Column space-1

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I.1 column space of A contains all vectors Ax

$$A=\begin{bmatrix}2&1&3\\3&1&4\\5&7&12\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=x_1\begin{bmatrix}2\\3\\5\end{bmatrix}+x_2\begin{bmatrix}1\\1\\7\end{bmatrix}+x_3\begin{bmatrix}3\\4\\12\end{bmatrix}$$

all Ax = column space $C(A)$

① dot product (row) x : $2x_1+x_2+3x_3$

$$A=\begin{bmatrix}1&3&8\\1&3&8\\1&3&8\end{bmatrix}=\begin{bmatrix}1\\1\\1\end{bmatrix}\begin{bmatrix}1&3&8\end{bmatrix}$$

$C(A)$ = line , $\underline{\text{rank}(A)=1}$

number of independent columns

basis for column space

basis for row space

$$=\begin{bmatrix}2&1\\3&1\\5&7\end{bmatrix}\begin{bmatrix}1&0&1\\0&1&1\end{bmatrix}$$

$C\ (3\times2)$ $R\ (2\times3)$

column rank = 2 = row rank

Column space-2

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$A = CR$

row space of A ? all combinations rows $\to$ column space of A^T $= C(A^T)$

column space of $A = C(A)$

$$\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ 5 & 7 & 12 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 1 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$[2\ 1\ 3] = 2[1\ 0\ 1] + 1[0\ 1\ 1]$

random sampling of A : $x = \text{rand}(m, 1)$

10^5

Ax, $ABCx \to A(BCx)$

column space of A? Yes

Matrix multiplication

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Matrix Multiplication

$m(np)$ multiplications

$$AB = \begin{bmatrix} \overset{n}{\text{row } A} \end{bmatrix} \begin{bmatrix} \text{column } B \;\; p \end{bmatrix} = \text{dot product}$$

(row) x (column)

$\downarrow$

$$AB = \begin{bmatrix} \; | \; \end{bmatrix} \begin{bmatrix} \; — \; \end{bmatrix} = (\text{col } 1)(\text{row } 1) + \cdots + (\text{col } k)(\text{row } k) + \cdots + (\text{col } n)(\text{row } n)$$

$n(mp)$

$(m \times 1) \; (1 \times p)$

$(m \times n) \; (n \times p) = (m \times p)$

Sum of **rank 1** matrices

Factorization 1

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① $A = LU$ elimination >> lu(A) $b = Ax = L(Ux)$ $[Ux] = [b]$ $Ux = \square$ $x = \square$

② $A = QR$ Gram-Schmidt (Q : orthogonal or orthonormal)

③ $S = Q\Lambda Q^T$ symmetric matrix (eigenvalue λ_i : real, eigenvector q_i : orthonormal)

$$= \begin{bmatrix} q_1 & \cdots & q_n \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} \begin{bmatrix} q_1^T \\ \vdots \\ q_n^T \end{bmatrix}$$

$$= (Q\Lambda)Q^T = \text{sum of rank 1} = \lambda_1 \underbrace{q_1 q_1^T}_{\text{symmetric}} + \lambda_2 q_2 q_2^T + \cdots + \lambda_n q_n q_n^T = S$$

spectral theorem

(column of $Q\Lambda$) (row of Q^T)

$$\begin{bmatrix} q_1 & \cdots & q_n \end{bmatrix} \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix}$$

$$\boxed{S q_1 = \lambda_1 q_1} \underbrace{q_1^T q_1}_{=1} + \lambda_2 q_2 \underbrace{q_2^T q_1}_{=0} + \cdots$$

eigenvalue problem

$Ax = \lambda x$

Factorization 2

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4 $A = X \Lambda X^{-1}$

5 $A = U \Sigma V^T = (\text{orth})(\text{diag})(\text{orth}) = \text{SVD}$

4 Fund. Subspaces

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A: m×n rank r

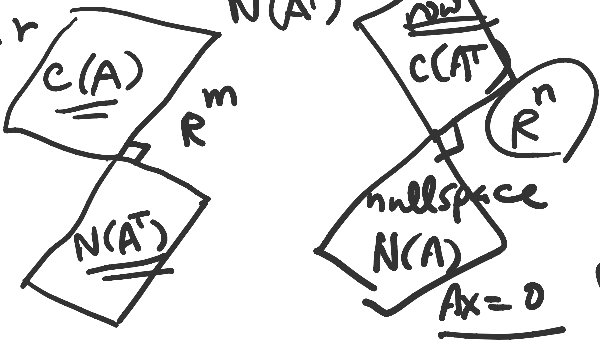
column space $C(A)$ dim r

row " $C(A^T)$ "

null space $N(A)$

$N(A^T)$

dim r



nullspace = all solutions of $Ax=0$

$A(cx)=0$

$A(x+y)=0$

$$A=\begin{bmatrix}1&2&4\\2&4&8\end{bmatrix}\begin{bmatrix}0&4\\-2&0\\1&-1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$m=2$

$n=3$

$r=1$

	row	nullspace
dim	r	n−r
	1	2

$Ax=0$

$\underline{x}$ is orthogonal to $\underline{\text{row of } A}$

n−r n