1.

The closest rank 1 approximations are $A=\begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $A=\begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$ $A=\frac{3}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

2.

A is invertible so $A^{-1} = V\Sigma^{-1}U^{\mathrm{T}}$ has singular values $1/\sigma_1$ and $1/\sigma_2$. Then $||A^{-1}||_2 =$ max singular value $= 1/\sigma_2$. And $||A^{-1}||_F^2 = (1/\sigma_1)^2 + (1/\sigma_2)^2$.

3.

$||v||_2^2 = v_1^2 + \cdots + v_n^2 \leq (\max|v_i|)\,(|v_1| + \cdots + |v_n|) = ||v||_\infty ||v||_1$

4.

The link from $A = U\Sigma V^{\mathrm{T}}$ to $A^+ = V\Sigma^+ U^{\mathrm{T}}$ shows that A and A^+ have the same number (the rank r) of nonzero singular values. If A is square and $Ax = \lambda x$ with $\lambda \neq 0$, then $A^+ x = \frac{1}{\lambda}x$. Eigenvectors are the same for A and A^+, eigenvalues are λ and $1/N$ (except that $\lambda = 0$ for A produces $\lambda = 0$ for A^+ !).

5.

6.

Complete the Gram-Schmidt process in Problem 8 by computing $q_1 = a/||a||$ and $A_2 = b - (b^T q_1)q_1$ and $q_2 = A_2/||A_2||$ and factoring into QR.

$$q_1 = a/||a|| = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix}$$

$$A_2 = b - (b^T q_1)q_1 = \begin{bmatrix}4\\0\end{bmatrix} - \left(\begin{bmatrix}4 & 0\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix}\right)\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}4\\0\end{bmatrix} - \frac{4}{\sqrt{2}}\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} = \begin{bmatrix}4\\0\end{bmatrix} - \begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}2\\-2\end{bmatrix}$$

$$q_2 = A_2/||A_2|| = \begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{bmatrix}$$

$$\begin{bmatrix}1 & 4\\1 & 0\end{bmatrix} = \begin{bmatrix}q_1 & q_2\end{bmatrix}\begin{bmatrix}||a|| & 2\sqrt{2}\\0 & ||A_2||\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}\sqrt{2} & 2\sqrt{2}\\0 & 2\sqrt{2}\end{bmatrix}$$

$$Q = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}$$

$$R = \begin{bmatrix}\sqrt{2} & 2\sqrt{2}\\0 & 2\sqrt{2}\end{bmatrix}$$

7. (1)

With $b = (0, 8, 8, 20)$ at $t = (0, 1, 3, 4)$, set up and solve the normal equations $A^T A\hat{x} = A^T b$. For the best straight line in Figure II.3a, find its four height p_i and four errors e_i. What is the minimum squared error $E = e_1^2 + e_2^2 + e_3^2 + e_4^2$?

$$A^T A\hat{x} = \begin{bmatrix}4 & 8\\8 & 26\end{bmatrix}\begin{bmatrix}\hat{C}\\\hat{D}\end{bmatrix} = \begin{bmatrix}36\\112\end{bmatrix}$$

$$\hat{x} = \begin{bmatrix}\hat{C}\\\hat{D}\end{bmatrix} = \begin{bmatrix}4 & 8\\8 & 26\end{bmatrix}^{-1}\begin{bmatrix}36\\112\end{bmatrix} = \frac{1}{20}\begin{bmatrix}13 & -4\\-4 & 2\end{bmatrix}\begin{bmatrix}36\\112\end{bmatrix} = \begin{bmatrix}1\\4\end{bmatrix}$$

$p_i = \hat{C} + \hat{D}t_i$ so our p_is will be 1, 5, 10, 13. This makes our e_is be 1, 3, 2, 7 (signs not given) for a total minimum squared error of $E_1 = 63$.

(2)