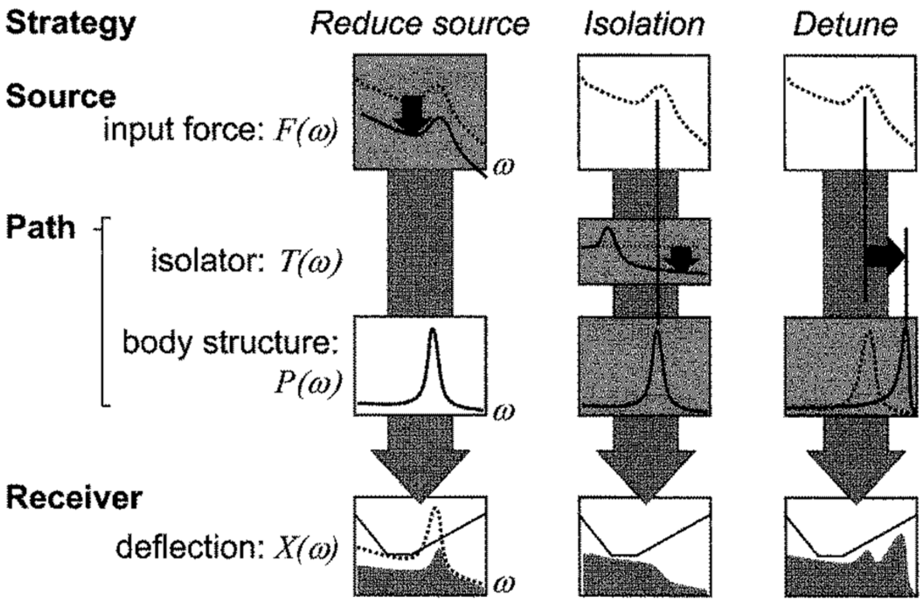
7.5 Strategies for Design

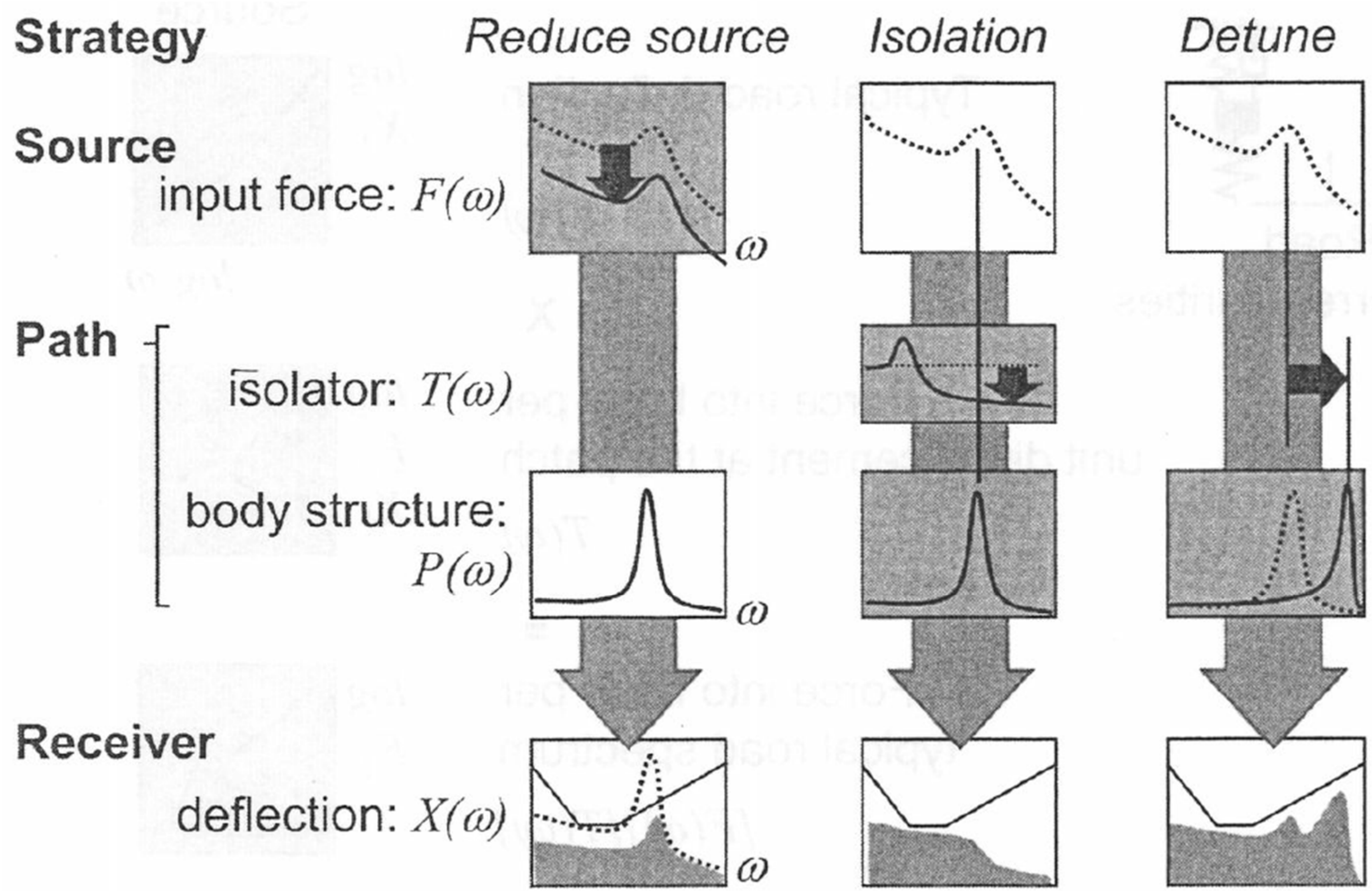
- Objective: minimize the source vibration energy flowing to the receiver with undesirable results
- Three of most important strategies
  - Reduce amplitude of the source
  - Block the flow of energy using isolators in the path
  - Detune resonances in the system



Design for Vibration Strategies

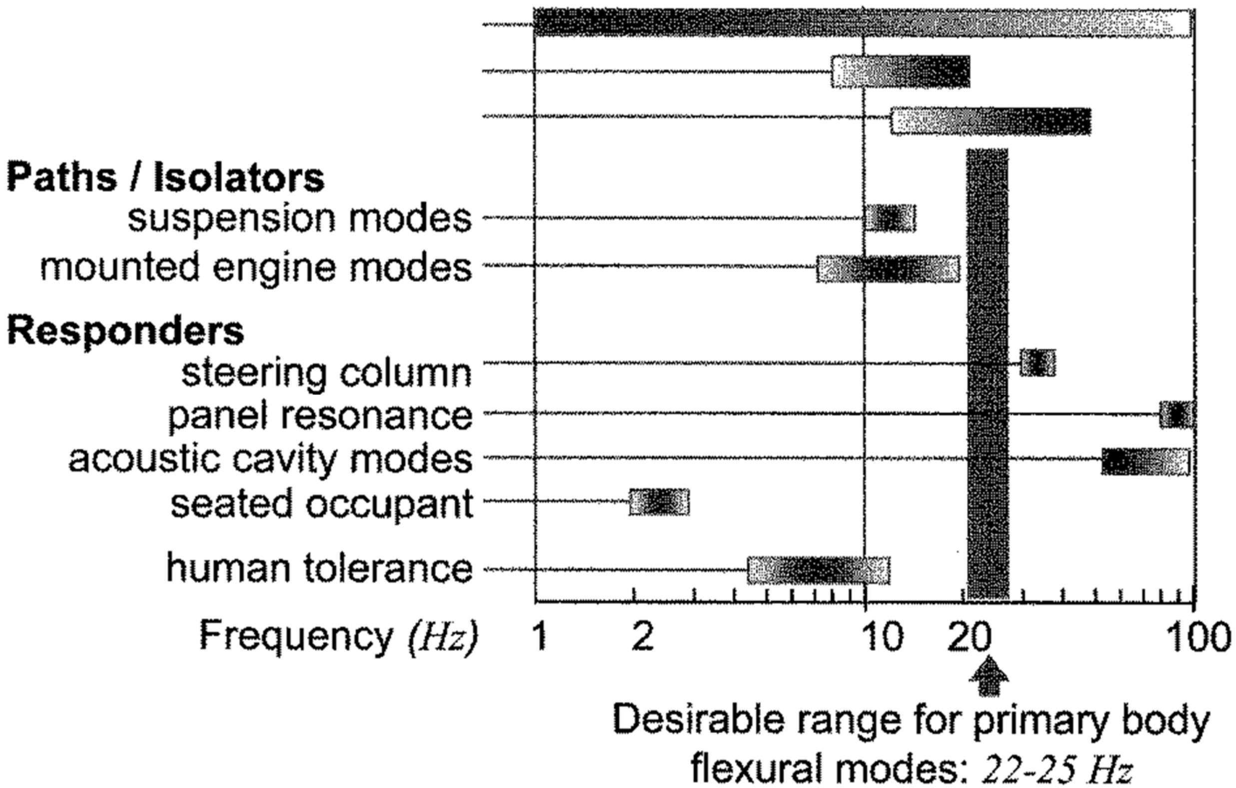
- Reduce amplitude of the source
  - Powertrain
    - Minimize reciprocating mass in engine
    - Add balance shafts to in-line 4 cylinder engine
  - Suspension
    - Balance tires
    - High quality tires with low radial force vibration
    - Minimize shock absorber forces using a linkage ratio ~ 1
- Block the flow of energy using isolators in the path
  - Mounted powertrain at isolator
  - Suspension as isolator
  - Rubber bushings in chassis links at acoustic frequencies
- Detune resonances in the system
  - Position body primary bending and torsion resonances

Vibration Control Strategies



Noise and Vibration Mode Map

- Detune resonances of the body from sources and responders
- Desirable structural resonance band: 22~25 Hz

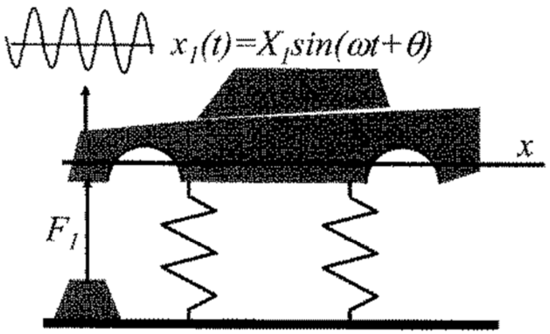


7.6 Body Structure Vibration Testing (1)

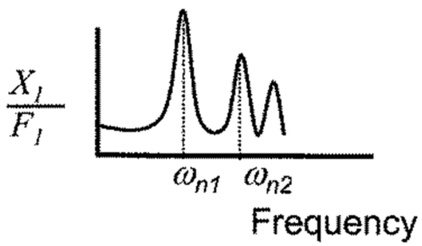
- Result of a vibration test
  - Transfer function: P(ω)
  - Deflected shape (mode shape) for each resonance
- Typical test set-up
  - Support soft springs: inflated inner tubes or elastic cords
    - Rigid body modes at low frequencies (< 3Hz)
  - Electromagnetic or hydraulic shaker: (forcing location) front bumper attachment
    - Excite major modes of vibration (not near a nodal point)
    - Locally stiff (not to locally flexing the structure)
  - Accelerometer: body at the shaker attachment
    - Measure the driving point frequency response
  - input(randomly varying force)→[Fourier Transform]→(out signals)

Body Structure Vibration Testing (2)

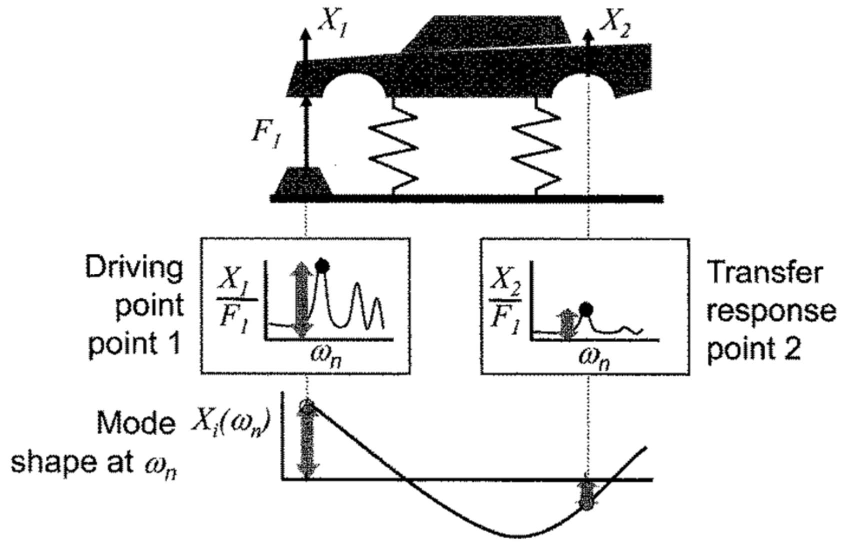
- Driving point response
  - Force amplitude fixed → frequency incremented
- Mode shape
  - Forcing frequency fixed (resonance) → amplitude measured
  - Node(no deflection) / Anti-node(greatest deflection)
  - Lightly damped structure: In-phase / 180° out-of-phase



Vibration test set up

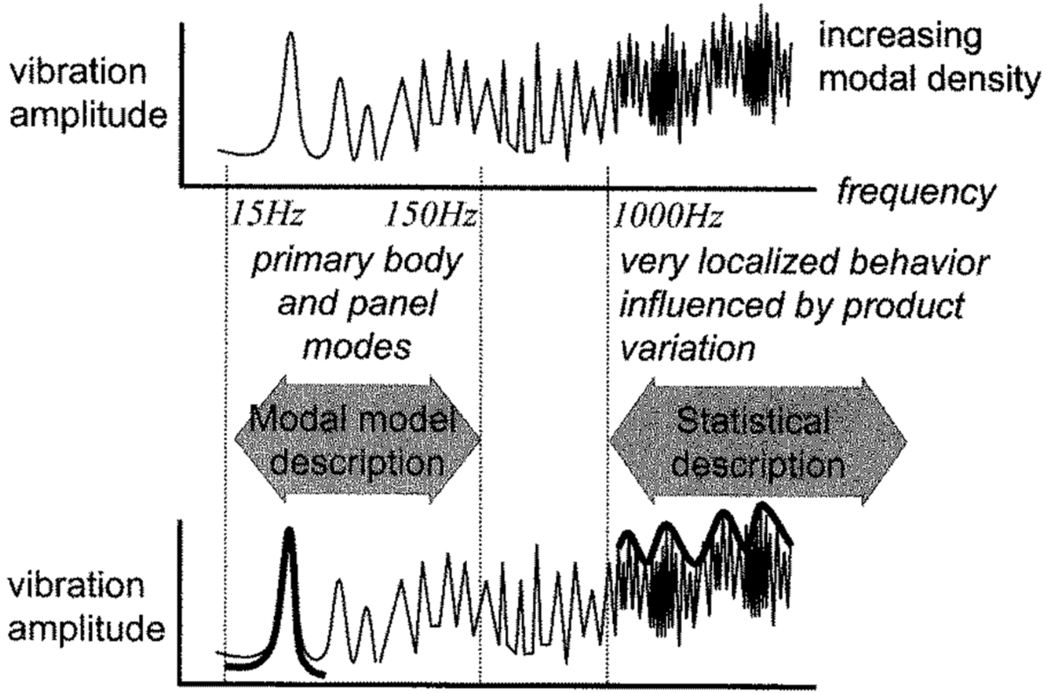


Driving point response



7.7 Modeling Resonant Behavior

- Structure's modal density
  - Number of modes occurring in a fixed bandwidth
  - Increase with increasing frequency
- Lower frequency
  - 10~150Hz
  - individual modes
  - modal model
- High frequency
  - 1000Hz~
  - high modal density
  - statistical approach



Modal Model (1)

- Primary modes of vibration

$$F_{\text{physical}} \rightarrow F_{\text{modal}} \rightarrow X_{\text{modal}} \rightarrow X_{\text{physical}}$$

$$F_{\text{modal}} = F_{\text{physical}} \phi_{\text{input}}$$

F_{physical}: force applied to the physical body structure at the input location

F_{modal}: force applied to the modal model

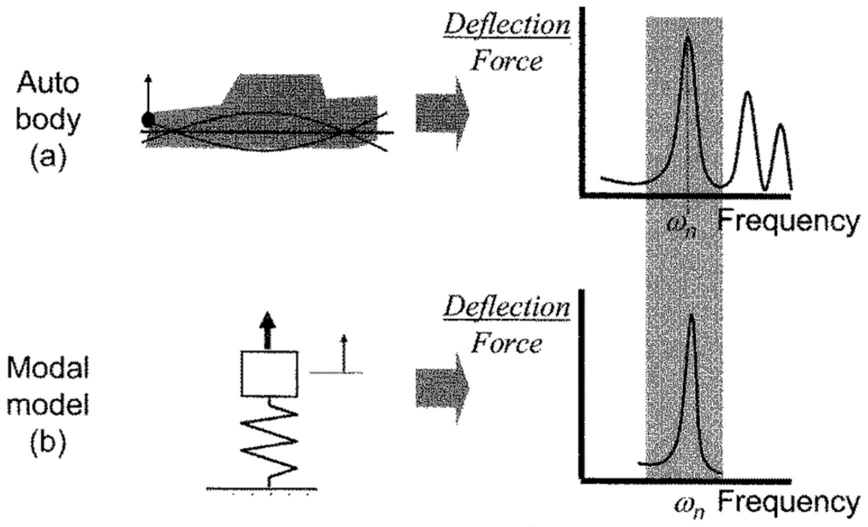
ϕ_{input}: influence coefficient at the input (determined from mode shape at resonance)

$$X_{\text{physical}} = X_{\text{modal}} \phi_{\text{output}}$$

X_{physical}: deflection of the physical body structure at the output location

X_{modal}: deflection of the modal model

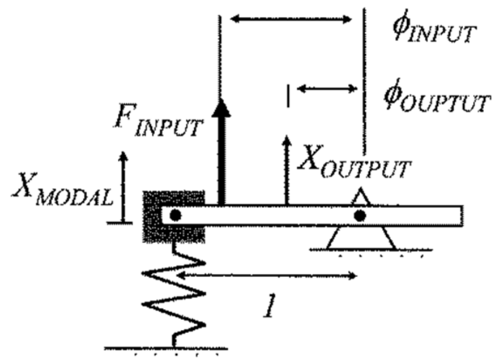
ϕ_{output}: influence coefficient at the output (determined from mode shape at resonance)

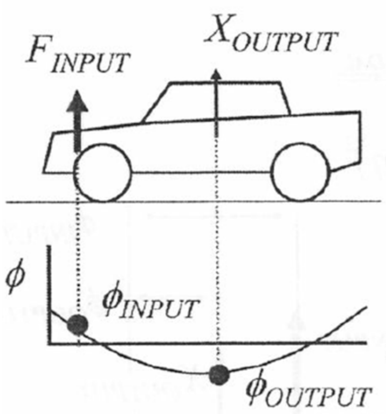


Modal Model (2)

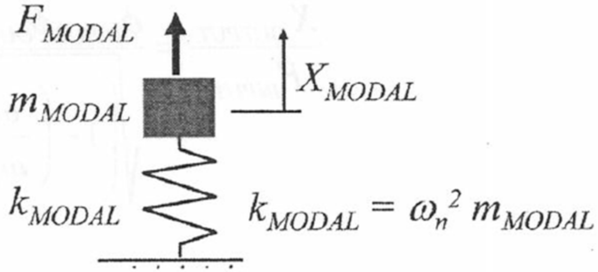
$$F = kX - m\omega^2 X \rightarrow \frac{X}{F} = \frac{1}{k - m\omega^2} = \frac{1/k}{1 - \left(\frac{m}{k}\right)\omega^2} = \frac{1/k}{1 - \left(\frac{\omega}{\omega_n}\right)^2} = P(\omega)$$

$$\frac{X_{\text{physical}}}{F_{\text{physical}}} = \frac{X_{\text{modal}}\phi_{\text{output}}}{F_{\text{modal}}/\phi_{\text{input}}} = \phi_{\text{input}}\phi_{\text{output}}\frac{X_{\text{modal}}}{F_{\text{modal}}} = \frac{\phi_{\text{input}}\phi_{\text{output}}/k_{\text{modal}}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \eta^2}}$$

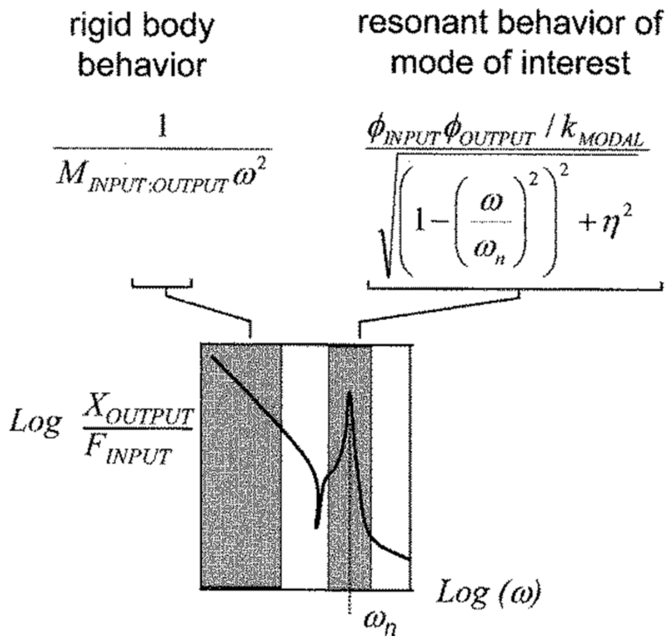




Test data



Modal model



Example: Effect of Mass Placement

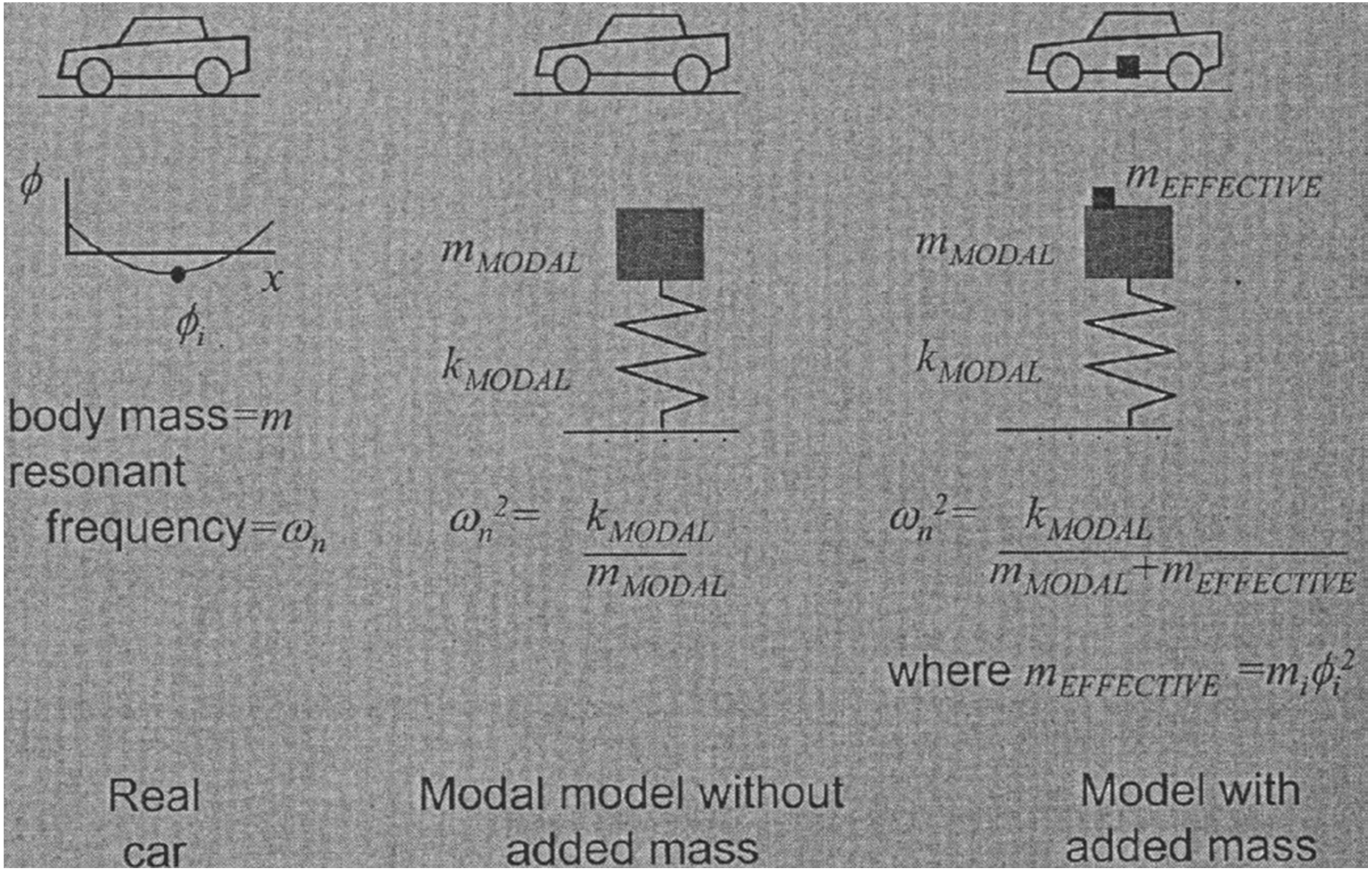
- Primary body resonance: 22~25 Hz
- Increase the resonant frequency
  - Increased body stiffness
  - Careful placement of subsystem masses
- Selection of battery location: front corner, dash, trunk

$$\text{primary bending resonance: } f_n = 47\text{Hz}\left(\omega_n = 295.3\,\text{rad/sec}\right) \rightarrow \phi_i = \begin{cases} 0.9 & \text{@front corner} \\ -0.2 & \text{@dash} \\ 0.15 & \text{@trunk} \end{cases}$$

$$\text{body shell mass: } M = 250\text{kg}\left(= M_{\text{modal}}\right)$$

$$\text{battery mass: } m = 16.8\text{kg}$$

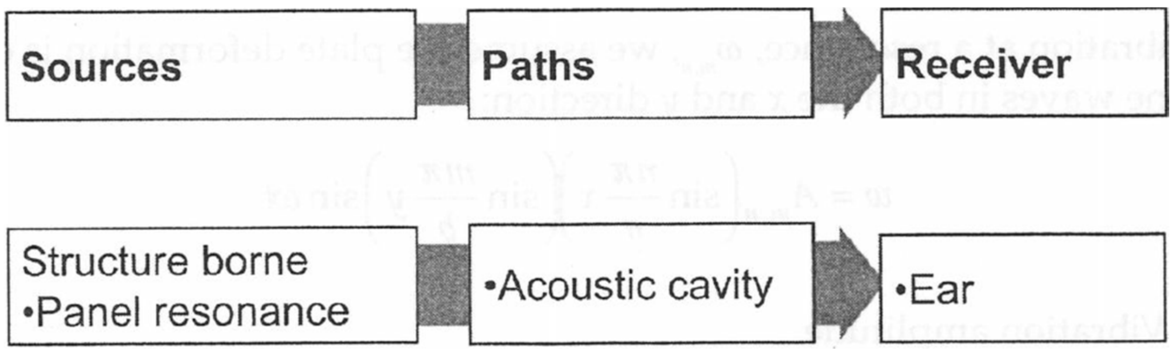
Effect on Resonant Frequency of an Added Mass



7.8 Vibration at High Frequency

- Primary body structure resonance: 18~50Hz
  - Vibration at the receiver: tactile
- Higher frequencies: 50~400Hz
  - More localized response of body structure, acoustic
- Structure-borne panel vibration system

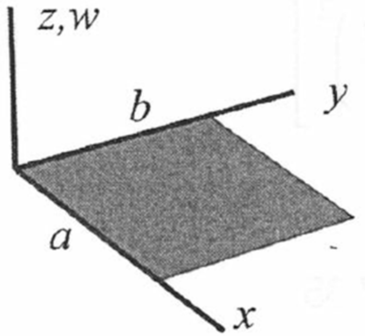
Source $F(\omega)$	Body structure transfer function $P(\omega)$	Acoustic deflection $X(\omega)$
Body panel vibrations	Passenger compartment acoustic resonances	Interior sound pressure



Body Panel Vibration (1)

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{q}{D} = 0$$

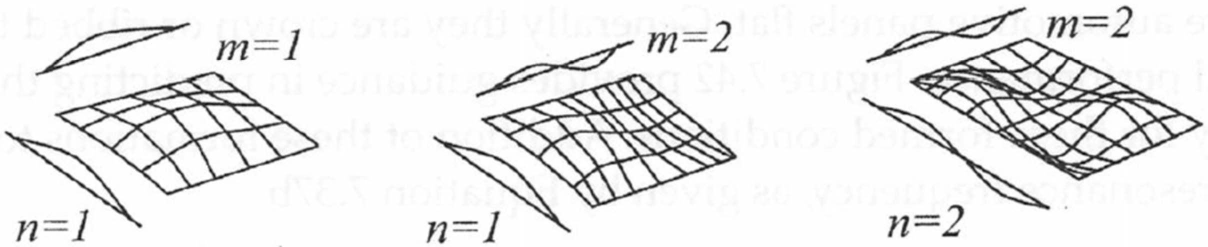
- $w(x, y)$: normal deflection of the plate
- $q(x, y)$: normal load per unit area
- $D = \dfrac{Et^3}{12\left(1-\nu^2\right)}$: plate bending stiffness



Let deflected shape be:

$w = A_o sin(n\pi x/a) sin(m\pi y/b)\ sin\ \omega_n t$

(note similarity to plate buckling shapes)



$$\omega_n = \sqrt{\frac{Et^3}{12(1-\mu^2)m''}}\left(\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right)$$

where

m''=mass per unit area

Body Panel Vibration (2)

$$\left.\begin{aligned} w_{m,n} &= A_{m,n}\left(\sin\frac{n\pi}{a}x\right)\left(\sin\frac{m\pi}{b}x\right)\sin\omega t \\ q &= m''\frac{\partial^2 w}{\partial t^2} \end{aligned}\right\} \rightarrow \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{q}{D} = 0$$

$$\left[\left(\frac{n\pi}{a}\right)^4 + 2\left(\frac{n\pi}{a}\right)^2\left(\frac{m\pi}{b}\right)^2 + \left(\frac{m\pi}{b}\right)^4 - \frac{\omega^2 m''}{D}\right]\left[A_{m,n}\left(\sin\frac{n\pi}{a}x\right)\left(\sin\frac{m\pi}{b}x\right)\sin\omega t\right] = 0$$

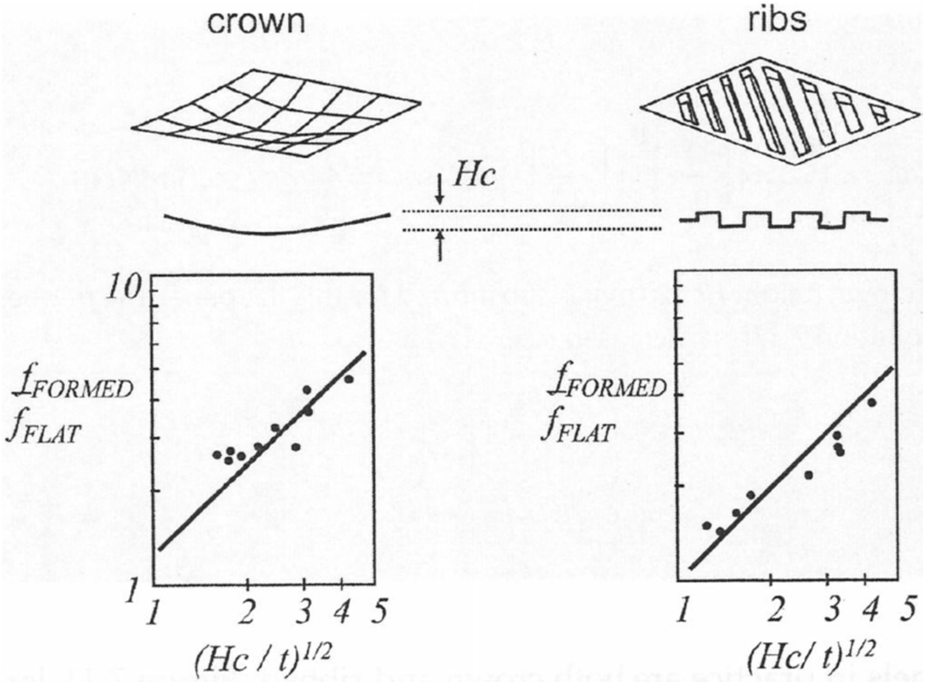
$$\rightarrow \omega_{m,n} = \pi^2\sqrt{\frac{D}{m''}}\left[\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2\right]$$

$$\frac{f_{\mathrm{FORMED}}}{f_{\mathrm{FLAT}}} = C\sqrt{\frac{H_C}{t}}$$

H_C : crown height

t : panel thickness

$$C = \begin{cases} 1.25 \ (\text{crown}) \\ 1 \ (\text{ribbed}) \end{cases}$$

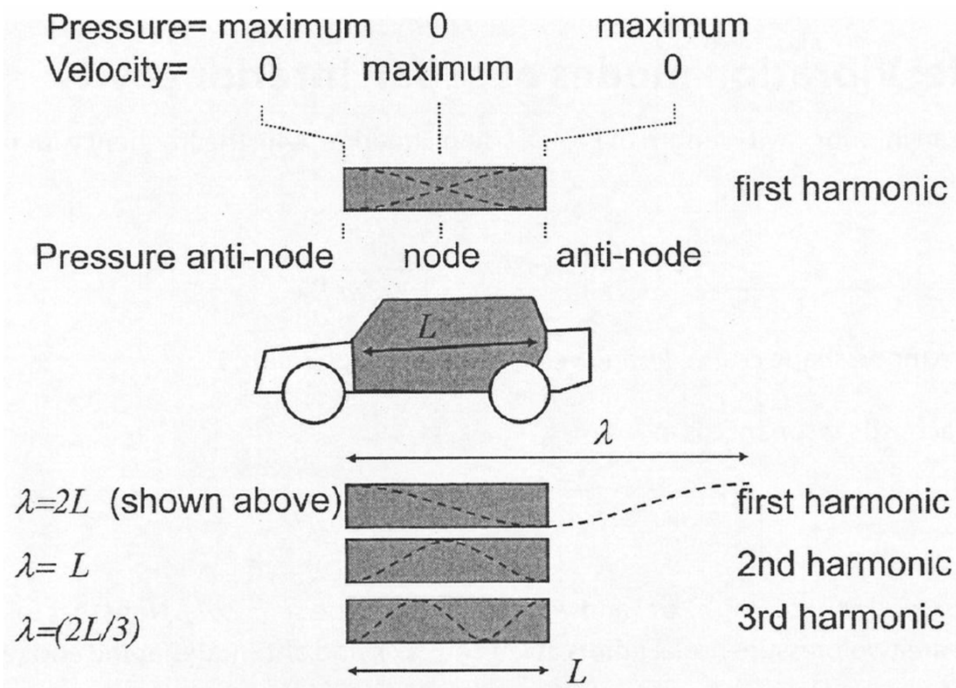


Acoustic Cavity Resonance

- Closed air cavity of passenger compartment
  - Resonate with a standing acoustic wave
  - Closed boundary conditions at either end

$$\left.\begin{aligned} f_n \lambda &= c \\ \lambda &= \frac{2L}{n} \end{aligned}\right\} \rightarrow f_n = c\left(\frac{n}{2L}\right)$$

$$\begin{cases} f_n: \text{ resonant frequency}\,(Hz) \\ \lambda: \text{ wavelength} \\ c: \text{ speed of sound in air}\,(330\, m/\sec) \\ L: \text{ cabin length} \\ n: \text{ number of half cosine waves along cabin length} \end{cases}$$



pressure mode shape @each resonance: $\cos\left(\frac{n\pi x}{L}\right)$

notion of sound level

air velocity mode shape @each resonance: $\sin\left(\frac{n\pi x}{L}\right)$

indication of sensitivity of cavity mode to excitation by a panel

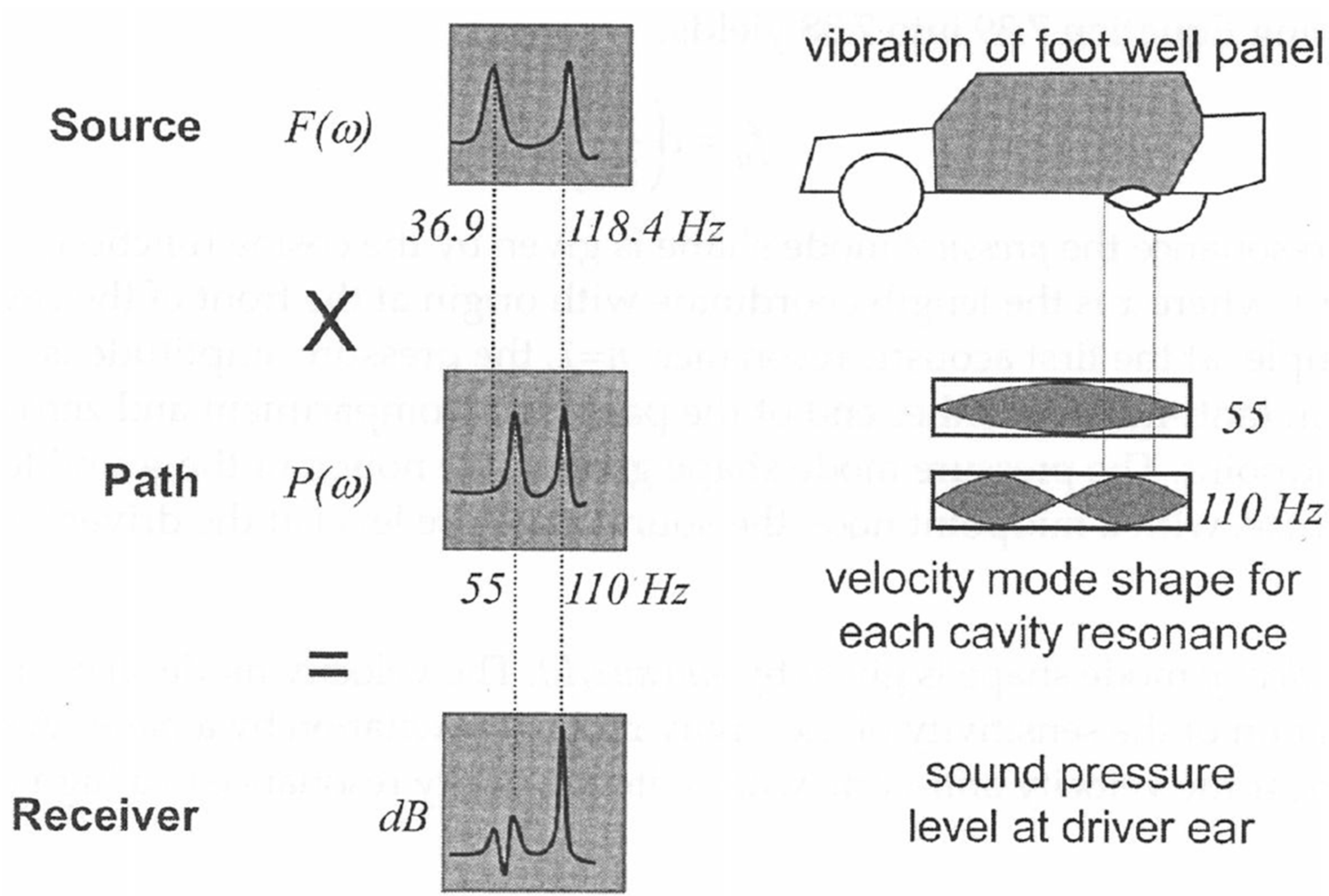
Example

- Vibration frequencies for floor pan
- Vibration modes of sedan interior cavity

$$\left.\begin{aligned} a &= 500mm \\ b &= 300mm \\ t &= 1mm \\ \rho &= 7.83\times10^{-6}\ kg/mm^2 \end{aligned}\right\} \rightarrow \omega ? \begin{cases} \omega_{1,1} \\ \omega_{1,2} \end{cases}$$

$$H_C = 20mm \rightarrow f_{\mathrm{FORMED}} ?$$

Panel and Acoustic Cavity



Vibration Isolation through Elastomeric Elements

- Suspension elements due to road impacts → high frequency deflections
- Isolation of higher frequency vibration
  - Elastomeric bushings at the body connections

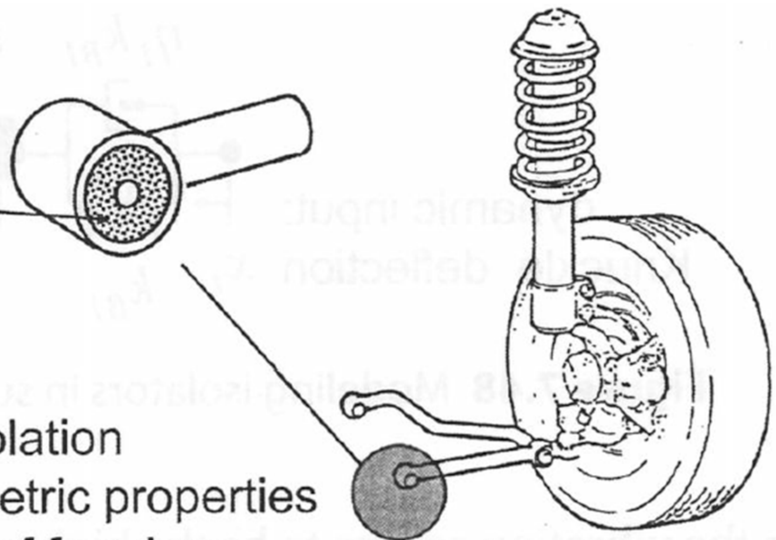
Source	**Isolator**	**Force into body**	**Body transfer function**	**Body deflection**
F(ω)	T(ω)	$F_T(\omega)$	P(ω)	X(ω)
High frequency chassis deflections	Chassis links with end bushings	Body panel vibrations	Passenger compartment acoustic resonances	Interior sound pressure

Suspension Lower Control Arm

High frequency displacement → Isolator → Panel (structure borne noise)

- Suspension noise and harshness
- Chassis noise and harshness

- Suspension links with rubber bushings
- Body mounts



Functions:
•noise isolation
•vibration harshness isolation
•tune suspension geometric properties
•allow linkage degrees of freedom

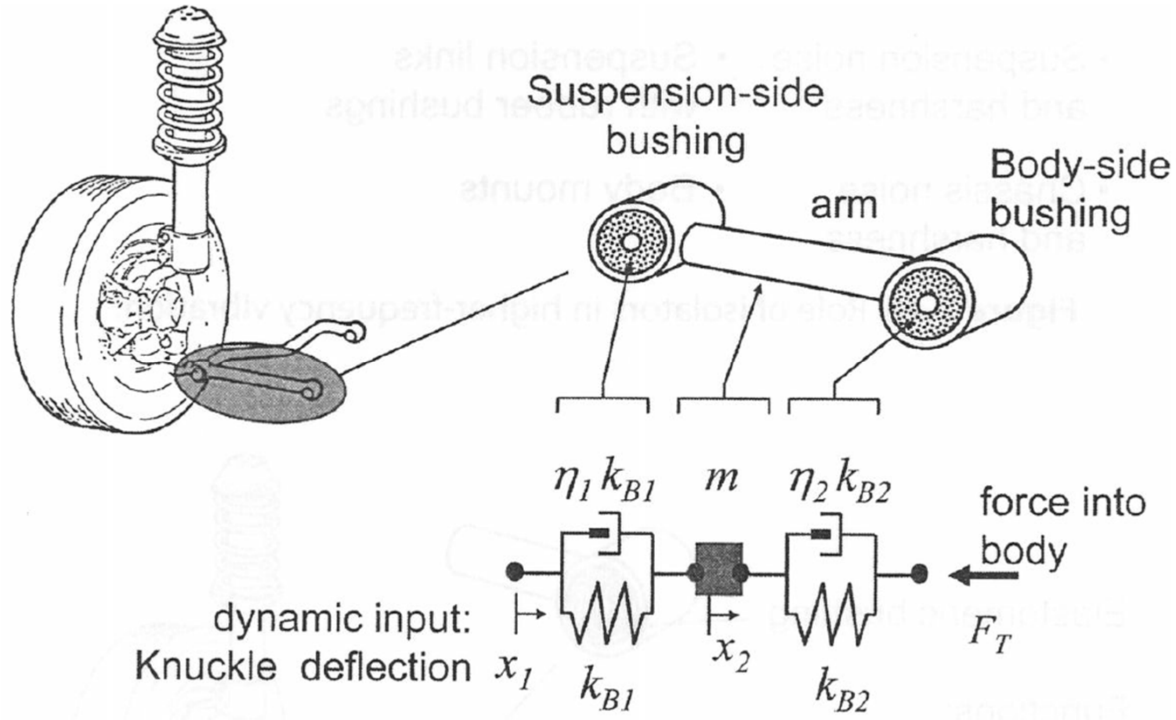
Modeling Isolators

$$F = kX + i\eta kX = k^* X \rightarrow \frac{F}{X} = \underbrace{k}_{\text{stiffness}} + i \underbrace{\eta k}_{\text{damping}} = k^*$$

F : force through the bushing

X : deflection across the bushing

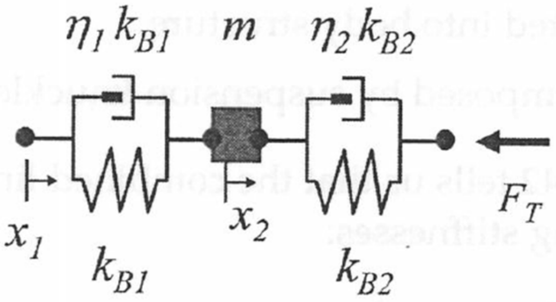
η : loss factor for the elastomeric material

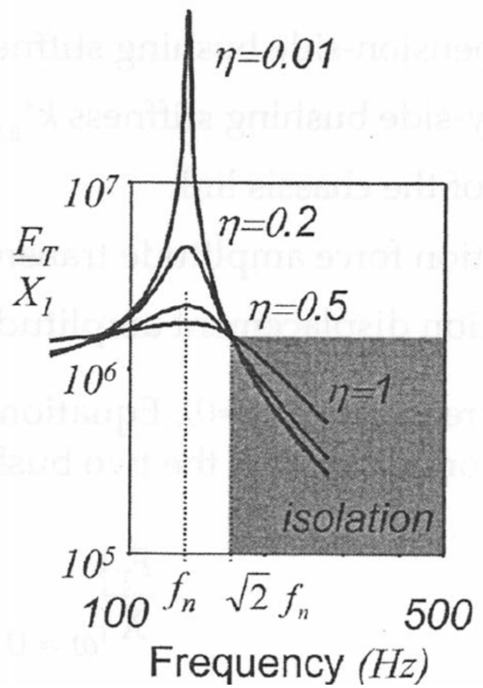


Response of Isolators

$$\frac{F_T}{X_1} = \frac{\dfrac{k_{B1}^{*}k_{B2}^{*}}{k_{B1}^{*}+k_{B2}^{*}}}{1-\omega^2\dfrac{m}{k_{B1}^{*}+k_{B2}^{*}}} \rightarrow \begin{cases} \omega \approx 0 : \left|\dfrac{F_T}{X_1}\right| = \dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}} \\ \omega_n = \sqrt{\dfrac{k_{B1}+k_{B2}}{m}} : \left|\dfrac{F_T}{X_1}\right| = \left(\dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}}\right)\dfrac{\sqrt{1+\eta^4}}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2+\eta^2}} = \left|T(\omega)\right| \end{cases}$$

$$\begin{cases} k_{B1}^{*} : \text{ suspension-side bushing stiffness, } k_{B1}^{*} = k_{B1} + i\eta_1 k_{B1} \\ k_{B2}^{*} : \text{ body-side bushing stiffness, } k_{B2}^{*} = k_{B2} + i\eta_2 k_{B2} \\ m : \text{ mass of the chassis link} \\ F_T : \text{ vibration force amplitude transmitted into body structure} \\ X_1 : \text{ vibration displacement amplitude imposed by suspension knuckle} \end{cases}$$





Example

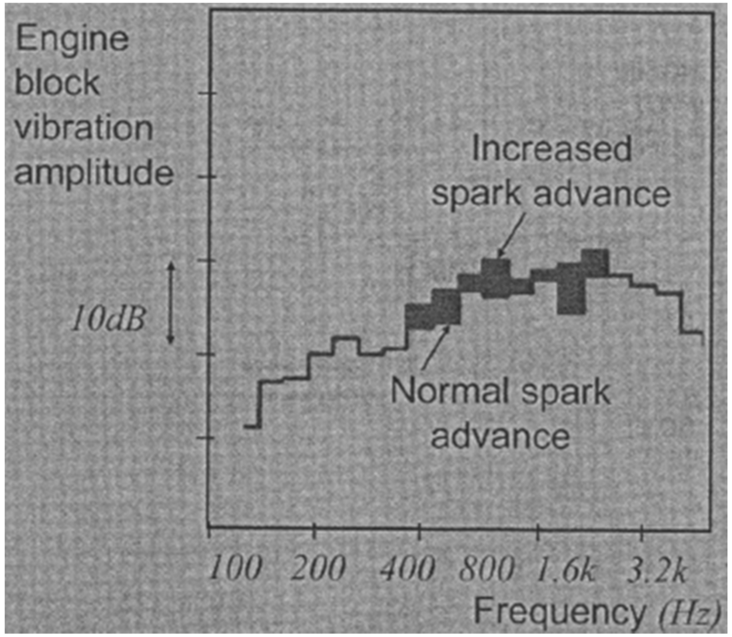
- gear meshing in the transmission → front wheel drive shaft → suspension knuckle → suspension control arm → body structure

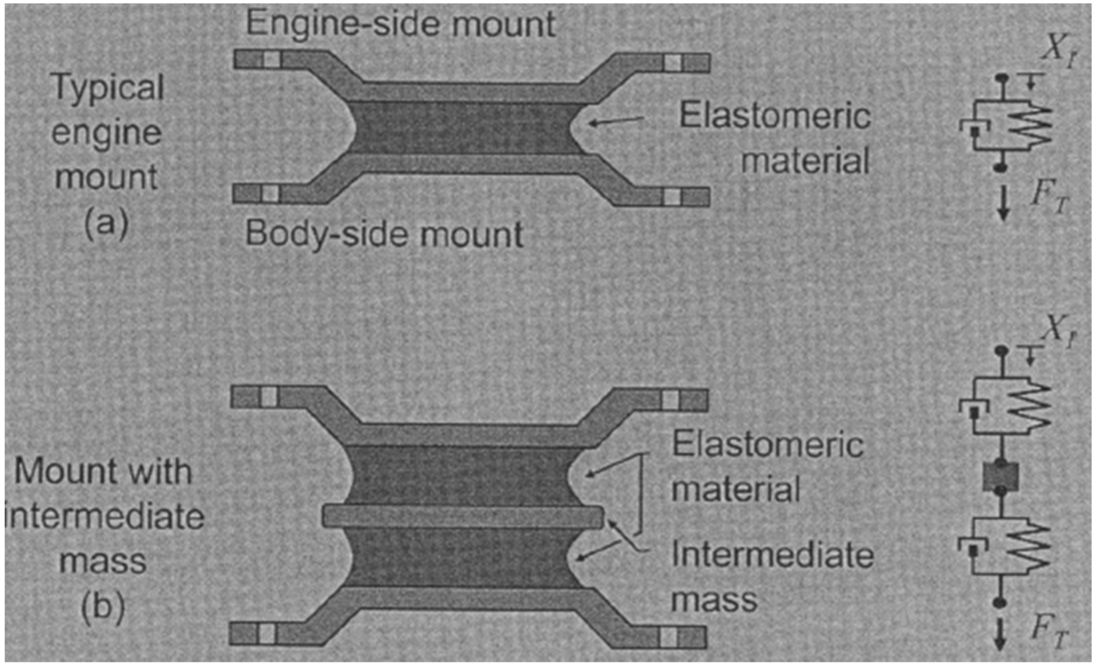
$$\text{mesh frequency: } f = 400Hz$$
$$k_{B1} = k_{B2} = 175000\,N/m$$
$$\eta = 0.2$$
$$m = 5kg$$

$$\begin{cases} \omega \approx 0: \left|\dfrac{F_T}{X_1}\right| = \dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}} = 875000\dfrac{N}{m} \\ \omega_n = \sqrt{\dfrac{k_{B1}+k_{B2}}{m}} = 836.7\dfrac{rad}{s}(133Hz): \\ \left|\dfrac{F_T}{X_1}\right| = \left(\dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}}\right)\dfrac{\sqrt{1+\eta^4}}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2+\eta^2}} = \left(\dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}}\right)\dfrac{\sqrt{1+0.2^4}}{\sqrt{\left[1-\left(\dfrac{400}{133}\right)^2\right]^2+0.2^2}} = 0.125\left(\dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}}\right) \end{cases}$$

Example: High-Frequency Powertrain Vibration through Engine Mount (1)

- Powertrain → engine mount → body structure: direct mount
- High frequency vibration of engine block: structure-borne noise
- Increase engine spark timing → improve fuel economy
  - Increase dynamic block deflections in 400~2000Hz range
  - To isolate acoustic vibrations, engine mount with free mass





Example: High-Frequency Powertrain Vibration through Engine Mount (2)

- Target static stiffness: 200 N/mm
- Isolation begins at 270 Hz
- Needed intermediate mass?

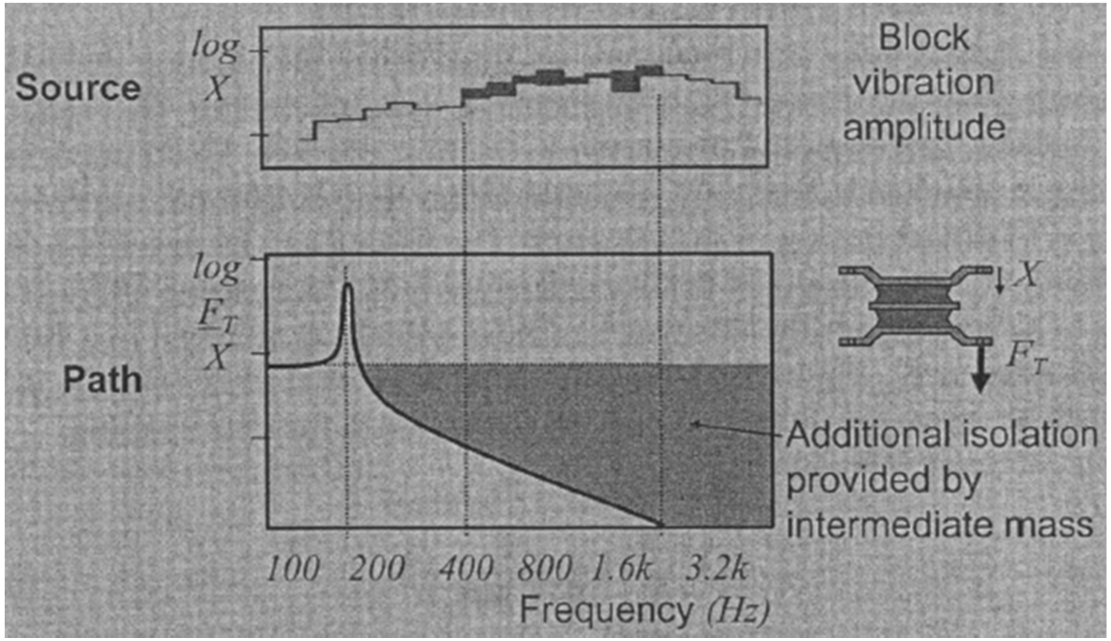
$$\omega \approx 0 : \left|\frac{F_T}{X_1}\right| = \frac{k_{B1}k_{B2}}{k_{B1}+k_{B2}} = 200\frac{N}{mm}$$

$$\rightarrow k_{B1} = k_{B2} = 400\frac{N}{mm}$$

$$f_n\sqrt{2} = 270Hz \rightarrow f_n = 190Hz$$

$$\omega_n = \sqrt{\frac{k_{B1}+k_{B2}}{m}}$$

$$\rightarrow m = \frac{k_{B1}+k_{B2}}{\omega_n^{\ 2}} = \frac{2(400)\dfrac{N}{10^{-3}m}}{\left[2\pi(190)\right]^2} = 0.56kg$$



Local Stiffness Effect on Vibration Isolators

- Desired high-frequency-isolation: bush material?
- Localized flexing of structure: local stiffness

$$X = X_{local} + X_{bushing} = \frac{F}{K_L} + \frac{F}{k_B + i(\eta k_B)} \rightarrow \frac{F}{X} = \frac{k_B K_L + i(K_L \eta k_B)}{K_L + k_B + i(\eta k_B)}$$

$$\frac{F}{X} = \frac{k_B\left[(k_B/K_L)+1+\eta^2(k_B/K_L)\right]+i(\eta k_B)}{\left[(k_B/K_L)+1\right]^2+\left[\eta(k_B/K_L)\right]^2} \xrightarrow{\eta^2\sim 0} \frac{F}{X} = \frac{k_B}{\left[(k_B/K_L)+1\right]} + i\frac{\eta k_B}{\left[(k_B/K_L)+1\right]^2} \Leftrightarrow \frac{F}{X} = k_B + i\eta k_B$$