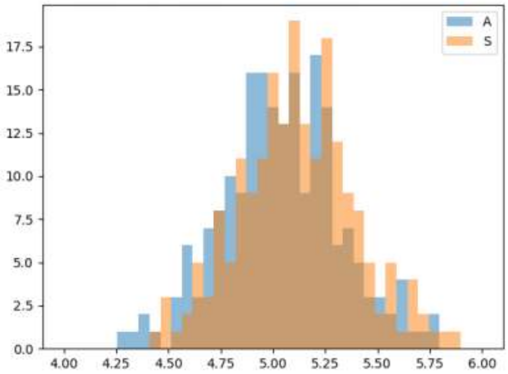
1.

Histogram of max eigenvalue values for 10x10 random matrices A and $S = \frac{1}{2}(A + A^T)$



It seems like the histogram of max eigenvalue values represents some distribution where 5.00 is the mean, and the values overall fall in a range not exceeding 6.00 or falling below 4. The code is below.

```
import numpy as np
from numpy import linalg as LA
import matplotlib.pyplot as plt

def generate_matrix():
        mat = np.random.rand(10,10)
        return mat

def get_max_eigenvalue(mat):
        w, v = LA.eig(mat)
        return np.amax(w)

def run_problem():
        A_maxes = []
        S_maxes = []
        for i in range(200):

                A = generate_matrix()
                S = 0.5 * (A + np.transpose(A))
                A_maxes.append(get_max_eigenvalue(A))
                S_maxes.append(get_max_eigenvalue(S))
        return [A_maxes, S_maxes]

A_maxes, S_maxes = run_problem()
np.histogram(A_maxes)

bins = np.linspace(4, 6, 40)
plt.hist(A_maxes, bins, alpha=0.5, label='A')
plt.hist(S_maxes, bins, alpha=0.5, label='S')
plt.legend(loc='upper right')
plt.show()
```

2.

```
A = triu(ones(1000));
G1 = normrnd(0,1,1000,10);  % 1000 by 10 Gaussian random matrix G1
G2 = normrnd(0,1,1000,100); % 1000 by 100 Gaussian random matrix G2

% Exact or Actual SVD
[u,s,v] = svd(A);

% Randomized or Approximate SVD with G1
Y1 = A*G1;
[Q1,R1] = qr(Y1);
[U1,D1,V1] = svd(Q1'*A); W1 = Q1*U1; % A = (Q1*U1)*D1*V1' = W1*D1*V1'

% Randomized or Approximate SVD with G2
Y2 = A*G2;
[Q2,R2] = qr(Y2);
[U2,D2,V2] = svd(Q2'*A); W2 = Q2*U2; % A = (Q2*U2)*D2*V2' = W2*D2*V2'

s11 = diag(s);  display(s11(1:10)) % The 10 Largest Singular Values From Actual SVD
```

```
 636.93814767091
212.312890336131
127.387943537033
90.9916125292626
70.7714867858701
57.9041816178516
48.9960875295752
42.4635200898666
37.4680581293865
 33.524299918613
```

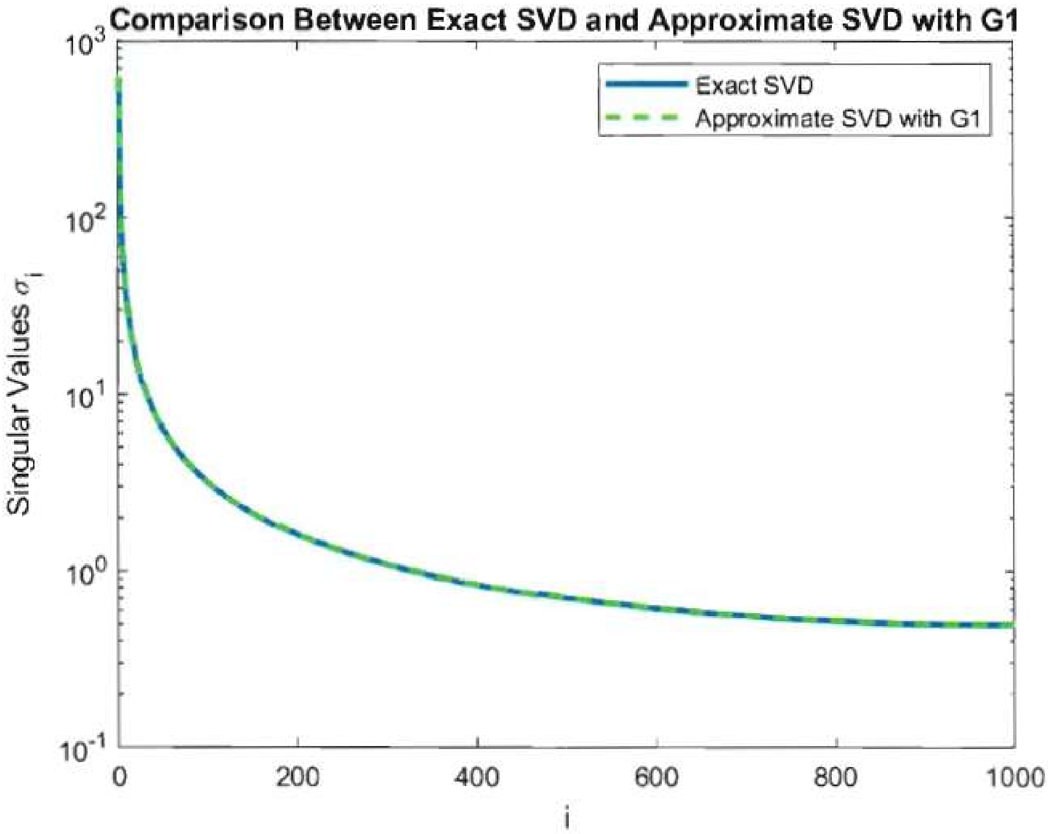
```
D11 = diag(D1); display(D11(1:10)) % The 10 Largest Singular Values From Approximate SVD with G1
```

```
636.938147670908
212.312890336131
127.387943537033
90.9916125292626
70.7714867858701
57.9041816178517
48.9960875295753
42.4635200898666
37.4680581293866
 33.524299918613
```

```
D22 = diag(D2); display(D22(1:10)) % The 10 Largest Singular Values From Approximate SVD with G2
```

```
636.938147670908
 212.31289033613
127.387943537033
90.9916125292626
70.7714867858701
57.9041816178516
48.9960875295752
42.4635200898666
37.4680581293866
 33.524299918613
```

```
% Comparison Between Actual SVD and Randomized SVD with G1
% Plot of the Singular Values of the Actual SVD and Randomized SVD with G1
figure();
semilogy(s11,'linewidth',2)
hold on
semilogy(D11,'--g','linewidth',1.5)
grid on
title('Comparison Between Exact SVD and Approximate SVD with G1')
legend('Exact SVD','Approximate SVD with G1','location','Northeast')
xlabel('i')
ylabel('Singular Values \sigma_i')
```



As we can see, both SVD's, the actual SVD and randomized SVD with G1, result in approximately same Singular Values.