

# Ch.4 Design for Body Bending

---

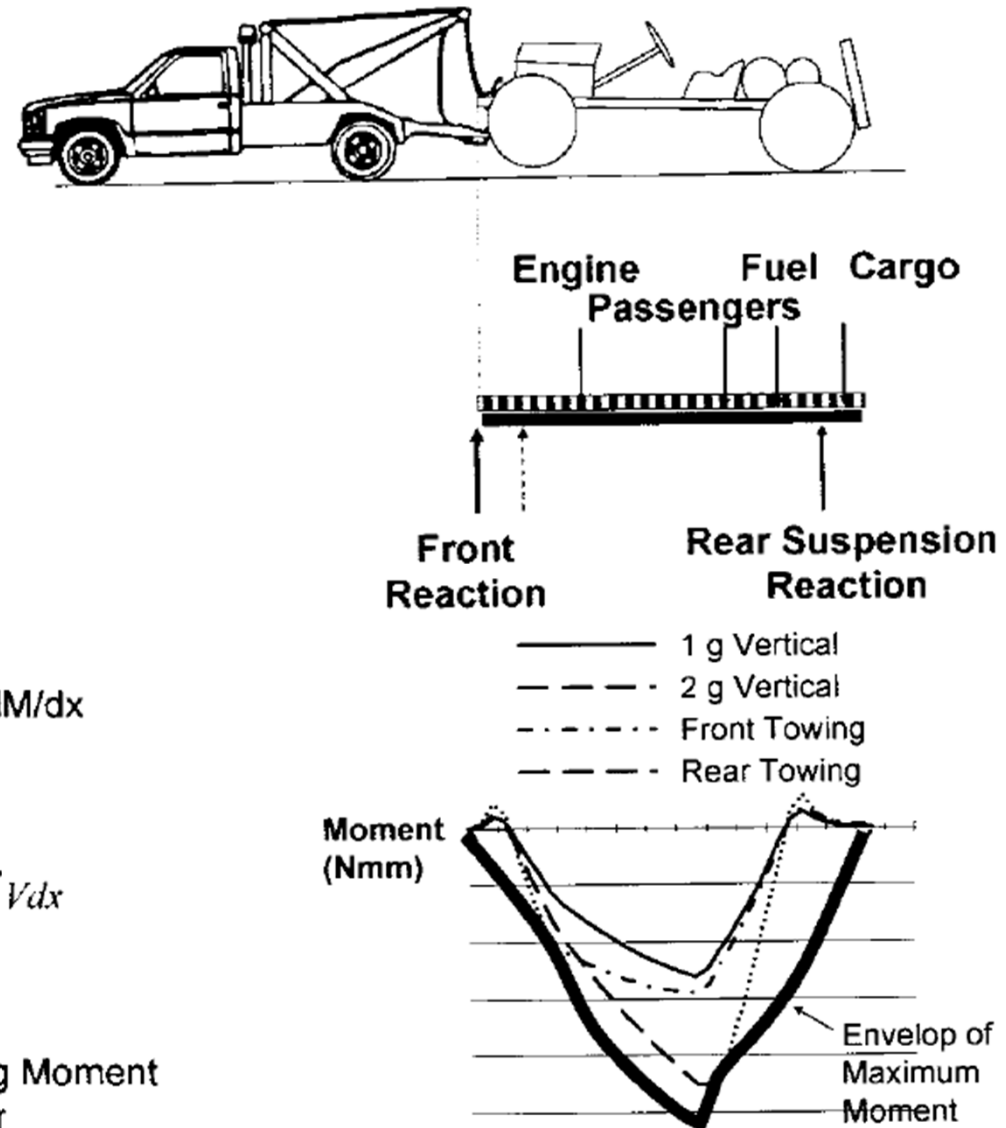
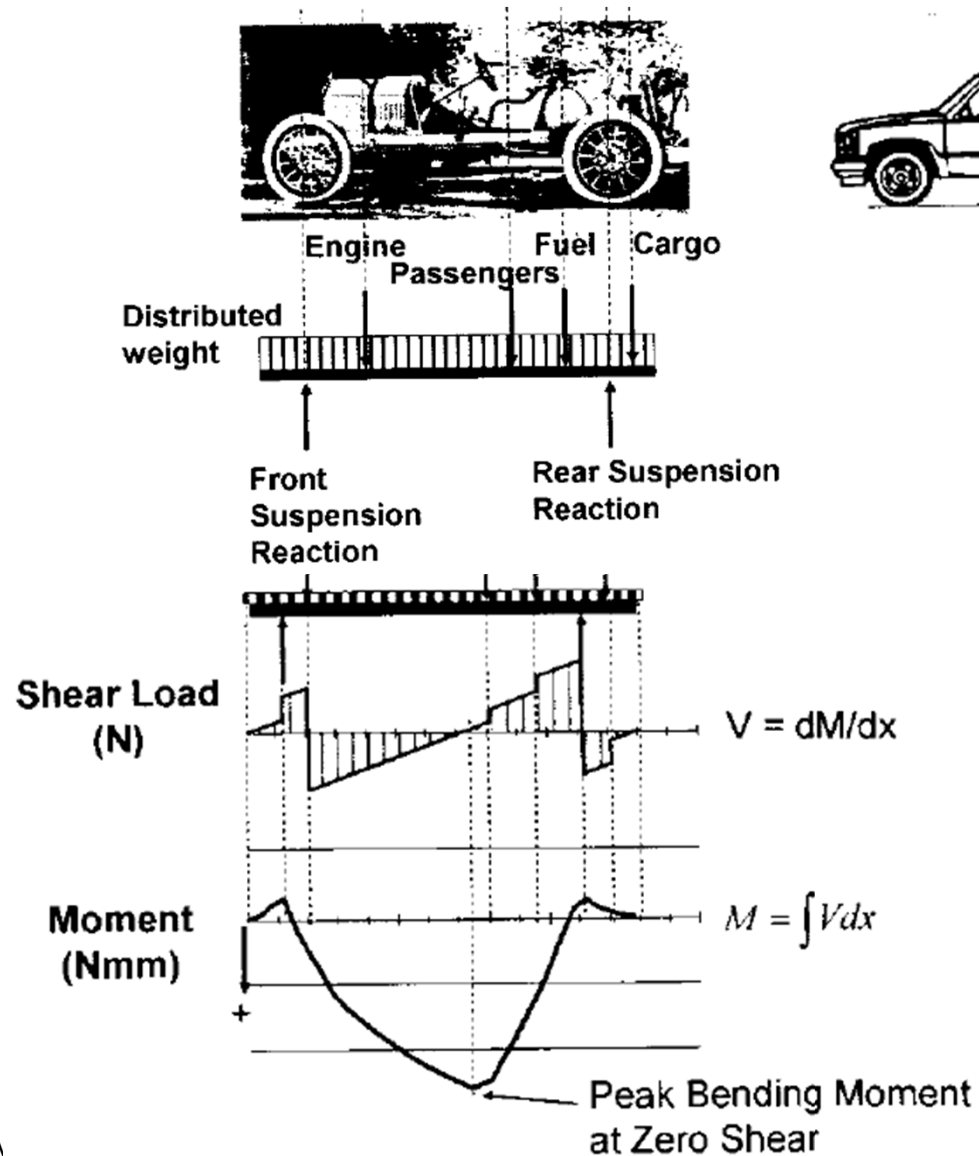
- Consider the overall body structure supported and loaded similar to a single beam
- Symmetrical supporting points and loads applied to the vehicle center line
- Body bending requirements
  - For midsize vehicle:  $K = 7000 \text{ N/mm}$ ,  $F = 6680 \text{ N}$
- Internal loads during global bending: load path analysis
- Analysis of body bending stiffness
- Principles of good joint design

# 4.1 Body Bending Strength Requirement

---

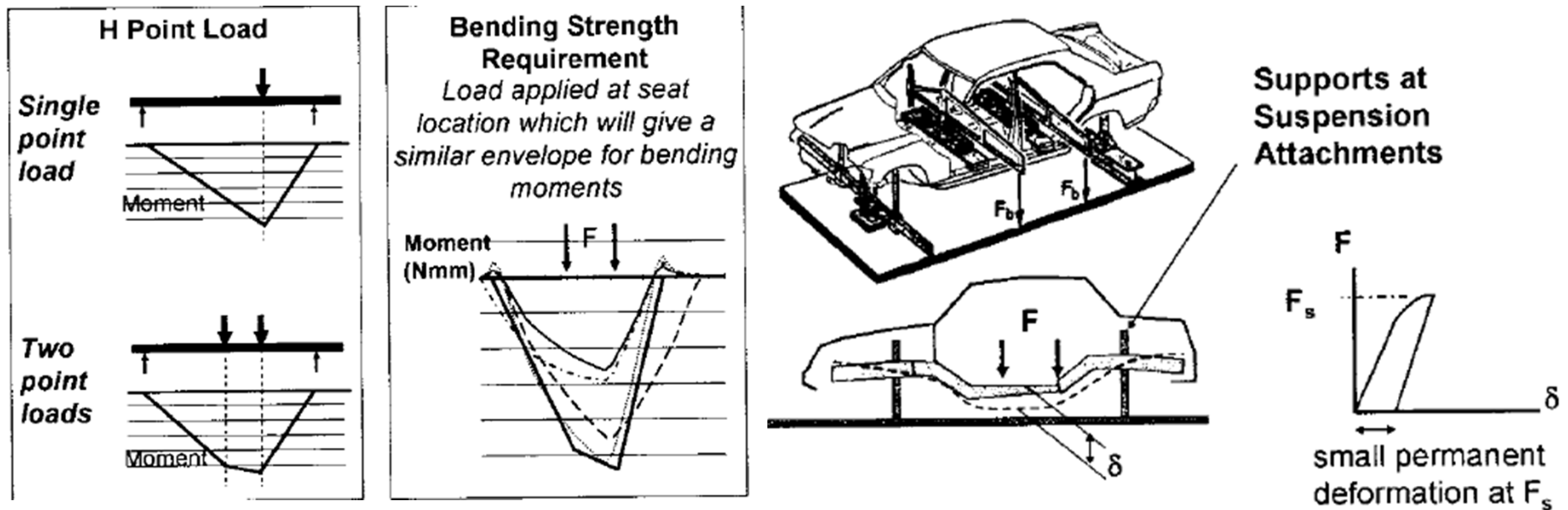
- Most basic structure requirement
  - Locate and retain the vehicle subsystems in correct positions
  - Support powertrain, occupants, suspension, etc.
- Envelop of maximum bending moments: NO failure
  - Static weight loading
  - Dynamic loading: inertia loads of the subsystem mass ( $\times 2g$ )
  - Front/Rear jacking or towing: one support point is moved to an end of the vehicle

# Body Loaded by Subsystem Weight



# H point Bending Test

- Consider body supported at the suspension points and loaded by just one or two loads at the seating position (H point)
- Superimpose the diagram over bending moments for the vehicle
- Vary the magnitude of the H point load
- $F_s$ : bending strength for the body

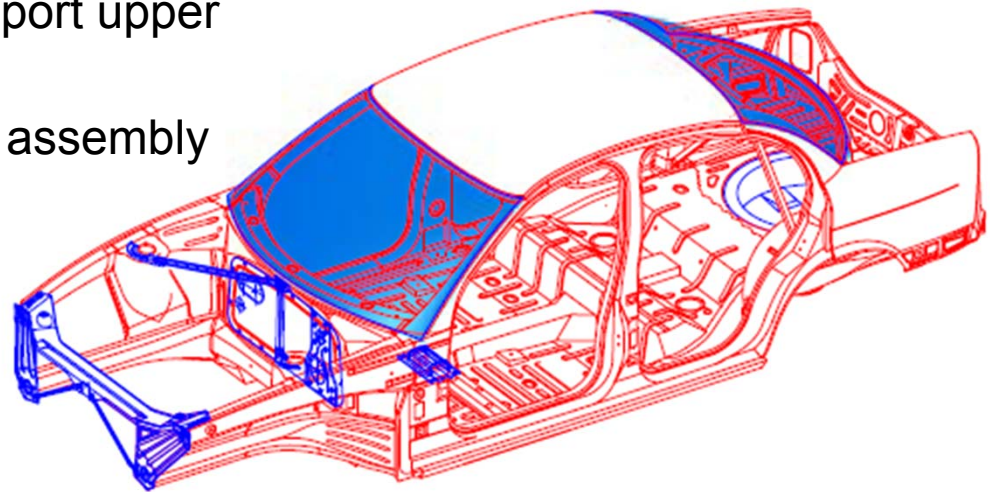




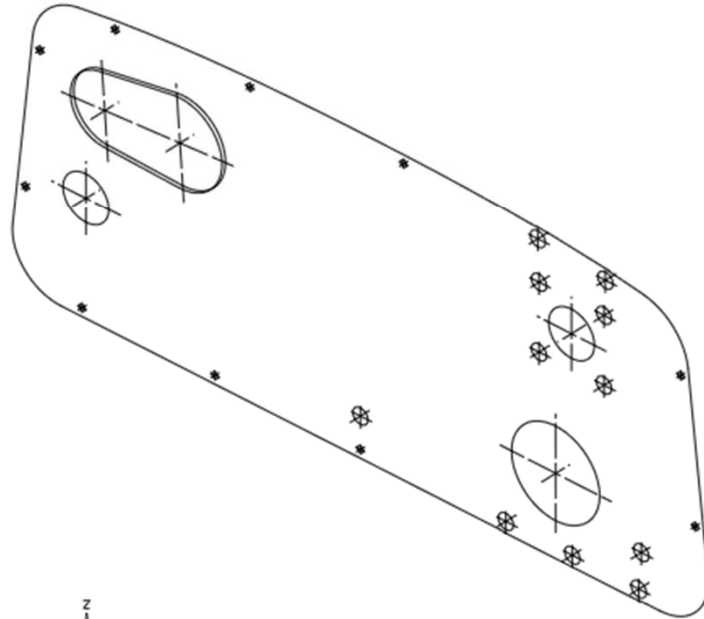
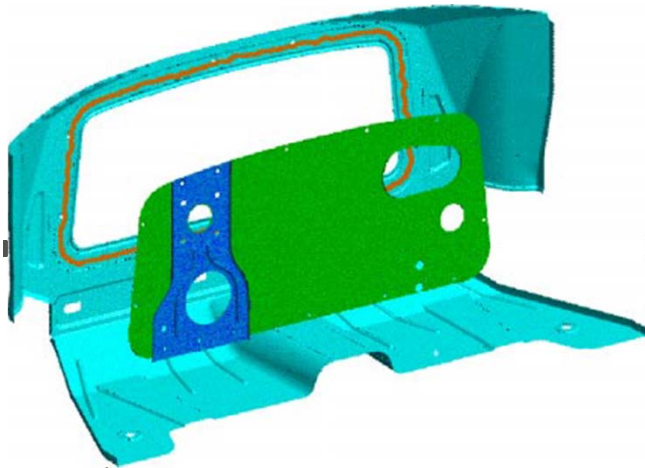
# Test: Configuration

---

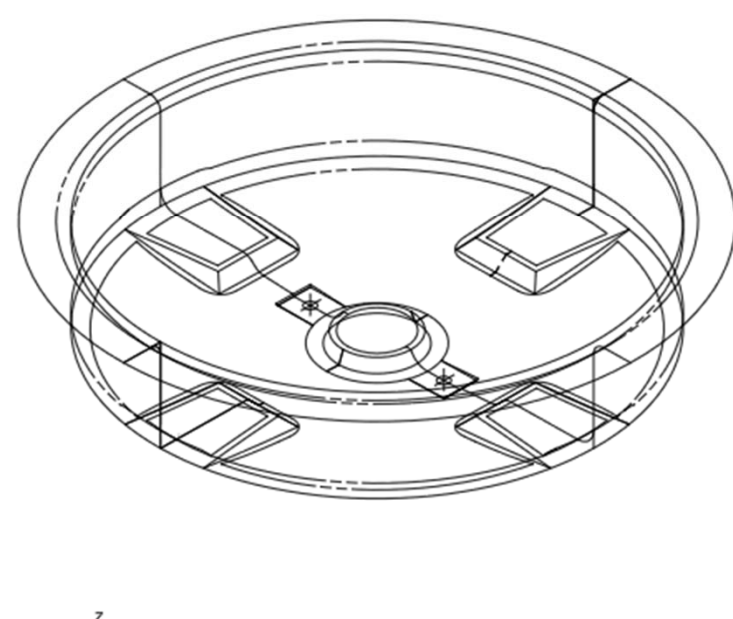
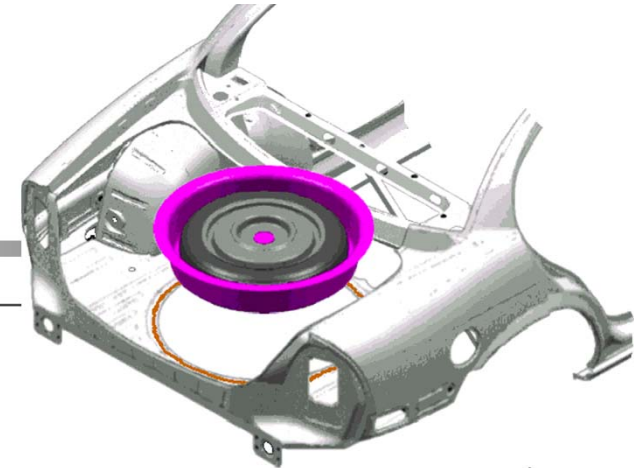
- Welded body structure
  - Bonded windshield and back light (aluminum panels)
  - Bonded and bolted panel dash insert
  - Bonded panel spare tire tub
  - Bolted reinforcement panel dash brake booster
  - Bolted braces radiator
  - Bolted reinforcement radiator rail closeout RH/LH
  - Bolted reinforcement radiator support upper
  - Bolted tunnel bridge lower/upper
  - Bolted brace cowl to shock tower assembly
- Holding
    - Front: at panel skirt RH/LH
    - Rear: at plate rear spring upper
  - Measurement
    - 12 stadia rods along the front rails, rockers, rear rails



# Parts (1)

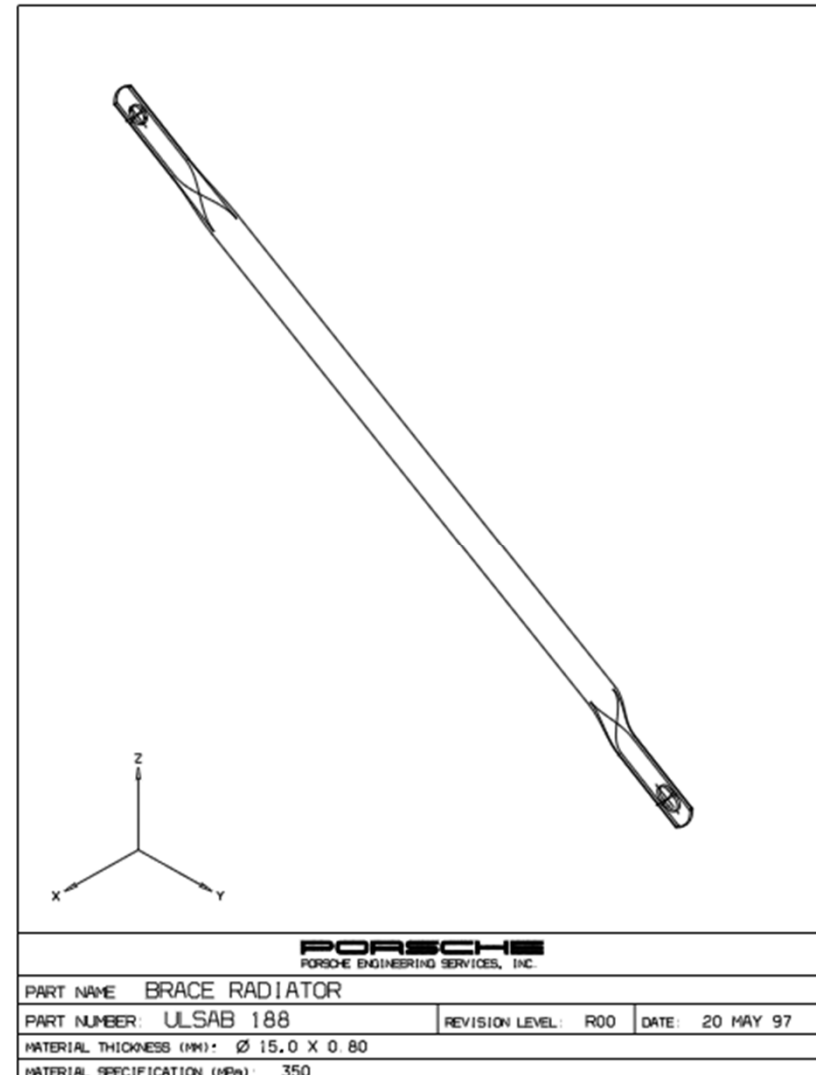
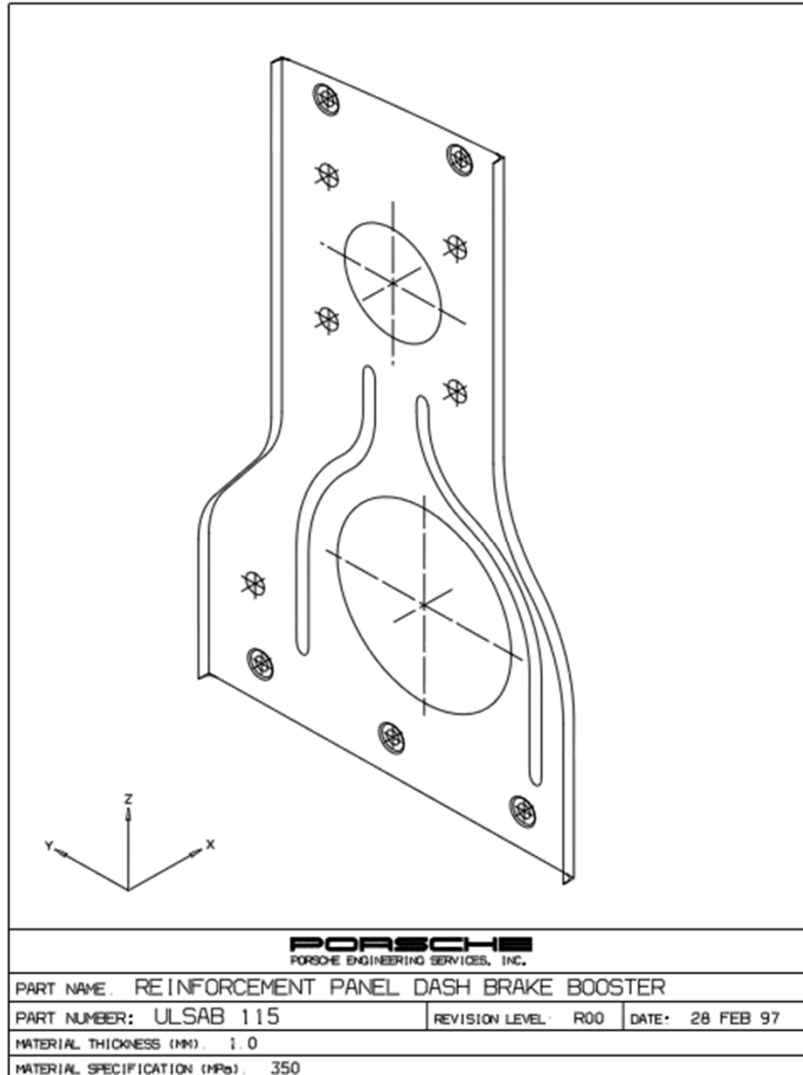


<div><div>PORSCHE</div><div>PORSCHE ENGINEERING SERVICES, INC.</div></div>			
PART NAME: PANEL DASH INSERT			
PART NUMBER: ULSAB 022		REVISION LEVEL: R02	DATE: 30 AUG 97
MATERIAL THICKNESS (MM): 0.95 - SKIN = 0.12 X 2 CORE = 0.67 COATING = 0.04			
MATERIAL SPECIFICATION (MPa): SANDWICH			

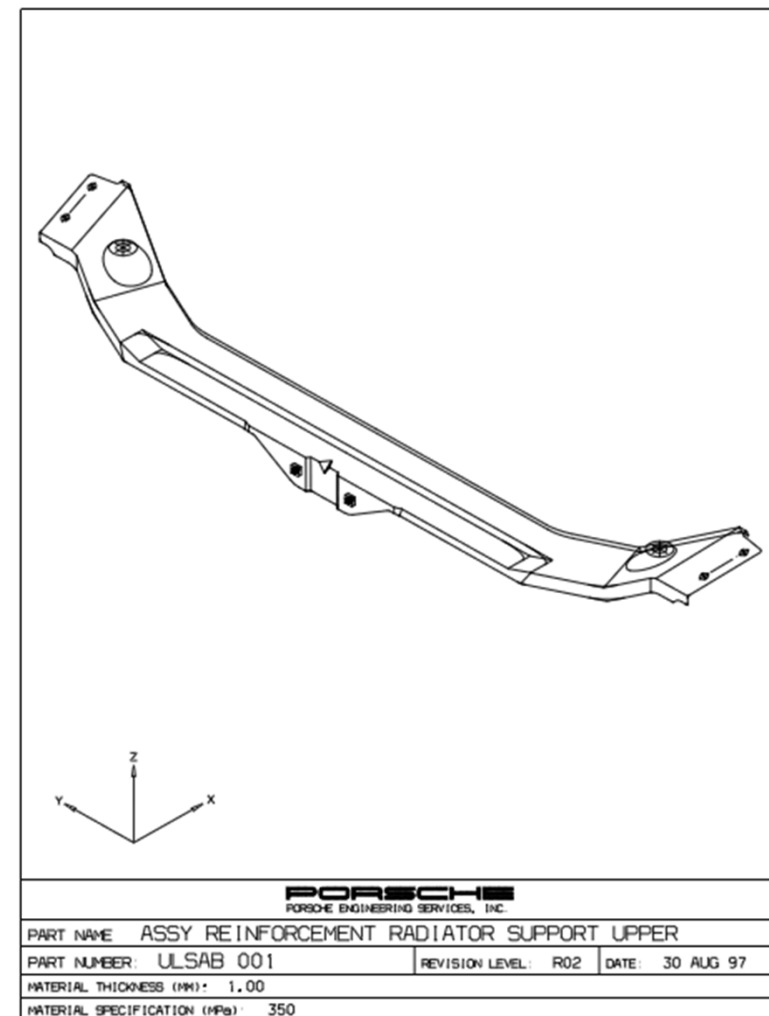
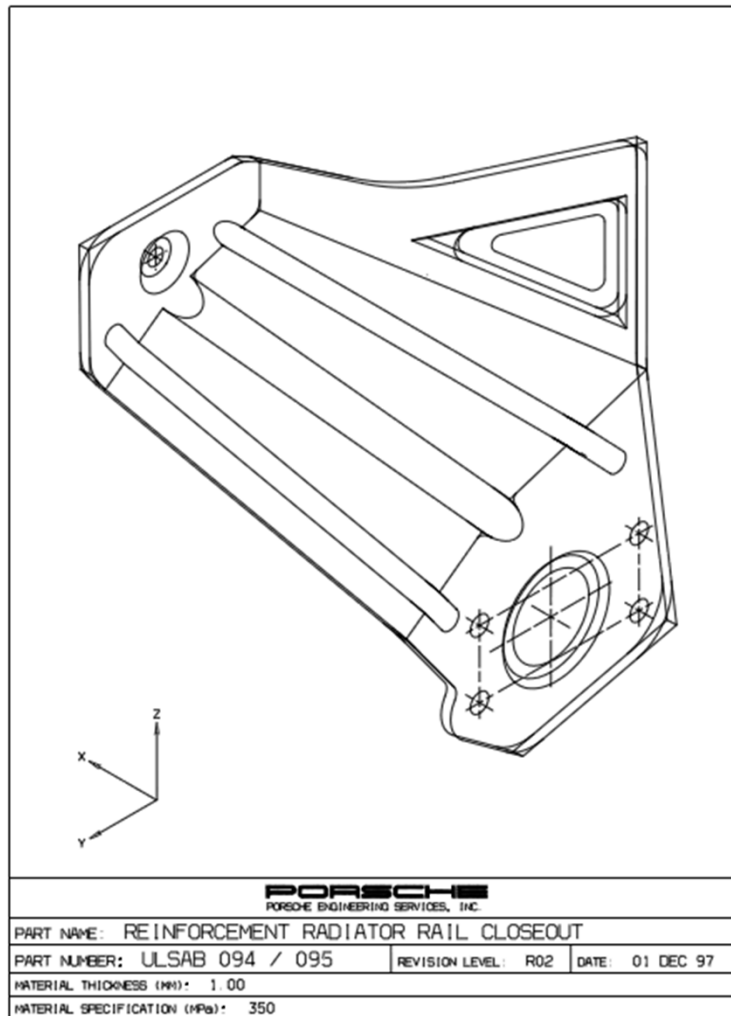


<div><b>PORSCHE</b> PORSCHE ENGINEERING SERVICES, INC.</div>			
PART NAME: PANEL SPARE TIRE TUB			
PART NUMBER: ULSAB 050		REVISION LEVEL: R02	DATE: 23 SEP 97
MATERIAL THICKNESS (MM): 0.96 - SKIN = 0.14 X 2 CORE = 0.67 COATING = 0.01			
MATERIAL SPECIFICATION (MPa): SANDWICH			

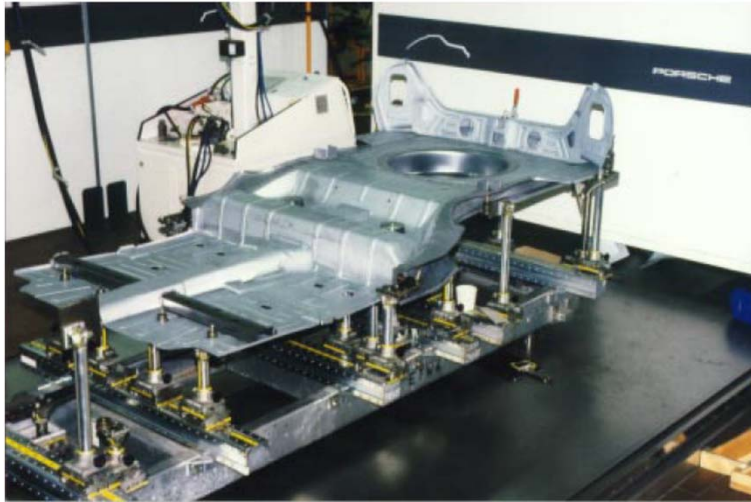
## Parts (2)



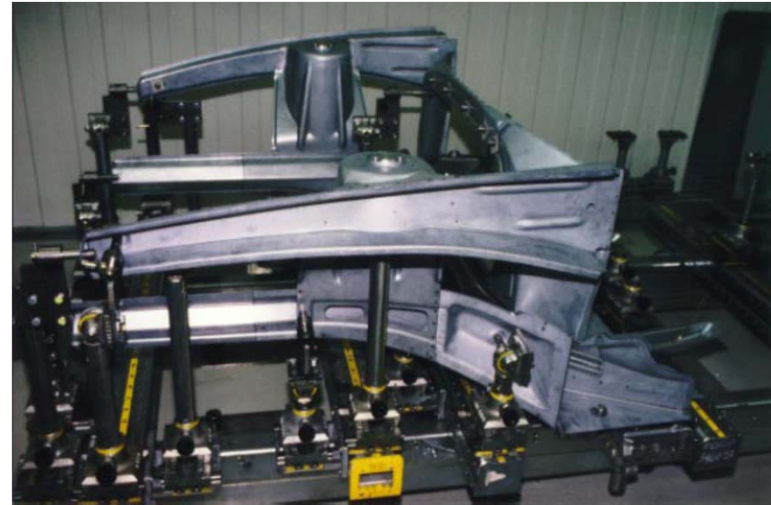
# Parts (3)



# Underbody



Rear floor



Front End



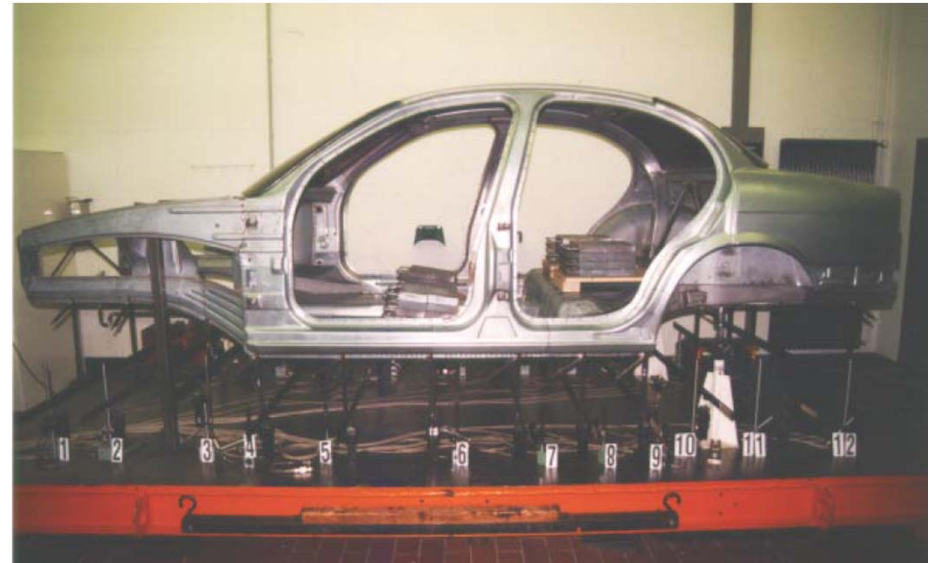
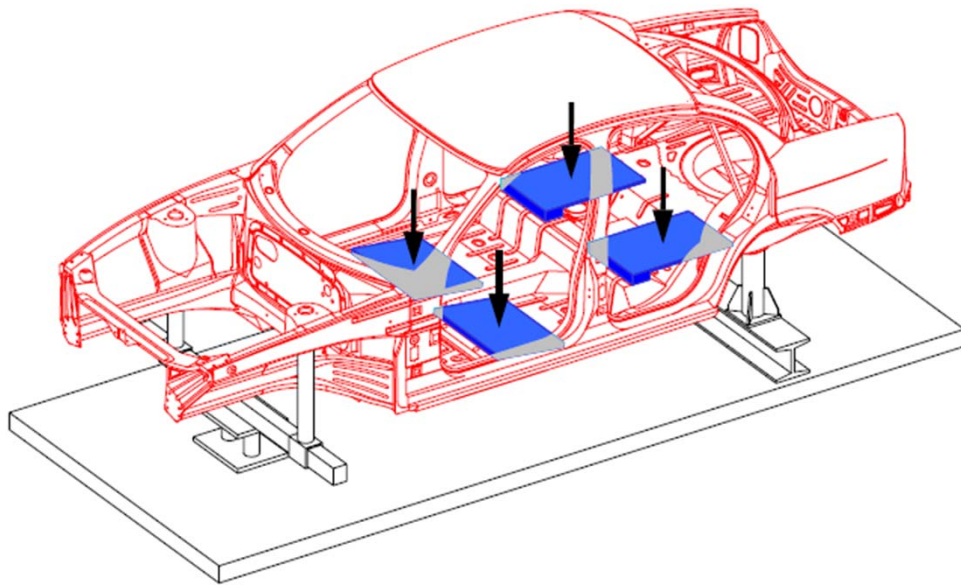


# Body Side Outer/Inner + Underbody



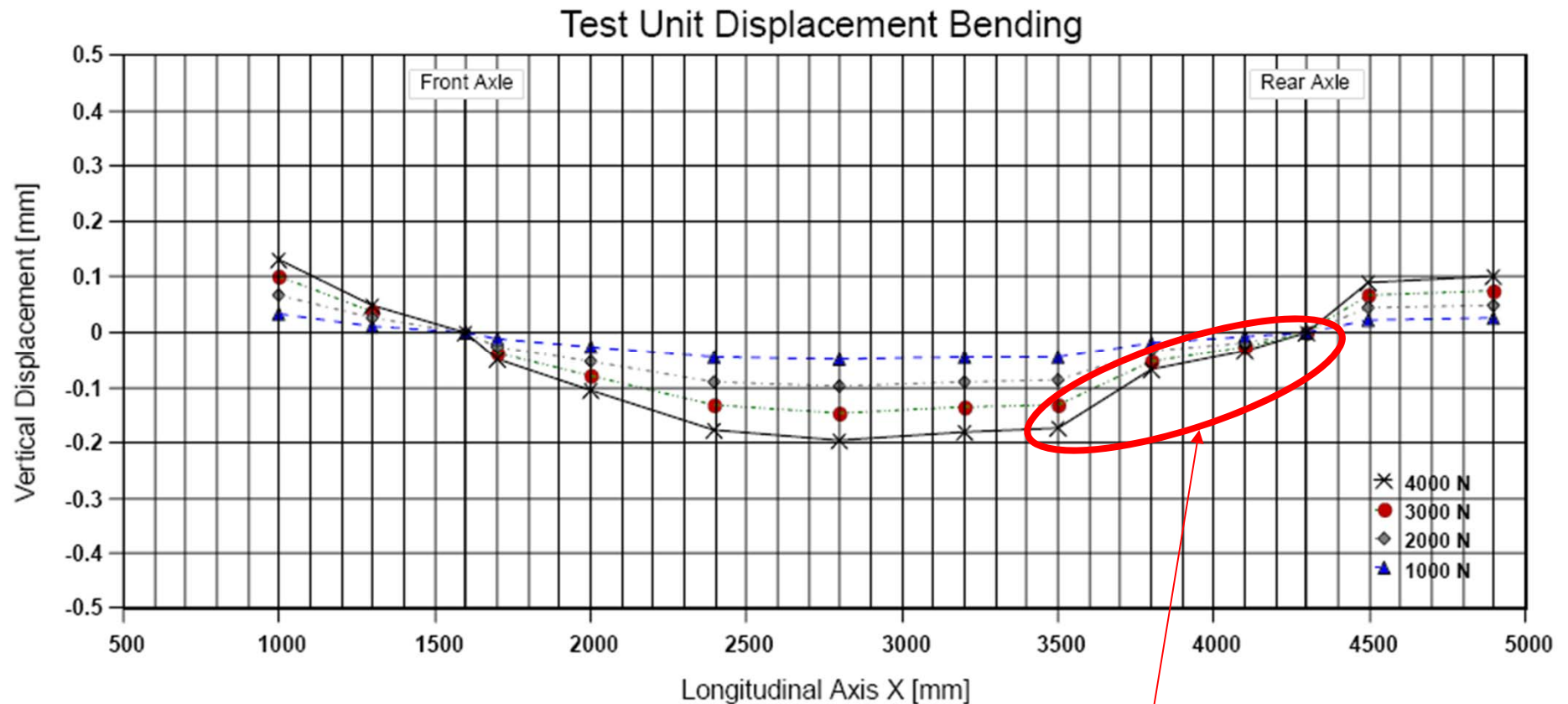
# Test: Static Bending

- Constraint: four fixing points
- Load: center of the front seats and center of the two outer rear seats, from  $F = 1000\text{N}$  ( $4 \times 250\text{N}$ ) to  $4000\text{N}$  ( $4 \times 1000\text{N}$ )



# Test: Results for Static Bending

- With glass: 20,460 N/mm
- Without glass: 17,150 N/mm

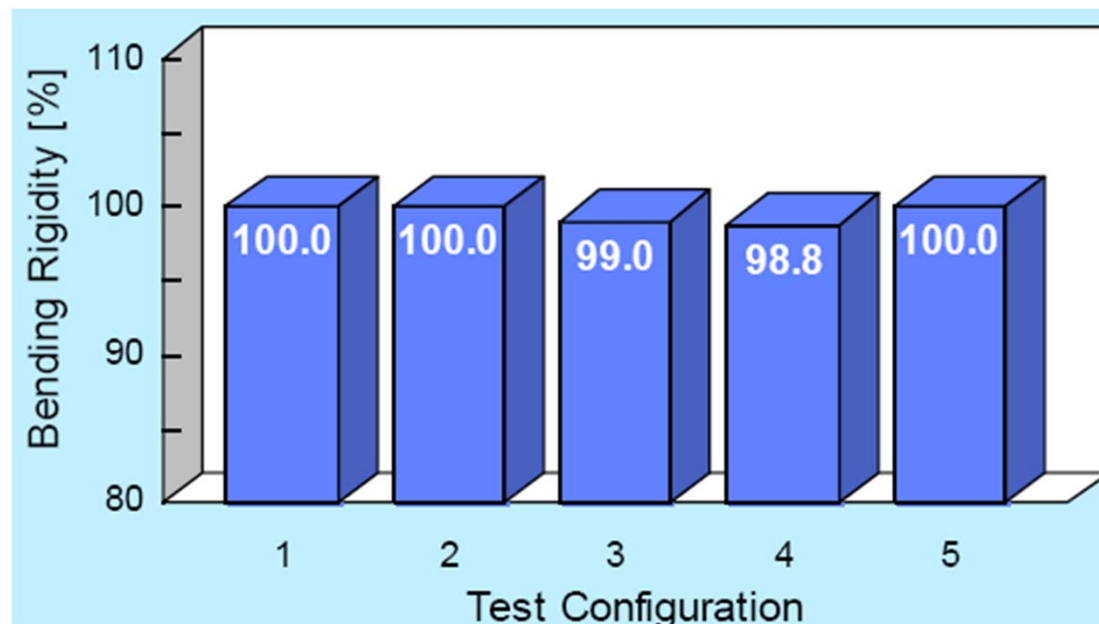




# Static Bending: impact of bonded and/or bolted parts

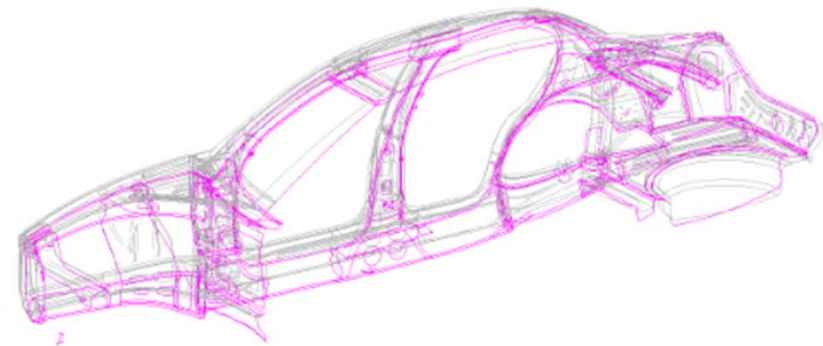
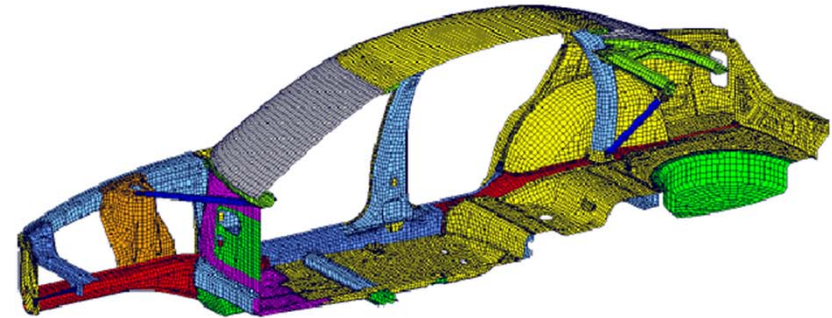
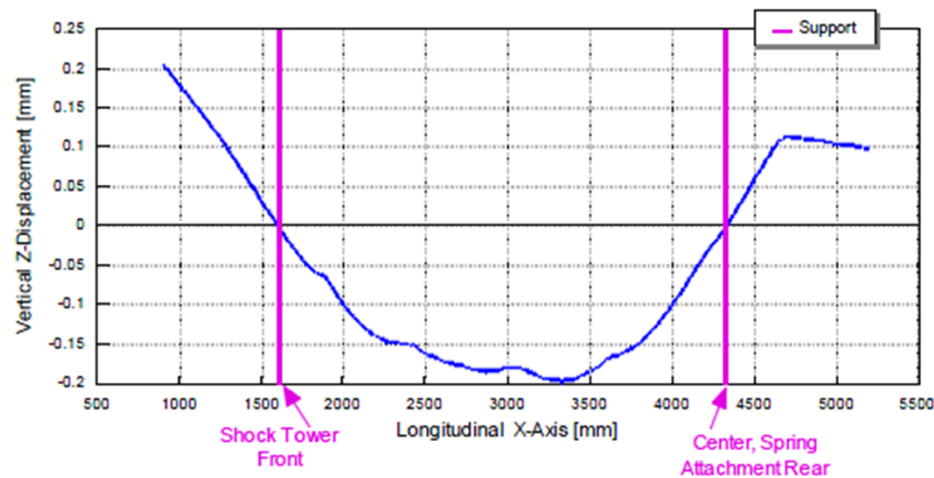
---

- 1. full configuration
- 2. as 1, but without braces radiator
- 3. as 2, but without radiator support upper
- 4. as 3, but without bolted brace cowl to shock tower assembly
- 5. as 4, but without tunnel bridge

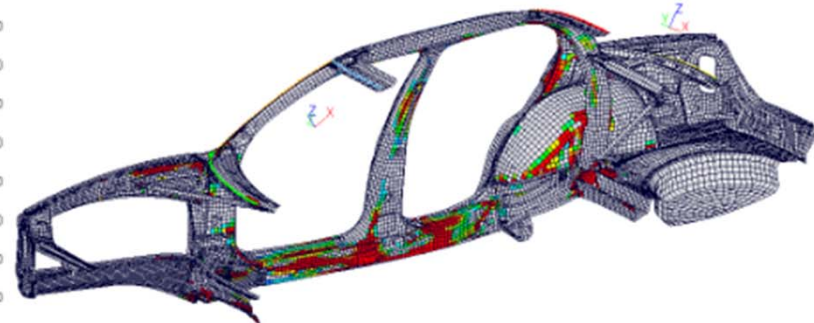
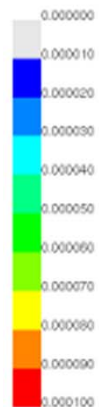


# CAE Analysis

- Half model
  - 54,521 shell elements
  - 53,460 nodes
- Static bending stiffness
  - 20,540 N/mm

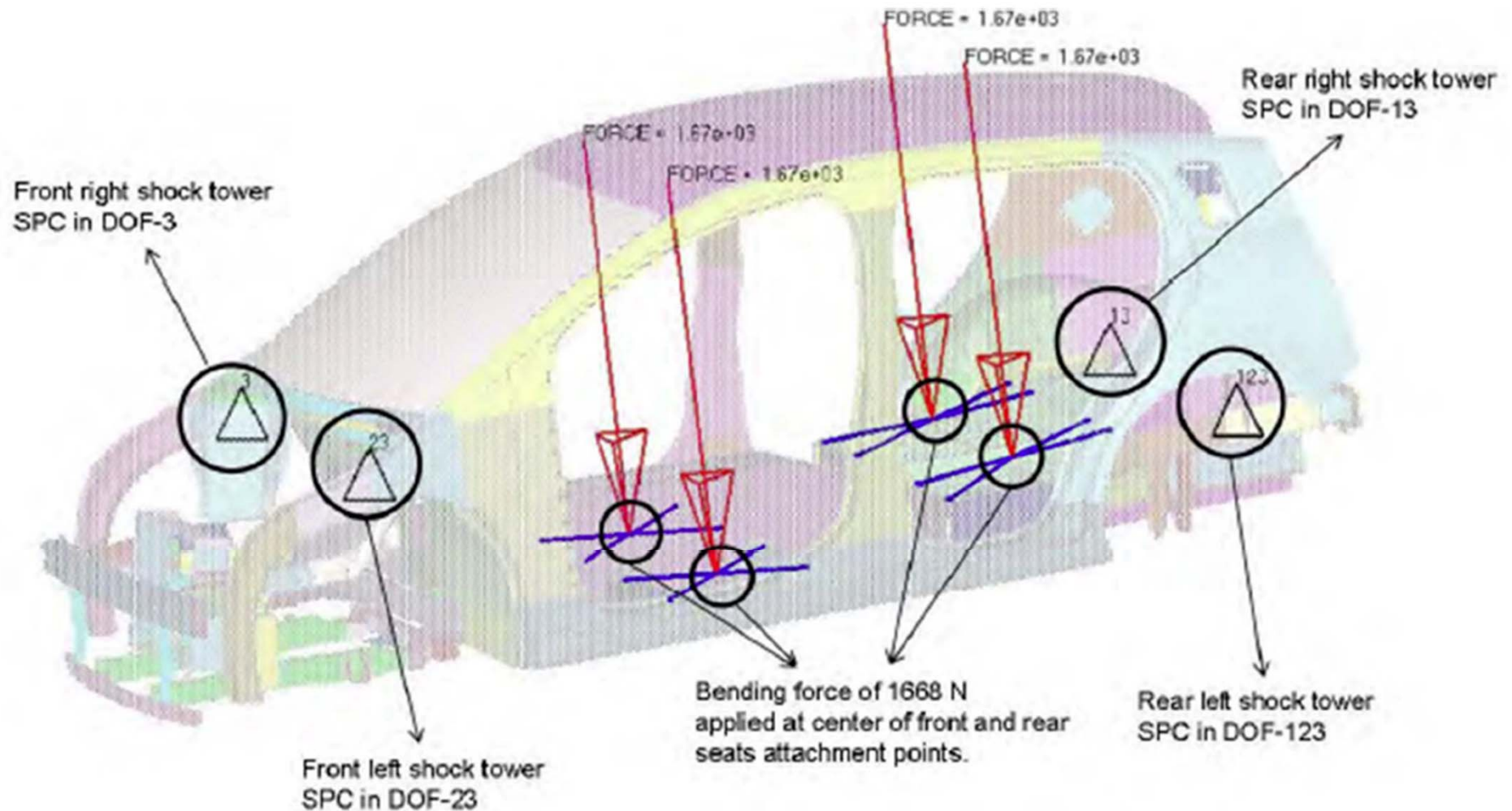


Energy density (Shell/Solid)



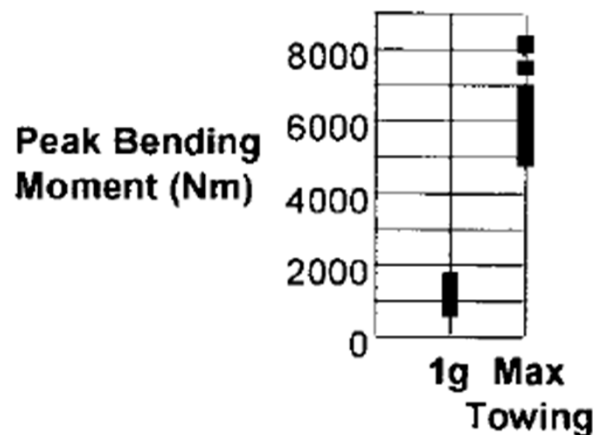
# Bending Stiffness

- Constraints and Loading in FSV Report

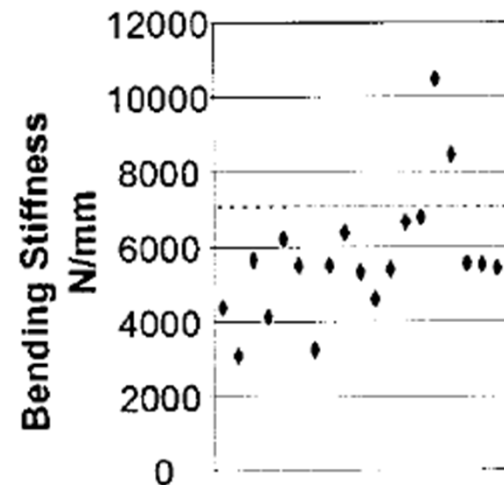


# Benchmark

- Maximum bending moments for a sampling of 20 vehicles
- Peak bending moments depend on
  - Placement of the subsystem mass
  - Longitudinal dimensions of the vehicle: wheelbase



Reference Vehicles- Honda Accord, Lexus LS400, Nissan 300ZX, Lumina, Century, Infinity Q45, Transport, Grand Prix, Toyota Camry. (AISI Data)

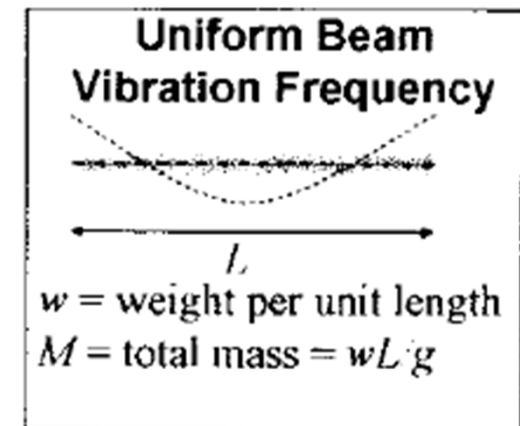


Reference Vehicles- Honda Accord, Lexus LS400, Nissan 300ZX, Lumina, Century, Infinity Q45, Transport, Grand Prix, Toyota Camry. (AISI Data)

## 4.2 Body Bending Stiffness Requirement

- Bending stiffness: slope of the load-deflection curve in the linear region from H point bending test
- Feeling of solidness: subjective → body vibration resonance
- Benchmarking (body bending resonant frequency)
  - body shell > full vehicle
- Desirable range for vehicle bending frequency: 22~25 Hz
- Uniform beam
  - $\omega_n$ : bending resonant frequency (rad/sec)
  - $w$ : weight per unit length
  - $M$ : total mass
  - $L$ : beam length

$$\omega_n = (2\pi f_n) = \frac{22.4}{L^2} \sqrt{\frac{EIg}{w}} \xrightarrow{M = \frac{wL}{g}} \omega_n = 22.4 L^{-(3/2)} \sqrt{\frac{EI}{M}}$$



# Lateral Vibration of Beam

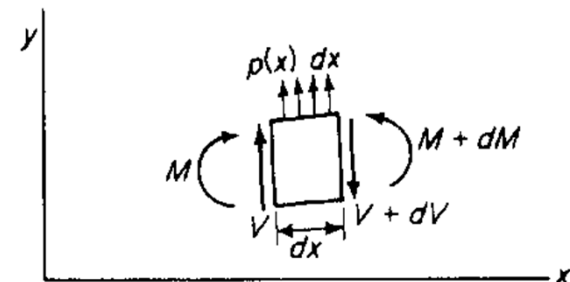
- Beam vibrating about its static equilibrium position under its own weight
  - Load per unit length = inertia load due to its mass and acceleration
  - Assuming harmonic motion

$$\left. \begin{aligned} \frac{d^2 M}{dx^2} &= \frac{dV}{dx} = p(x) \\ M &= EI \frac{d^2 y}{dx^2} \end{aligned} \right\} \rightarrow \frac{d^2}{dx^2} \left( EI \frac{d^2 y}{dx^2} \right) = p(x) = \rho \omega^2 y \xrightarrow{EI=const} EI \frac{d^4 y}{dx^4} - \rho \omega^2 y = 0$$



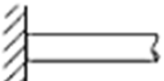







$$\xrightarrow{\beta^4 = \rho \frac{\omega^2}{EI}} \frac{d^4 y}{dx^4} - \beta^4 y = 0 \rightarrow y = e^{ax} \rightarrow \begin{cases} a = \pm \beta \\ a = \pm i\beta \end{cases} \rightarrow \begin{cases} e^{\pm \beta x} = \cosh \beta x \pm \sinh \beta x \\ e^{\pm i\beta x} = \cos \beta x \pm i \sin \beta x \end{cases}$$

$$\rightarrow \begin{cases} y = C_1 e^{\beta x} + C_2 e^{-\beta x} + C_3 e^{i\beta x} + C_4 e^{-i\beta x} \\ y = C_1 \cos \beta x + C_2 \sin \beta x + C_3 \cosh \beta x + C_4 \sinh \beta x \end{cases}$$

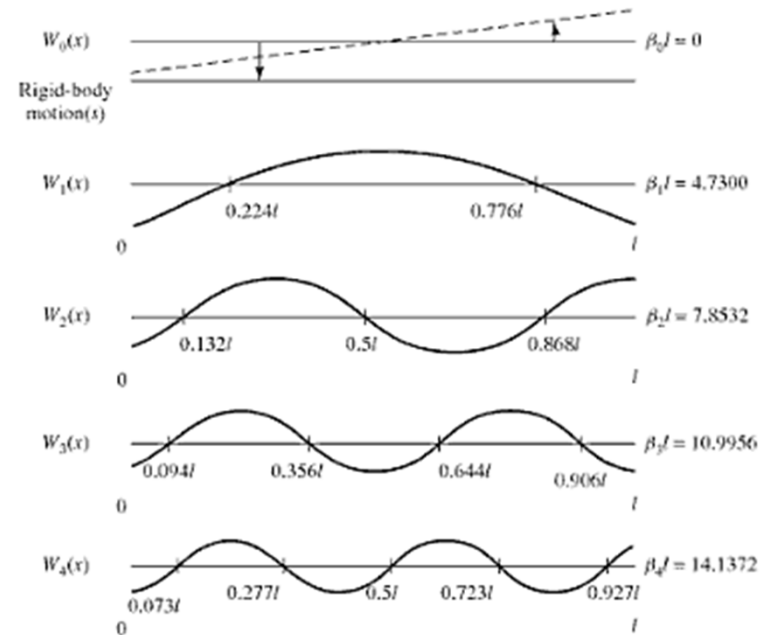
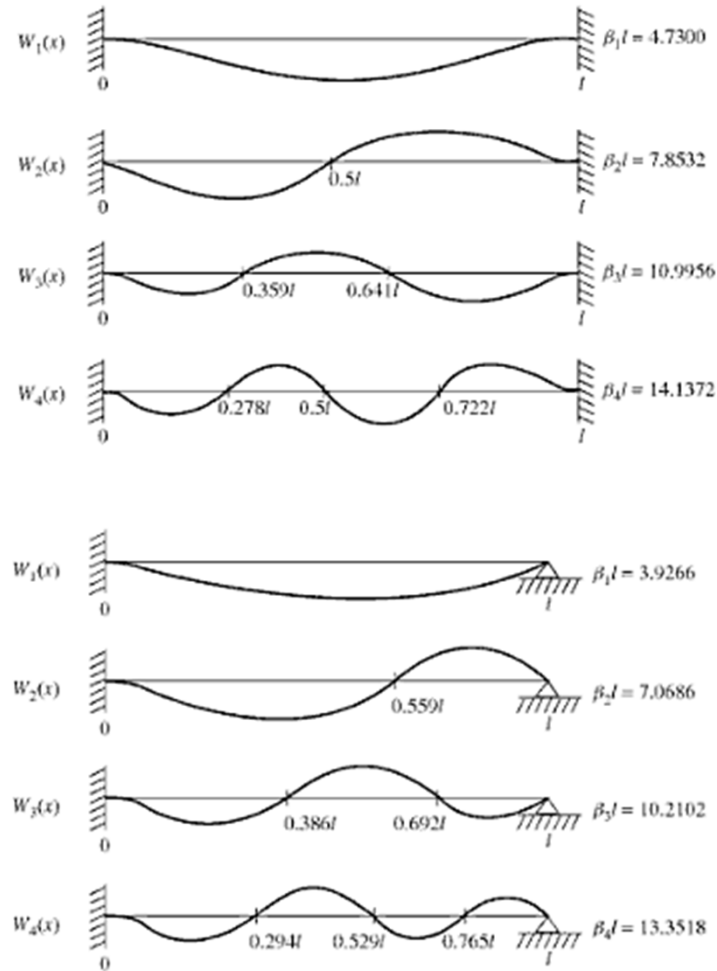
$$\omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho}} = (\beta_n l)^2 \sqrt{\frac{EI}{\rho l^4}}$$



# Boundary Conditions

Boundary condition		At left end ( $x = 0$ )		At right end ( $x = l$ )
1. Free end (bending moment = 0, shear force = 0)		$EI \frac{\partial^2 w}{\partial x^2}(0, t) = 0$ $\frac{\partial}{\partial x} \left( EI \frac{\partial^2 w}{\partial x^2} \right) \Big _{(0,t)} = 0$		$EI \frac{\partial^2 w}{\partial x^2}(l, t) = 0$ $\frac{\partial}{\partial x} \left( EI \frac{\partial^2 w}{\partial x^2} \right) \Big _{(l,t)} = 0$
2. Fixed end (deflection = 0, slope = 0)		$w(0, t) = 0$ $\frac{\partial w}{\partial x}(0, t) = 0$		$w(l, t) = 0$ $\frac{\partial w}{\partial x}(l, t) = 0$
3. Simply supported end (deflection = 0, bending moment = 0)		$w(0, t) = 0$ $EI \frac{\partial^2 w}{\partial x^2}(0, t) = 0$		$w(l, t) = 0$ $EI \frac{\partial^2 w}{\partial x^2}(l, t) = 0$
4. Sliding end (slope = 0, shear force = 0)		$\frac{\partial w}{\partial x}(0, t) = 0$ $\frac{\partial}{\partial x} \left( EI \frac{\partial^2 w}{\partial x^2} \right) \Big _{(0,t)} = 0$		$\frac{\partial w}{\partial x}(l, t) = 0$ $\frac{\partial}{\partial x} \left( EI \frac{\partial^2 w}{\partial x^2} \right) \Big _{(l,t)} = 0$
5. End spring (spring constant = $k$ )		$\frac{\partial}{\partial x} \left( EI \frac{\partial^2 w}{\partial x^2} \right) \Big _{(0,t)} = -kw(0, t)$ $EI \frac{\partial^2 w}{\partial x^2}(0, t) = 0$		$\frac{\partial}{\partial x} \left( EI \frac{\partial^2 w}{\partial x^2} \right) \Big _{(l,t)} = kw(l, t)$ $EI \frac{\partial^2 w}{\partial x^2}(l, t) = 0$

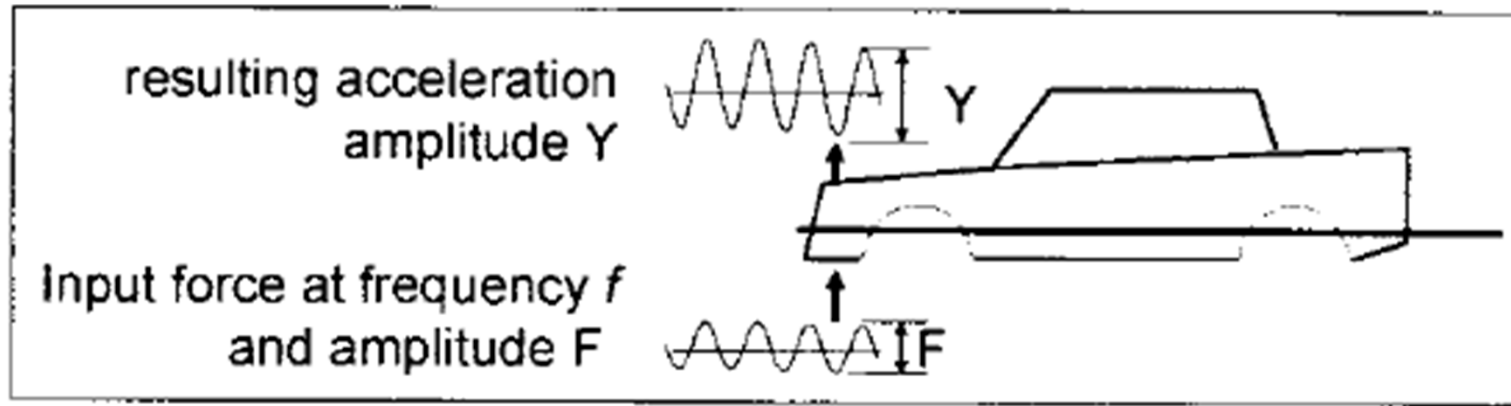
# Natural Frequencies and Mode Shape



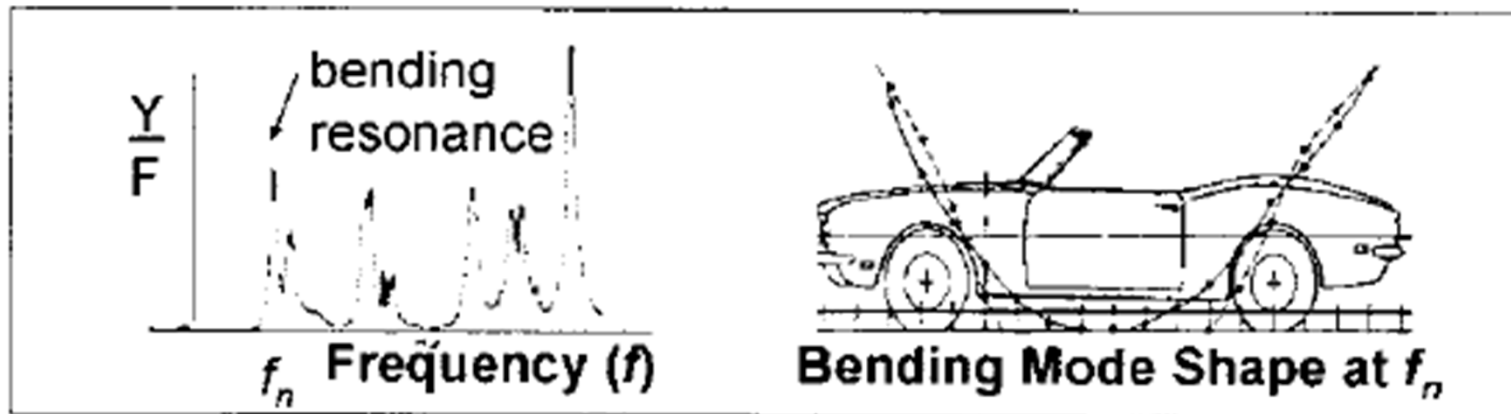
Beam Configuration	$(\beta_1 l)^2$ Fundamental	$(\beta_2 l)^2$ Second Mode	$(\beta_3 l)^2$ Third Mode
Simply supported	9.87	39.5	88.9
Cantilever	3.52	22.0	61.7
Free-free	22.4	61.7	121.0
Clamped-clamped	22.4	61.7	121.0
Clamped-hinged	15.4	50.0	104.0
Hinged-free	0	15.4	50.0



# Body Vibration Test and Behavior



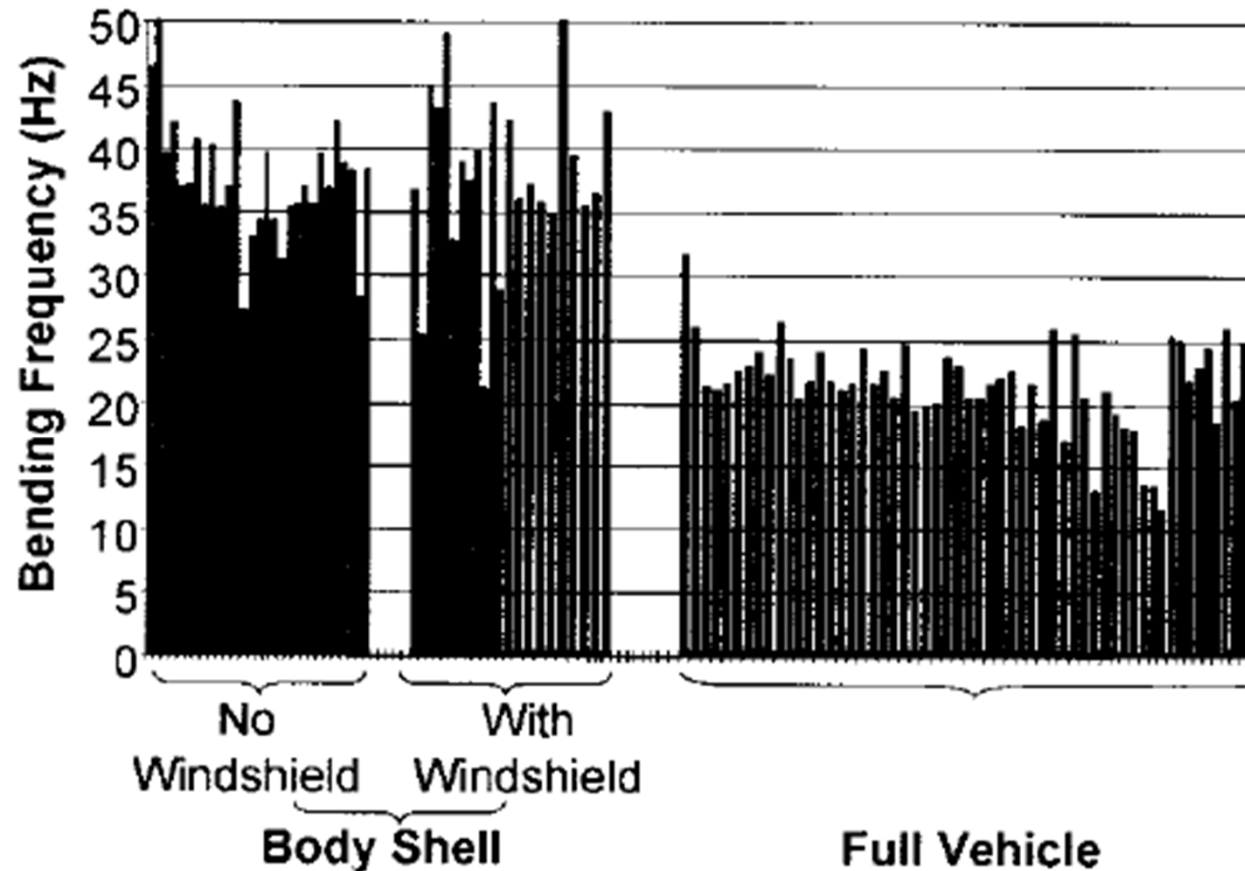
(a) Vibration Test Set up



(b) Typical Frequency Response and Mode Shape

# Bending Resonant Frequency Benchmark

---



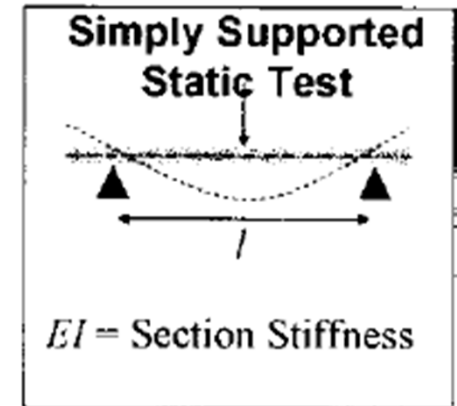
# Body Bending Stiffness Requirement

- Simply supported static test loaded at its center

- $l$ : wheelbase
- $M$ : rigidly mounted mass
- $K$ : required bending stiffness

$$K = \frac{48EI}{l^3} \rightarrow EI = \frac{Kl^3}{48}$$

$$\omega_n = 22.4L^{-(3/2)} \sqrt{\frac{Kl^3}{48M}} = 3.2332 \left( \frac{l}{L} \right)^{(3/2)} \sqrt{\frac{K}{M}} \rightarrow K = 0.096\omega_n^2 M \left( \frac{L}{l} \right)^3$$



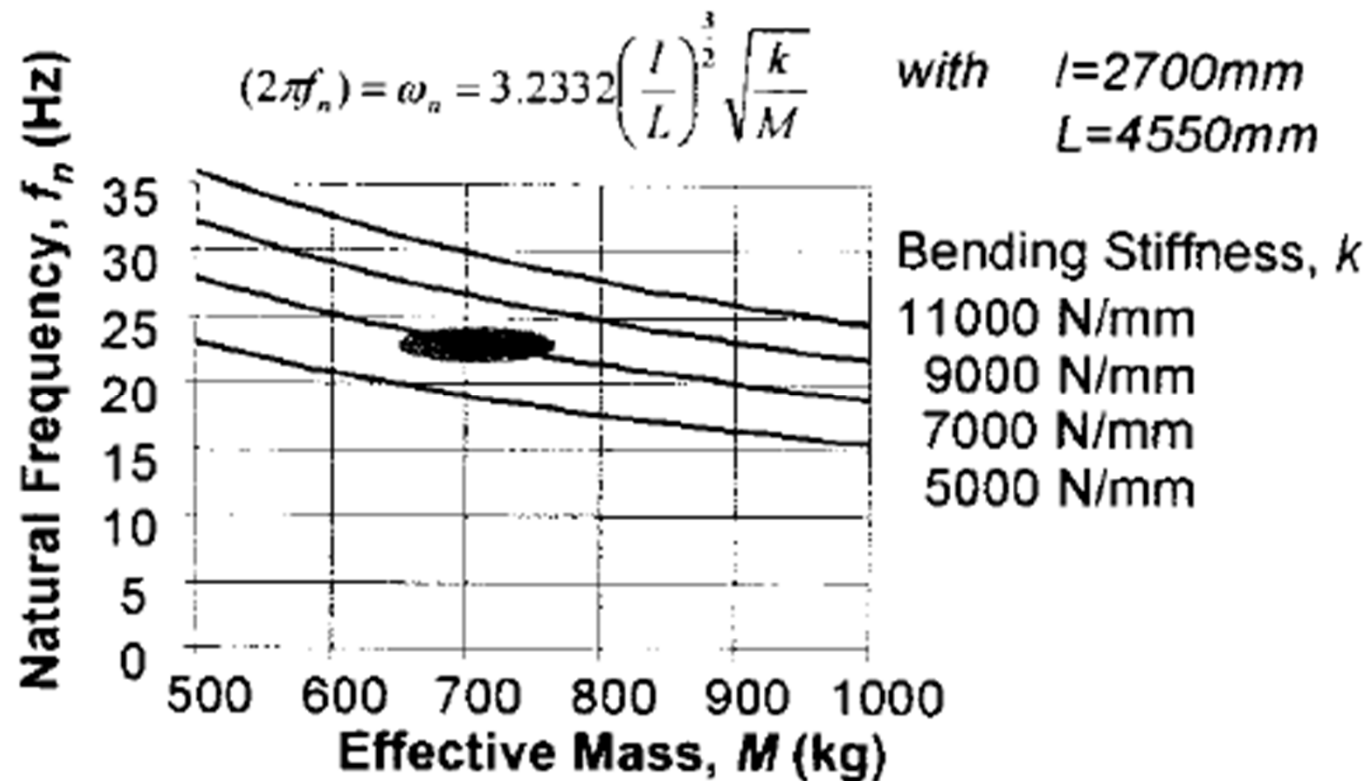
- Typical midsize vehicle

$$l = 2700mm, L = 4550mm, M \approx (0.4 \sim 0.6) \times (\text{curb mass} = 1446kg)$$

$$578 \leq M \leq 868, f_n = 22 \sim 25Hz \rightarrow K = 7000N/mm$$

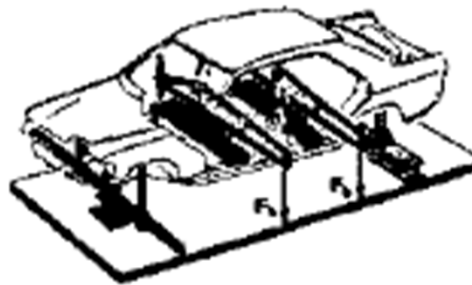
- High bending stiffness required to achieve the same  $\omega_n$ 
  - Higher mass loading (optioned luxury car), long overall length (four door sedans vs. two seat sport coupes)
  - Reduce relative deformations which cause squeaks and rattles

# First Order Estimation of Bending Stiffness



# Typical Bending Requirements: Midsize Vehicle

---



Restraints at  
Suspension  
Attachments

## **Bending Stiffness**

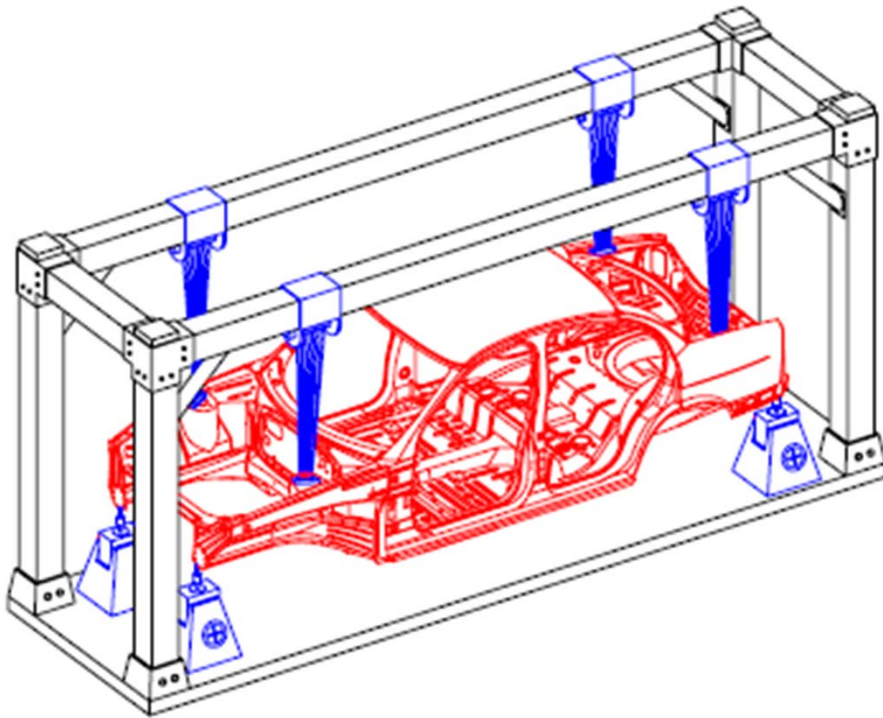
Nominal Value  
Stiffness = 7000 N/mm

## **Bending Strength**

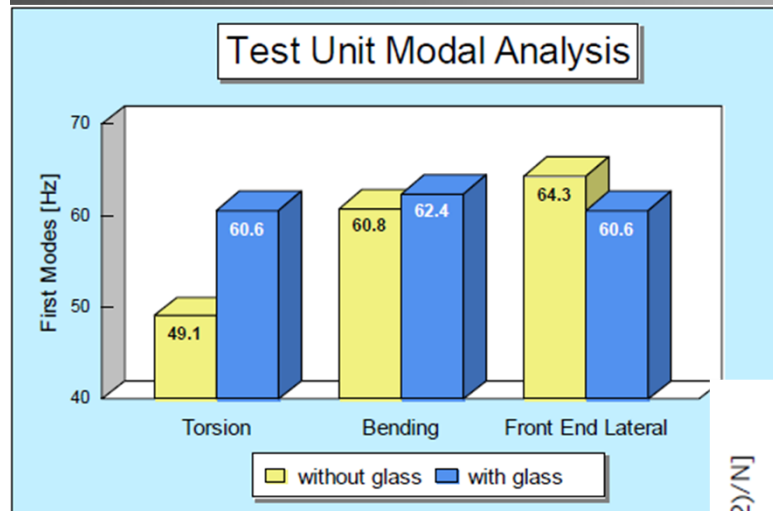
Nominal Value  
F = 6680 N  
no permanent deformation

# Test: Modal Analysis

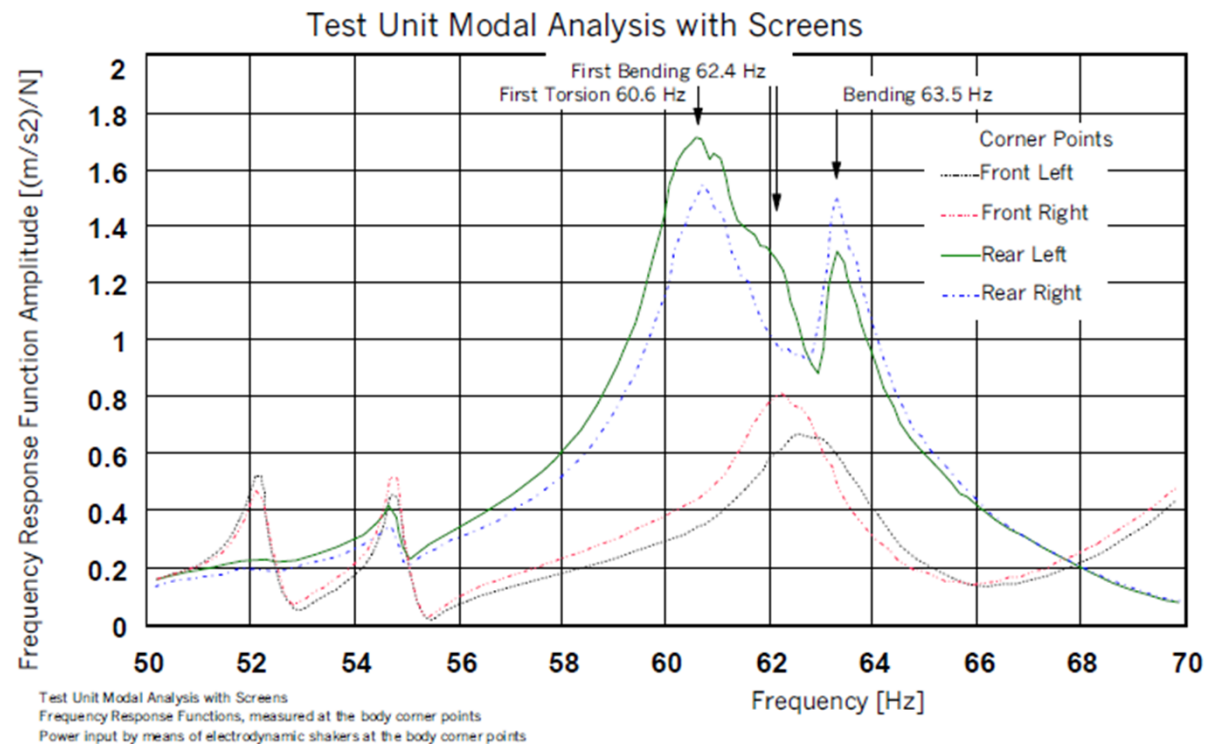
- resonance frequencies of the specific structure and the corresponding mode shapes (how the structure vibrates)
  - Input: applied energy (electrodynamic shakers at corner points)
  - Output: acceleration at different points



# Test: Results for Modal Analysis

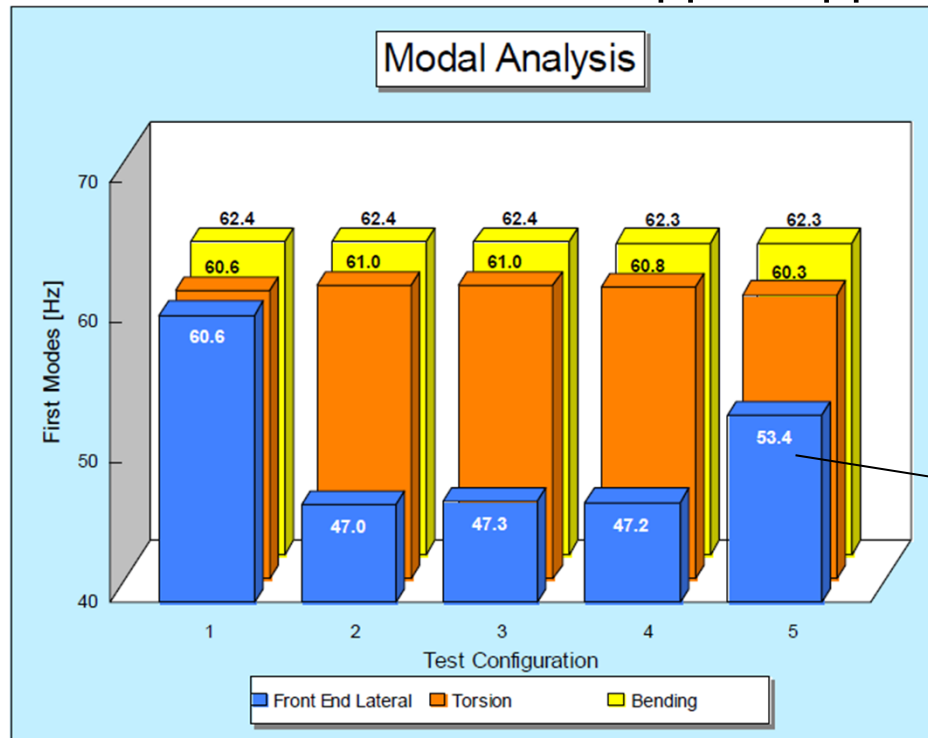


Second bending mode at 63.5 Hz occurs mainly in the rear; whereas the first bending mode occurs in the front and rear of the structure



# Modal Analysis: impact of bonded and/or bolted parts

- 1. full configuration
- 2. as 1, but without bolted brace cowl to shock tower assembly
- 3. as 2, but without braces radiator
- 4. as 3, but without tunnel bridge
- 5. as 4, but without radiator support upper



influence of the mass of assembly radiator support



# ULSAB Testing Results Overview vs. CAE Results

Testing	Testing		CAE		Benchmark Average	Targets
	DH #2	Test Unit	Final Version	Test Unit		
Static Rigidity						
Torsion (Nm/deg)	20,800	21,620	20,350	19,020	11,531	≥ 13,000
Bending (N/mm)	18,100	20,460	20,540	20,410	11,902	≥ 12,200
Modal Analysis						
Torsion (Hz)	60.1	60.6	61.4	61.1	38*	≥ 40
Bending (Hz)	63.9	62.4	61.8	64.1	38*	≥ 40
Front End Lateral (Hz)	64.9	60.6	60.3	58.5	38*	≥ 40

# FSV: Body Structure Performance

- Static Stiffness

Analysis Type	Target	FSV Model Results
Torsion stiffness (KN-m/deg)	20.0	19.604
Bending stiffness (N/mm)	12.0	15.552
Global Modes	Target	Frequency (Hz)
Torsion	>40 Hz (both modes), separated by 3 Hz	54.8
Vertical bending		60.6

- Durability

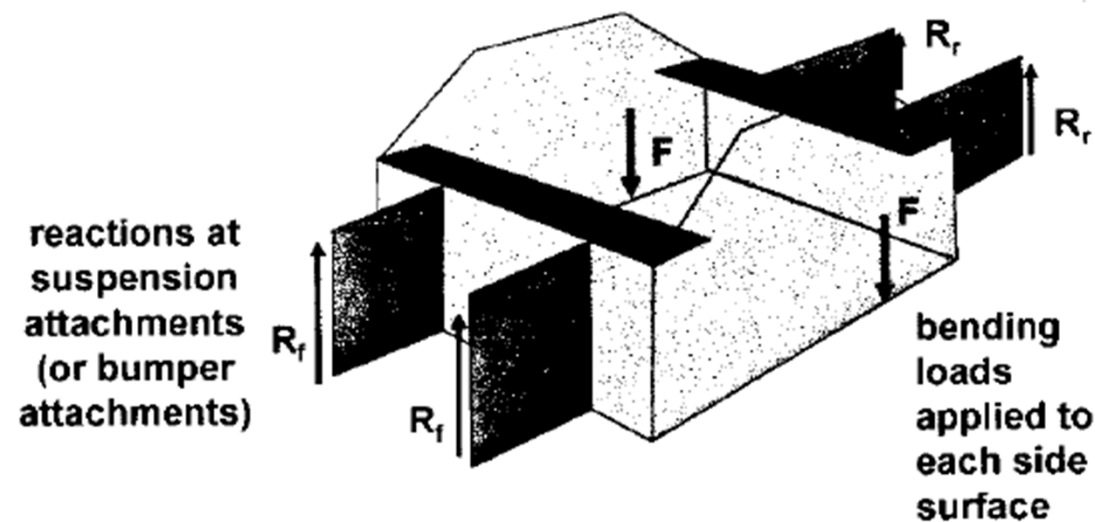
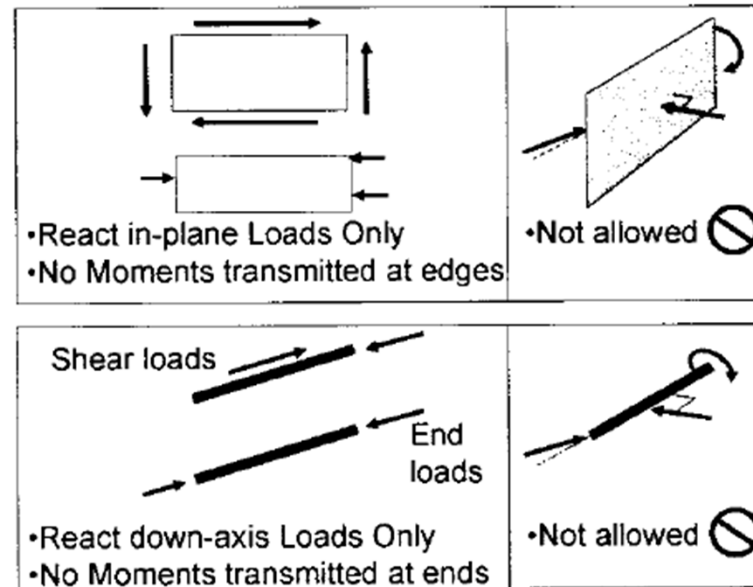
Analysis Type	Target life cycles	Predicted life cycles (FSV Model)
3 g pot hole	200,000	927,100
0.7 g cornering	100,000	1,676,000
0.8 g forward braking	100,000	274,700 (engine cradle life), 17,340,000(body life)

## 4.3 Load Path Analysis: Global Bending

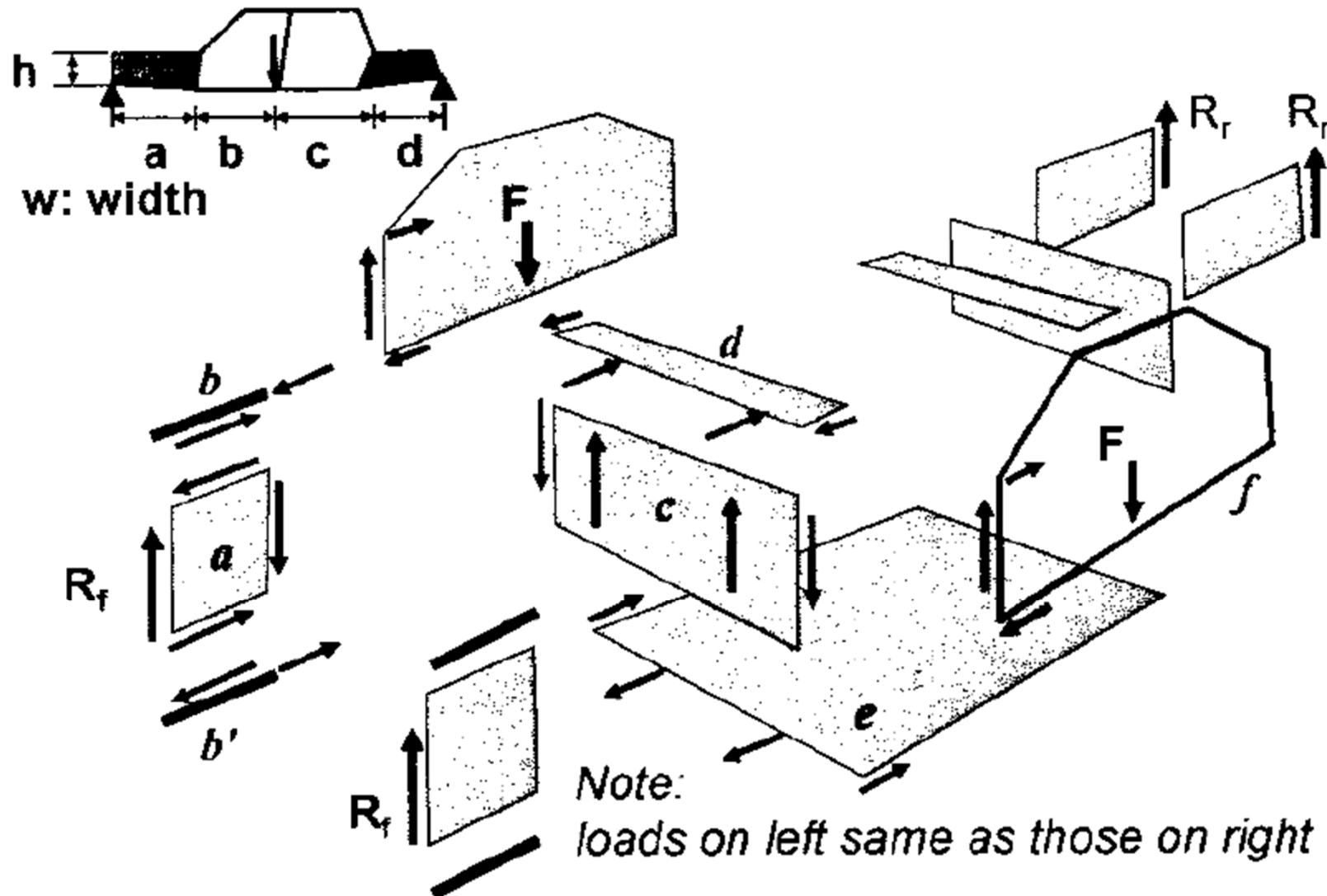
---

- Understand how global body requirements flow down to loads on structural elements
- Idealize the body as a set of structural surface and bar
  - Structural surface: flat element loaded in shear along edges
  - Bar element: linear element only reacts loads along it's axis either end loads or shearing loads along the length
- Apply loads in the global bending strength test
- Find loads on each individual structural subsystem
- Find internal element loads using static equilibrium
- Find appropriate sections to react these loads

# Structural Surface and Bar Body Model

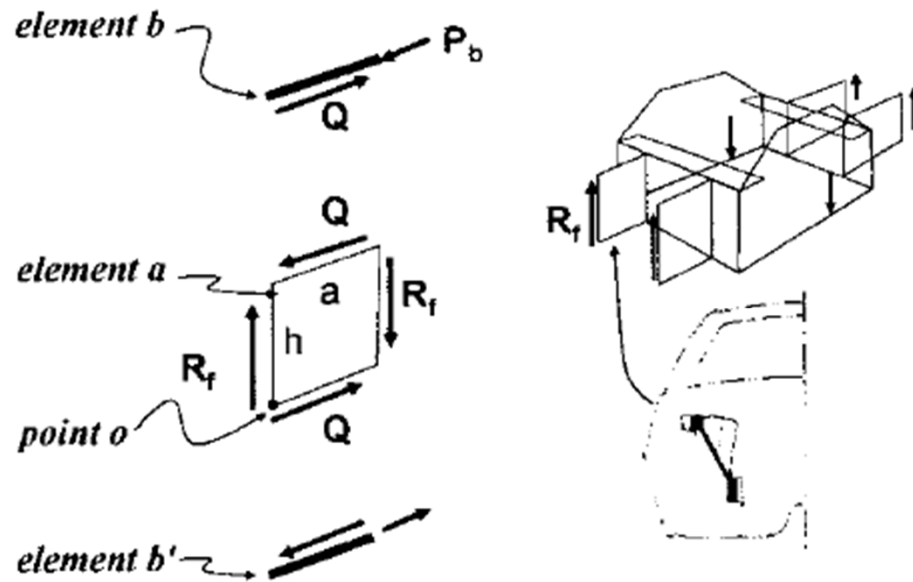


# Internal Loads on Structural Surface Model

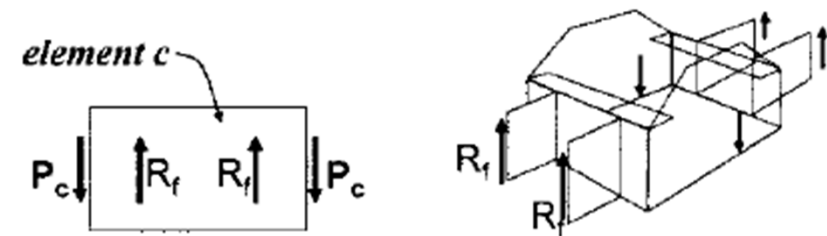


# Internal Loads (1)

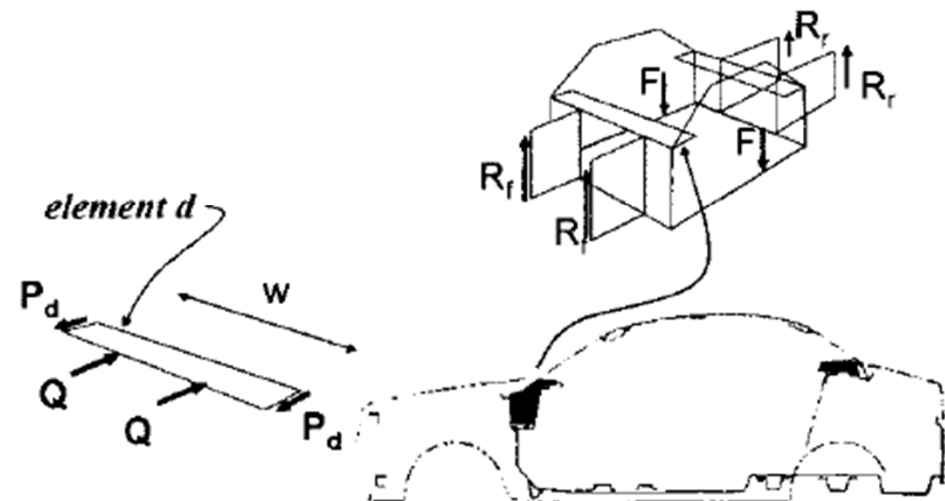
- Motor compartment panel



- Structural surface model

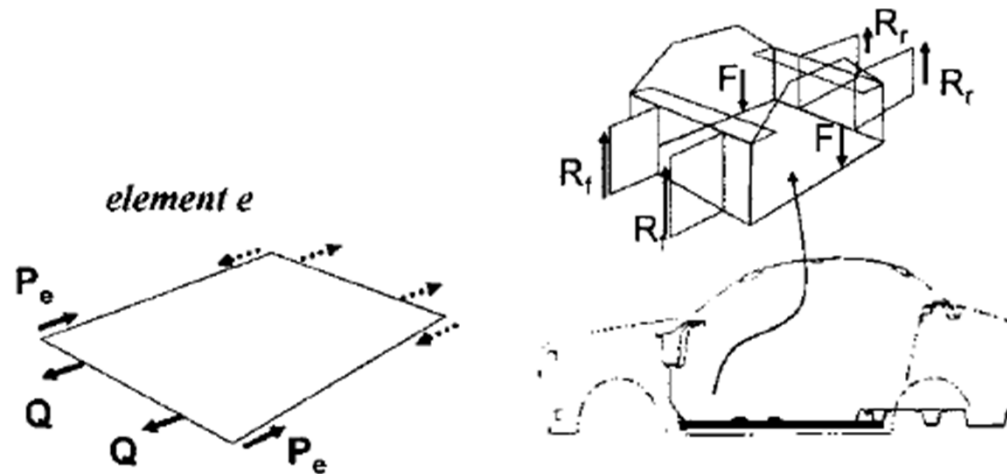


- Cowl or package shelf

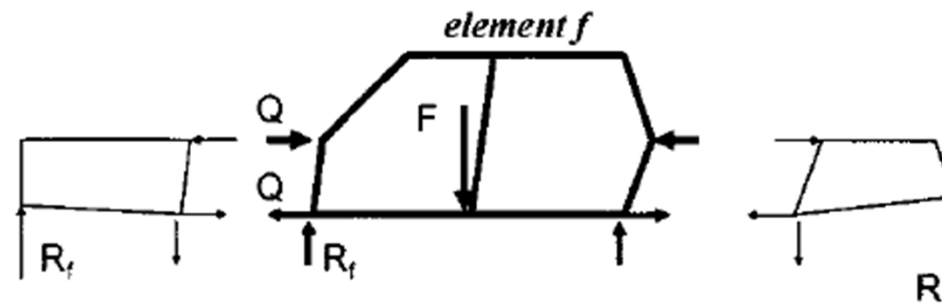
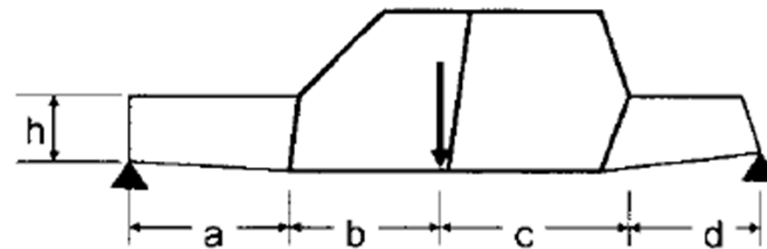


# Internal Loads (2)

- Floor pan



- Side frame

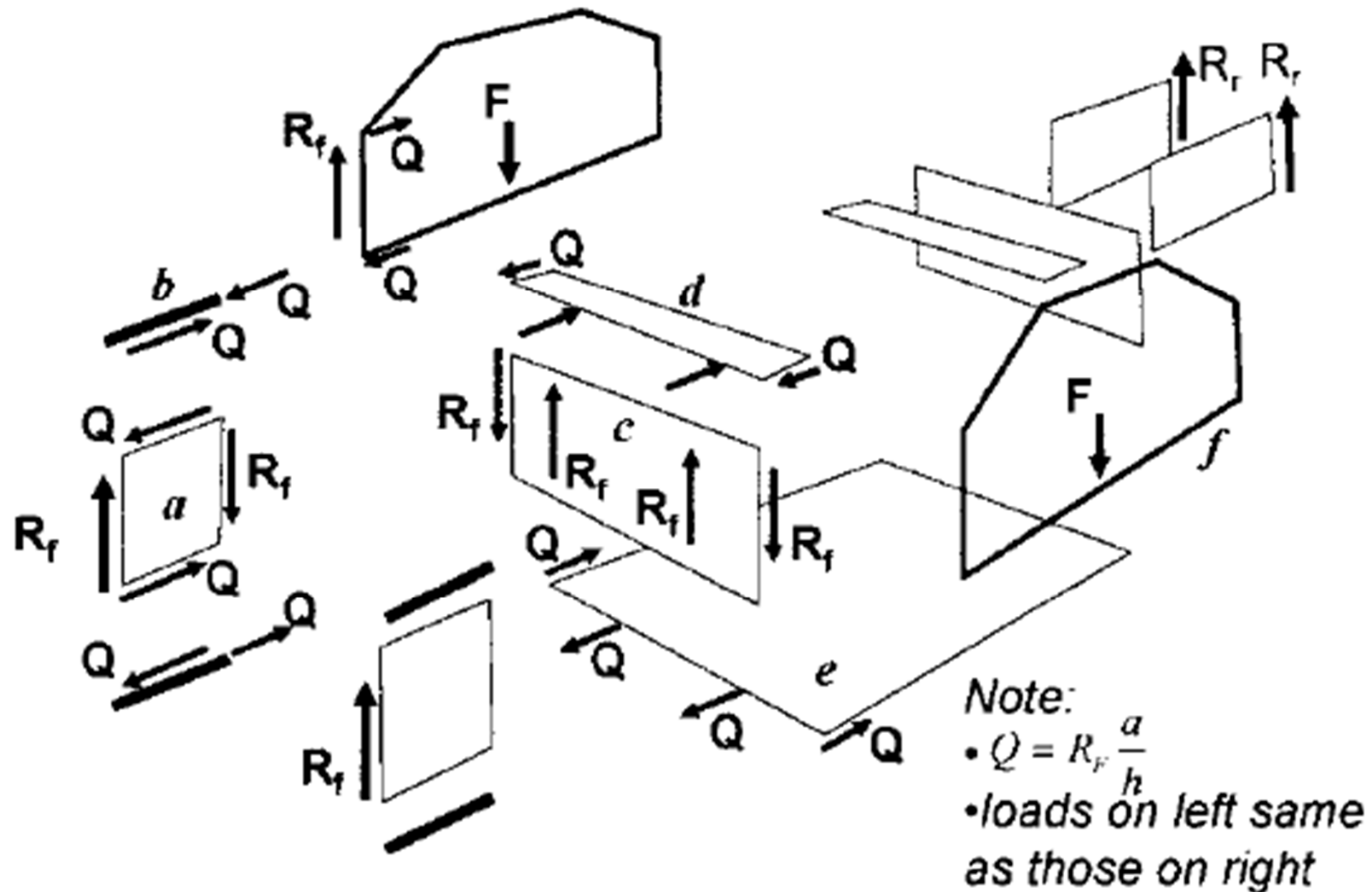


$$R_f = F(c+d)/(a+b+c+d)$$

$$R_f = F(a+b)/(a+b+c+d)$$

n for Body Bending - 35

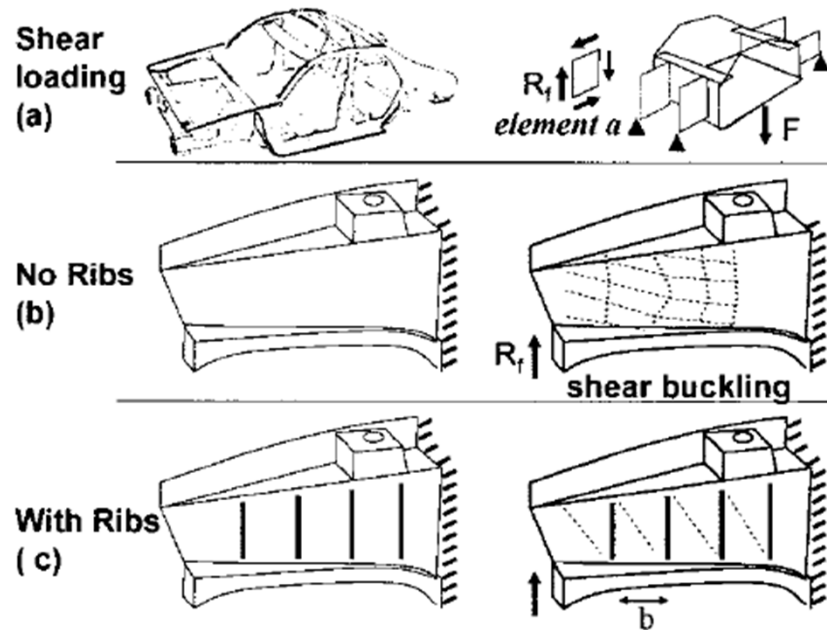
# Internal Loads on Structural Surface Model



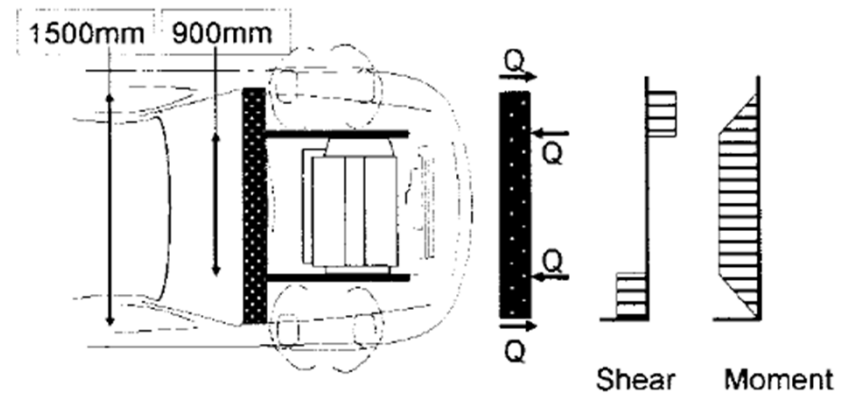


# Structural Subsystem

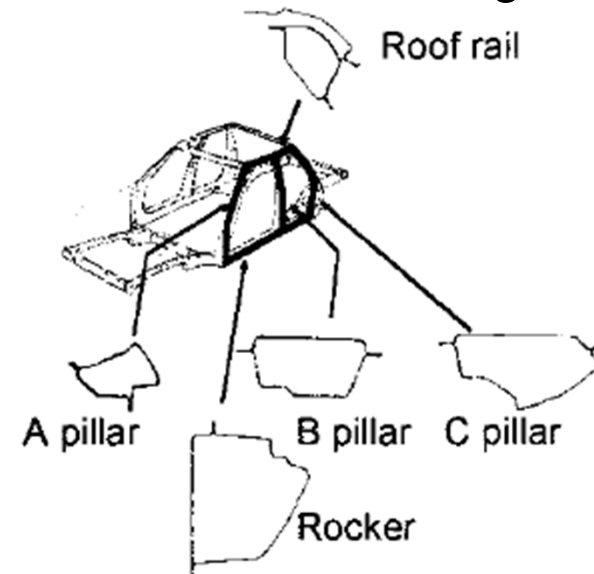
- Motor compartment side panel
  - Plate buckling



- Cowl: bending moment

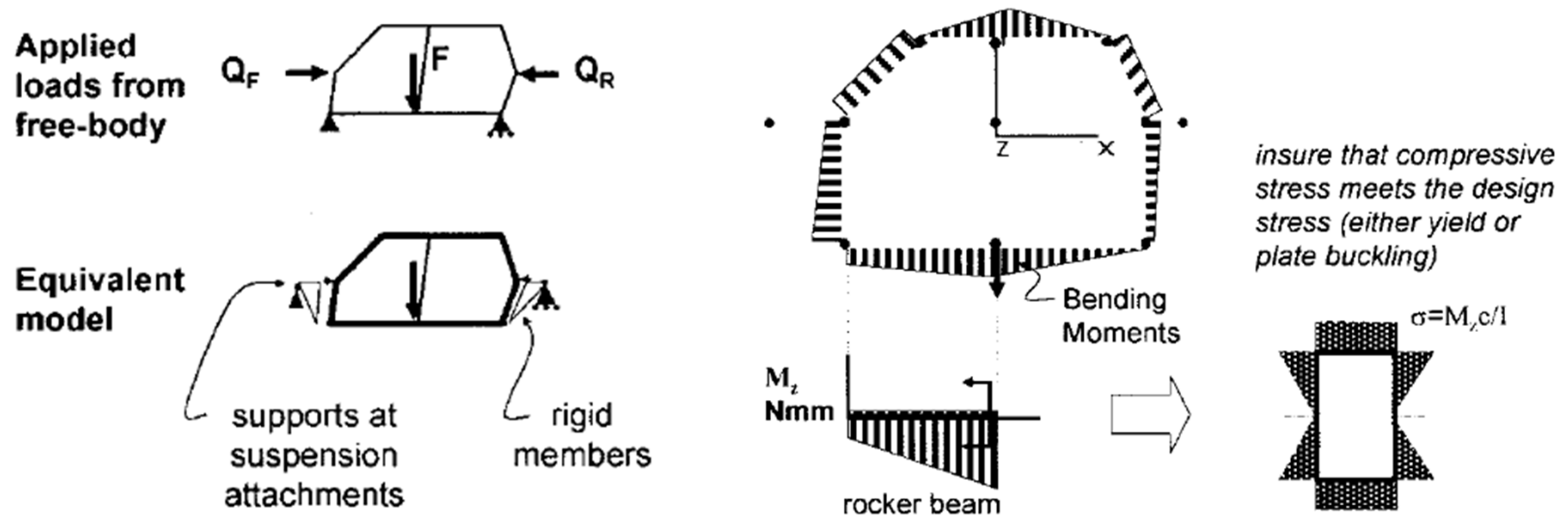


- Side frame: bending moment



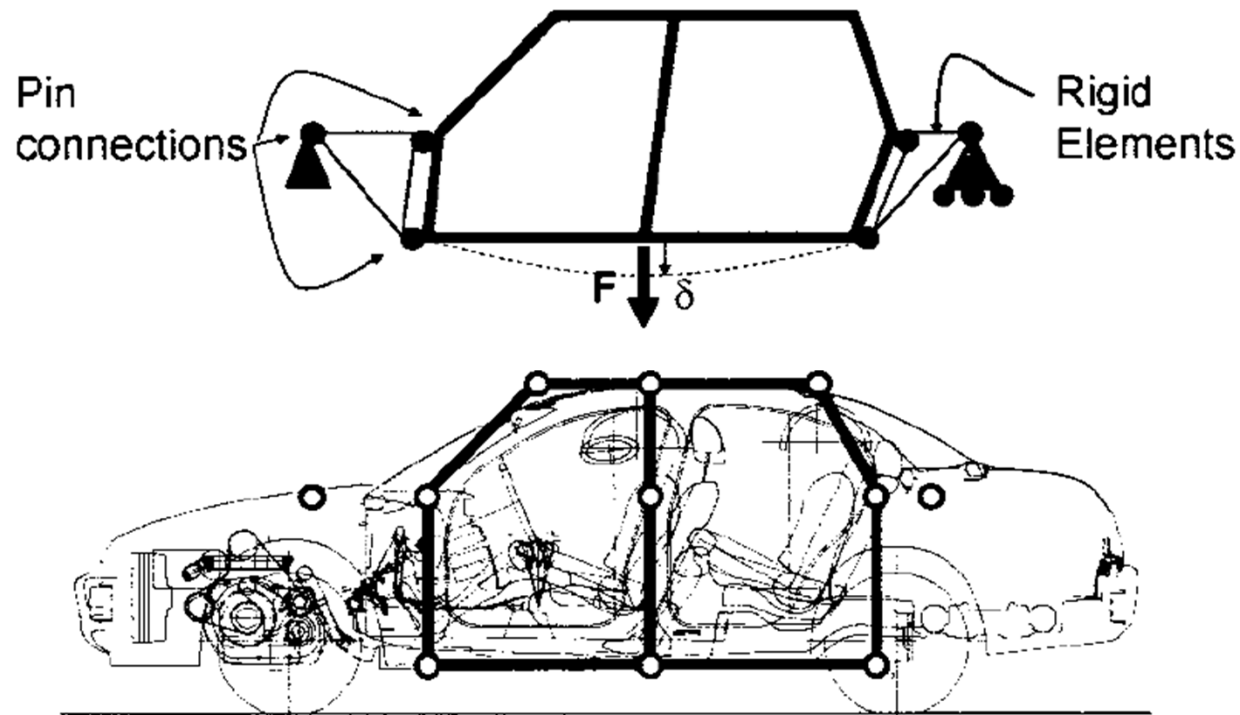
# Side Frame Beam Model

- What moments are applied to each beam by these loads so that we may design the beam section
  - Statically indeterminate
  - Relative stiffness of each beam



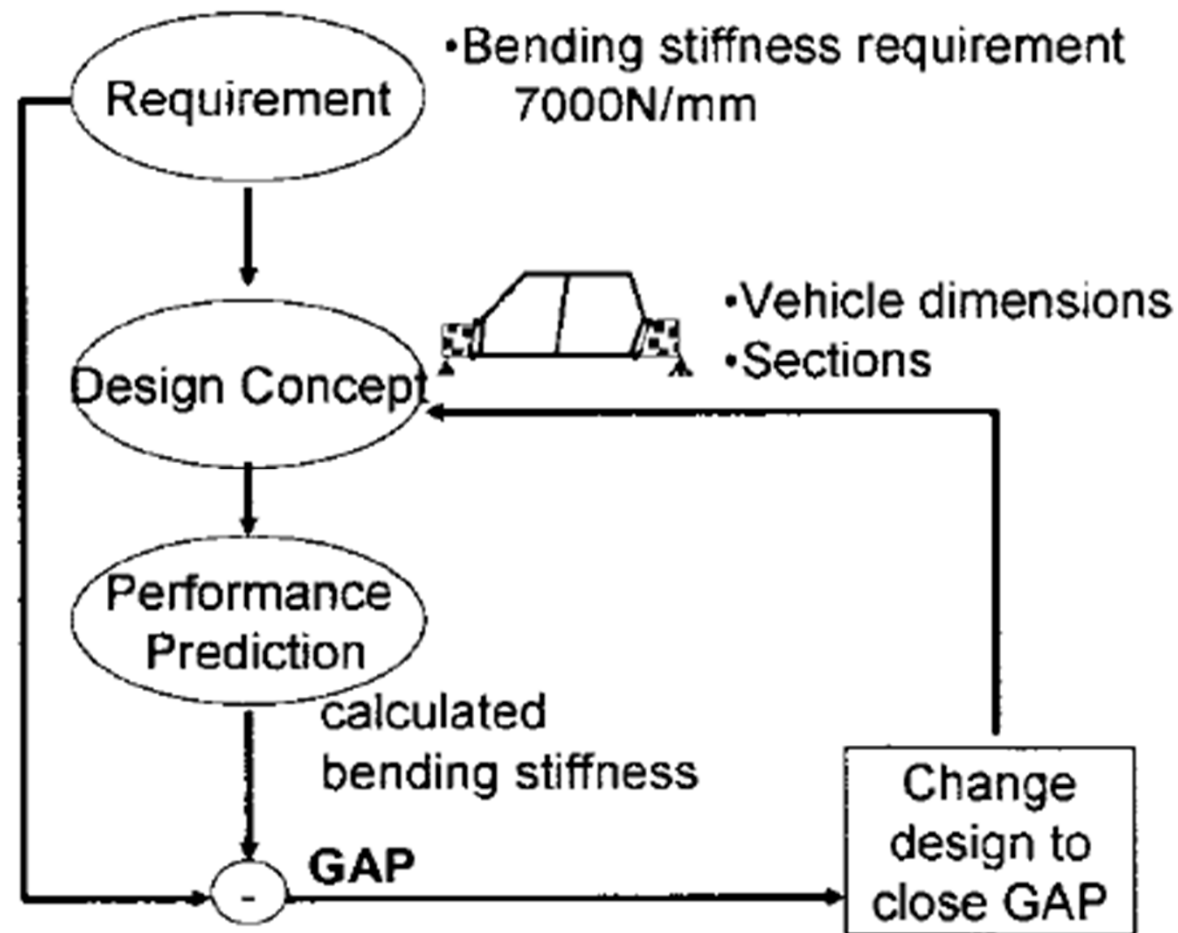
## 4.4 Analysis of Bending Stiffness

- Focus on side frame due to its dominant effect
  - Basic beam finite element model
  - Bending stiffness: ratio of applied load to deflection at the node of load application

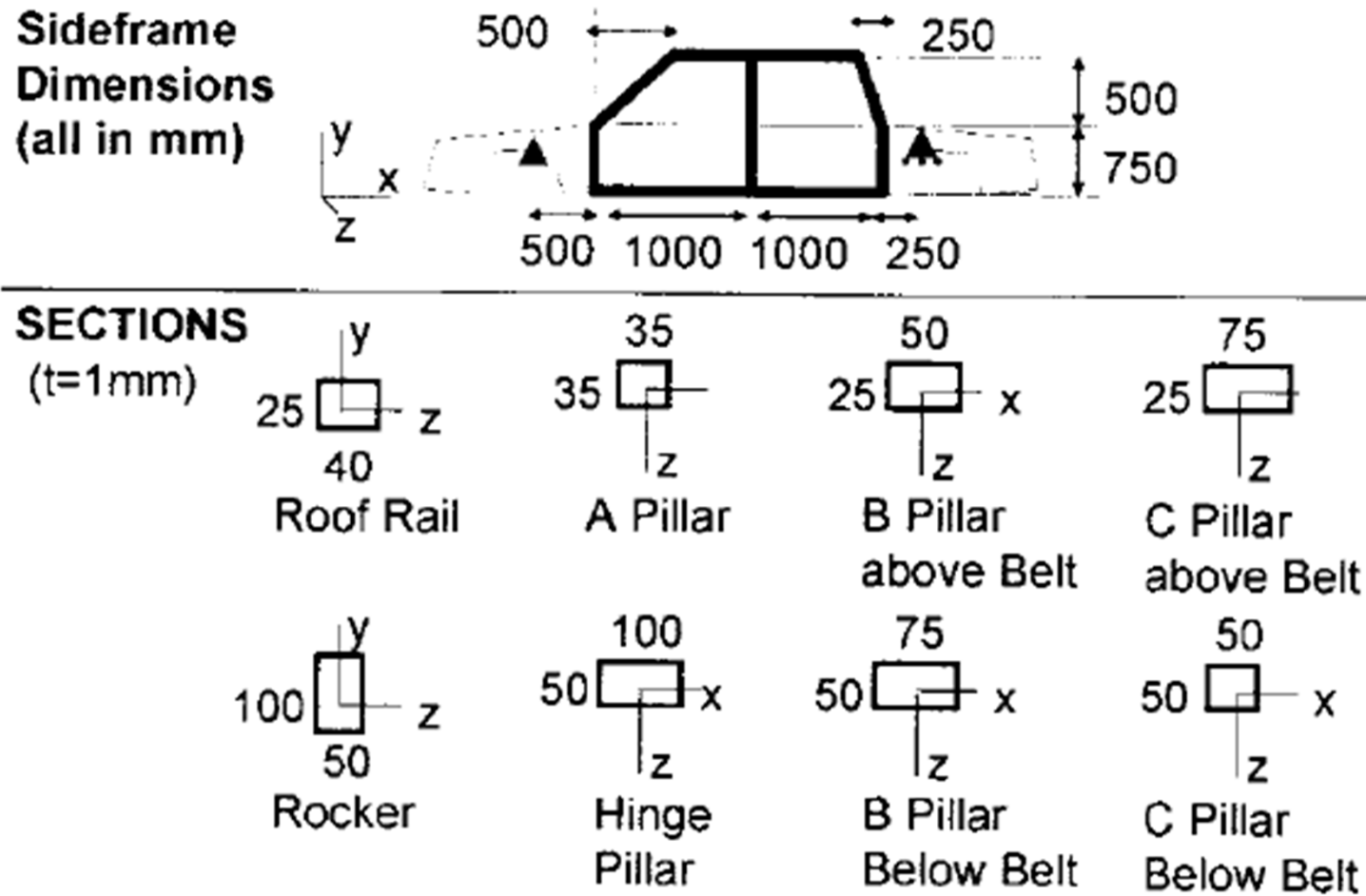


# System Design Procedure

---



# Example: Side Frame Model



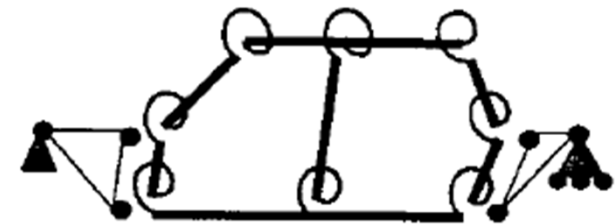
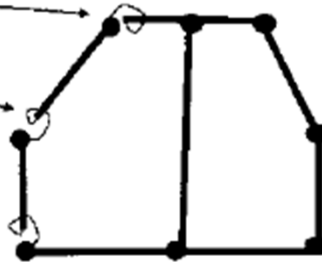
# FEA Results

- $F = 6680 \text{ N} \rightarrow \delta = 6.4 \text{ mm}$ 
  - $K = 1044 \text{ N/mm}$  per side =  $2088 \text{ N/mm}$  bending stiffness
  - 30% of  $7000 \text{ N/mm}$  target
  - Twice the actual stiffness: too stiff ?
- Modified model with flexible joints

$K = 3.8 \times 10^7 \text{ Nmm/rad}$  top A pillar

$K = 2.8 \times 10^7 \text{ Nmm/rad}$  bottom A pillar

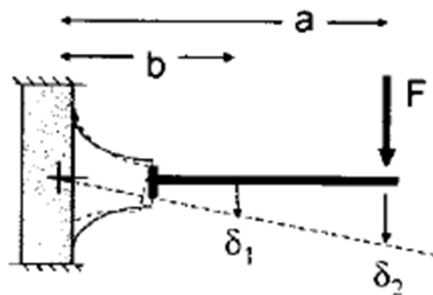
$K = 3.5 \times 10^7 \text{ Nmm/rad}$  on hinge pillar



- $F = 6680 \text{ N} \rightarrow \delta = 7.7 \text{ mm}$ 
  - $K = 1735 \text{ N/mm}$  bending stiffness

# Joint Flexibility

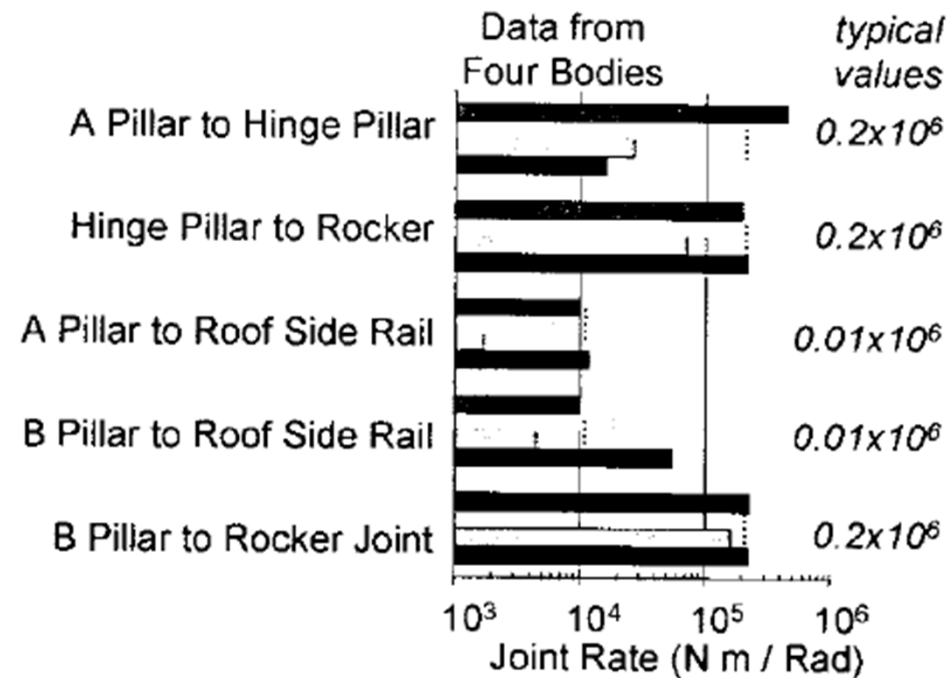
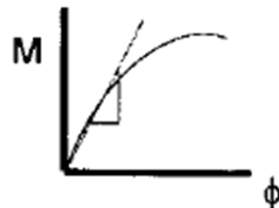
- Two or more thin walled beams are joined → considerable local deformation
- Rigid joints → flexible joints: rotational stiffness



$$\phi = (\delta_2 - \delta_1) / (a - b)$$

$$M = a F$$

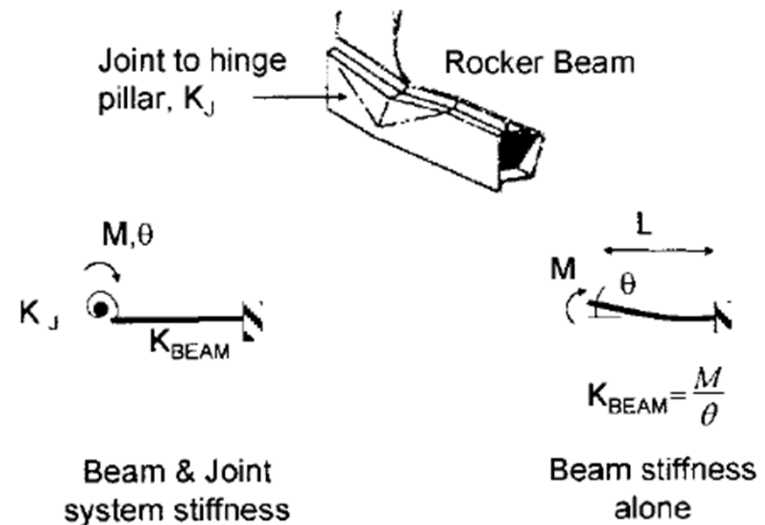
$K$  = slope of  $M$  vs.  $\phi$  plot:



# Joint Stiffness

- Is  $0.2 \times 10^6$  Nm/rad very stiff or very flexible?
- Joint efficiency
  - Ratio of the combined of the beam with joint to the stiffness of the beam alone (assuming a rigid joint)
  - $K_{\text{joint}} \gg K_{\text{beam}} : f \approx 1$
  - Very low efficiency: joint deformation dominant

$$f = \frac{K_{\text{beam+joint}}}{K_{\text{beam}}} \quad \left. \begin{array}{l} K_{\text{beam+joint}} = \frac{K_{\text{joint}} K_{\text{beam}}}{K_{\text{joint}} + K_{\text{beam}}} \\ K_{\text{beam}} = \frac{M}{\theta} = \frac{2EI}{L} \end{array} \right\} \rightarrow f =$$





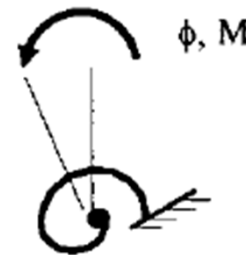
# Example: Hinge Pillar-to-Rocker Joint

---

- $I = 4.15 \times 10^6 \text{ mm}^4$ :  $h=100$ ,  $w=50$ ,  $t=1$
- $L = 1000 \text{ mm}$
- $K_{\text{joint}} = 0.2 \times 10^6 \text{ Nm/rad} = 0.2 \times 10^9 \text{ Nmm/rad}$

# Strain Energy and Stiffness

- If initial guess does not meet the required bending stiffness, which beams to adjust first and is it the minimum mass solution?
  - Increase the performance of the structural element with the highest fraction of strain energy
- Strain energy



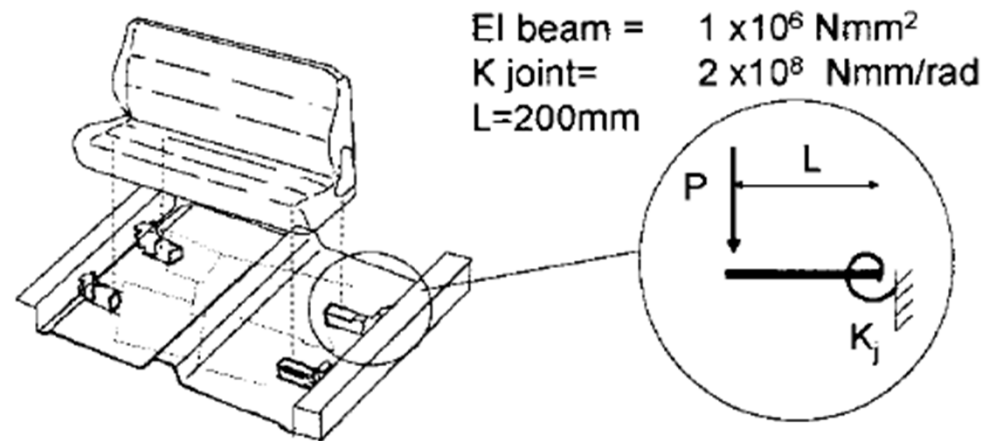
$$M = M_1 + \frac{M_2 - M_1}{L} x$$

$$e_{\text{beam}} =$$

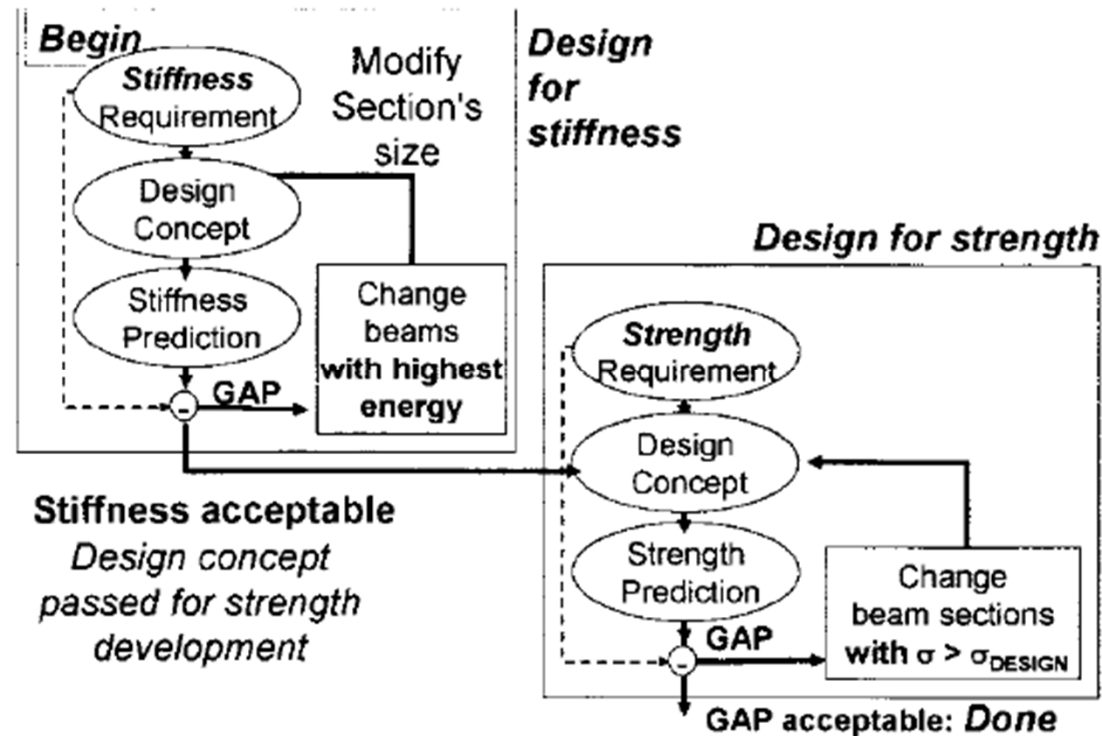
$$e_{\text{joint}} =$$

# Example: Seat Mount System

- Assume that the current system design does not meet the (vertical) stiffness requirement, and we wish to know which element to change (beam or joint) to increase the stiffness of the system.



# Iterative Process



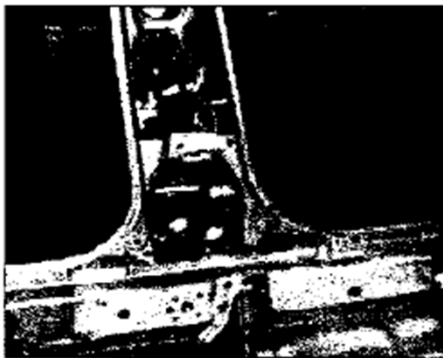
- If stress is greater than the design stress
  - Increase the buckling design stress by inhibiting elastic plate buckling
  - Choose a material with increased yield
  - Reduce the stress by increasing the section properties

## 4.5 Principles of Good Joint Design

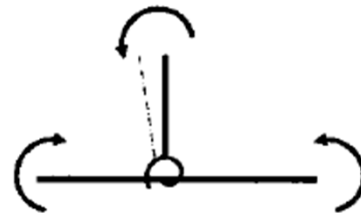
- Two cases for joint bending stiffness
  - In-plane bending: global body bending
  - out-of-plane bending: global body torsion



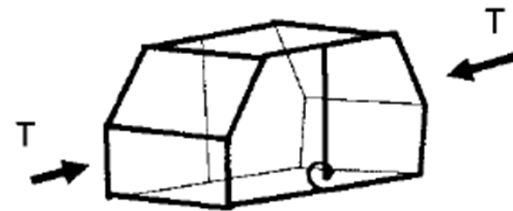
B Pillar to  
Rocker  
Joint



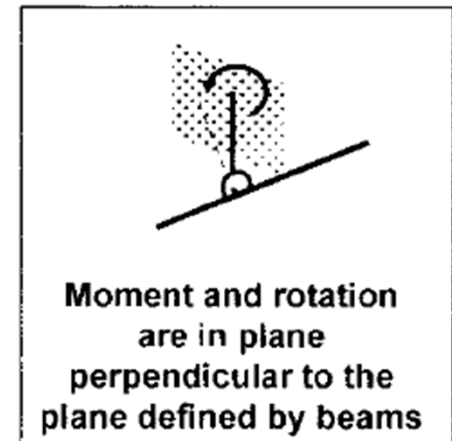
In-Plane Bending



Moments and  
rotations are in plane  
defined by beams



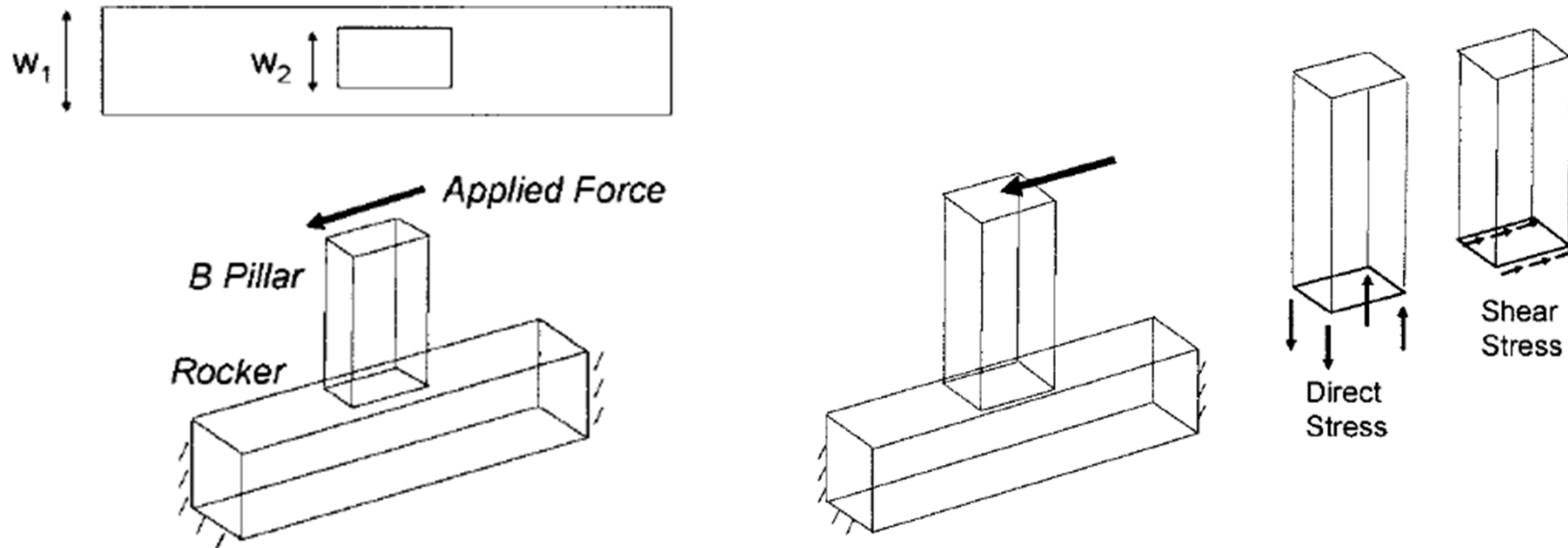
Out-of-Plane Bending



Moment and rotation  
are in plane  
perpendicular to the  
plane defined by beams

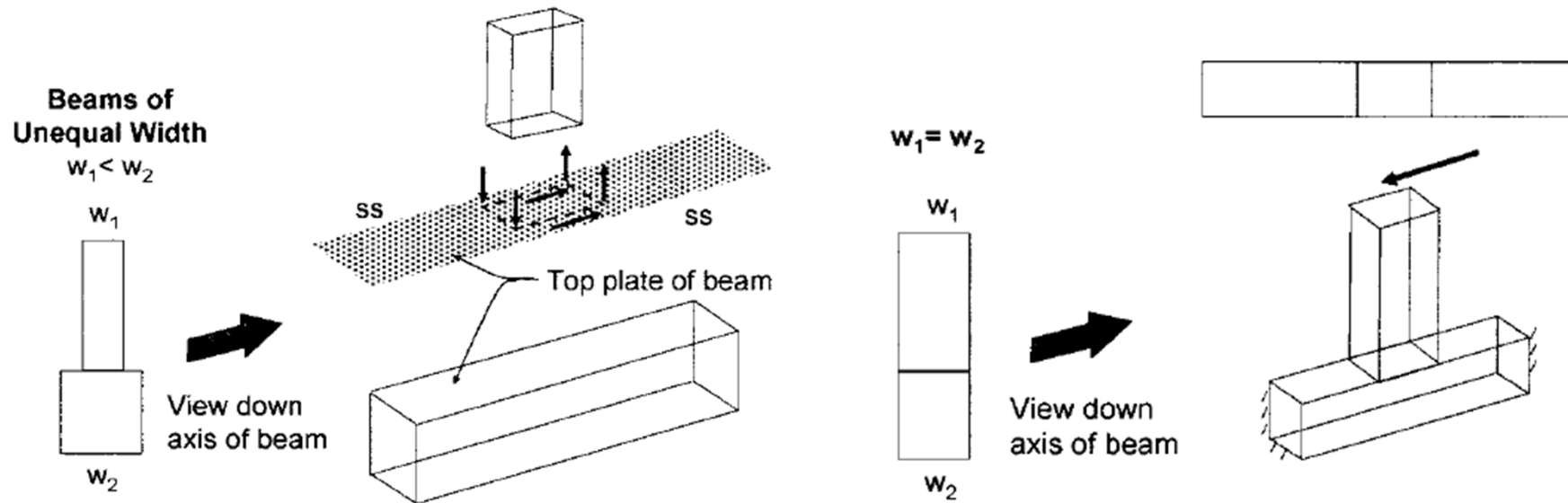
# Simplified Joint of Thin-Walled Sections

- Under in-plane bending
  - Thin walled section: relatively high stress at the corners of the section than the center of the walls
  - Assuming that all of the stress is taken by the corners of the section



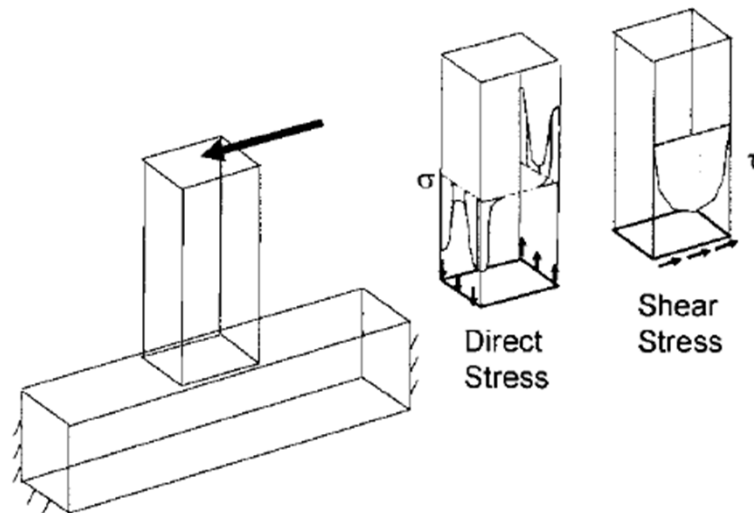
# Guideline for Stiff Joint Construction

- For high joint stiffness, the shear walls of the connected beam should be aligned at the joint and flow smoothly from one beam to another

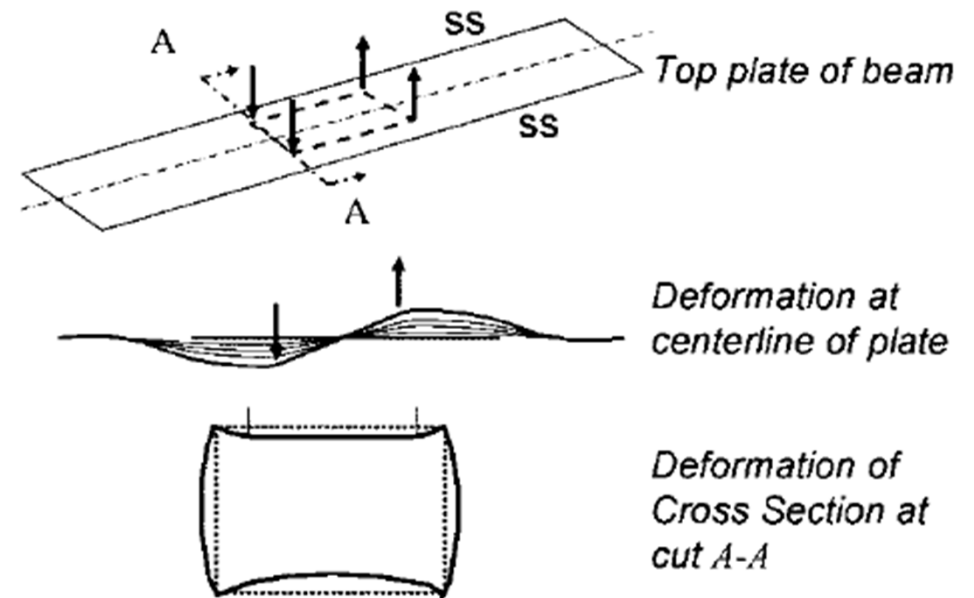


# Joint Bending Stiffness

- Stress distribution

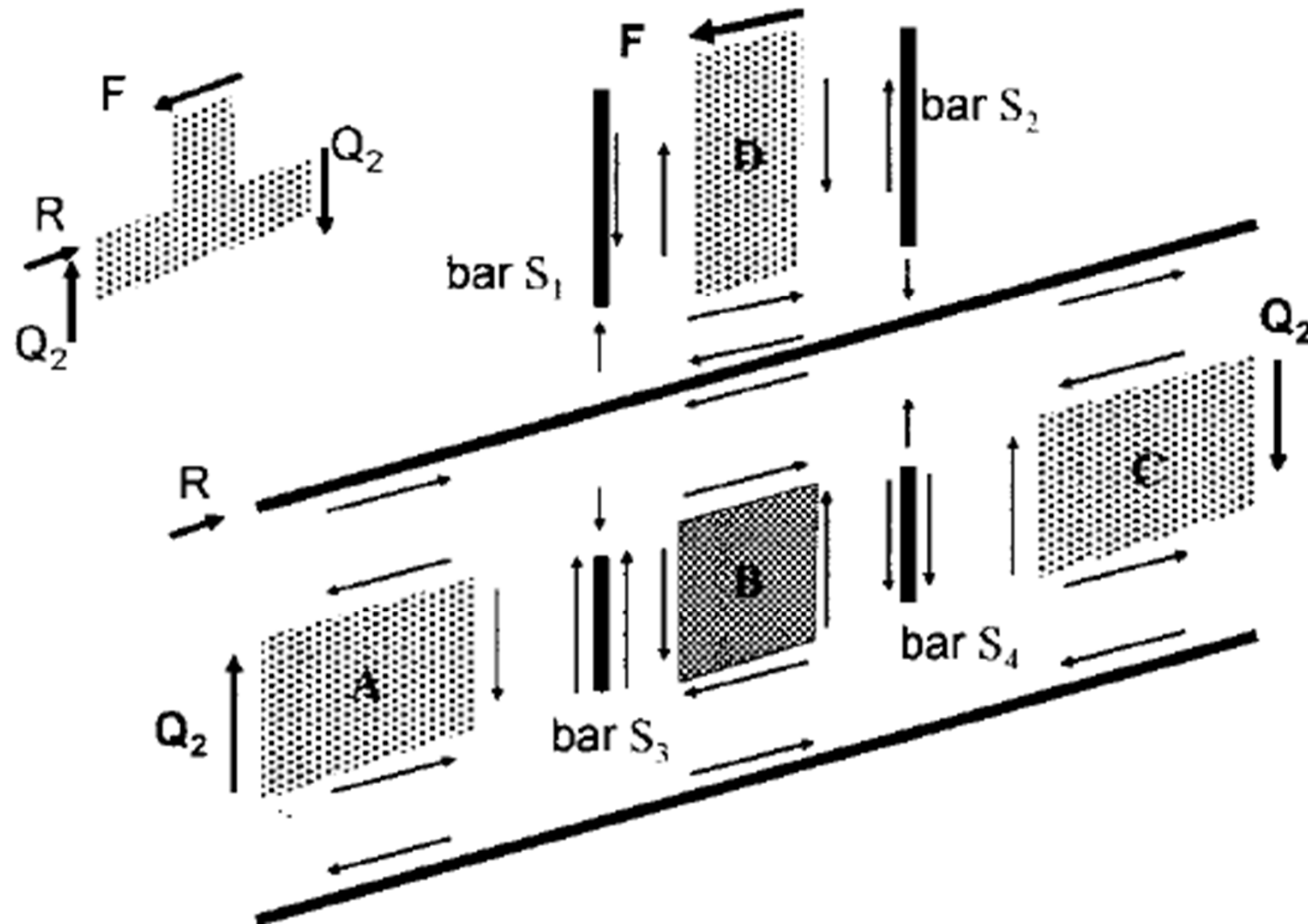


- Deformation



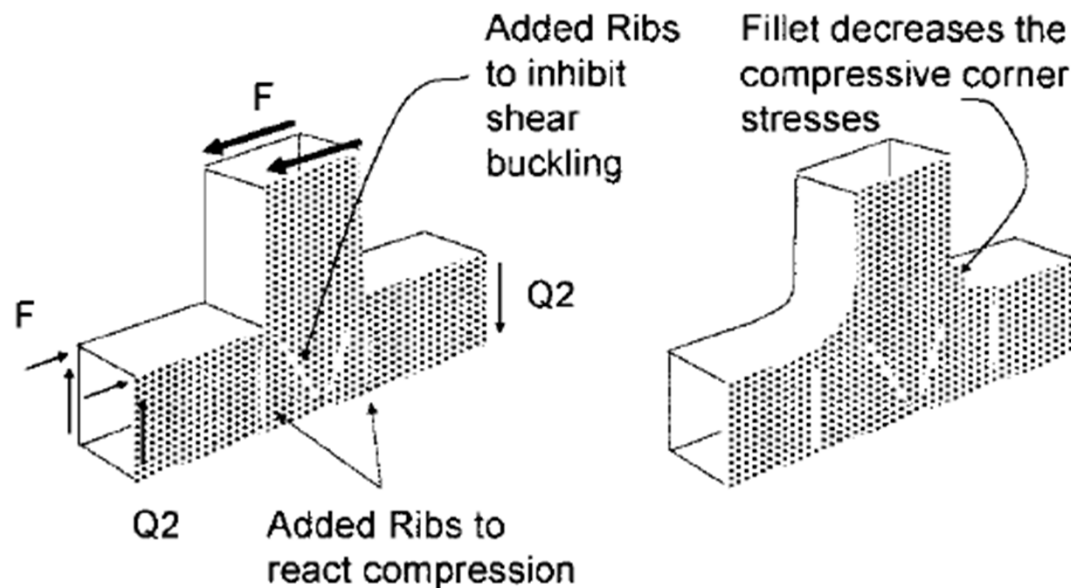


# In-Plane Bending



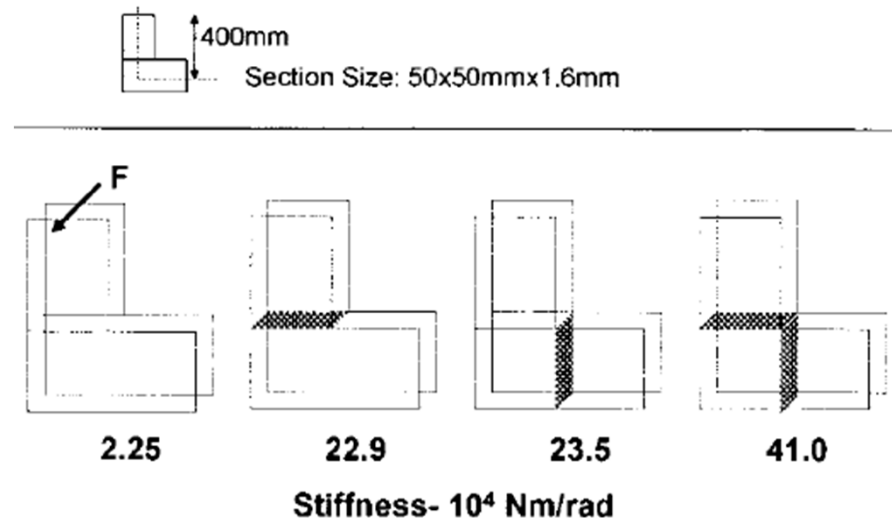
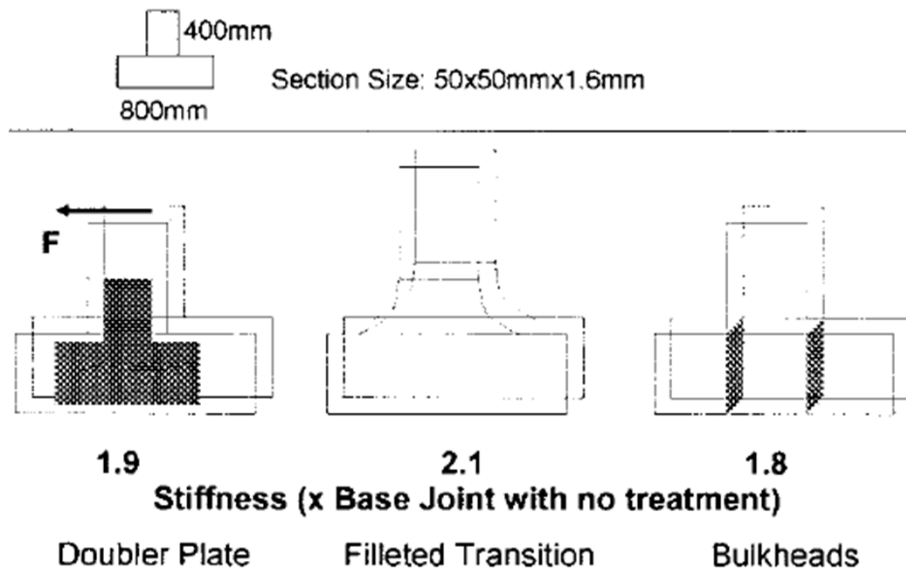
# Modification to Joint Shear Wall

- Concerns: high compression, buckling
  - Rib pattern
    - V rib pattern: increase the shear buckling stress of the panel
    - Vertical rib: provide a path for the compressive load
  - Increase of the span: provide a filleted transition



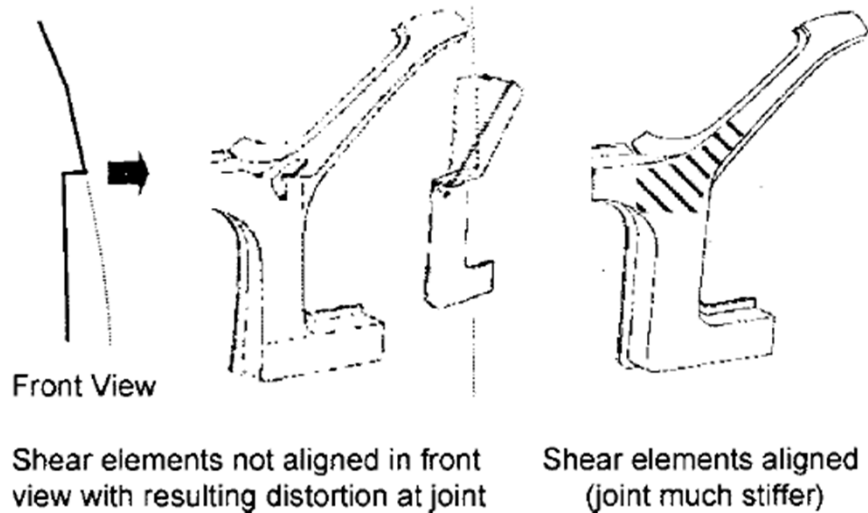
# Experimental Data

- Joint rigidity in-plane bending
  - Mass penalty
  - Additional load path
- Effect of added bulkhead on out-of-plane joint rigidity

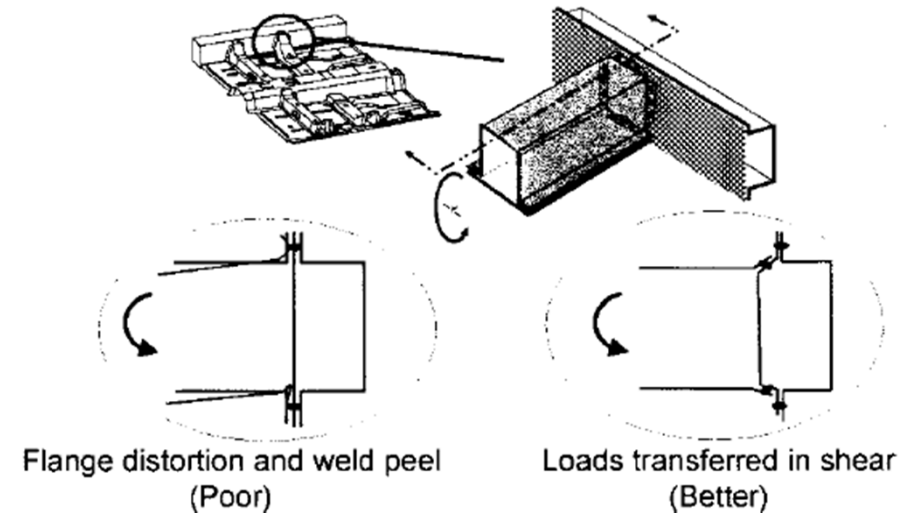


# Example: Body Joint Design (1)

- A pillar to Hinge pillar joint

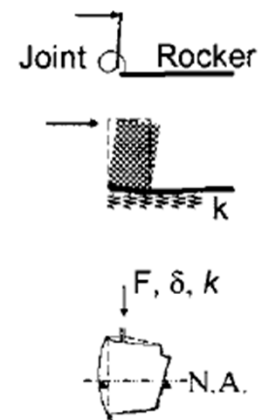
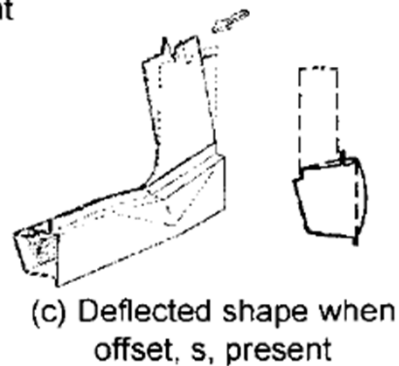
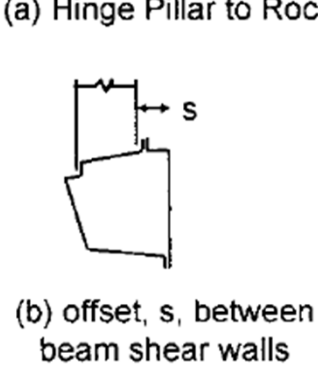
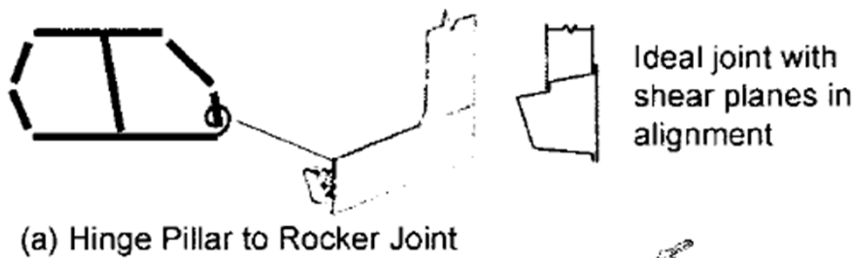


- Floor Cross Member to Rocker joint

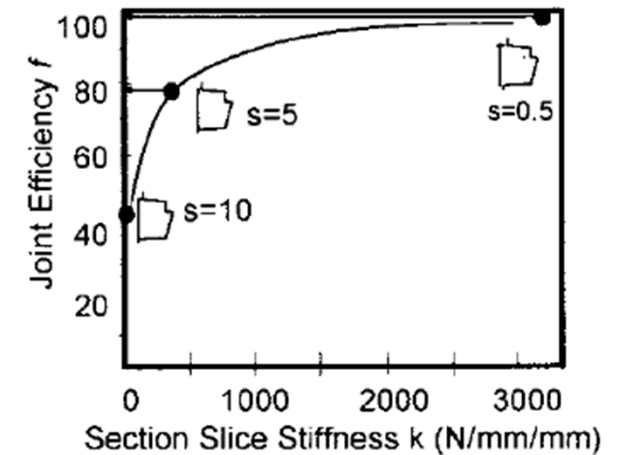


# Example: Body Joint Design (2)

- Hinge pillar to Rocker joint



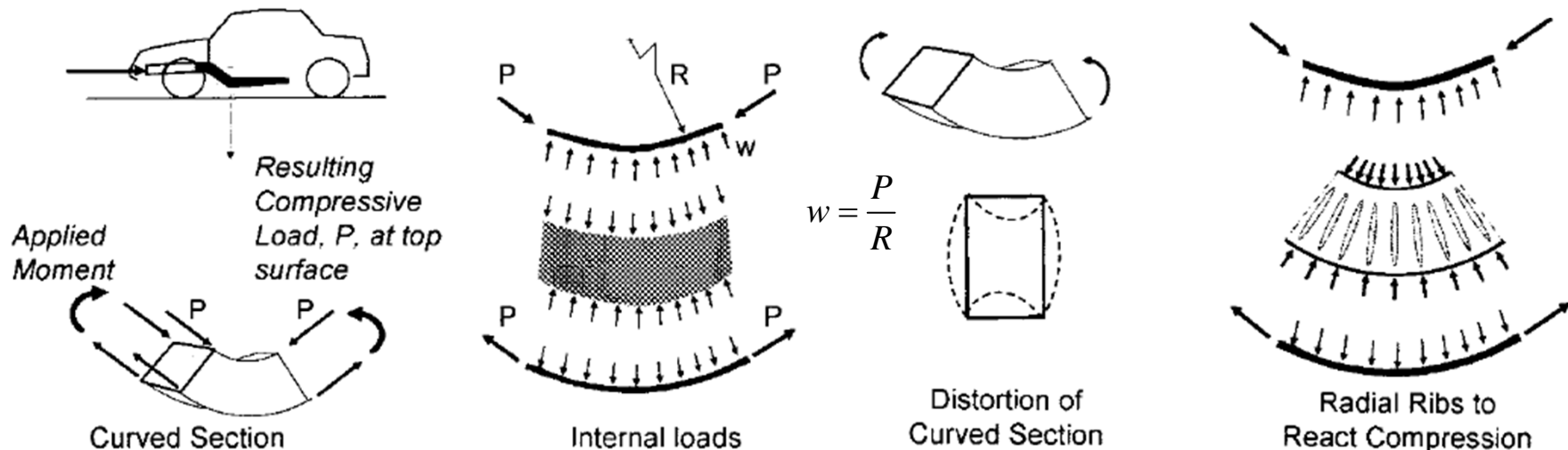
Local distortion of rocker behaving as a joint



Joint efficiency as a function of section stiffness

# Joint Behavior at Geometric Transitions

- Large localized deflection
  - Beam with two relatively straight portions and a central curved portion
- Treat the transition area as a joint stiffness
  - Increase the radius of curvature
  - Include radial ribs which react the compressive web loads
  - Smooth and gradual transition



# Geometric Transition in a Straight Beam

- Clearing some component of the vehicle
- Smooth transition: stiffer effective joint

