Automotive Body Structural Elements (1)

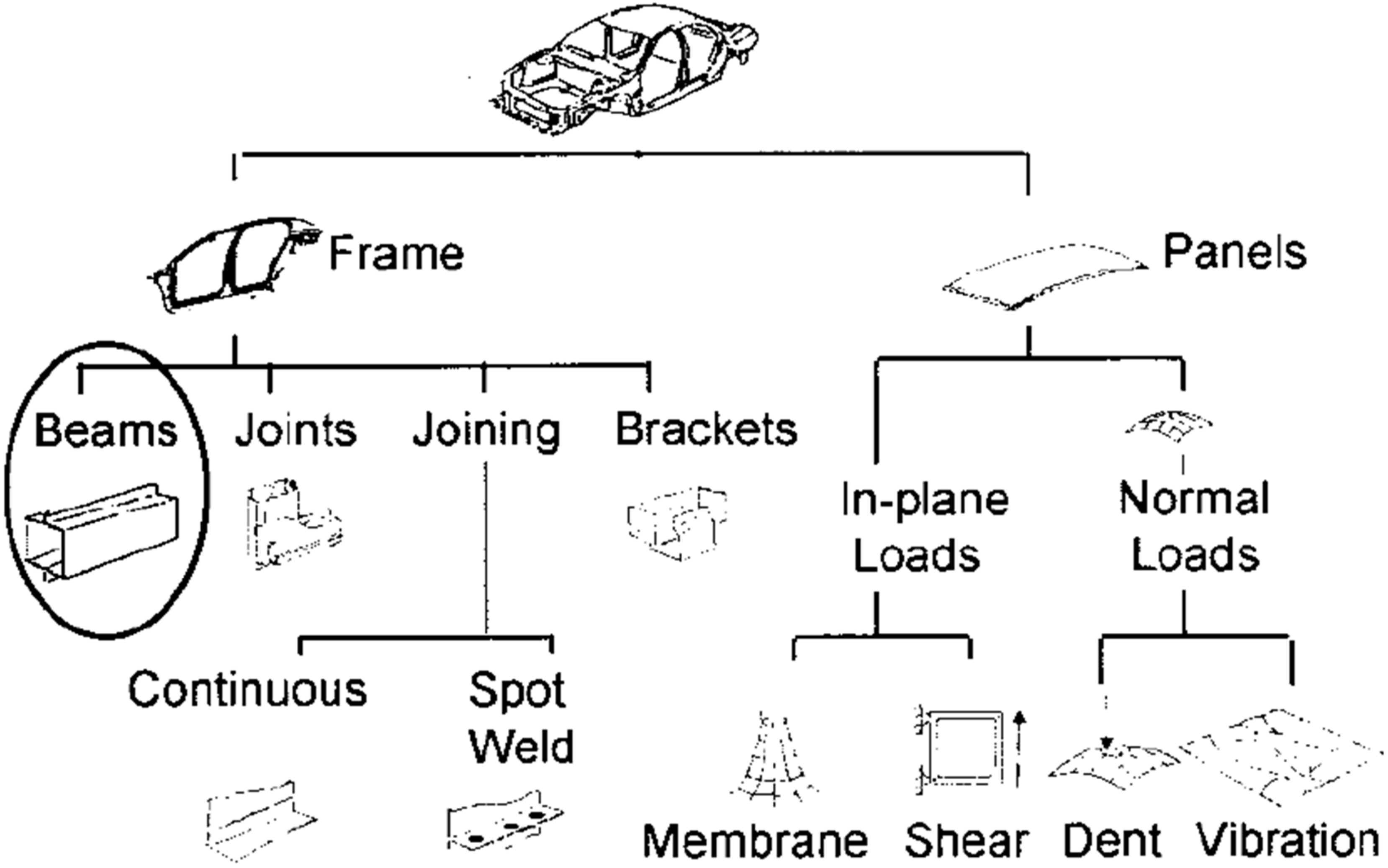
- Section design tools
  - How automotive structural elements respond to loading?
  - How they deflect? How they fail?
  - Predict stiffness and strength given the section geometry, the material and the bending moment, torque or applied force

- Classical beam behavior
- Design of automotive beam sections
  - Bending of non-symmetric beams
  - Point loading of thin walled sections

Automotive Body Structural Elements (2)

- Torsion of thin wall members
  - Torsion of member with closed/open section
  - Warping of open sections
  - Effect of spot welds on structural performance
  - Longitudinal stiffness of a shear loaded weld flange
- Thin wall beam section design
- Buckling of thin wall members
  - Plate buckling
  - Effective width
  - Techniques to inhibit buckling
- Panels: plates and membranes
  - Curved panel with normal loading
  - In-plane loading of panels
  - Membrane shaped panels

Structural Elements Classification



Frame

Panels

Beams

Joints

Joining

Brackets

In-plane Loads

Normal Loads

Continuous

Spot Weld

Membrane

Shear

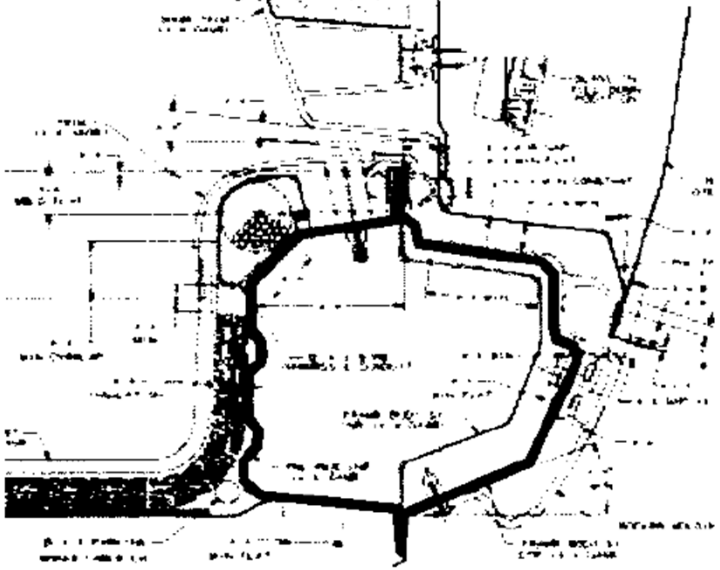
Dent

Vibration

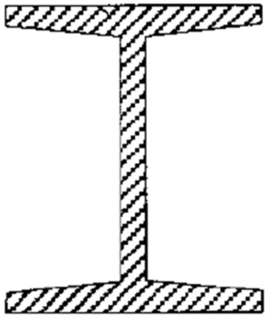
Beam Sections

- Thin walled structural elements
  - Relatively large width to thickness ratio
  - Non-symmetrical sections
  - Fabrication of several formed pieces spot welded

Automotive Rocker
Typical Section



Civil Engineering
Typical Section



3.1 Classical Beam Behavior

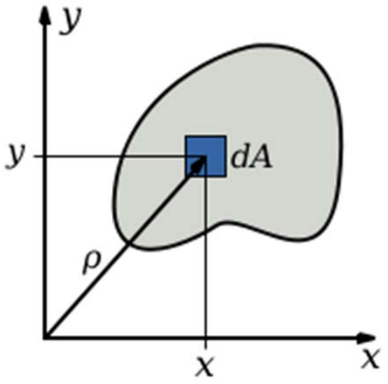
- Long straight beam with an I beam section
- Assumptions
  - Section is symmetric
  - Applied forces are down the axis of symmetry for the section
  - Section will not change shape upon loading
  - Deformation will be in the plane and in the direction of the applied load
  - Internal stresses vary in direct proportion with the strain
  - Failure: yielding of the outmost fiber
- Static equilibrium at a beam section: $M(x) = \int_0^x V dx$
- Stress over a beam section: $\sigma = -\frac{Mz}{I}$ where $I = \int_{\text{section}} z^2 dA$
- Beam deflection: $y = f(x),\ y'' = \frac{M(x)}{EI}$

Moment of Inertia

- Mass moment of inertia (관성모멘트)

$$I = kmr^2 = \sum_{i=1}^{n} m_i r_i^2 = \int r^2 dm = \iiint_V r^2 \rho(r)\, dV \rightarrow I = I_{cm} + md^2$$

- Area moment of inertia
  - Second moment of area (단면이차모멘트): bending
  - Polar moment of inertia (극관성모멘트): torsion
  - Product of inertia: unsymmetric geometry



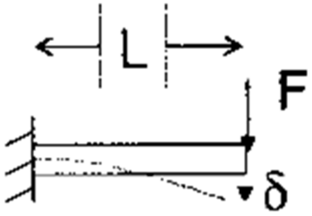
$$I_{xx} = \int_A y^2 dA \rightarrow I_{xx} = I_{xx\_c} + \overline{x}^2 A \text{ where } \overline{x}A = \int_A x dA$$

$$I_{yy} = \int_A x^2 dA$$

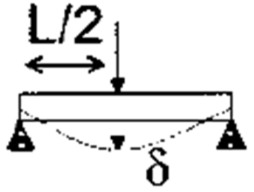
$$J\left(= I_z\right) = \int_A \rho^2 dA = \int_A \left(x^2 + y^2\right) dA = \int_A x^2 dA + \int_A y^2 dA = I_{xx} + I_{yy}$$

$$I_{xy} = \int_A xy dA$$

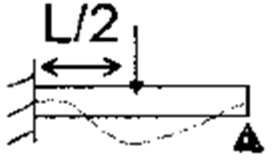
Beam Stiffness Equations



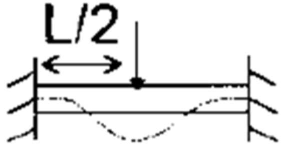
$$K = \frac{F}{\delta} = \frac{3EI}{L^3}$$



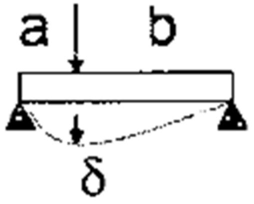
$$K = \frac{48EI}{L^3}$$



$$K = \frac{109.7EI}{L^3}$$



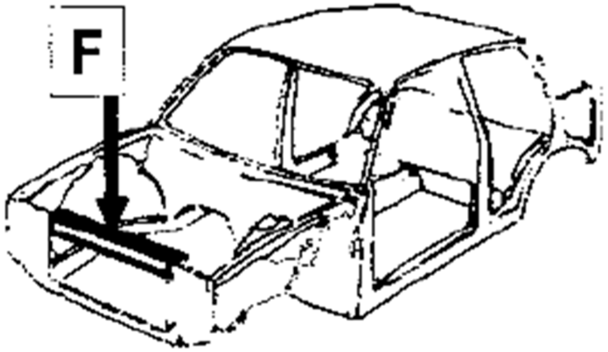
$$K = \frac{192EI}{L^3}$$

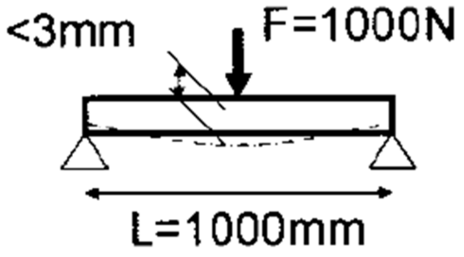


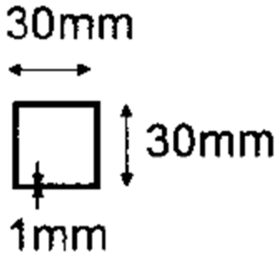
$$K = \frac{3EIL}{a^2b^2}$$

Example: Cross Member Beam

- Front motor compartment cross member holds the hood latch
- Under use, aerodynamic loading places a vertical load of 1000 N at the center of this beam
- Design requirements: section size ?
  - No yielding (σ_y = 210 N/mm^2) in the cross member
  - Maximum linear deflection at the hood latch of 3 mm







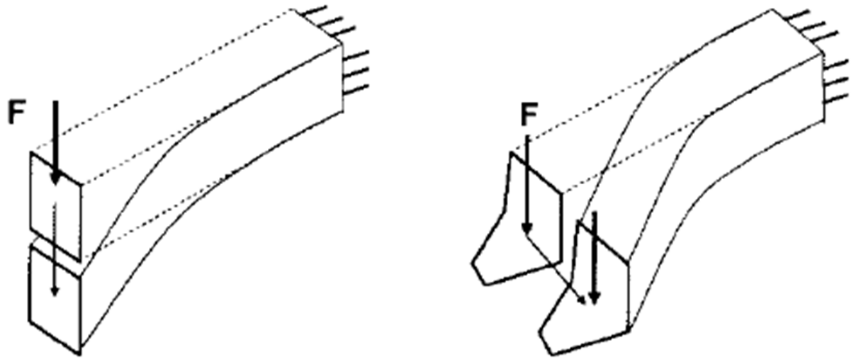
3.2 Design of Automotive Beam Sections

- Characteristics of automotive beams
  - Non-symmetrical nature of automotive beams
  - Local distortion of the section at the point of loading
  - Twisting of thin walled members
  - Effect of spot welds on structural performance

Bending of Non-Symmetric Beams

- Deflection
  - Resolve the load into components along each principle axis
  - Solve for the resulting deflection for each of these components
    - Moment of inertia is taken about the axis perpendicular to the load
    - Each of these deflections will be along the respective principle axis
  - Take the vector sum of the two deflections
- Stress
  - Resolve the moment into components along each principle axis
  - Solve for the resulting stress for each of these components
    - Dimension z is the distance to the point of interest from the axis which is colinear with the moment vector
  - Take the algebraic sum of two stresses for the resultant stress

Non-Symmetric Beams

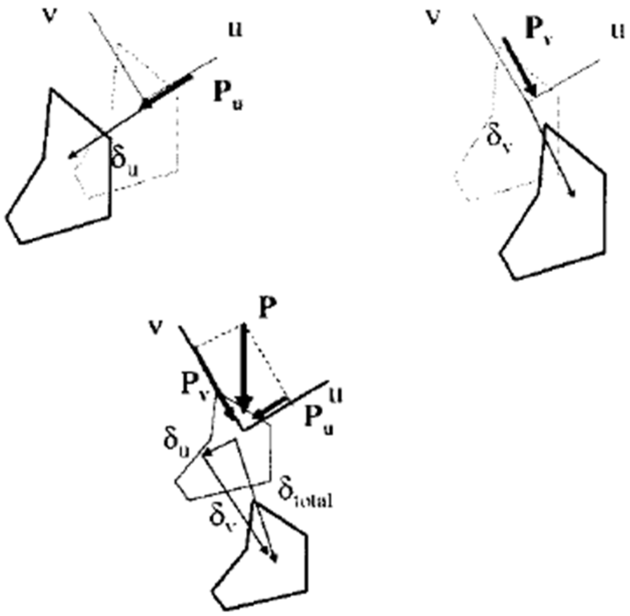


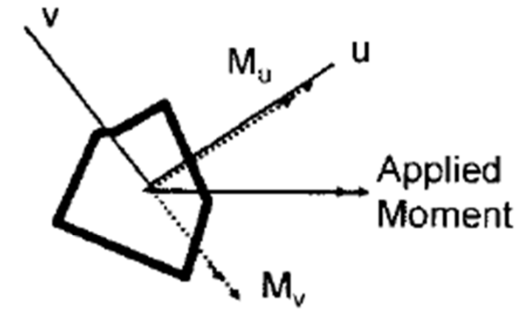
(a)
Symmetrical Beam

(b)
Non-Symmetrical Beam

Deflections along principle axes

Vector sum of deflections along principle axes



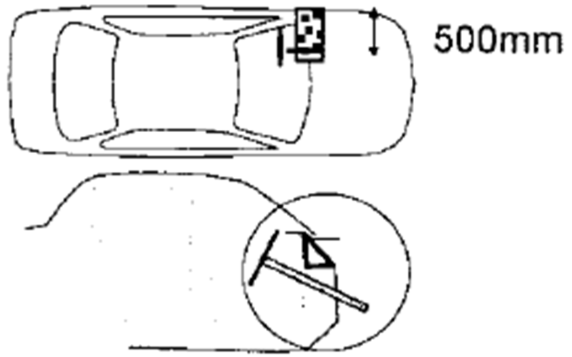


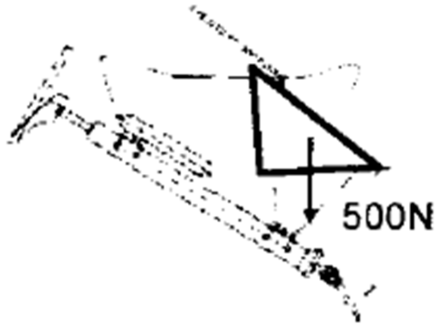
$$\sigma_{Mv} = -\frac{uM_v}{I_v}, \qquad \sigma_{Mu} = -\frac{vM_u}{I_u}$$

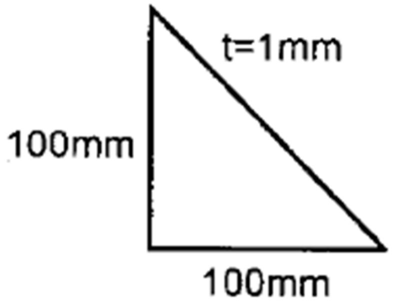
$$\sigma\ (u, v) = \sigma_{Mv} + \sigma_{Mu}$$

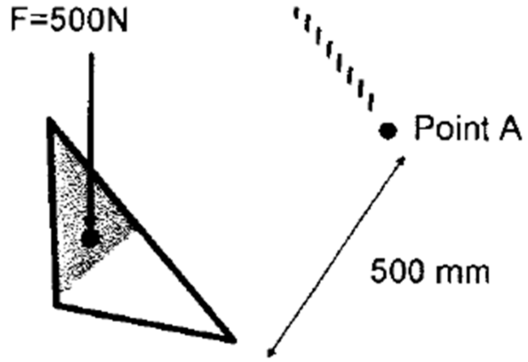
Example: Steering Column Mounting Beam

- Determine the tip deflection.
- Determine the stress at a specific point A where the beam joins the restraining structure.
- E = 207x10^3 N/mm^2

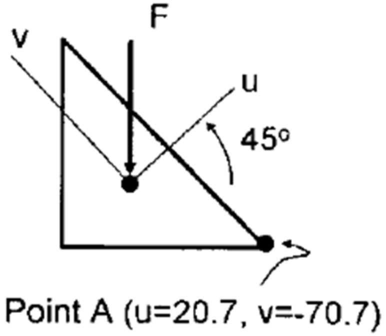








First Order Model



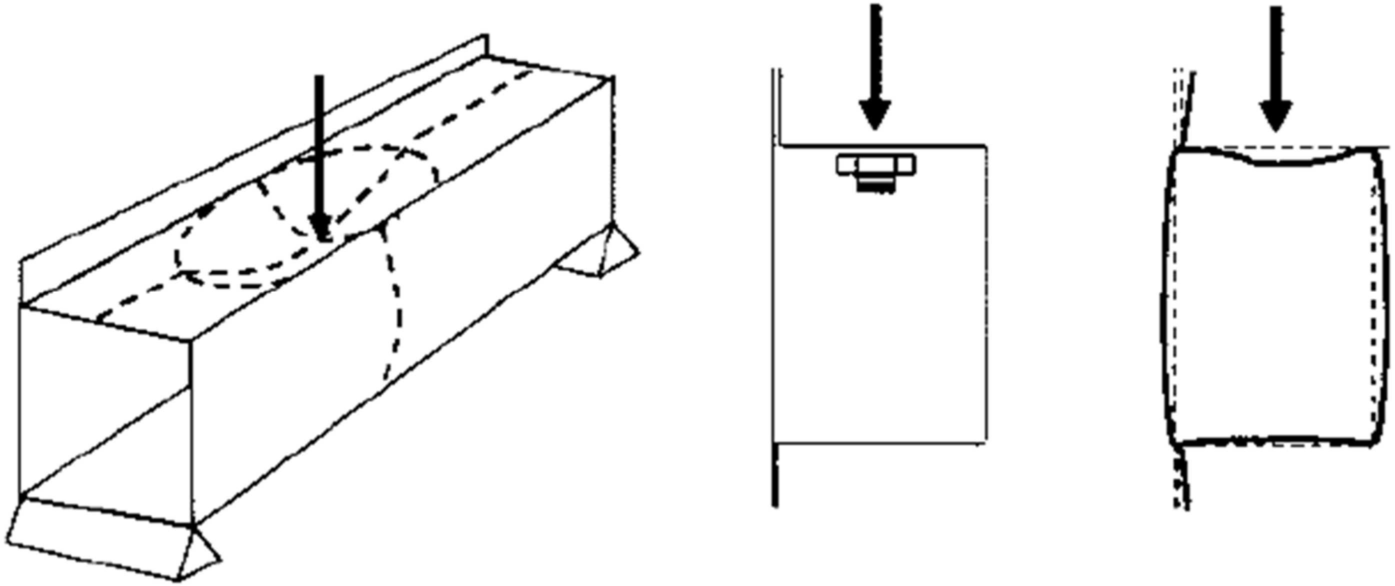
I_u= 5.69 x $10^5$$mm^4$

I_v= 1.87 x $10^5$$mm^4$

From section analysis

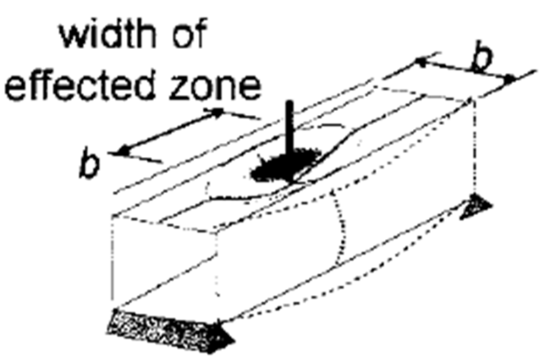
Point Loading of Thin Walled Sections

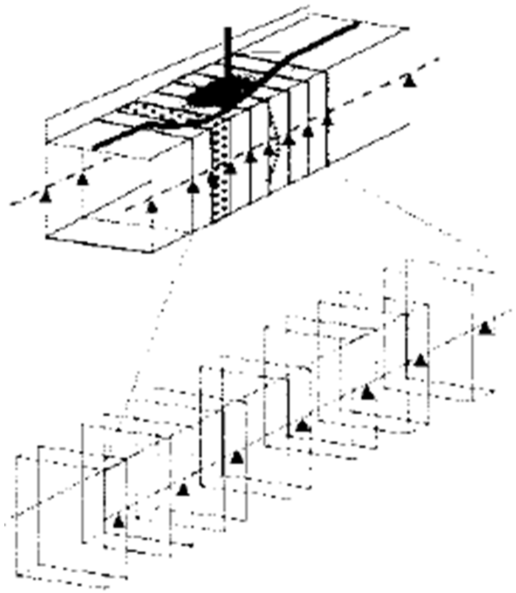
- Undesirable distortion in the vicinity of the load
  - Reduce apparent beam stiffness
  - Increase local stress

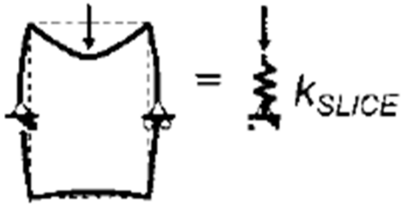


Prediction of Local Distortion

- Physical behavior: both beam deformation and local deformation
- Beam deformation eliminated by supporting beam along neutral axis leaving only local deformation: Local behavior isolated supporting beam along neutral axis
- Beam divided into slices of unit width over effective zone
- Slice characterized by a framework with stiffness k_{slice}

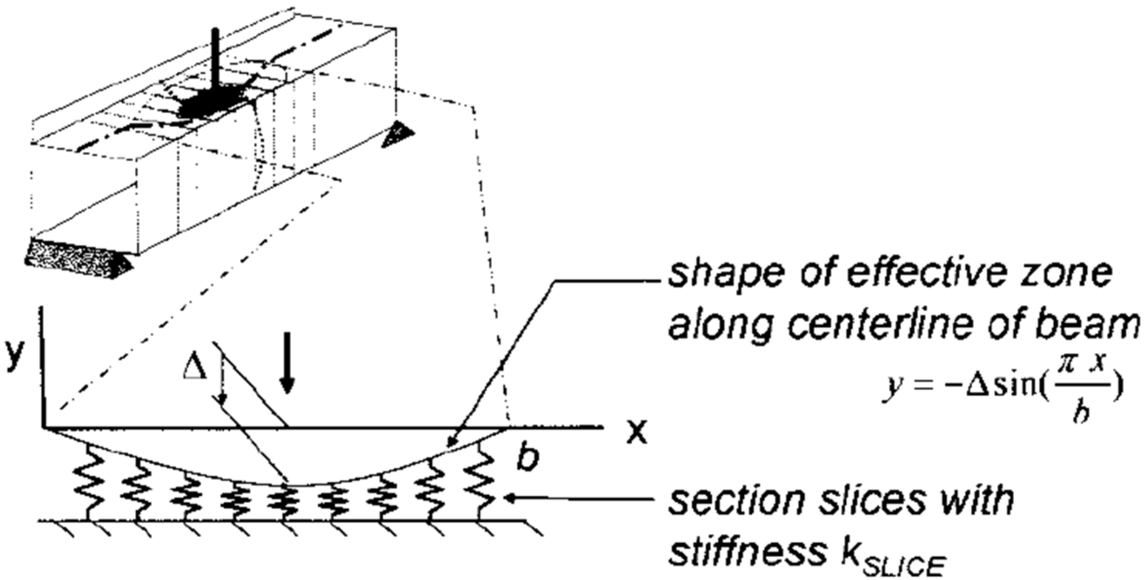


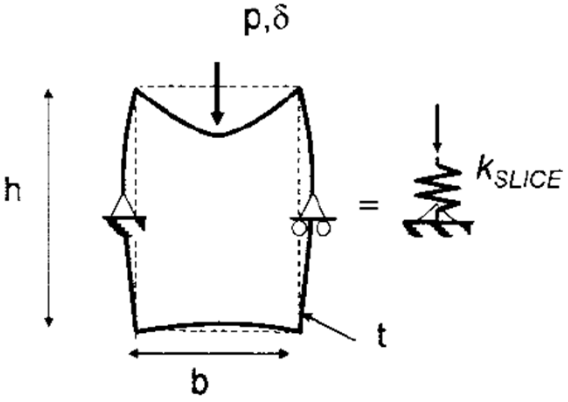




Idealized Beam Analysis

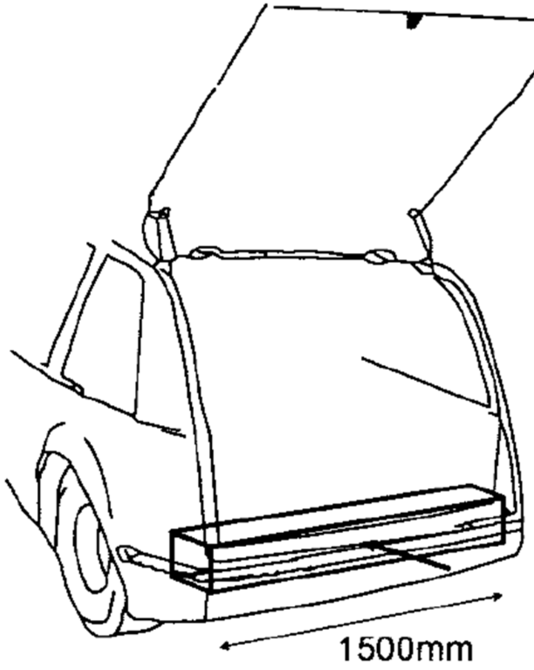
- (Energy stored by local stiffness at point of load application) = (Energy stored by distortion of all section slices)
- $K_{local} = F/\Delta$?





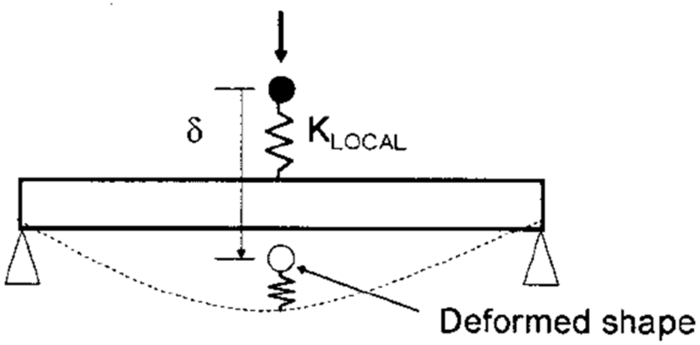
Example: Van Cross Member

- Two springs in series
  - Idealized beam stiffness
  - Stiffness of the local distortion of the section



Van Latch Structure
Stiffness requirement
K>1500N/mm

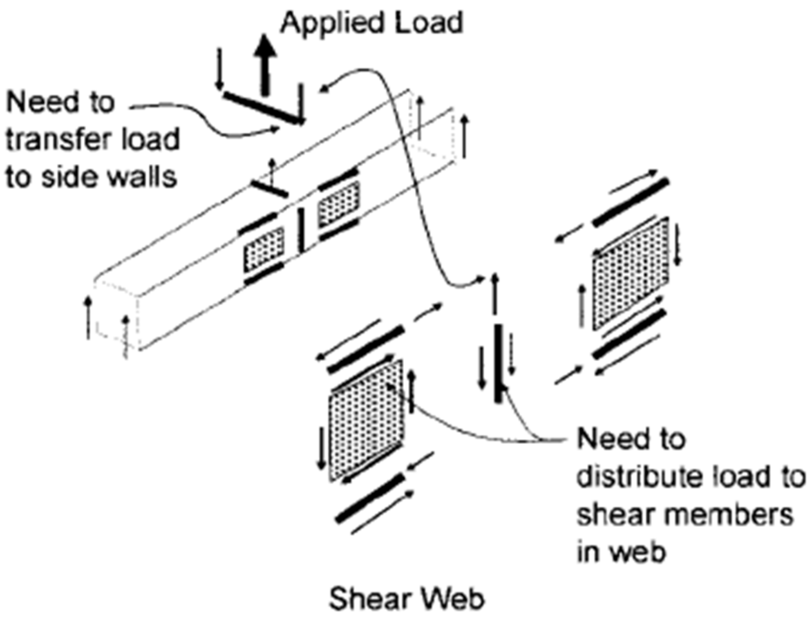
100mm square, t=0.8mm
Steel

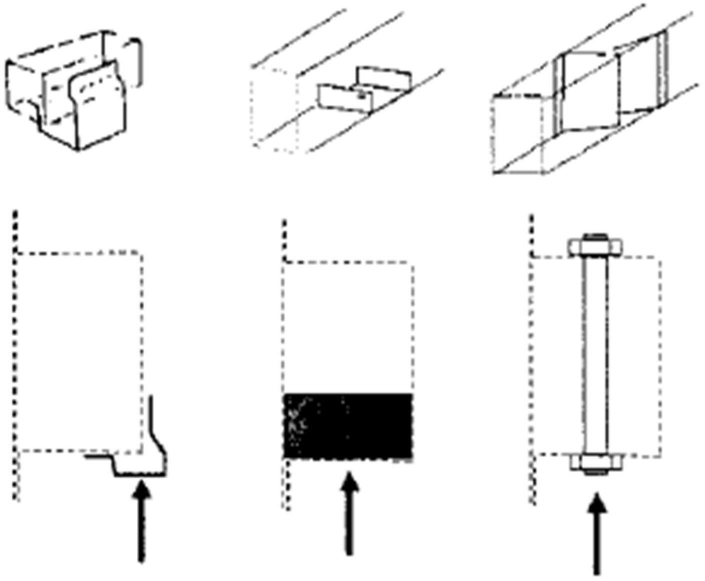


$$K_{system} = \frac{K_{ideal} K_{local}}{K_{ideal} + K_{local}}$$

Strategy to Reduce Local Distortion

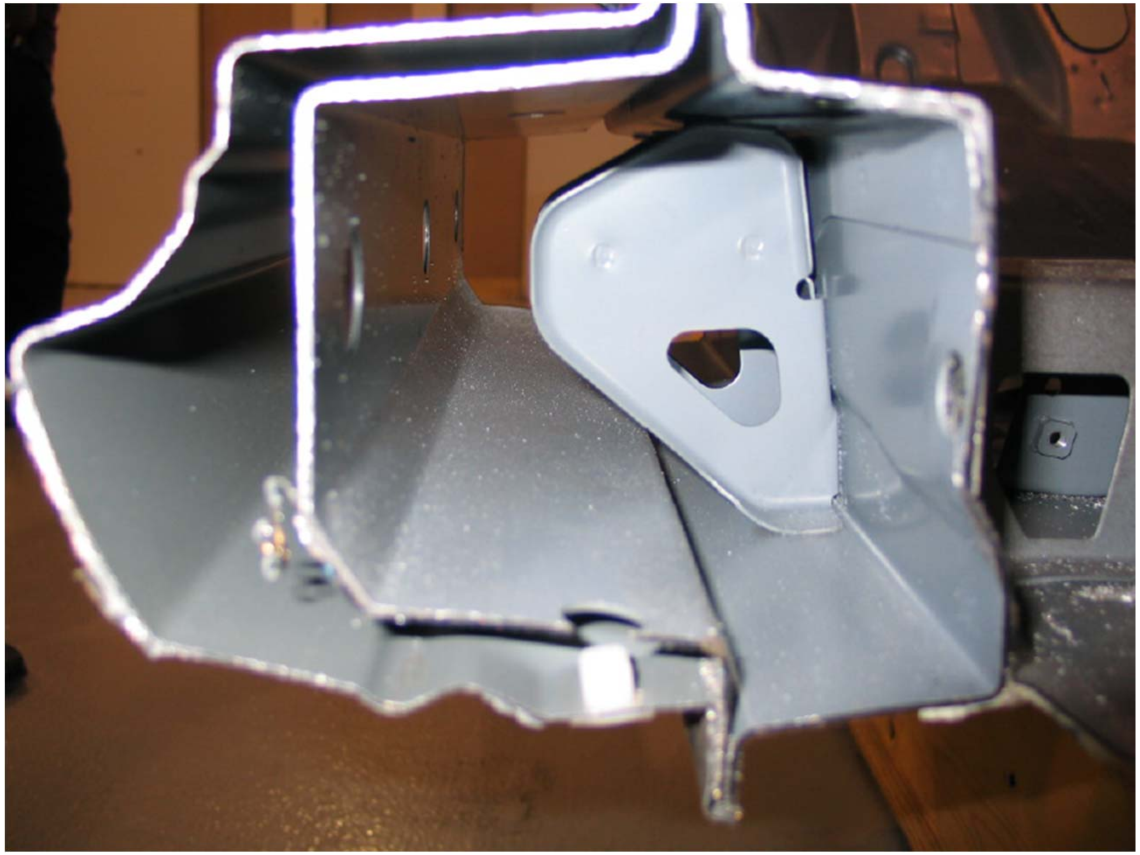
- Point load must load the shear web of the section directly
  - Moving the load point to align with the web
  - Adding stiff structural element to the section which reacts the load to the webs (local reinforcement)
  - Using through-section attachment with bulkhead to transfer the load to the web





2003 Toyota Camry SE

- Local Stiffeners Inside Rocker To B-Pillar Joint
  - Bulkheads Are Used For Local Buckling Prevention & FMVSS 214 Side Impact



3.3 Torsion of Thin Wall Members

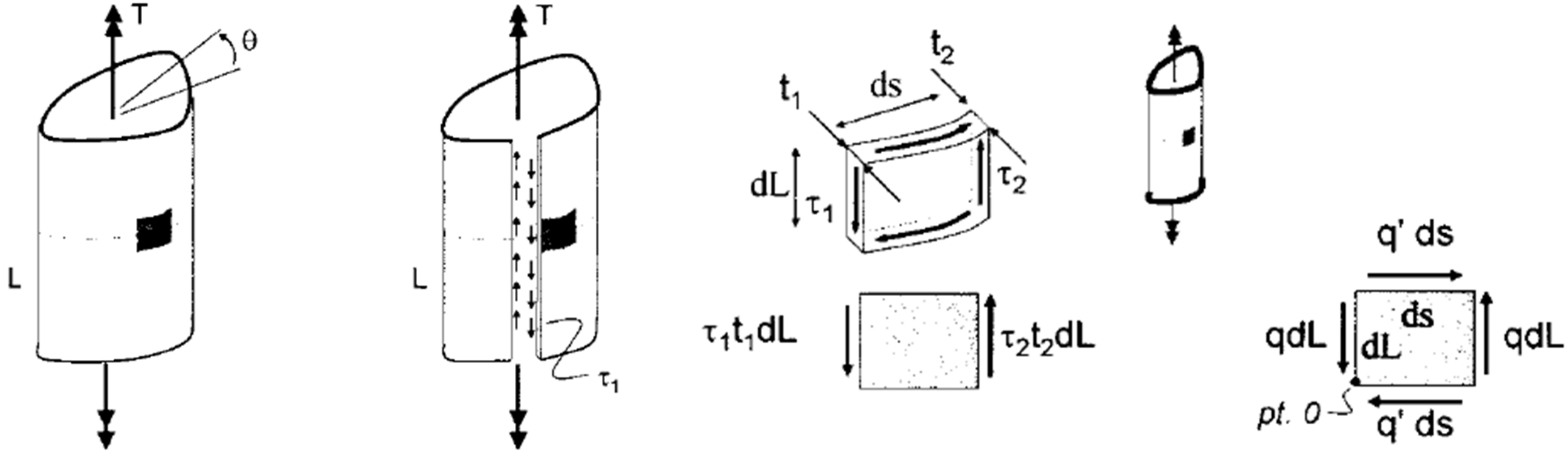
- For solid circular bar $\theta = \frac{TL}{GJ}, \quad \tau = \frac{Tr}{J}$

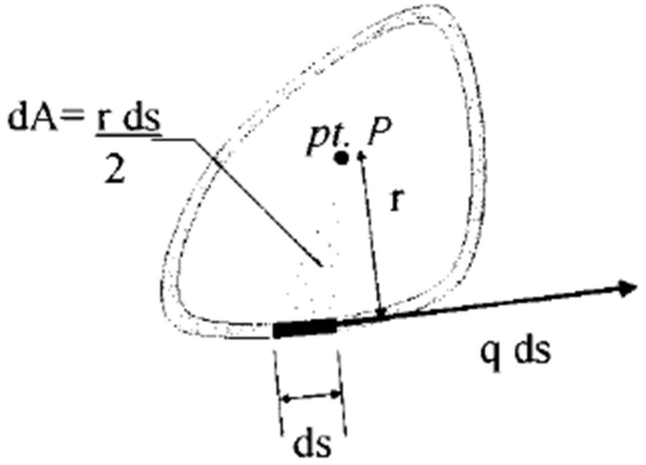
- Torsion of members with closed / open section

	closed section	open section
Angle of rotation	$\theta = \frac{TL}{GJ_{eff}}$	
Shear stress	$\tau = \frac{T}{2At}$	$\tau = \frac{Tt}{J_{eff}}$
Constant thickness	$J_{eff} = \frac{4A^2 t}{S}$	$J_{eff} = \frac{1}{3} t^3 S$
Non-uniform thickness	$J_{eff} = 4A^2 \Big/ \sum_i \frac{S_i}{t_i}$	$J_{eff} = \frac{1}{3} \sum_i {t_i}^3 S_i$

- Warping of open sections under torsion
  - Warping constant

Torsion of Members with Closed Section





$\tau_1 t_1 = \tau_2 t_2 \rightarrow q = \tau t$: shear flow (shearing force per unit length)

$dT = rdF = rqdS$

τ?

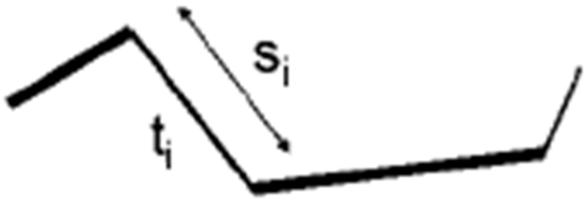
θ?

Torsion of Members with Open Section

Uniform Thickness

$J_{eff}={}^1/_3st^3$

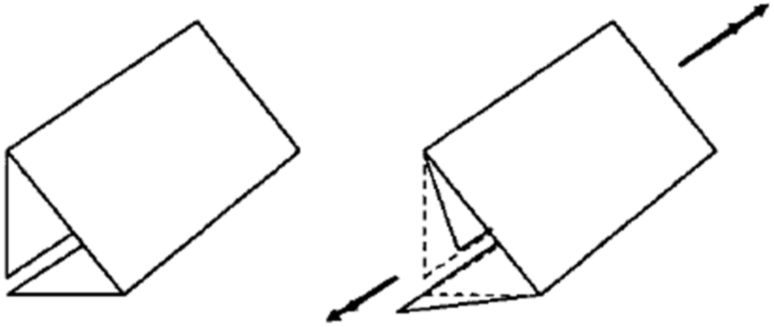
Variable Thickness



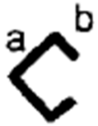
$J_{eff}={}^1/_3\Sigma_is_it_i^3$

Warping of Open Sections under Torsion

- Warping in the longitudinal direction
  - Rigidly hold an end of an open tube and prevent warping, stiffness of the tube ↑

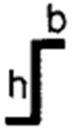


- Warping constant C_w
  - Depends on the geometry of the section
  - $C_w = 0$: section remains planar
  - Large C_w : greater out of plane deformation



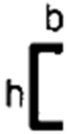
$$Cw = \frac{ta^4b^3}{6}\left(\frac{4a+3b}{2a^3-(a-b)^3}\right)$$

$$Cw = \frac{2tr^5}{3}\left(\alpha^3 - 6\frac{(\sin\alpha - \alpha\cos\alpha)^2}{\alpha - \sin\alpha\cos\alpha}\right)$$



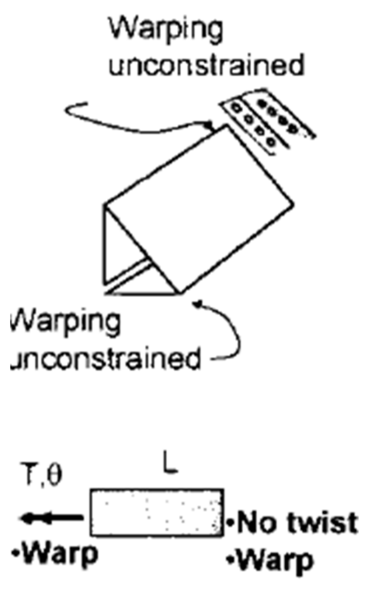
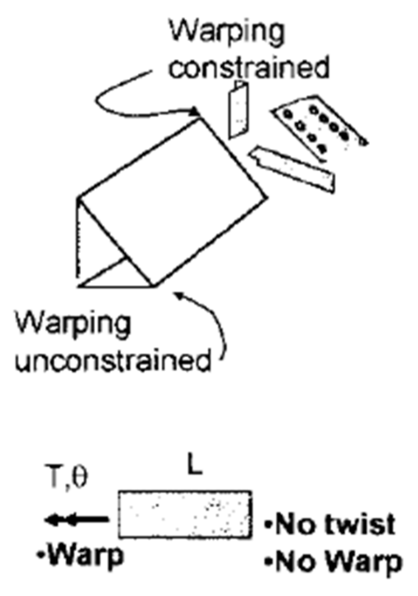
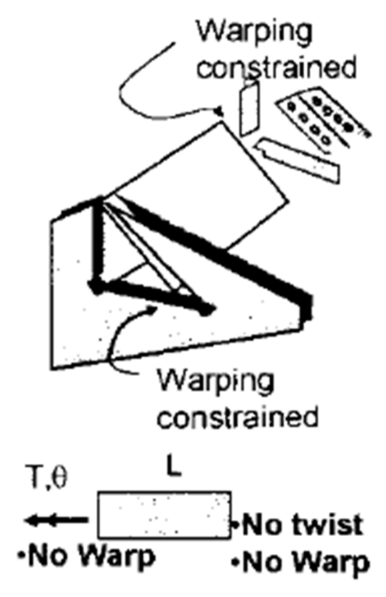
$$Cw = \frac{th^2b^3}{12}\left(\frac{2h+b}{h+2b}\right)$$

$$Cw = 0$$



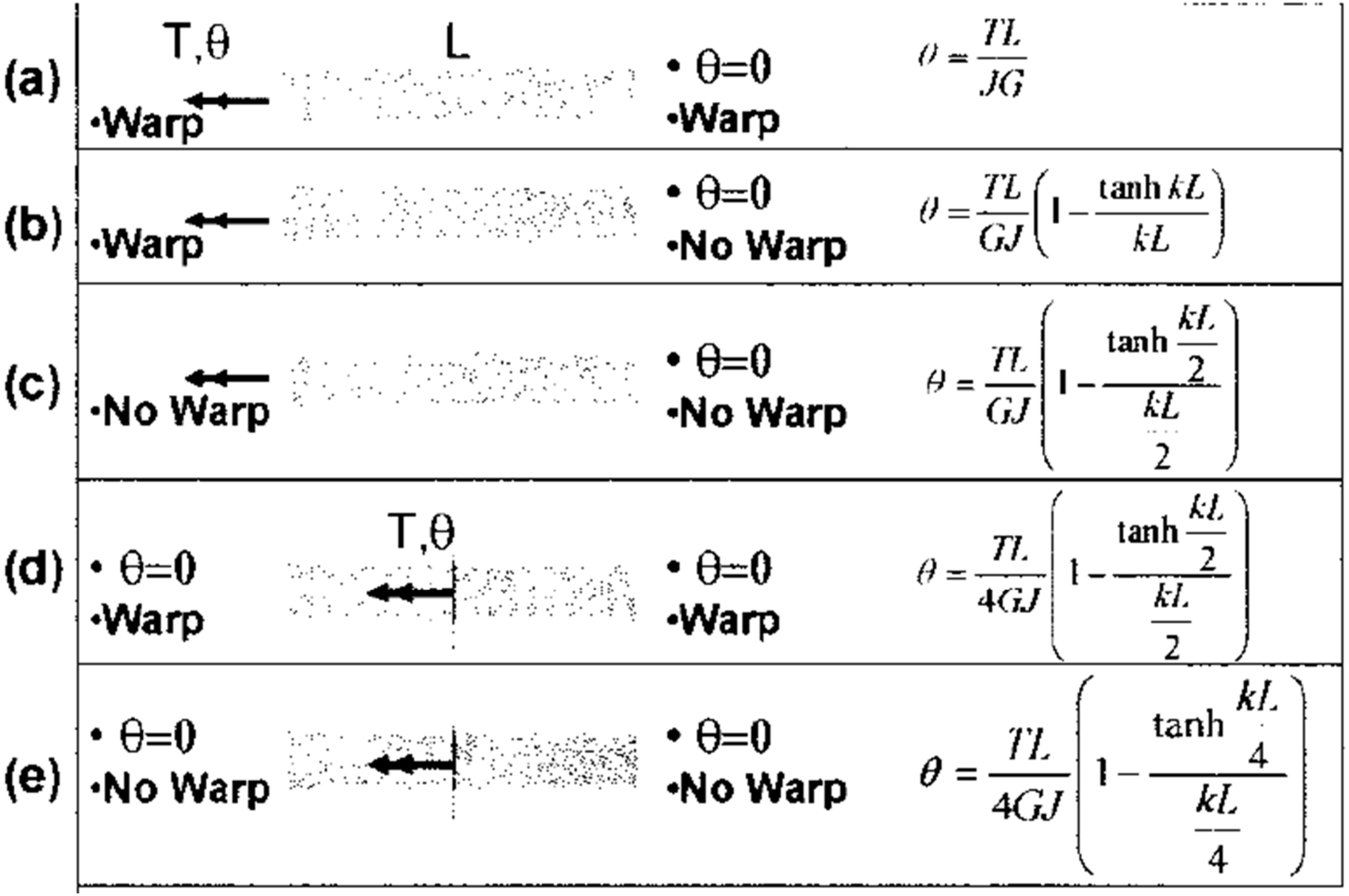
$$Cw = \frac{th^2b^3}{12}\left(\frac{2h+3b}{h+6b}\right)$$

Constrained Warping

		
$\theta = \frac{TL}{GJ}$	$\theta = \frac{TL}{GJ}\left(1 - \frac{\tanh kL}{kL}\right)$	$\theta = \frac{TL}{GJ}\left(1 - \frac{\tanh kL/2}{kL/2}\right)$

$$k = \sqrt{\frac{JG}{C_w E}}$$

Formulae for Twist of Warping Tubes

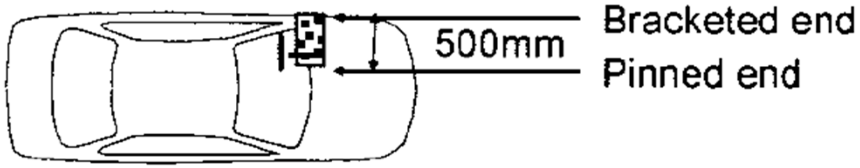


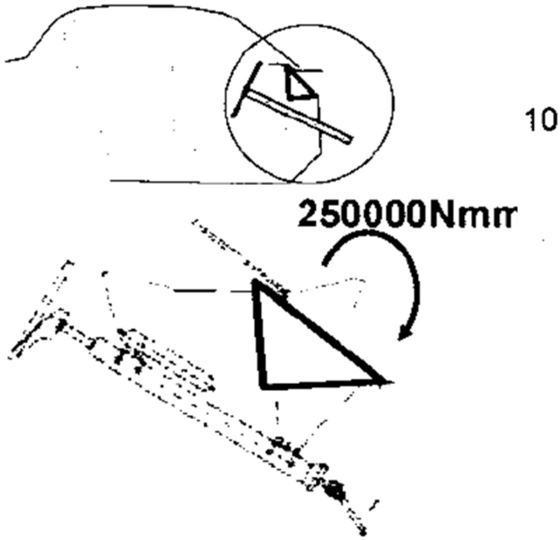
(a)	T,θ •Warp	L	• θ=0 •Warp	$\theta = \frac{TL}{JG}$
(b)	•Warp		• θ=0 •No Warp	$\theta = \frac{TL}{GJ}\left(1 - \frac{\tanh kL}{kL}\right)$
(c)	•No Warp		• θ=0 •No Warp	$\theta = \frac{TL}{GJ}\left(1 - \frac{\tanh \frac{kL}{2}}{\frac{kL}{2}}\right)$
(d)	• θ=0 •Warp	T,θ	• θ=0 •Warp	$\theta = \frac{TL}{4GJ}\left(1 - \frac{\tanh \frac{kL}{2}}{\frac{kL}{2}}\right)$
(e)	• θ=0 •No Warp		• θ=0 •No Warp	$\theta = \frac{TL}{4GJ}\left(1 - \frac{\tanh \frac{kL}{4}}{\frac{kL}{4}}\right)$

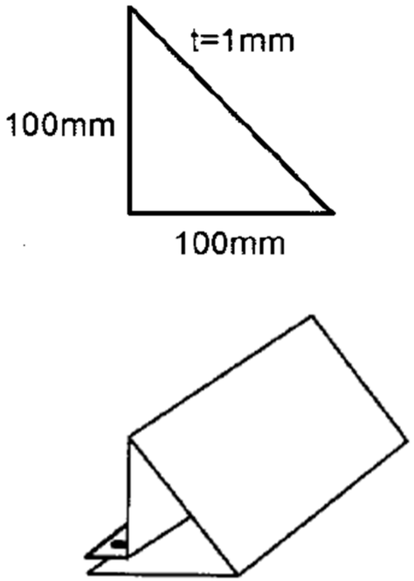
$$k = \sqrt{\frac{JG}{C_w E}}$$

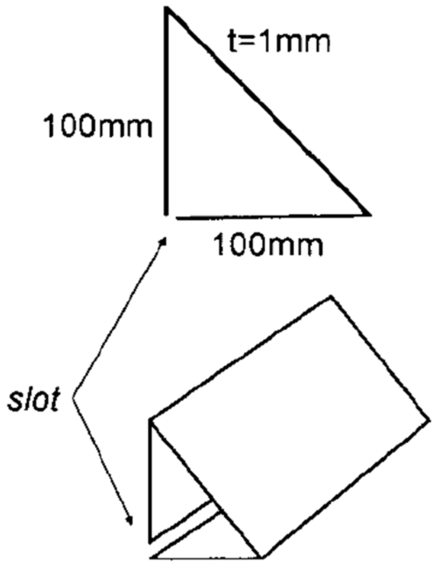
Example: Steering Column Mounting Beam

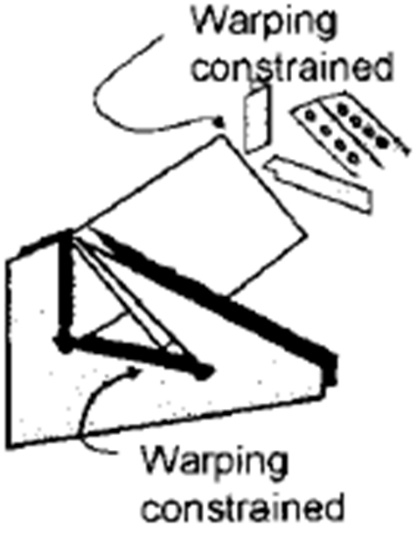
section	closed	open	No warping
Thin-wall torsion constant (mm^4)			
Angle of rotation (rad/degree)			
Shear stress (N/mm^2)			



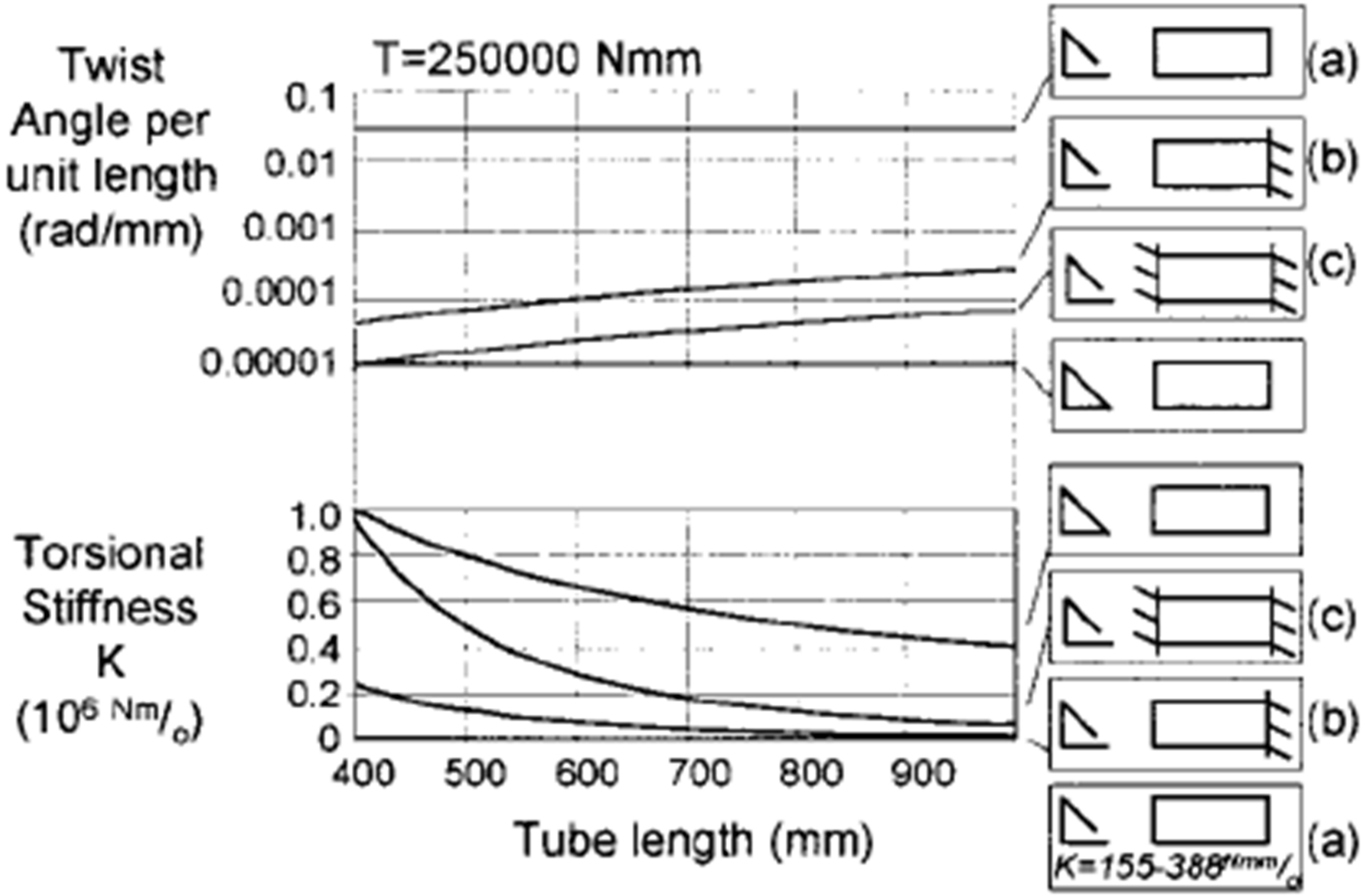






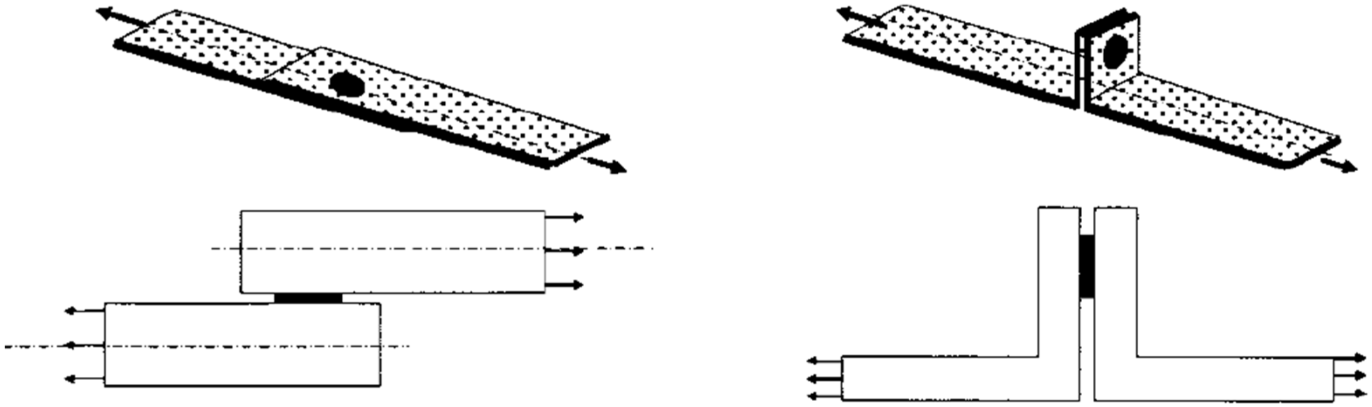


Effect of Beam Length on Angle of Rotation



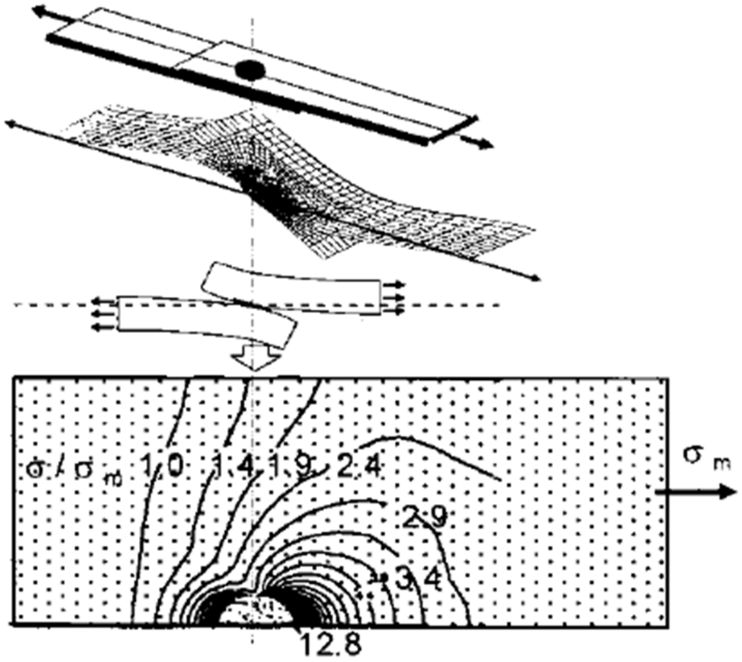
Effect of Spot Welds on Structural Performance

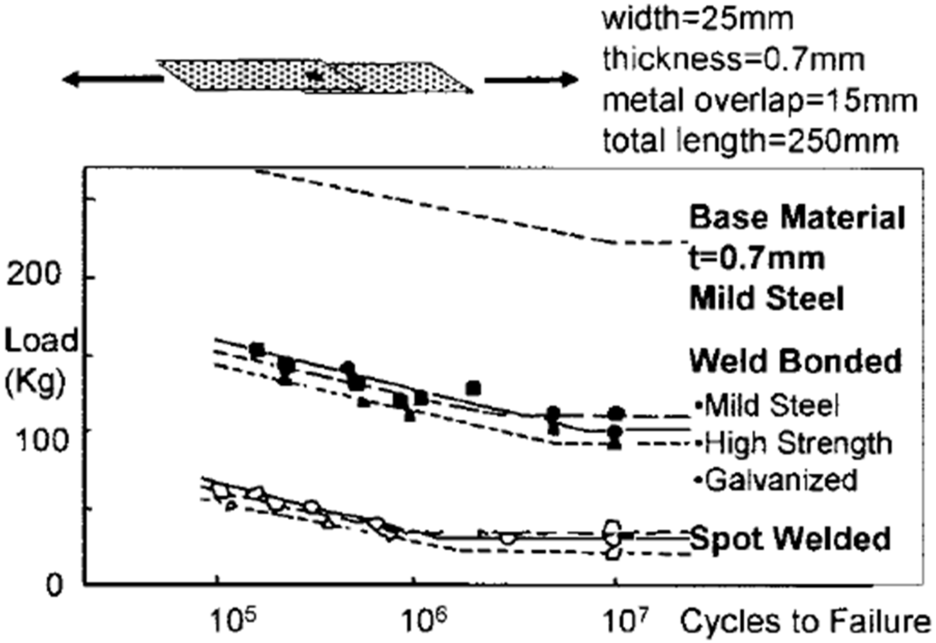
- Body sections
  - Fabrication of several formed element using spot welds
- Addition of shear flexibility in the section during torsion of fabricated sections
  - Tools to predict the degree of shear flexibility
  - Strategies to minimize the flexibility
- Shear vs. Peel loading



Shear Loading

- Create a moment at the weld
- Reduce fatigue limit by a factor of seven
  - Adhesive: more evenly distributed stress → fatigue performance

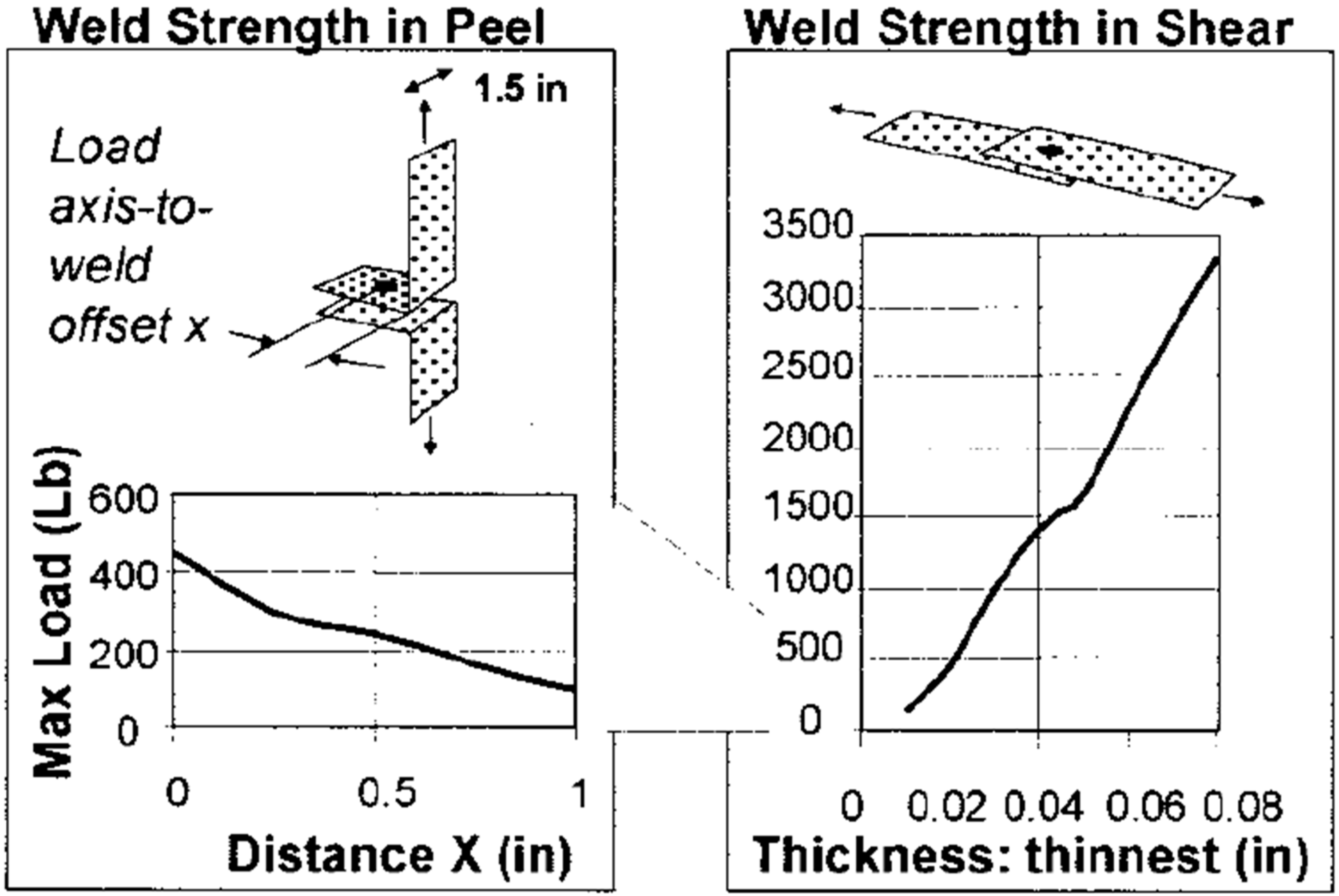




Peel Loading

- Increase the detrimental offset
- Effect of increasing the loading offset beyond the sheet thickness
- Design practice
  - Assumption: tensile load within the plane of the thin wall material
  - Minimize the offset of this tensile load from the weld
  - Use part geometry to put welds into shear loading rather than peel loading

Offset Effect on Spot Welded Joint Strength

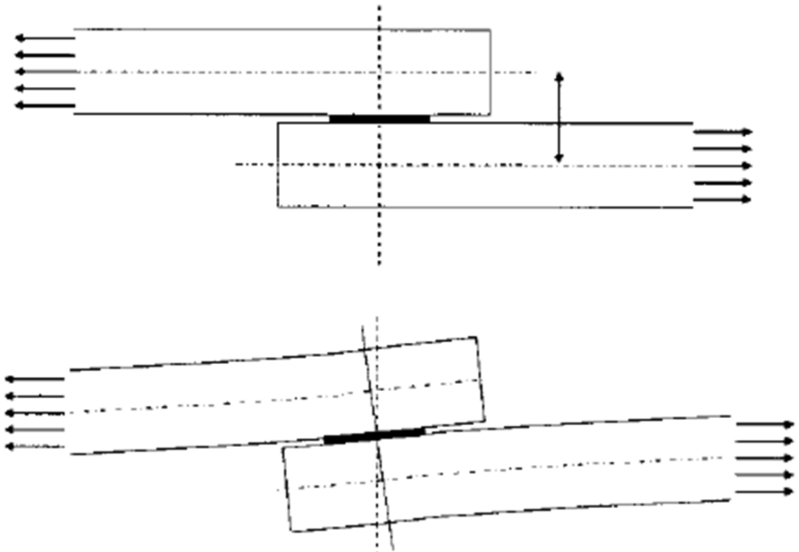


Weld Strength in Peel

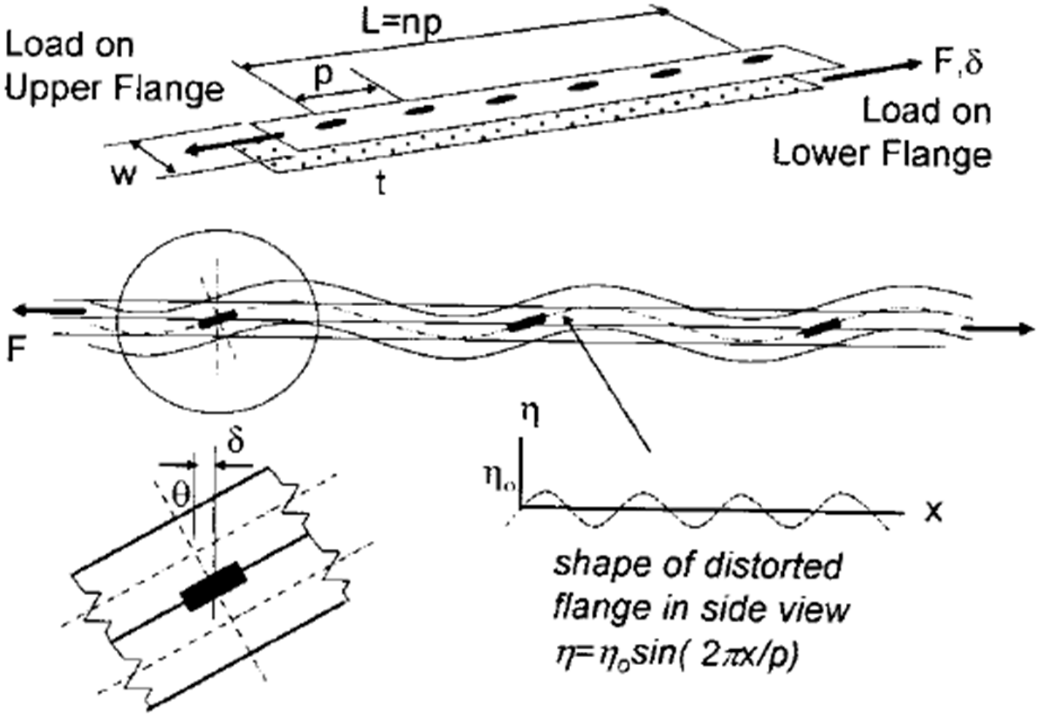
Weld Strength in Shear

Longitudinal Stiffness of a Shear Loaded Weld Flange

- Local deformation → reduce the apparent stiffness of a section
- Distortion under a shear load: rotation with the center at the interface of the weld



Distortion at Spot Weld



Longitudinal Deflection

- Deflected shape of the flange η at each weld
- (work done by an external elastic shearing force through distance δ) = (bending strain energy in the distorted flange)
  - Deflection $\propto$ square of the weld pitch

$$\left.\begin{aligned} work &= \frac{1}{2} F\delta \\ energy &= \int_0^L \frac{1}{2} EI\left(\eta''\right)^2 dx \end{aligned}\right\} \rightarrow \delta = \frac{3p^2}{2E\pi^2 wt} q$$

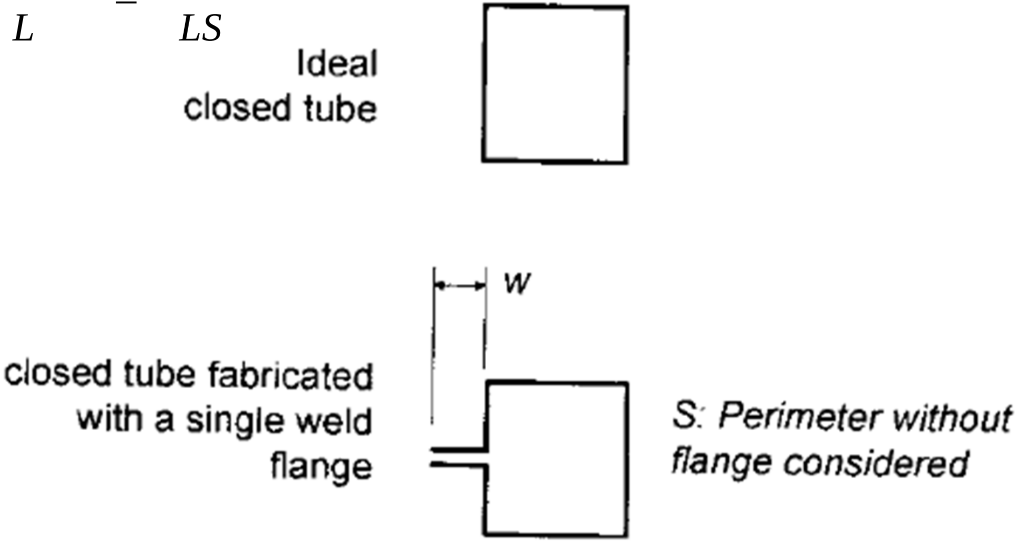
Tube Closed by a Single Spot Weld Flange

- Reduced stiffness in a twisted section by torque T
- (external energy) = (shear strain energy in tube wall) + (strain energy in distorted flange)
  - Estimate of the reduced stiffness in a twisted section when a single spot welded flange is present

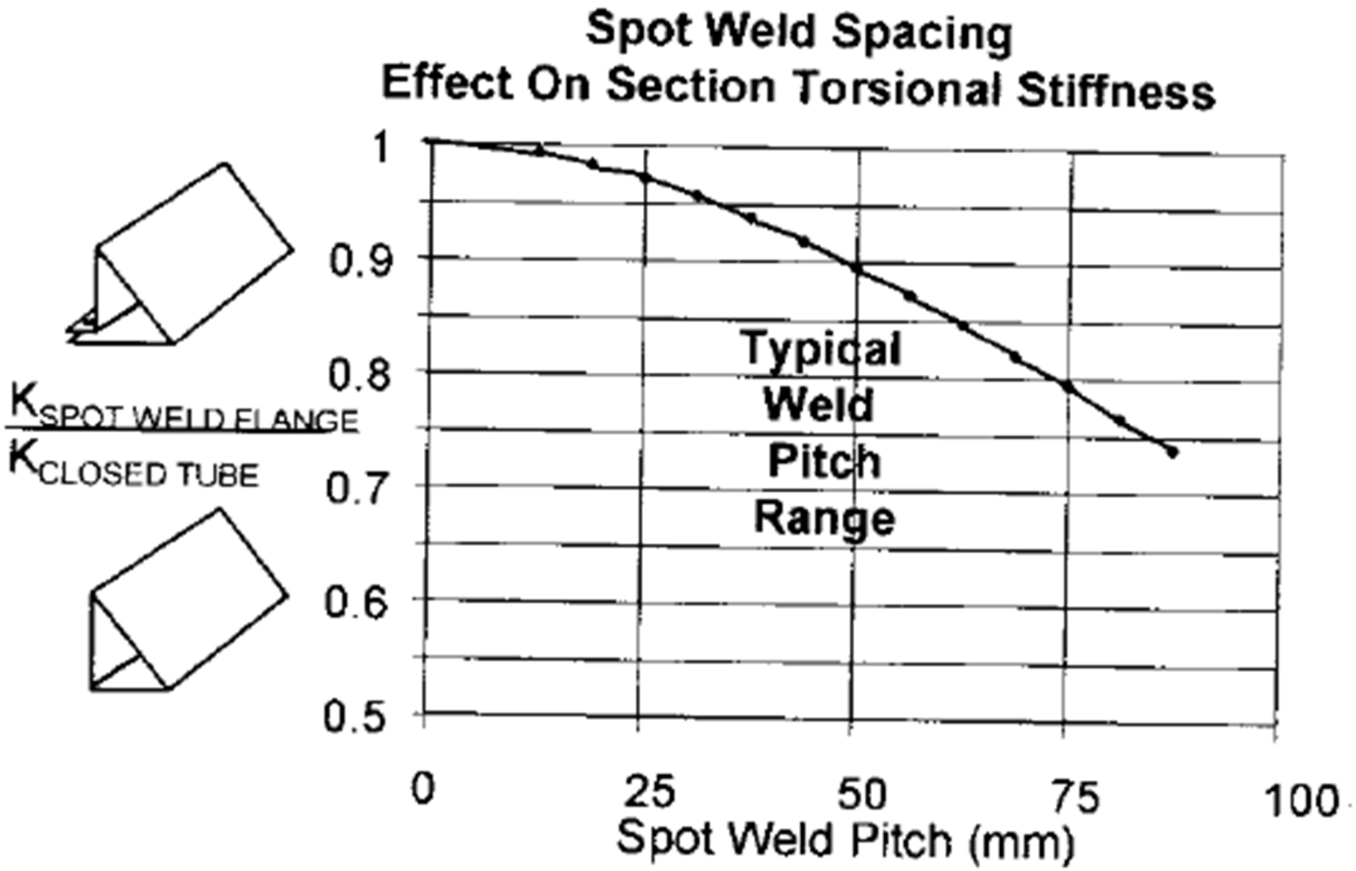
$$\left(\text{stiffness of closed tube w/o weld flange}\right) = \frac{GJ}{L} = \frac{G\left(\frac{4A^2t}{S}\right)}{L} = \frac{4GA^2t}{LS}$$

$$\begin{cases} work = \frac{1}{2}T\theta \\ energy = \frac{SL}{2Gt}q^2 + \frac{1}{2}F\delta \end{cases}$$

$$\rightarrow \frac{T}{\theta} = \frac{\left(\text{stiffness of closed tube w/o weld flange}\right)}{\left[1 + \frac{3}{4\pi^2\left(1+\nu\right)}\frac{p^2}{wS}\right]}$$



Spot Weld Spacing Effect



w (weld flange) = 8 mm, t = 1 mm
→ 40 mm < p < 60 mm