

Geometric Modeling

기하

- 기하학적 형상을 만드는 행위

- Modeling

- Model

대상

complete

precise

– 물체의 특성을 완전하고 정확하게 표현해야 함 ✓

– Physical Model vs. Mathematical Model

clay model

모형

VR/AR

$$\vec{F} = m \vec{a}$$

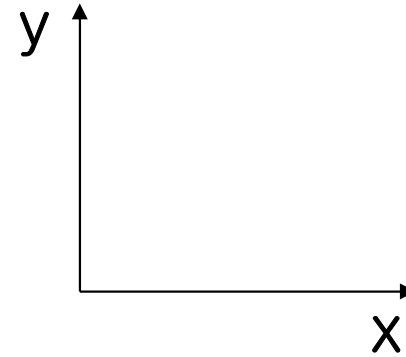
자연법칙

nature law

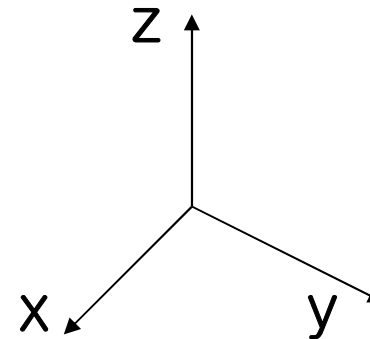
* 공학이론? 예측 prediction

Modeling Space

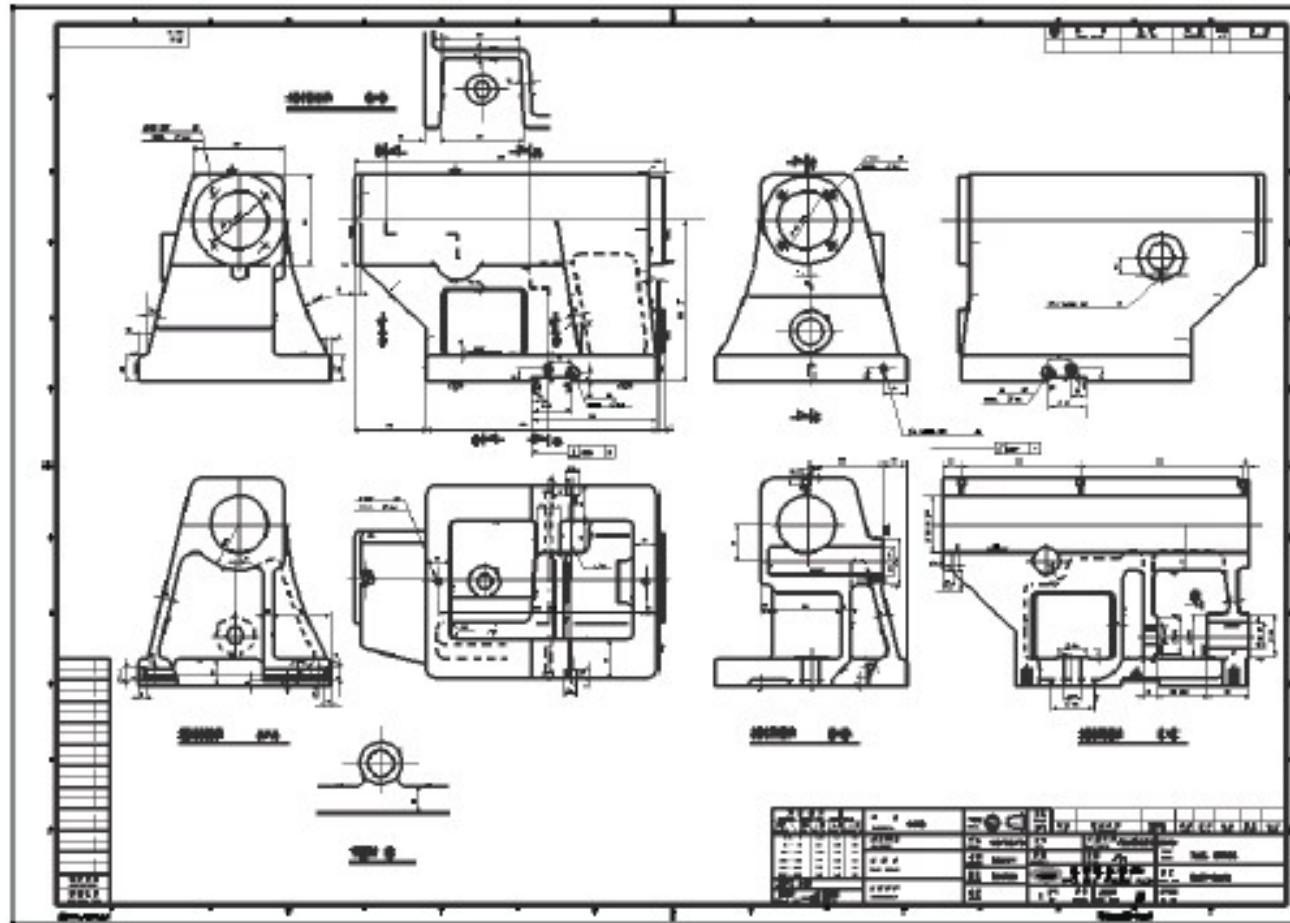
- 2D space (x,y)
 - Computer aided drafting



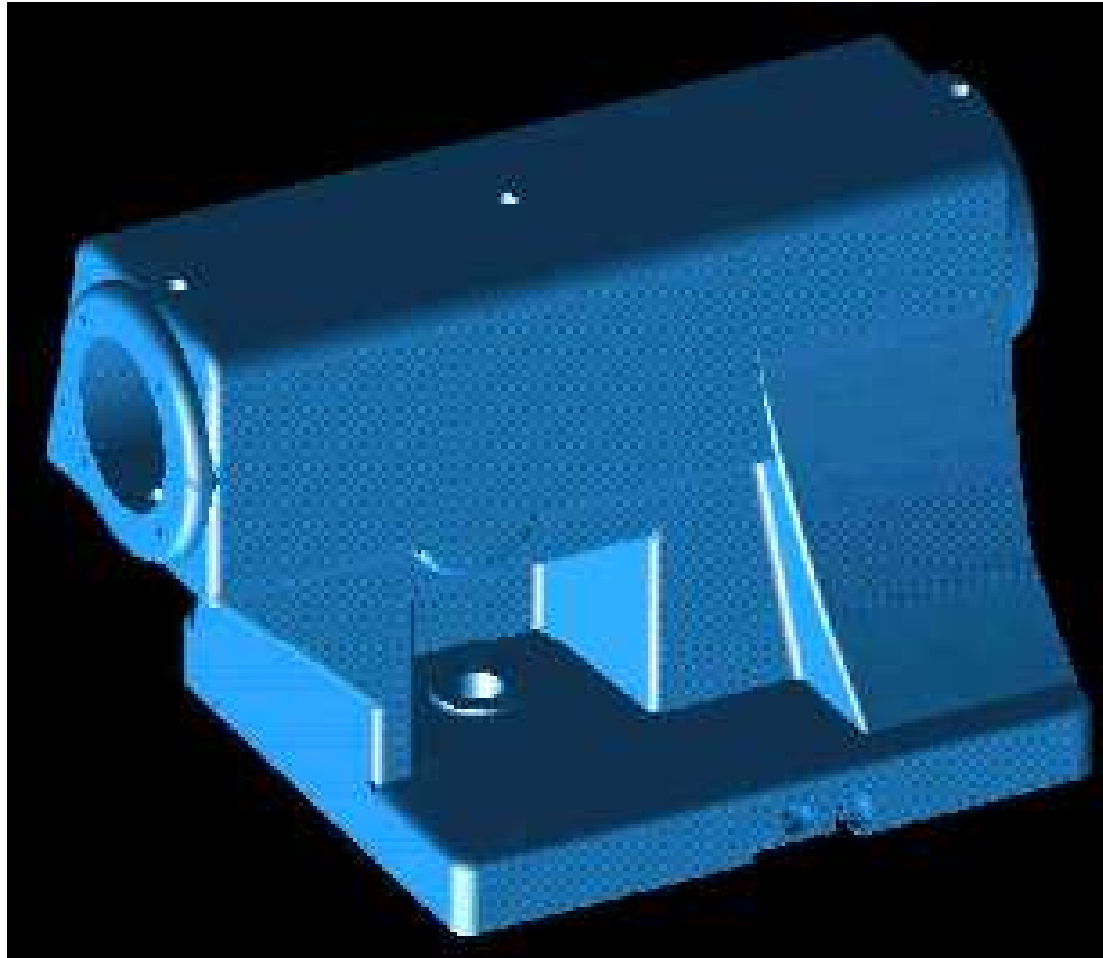
- 3D space (x,y,z)



Why 3D Model? (1)



Why 3D Model? (2)



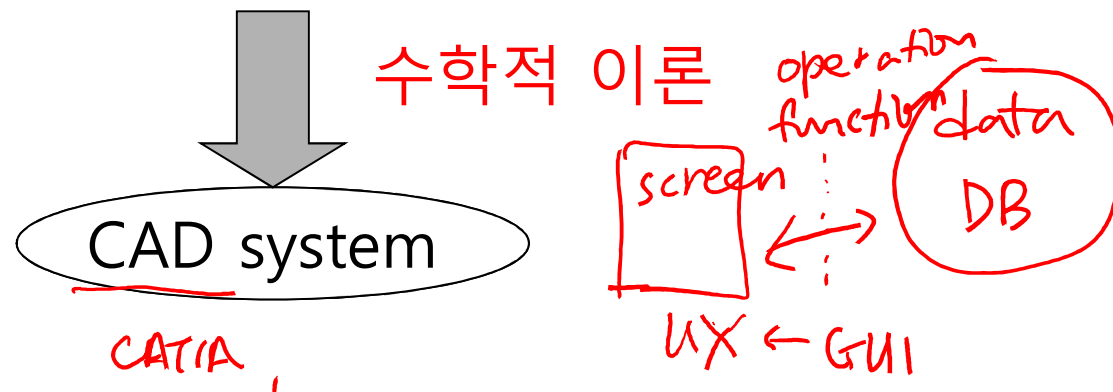
Geometric Entity

- Point : 좌표값
- Curve : 곡선의 방정식 (planar/space/freeform)
- Surface : 곡면의 방정식 (plane/sculptured)
- Solid : 부피를 둘러싸고 있는 곡면들의 방정식

Geometric modeling

↓
Solid modeling

Geometric entity를 편리하게 정의하는 기능
해당좌표나 방정식을 유도하고 저장하는 기능



점(point) 정의 $(x, y, z) \rightarrow (u, v, w)$ (v, q, p)

직접입력

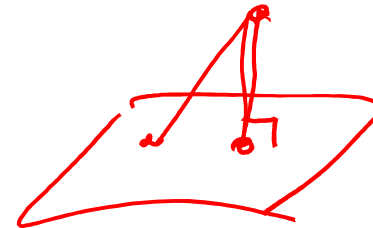
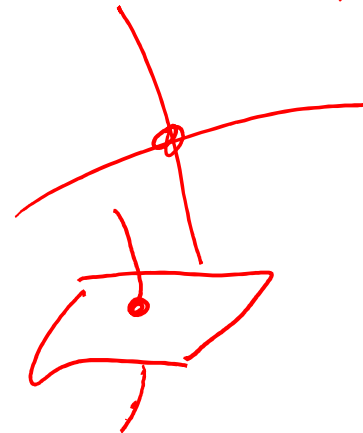
- 좌표값 입력

x, y, z

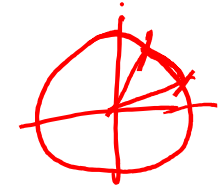
간접입력

- 두 곡선의 교점 이용
- 점을 곡면 위에 투영
- 곡선과 곡면의 교점 이용
- 주어진 곡선 위에 일정한 간격으로 n 개의 점 생성

parametric



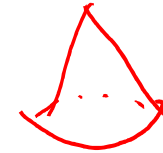
$$x^2 + y^2 = 1$$



2D 곡선(curve) 정의

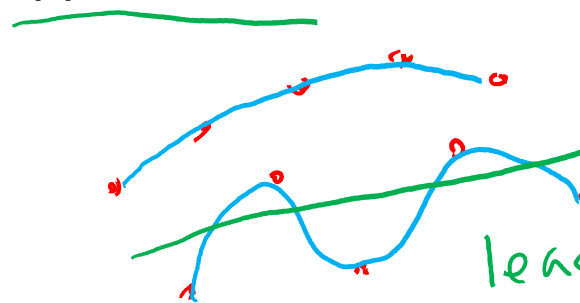
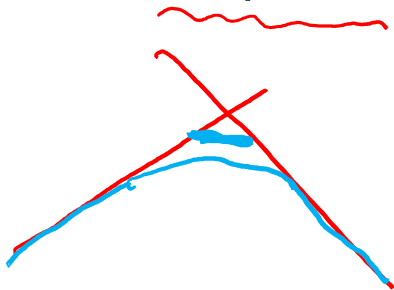
- 기본 2D 곡선 이용
 - 직선(line), 원(circle), 타원(ellipse), 포물선(parabola), 쌍곡선(hyperbola)
 - 몇 개의 파라미터값으로 간단하고 정확하게 표현
- 두 곡선 사이를 Blending
 - Rounding/Chamfer
- 순서가 정해진 여러 개의 2D 점 이용
 - 자유곡선을 정의
 - Interpolation/Approximation

2차 곡선



sketch

spline 3차 곡선
(미지수 4개)

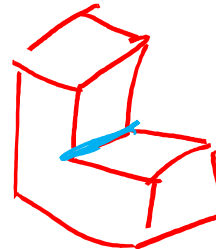
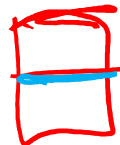
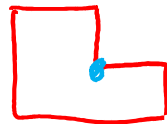
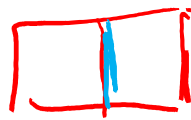
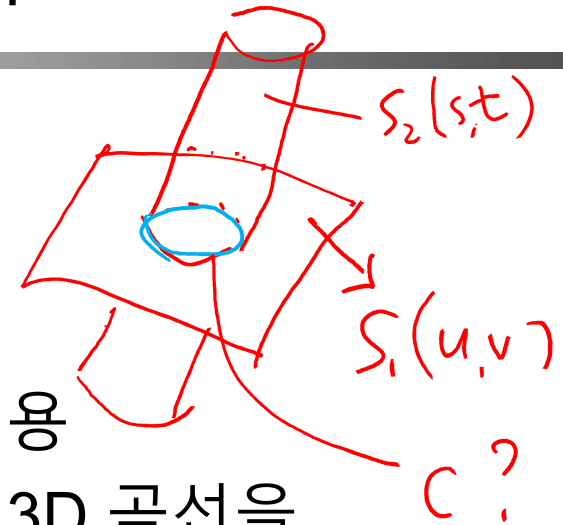


least square

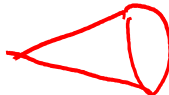
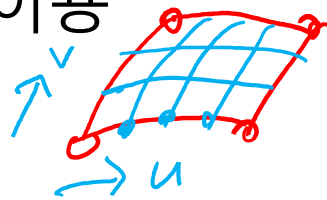


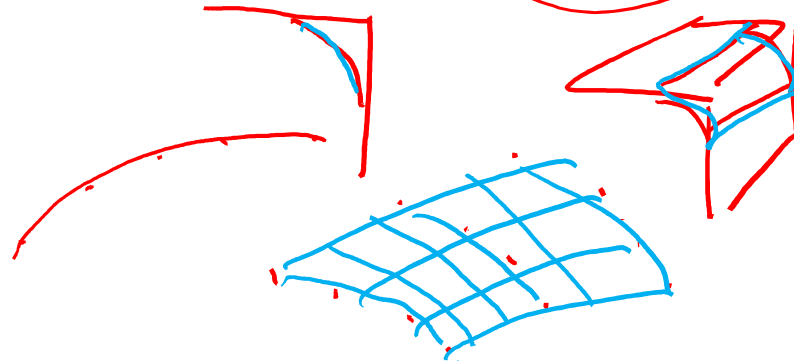
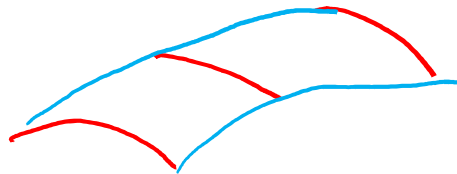
3D 곡선(curve) 정의

- 순서가 정해진 여러 개의 3D 점 이용
 - Interpolation/Approximation
- 두 곡면의 교선 이용 *intersection curve*
- 3D 곡선을 곡면 위에 투영시킨 곡선 이용
- 여러 view에 그려진 2D 곡선들로 부터 3D 곡선을 역으로 계산



곡면(surface) 정의

- 기본곡면 이용 
← 기본곡선
– 원통(cylinder), 원추(cone), 구(sphere), 타원체(ellipsoid),
원환체(torus)
- 두 곡면 사이를 Blending
- 윤곽곡선이나 단면곡선 Sweeping
- 그물과 같은 곡선망을 이용
– Curve-net fitting 
- 3D 점들의 집합을 이용
– Surface fitting



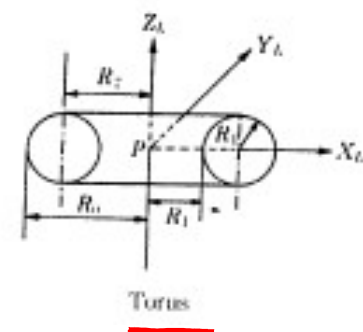
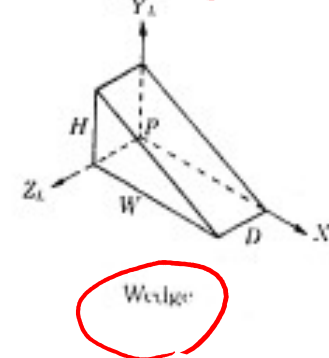
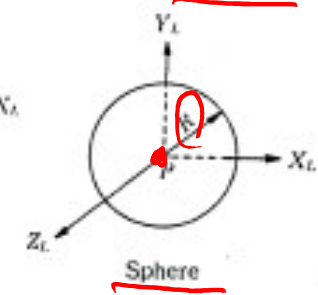
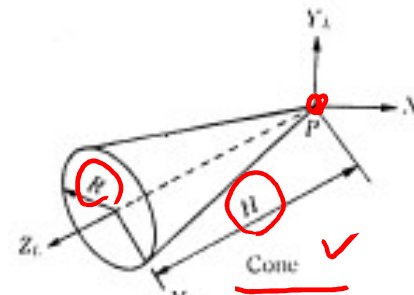
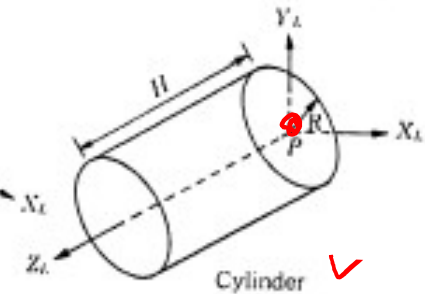
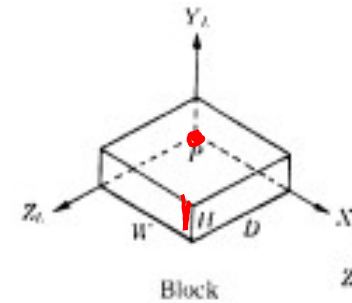
솔리드(solid) 정의

- 곡면 + 부피(닫힌 형태)
- Primitives *기본형태/솔리드*
- Complex Solids
 - Boolean operations of primitives

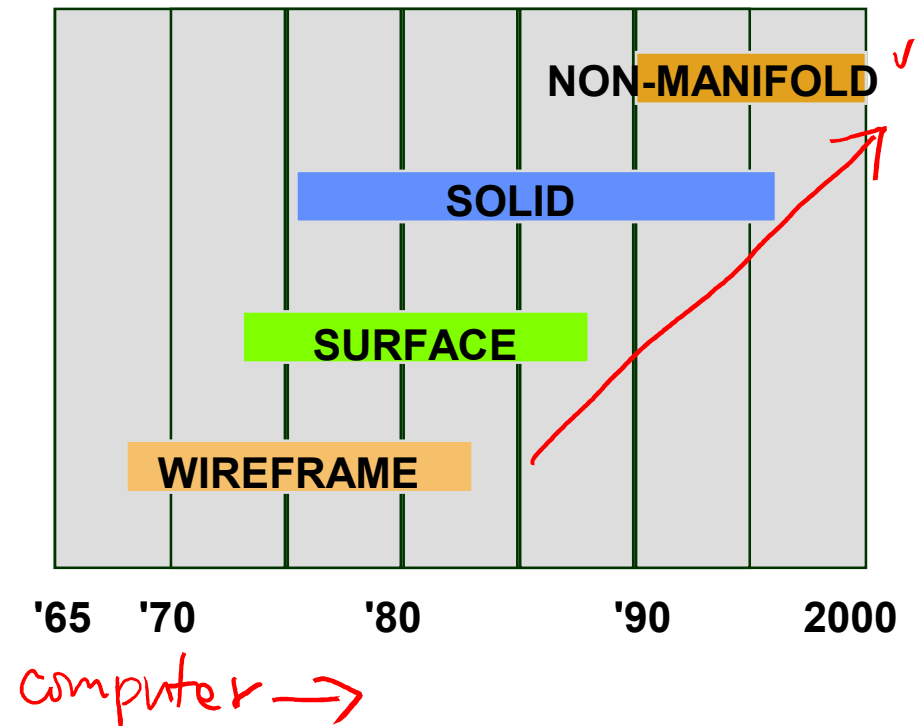
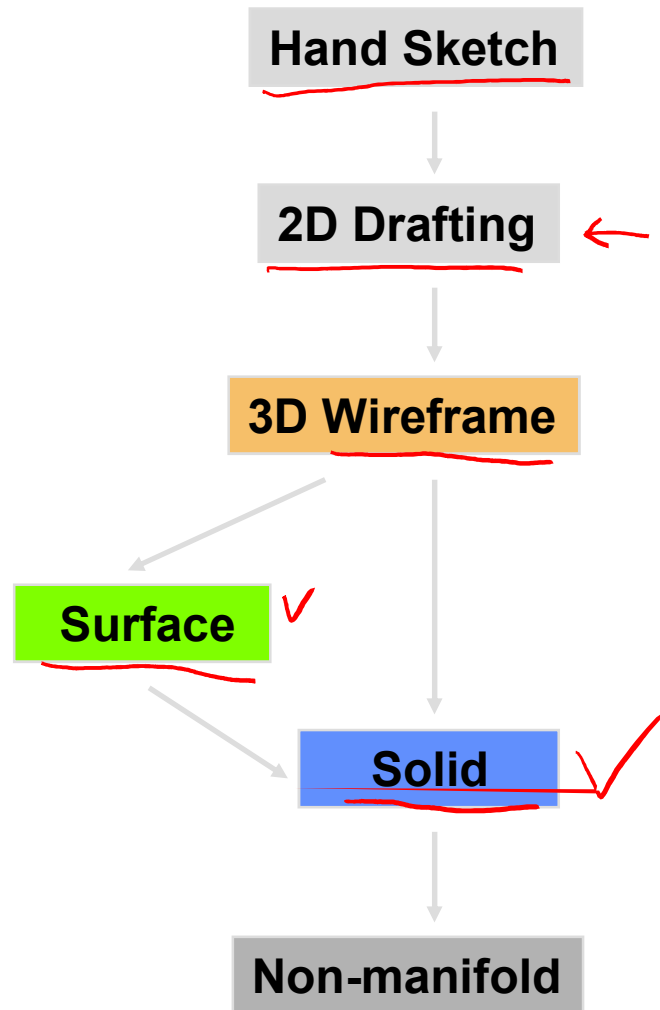
AND, OR, XOR
집합

origin, W, D, H

(R, H)

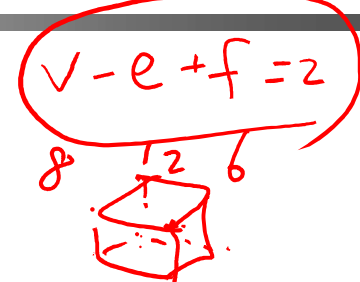


Evolution of Geometric Modeling Systems

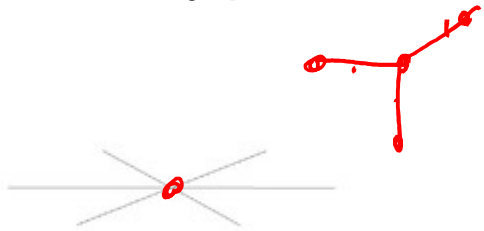


Manifold vs. Non-manifold

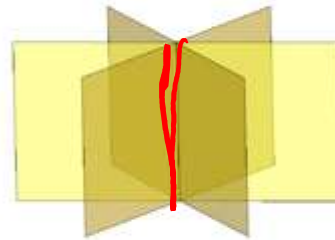
Euler eqn
 $V - E + F = 2$



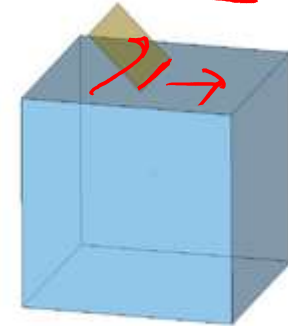
- Manufacturable vs. Non-manufacturable
- In manifold model,
 - Every point on a surface is two-dimensional
 - Every point has a neighborhood that is homeomorphic to 2D disk



Vertex in a wire body

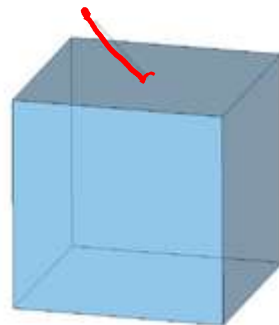


Edge in a sheet body



Edge

dangling



Vertex



- 



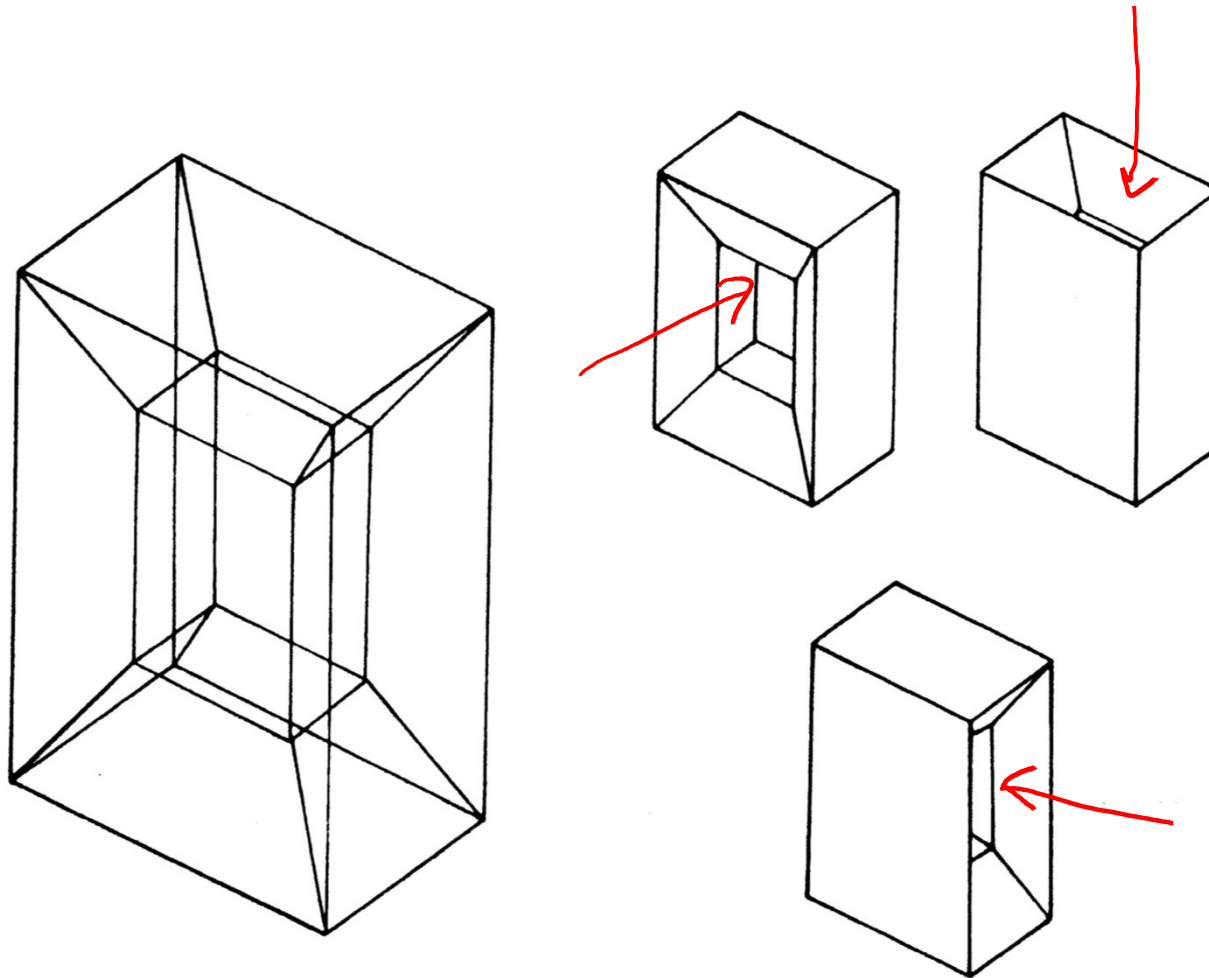
Contents

- Wireframe Modeling Systems
- Surface Modeling Systems
- Solid Modeling Systems
 - Modeling Functions ✓ Part Design
 - Data Structure ✓
 - Euler Operators ✓
 - Boolean Operations ✓
 - Calculation of Volumetric Properties
- Nonmanifold Modeling Systems

Wireframe Model

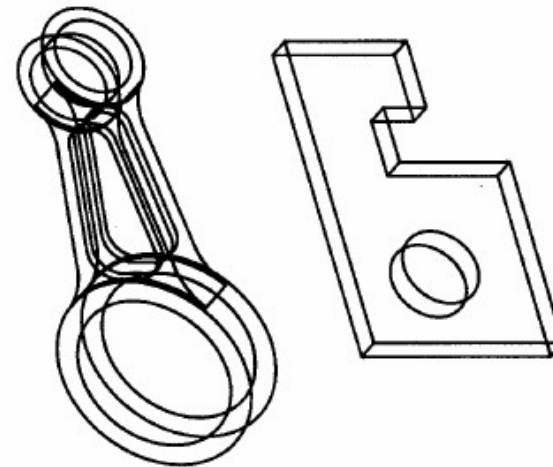
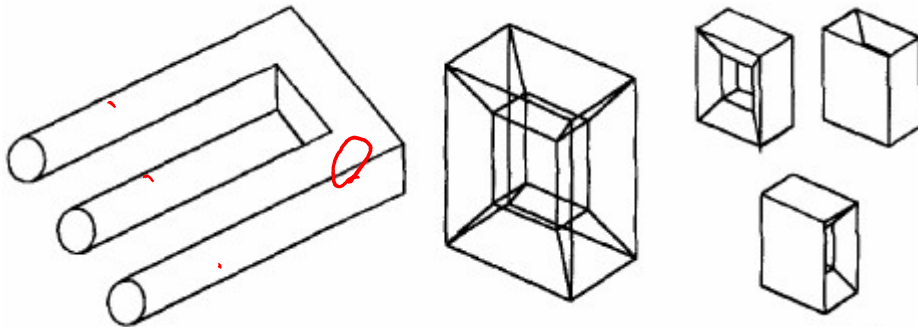
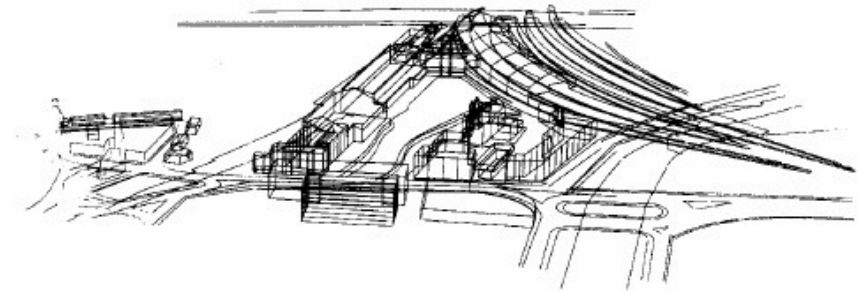
- Database
 - Represent a shape by its characteristic lines and end points
- Advantages
 - Require simple user input to create a shape
 - Easy to develop systems
- Disadvantages
 - Models can be ambiguous
 - No boundary surfaces and volume information
 - Impossible to calculate mass properties, drive NC tool paths, generate FEM meshes
- Products
 - Sketchpad, Steerbear

Ambiguous Wireframe Model

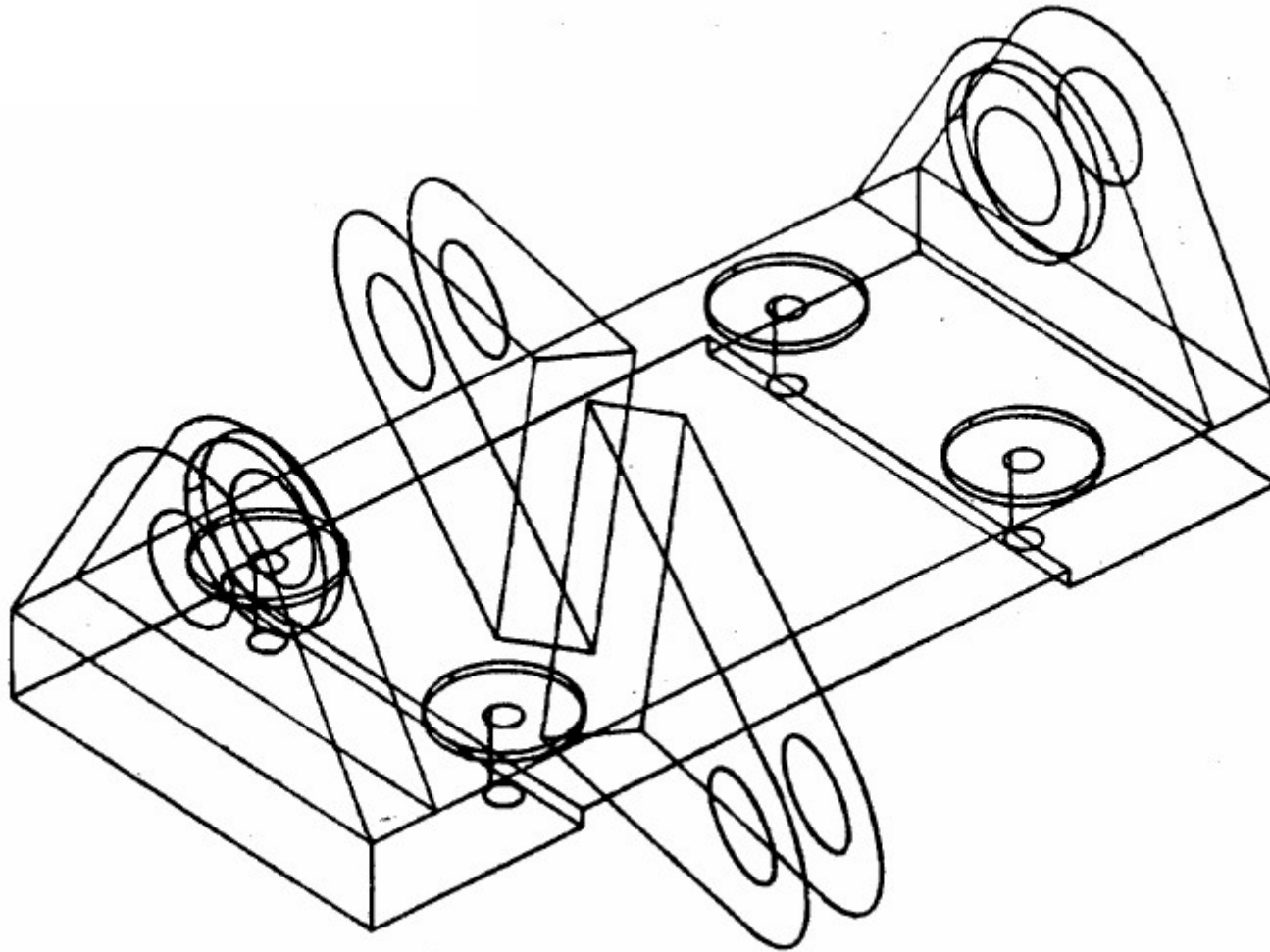


Basic 3D Models: Wireframe (1)

- Wireframes
 - easiest of all to create
 - nothing hidden
 - visually ambiguous
 - topological problems?

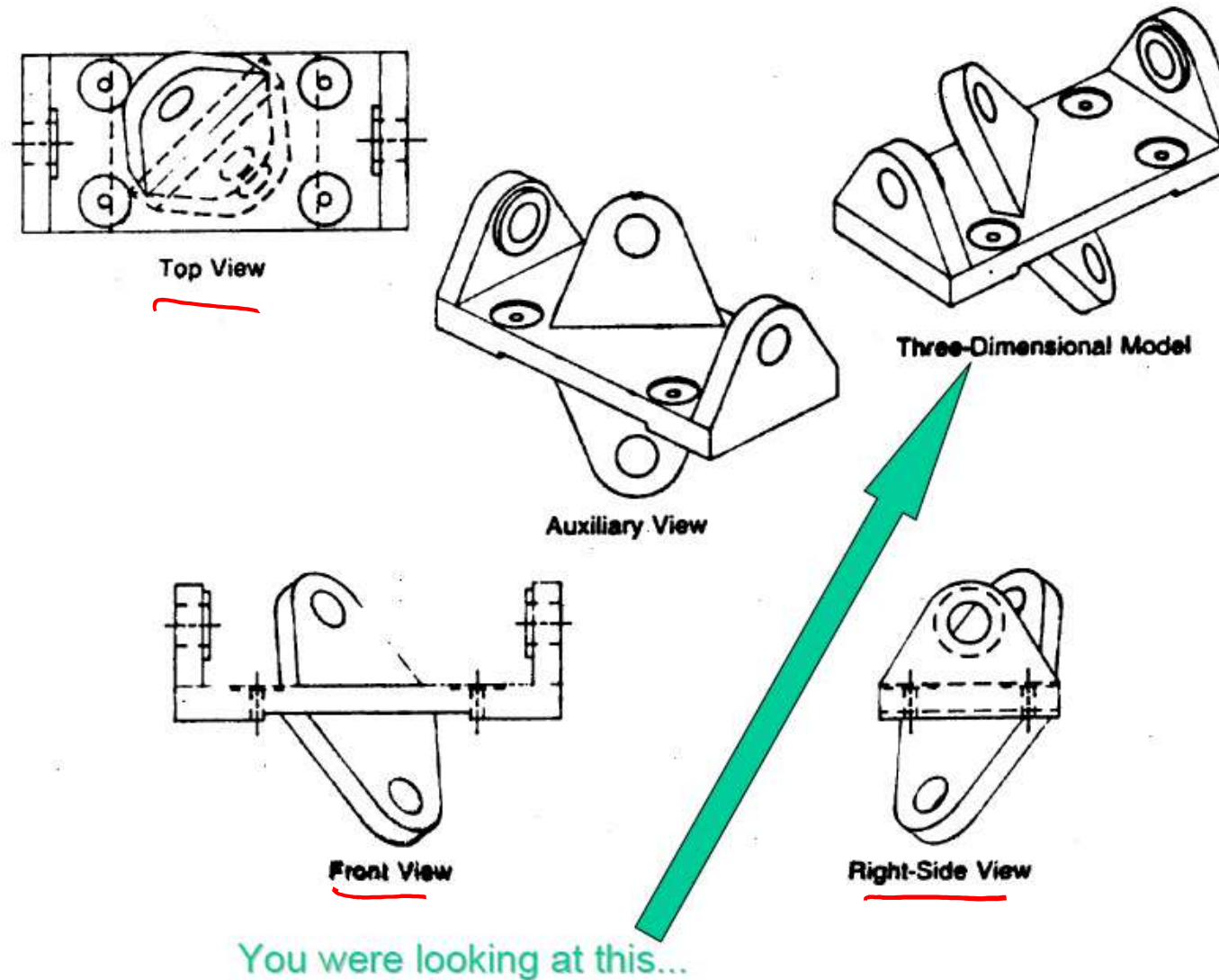


Basic 3D Models: Wireframe (2)



Can you figure out what this is? (It's really a valid wireframe model...)

Basic 3D Models: Wireframe (3)



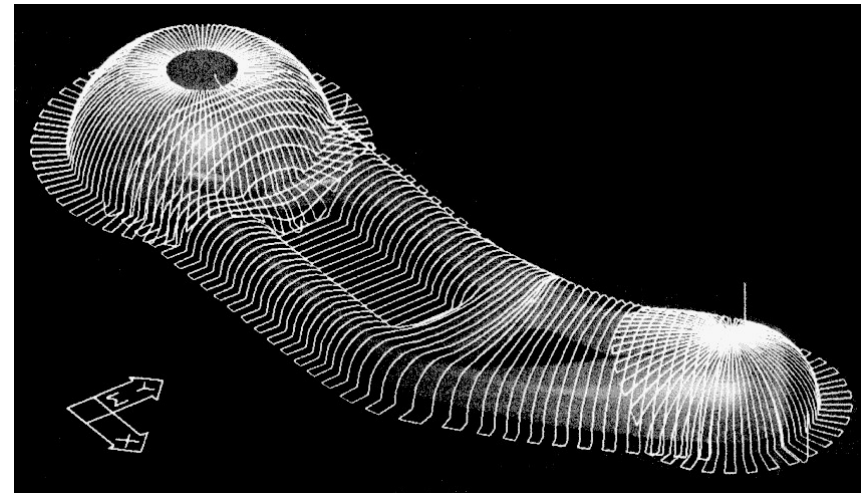
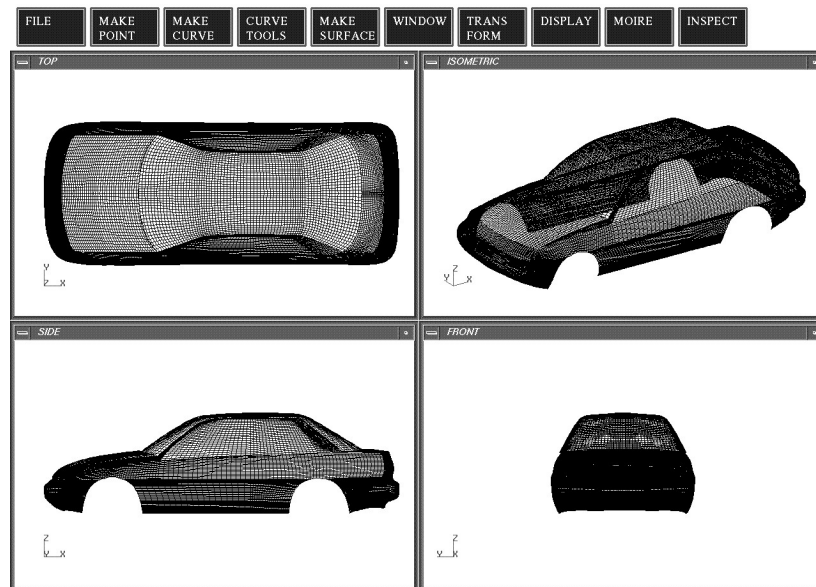
Surface Model (1)

- Purpose

- Visual model for aesthetical evaluation
- Mathematical description to generate the NC Tool Paths

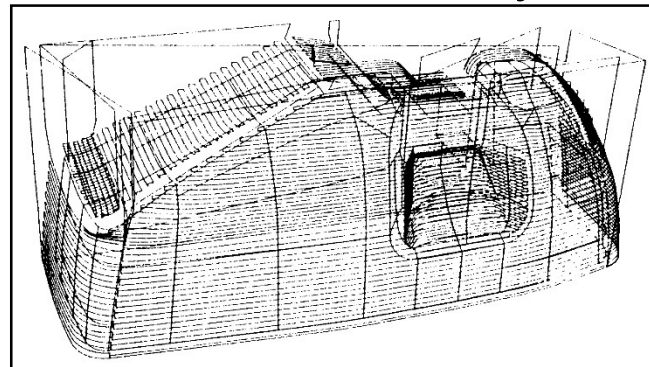
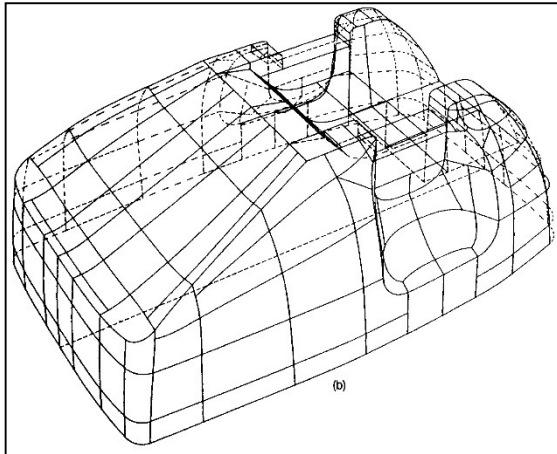
} $\frac{\text{이제까지}}{\text{이제까지}} \rightarrow \text{CATIA}$
PT (오리/바늘) \rightarrow ProE (Creo)
/ Solid

Surface



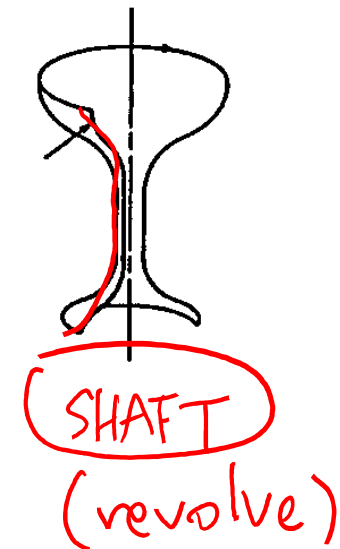
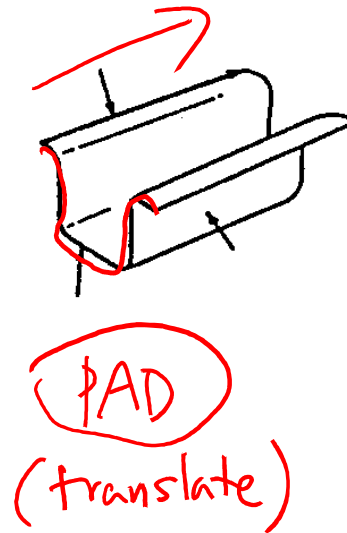
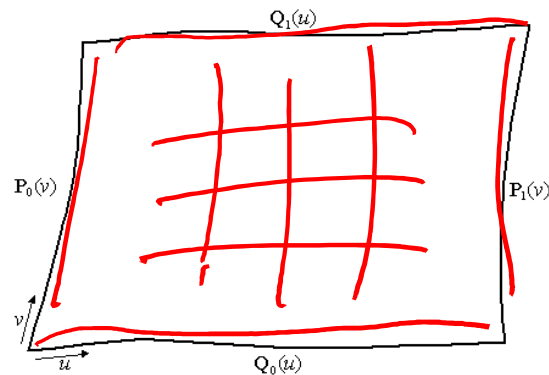
Surface Model (2)

- Database *wireframe*
 - (Characteristic lines and end points) + surface (+surface connectivity) information
 - surface connectivity information
 - Useful for checking gouging of a surface adjacent to the surface being machined
 - If the system includes only a list of surface equations of infinite surfaces without connectivity information, the application should derive the surface boundaries and their connectivity information
 - NC Tool Path Generation with Surface Connectivity Information



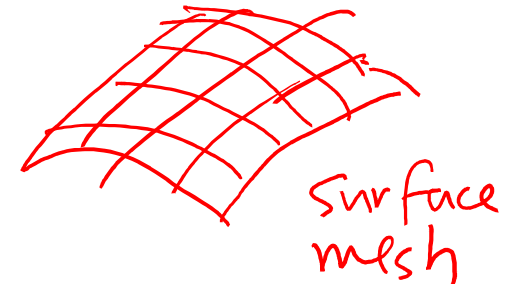
Surface Model (3)

- Creating methods
 - Interpolating the input points
 - Interpolating the curve nets
 - Translating or Revolving a specified curve



Surface Model (4)

- Advantages
 - Automatic NC tool path generation
 - Visual model colored and shaded
- Disadvantages
 - Cannot calculate mass properties
 - Cannot generate FEM meshes
- Products
 - CATIA, ALIAS, OMEGA, SPEED+
 - Applications for CAM and CG



부피/중심

무게중심 CG. 계산

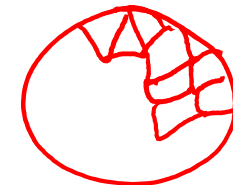
3-2차원 CAE

2차원/3차원 → 3차원/2차원

3차원/2차원? 수치해석

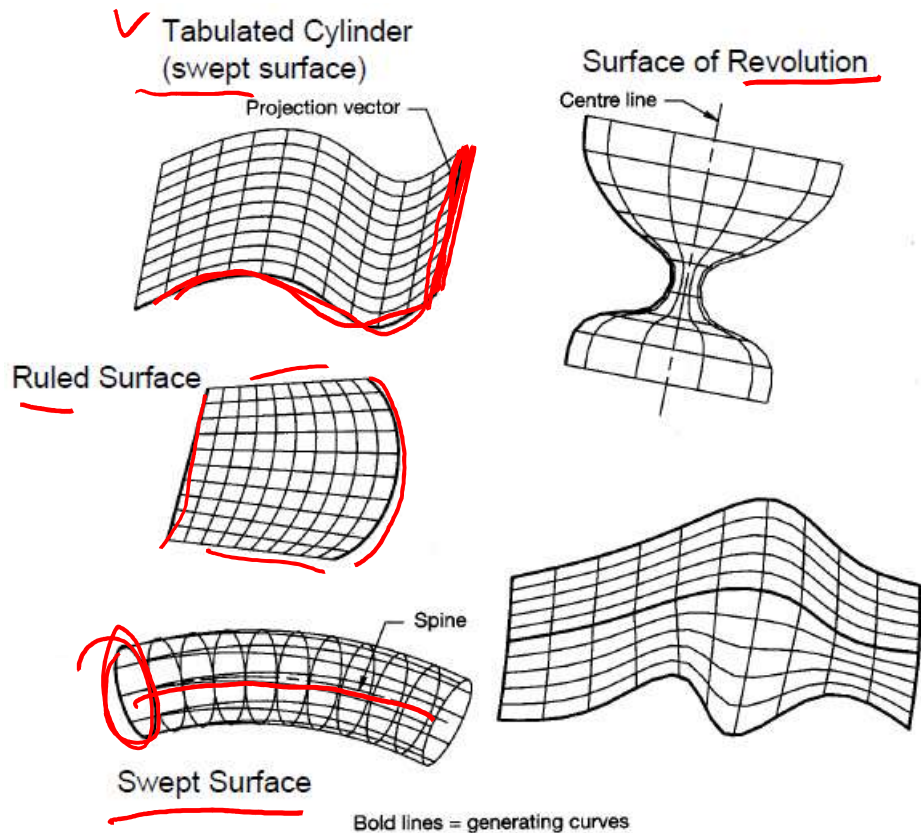
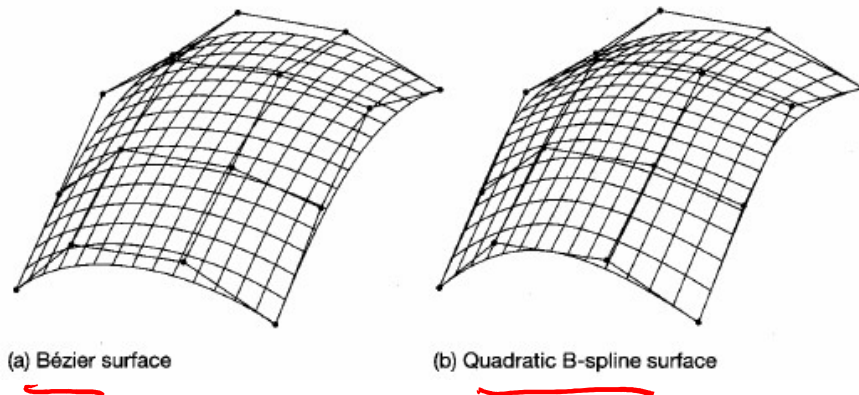
node, element

Continuum
→ finite element



Basic 3D Models: Surface (1)

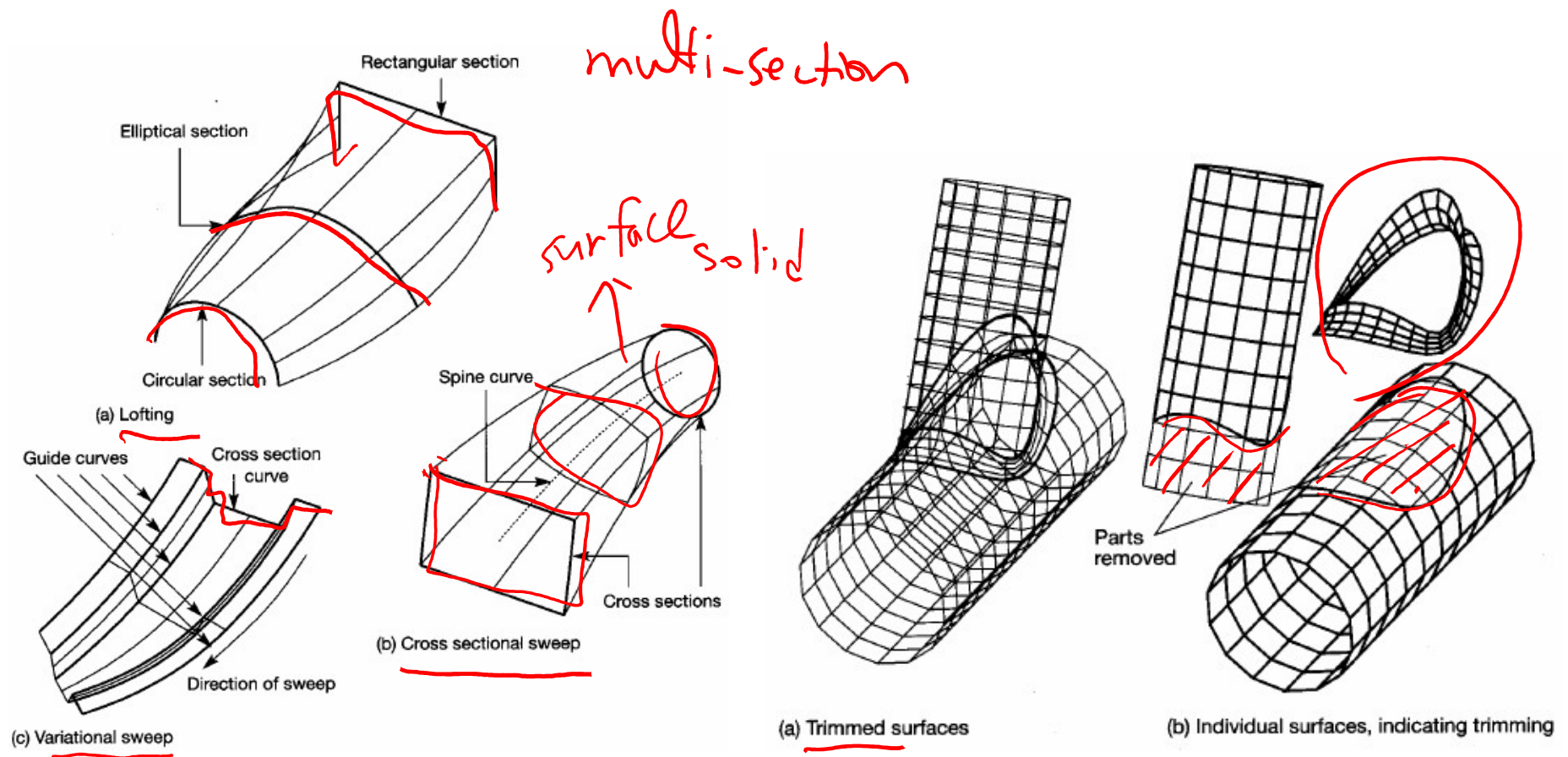
- Surface models
 - accurate surface definition
 - enclose a volume
 - topologically difficult to handle...



NOTE: control points define shape of surface

Basic 3D Models: Surface (2)

- More complex surfaces



Solid Model (1)

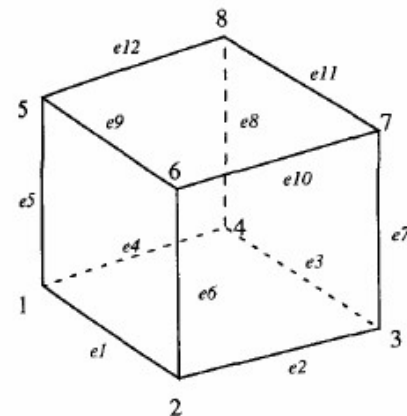
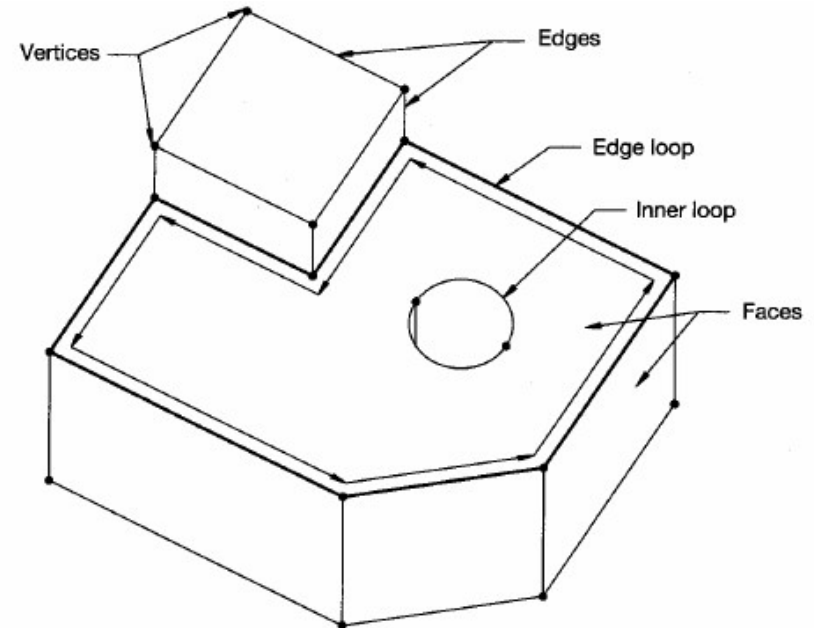
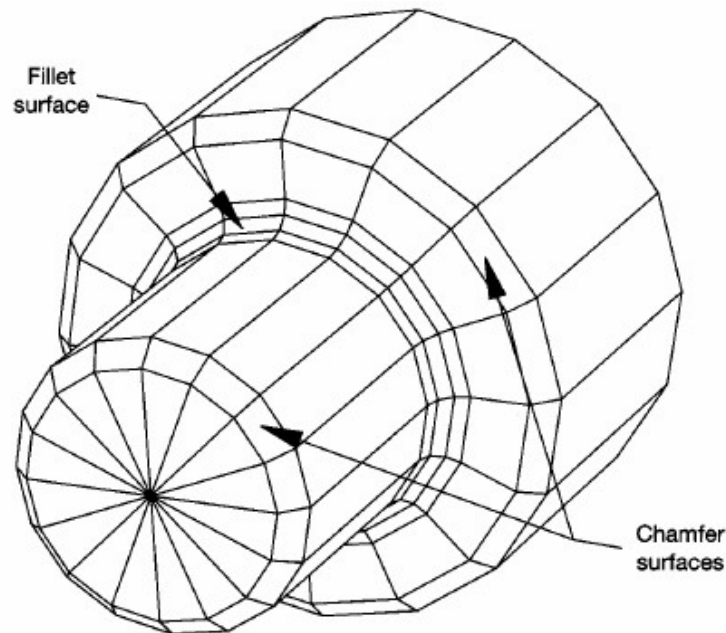
- Database
 - Store a closed volume
 - Surface information + In/out information
 - Not allow a simple set of surfaces or characteristic lines if it can not form a closed volume
- Products
 - TIPS, PADL-2, BUILD2, ROMULUS, DESIGNBASE, Pro/Engineer, SolidWorks, SolidEdge, CATIA, ParaSolid

Solid Model (2)

- Advantages
 - Calculate mass properties
 - Generate FEM solid meshes
 - Interference checking between objects
 - 3D visual model colored and shaded
 - NC tool path generation and simulation
- Disadvantages
 - Permit only a complete solid model *Euler eqn $\frac{b^2}{r}$*
 - Require a large amount of input data (complicated and difficult)
 - Large amount of data storage

Basic 3D Models: Solid (1)

- Volumes
 - combine surfaces together
 - topology is a problem
 - boundary representation models (B-rep)



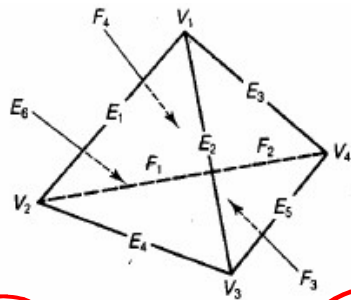
A vertex-based boundary model

v1	x1	y1	z1	f1	v1	v2	v3	v4
v2	x2	y2	z2	f2	v6	v2	v1	v5
v3	x3	y3	z3	f3	v7	v3	v2	v6
v4	x4	y4	z4	f4	v8	v4	v3	v7
v5	x5	y5	z5	f5	v5	v1	v4	v8
v6	x6	y6	z6	f6	v8	v7	v6	v5
v7	x7	y7	z7					
v8	x8	y8	z8					

Basic 3D Models: Solid (2)

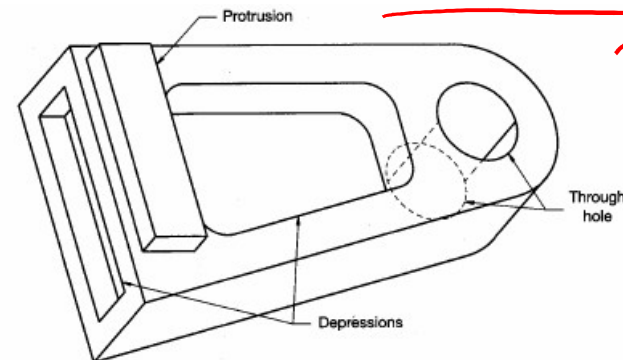
• Boundary models (b-rep) • Rules

- aka: graph-based models
- graph **nodes** & **edges**



Vertices	Edges	Faces
V ₁	E ₁	F ₁
V ₂ , V ₃ , V ₄ E ₁ , E ₂ , E ₃ F ₁ , F ₂ , F ₄	V ₁ , V ₂ E ₂ , E ₃ , E ₄ , E ₆ F ₁ , F ₄	V ₁ , V ₂ , V ₃ E ₁ , E ₄ , E ₂ F ₂ , F ₃ , F ₄
V ₂	E ₂	F ₂
V ₁ , V ₃ , V ₄ E ₁ , E ₄ , E ₆ F ₁ , F ₃ , F ₄	V ₁ , V ₃ E ₁ , E ₃ , E ₄ , E ₅ F ₁ , F ₂	V ₁ , V ₃ , V ₄ E ₂ , E ₅ , E ₃ F ₁ , F ₃ , F ₄
V ₃	E ₃	F ₃
V ₁ , V ₂ , V ₄ E ₂ , ...	V ₁ , V ₄ E ₁ , E ₂ , ...	V ₄ , V ₃ , V ₂ E ₄ , ...

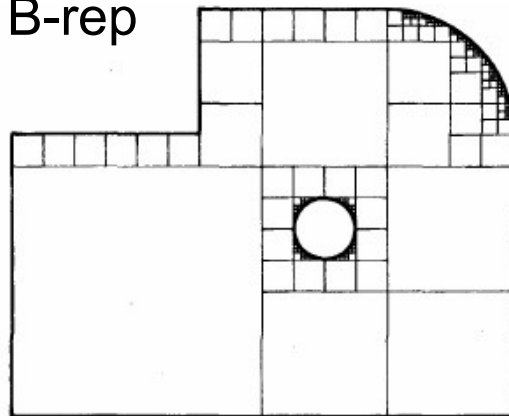
- faces bounded by single loop or ring of edges
- edge joins exactly 2 faces and is terminated by vertices
- at least 3 edges meet at each vertex
- Euler's Rule applies: $V-E+F=2$ (extended: $V-E+F-H=2(S-P)$)



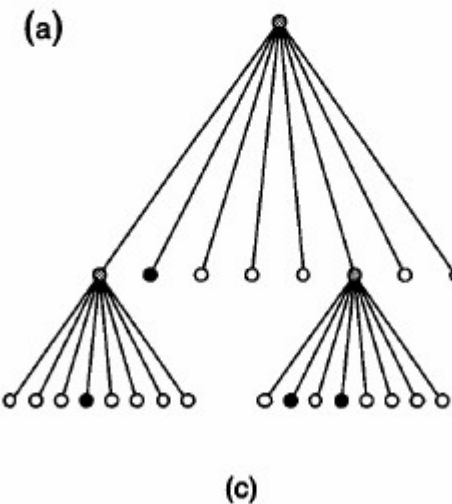
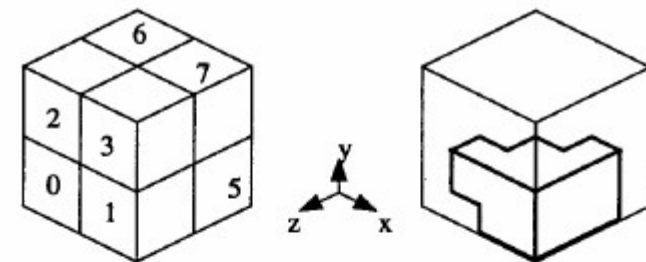
	Vertices	Edges	Faces	Holes	Passages
Basic shape	8	12	6		
Protrusion adds	8	12	5	1	
Depression with sharp corners adds	8	12	5	1	
Depression with filleted corners adds	16	24	9	1	
Through hole adds	4	6	2	2	1

Basic 3D Models: Solid (3)

- Solid models
 - Notion of inside vs. outside
 - Analytical models (extend surface to 3-parameters)
 - ~~Spatial decomposition~~ or cell enumeration
 - Constructive solid geometry (CSG)
 - B-rep



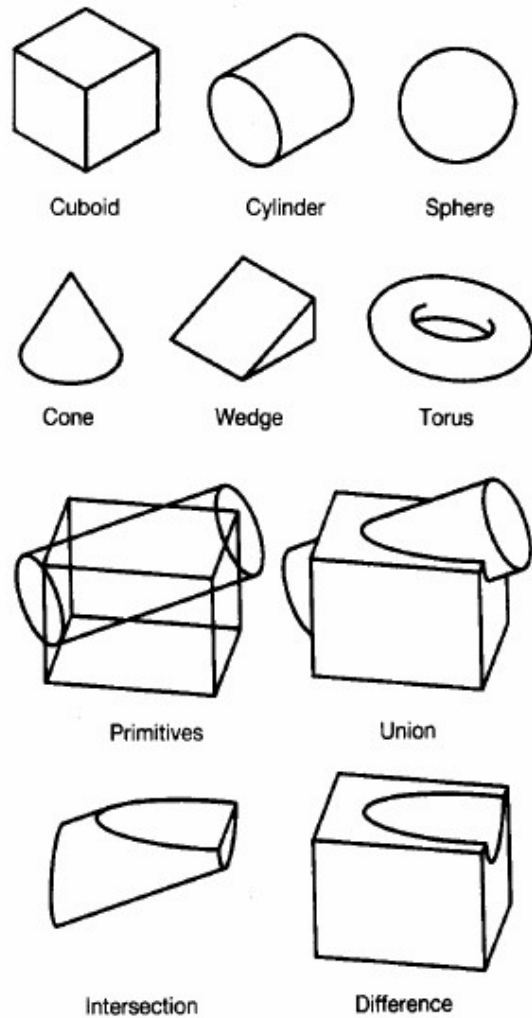
Quadtree (2D)



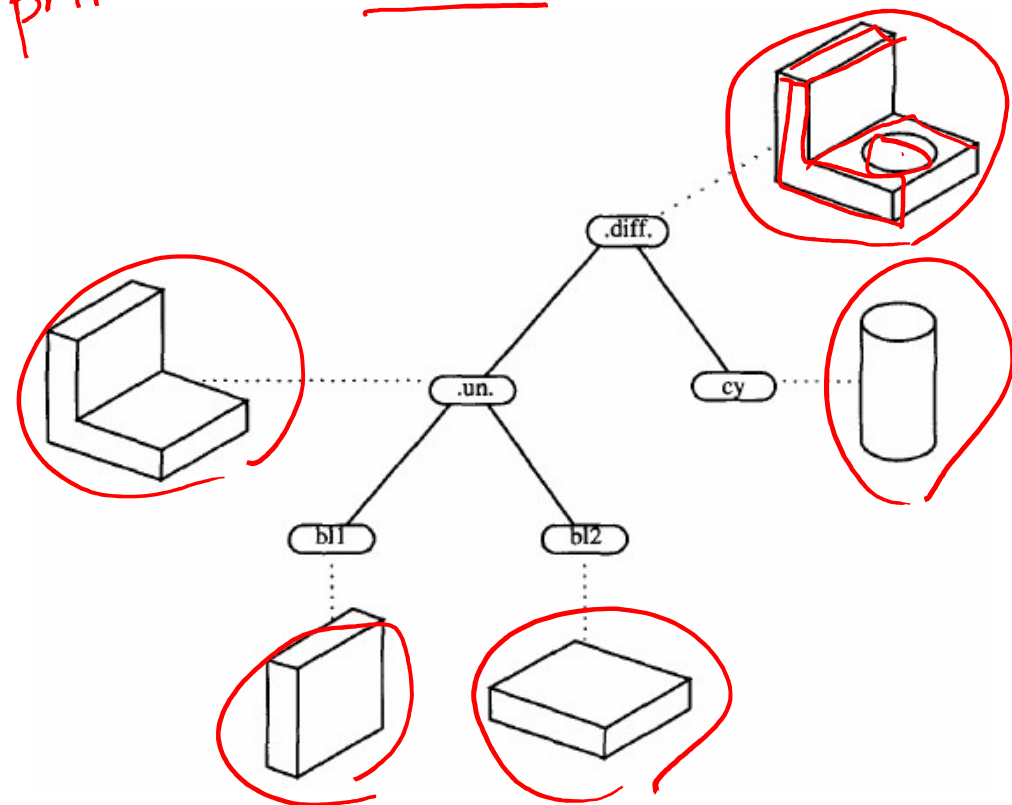
Octree concepts (Mäntylä 1988)

Octree (3D)

Basic 3D Models: Solid (4)



primitive + boolean operation

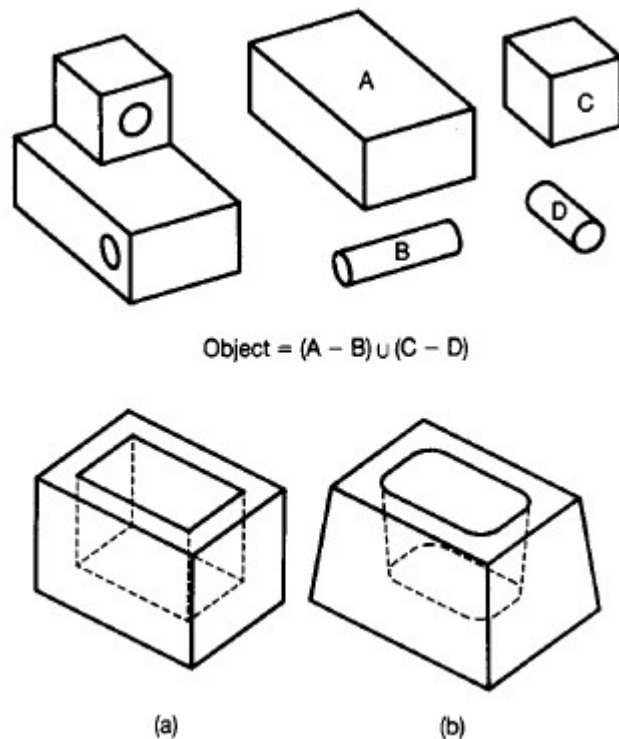


CSG Primitives and Boolean Operations

Graph-based CSG representation

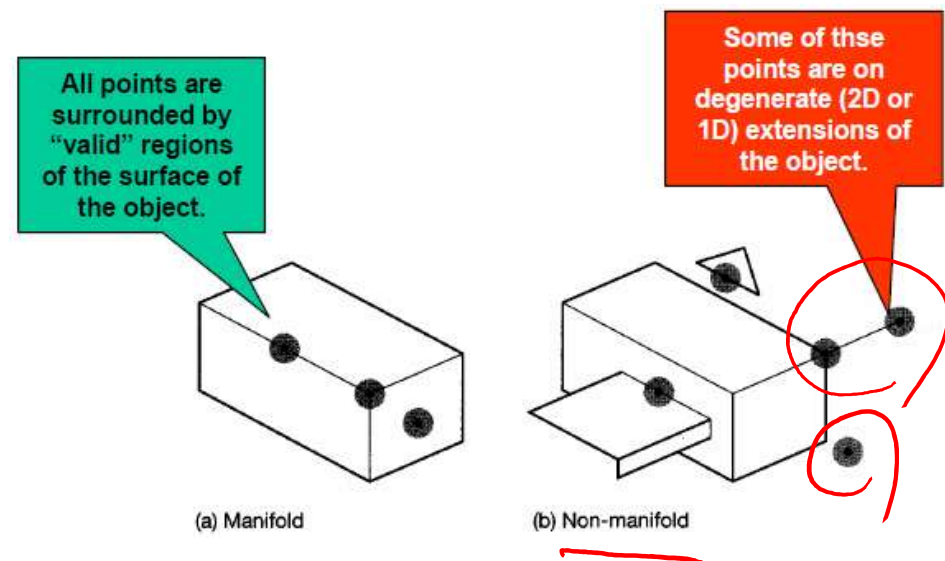
Basic 3D Models: Solid (5)

This is a “simple” CSG object...

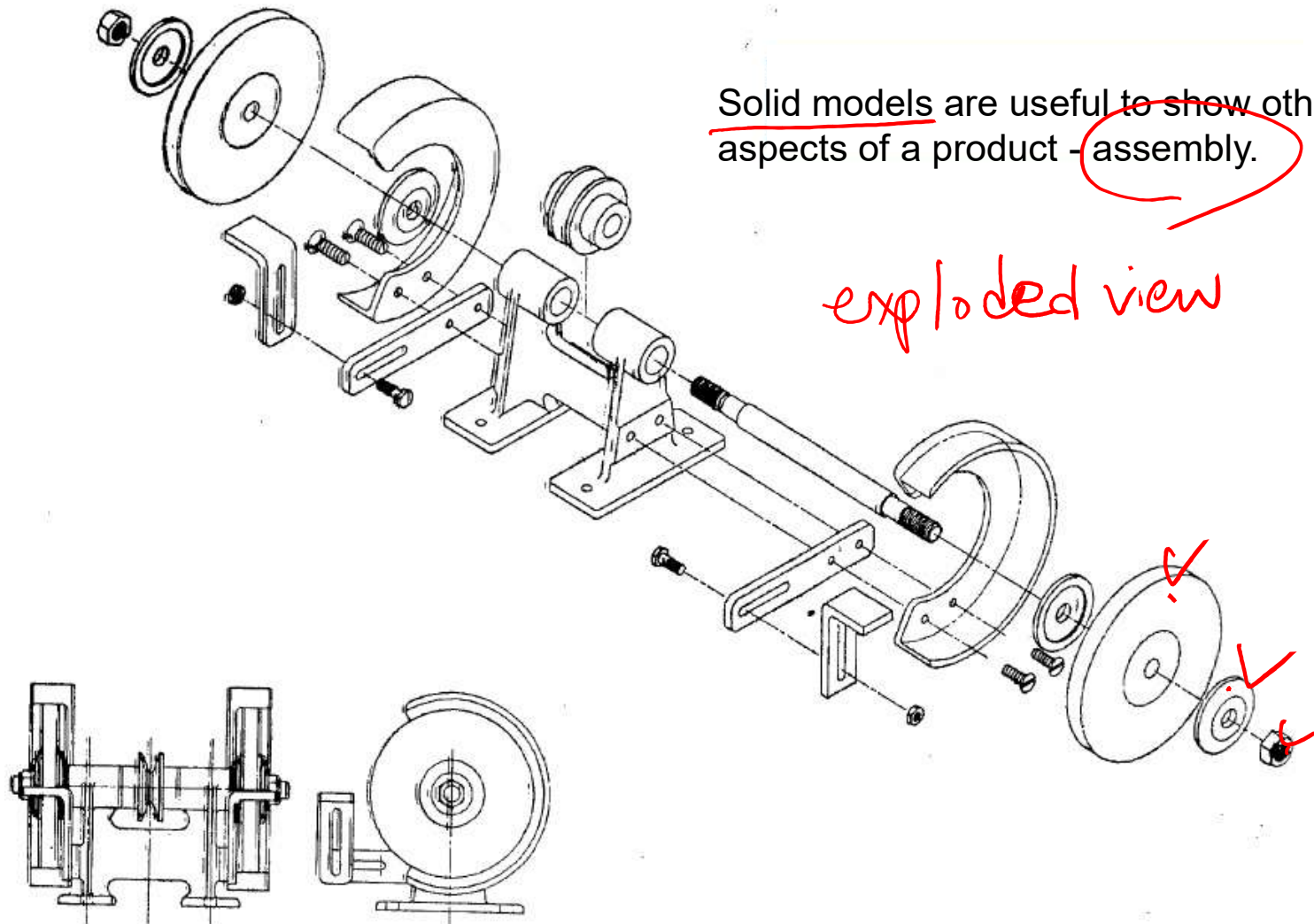


This is a much more complex CSG object!
(can you see why this is so?)

Solid models can have problems...



NOTE: By restricting the types of Boolean operations that are allowed, we can avoid most of these degeneracies.

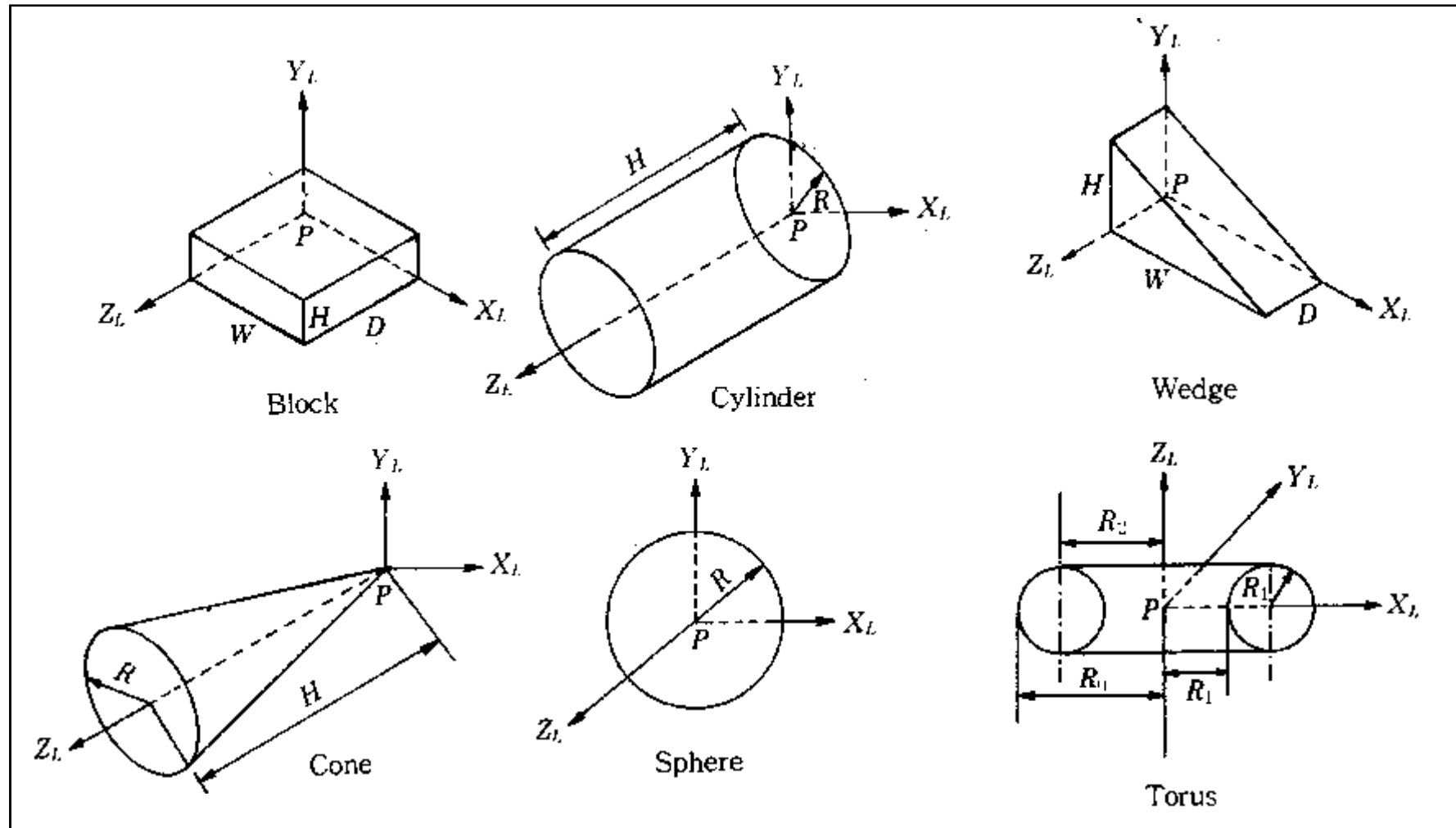


Solid Modeling Functions

- Primitive Creation Functions + Boolean Operations
- Surface Moving Functions
 - Sweeping, Swinging (used for Parametric Modeling)
 - Skinning
- Local Modification Functions
 - Rounding(or Blending, or Filleting), Lifting
- Boundary Modeling
- Feature-Based Modeling
- Parametric Modeling

CATIA
Dressed-up

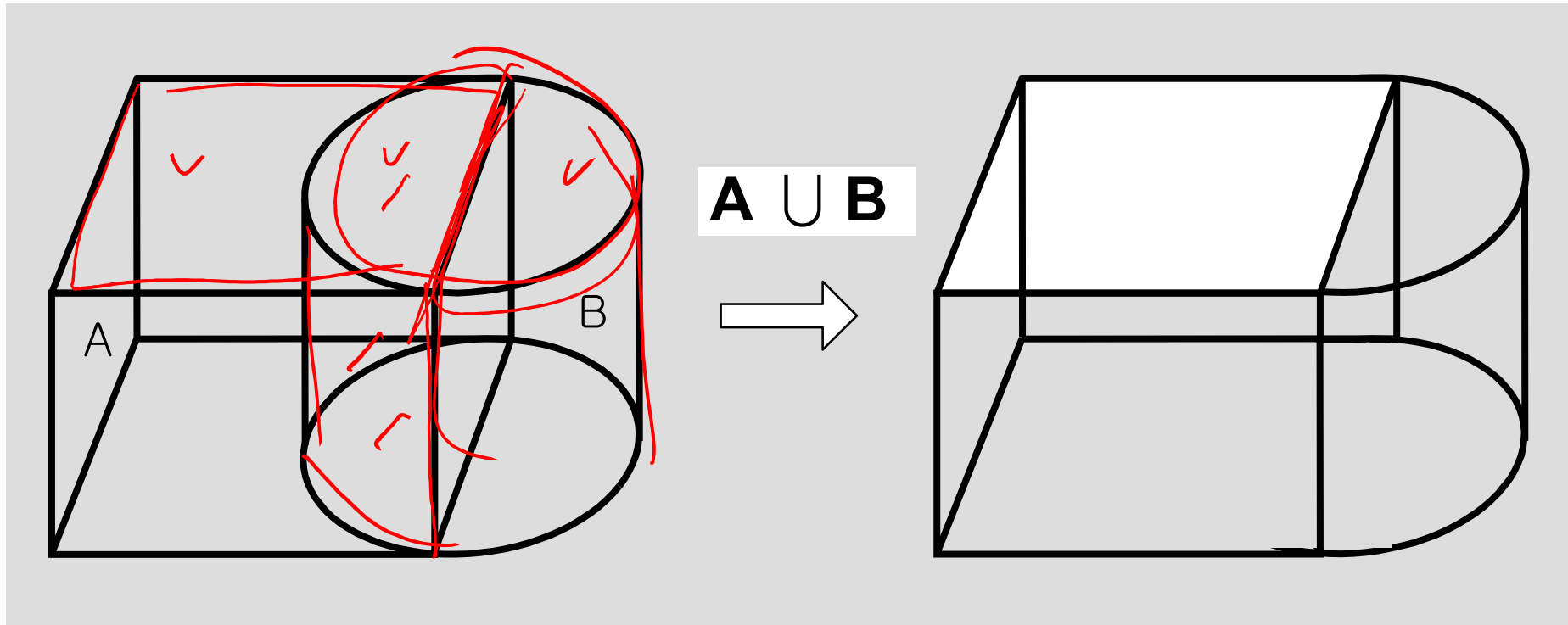
Primitive Creation Functions



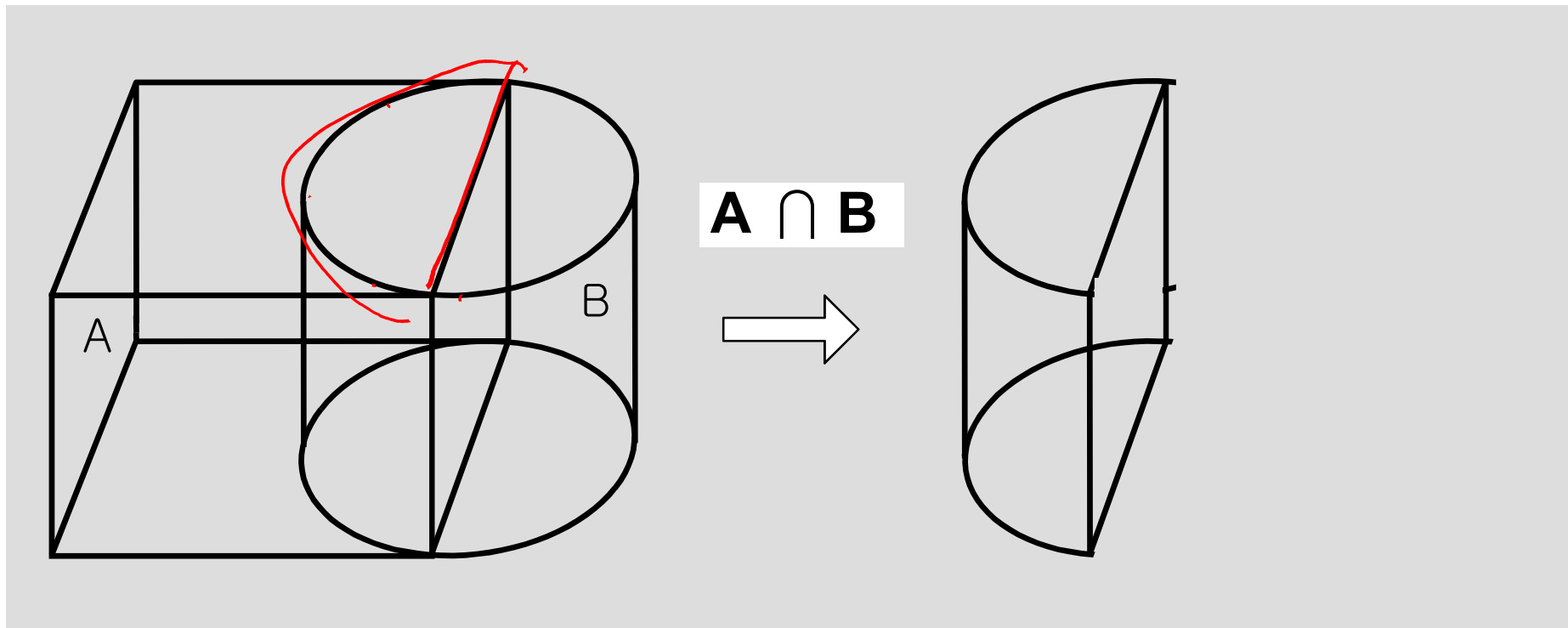
Boolean Operations

- Basic Idea:
 - Each primitive solid is assumed to be a set of points, a Boolean operation is performed on point sets, and the result is a solid composed of the points resulting from the operation.
- Boolean Operations
 - Union ✓
 - Intersection ✓
 - Difference ✓
 - (Similar Operations: Sectioning and Gluing)

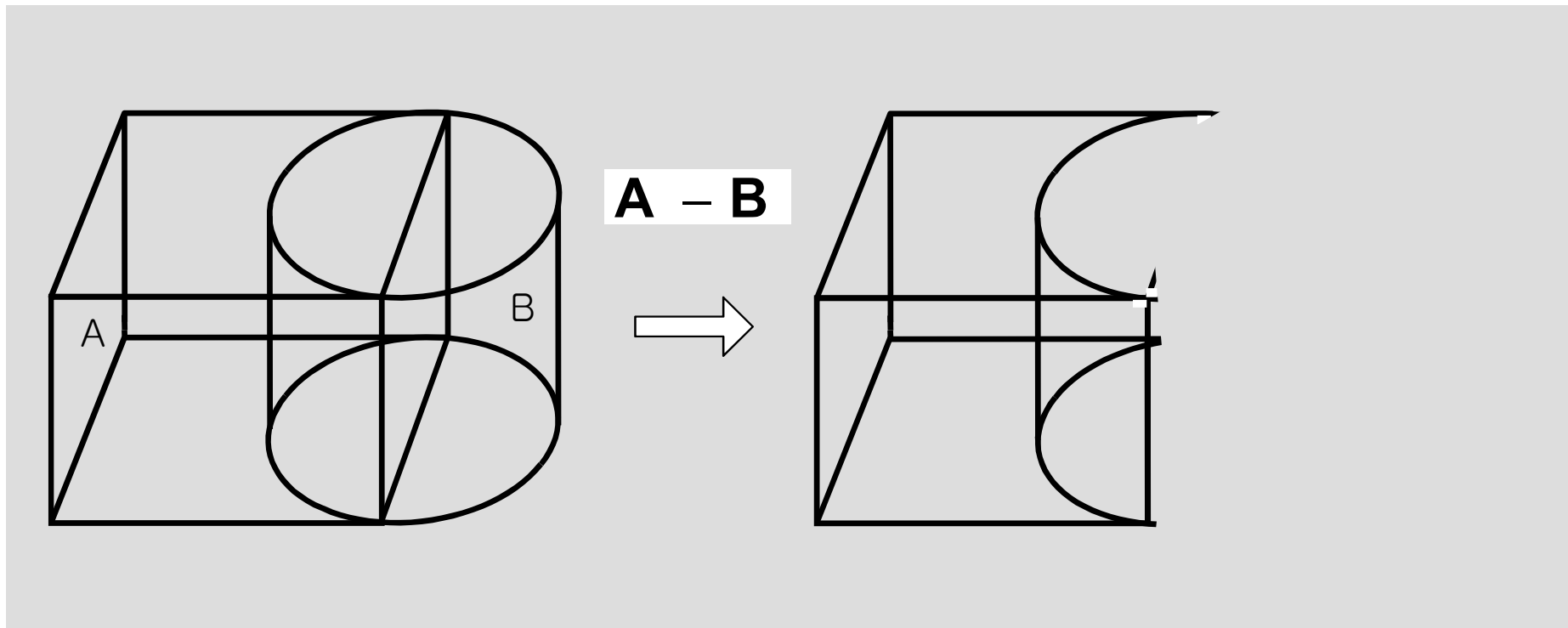
Union Operation



Intersection Operation

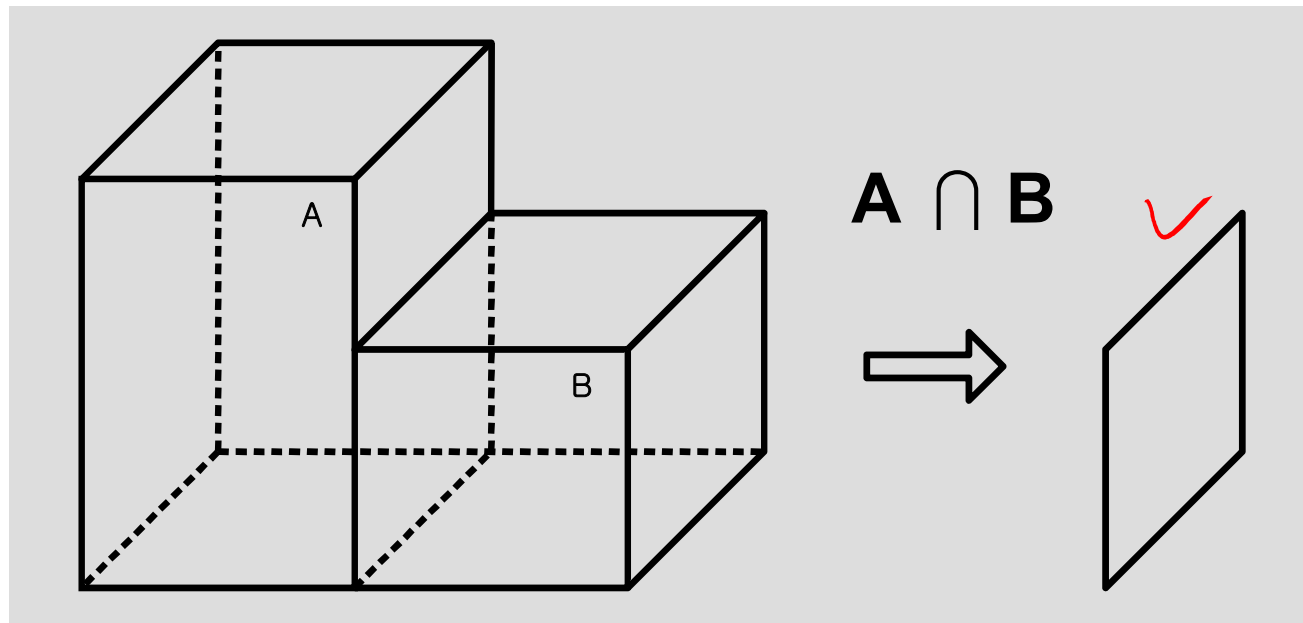


Difference Operations



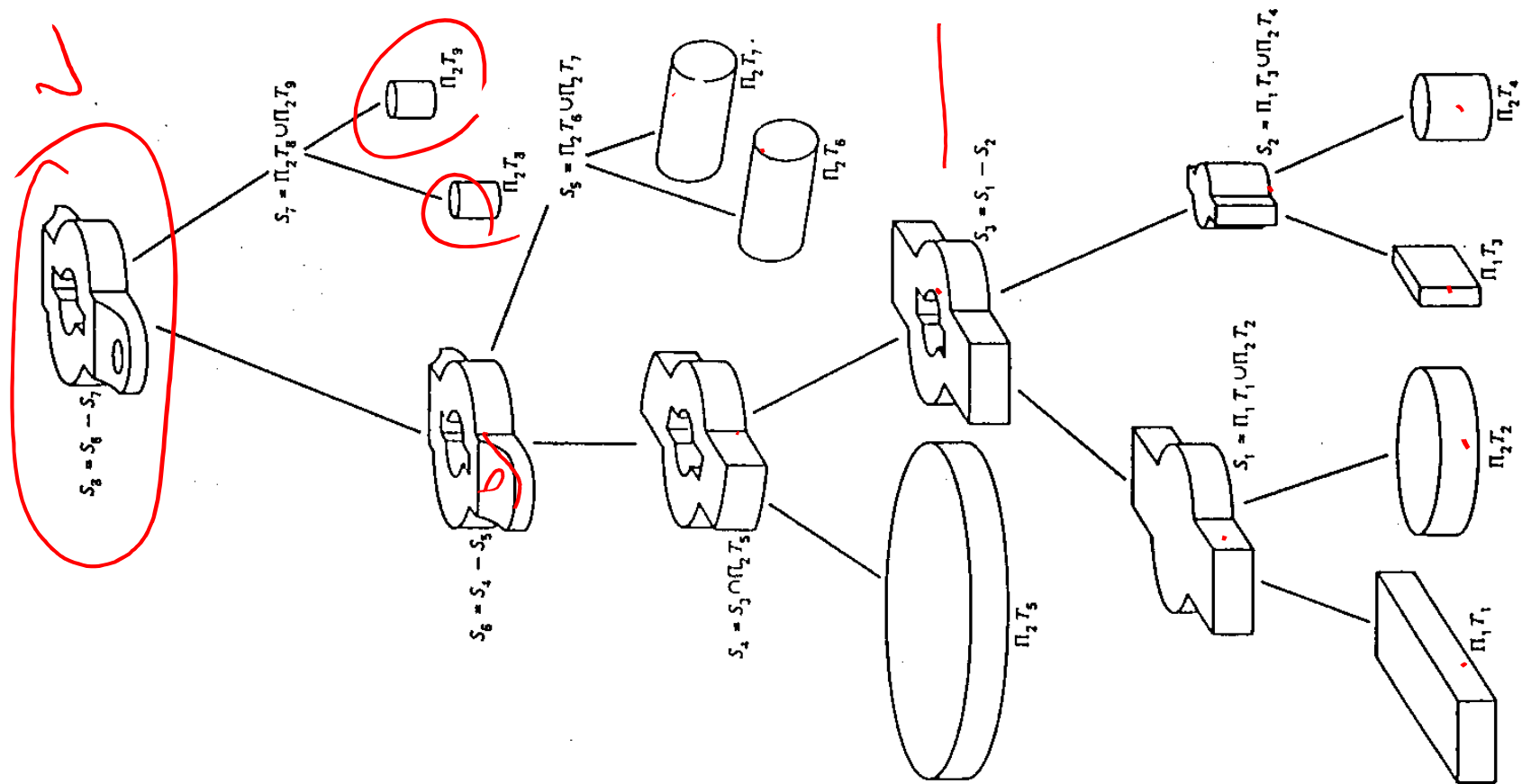
Limitation of Solid Models in Boolean Operations

- Solid models are not closed to Boolean operations

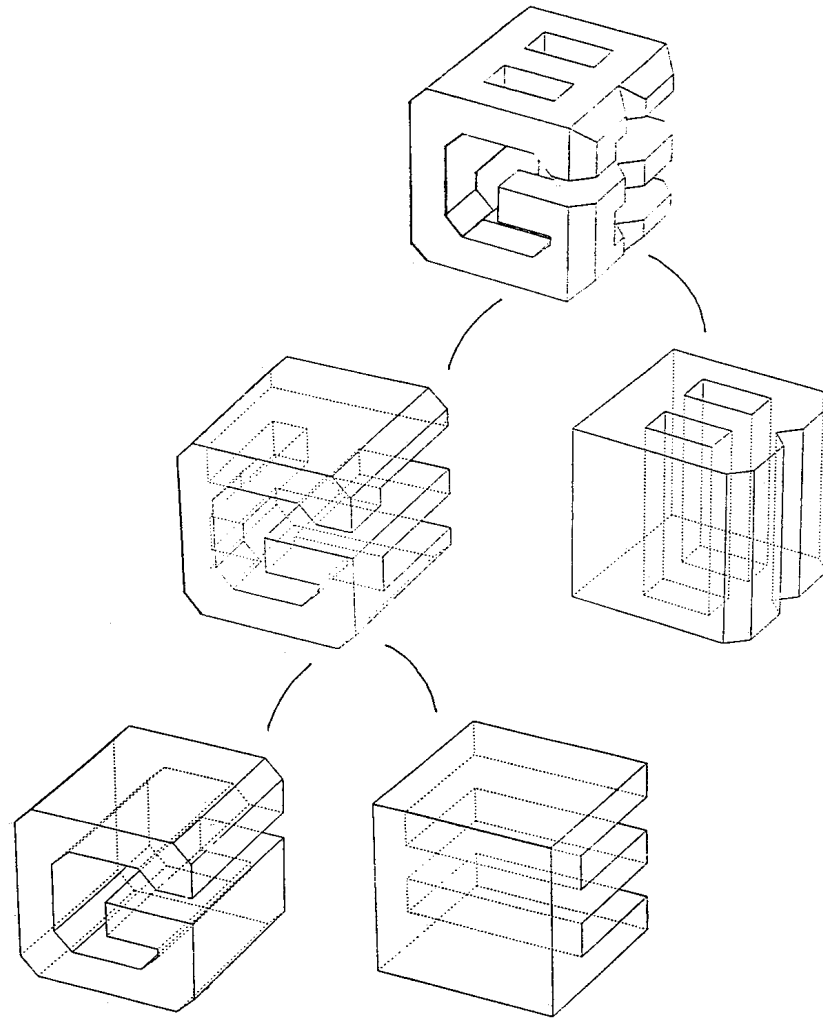


Result is \emptyset in Solid Modeler

Example of Boolean Operations (1)

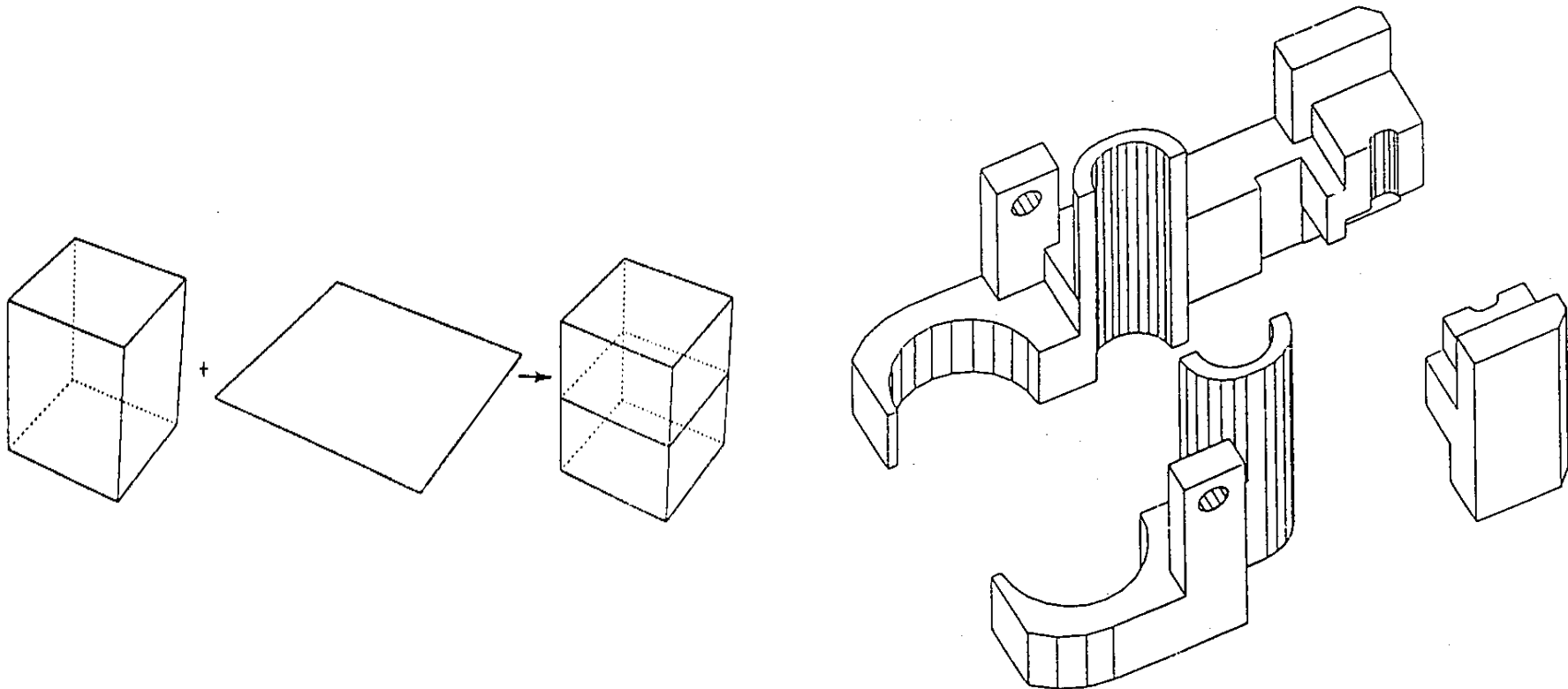


Example of Boolean Operations (2)

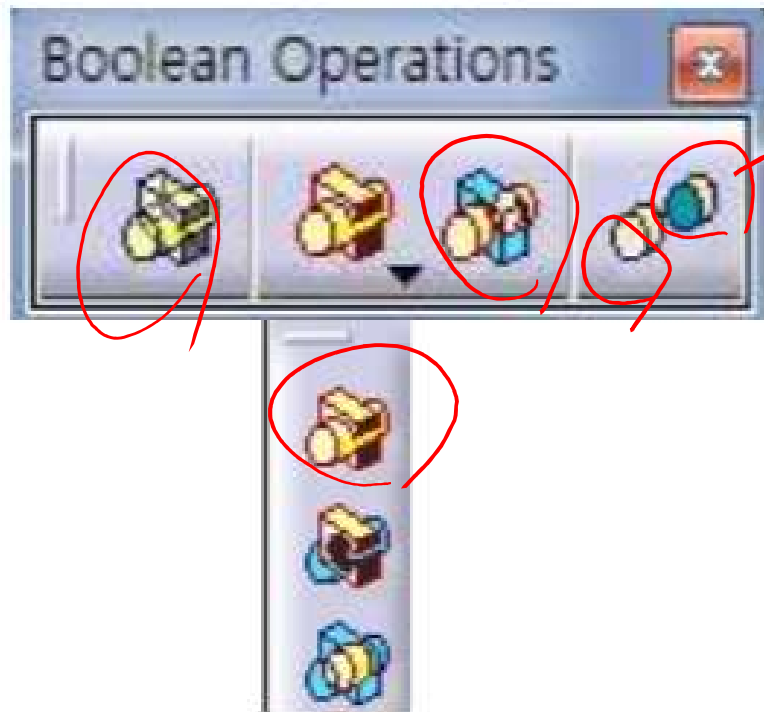


Sectioning (or Cutting)

- Useful for Cross-Sectional View

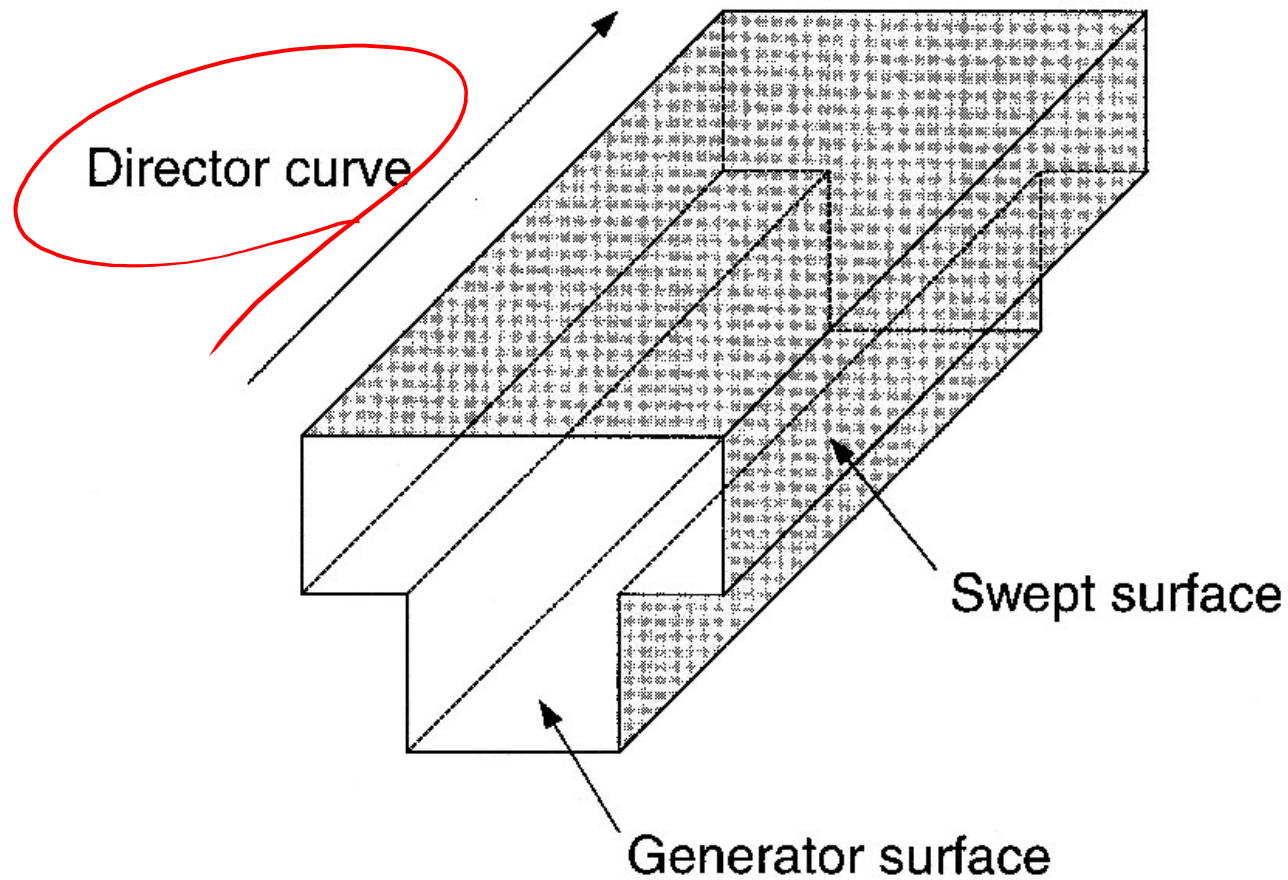


CATIA: Part Design

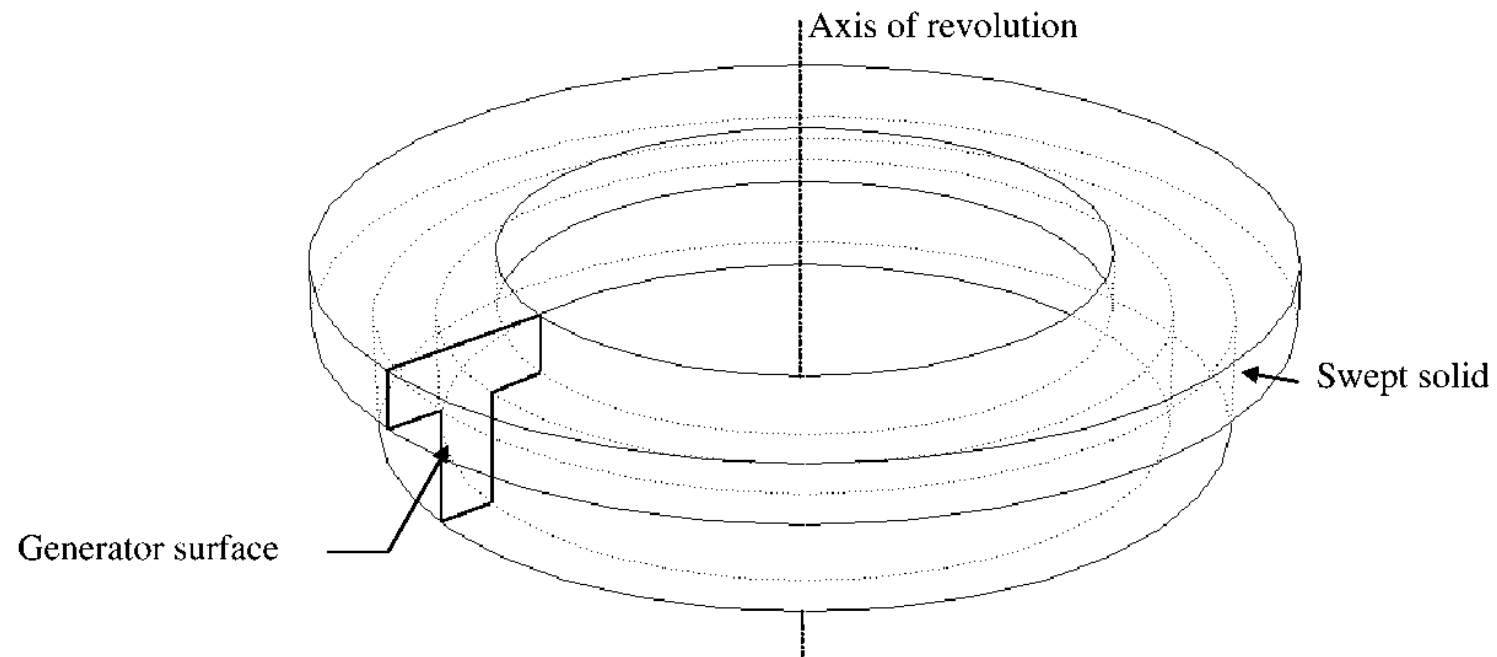


- Assemble (body속성 유지)
- Add *형상 only* : *union*
- Remove : *difference*
- Intersect : *intersect*
- ✓ • Union Trim
- ✗ • Remove Lump

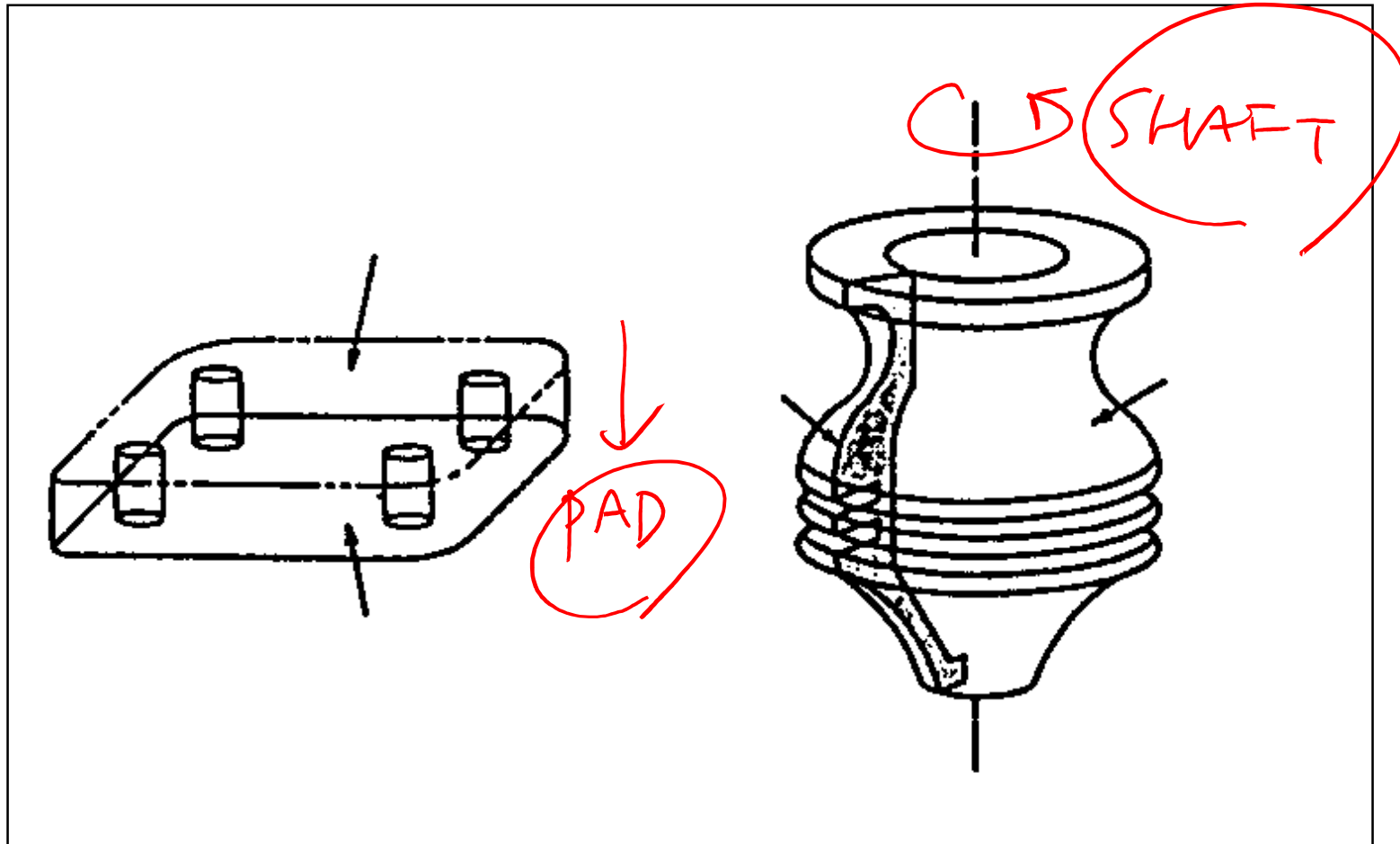
Translational Sweeping



Rotational Sweeping (Swing)

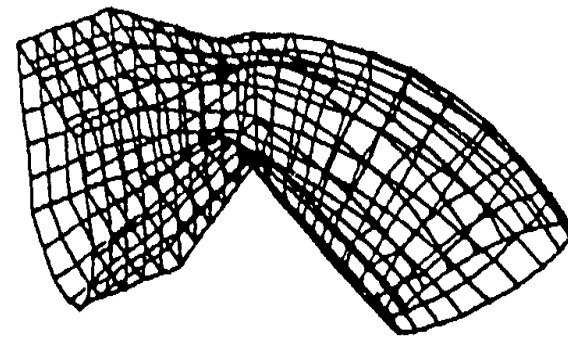
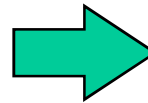
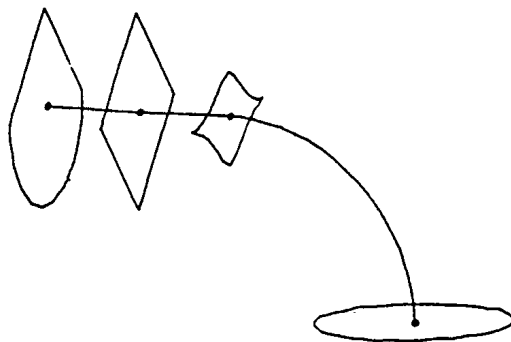
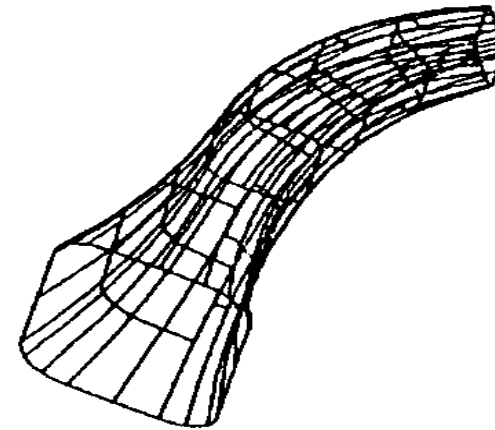
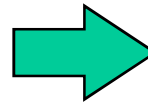
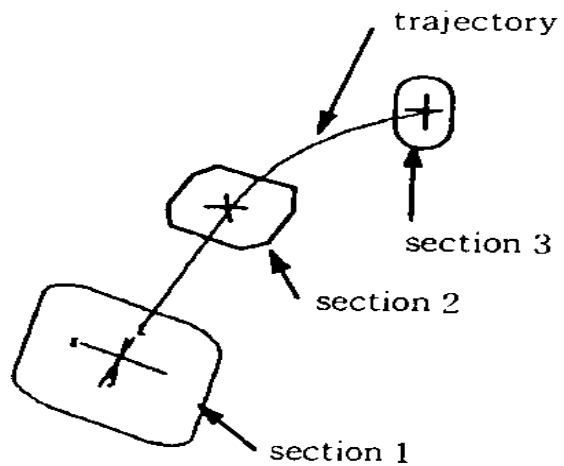


Examples of Sweeping Operations

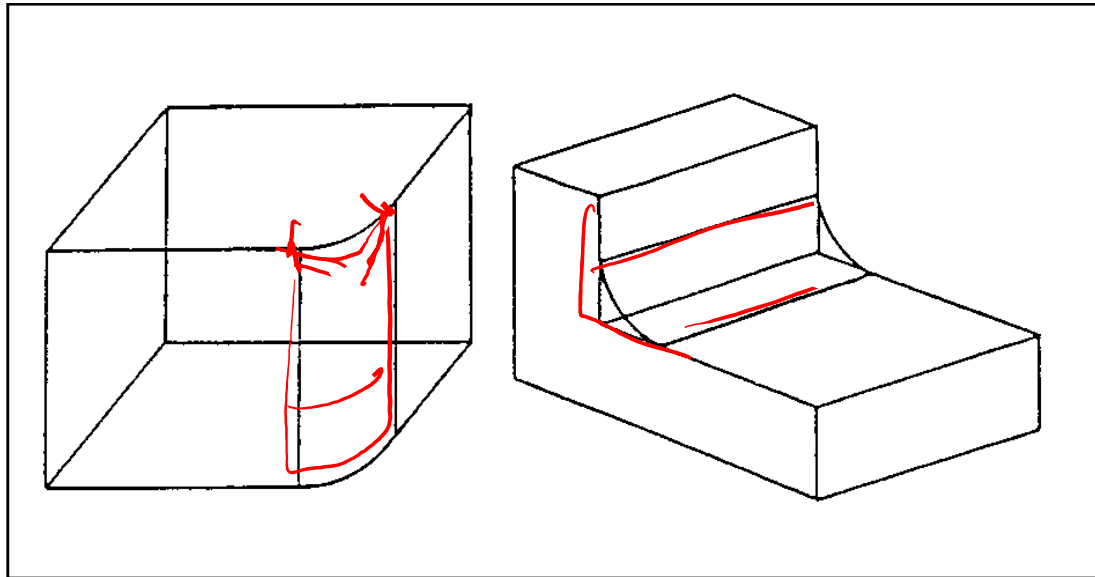


Skinning

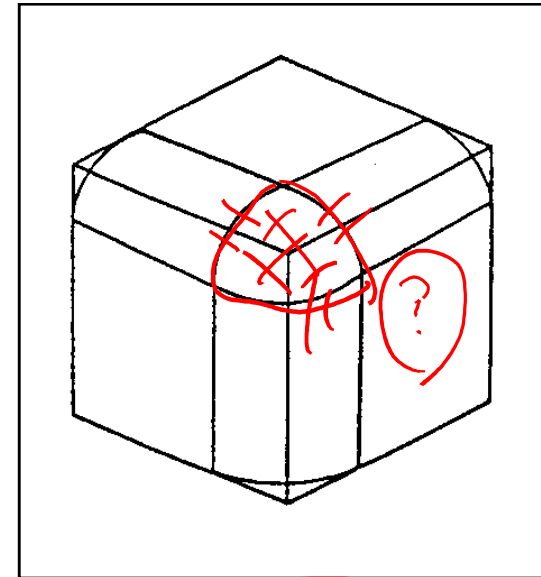
multi-section
sweeping



Rounding

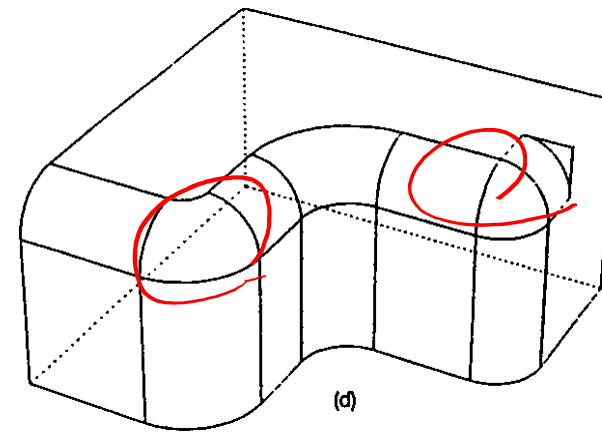
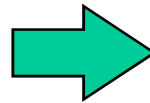
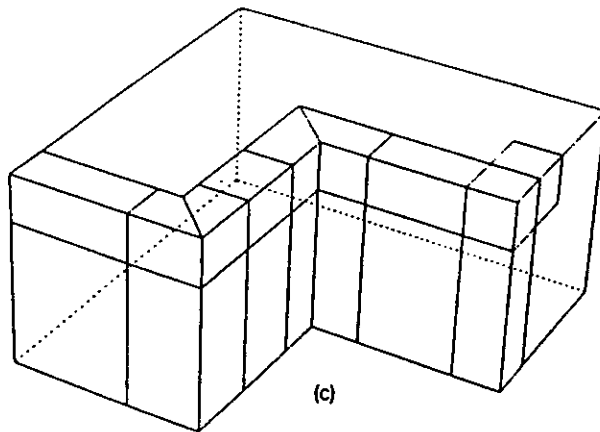
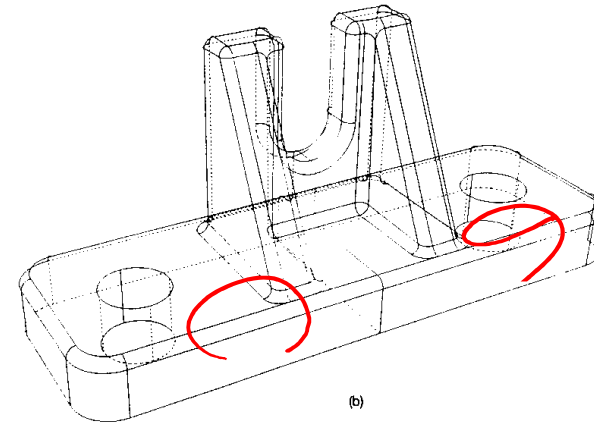
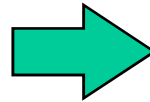
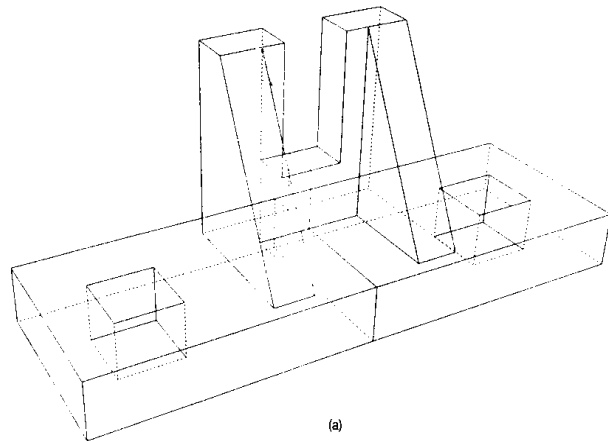


Edge Rounding

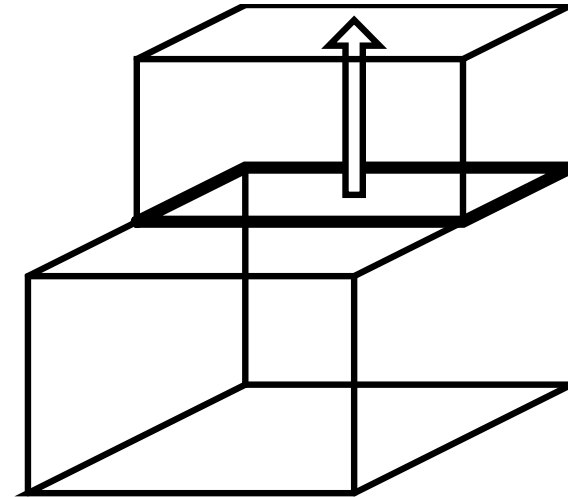
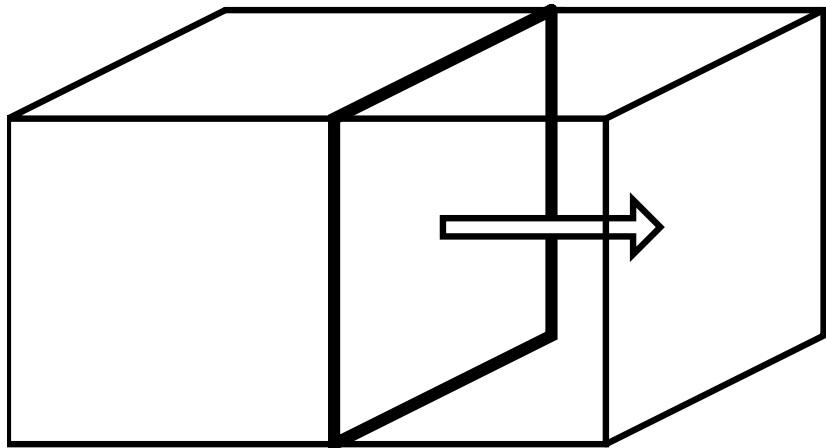


Vertex Rounding

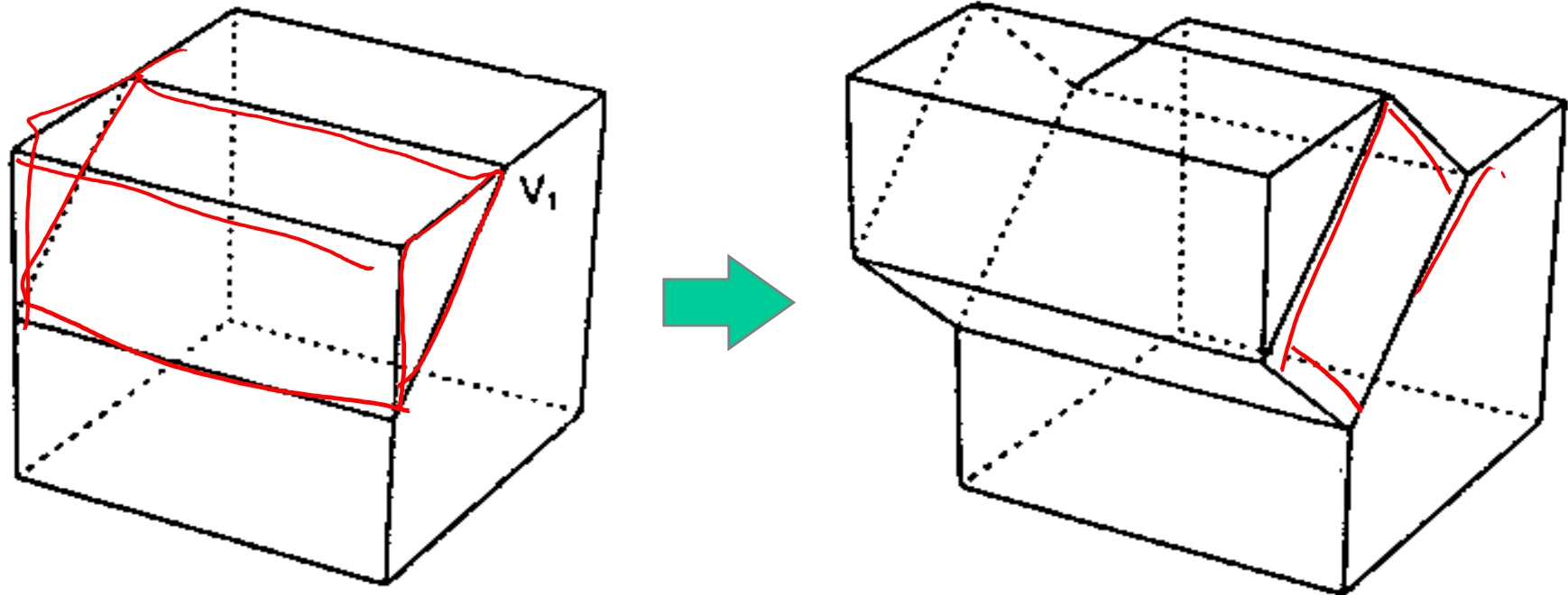
Examples of Rounding Operation



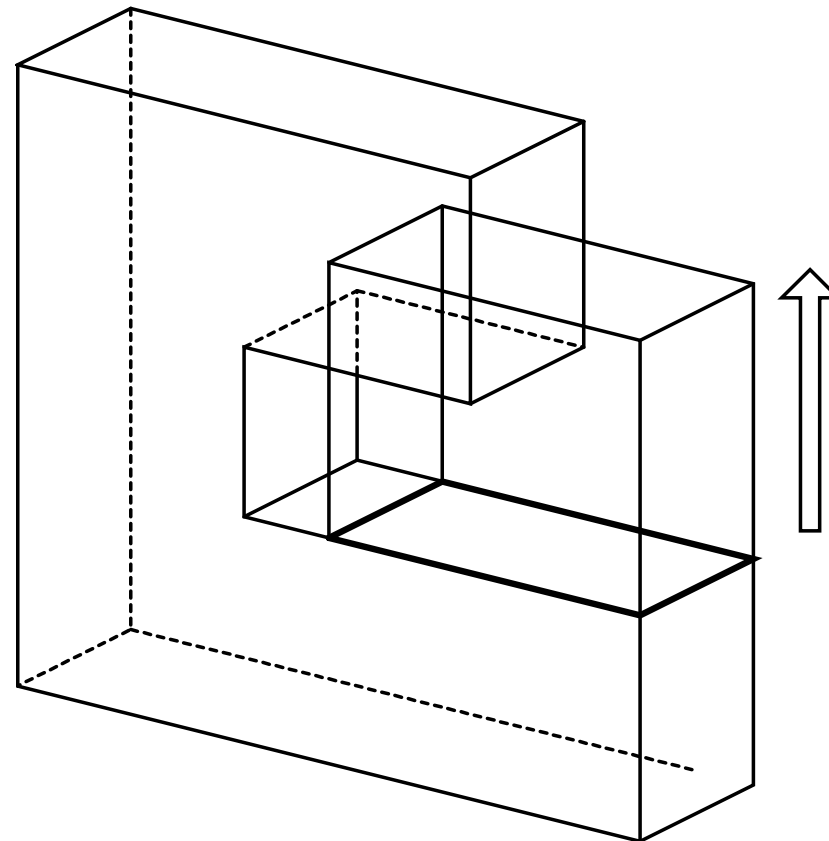
Lifting



Lifting a Face Group

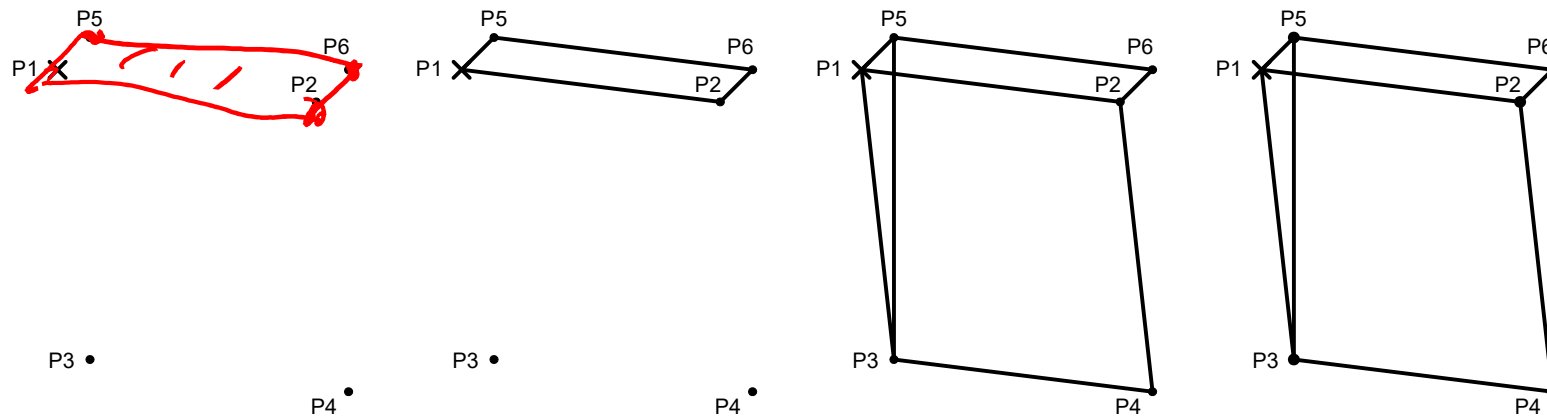


Self-Intersection Caused by Lifting



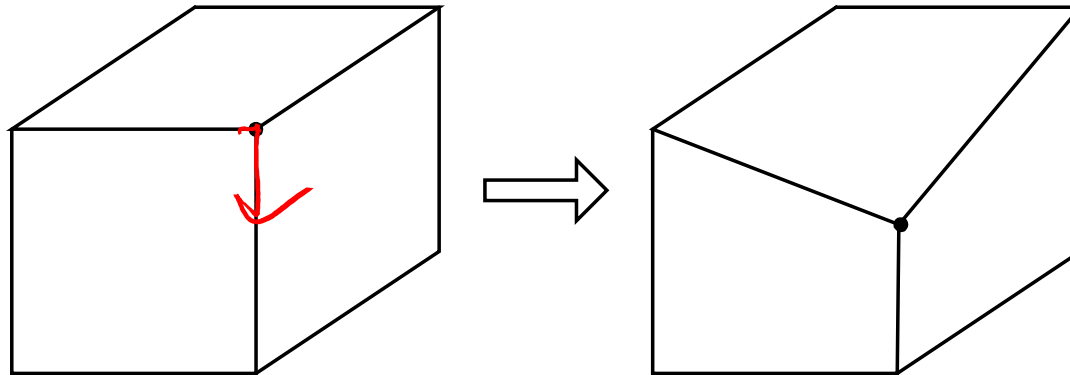
Boundary Modeling Functions

- Add, delete, or modify the lower entities of a solid, such as vertices, edges, and faces directly



Tweaking

- Vertex Moving



- Surface Replacement

