<u>Homework #4 (due 5/27/2020)</u>

1. Given positive numbers $a_1, \ldots, a_n$ find positive numbers $p_1 \ldots, p_n$ so that

$$p_1 + \cdots + p_n = 1 \quad \text{and} \quad V = \frac{a_1^2}{p_1} + \cdots + \frac{a_n^2}{p_n} \text{ reaches its minimum } (a_1 + \cdots + a_n)^2.$$

2. If $M = \mathbf{1}\,\mathbf{1}^{\mathrm{T}}$ is the n by n matrix of 1's, prove that $nI - M$ is positive semidefinite. Problem 3 was the energy test. For Problem 4, find the eigenvalues of $nI - M$.

3. What is the gradient descent equation $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k - s_k \nabla f(\boldsymbol{x}_k)$ for the least squares problem of minimizing $f(\boldsymbol{x}) = \frac{1}{2}||A\boldsymbol{x} - \boldsymbol{b}||^2$?