3.4 Thin Wall Beam Section Design

- Why are automotive sections so often thin walled?
 - Steel cantilever beam with a tip load
 - Cross section area is fixed: maximize strength and stiffness
 - Existence of new failure mode



Elastic Plate Buckling

- General behavior of a compressively loaded plate
 - Bifurcate into the buckled shape if the plate is sufficiently thin
 - Compressive stress: plate width to thickness ratio
- Section design: trade-off
 - Thick walled section: higher strength but lower stiffness performance
 - Thin walled section: higher stiffness but lower strength performance due to plate buckling
 - Selection of the best section proportion: relationship of strength requirement to stiffness requirement

Plate Buckling Stress



Example: Rocker Sizing in Convertible

Determine b/t to minimize rocker mass while meeting requirements



Vehicle Structure

Mass Savings of Thin Walled Section



Dominant Structural Requirement

- Many structural elements in automobile body design are dominated by stiffness requirements
 - Thin wall sections: mass effective
 - Failure mode of buckling



Section Proportion

- Mass effective means to design for stiffness performance
 - Thin wall sections



Reacting loads in a crash: roof crush, side impact, maintaining cabin integrity

Major subsystem attachment: suspension, powertrain

Overall stiffness of the body

3.5 Buckling of Thin Walled Members

- Significant difference between automotive sections and others: failure mode by plate buckling
 - Plate buckling stress in section elements
 - Strength of a buckled section
- Plate buckling
- Identifying plate boundary conditions in practice
- Post buckling behavior of plates
- Effective width
- Thin walled section failure criteria
- Techniques to inhibit buckling

Plate Buckling (1)

- Static equilibrium of the element under loads
- Compatibility of deformations within the plate
- Material stress-strain relationship



bending moments $M_x, M_y \to \sigma_x, \sigma_y$ twisting moment $M_{xy} \to \tau_{xy}$ shear loads $Q_x, Q_y \to \tau_{zx}, \tau_{yz}$ normal load, compressive stress q, f_x plate bending stiffness $D = \frac{Et^3}{12(1-v^2)}$

Plate Buckling (2)

$$\begin{split} \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{f_x t}{D} \frac{\partial^2 w}{\partial x^2} + \frac{q}{D} &= 0 \\ \begin{cases} M_x = -D \left(\frac{\partial^2 w}{\partial x^2} + v \frac{\partial^2 w}{\partial y^2} \right) \\ M_y = -D \left(v \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \\ M_{xy} = -D \left(1 - v \right) \frac{\partial^2 w}{\partial x \partial y} \end{split}$$

reasonable guess at the deflected shape

$$w(x, y) = A_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \text{ where } m, n = 1, 2, \dots$$

$$\left[\pi^4 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2 - \frac{f_x t}{D} \frac{m^2 \pi^2}{a^2}\right] A_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) = 0$$

$$\rightarrow f_x = \frac{D\pi^2}{tb^2} \left[m\left(\frac{b}{a}\right) + \frac{n^2}{m}\left(\frac{a}{b}\right)\right]^2$$

simply supported plate

Note that lowest Buckling load occurs when n=1 or:



Buckling Constant for Various B.C.



Compressive Stress in a Shear Panel



Identifying Plate Boundary Conditions



Example of Plate Edge Conditions



Post Buckling Behavior

- Beam
 - Once buckled, a beam loses the ability to carry increased load
- Plate
 - Even after buckling, a plate can carry increased load



Effective Width (1)



Effective Width (2)

– Width of an imaginary effective plate which has a uniform stress of σ_{s} across it



alternative empirical relationships for effective width

$$w = \begin{cases} 0.894b \sqrt{\frac{\sigma_{cr}}{\sigma_s}} \\ b \sqrt{\frac{\sigma_{cr}}{\sigma_s}} \left(1 - 0.22 \sqrt{\frac{\sigma_{cr}}{\sigma_s}} \right) \end{cases}$$

Vehicle Structure

Example

- Consider the 100 mm square thin wall steel beam of thickness 0.86 mm is loaded in compression. We determined the critical buckling stress for each side plate to be 55.35 N/mm² and the resulting compressive load to cause plate buckling to be 19040 N. What load will cause a maximum stress of 111 N/mm² in each plate?
- A 100 mm square thin walled steel beam of thickness 0.86 mm is loaded by a bending moment in the +x direction using the right hand rule. Under this moment, the maximum compressive stress in the top plate is 111 N/mm². What is the effective moment of inertia for the section under this moment loading?



Thin Walled Section Failure Criteria

- Ultimate failure load for the thin walled plate (P_{ult})



Techniques to Inhibit Buckling (1)

$$\sigma_{cr} = \underbrace{k}_{cr} \frac{E\pi^2}{12\left(1-\nu^2\right)} \frac{1}{\left(\frac{b}{t}\right)^2}$$

- Increase the critical plate buckling stress
 - Boundary conditions: flange curls, flanged holes
 - Normal stiffness of the plate: material, curved elements, foam filling
 - Width-to-thickness ratio: reducing width with beads and added edges (while maintaining moment of inertia)

Techniques to Inhibit Buckling (2)

- Boundary conditions: flange curls, flanged holes



Normal stiffness of the plate: material, curved elements, foam filling



Techniques to Inhibit Buckling (3)

 Width-to-thickness ratio: reducing width with beads and added edges (while maintaining moment of inertia)



Example: Z section

- Part of a bumper reaction structure
- Calculate the ultimate compressive load
 - For the section (a)
 - For the section (b) with two buckling inhibiting techniques: a flange curl and a central bead on the web
 - What if high strength steel (σ_{Y} = 650 N/mm²) is replaced?



3.6 Automotive Body Panel: Plate/Membrane

- Flat or curved surface with thin thickness
 - Bending stiffness: quite low
 - In-plane stiffness: quite high
 - Highly curved panel: stiffness to out-of-plane loads
- Type of load acting on
 - Normal loading of curved panels
 - In-plane loading of flat or curved panels



Curved Panel with Normal Loading

- Exterior panels
 - Influenced by overall styling
 - Structural performance is not the shape defining function
 - Reaction to normal point loading
 - Stiffness
 - Critical oil-canning load
 - Dent resistance
 - Solidness: pushing with a thumb (K, P_{cr})
- AISI Automotive Steel Design Manual
 - Simply supported boundary conditions
 - Combination of analytical and empirical considerations



Normal Stiffness of Panels

• Theoretical stiffness of a spherical shape under a concentrated load $\left(\frac{L_1^2}{R_1}\right) + \left(\frac{L_2^2}{R_2}\right)$

$$K = \frac{CEt^2}{R\sqrt{1-\nu^2}} \quad \text{where curvature } \frac{1}{R} = \frac{\left(\frac{1}{R_1}\right)^+ \left(\frac{1}{R_2}\right)^2}{2L_1L_2}$$

$$C:$$
 constant

- *t* : panel thickness
- R: spherical radius
- L_1, L_2 : rectangluar panel dimensions
- R_1, R_2 : panel radii of curvature in orthogonal directions

Theoretical shell stiffness

$$K = 1.466 \frac{\pi^2 E t^2}{\sqrt{1 - v^2}} \frac{H_c}{L_1 L_2} \text{ for } 20 \le \frac{H_c}{t} \le 60 \text{ where crown height } H_c = \left(\frac{L_1^2}{8R_1}\right) + \left(\frac{L_2^2}{8R_2}\right)$$

valid over the range $\frac{R_1}{L_1}$ and $\frac{R_2}{L_2} > 2$, $\frac{1}{3} < \frac{L_2}{L_1} < 3$, $L_1 L_2 < 0.774 m^2$

Oil-can Load

- Load where a hard snap over occurs
- Curvature inversion
 - Soft: surface stays in contact with the load applicator
 - Hard: surface snaps over and looses contact with the load applicator

$$P_{cr} = \frac{CR_{cr}\pi^{2}Et^{4}}{L_{1}L_{2}(1-\nu^{2})} \text{ where } \begin{cases} R_{cr} = 45.929 - 34.183\lambda + 6.397\lambda^{2} \\ \lambda = 0.5\sqrt{\frac{L_{1}L_{2}}{t}\sqrt{\frac{12(1-\nu^{2})}{R_{1}R_{2}}}} \\ C = 0.645 - 7.75 \times 10^{-7}L_{1}L_{2} \end{cases}$$
valid over the range $\frac{R_{1}}{L_{1}}$ and $\frac{R_{2}}{L_{2}} > 2$, $\frac{1}{3} < \frac{L_{2}}{L_{1}} < 3$, $L_{1}L_{2} < 0.774m^{2}$

Dent Resistance

- Kinetic energy of a dart, directed normal to a surface which leaves a permanent dent in the panel
 - W: minimum energy to dent the surface (0.025 mm permanent deformation in the panel)
 - Yield at a dynamic strain rate (10~100/sec)
 - Static tensile test strain rate (0.001/sec)

$$W = 56.8 \frac{\left(\sigma_{yd} t^2\right)^2}{K}$$

$$\begin{cases} K: \text{ panel normal stiffness (theoretical shell stiffness)} \\ \sigma_{yd}: \text{ yield strength at a dynamic strain rate } (298 \text{ N/mm}^2) \end{cases}$$

Example: Automobile Hood Outer Panel

- Simply supported boundary conditions
- Dynamic yield stress: σ_{yd} = 298 N/mm²
- Panel stiffness
- Oil-can load
- Denting energy



Membrane Shaped Panels

- Structural elements of the underbody
 - Floor pan, motor compartment sides, dash, wheel house inner panel
 - Shaped by structural requirements
 - Given a set of loads applied to a panel, what is the best panel shape to react those loads? (stiff, strong, light)
 - Very low panel stiffness in bending: very small thickness
- Given fully stressed quality of a membrane, design the panel shape only membrane loading is present
 - Define the loading
 - Substitute the loading into the force balance
 - Geometric relationships
 - Differential equation: y = f(x)

Axially Symmetrical Membrane (1)

$$p(ds)(ds') - 2\sigma_{\tan}(tds)\frac{d\theta}{2} + 2\sigma_{\log}(tds')\frac{d\phi}{2} = 0$$
$$\xrightarrow{ds' = R_T d\theta, ds = R_L d\phi} \frac{p}{t} = \frac{\sigma_{\tan}}{R_T} + \frac{\sigma_{\log}}{R_L}$$



Vehicle Structure

Axially Symmetrical Membrane (2)



Example: loaded ring on a circular membrane

- Idealization of a suspension attachment point
- What is the shape of a panel which will react this loading with only membrane stress?
 - Horn-shaped surface



Membrane Analogy

- Uniformly stressed membrane panel: light, stiff panel
- Visualizing panel shape
 - Thin rubber membrane or soap film stretched under the action of the loads and constraints
 - Predominant internal loads: tangent to the surface (weld flanges)

	Idealized Membrane	Actual Sheet Metal
1	Can accept either tensile or compressive stress	Buckling can occur under compressive stress
2	All stresses lies in plane of surface	Bending stresses can be reacted
3	Ideal boundary conditions: boundary loads are tangent to surface	Boundary conditions do not agree completely with theoretical requirements

- 1. seek proper design to generate tensile loads only or assure the buckling stress will not be exceeded
- 2. highly beneficial property of real panels
- 3. require bending moments to be generated in the reaction structure
- Highly efficient means to react both distributed and point loads

Example: Rear Load Floor Panel

- Vibration of the rear load floor panel: source of noise
- High resonance frequency required: high normal stiffness
- Inertia loads during vibration → uniformly distributed normal load across the panel
- Heated plastic film constrained at the boundary and loaded by its weight: react tensile loads and behave as membrane
 → cooling



Flow of Body Strength and Stiffness Requirements

- Relationship between loads applied to the body system and the resulting loading on a particular section
- Flow down structural requirements from the global body level to the individual section

