

Homework #2 (due 4/8/2020)

1. Compute the eigenvalues and eigenvectors of A and A^{-1} . Check the trace !

$$A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad A^{-1} = \begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix}.$$

A^{-1} has the _____ eigenvectors as A . When A has eigenvalues λ_1 and λ_2 , its inverse has eigenvalues _____.

2. **The eigenvalues of A equal the eigenvalues of A^T .** This is because $\det(A - \lambda I)$ equals $\det(A^T - \lambda I)$. That is true because _____. Show by an example that the eigenvectors of A and A^T are *not* the same.
3. (a) Factor these two matrices into $A = X\Lambda X^{-1}$:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}.$$

(b) If $A = X\Lambda X^{-1}$ then $A^3 = (\quad)(\quad)(\quad)$ and $A^{-1} = (\quad)(\quad)(\quad)$.

4. For which numbers b and c are these matrices positive definite?

$$S = \begin{bmatrix} 1 & b \\ b & 9 \end{bmatrix} \quad S = \begin{bmatrix} 2 & 4 \\ 4 & c \end{bmatrix} \quad S = \begin{bmatrix} c & b \\ b & c \end{bmatrix}.$$

With the pivots in D and multiplier in L , factor each A into LDL^T .

5. Find the 3 by 3 matrix S and its pivots, rank, eigenvalues, and determinant:

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} S \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 4(x_1 - x_2 + 2x_3)^2.$$

6. Compute the three upper left determinants of S to establish positive definiteness. Verify that their ratios give the second and third pivots.

$$\text{Pivots} = \text{ratios of determinants} \quad S = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 3 \\ 0 & 3 & 8 \end{bmatrix}.$$

7. A symmetric matrix $S = S^T$ has orthonormal eigenvectors v_1 to v_n . Then any vector x can be written as a combination $x = c_1 v_1 + \cdots + c_n v_n$. Explain these two formulas :

$$x^T x = c_1^2 + \cdots + c_n^2 \quad x^T S x = \lambda_1 c_1^2 + \cdots + \lambda_n c_n^2.$$

8. Find the σ 's and v 's and u 's in the SVD for $A = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix}$.