

Part-I

$$(1) u(x) = \begin{cases} -2x^2 + \frac{4}{3}, & 0 \leq x \leq \frac{1}{3} \\ -\frac{x^2}{2} - x + \frac{3}{2}, & \frac{1}{3} < x \leq 1 \end{cases}$$

$$(2) F = \frac{1}{2} \begin{bmatrix} hf\left(\frac{h}{2}\right) \\ hf\left(\frac{h}{2}\right) + Hf\left(\frac{2h+H}{2}\right) \\ Hf\left(\frac{2h+H}{2}\right) \end{bmatrix}$$

$$(3) h = H = \frac{1}{2} \rightarrow \begin{bmatrix} U_0 \\ U_1 \end{bmatrix} = \begin{bmatrix} 13/8 \\ 9/8 \end{bmatrix}, h = \frac{1}{3} H = \frac{2}{3} \rightarrow \begin{bmatrix} U_0 \\ U_1 \end{bmatrix} = \begin{bmatrix} 4/3 \\ 10/9 \end{bmatrix}$$

Part-II

$$[K, F, K_b, F_b, U] = \text{hw10}(\underbrace{[1/2 \ 1/2]}_{\text{sub-intervals}}, \underbrace{3}_{\text{nodes at boundary}})$$