Contents
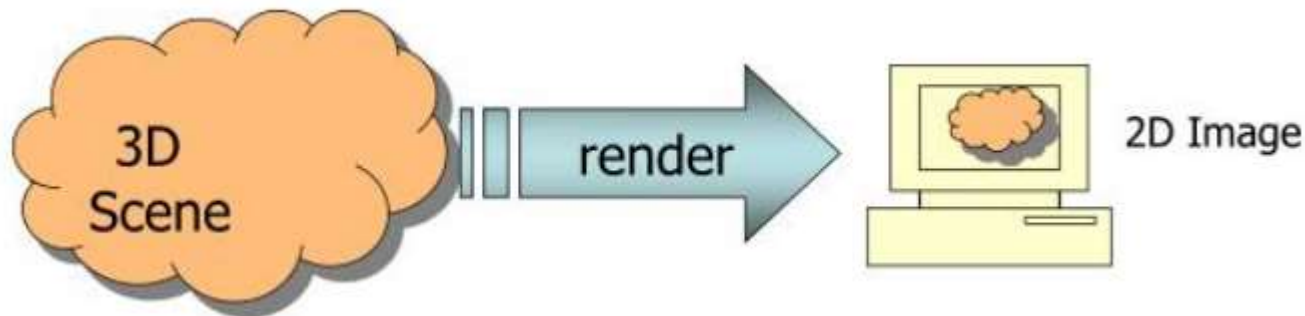
- Graphics Library
- Display 2D/3D objects
- Coordinate Systems
- Window and Viewport
- Transformation

Graphics Rendering Pipeline

- Rendering: the conversion of a scene into an image
  - Scene is composed of "models" in 3D space
  - Models are composed of "primitives" supported by graphics packages such as OpenGL
  - Models entered by hand or created by a program

Graphics Rendering Pipeline

- Modeling (local) coordinates
- World coordinates
- Normalized (device-independent) coordinates
- Device (screen) coordinate
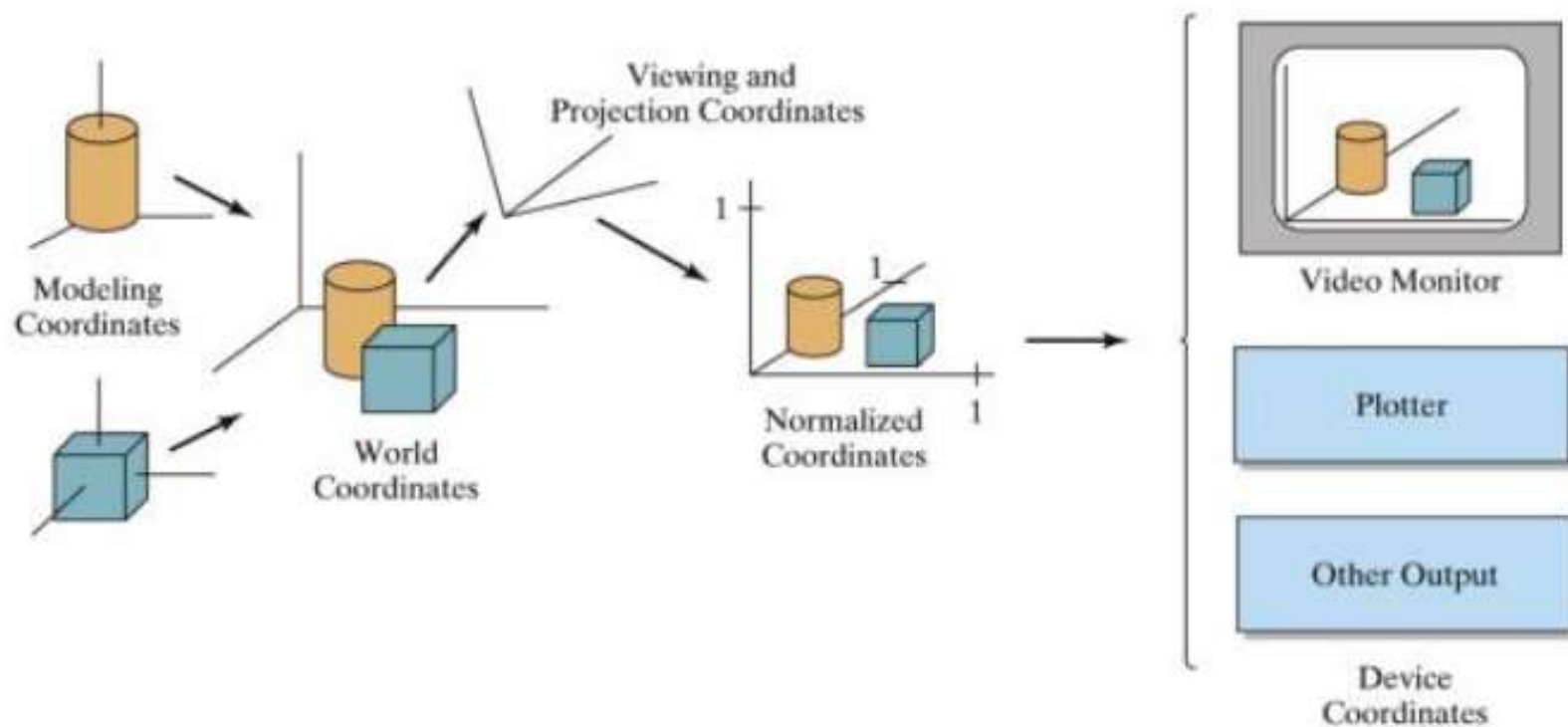
Functions of a Graphic Package

- **Graphics Library such as Core, OpenGL, DirectX**
  - Provide primitives for graphic description
  - Build and maintain graphic representation models
  - Provide primitives for viewing operations
  - Support user interaction with application program
  - Interact directly with users to allow them modify viewing parameters, if possible

Graphic System
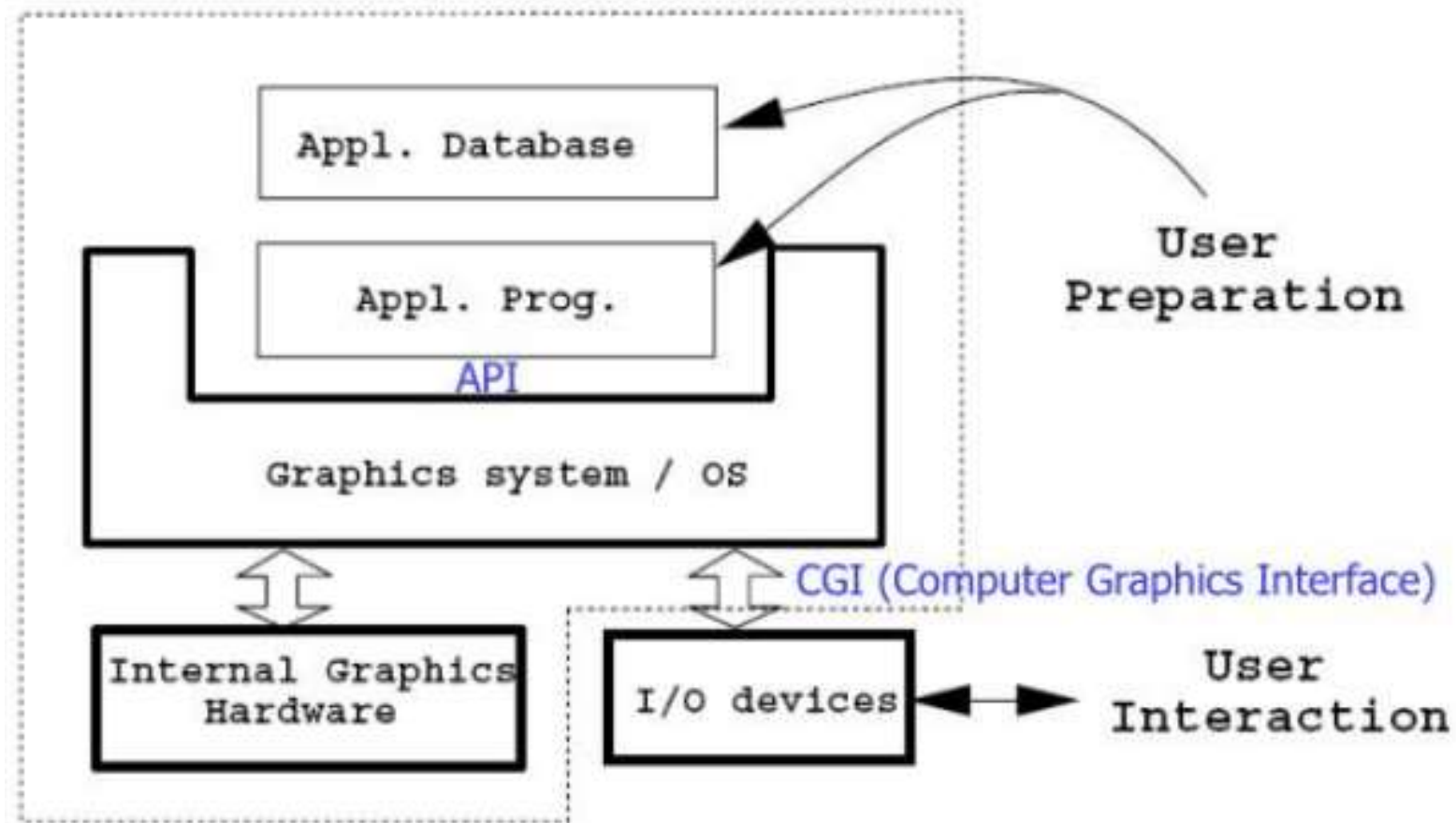
Graphics system: a library of graphics functions

General Graphics Packages

- Graphics packages are device-independent
- Official standards
  - Core: ACM SIGGRAPH 1977, US
  - GKS: ANSI85, 2D, Europe
  - GKS-3D: ANSI88
  - PHIGS: ANSI88 – Hierarchical structures
  - PHIGS+: ISO92
- Non-official standards
  - X Window System, PEX
  - Silicon Graphics OpenGL (1992)
  - Open Inventor
  - Microsoft DirectX
  - Sun Microsystems VRML

Bit-Mapped vs. Object-Based Graphics (1)

- **Painting programs**
  - Bit-mapped representation
  - Adobe Photoshop, bit-map fonts
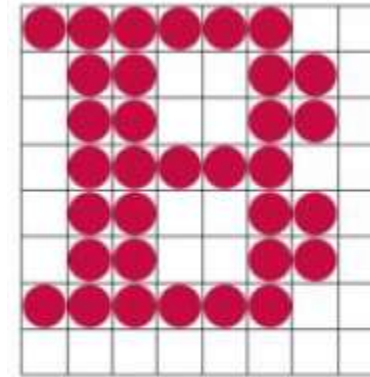
- **Drawing programs**
  - Object-base representation
  - Powerpoint, outline fonts

Bit-Mapped vs. Object-Based Graphics (2)

- Bit-mapped memory
  - Represent graphical images and patterns by assigning a block of memory for the direct storage of the intensity patterns
- Objectiveness of a line, polygon or brush stroke is lost
- Limitations
  - Pixel democracy: all pixels are created equal once, they are set in a bit-mapped image
  - No support of images requiring precision or numerical dimensioning device dependency

- Mathematical representation of lines, rectangles, ovals, polynomial curves, etc.
- Objectiveness is maintained
  - Objects designed may have a hierarchical structure
  - Easily editable
- Difficulties
  - Representation is more complex

Device Driver vs. Graphics Library

- ## Device Driver
  - A set of codes that controls a physical device directly with device-specific commands
  - Device dependent

Application Program	⇒	Driver	⇒	Input/Output Devices

- ## Graphics Library (cf. math library, e.g. draw a line)
  - A set of subroutines for graphics input/output functionality
  - Each subroutine == a series of a device driver commands
  - High level language binding (C, C++, FORTRAN, JAVA, etc)

Application Program	⇒	Graphic Library	⇒	Driver	⇒	Input/Output Devices
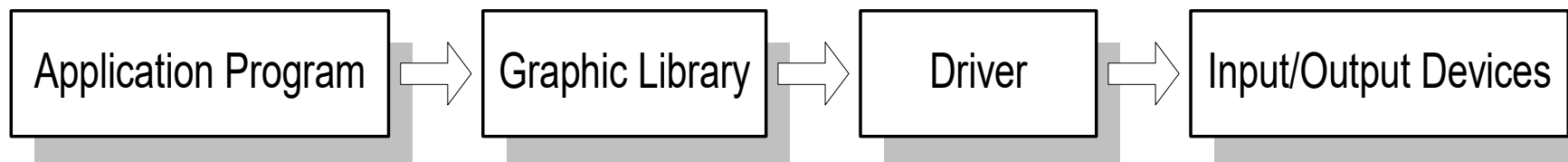
Standard Graphics Library (1)

- CORE (1977)
  - ACM SIGGRAPH (revised '79)
  - Poor support to raster concept
- GKS (Graphical Kernel System, 1984)

  - ISO

  - GKS-3D : Extension to 3D
- PHIGS (Programmer's Hierarchical Graphics System, 1984)
  - ISO
  - Poor capabilities for dynamic graphics and user interaction
  - PHIGS+: Raster concepts, rendering capabilities, parametric curves and surfaces
  - PEX(1987) = PHIGS + X-Window (PHIGS extensions to X)

Standard Graphics Library (2)

- **OpenGL** (1996)
  - *de facto* standard in the industry
  - Originated from IRIS GL developed by Silicon Graphics, Inc.
  - Governed by ARB (Architecture Review Board)
    - DEC, Evans & Sutherland, HP, IBM, Intel, Intergraph, Microsoft, SGI, Sun Microsystems
  - Available on all UNIX workstations as well as the Windows 95 and NT PC's
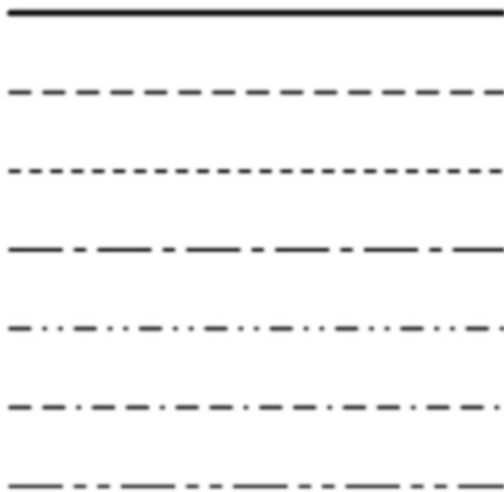 - C, C++, Ada and FORTRAN language bindings

Output Primitives

- Graphics elements displayed by a graphics library
- Common output primitives
  - Line / Polyline
  - Polygon
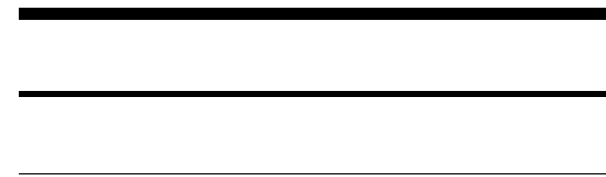 - Marker / Polymarker
  - Text

Line / Polyline

- A straight line segment defined by two end points
- Attributes : type, thickness, color, etc.
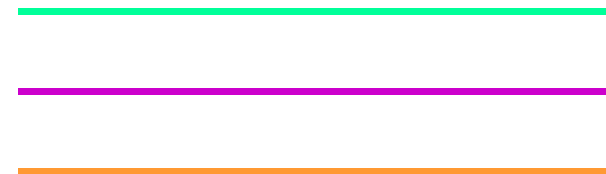- Polyline is default in GKS, PHIGS, and OpenGL
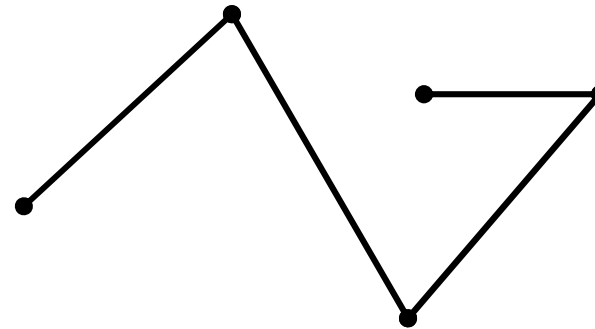
Type

Thickness

Color

Polyline Capability

- A set of connected line segments defined by a point sequence
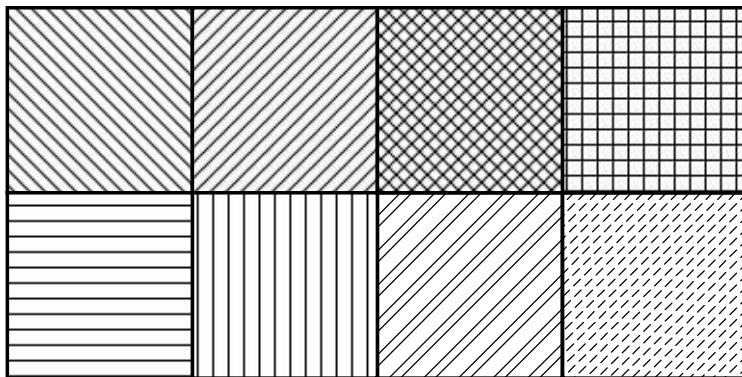- Default line drawing in GKS, PHIGS and OpenGL

$$[p] = \begin{bmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ X_n & Y_n & Z_n \end{bmatrix}$$
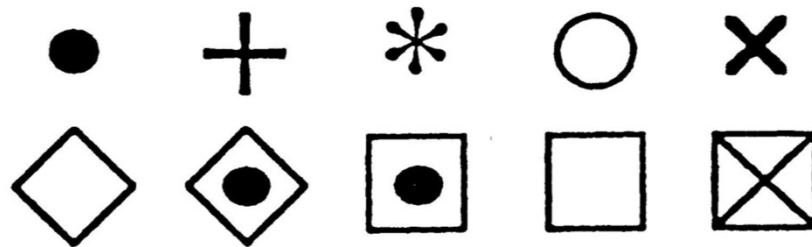
Polygon

- A set of straight line segments closing a region on a plane (inside and outside information)

- The first and the last points for the polyline function are coincident

- Attribute: fill type

Marker / Polymarker

- Symbol placed at specific location
- Attribute: type

- Polymarker is a default in GKS and PHIGS
- OpenGL: not support explicitly, but any marker can be defined in a bitmap

Text

- A character string placed at a given location
- Annotation text (screen text or 2-D text) and 3-D Text
- Attribute: size, font (HW and SW font), ratio of height to width, slant angle, direction of text line, color, thickness, etc.

A B C **ABC** *ABC* **ABC** ABC
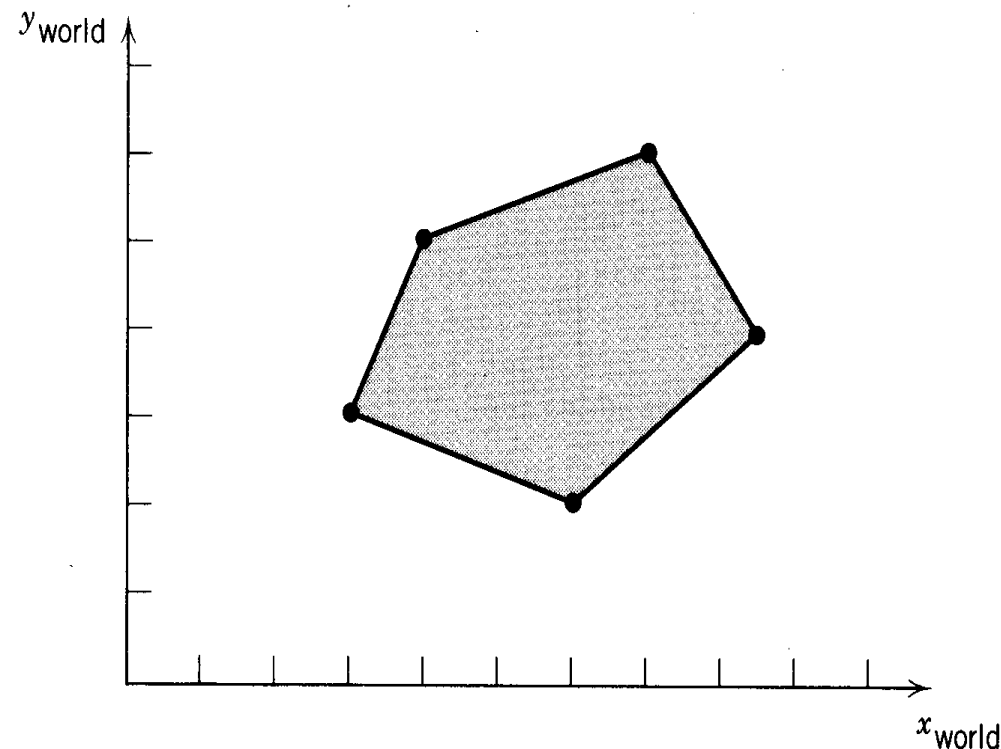
ABC

ABC

Display 2D Object

- Geometric Model $\rightarrow$ Graphical Image
  - (1) Convert the model into a screen coordinate system
  - (2) Decide how much the model is to appear and where it should appear on the screen (2D Clipping)

- 2D Coordinate Systems
  - World coordinate (WC) system
  - Device coordinate (DC) system
  - Normalized (Virtual) coordinate (NC) system
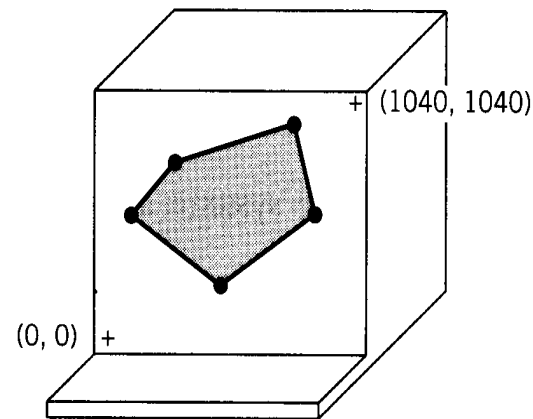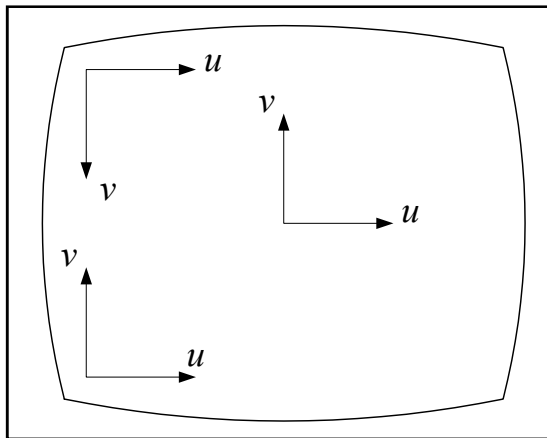
World Coordinate System

- Right-handed Cartesian system used by application program
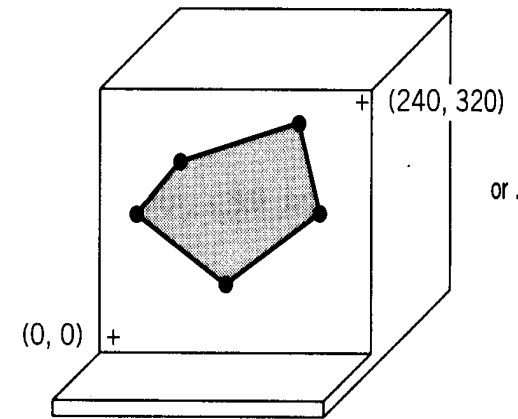- Actual coordinate of an object

Device Coordinate System

- Corresponds to the actual device used
- Device dependent ➔ Lack of portability

(a)   (b)
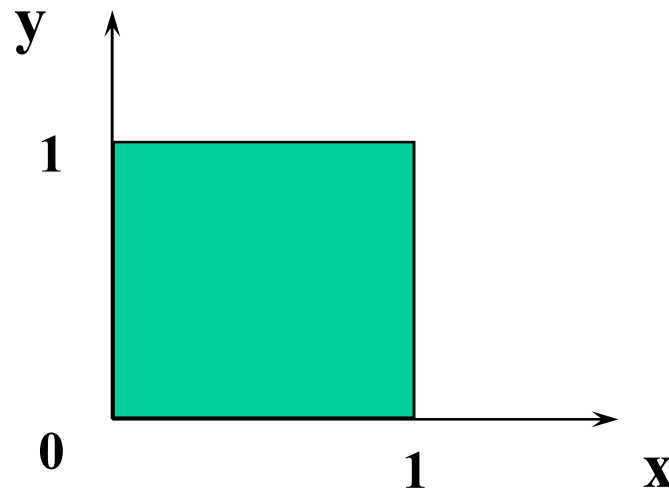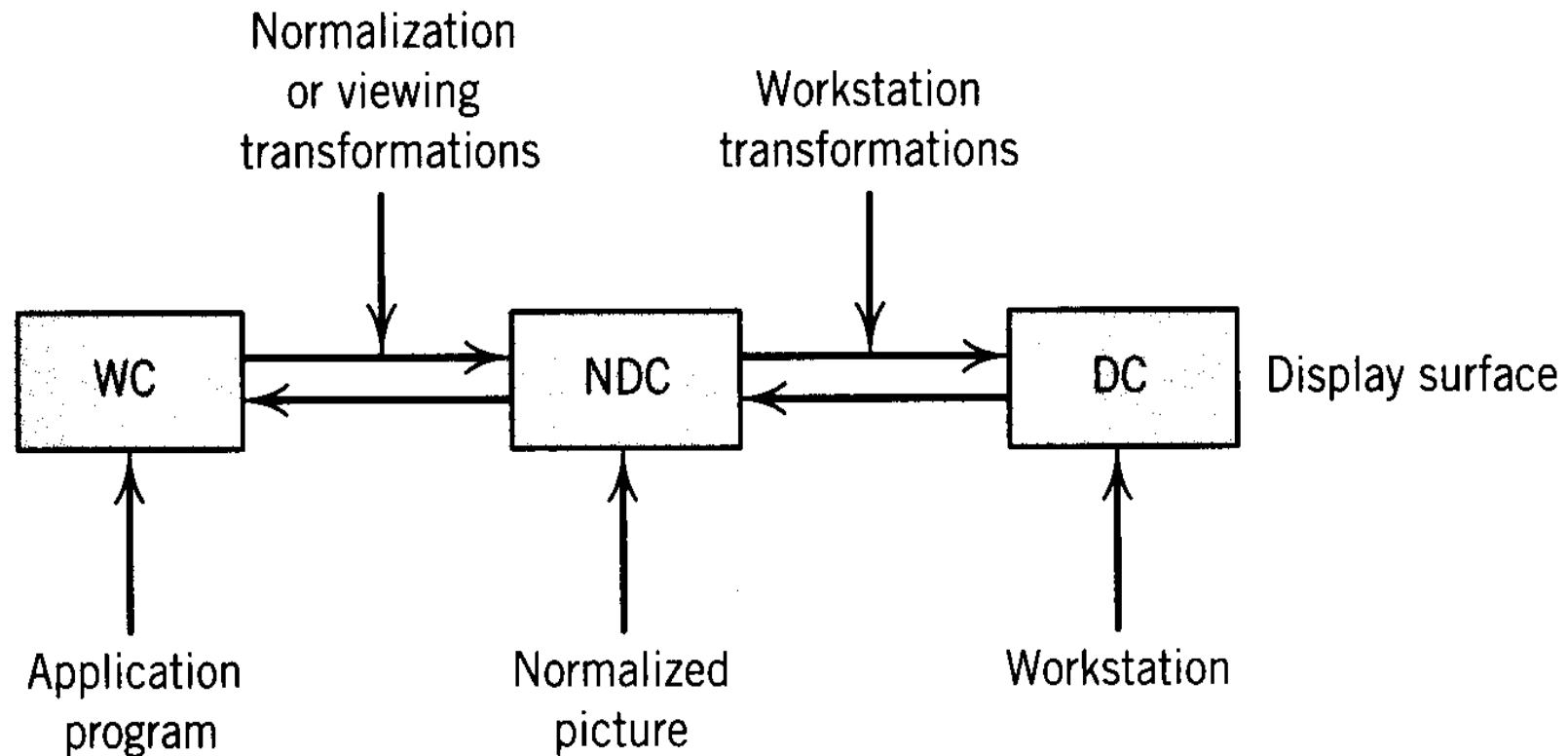
Normalized (Virtual) Device Coordinate

- Device independent
- Unit square represents display surface (screen)
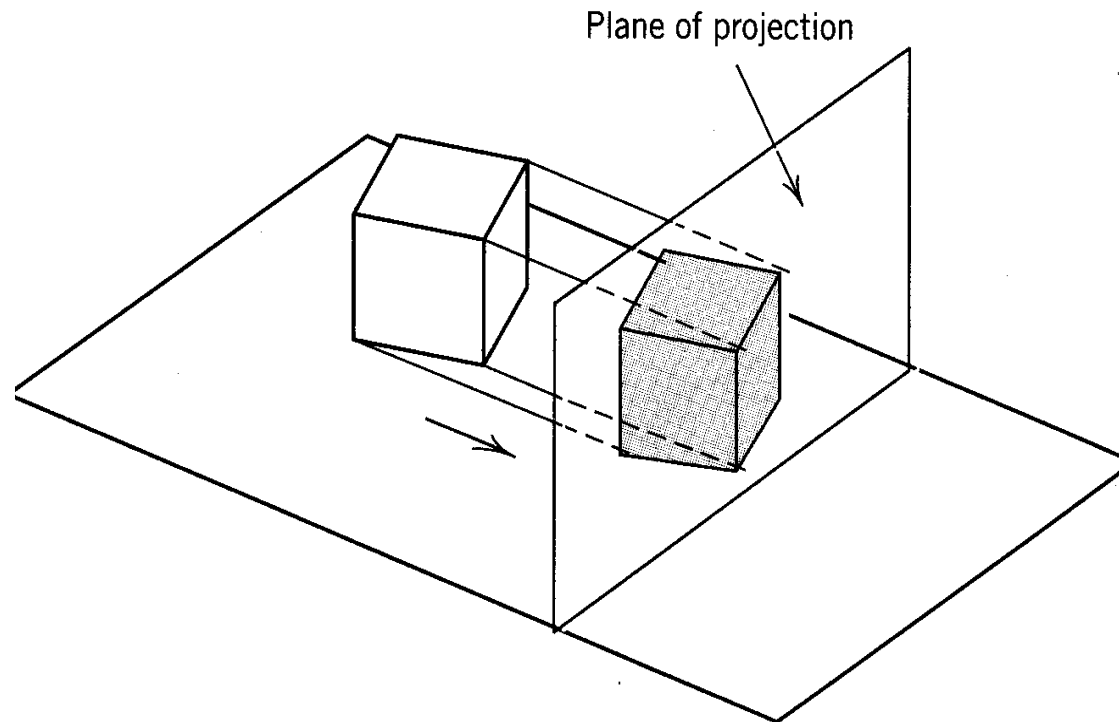
Coordinate Transformations (GKS)

Display 3D Object

- 3D Geometric Model $\rightarrow$ 2D Graphical Image
  - Convert the model into a screen coordinate system
  - Decide how much the model is to appear and where it should appear on the screen (Clipping)

Plane of projection

3D Viewing Operations

- **Coordinate Systems**
  - World coordinate system
  - Device coordinate system
  - Virtual (Normalized) device coordinate system
  - Model coordinate system
  - Viewing coordinate system

- **Viewpoint and Viewsite**
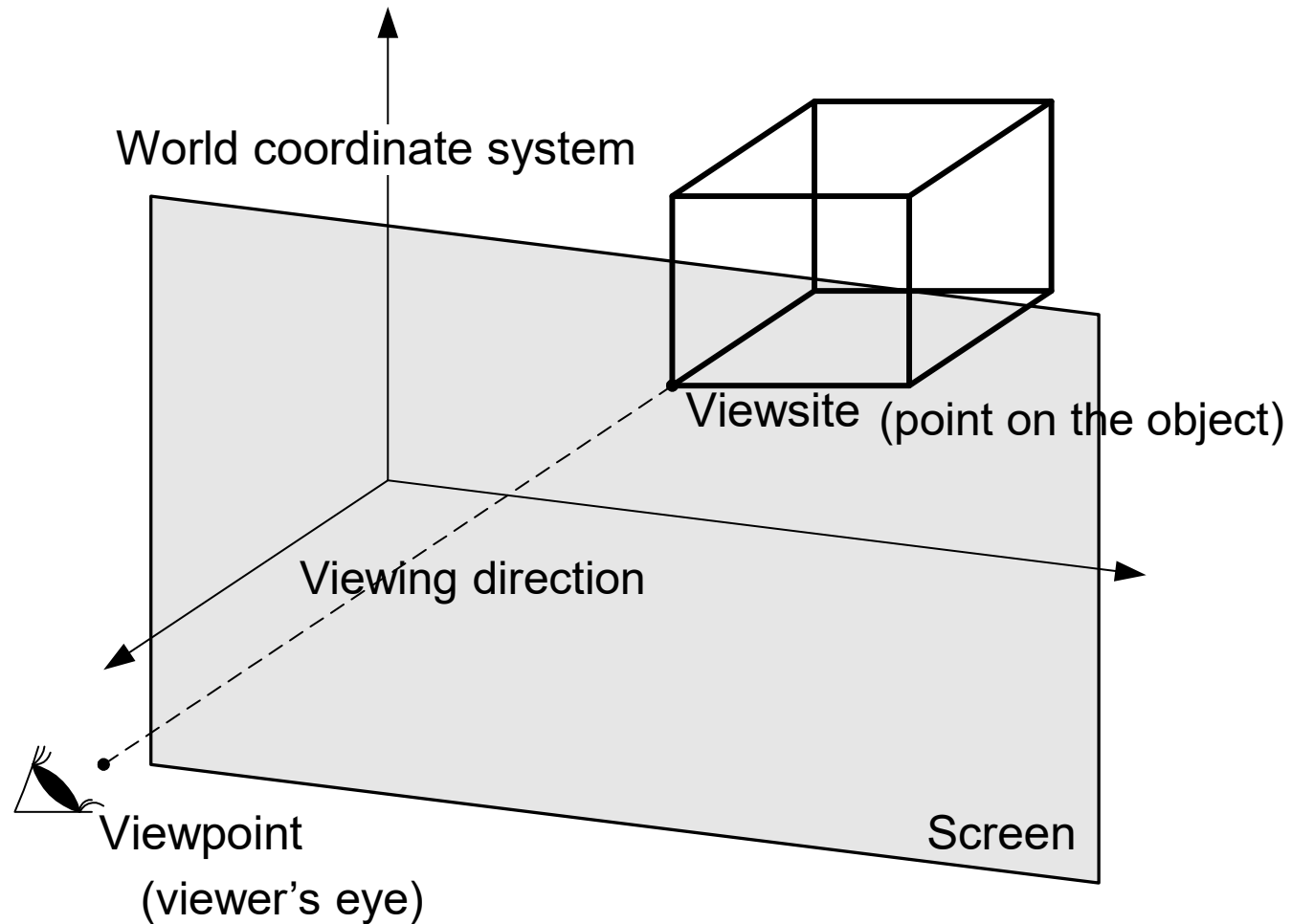
- **Projections**

- **The Viewing Pipeline**

Model Coordinate System

- A local coordinate system attached to the object
    - The values of point coordinates w.r.t model coordinate system do not changed for translation and rotation of the object
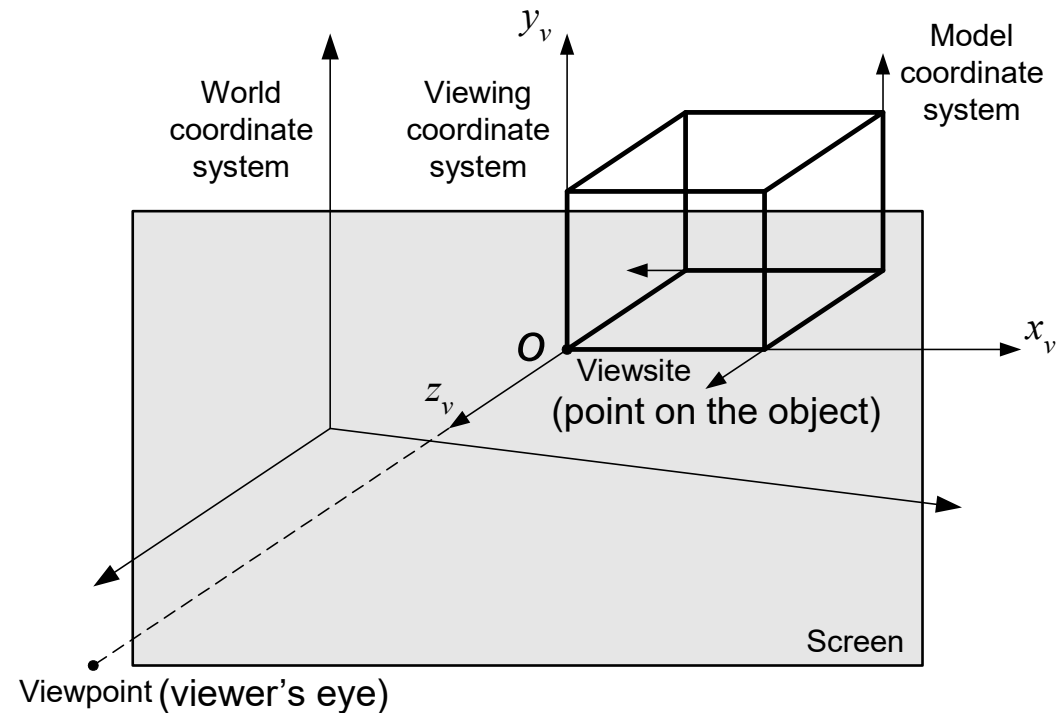
Viewpoint and Viewsite

World coordinate system

Viewsite (point on the object)

Viewing direction
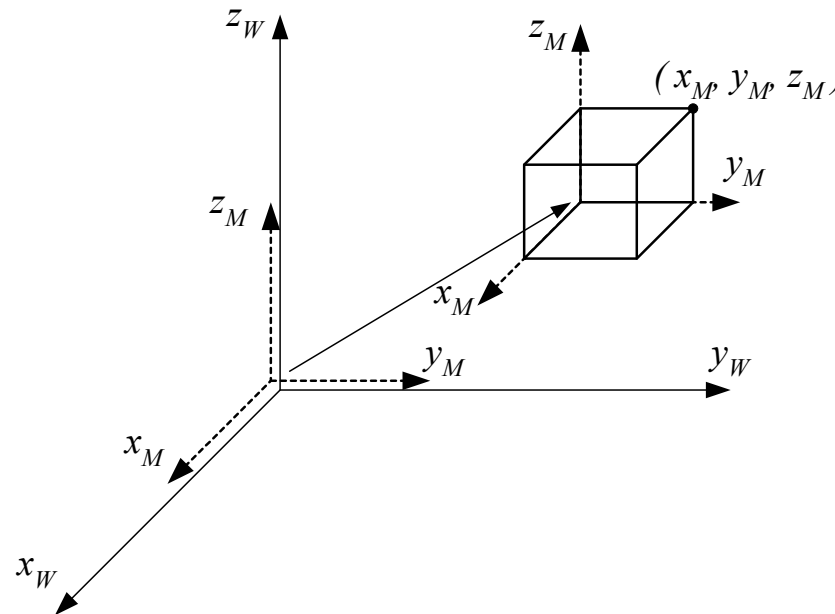
Viewpoint
(viewer's eye)

Screen

Viewing Coordinate System

For easy calculation
of projection

y_v

World
coordinate
system

Viewing
coordinate
system

Model
coordinate
system

O

Viewsite
(point on the object)
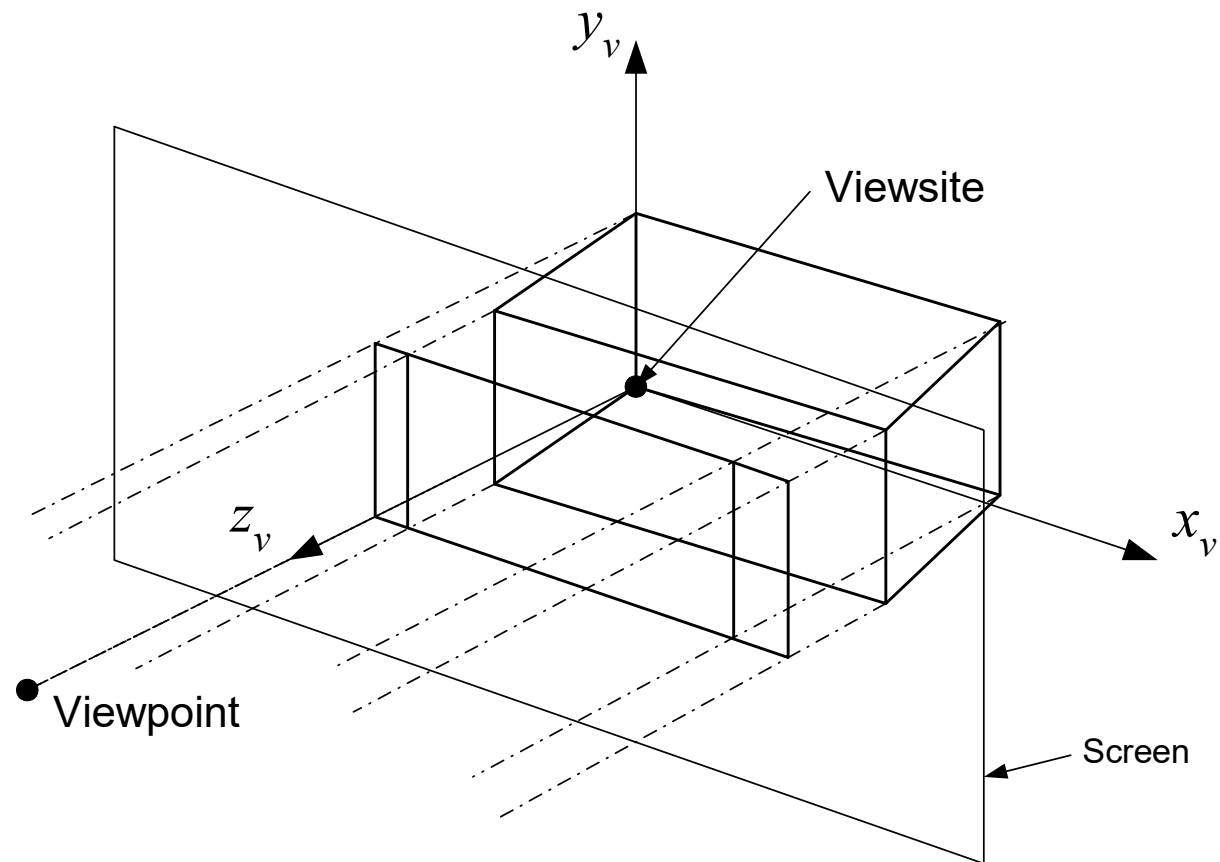
z_v

x_v

Screen

Viewpoint (viewer's eye)

o : viewsite

z_v : viewsite --> viewpoint

y_v : vertical direction of the screen

x_v : cross product of y_v and z_v

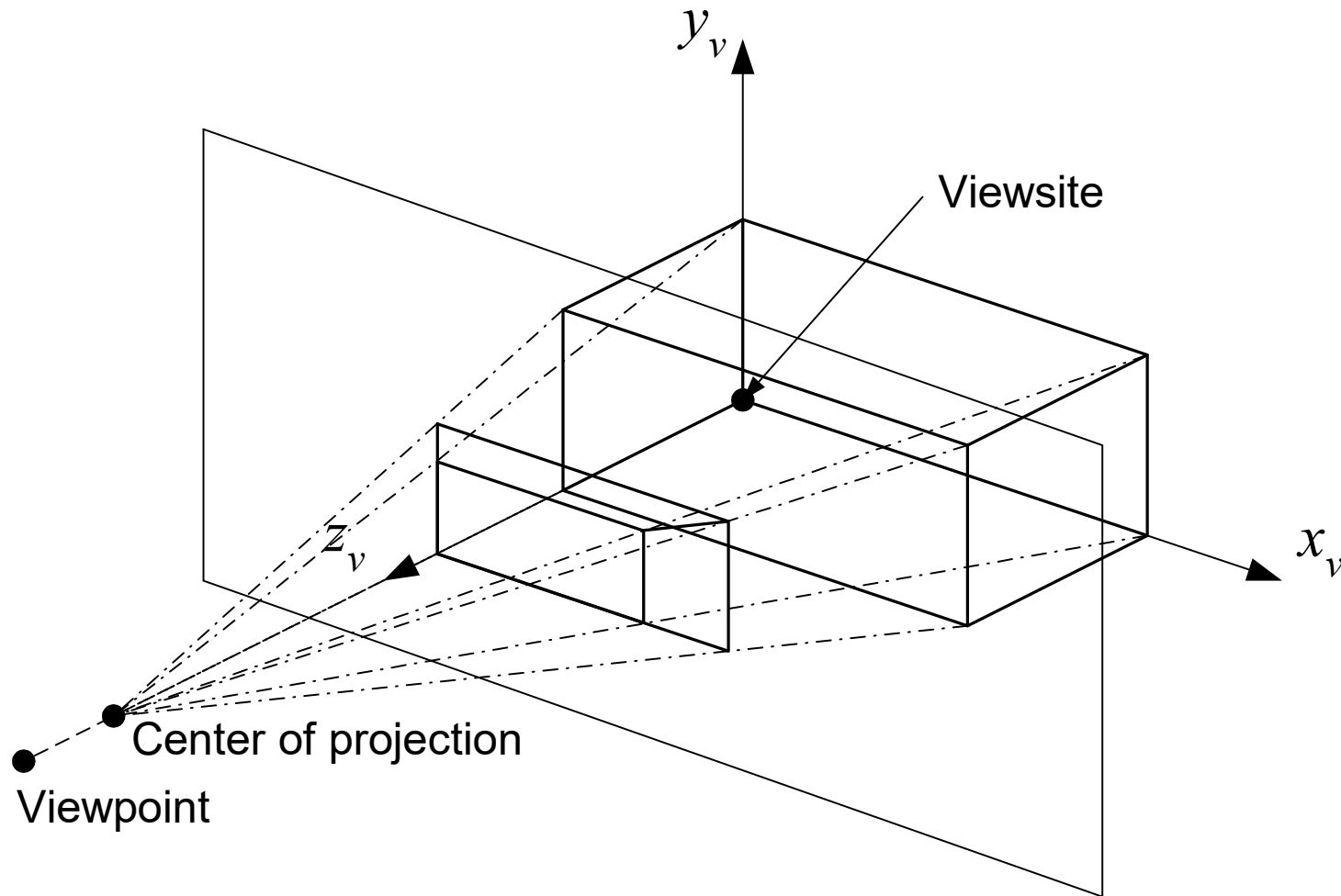# Parallel Projections (평행 투상)

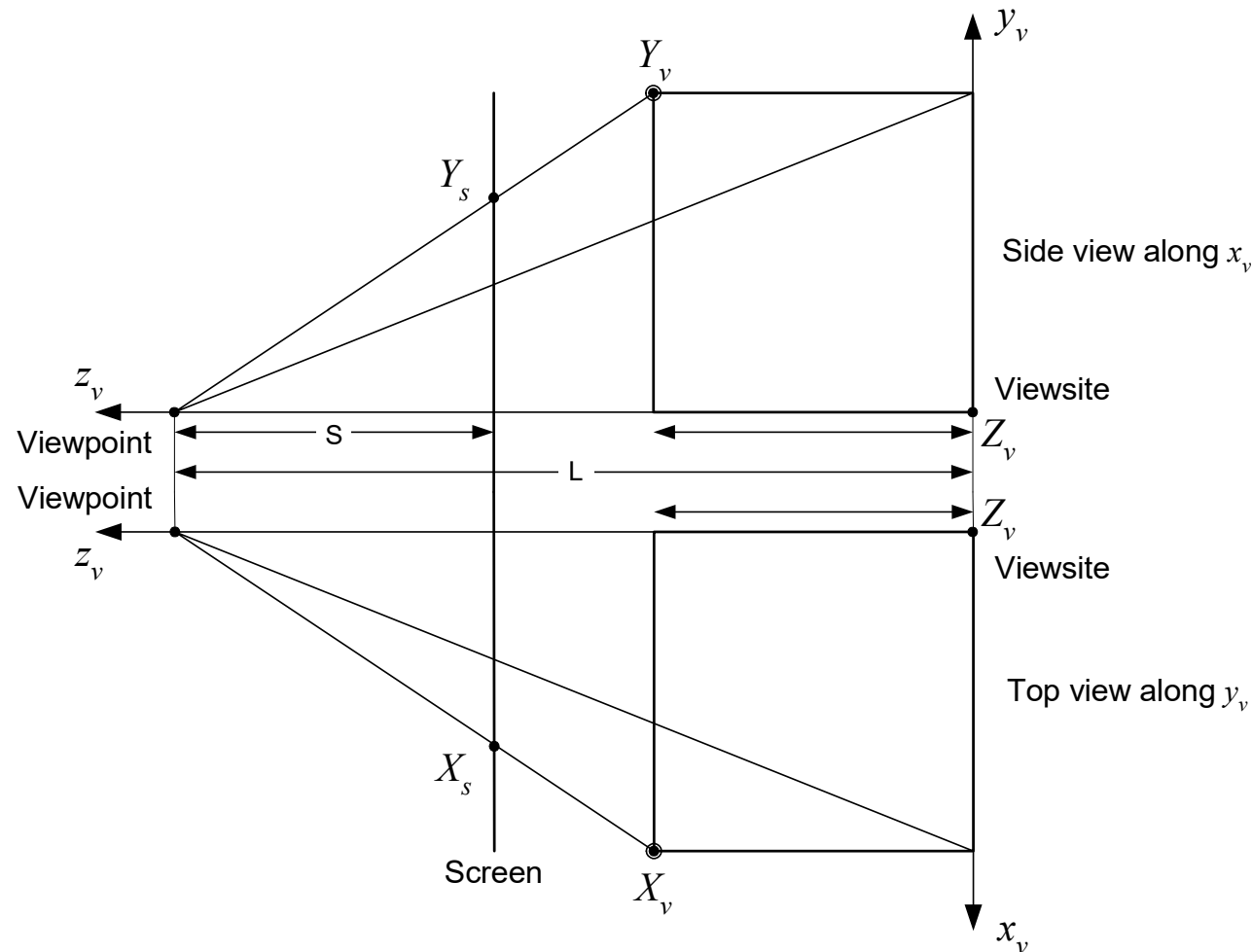# 1-Point Perspective Projection (투시 투상)

y_v

Viewsite

z_v

x_v

Center of projection

Viewpoint

Calculation of Perspective Projection Point

Side view along x_v

Viewsite

Viewpoint

Viewpoint

Viewsite

Top view along y_v

Screen

$$\begin{cases} S : X_s = (L - Z_v) : X_v \\ S : Y_s = (L - Z_v) : Y_v \end{cases}$$

$$\begin{cases} X_s = \dfrac{S}{L - Z_v} X_v \\ Y_s = \dfrac{S}{L - Z_v} Y_v \end{cases}$$

Viewing Pipeline

Coordinate value
in model coordinates

$\downarrow$

Model transformation ●————

Coordinate value
in world coordinates

(Translations and rotations of objects)

$\downarrow$

Viewing transformation ●————

Coordinate value
in viewing coordinates

(viewpoint, viewsite, up vector)

$\downarrow$

Projection transformation ●————

Virtual device coordinates

(type of projection, center of projection and screen)
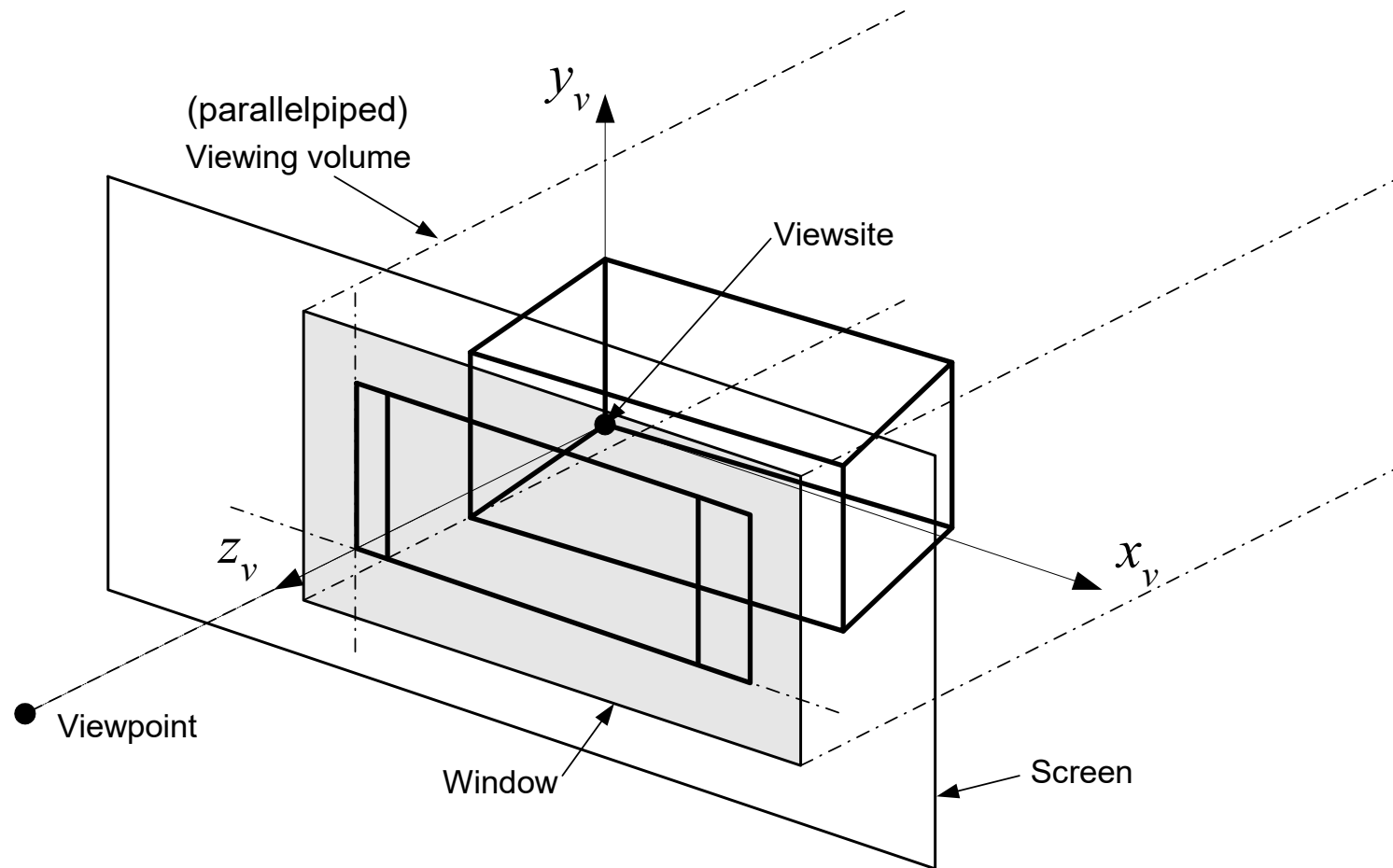
$\downarrow$

Device coordinates

Window and Viewport Definitions

- **Window : rectangular region of WC space**
  - Region in space that will be projected onto the display monitor
- **Viewport : rectangular region of NDC space**
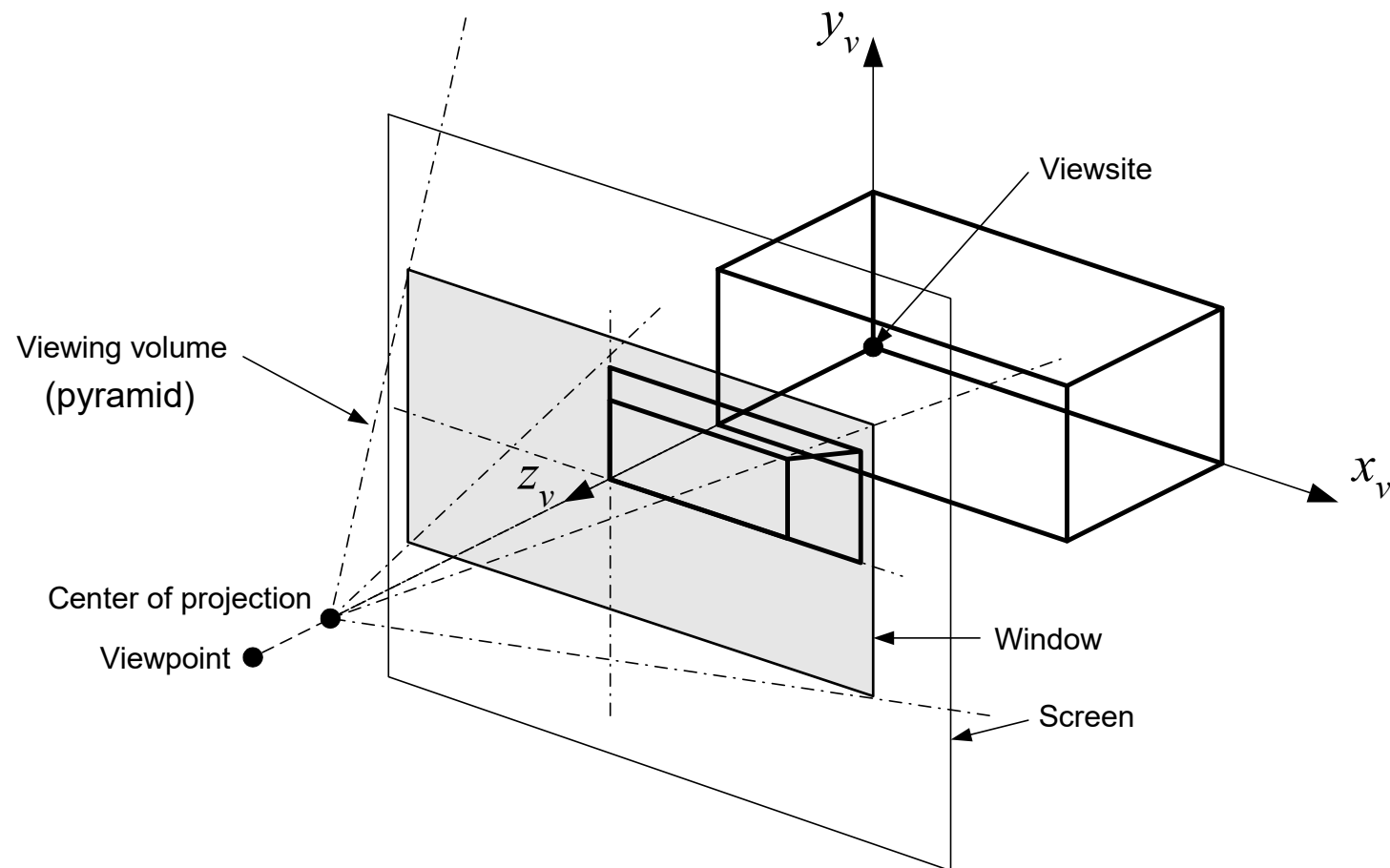 - Area in the display monitor where we want the projected image to appear
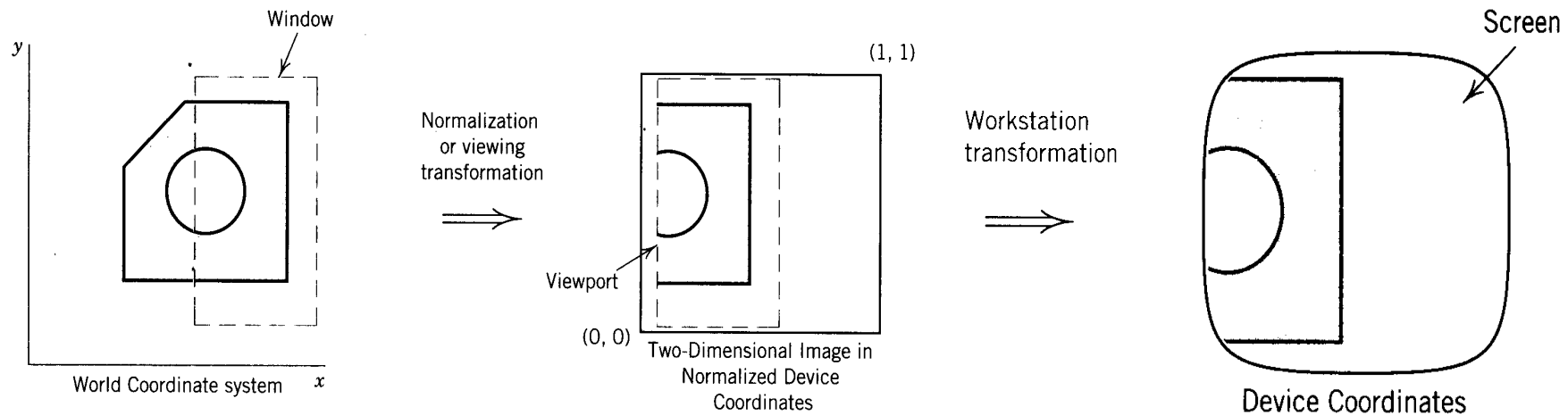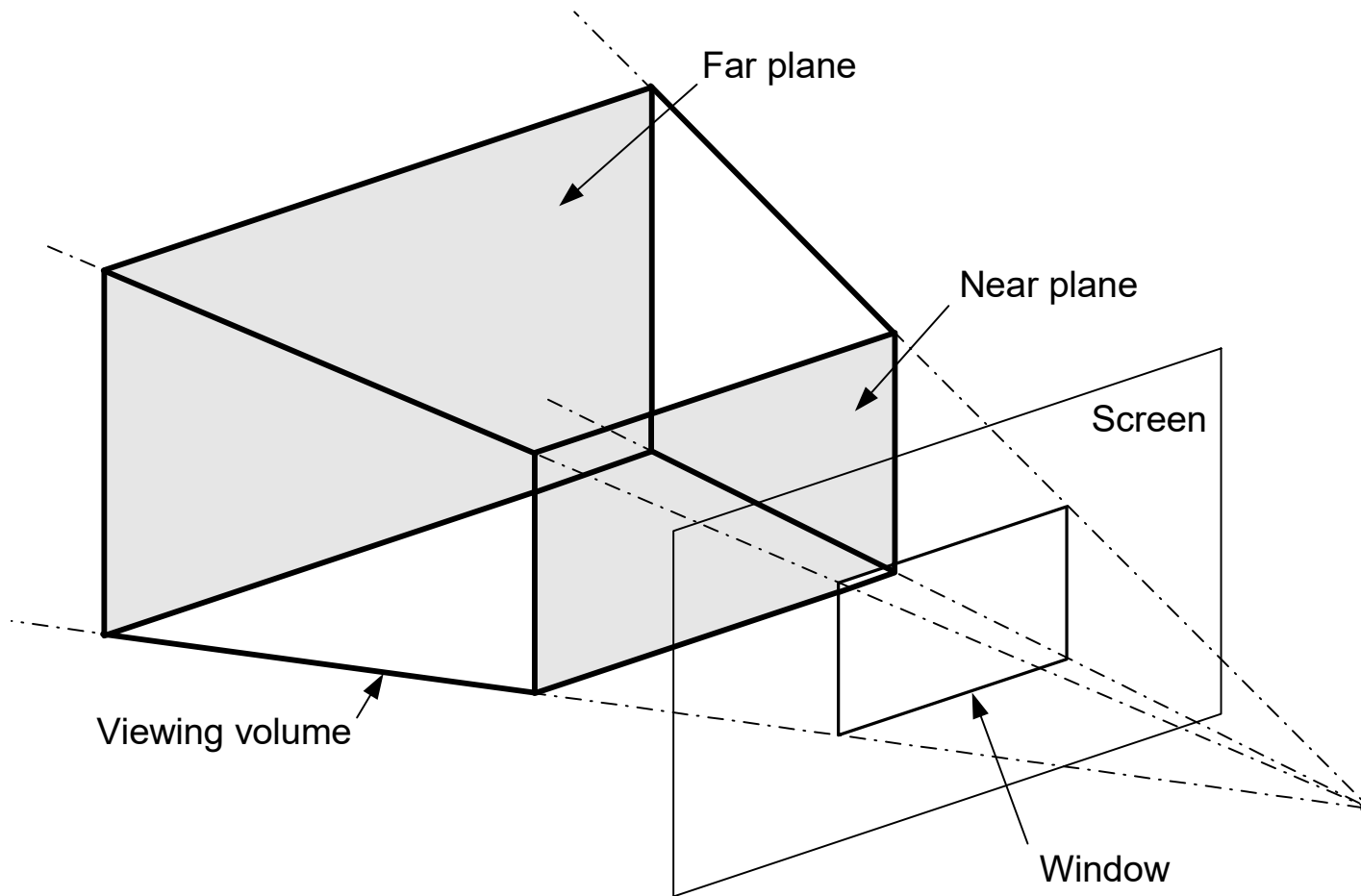
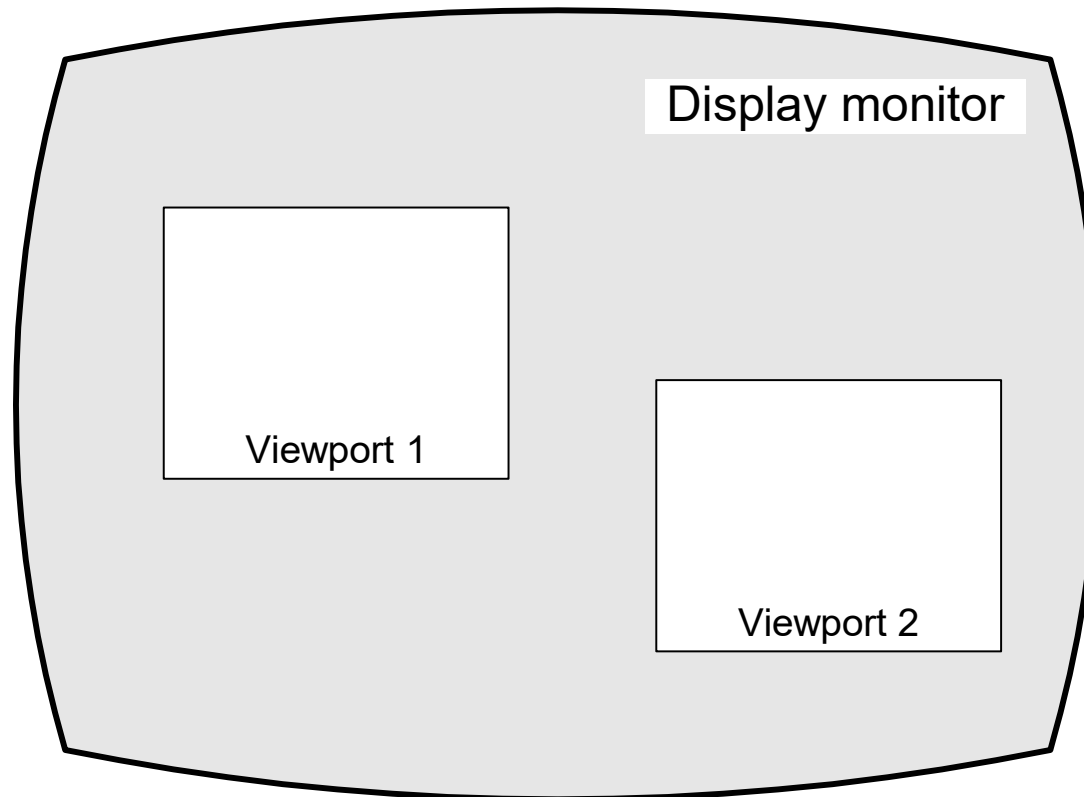# Viewing Volume for Parallel Projection

(parallelpiped)
Viewing volume

Viewsite

y_v

z_v

x_v

Viewpoint

Window

Screen

Viewing Volume for Perspective Projection

Viewing volume (pyramid)

Viewsite

y_v

x_v

z_v

Center of projection

Viewpoint

Window

Screen

Near and Far Plane

Far plane

Near plane

Screen

Viewing volume

Window

Multiple Viewports

Display monitor

Viewport 1
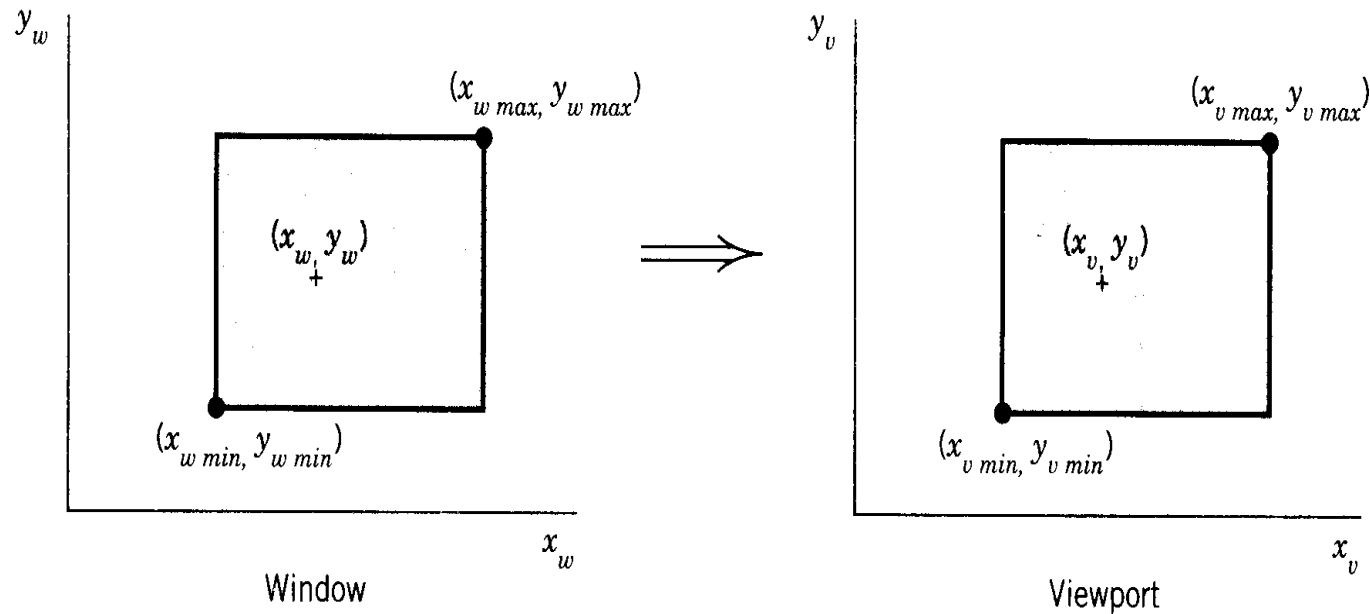
Viewport 2

Window-to-Viewport Mapping

$$\frac{x_v - x_{v\,\min}}{x_{v\,\max} - x_{v\,\min}} = \frac{x_w - x_{w\,\min}}{x_{w\,\max} - x_{w\,\min}}$$

$$\frac{y_v - y_{v\,\min}}{y_{v\,\max} - y_{v\,\min}} = \frac{y_w - y_{w\,\min}}{y_{w\,\max} - y_{w\,\min}}$$

$$x_v = \left(x_w - x_{w\,\min}\right)\left(\frac{x_{v\,\max} - x_{v\,\min}}{x_{w\,\max} - x_{w\,\min}}\right) + x_{v\,\min}$$

$$y_v = \left(y_w - y_{w\,\min}\right)\left(\frac{y_{v\,\max} - y_{v\,\min}}{y_{w\,\max} - y_{w\,\min}}\right) + y_{v\,\min}$$
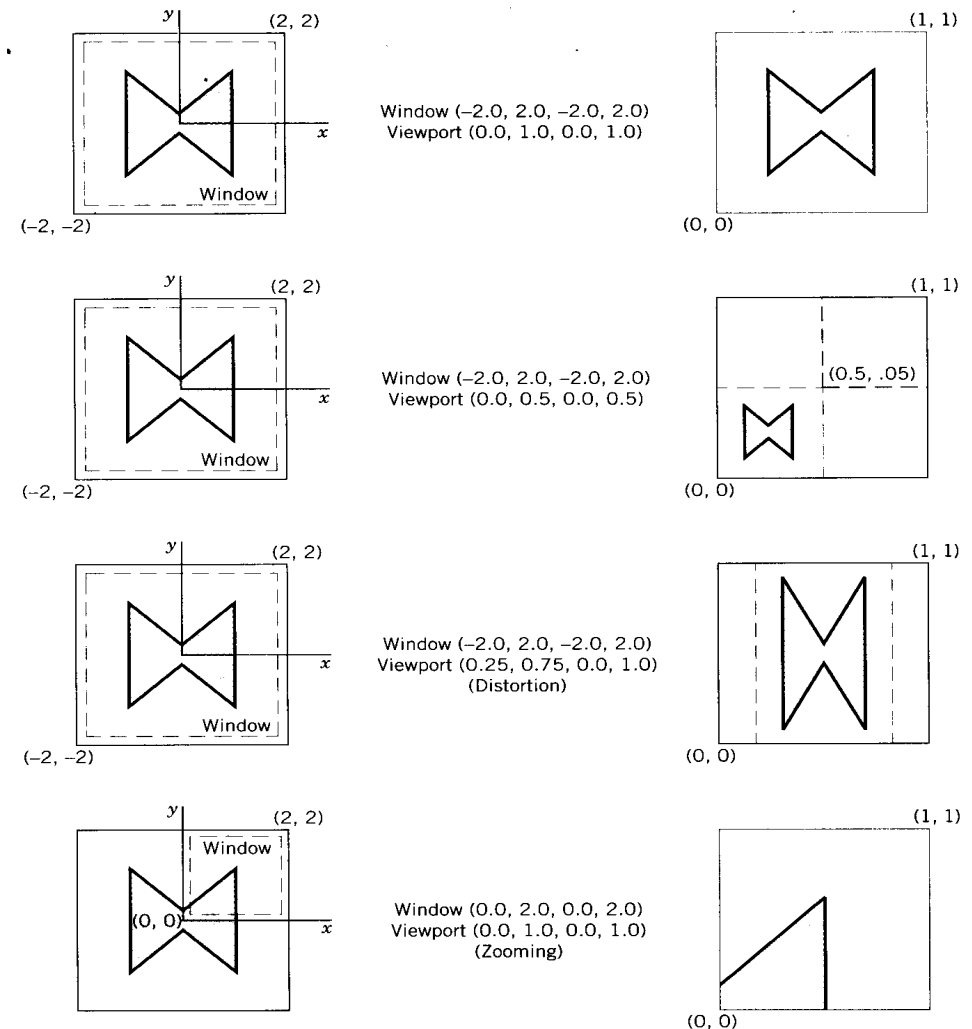
Window-to-Viewport Mapping

- ## Scaling factors : S_x, S_y
  - If S_x != S_y, distortion occurs

$$S_x = \left(\frac{x_{v\,max} - x_{v\,min}}{x_{w\,max} - x_{w\,min}} \right)$$

$$S_y = \left(\frac{y_{v\,max} - y_{v\,min}}{y_{w\,max} - y_{w\,min}} \right)$$

- ## Aspect ratio (AR)

$$AR = \left(\frac{x_{max} - x_{min}}{y_{max} - y_{min}} \right)$$

Window (−2.0, 2.0, −2.0, 2.0)
Viewport (0.0, 1.0, 0.0, 1.0)

Window (−2.0, 2.0, −2.0, 2.0)
Viewport (0.0, 0.5, 0.0, 0.5)

Window (−2.0, 2.0, −2.0, 2.0)
Viewport (0.25, 0.75, 0.0, 1.0)
(Distortion)

Window (0.0, 2.0, 0.0, 2.0)
Viewport (0.0, 1.0, 0.0, 1.0)
(Zooming)

CATIA V5: View → Render Style

Camera Properties

Name:	Camera 1	
Type:	Perspective	▼
Origin:	247.15	255.25	251.25
Target:	12.204	-1.4065	-19.074
View Angle:	1.5	
Zoom:	0.0094533	

OK   Apply   Cancel
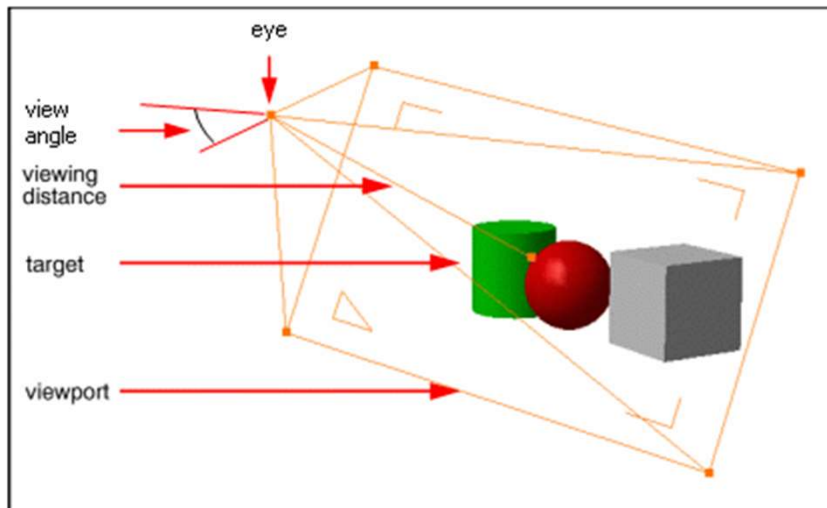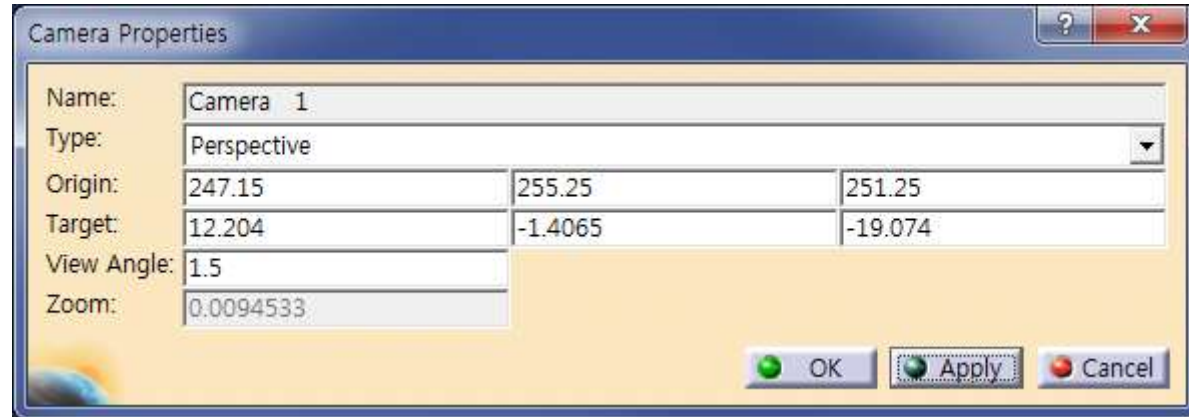
Type: lets you set a Parallel or Perspective view projection
Origin: coordinates of your eye position
Target: coordinates of the center of rotation of the camera (the point located at the center of the viewport).
You can set the center of rotation by clicking the middle mouse button on the desired point: the coordinated are memorized with the camera
View angle: sets the angle of the pyramid-like shape with which you look at the geometry (available in perspective views only).
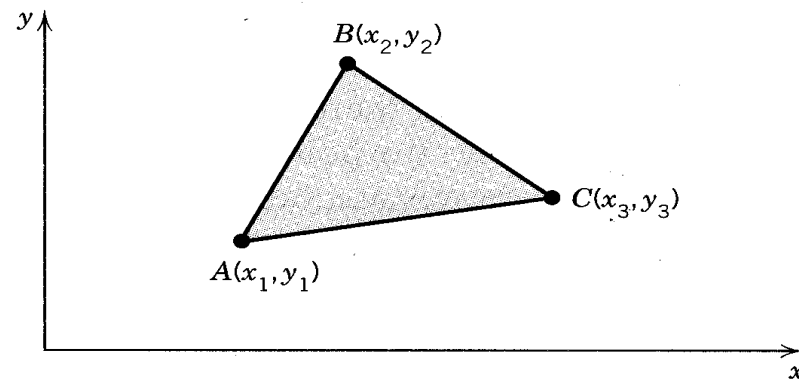A large angle has the effect of zooming out to make the geometry look small; a small angle has the reverse effect
Zoom: zoom factor (available in parallel views only).

Representation of 2D Geometry

- **Ordinary Cartesian Coordinates**

- – (Note) Some geometric transformation are obtained by matrix multiplications and others by vector additions
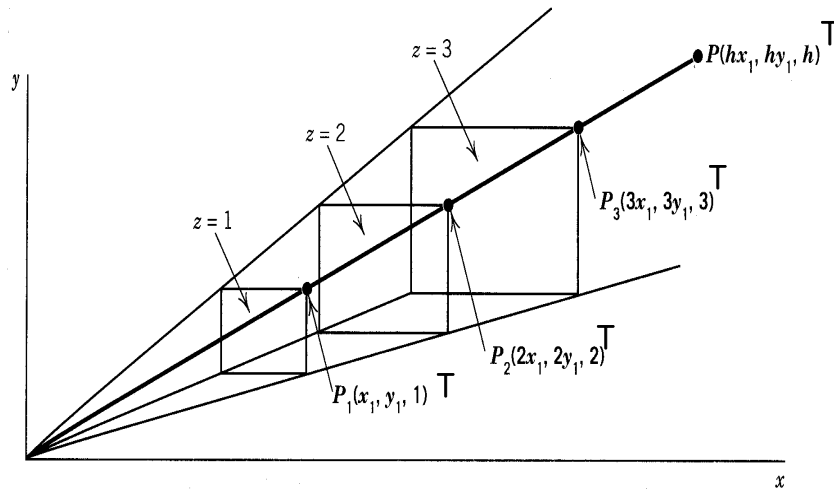
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous Coordinates (1)

- Unified approach to the description of geometric transformations

- homogeneous space: ray tracing

$$P(x, y, z)^T = P(hx_1, hy_1, h)^T$$

General form: $P(hx, hy, h)^T$

$$P(m, n, h)^T \rightarrow P\left(\frac{m}{h}, \frac{n}{h}, 1\right)^T$$

triangle: $[P] = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}^T$

Homogeneous Coordinates (2)

- All geometric transformations are obtained by matrix multiplications

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix} \rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$
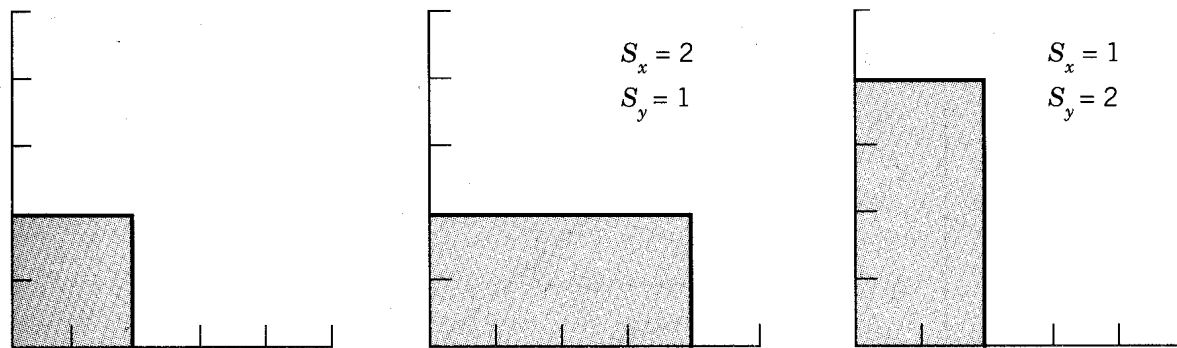
2D Transformations

- **Calculation of new coordinates**
  - Rigid body transformation
    - Scaling
    - Translation
    - Rotation

- **Object transformation**

- **Coordinate system transformation**
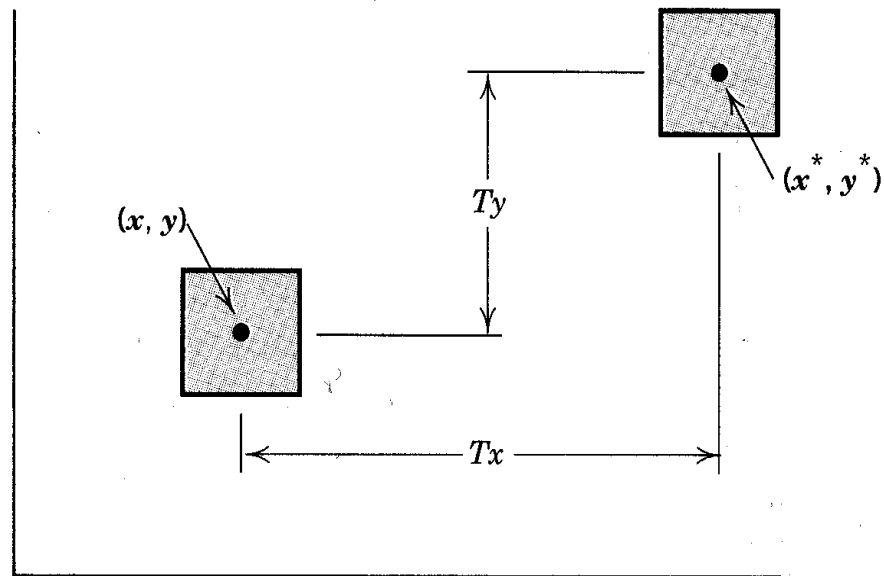
Scaling

- Scaling about the origin
- S_x, S_y : positive
  - if negative, the transformation is reflection

$S_x = 2$
$S_y = 1$

$S_x = 1$
$S_y = 2$

$$[P'] = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x x \\ S_y y \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = Scale\left(S_x, S_y\right)[P]$$

Translation

$$[P'] = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x + T_x \\ y + T_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = Trans\left(T_x, T_y\right)[P]$$

Rotation

$$\begin{cases} x = r\cos\phi \\ y = r\sin\phi \end{cases}$$

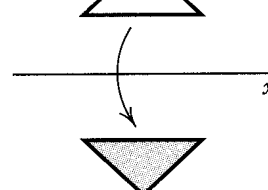$$\begin{cases} x' = \\ \quad = \\ y' = \\ \quad = \end{cases}$$

P' (x', y')

y'

r

y

r

P(x, y)

θ

φ

x'

x
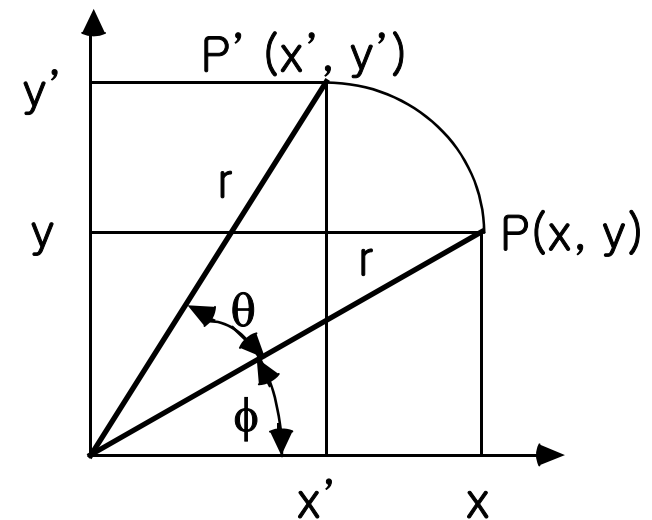
$$[P'] = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = Rot(\theta)[P]$$
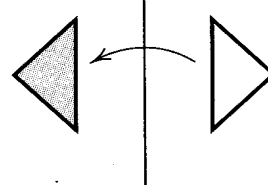
Reflection

$$Ref(a,b) = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- A special case of scaling: negative scaling
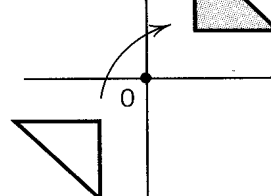
About the x axis (x values are kept and y values are flipped:)

$$[T_{\mathrm{RFL}}]_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
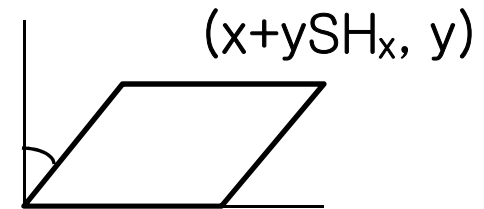
About the y axis (y values are kept and x values are flipped:)

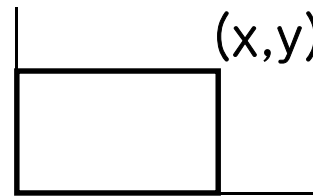$$[T_{\mathrm{RFL}}]_y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

About the origin (both x and y values are flipped:)

$$[T_{\mathrm{RFL}}]_\mathrm{o} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
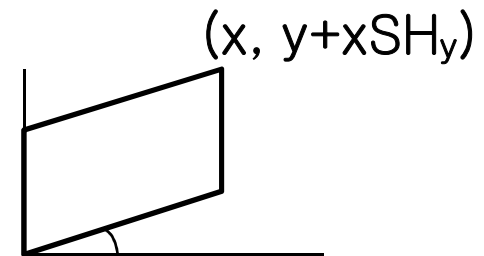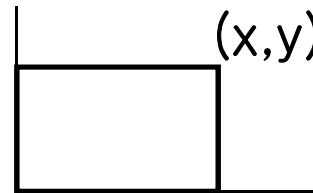
Shearing

$$Shear\left(SH_x, SH_y\right) = \begin{bmatrix} 1 & SH_x & 0 \\ SH_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$Shear\left(SH_x, 0\right) = \begin{bmatrix} 1 & SH_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} x + ySH_x \\ y \\ 1 \end{bmatrix}$$
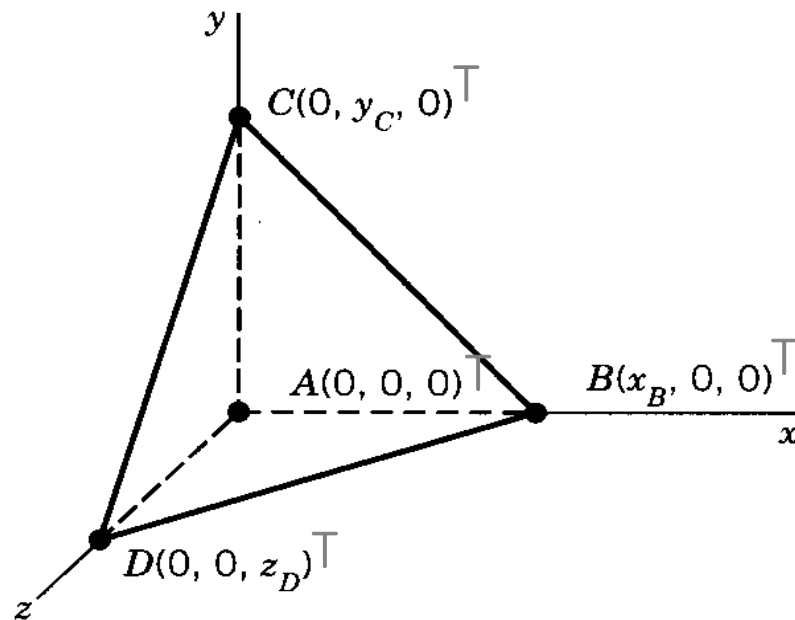
(x,y)

(x+ySH$_x$, y)

$$Shear\left(0, SH_y\right) = \begin{bmatrix} 1 & 0 & 0 \\ SH_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y + xSH_y \\ 1 \end{bmatrix}$$

(x,y)

(x, y+xSH$_y$)

Representation of 3D Geometry

$$P = \begin{bmatrix} 0 & x_B & 0 & 0 \\ 0 & 0 & y_C & 0 \\ 0 & 0 & 0 & z_D \\ 1 & 1 & 1 & 1 \end{bmatrix}$$
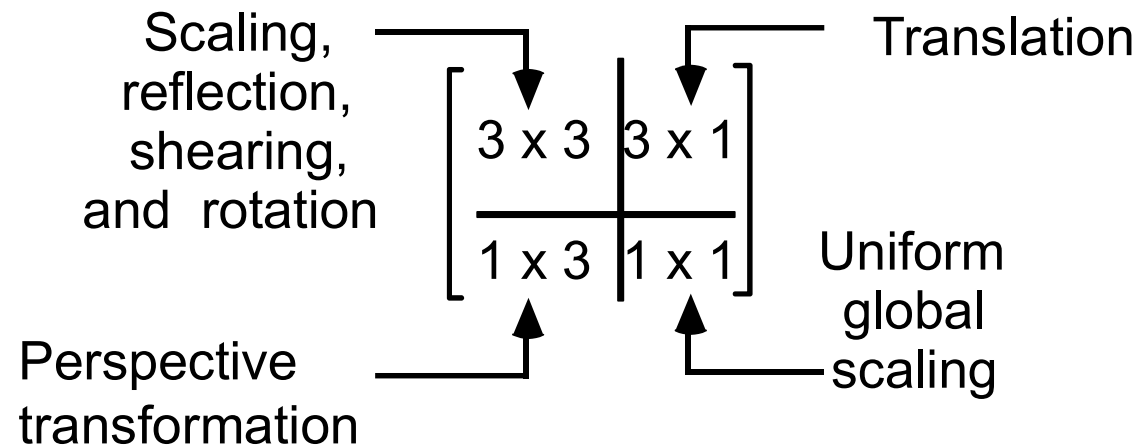
3D Transformation Matrix (1)

- **Translation and Rotation**
  - To calculate the world coordinates from the model coordinates
  - The model has been translated and rotated from the initial position

- **Mapping**
  - To map the world coordinate system to the viewing coordinate system
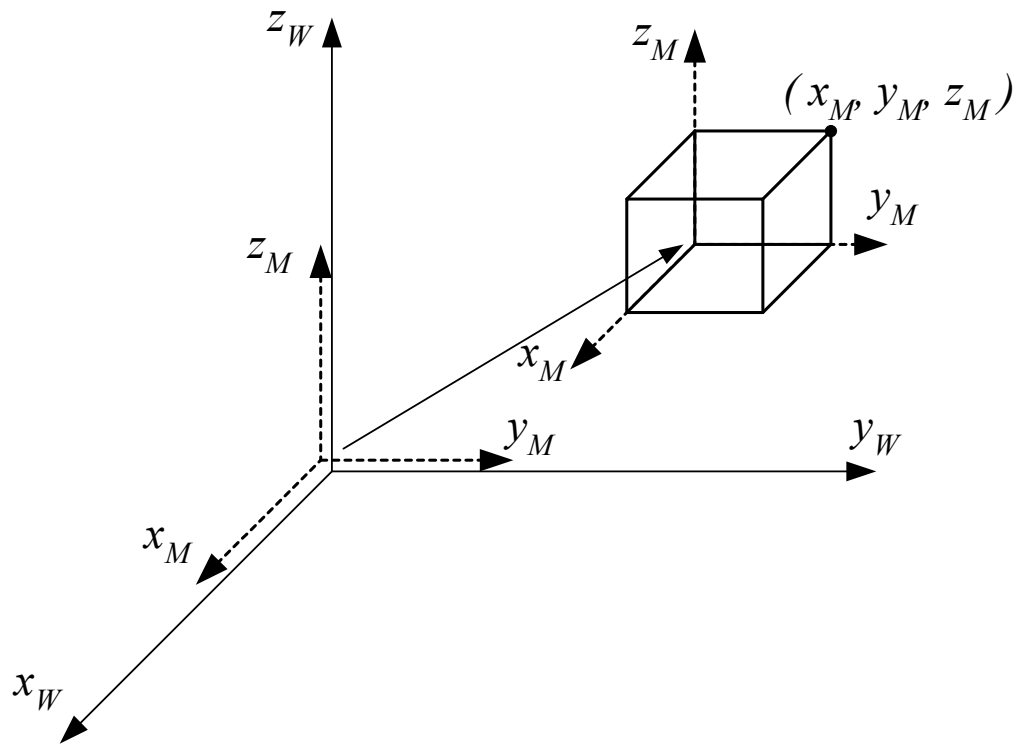 - Calculate the viewing coordinates of the same points from the world coordinates

3D Transformation Matrix (2)

- 4x4 Transformation Matrix

$$P = \begin{bmatrix} A & B & C & J \\ D & E & F & K \\ G & H & I & L \\ 0 & 0 & 0 & S \end{bmatrix}$$

Scaling, reflection, shearing, and rotation

Translation

$$\begin{bmatrix} 3 \times 3 & 3 \times 1 \\ 1 \times 3 & 1 \times 1 \end{bmatrix}$$

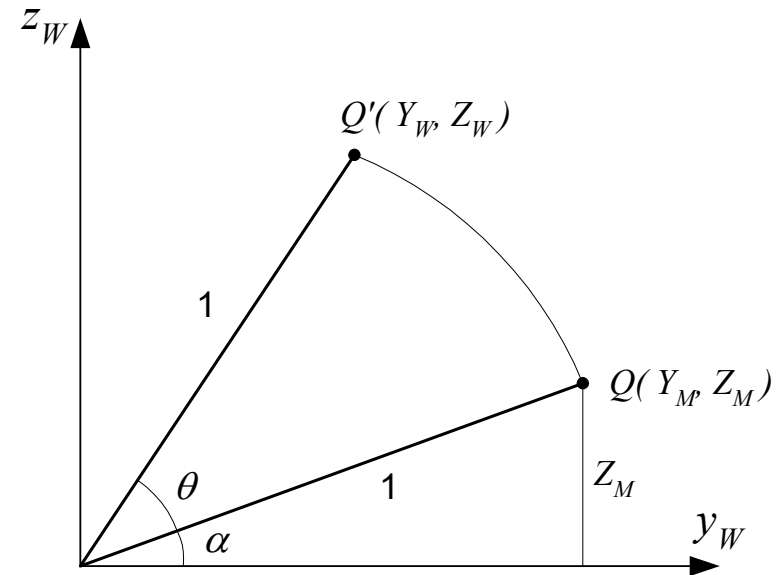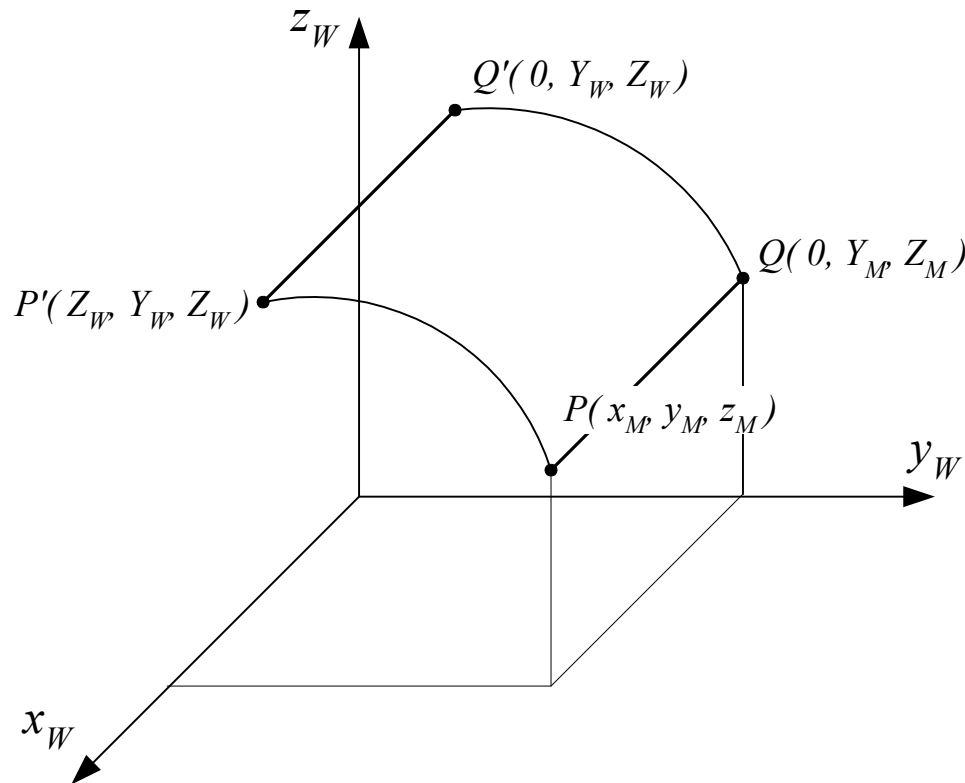Perspective transformation

Uniform global scaling

Translation

$$\begin{cases} X_w = X_M + a \\ Y_w = Y_M + b \\ Z_w = Z_M + c \end{cases}$$

$$\begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_M \\ Y_M \\ Z_M \\ 1 \end{bmatrix}$$
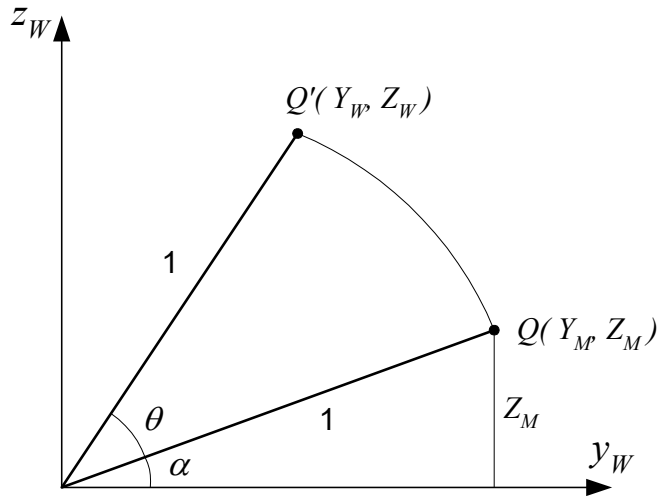
$$Trans(a,b,c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation about X Axis (1)

Projection onto the *yz* plane

Rotation about X Axis (2)

$$X_W = X_M$$

$$Y_W = \ell \cos(\theta + \alpha)$$

$$= \ell(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$$

$$= \ell\cos\alpha\cos\theta - \ell\sin\alpha\sin\theta$$

$$= Y_M\cos\theta - Z_M\sin\theta$$

$$Z_W = \ell\sin(\theta + \alpha)$$

$$= \ell(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$$

$$= \ell\cos\alpha\sin\theta + \ell\sin\alpha\cos\theta$$

$$= Y_M\sin\theta + Z_M\cos\theta$$

$$\begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_M \\ Y_M \\ Z_M \\ 1 \end{bmatrix}$$

$$Rot(x,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotation Matrix

$$Rot(x, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
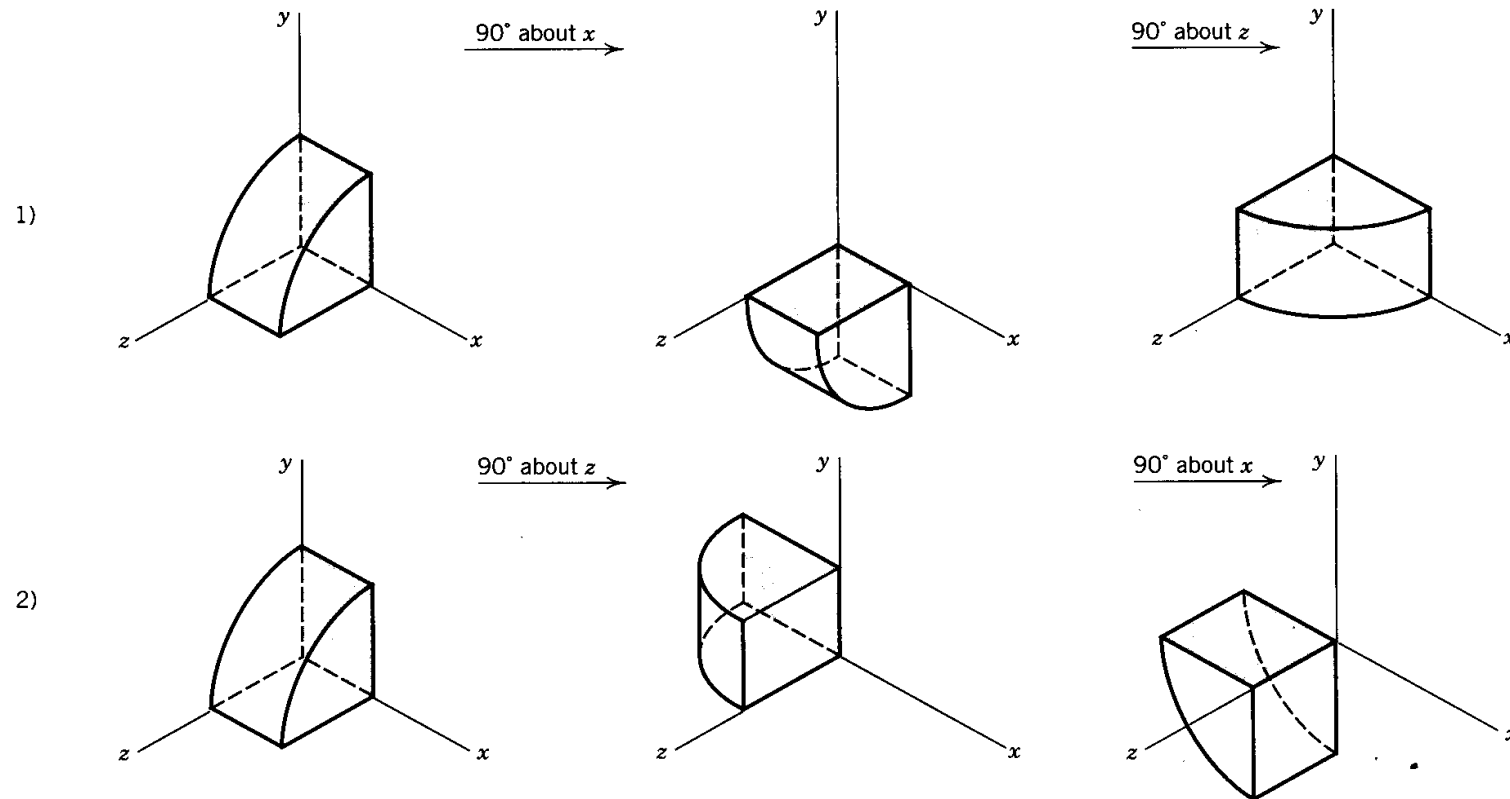
$$Rot(y, \theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot(z, \theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
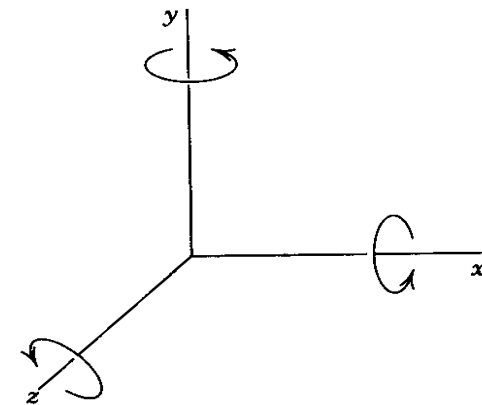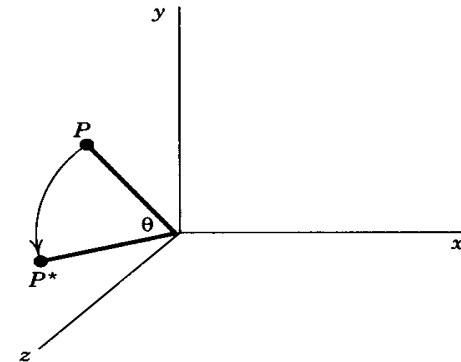
# Order of Rotation

- Affects the final position of an object

Examples

– An object in space is translated by 5 units in the y-direction of the world coordinate system and then rotated by 90 degrees about the x-axis of the world coordinate system. If a point on the object has the coordinates (0,0,1) with respect to its model coordinate system, what will be the world coordinates of the same point after the translation and rotation? Ans. (0,-1,5)

– An object in space is rotated by 90 degrees about an axis that is parallel to the x-axis of the world coordinate system and passes through a point having world coordinates (0,3,2). If a point on an object has model coordinates (0,0,1), what will be the world coordinates of the same point after the rotation? Ans. (0,4,-1)

Mapping between Two Coordinate Systems (1)

- Known: (X_1, Y_1, Z_1), $x_2 y_2 z_2$ coordinate system w.r.t. $x_1 y_1 z_1$ system
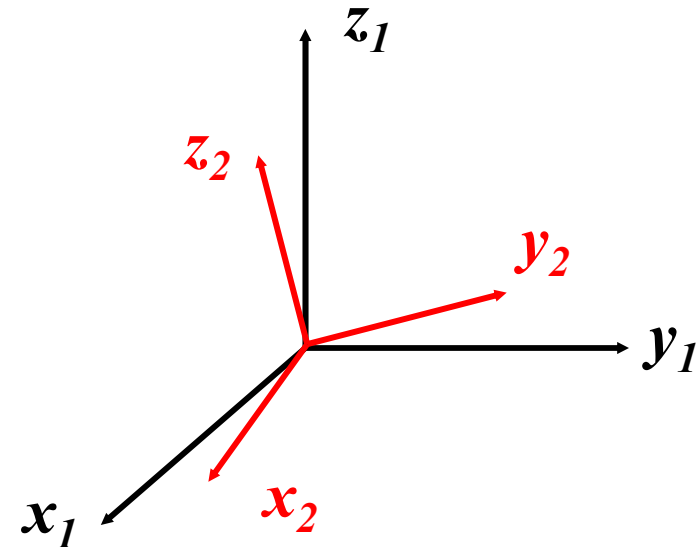- Find: (X_2, Y_2, Z_2)
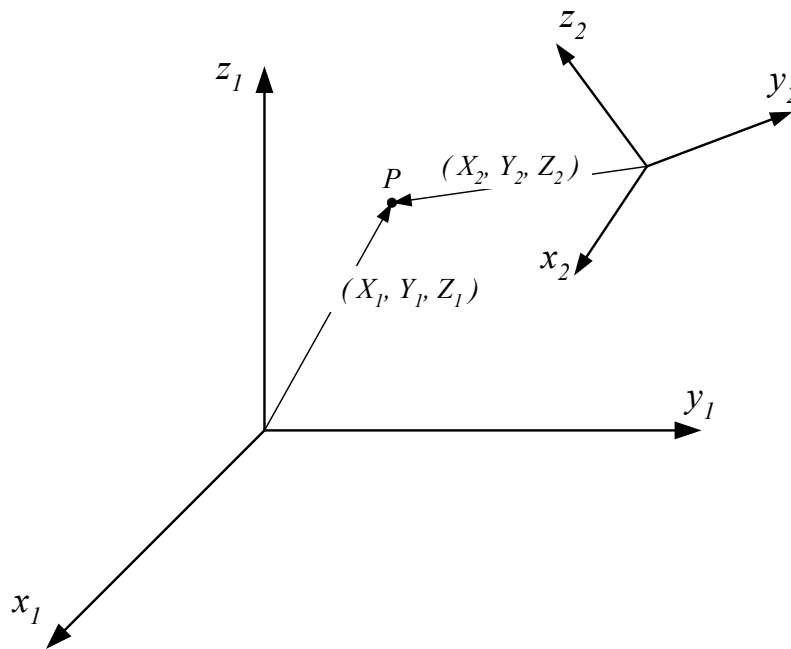
$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = T_{1-2} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$T_{1-2} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Mapping between Two Coordinate Systems (2)

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

- p_x, p_y, p_z : x_2, y_2, z_2 components of the origin of the $x_1 y_1 z_1$ system $(X_1 = 0,\ Y_1 = 0,\ Z_1 = 0)$
- n_x, n_y, n_z : x_2, y_2, z_2 components of the x_1 axis $(X_1 = 1,\ Y_1 = 0,\ Z_1 = 0)$
- o_x, o_y, o_z : x_2, y_2, z_2 components of the y_1 axis $(X_1 = 0,\ Y_1 = 1,\ Z_1 = 0)$
- a_x, a_y, a_z : x_2, y_2, z_2 components of the z_1 axis $(X_1 = 0,\ Y_1 = 0,\ Z_1 = 1)$

CAD

Examples
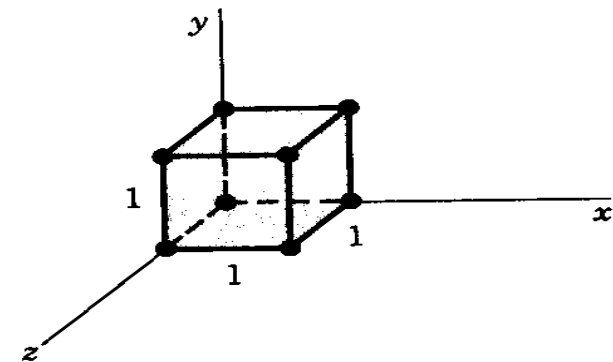
- Corresponding to the viewpoint (-10,0,1), the viewsite (0,0,1) and the up vector (0,0,1), the viewing coordinate system is drawn. Note that all the coordinate and component values are given in world coordinates. From the relative position between the viewing coordinate system and the world coordinate system, (1) calculate the mapping transformation T_{w-v} and (2) calculate the coordinates of a point in viewing coordinates if it has world coordinates (5,0,1). <u>Ans. (0,0,-5)</u>

- The viewpoint and the viewsite are set at (5,5,5) and (0,0,0), respectively, to draw an isometric view, and the up vector is chosen to be (0,0,1). Derive the mapping transformation matrix T_{w-v}, and the viewing coordinates of a point represented by (0,0,5) in world coordinates. <u>Ans. (0,5√6/3)</u>
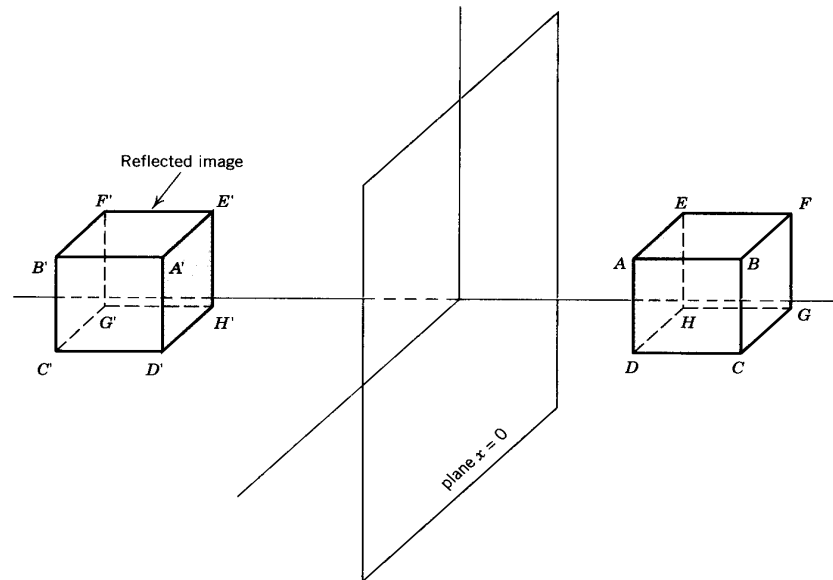
Scaling

$$\begin{bmatrix} X' \\ Y' \\ Z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$
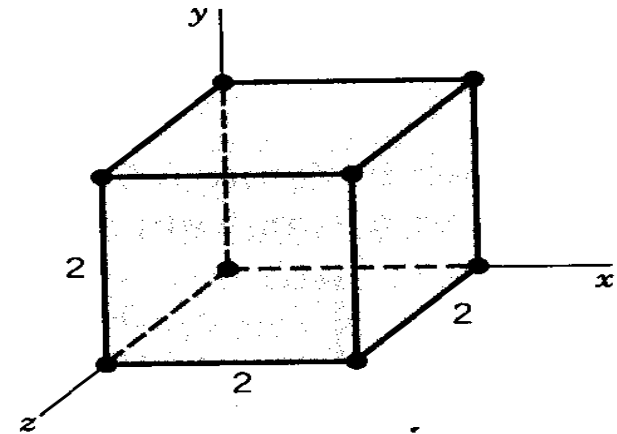
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \Downarrow \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Mirror Reflection

Reflected image

F' E'
B' A'
G' H'
C' D'

E F
A B
H G
D C

plane $x = 0$

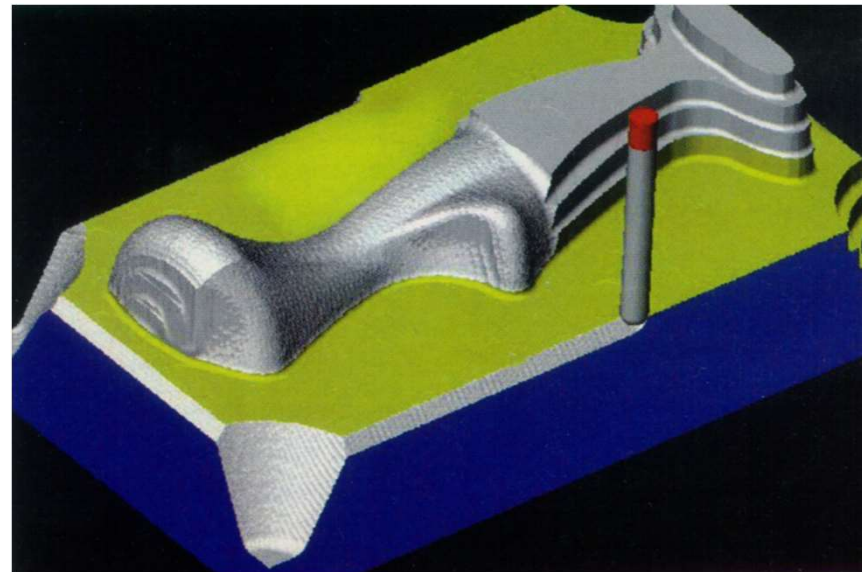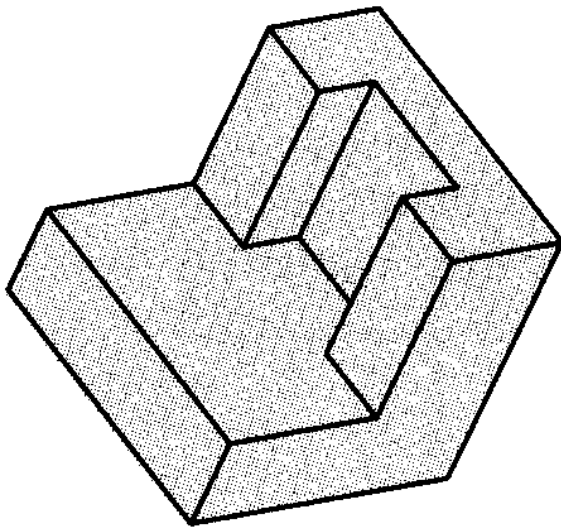| Plane	$x = 0$	$y = 0$	$z = 0$	Point (0,0,0)
$[T]_{RFL}$	$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Contents

- **Hidden line removal**
  - Object space
  - Image space

- **Rendering**
  - Shading
  - Ray tracing

- **Color model**
  - RGB Model
  - CMY Model
  - HSV Model

Visual Realism

- Hidden Line/Surface Removal
- Rendering

Hidden Line/Surface Removal

- **Hidden Line / Surface Removal**
  - The task of deciding which edges or faces (or portions thereof) must be removed
  - Requires substantial computer time and memory

- **Dependent on Type of Display**
  - Vector display: removal of hidden lines
  - Raster display: removal of lines and faces showing shaded surfaces

Approaches for Hidden Line / Surface Removal

- **Object space approach**
  - Determines which parts of object are visible
  - Using spatial and geometrical relationships
  - Operates with object database precision
  - Software-based approach

- **Image space approach**
  - Determines what is visible at each image pixel
  - Concentrating on the final image
  - Operates with image resolution precision
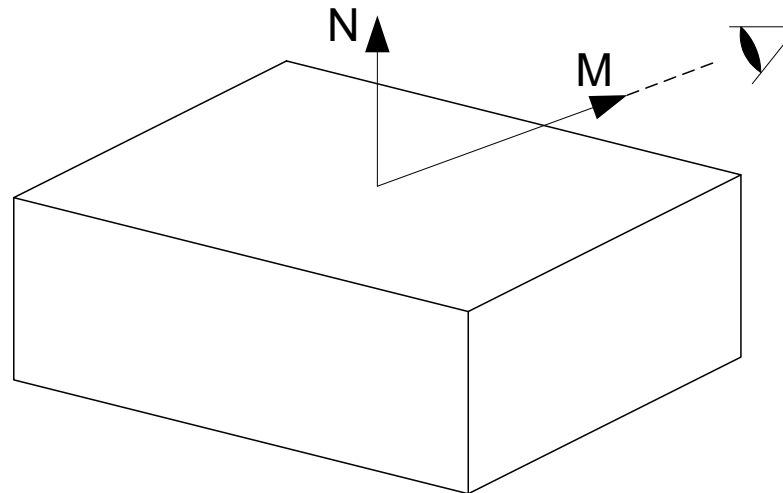 - Very adaptable for use in raster display
  - Hardware-based approach : z-buffer

Back-Face Removal Algorithm (1)

N : outward normal vector of a face
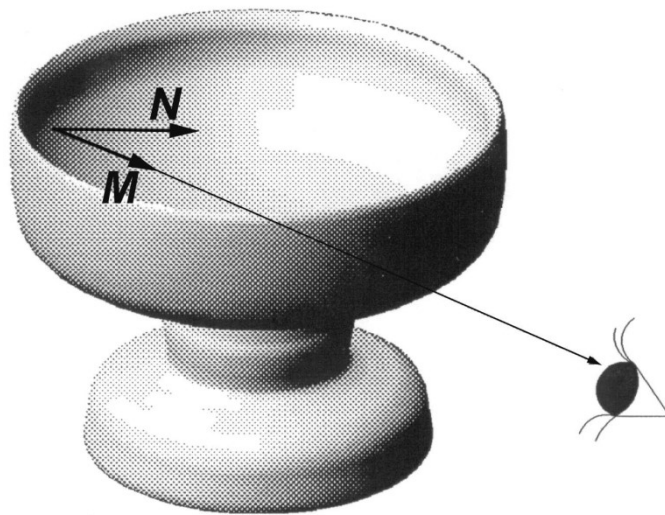
M : a view vector from a point on the face to the viewer

$$\text{if } \begin{cases} M \cdot N > 0, \ \text{the face is visible} \\ M \cdot N = 0, \ \text{the face is displayed as a line } \left(\text{silhouette line}\right) \\ M \cdot N < 0, \ \text{the face is invisible} \end{cases}$$

Back-Face Removal Algorithm (2)

- Limitations
  - Can not be applied to a CONCAVE object
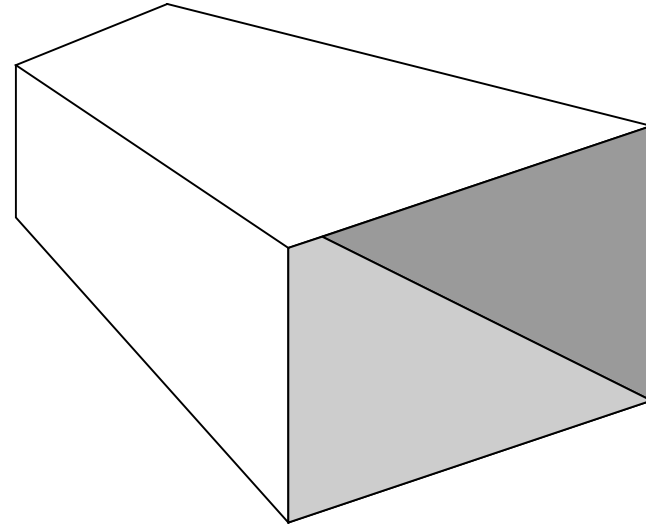
Depth-Sorting or Painter's Algorithm

- (Step 1) The faces of the objects are sorted based on their distances from the viewer

- (Step 2) Paint from the farthest face to nearer

  - from the face of minimum Z_v to the face of maximum Z_v in the Viewing Coordinate System

- NOTE:

  - If the range of Z_v values of all the points on a surface overlaps the Z_v range of the other surface, splitting each of the surfaces into two or several pieces until the Z_v ranges do not overlap

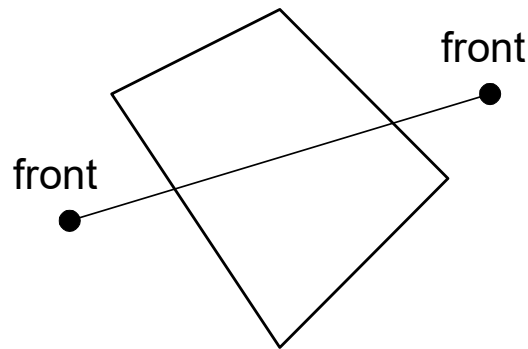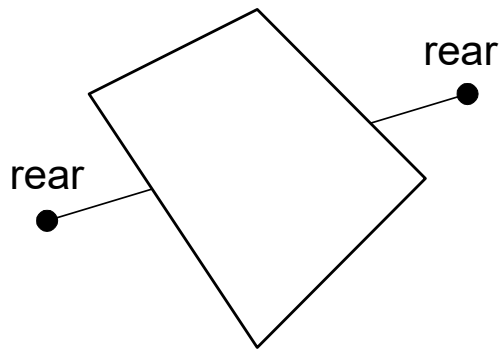# Hidden-Line Removal Algorithm (1)
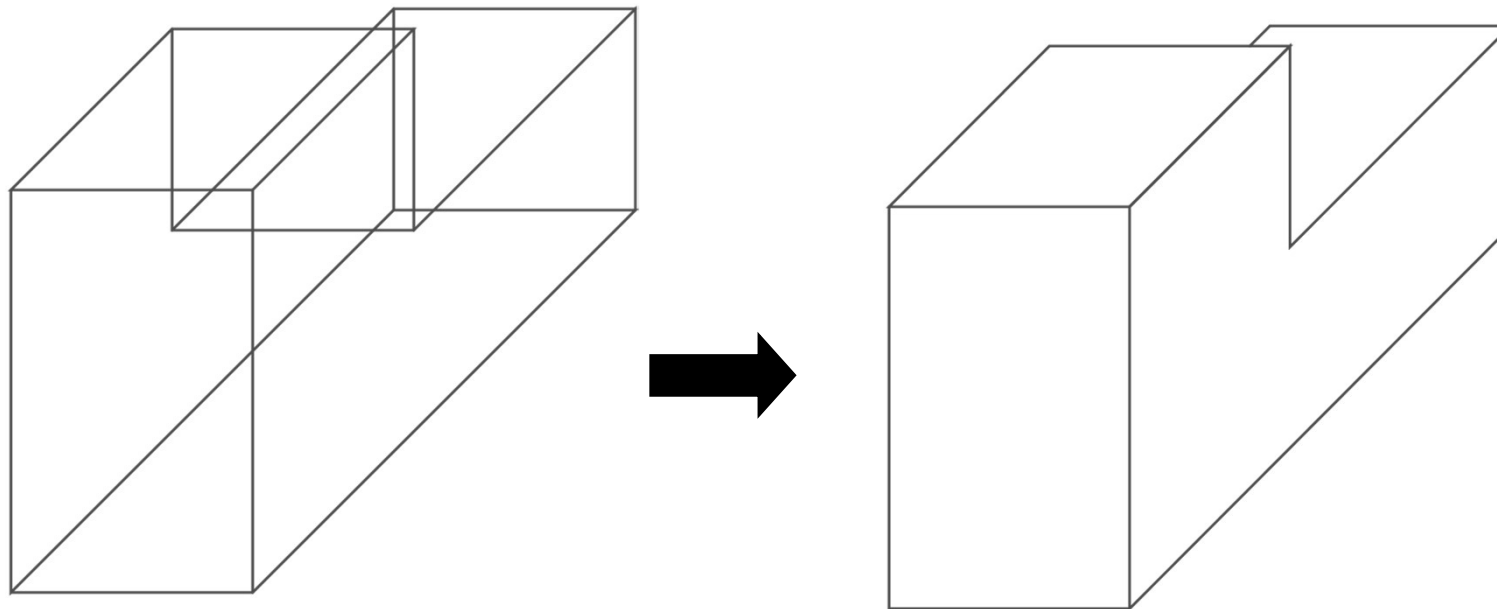
- **Step 1.** The faces pointing toward the viewer are collected by applying back-face algorithm and stored in an array FACE-TABLE with the min/max Z_v values
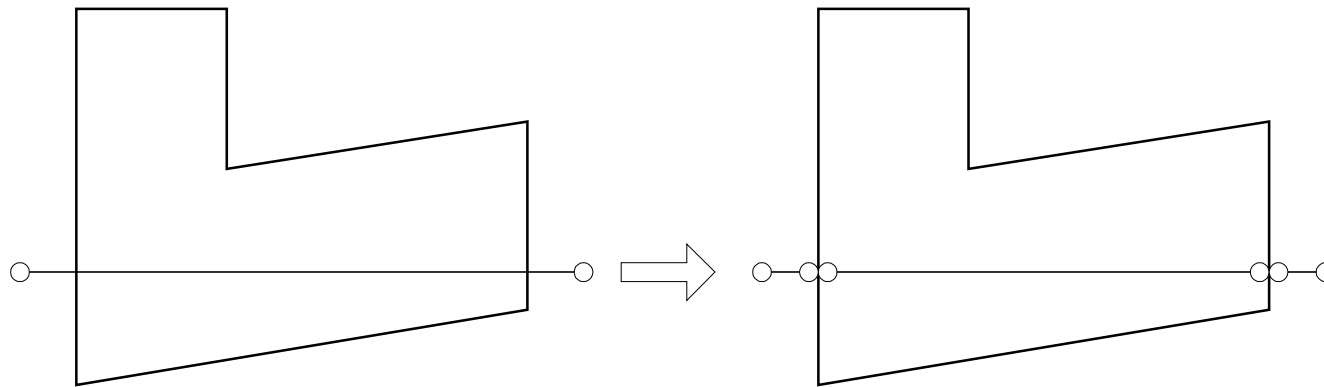
Hidden-Line Removal Algorithm (2)

- **Step 2.** Collect the edges of the faces in FACE-TABLE and test whether they are obscured by each face in FACE-TABLE in sequence

- **Step 3.** Classify the edges as three situations
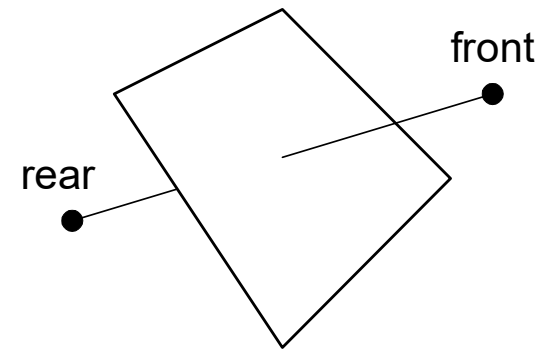
Hidden-Line Removal Algorithm (3)

- **Step 4.** If the projected entities overlap, the edge is split at the intersection points with the face. Then repeat Step 3.
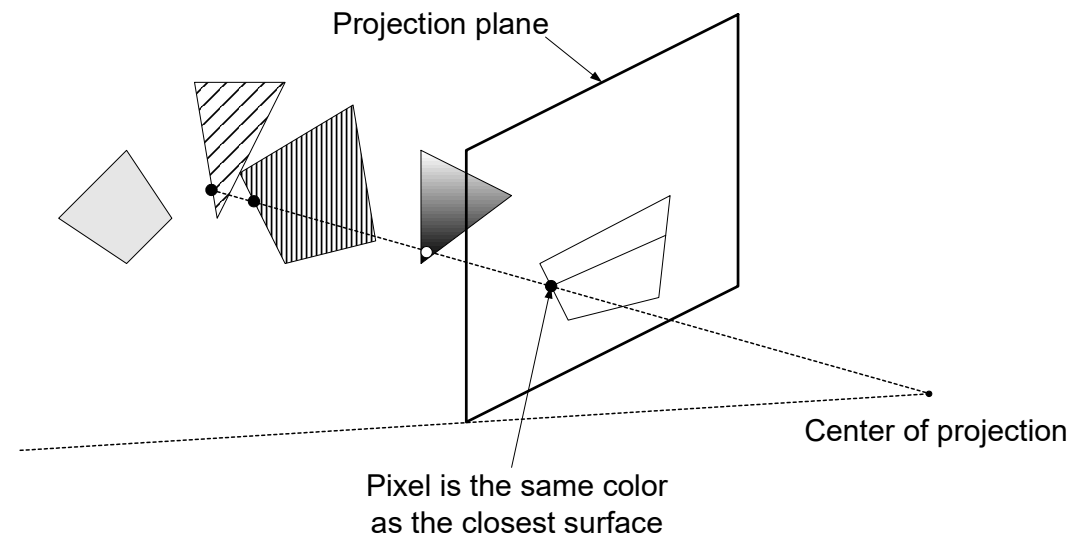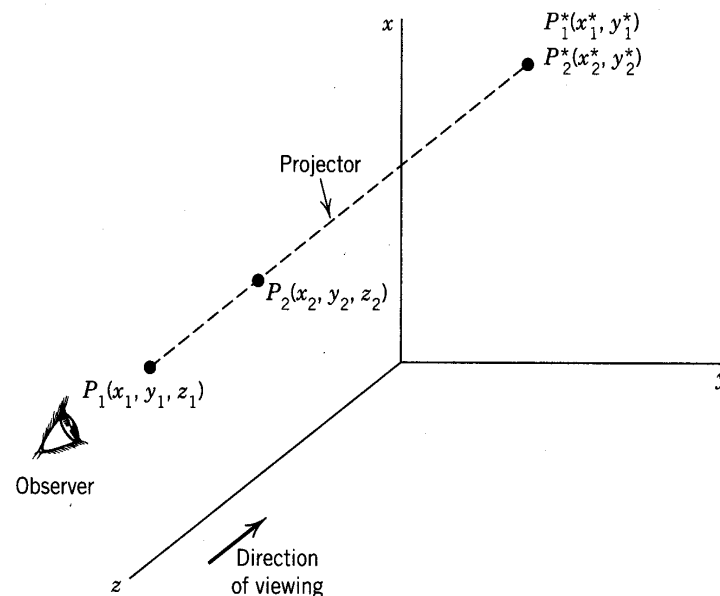
- **Step 5.** The edges which pass through the test with all the faces in the FACE-TABLE are collected to form the visible edges

Z-Buffer Method

- Frame buffer stores the z coordinate or depth of every pixel in the image
- Step 1. Initialize z-buffer with the smallest z value
- Step 2. If (new_z > existing_z) then, place new_z into the z-buffer at (x,y)

Projection plane

Projector

$P_1^*(x_1^*, y_1^*)$
$P_2^*(x_2^*, y_2^*)$

$P_2(x_2, y_2, z_2)$

$P_1(x_1, y_1, z_1)$

Observer

Direction of viewing

Center of projection

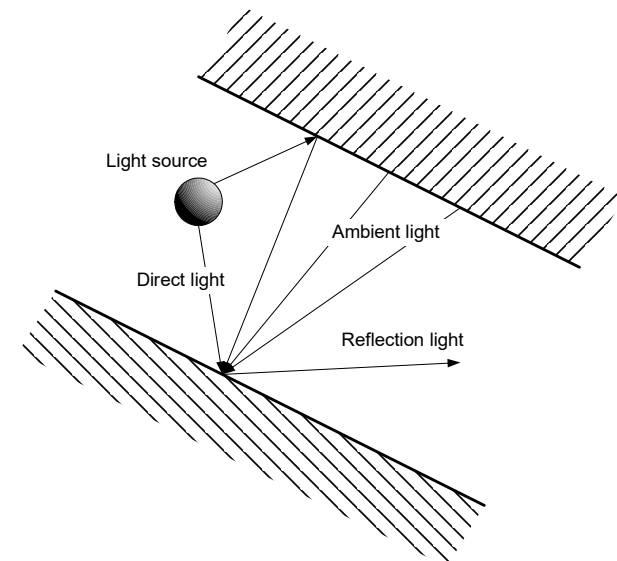Pixel is the same color as the closest surface

Rendering

- The process to recreate the effects of light on surfaces of objects for simulation of a real scene
- Two major rendering techniques
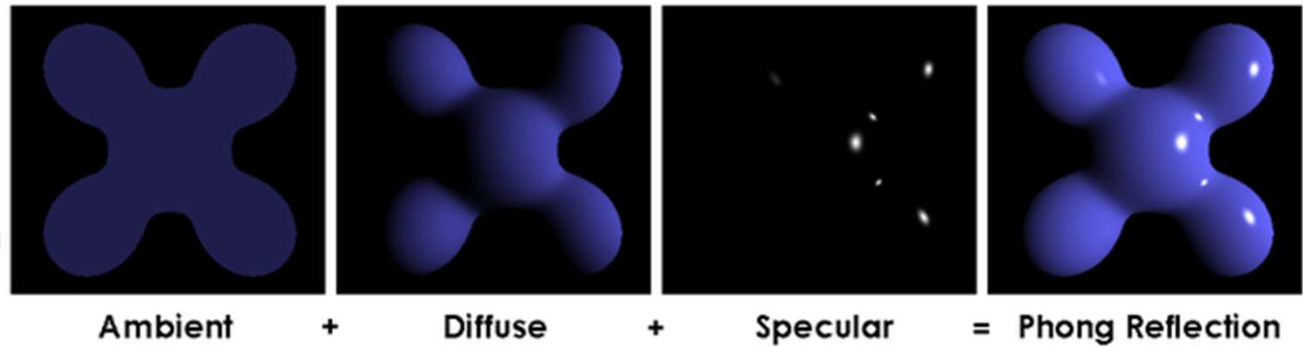 - Shading
  - Ray Tracing
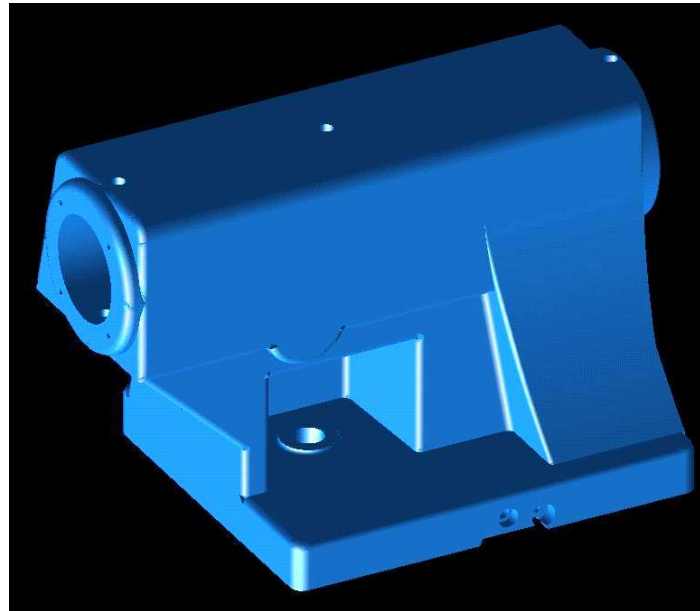
Shading (1)

- **Shading Parameters**
  - Light sources
  - Characteristics of the surface
  - Relative position of the light sources w.r.t. the surface

- **Light Source**
  - Position, intensity, direction, etc.
  - Classification of Light Source
    - Point light : distributes the light from a single point in specific directions
    - Ambient light : distributes uniformly in all directions, without regard to location
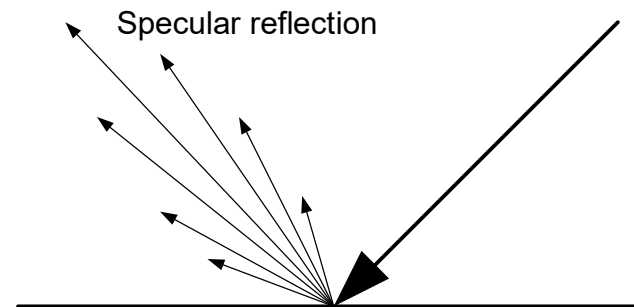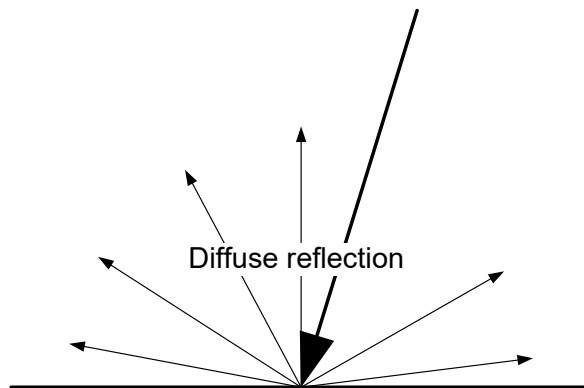
Light source

Ambient light

Direct light

Reflection light

Shading (2)

Ambient + Diffuse + Specular = Phong Reflection

- **Surface Characteristics (Ways of Reflection)**
  - Diffuse reflection: light is scattered equally in all directions - commonly used
  - Specular reflection: reflects light in only one direction
  - *Real surfaces reflect light by a combination of these two effects*

$$I = k_d I_a + \frac{I_s}{D + D_0} \left[k_d \left(N \cdot L \right) + k_s (V \cdot R)^n \right]$$

Diffuse reflection

Specular reflection

Intensity of Diffuse Reflection

- **Lambert's Cosine Law**
  - Intensity Value

$$I = I_s k_d \cos\theta = I_s k_d (\mathbf{N} \cdot \mathbf{L}) \rightarrow I = \frac{I_s k_d}{D + D_0}(\mathbf{N} \cdot \mathbf{L})$$

I_s : intensity of the point light source

k_d : diffuse coefficient indicating the surface reflectivity $(0 < k_d < 1)$

$\mathbf{N}$: surface normal (unit vector)

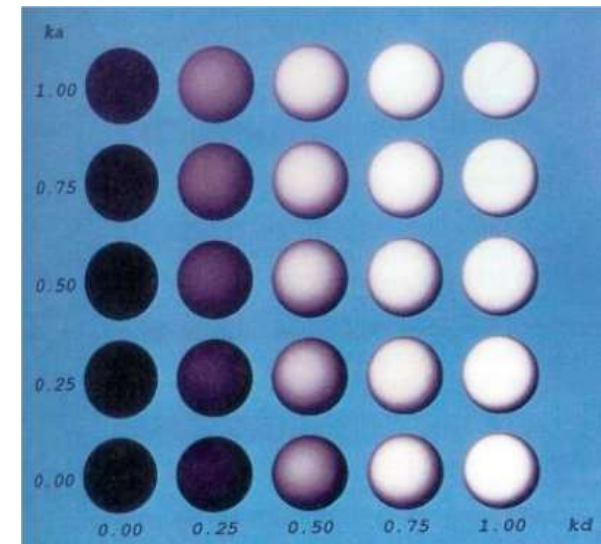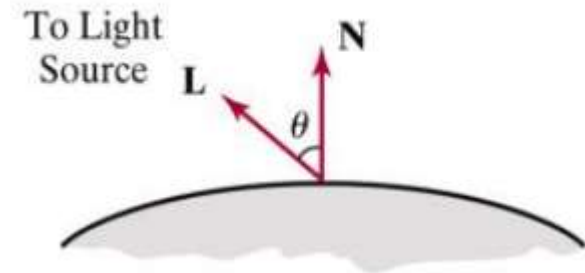$\mathbf{L}$: direction of light source (unit vector)

D : distance between the point of interest and the viewpoint

D_0 : to avoid division by zero

  - Several Light Sources and Ambient Light

$$I = k_d I_a + \sum_j I_j k_d (n \cdot i_j)$$

I_a : contribution from ambient light

Intensity of Specular Reflection

- **Phong (specular-reflection) Model**
  - For a directional light, we can attenuate the light intensity angularly as well as radially

$$f(\alpha) = \cos^n \alpha$$

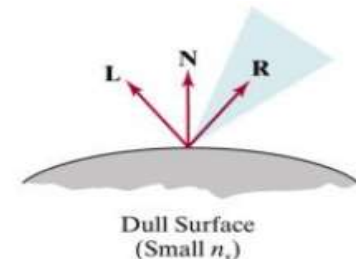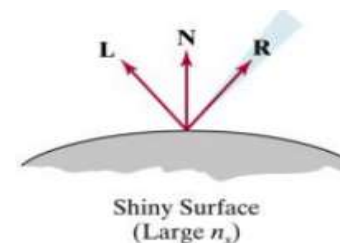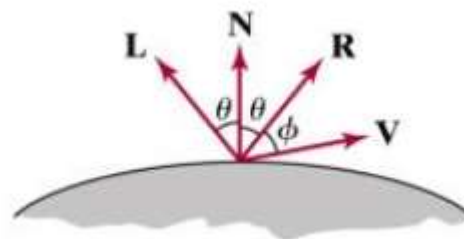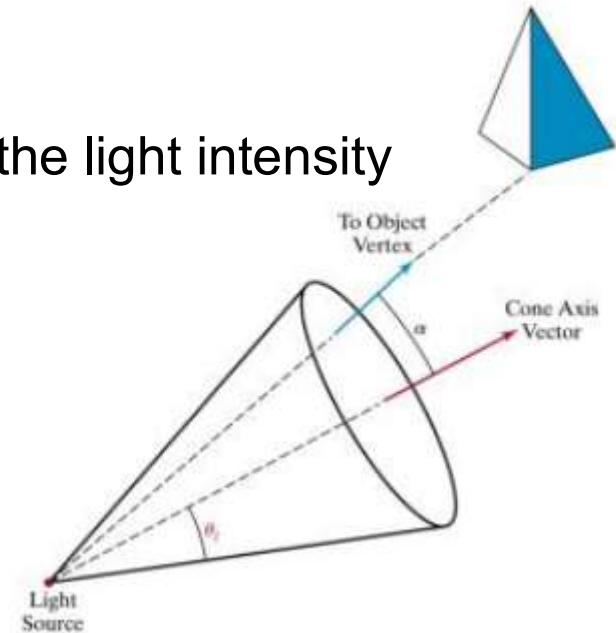$$I = \frac{I_s}{D + D_0} k_s \cos^n \phi = \frac{I_s}{D + D_0} k_s (V \cdot R)^n$$

I_s : intensity of the point source

k_s : specular reflectance, coefficient indicating the surface reflectivity $(0 < k_s < 1)$

D : distance between the point of interest and the viewpoint

V : line of sight vector

n: gloss of the surface

To Object Vertex

Cone Axis Vector

Light Source

N L R θ θ ϕ V

Shiny Surface (Large n_s)

Dull Surface (Small n_s)
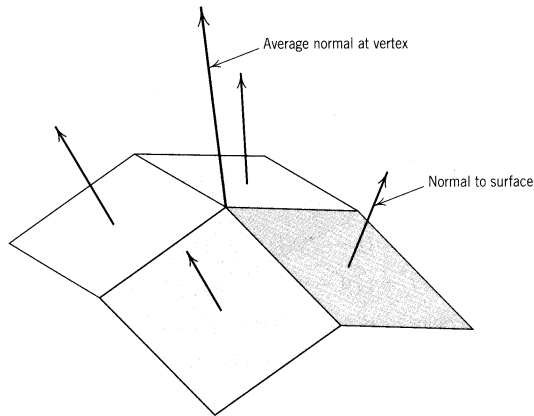
Faceted Shading

- **For Polygonal Models**
  - Assign the same light intensity to all points on a facet
  - Creating banded appearance

- **Quickest shading algorithm**

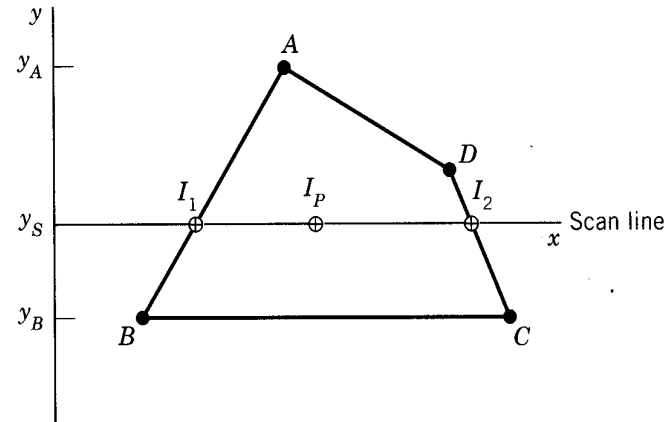- **Commonly used as a preliminary design check**
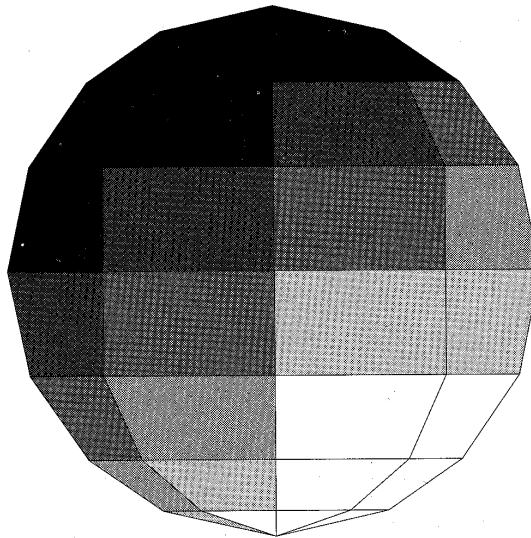
Gouraud (Smooth) Shading

- (1) Light intensities are calculated at each vertex
- (2) They are blended across the overall surface by linear interpolation
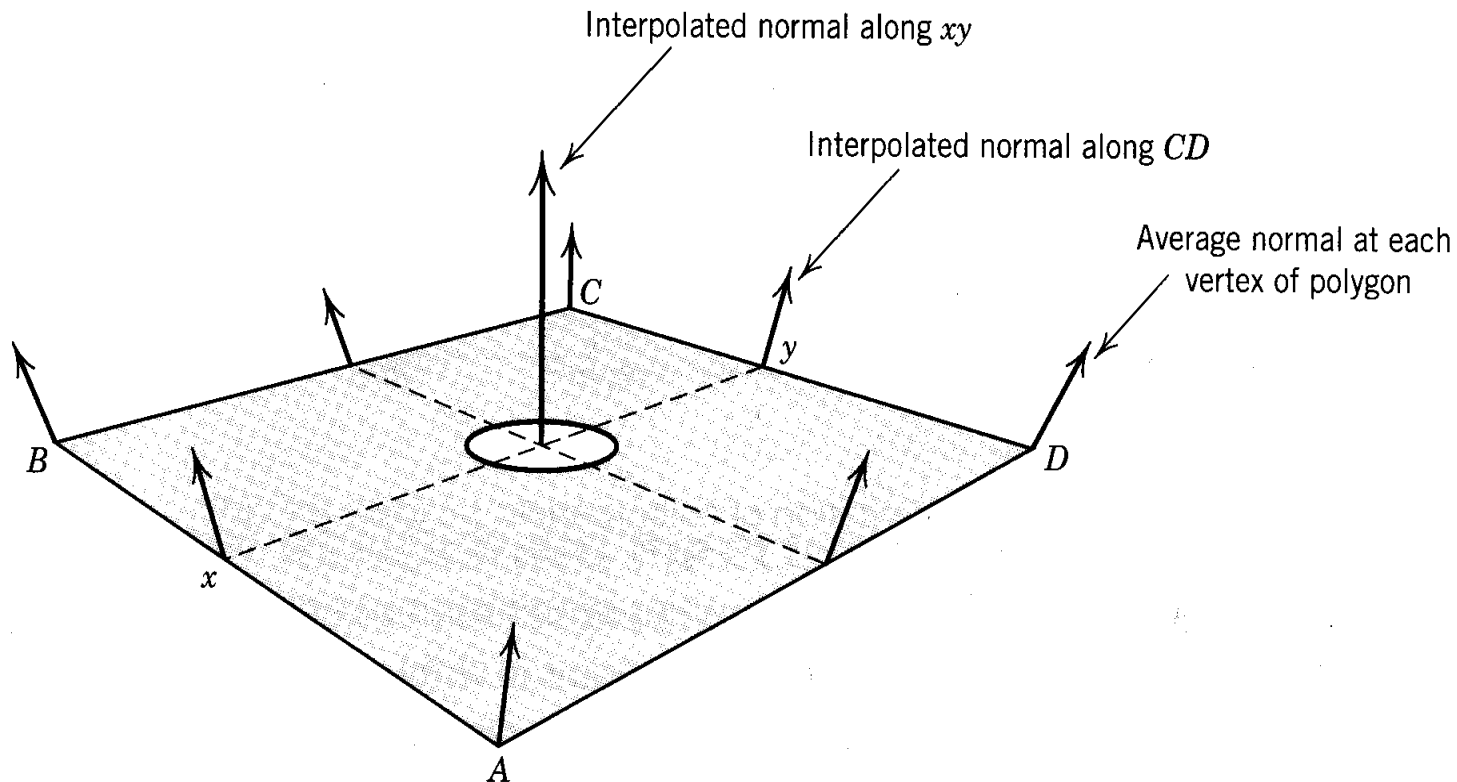

$$I_1 = \frac{(I_B - I_A)(y_A - y_S)}{(y_A - y_B)} + I_A$$

$$I_2 = \frac{(I_C - I_D)(y_D - y_S)}{(y_D - y_C)} + I_D$$

$$I_P = \frac{(I_2 - I_1)(x_P - x_1)}{(x_2 - x_1)} + I_1$$

$$n_{avg} = \sum_i n_i \Big/ \left| \sum_i n_i \right|$$

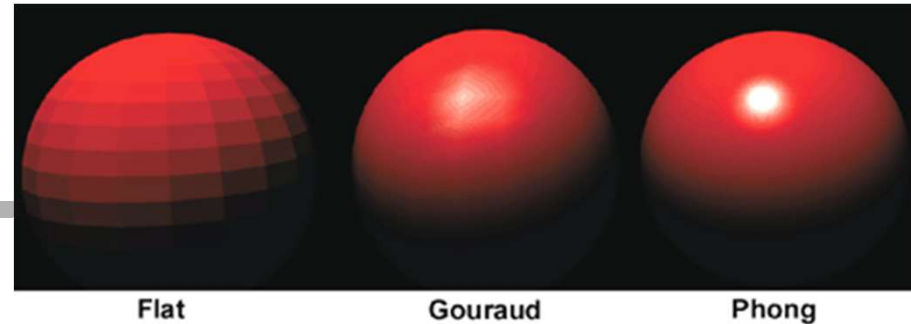# Phong Shading

- ## Normal Interpolation Shading
  - Interpolates normal vectors instead of shade intensities

Interpolated normal along xy

Interpolated normal along CD

Average normal at each vertex of polygon
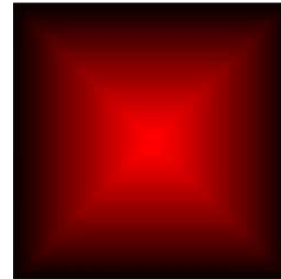
C

y

B

D

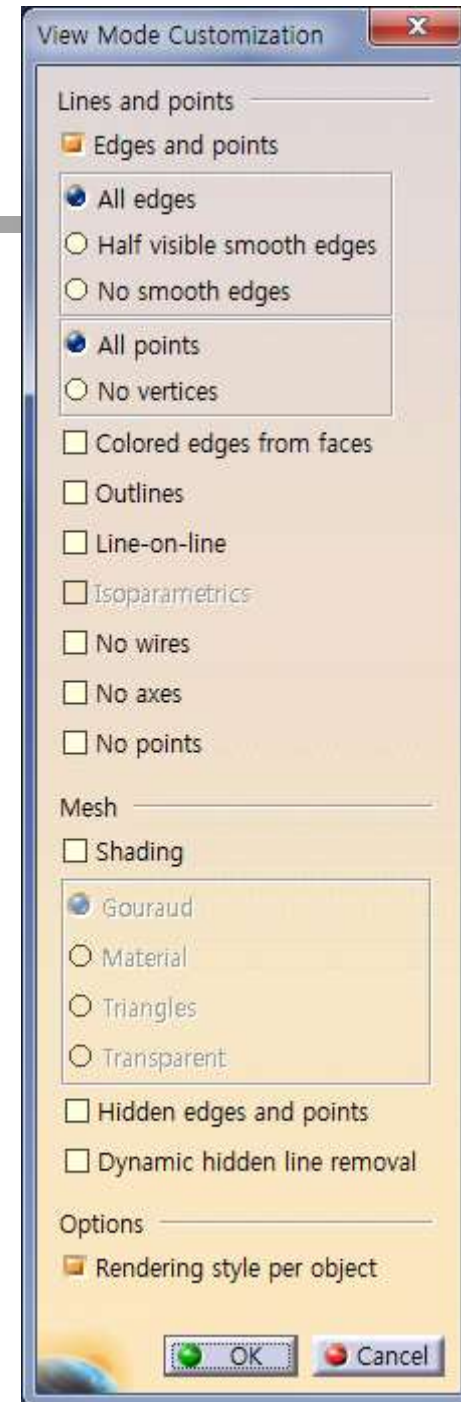x

A

Comparison

- **Gouraud Shading**
  - Highlights can be seen only at the vertices and can be distorted
  - Mach bands (rapid brightness change) can occasionally be produced

- **Phong Shading**
  - Much slower than Gouraud shading but create very realistic pictures
  - Constitute the core of most rendering systems
  - Silhouettes of curved surface are always shown in a polygonal form (unrealistic)
  - Variety of methods can add texture, shadows, transparency, and the like, to the shaded surfaces

CATIA V5

View Mode Customization

Lines and points
- Edges and points
  - All edges
  - Half visible smooth edges
  - No smooth edges
  - All points
  - No vertices
- Colored edges from faces
- Outlines
- Line-on-line
- Isoparametrics
- No wires
- No axes
- No points

Mesh
- Shading
  - Gouraud
  - Material
  - Triangles
  - Transparent
- Hidden edges and points
- Dynamic hidden line removal

Options
- Rendering style per object

OK   Cancel

Options
- Options
  - General
    - Display
    - Compatibility
    - Parameters and Measure
    - Devices and Virtual Reality
  - Infrastructure
    - Product Structure
    - Material Library
    - Catalog Editor
    - Photo Studio

Material | Shading Display

General parameters
- Deactivate ambient lighting

Default Shading Material parameters
- Ambient & Diffuse coefficient: 1.00
- Specular coefficient: 1.00
- Roughness coefficient: 0.41

View mode

Ray Tracing

- Support multi-object rendering unlike Shading
- Trace back the rays passing through the screen pixels
  - Pass out of the viewing volume: color of the light source
  - Strike a diffuse surface: background color
  - Hit the light source: color of the reflection on the surface
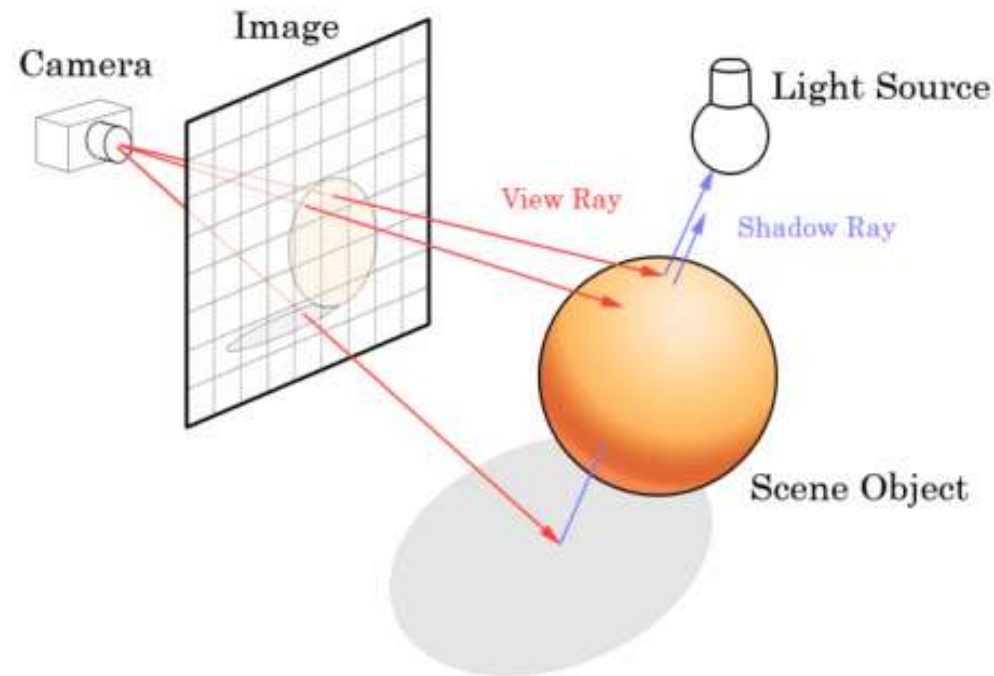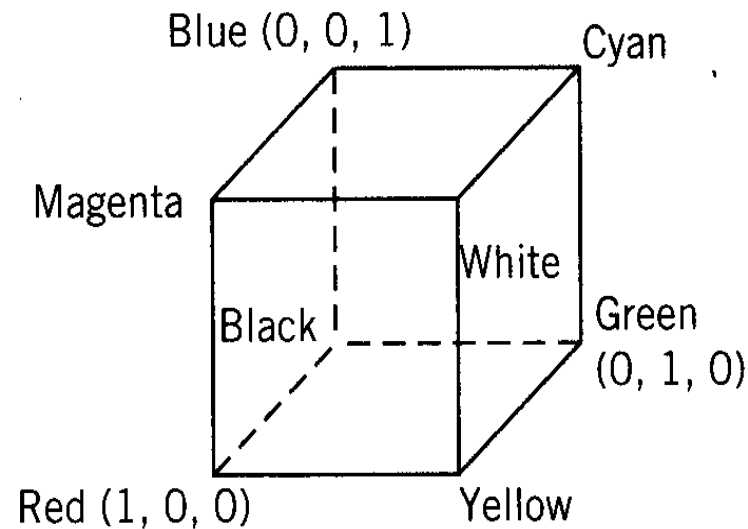
Color Models

- RGB Model
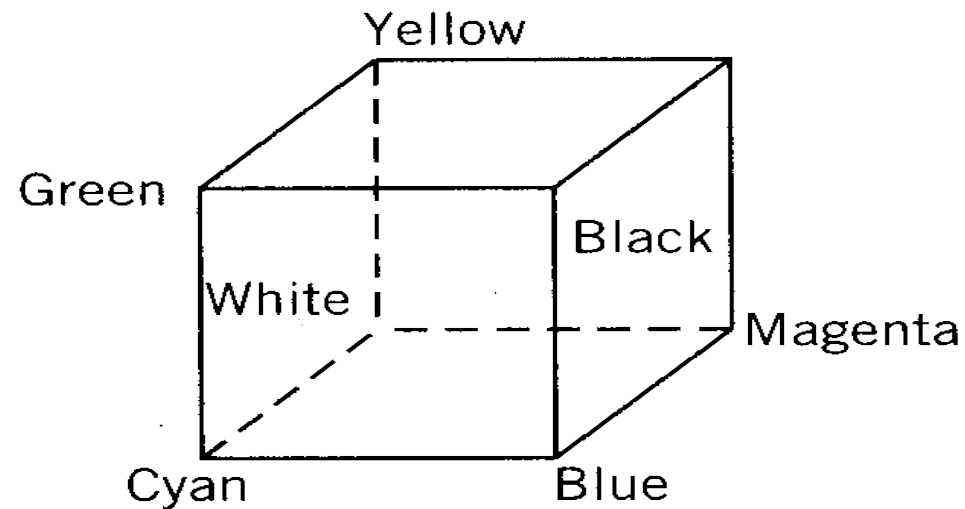- CMY Model
- HSV Model

RGB Model

- RGB primaries - red, green, and blue
- Additive model
- Black - origin (0,0,0) : White - at the point (1, 1, 1)
- All other colors are represented by points inside the cube
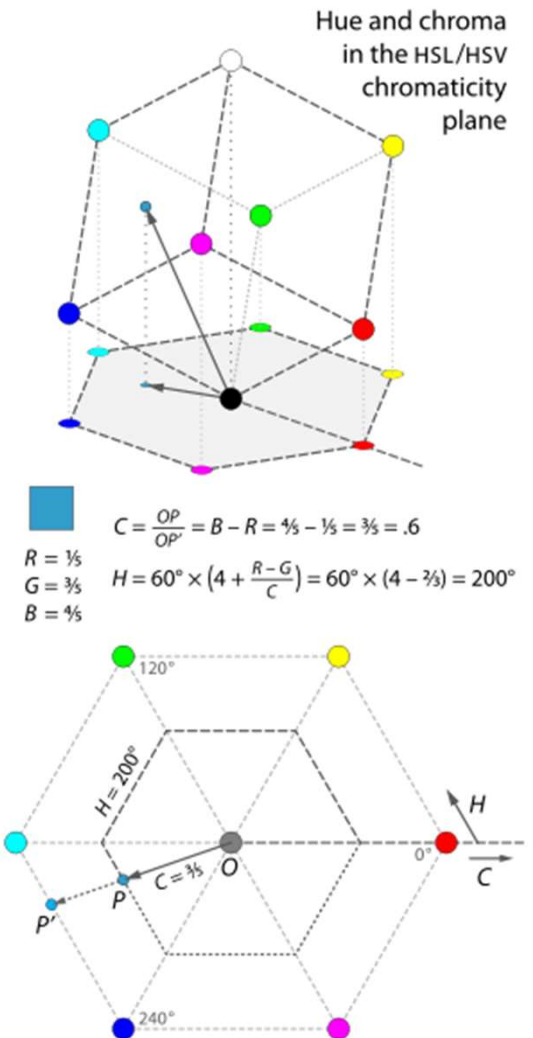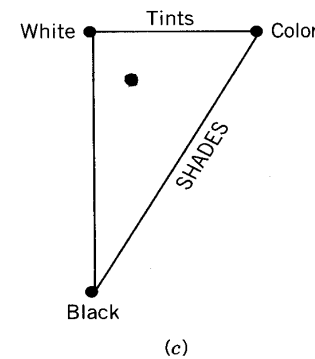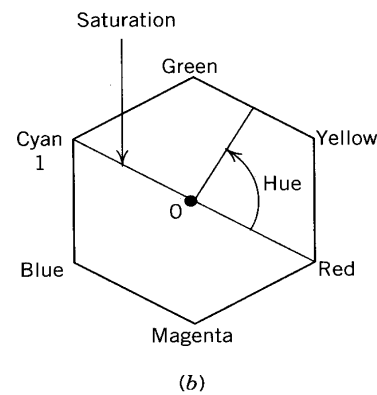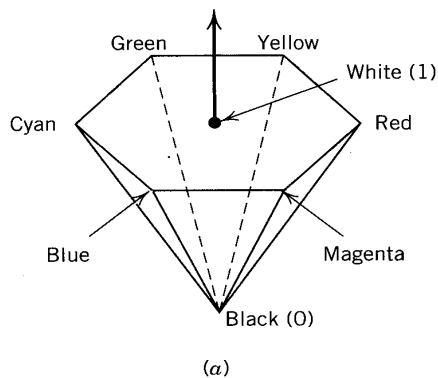
CMY(K) Model

- Cyan, Magenta, and Yellow, (Key: Black)
- Subtractive model
- White - Origin (0,0,0) : Black - at (1,1,1)

HSV Model

- **cylindrical-coordinate representations of points in an RGB color model**
  - Hue - color spectrum : angle (0° to 360°)
  - Saturation - purity : (0 to 1)
  - Value of a color – lightness, brightness (0 to 1)

Hue and chroma in the HSL/HSV chromaticity plane

$$C = \frac{OP}{OP'} = B - R = 4/5 - 1/5 = 3/5 = .6$$

$R = 1/5$
$G = 3/5$
$B = 4/5$

$$H = 60° \times \left(4 + \frac{R-G}{C}\right) = 60° \times (4 - 2/3) = 200°$$

(a)

(b)

(c)

CATIA V5

– The HSL (Hue, Saturation and Luminance) and RGB (Red, Green and Blue) values vary according to where the cross is located. You can also enter HSL and RGB values in the fields provided to suit your exact color specifications.
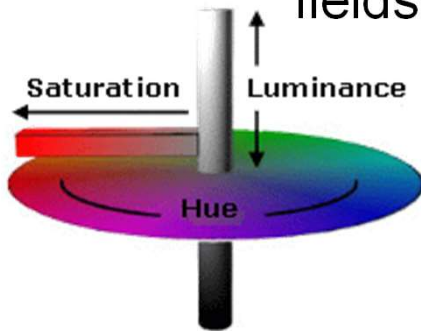
View →Toolbars →Graphic properties