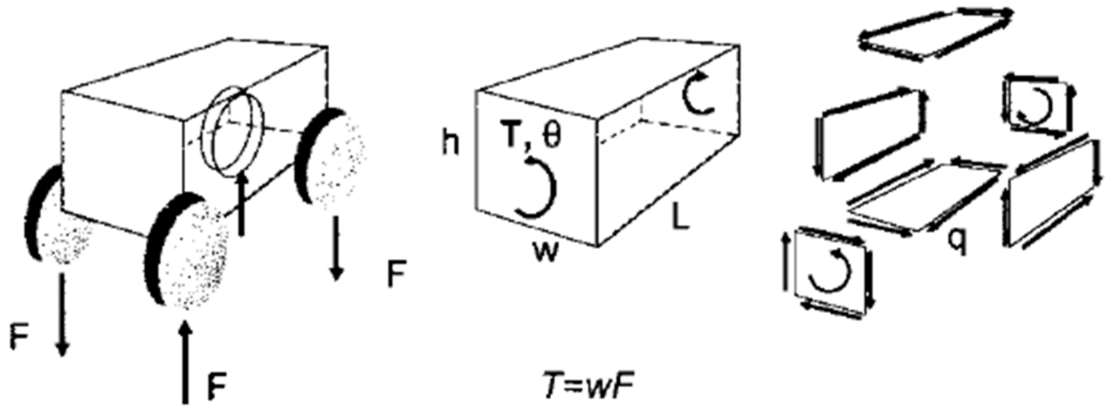
A							T	Q	q
750	1560	0	0	0	0	0	7730000	3343	2.142758
707	0	-1560	0	0	0	0	0	3348	
1250	0	0	-1560	0	0	0	0	1515	2.142758
559	0	0	0	-1560	0	0	0	2678	2.142758
750	0	0	0	0	1560	0	7730000	1198	2.142758
2000	0	0	0	0	0	-1560	0	3348	
0	0	0	500	1789	-2000	750	0	4286	2.142758

5.4 Analysis of Body Torsional Stiffness

- Shear strain energy of a surface

$$e = \int \frac{\tau\gamma}{2} dV = \int \frac{\tau^2}{2G} dV =$$

- Energy balance for torque loaded box

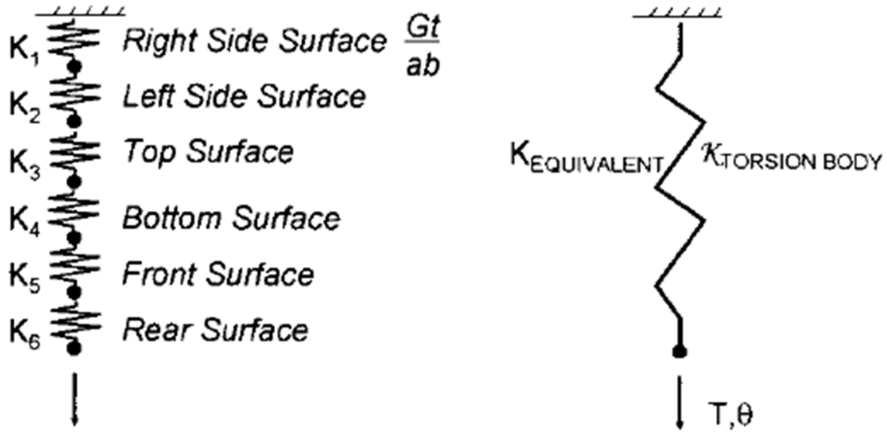


Series Spring Analogy

- Consider a set of six linear springs in series

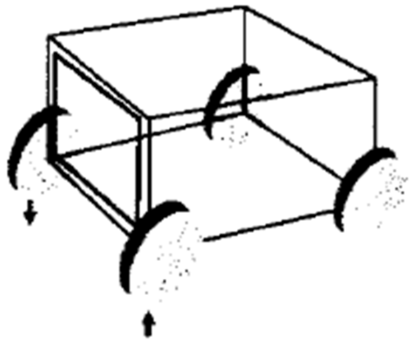
$$K_{eq} = \frac{1}{\sum_{i=1}^{6}\left[1/K_i\right]} \leftrightarrow K =$$

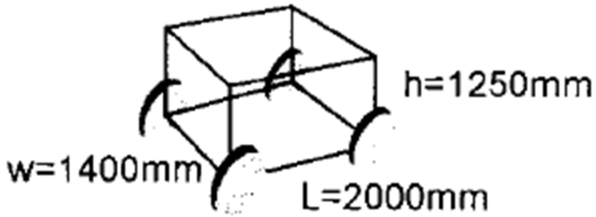
$$\Rightarrow K_i \leftrightarrow \left(\frac{Gt}{ab}\right)_{\text{surface } i}$$



- How to increase torsional stiffness?
  - Identify which surfaces is the most flexible: lowest $\left[\frac{(Gt)}{ab}\right]$
  - Increase the stiffness of the least stiff spring

Example: Box Van





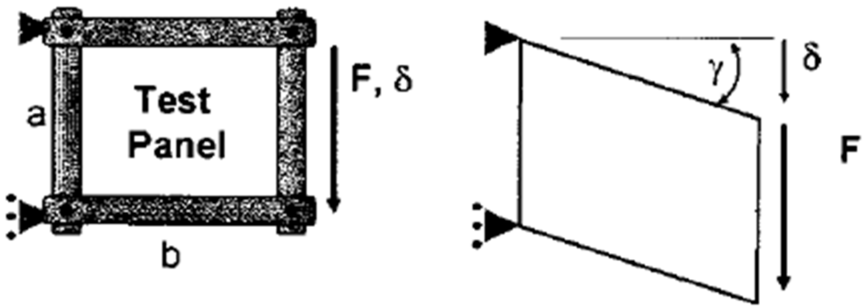
All Panels Steel
G=80,000N/mm²
t=1 mm

– K = 6.95x10^{10} Nm/rad = 1,200,000 Nm/°
– About 100 times stiffer than measured data
– Why?
  - Ideal flat plate assumption: surfaces remain perfectly flat during loading
  - Reality: out-of-plane shape, holes and cut-outs, framework of beams with flexible joints
– Effective shear stiffness: $(Gt)_{eff}$

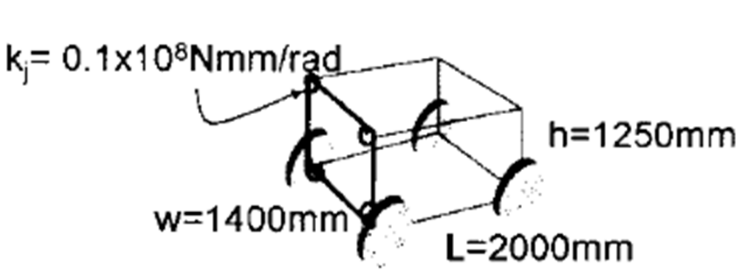
Effective Shear Stiffness

- Test panel in a pinned frame fixture
  - $(Gt)_{eff}$: shear stiffness for a panel
  - S: measured stiffness (slope of load vs. deflection curve)
    - Physical test / FEM
  - a: panel dimension of the side load is applied
  - b: adjacent side dimension

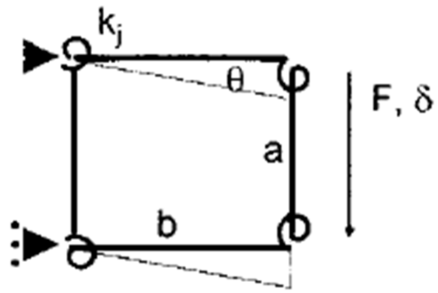
$$G = \frac{\tau}{\gamma} \xrightarrow{\tau=\frac{F}{at},\ \gamma=\frac{\delta}{b}} (Gt)_{eff} =$$



Example: Van Hatch Opening (1)



All Panels Steel
G=80,000N/mm²
t=1 mm



$$\underbrace{\frac{1}{2}F\delta}_{\text{work done}} = \underbrace{4\left(\frac{1}{2}K_j\theta^2\right)}_{\text{energy stored in the joints}} \xrightarrow{\theta=\frac{\delta}{b}} S = \frac{F}{\delta} = \frac{4K_j}{b^2} \rightarrow (Gt)_{eff} = \frac{4K_j}{ab}$$

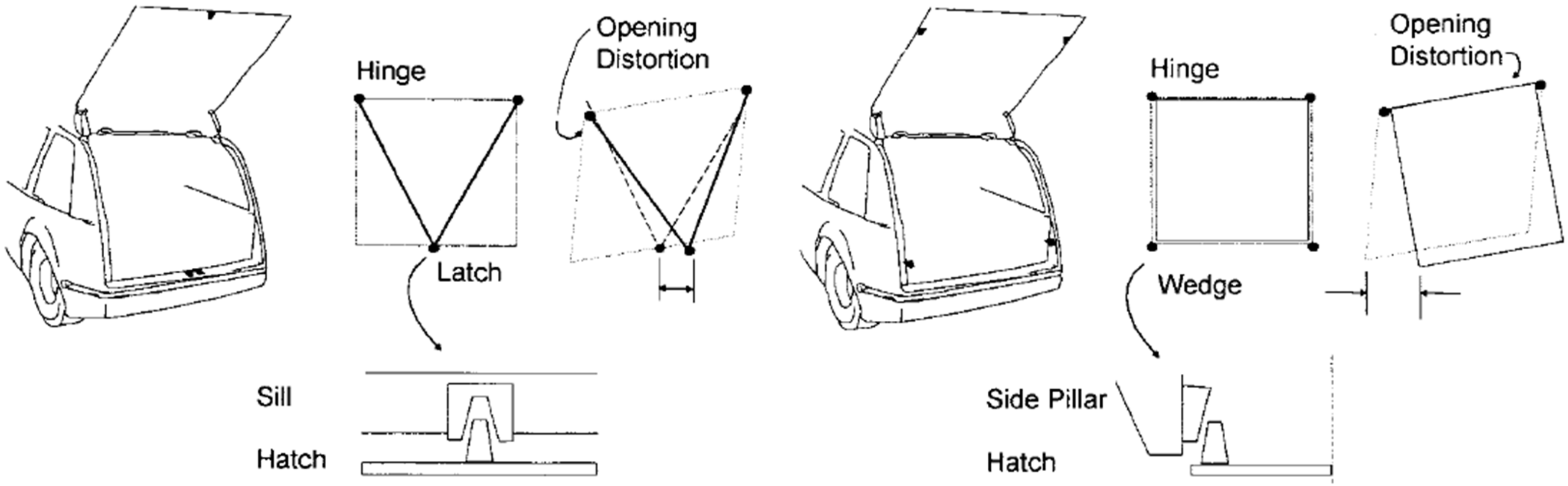
– Replace the rear panel with an open frame of rigid links with a typical joint stiffness
  - Typical joint stiffness: K_j = 0.1x10^8 Nm/rad
– Much more flexible frame than the original assumption of a flat panel

Example: Van Hatch Opening (2)

- K = 1.6×10^8 Nm/rad = 2807 Nm/°
- Influence of the hatch opening: rear surface
  - Only one surface of the closed box need be flexible to reduce the stiffness for the whole box

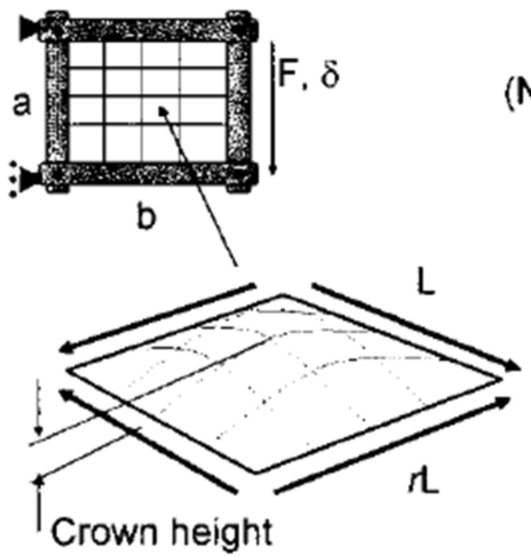
Example: Van Hatch Opening (3)

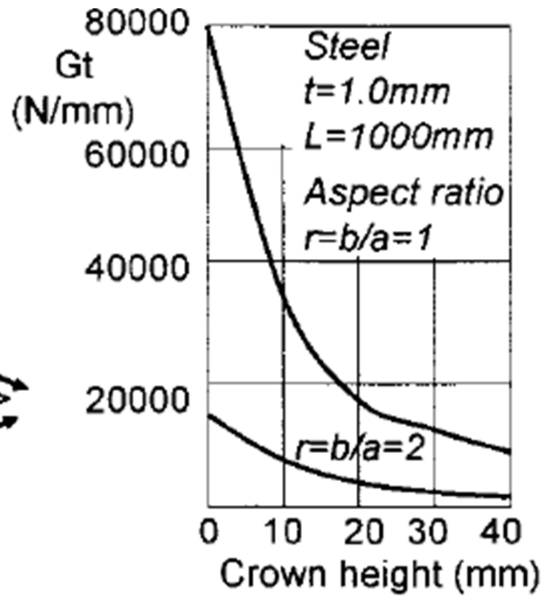
- In practice, increase the shear stiffness of the rear surface
- Typically the hinge and latch are not sufficiently stiff
- Mechanisms to wedge the door into the opening



Example: Crowned Panels

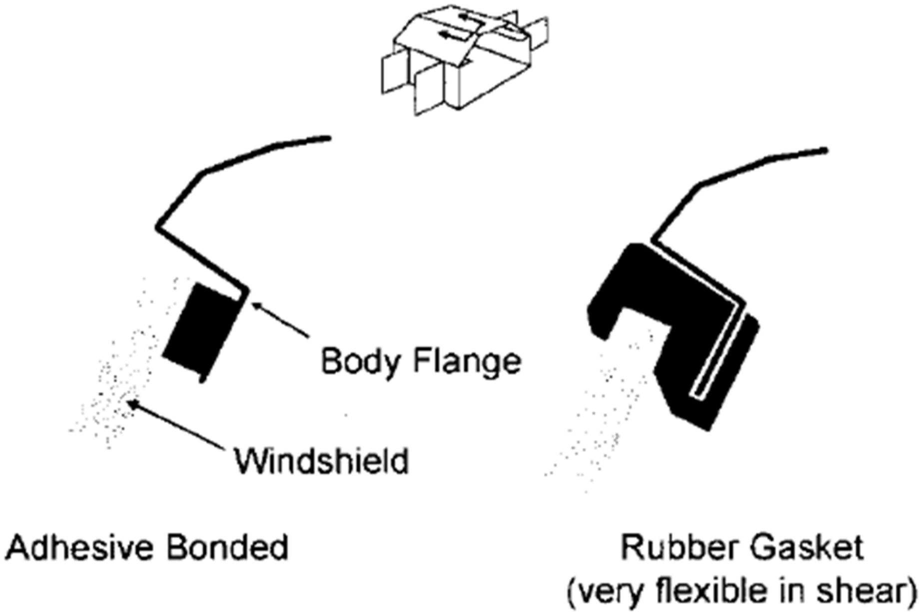
- Improve panel stiffness for normal loading such as dent resistance and panel vibration
- Effective shear stiffness?
  - FEA model of the shear test fixture
  - Much smaller than a flat panel (unrealistically high)



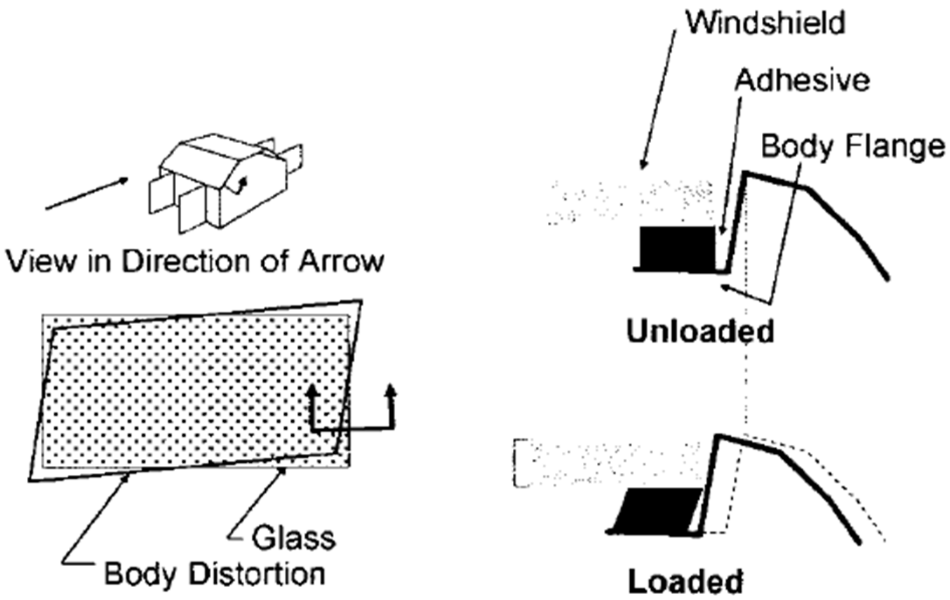


Example: Windshield (1)

– All surfaces enclosing the cabin must act as shear resistant members
– Most effective for shear resistance: adhesive bonding



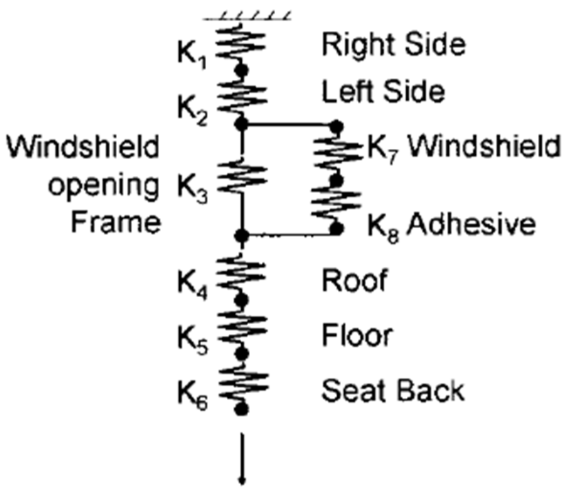
windshield retention alternatives



effective shear stiffness of windshield

Example: Windshield (2)

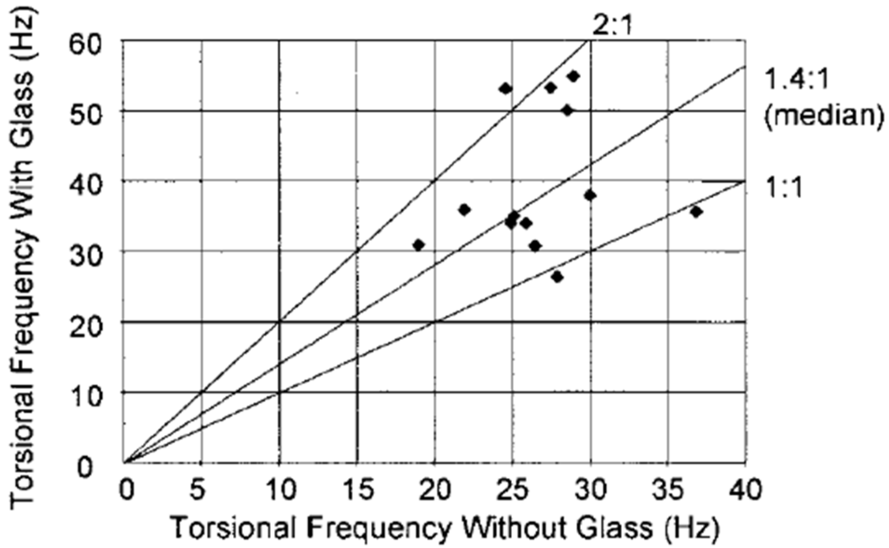
- Windshield model for torsional stiffness



Spring Model of Windshield / Adhesive

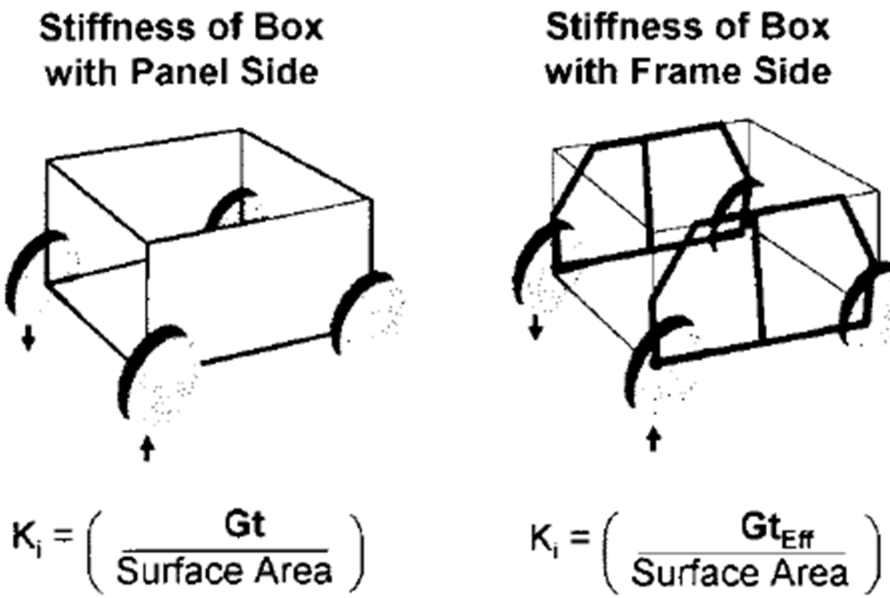
Relative Shear Stiffness of Windshield Elements

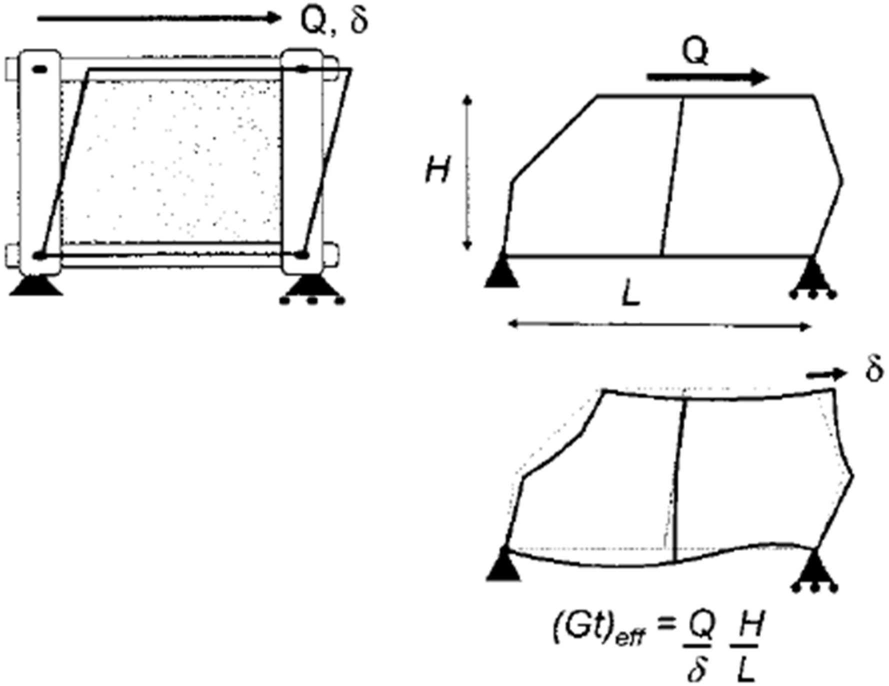
- Effect of windshield on torsional frequency
  – Increase with glass: torsionally stiffer body
  – No increase: very stiff body windshield opening perimeter



Example: Side Frame Model (1)

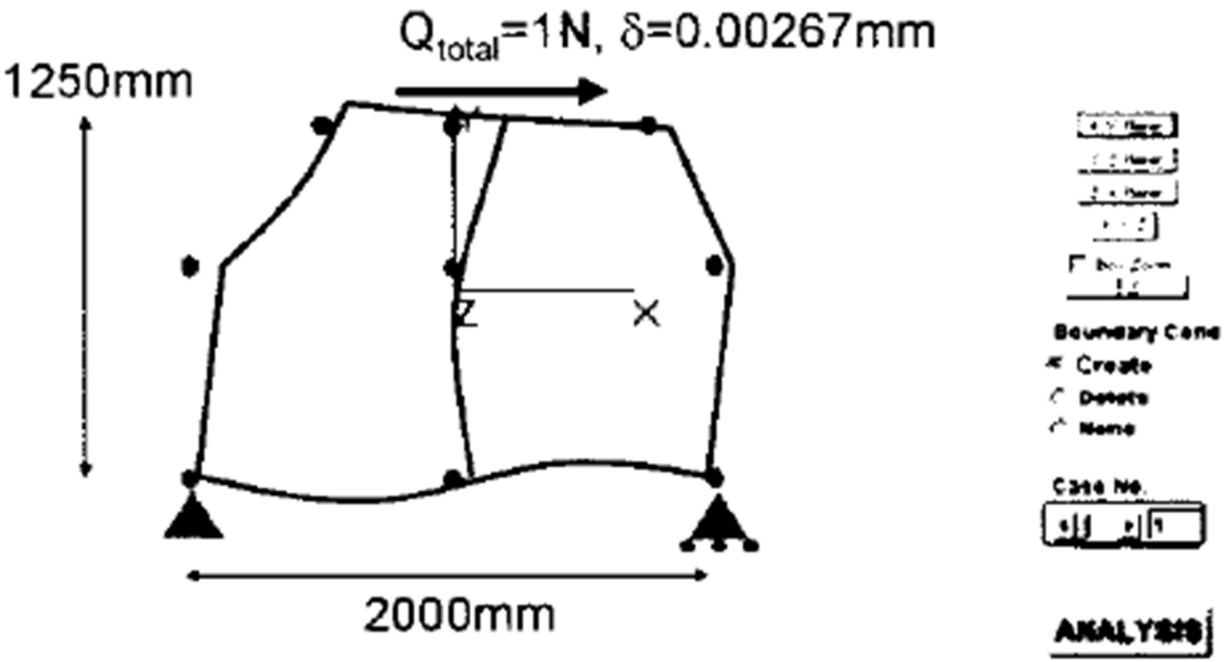
- Contribution to torsional stiffness
- Effective shear stiffness





Example: Side Frame Model (2)

- FEA under shear loading



$$(Gt)_{Eff} = \left(\frac{Q}{\delta}\right)\frac{H}{L} = \left(\frac{1N}{.00267mm}\right)\frac{1250mm}{2000mm} = 234N/mm$$

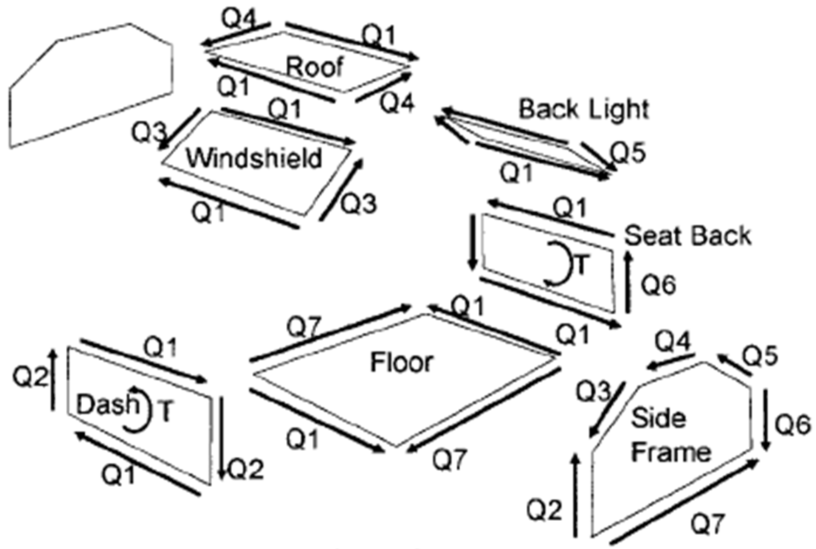
Torsional Stiffness of a Vehicle Cabin

- Solve for internal shear loads: $Q = A^{-1}T$
- Find the resulting shear flow on any non-loaded surface: q = Q/(side length)
- Determine the effective shear stiffness: $(Gt)_{eff}$
- Determine the torsional stiffness of the cabin: (q/T), $(Gt)_{eff}$, surface area

$$\frac{1}{2}T\theta = \sum_{\text{all surfaces}} \frac{1}{2}q^2\left[\frac{ab}{(Gt)}\right]_i$$

$$\frac{\theta}{T} = \left(\frac{q}{T}\right)^2 \sum_{\text{all surfaces}} \left[\frac{\text{area of surface}}{(Gt)_{eff}}\right]_i$$

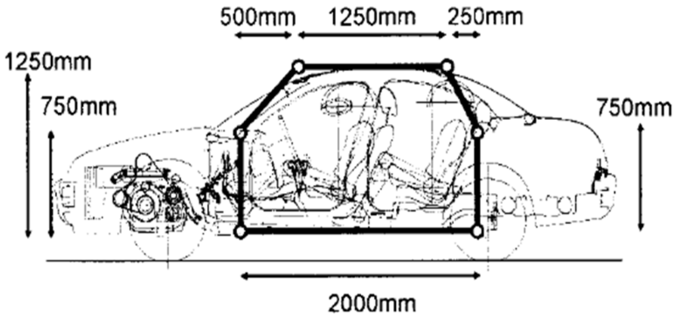
$$K = \frac{1}{\left(\frac{q}{T}\right)^2 \sum_{\text{all surfaces}} \left[\frac{\text{area of surface}}{(Gt)_{eff}}\right]_i}$$

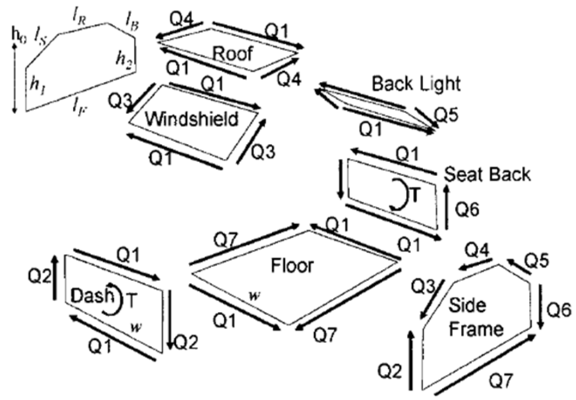


Cabin Surfaces

Example: Sedan

Torque	7730000
track	1560
h0	1250
h1	750
h2	750
Ls	707
Lr	1250
Lb	559
Lf	2000





A							T	Q	q
750	1560	0	0	0	0	0	7730000	3343	2.142758
707	0	-1560	0	0	0	0	0	3348	
1250	0	0	-1560	0	0	0	0	1515	2.142758
559	0	0	0	-1560	0	0	0	2678	2.142758
750	0	0	0	0	1560	0	7730000	1198	2.142758
2000	0	0	0	0	0	-1560	0	3348	
0	0	0	500	1789	-2000	750	0	4286	2.142758

panel	area	(Gt)eff	area/(Gt)eff
dash	1170000	80000	14.6
windshield	1103087	80000	13.8
roof	1950000	80000	24.4
back light	872067	80000	10.9
seat back	1170000	80000	14.6
floor	3120000	80000	39.0
side frame:L	2312500	234	9882.5
side frame:R	2312500	234	9882.5
SUM			19882.3

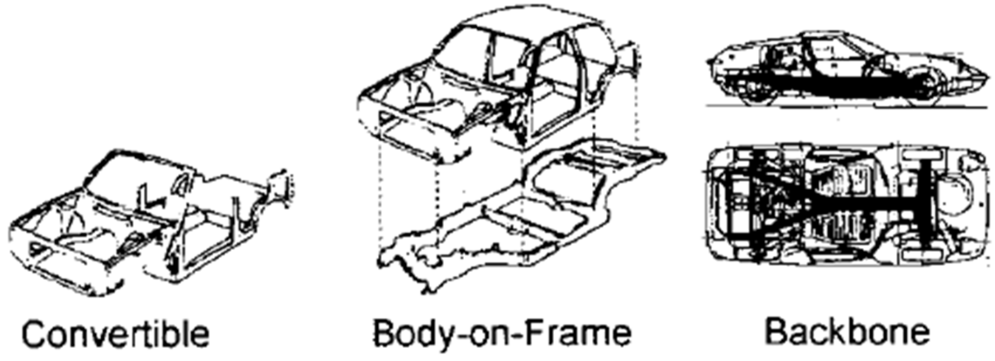
$$K = \frac{1}{\left(\frac{q}{T}\right)^2 \sum_{\text{all surfaces}} \left[\frac{\text{area of surface}}{(Gt)_{eff}}\right]_i}$$

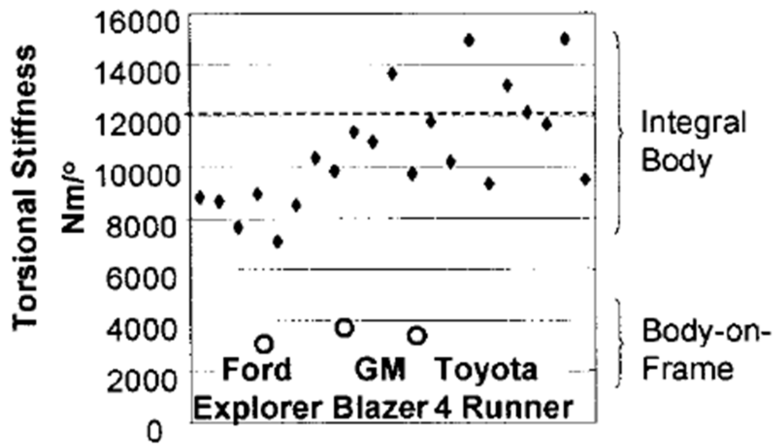
$$= 6.55E+08\text{Nmm/rad}$$

$$=11{,}423\text{Nm/}^\circ$$

5.5 Torsional Stiffness of Framed Vehicle

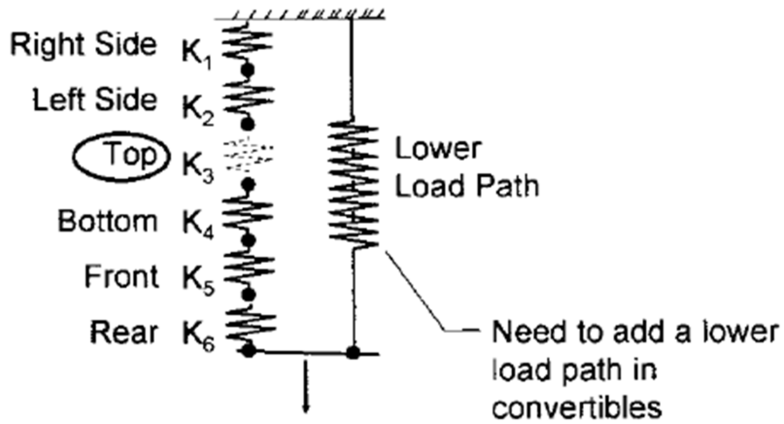
  - Effective structure for torsional stiffness: large central closed section, but limited to seating arrangements
- Monocoque structure
  - efficient in reacting torsional loading
- Alternatives
  - Convertible: absence of top surface
  - Body-on-frame: common for passenger and utility vehicle
  - Backbone frame: closed thin walled sections → shear buckling

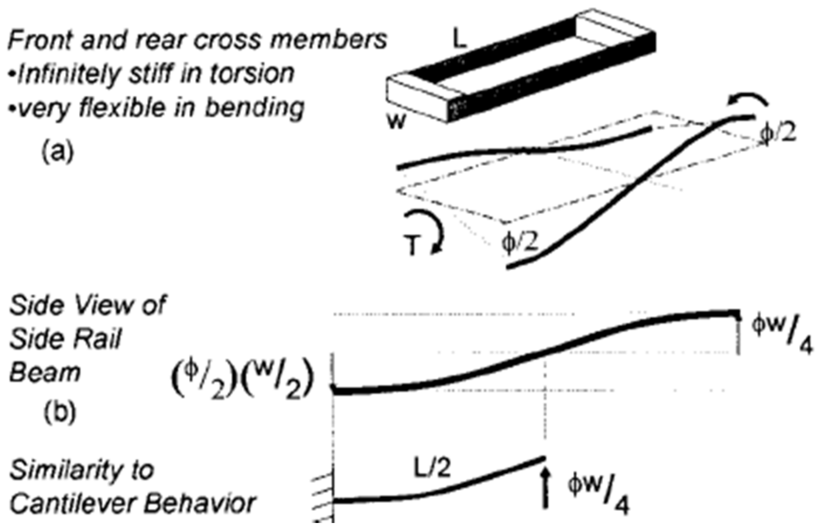




Convertibles

– Lower load path to resist torsional loads: differential bending of the rocker beams
– Lower structure: two cross members (front: dash, rear: rear seat back), two side beams
  - Cross member: infinitely rigid in torsion → zero twist along cross-car axes, flexibility in bending → side rails are not twisted down
  - Side rail: pure bending with zero slope at its ends





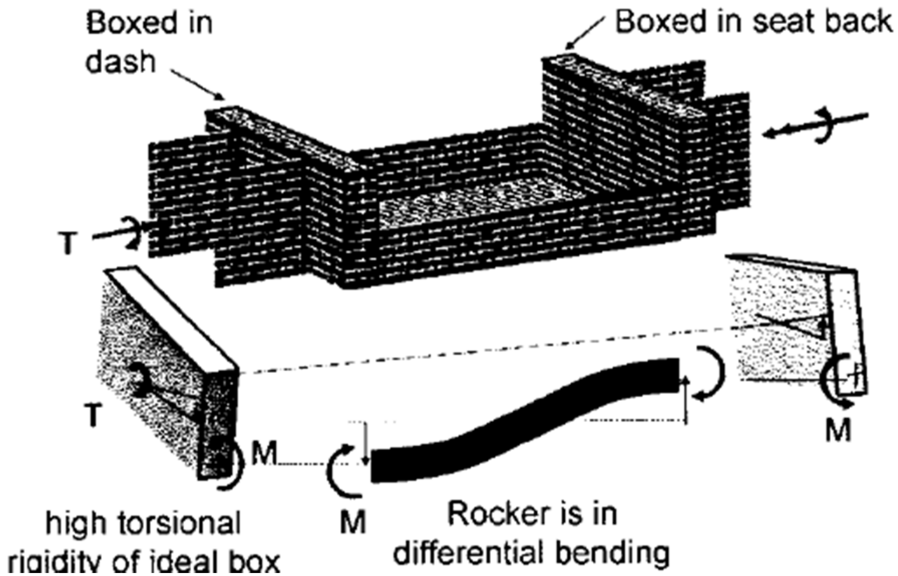
Effect of Differential Bending

- Behavior for the front half: cantilever beam of length L/2

$$\delta = \frac{Fl^3}{3EI} = \left(\frac{w}{2}\right)\left(\frac{\phi}{2}\right) \rightarrow F = 3EI\left(\frac{w\phi}{4}\right)\left(\frac{2}{L}\right)^3$$

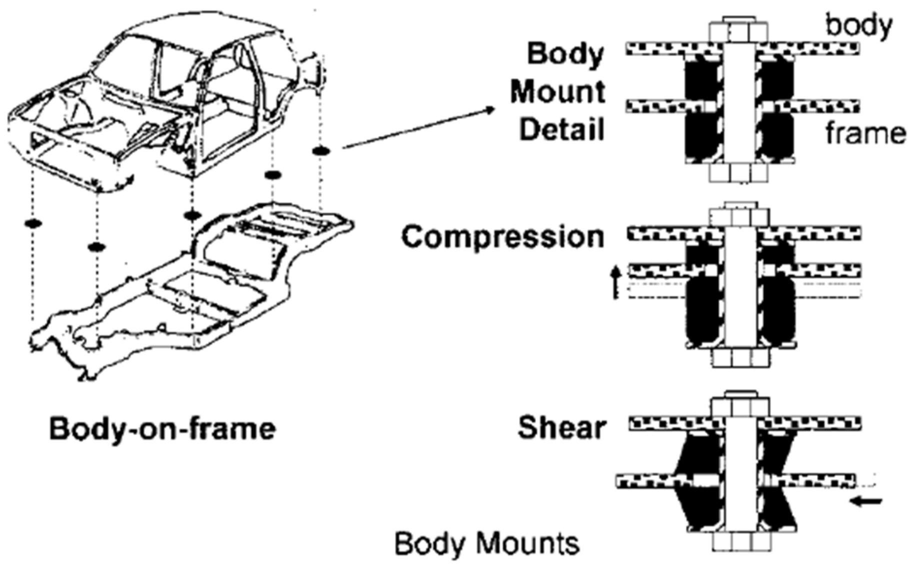
$$K = \frac{T}{\phi} = \frac{wF}{\phi} = \frac{w}{\phi}\left[3EI\left(\frac{w\phi}{4}\right)\left(\frac{2}{L}\right)^3\right] = \frac{6w^2EI}{L^3}$$

- In practice
  - large closed box section at dash and rear seat back
  - Difficulty: cross member to side rail joint
    - Zero slope: very large bending moment
    - Stress concentration at the joint



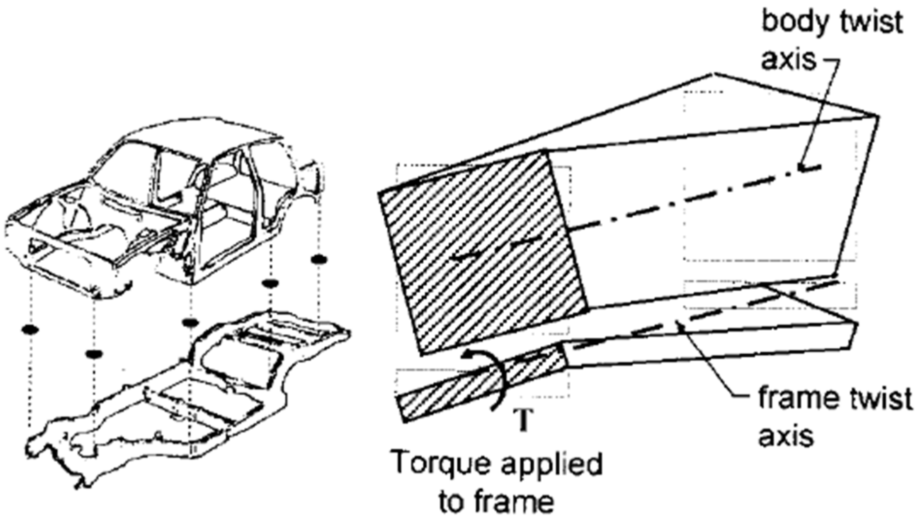
Body-on-Frame

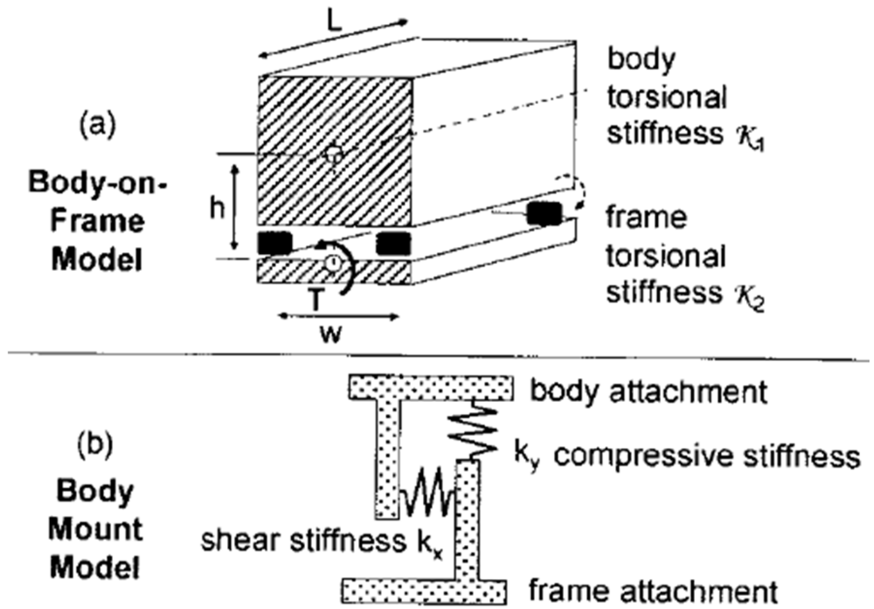
- Body shell, ladder frame, elastomeric body mounts
- Body mounts
  - Relative motion between the frame and body both in vertical direction (compression) and in lateral direction (shear)
  - Isolation of structure borne noise and vibration from the frame into the body



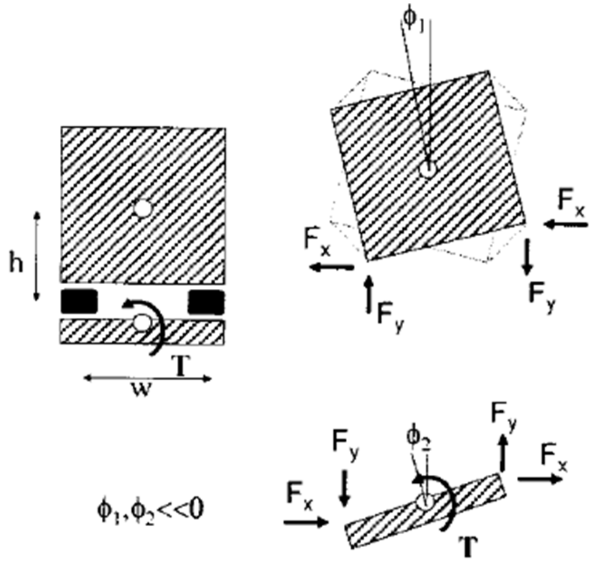
Body-on-Frame: Torsion Model

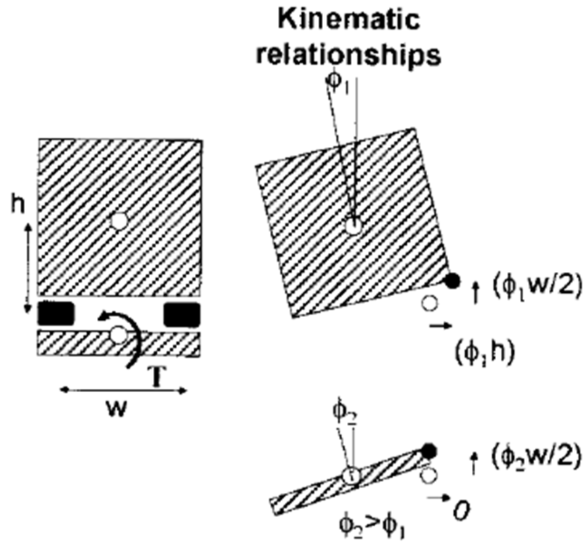
- Torque applied to the frame through the suspension
  - Twist about different longitudinal axes
  - Shearing deformation in the body mounts
    - Reduce the stiffness of the system: K < K1 + K2





Body-on-Frame: Torsional Stiffness (1)





$$\left.\begin{array}{l}\begin{cases} wF_y - 2hF_x - K_1\phi_1 = 0 \\ wF_y + K_2\phi_2 = T \end{cases} \\ \left.\begin{array}{l} \delta_H = h\phi_1 \\ \delta_V = \dfrac{w}{2}\phi_2 - \dfrac{w}{2}\phi_1 \end{array}\right\} \rightarrow \begin{cases} F_x = k_x h\phi_1 \\ F_y = k_y\left(\dfrac{w}{2}\phi_2 - \dfrac{w}{2}\phi_1\right) \end{cases}\end{array}\right\} \Rightarrow \begin{bmatrix} -2h & w & -K_1 & 0 \\ 0 & w & 0 & K_2 \\ 1 & 0 & -k_x h & 0 \\ 0 & 1 & k_y\dfrac{w}{2} & -k_y\dfrac{w}{2} \end{bmatrix}\begin{bmatrix} F_x \\ F_y \\ \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ T \\ 0 \\ 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} F_x \\ F_y \\ \phi_1 \\ \phi_2 \end{bmatrix} = \frac{T}{K_2\left(w^2k_y + 4h^2k_x + 2K_1\right) + w^2k_y\left(K_1 + 2h^2k_x\right)}\begin{bmatrix} hk_x + w^2k_y \\ \left(2h^2 + k_xK_1\right)wk_y \\ w^2k_y \\ w^2k_y + 4h^2k_x + 2K_1 \end{bmatrix}$$

Body-on-Frame: Torsional Stiffness (2)

$$K_{vehicle} = \frac{T}{\phi_2} = K_2 + K_1\psi + 2h^2 k_x \psi$$

$$\psi = \frac{1}{1 + \dfrac{2h^2 k_x}{\left(\dfrac{w^2}{2} k_y\right)} + \dfrac{K_1}{\left(\dfrac{w^2}{2} k_y\right)}}$$

$K_1,\ K_2$: torsional stiffness of the body and frame

$k_x,\ k_y$: mount stiffness in the horizontal and vertical directions

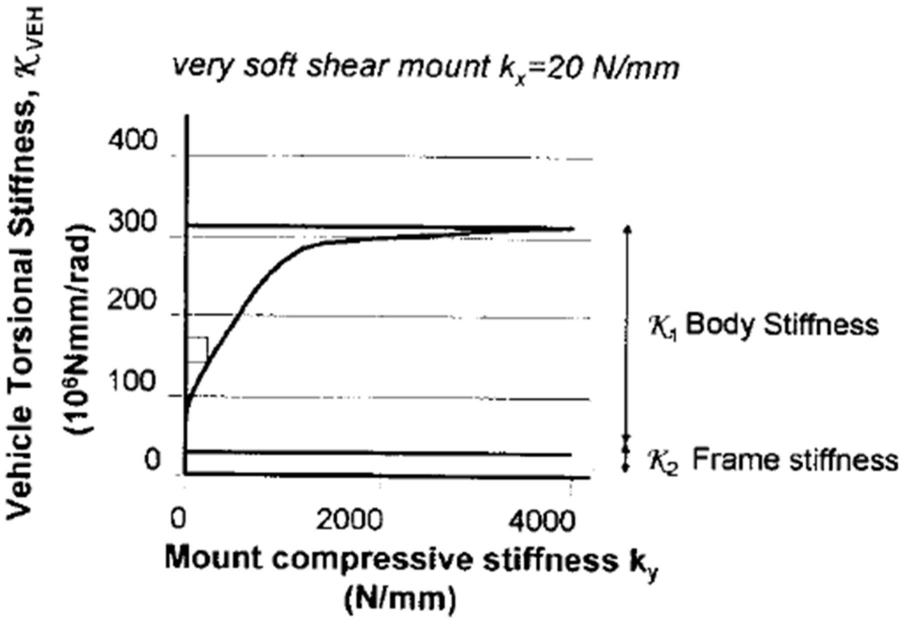
h : height of the body twist axis above the plane of the frame

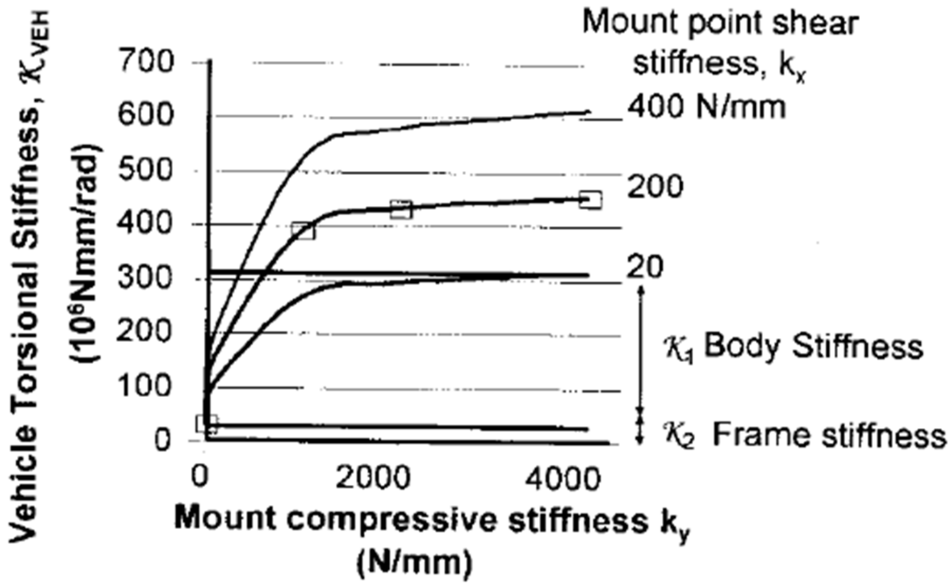
w : width between body mounts

ψ : body-frame coupling term which indicates how tightly coupled are the twisting actions of the frame and body (larger ψ, greater coupling)

Effect of Body Mount (1)

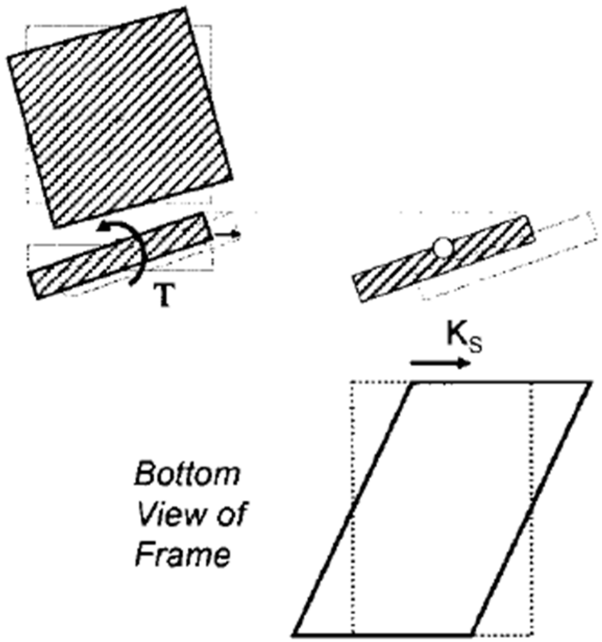
- Compressive stiffness (k_y)
  - Soft mount: body is not coupled to the twisting motion of the frame ($K_{VEH} \rightarrow K_2$)
  - Stiff mount: body and frame are highly coupled ($K_{VEH} \rightarrow K_1 + K_2$)
- Shear stiffness (k_x)
  - $k_x \uparrow \rightarrow K_{VEH} > K_1 + K_2$: why?





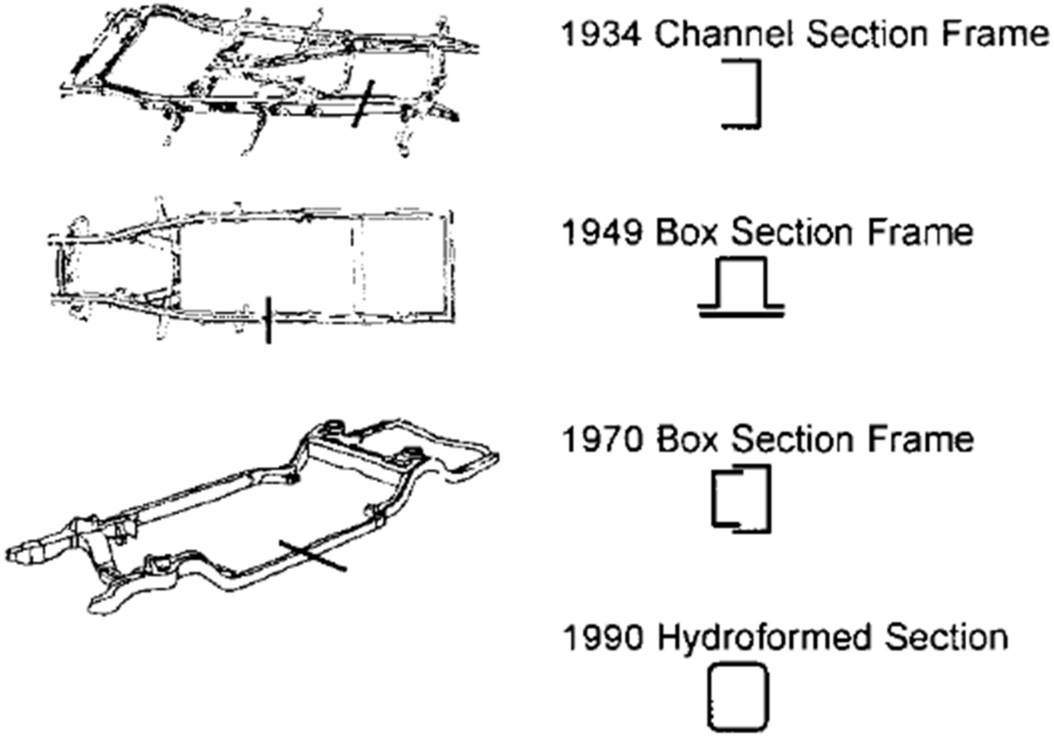
Effect of Body Mount (2)

- Body and frame have different twist axes
  - Body and frame fight against one another for the axis to twist about by increasing the mount shear stiffness
- Combined twist axis locates above the frame
  - Frame becomes a shear resistant member
  - Shear stiffness of the frame: important design consideration



Evolution of Automobile Frame

- Closed sections for both side rails and cross member
  - Improve torsional stiffness
- Improved joints at cross member to side rail
  - Improve both torsional and shear stiffness

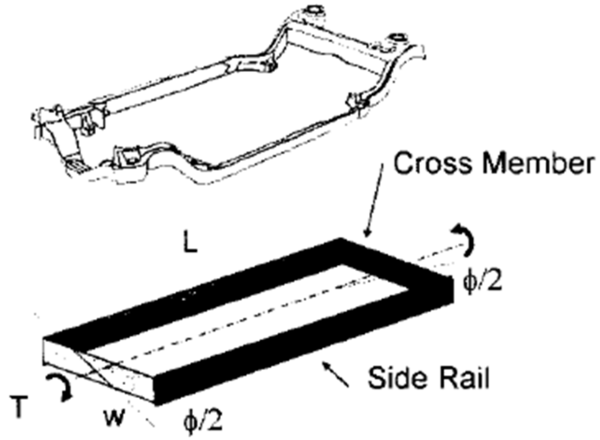


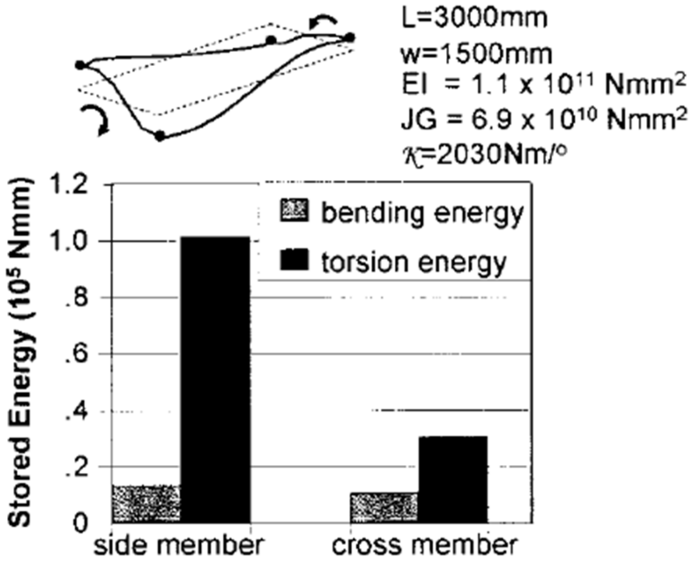
Typical Frame Configurations

Ladder Frame

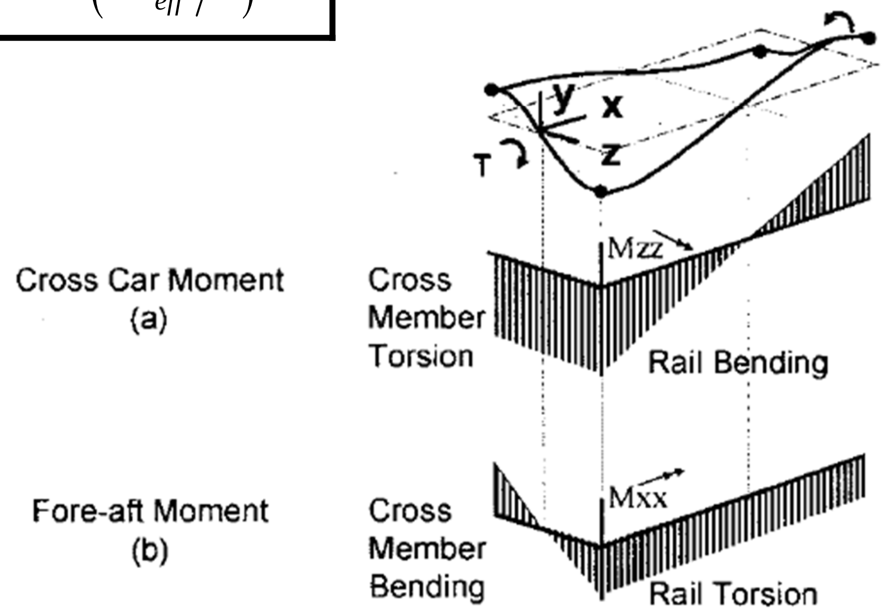
- Two limiting cases for simple frame
  - Cross member

Torsion	Bending	Torsional Stiffness
infinitely rigid	very flexible	$K = 6w^2 EI / L^3$
very flexible	infinitely rigid	$K = 2\left(GJ_{eff} / L\right)$





Strain Energy in Frame Under Torsion

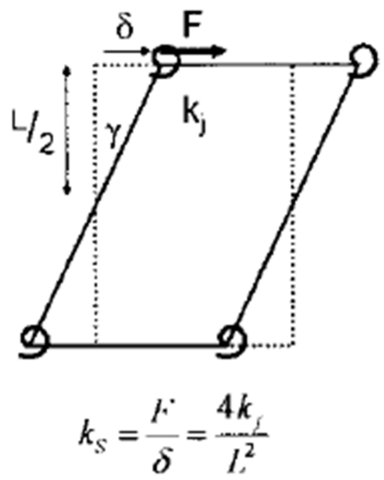


Frame: General Case

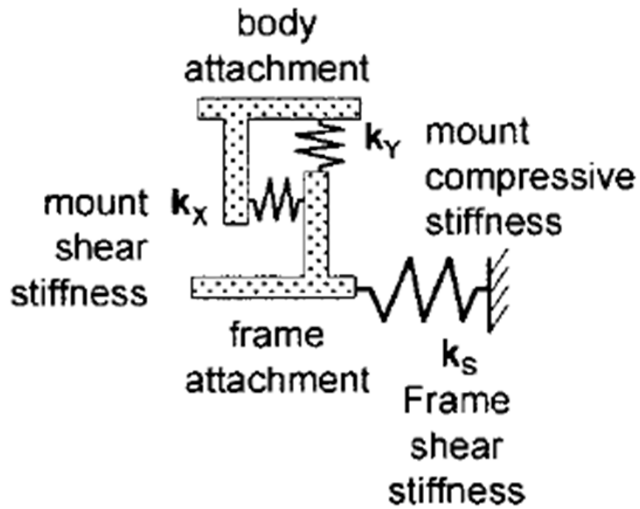
Frame Shear Stiffness

$$\underbrace{\frac{1}{2}F\delta}_{\text{work done}} = \underbrace{4\left(\frac{1}{2}k_J\theta^2\right)}_{\text{energy stored in the joints}} \xrightarrow{\theta=\frac{\delta}{L}} k_S = \frac{F}{\delta} = \frac{4k_J}{L^2}$$

– $k_J = 1\text{x}10^9$Nmm/rad, L = 4500mm, $k_S \approx 200$N/mm
– Very near the shear stiffness for a body mount
– Increase the frame shear stiffness
  - increase the joint rate with gusset, X configuration of rails



Frame Plan View with Shear Load

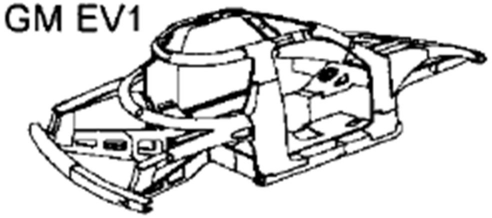


Body Mount and Frame shear Stiffness in Parallel

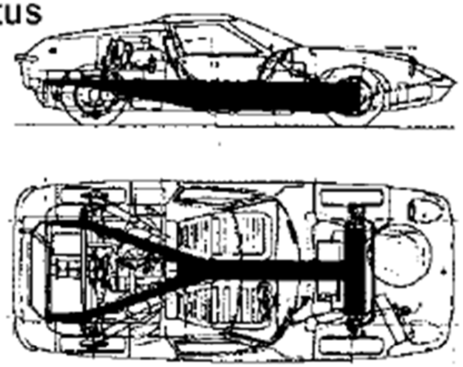
Backbone Frame

- Large central closed section
  - Open two seat sport cars
- Large width to thickness ratio: elastic shear buckling of walls
  - Diagonal rib patterns on the backbone sides

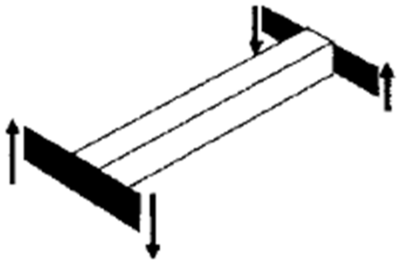
GM EV1



Lotus



Corvette



$$\theta = \frac{TL}{GJ_{eff}}$$

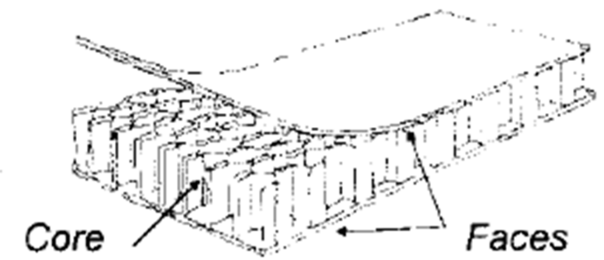
$$J_{eff} = \frac{4A_{enclolsed}^{2}t}{S}$$

$$\tau = \frac{T}{2At}$$

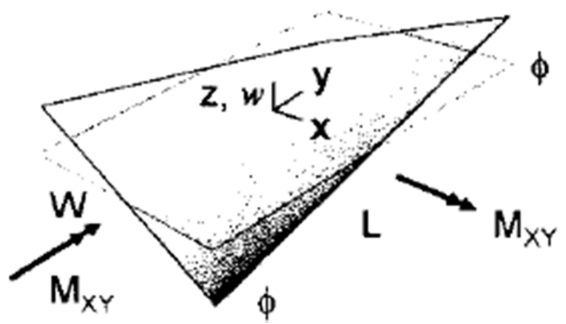
Torsional Resistance of Sandwich Plates

- Thick plate: effective means to resist torsional loads
  - Between the occupant's foot and the ground clearance plane
  - Laminate with thin outer faces of a stiff material and a shear resisting core of low density: mass efficiency

(a) Sandwich Plate



(b) Twisting Plate



Torsional Resistance of Sandwich Plates

$$w = Cxy$$

$$\frac{\partial w}{\partial x} = \phi = Cy = \pm C\frac{L}{2} \rightarrow \theta = CL \text{ (total angle of rotation)}$$

$$\frac{\partial^2 w}{\partial x^2} = 0, \frac{\partial^2 w}{\partial y^2} = 0, \frac{\partial^2 w}{\partial x \partial y} = C$$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) = 0$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) = 0$$

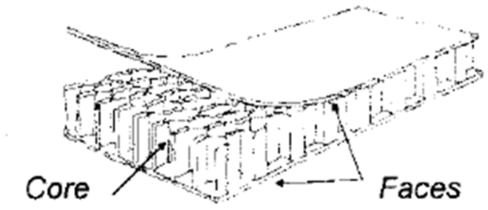
$$M_{xy} = -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y} = -DC(1-\nu)$$

$$F = M_{xy}$$

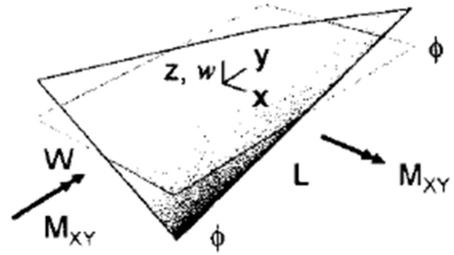
$$T = (2F)W = 2DC(1-\nu)W$$

$$K = \frac{T}{\theta} = \frac{2DC(1-\nu)W}{CL} = 2D(1-\nu)\frac{W}{L}$$

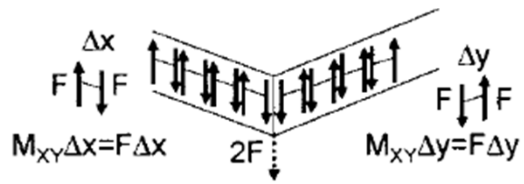
(a) Sandwich Plate



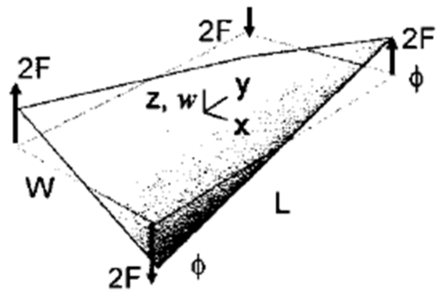
(b) Twisting Plate



(a) Applying Twisting Moment

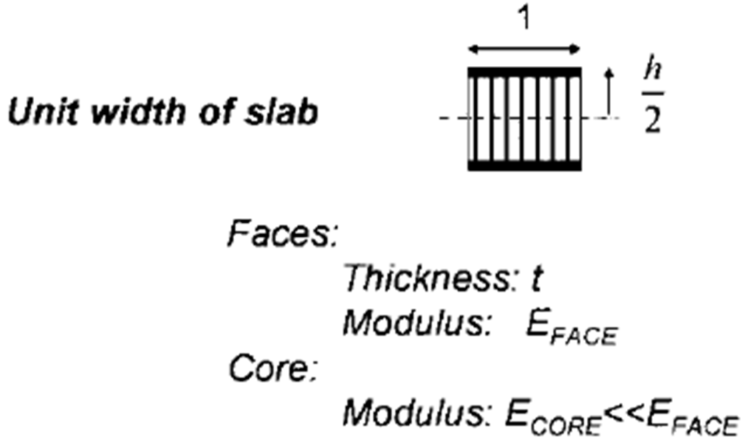


(b) Equivalent corner loading



Example: Sandwich Floor Pan

- Laminate of steel faces (t = 0.5 mm) with a shear resistant core
- Total sandwich height: 100 mm
- Dimensions of the floor pan
  - L=3500 mm, W=1500 mm



$$K = \frac{E_{FACE} t h^2}{1+\nu} \frac{W}{L}$$

$$= \frac{\left(207000 N/mm^2\right)\left(0.5mm\right)\left(100mm\right)^2}{1+0.3} \frac{1500mm}{3500mm}$$

$$= 341,200,000 Nmm/rad$$

$$= 5986 Nm/\deg$$

$$EI = E_{FACE}\left[2\left(t\cdot 1\right)\left(\frac{h}{2}\right)^2\right] = \frac{E_{FACE} t h^2}{2}$$

$$K = 2D\left(1-\nu\right)\frac{W}{L} = 2\left(\frac{EI}{1-\nu^2}\right)\left(1-\nu\right)\frac{W}{L}$$

$$= \frac{E_{FACE} t h^2}{1+\nu} \frac{W}{L}$$