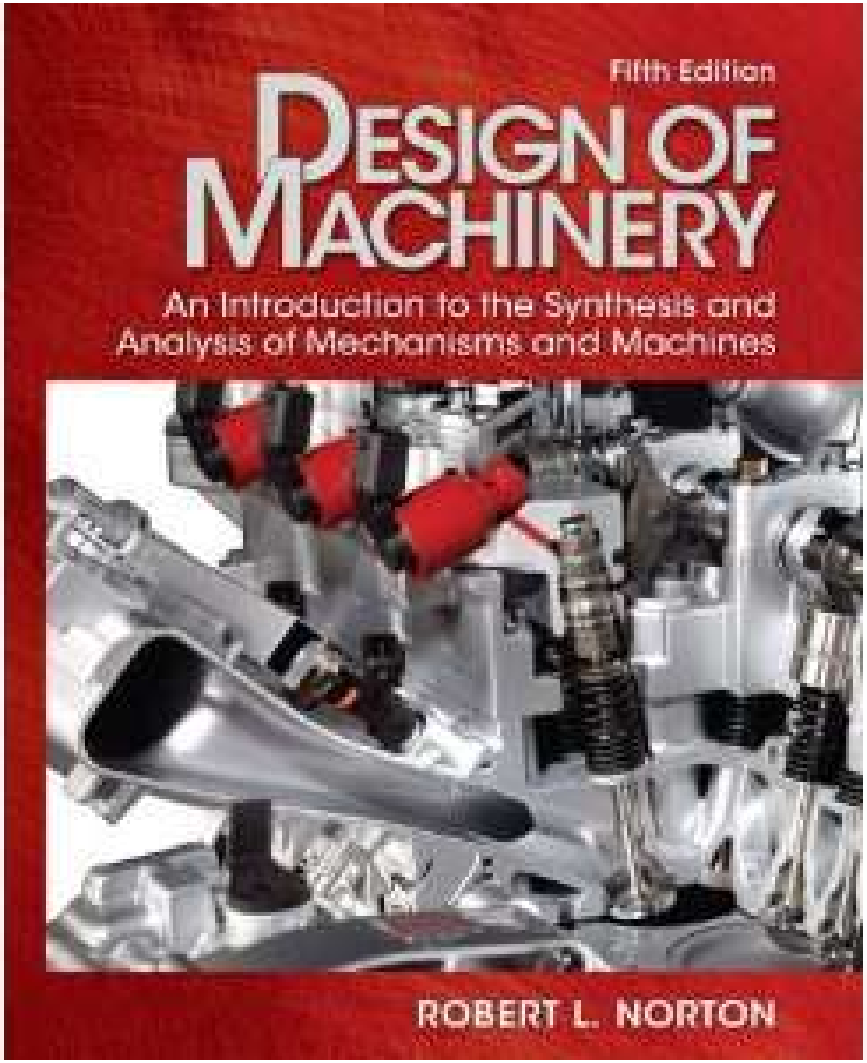

Fifth Edition
DESIGN OF MACHINERY
An Introduction to the Synthesis and Analysis of Mechanisms and Machines
ROBERT L. NORTON

Kinematics and Kinetics

- Kinematics
  - The study of motion without concern of what forces cause the motion, first step in the analysis or design of a mechanism
- Kinetics
  - The study of forces on systems in motion
- Design requires the knowledge both
- Dynamics
  - Combination of kinematics and kinetics

- However, first must know the acceleration before applying Newton's 2nd Law (F=ma)

- Designs require some finite service life, with this in mind the following must be taken into consideration:
  - The stresses on the system must be within some allowable range
  - The maximum forces on the system must be within some allowable range
  - The acceleration must be known
- How do we accomplish this? Kinematics

Mechanism and Machine

- Mechanism
  - A device that transforms motion into some desirable pattern and typically develops very low forces and transmits very little power
  - Combination of rigid or resilient bodies joined together to provide a specific absolute motion
- Machine
  - Contains mechanisms which are designed to provide significant forces and transmit significant power
  - Mechanism capable of performing useful work or capable of transmitting significant forces

Degrees of Freedom (DOF)

- Number of independent parameters (coordinates) which are needed to uniquely define position in space at any instant in time
  - In a 3-D frame: x, y, z, θ, ϕ, ρ
- Rigid body: essentially nondeformable
  - Assume: rigid & massless
- Types of motion
  - Rotation + Translation = Complex Motion

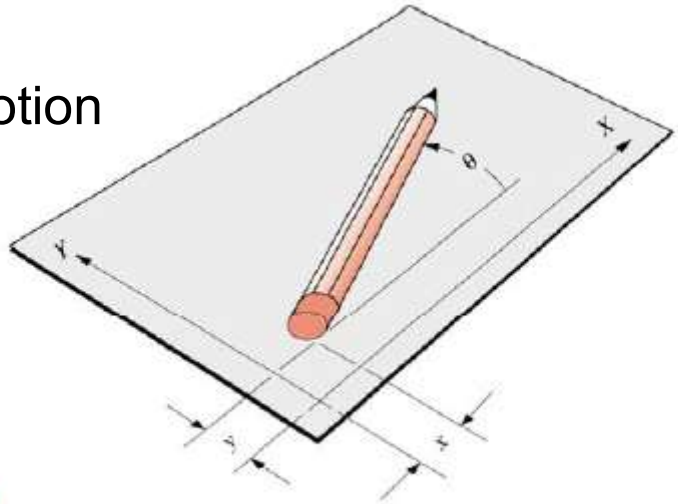
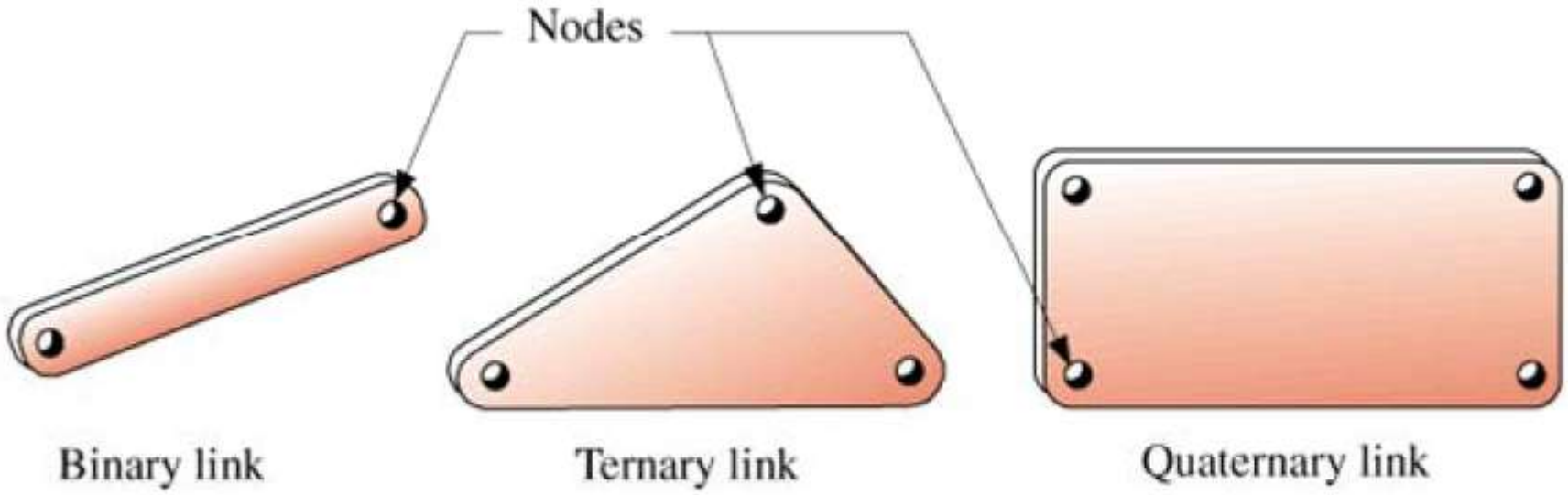


FIGURE 2-1
A rigid body in a plane has three *DOF*

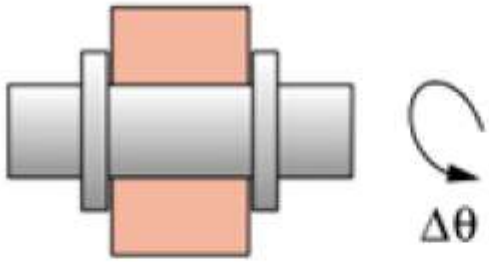
Links, Joints and Kinematic Chains

- Link: rigid or flexible members have at least two nodes (points of attachment)
- Linkage: basic building block of all mechanisms
- Links of different order

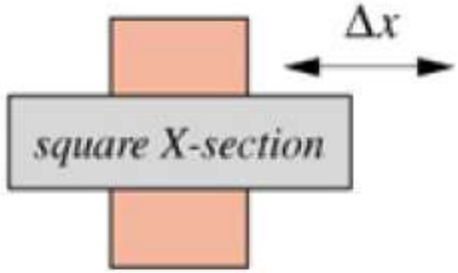


Six Lower Pairs

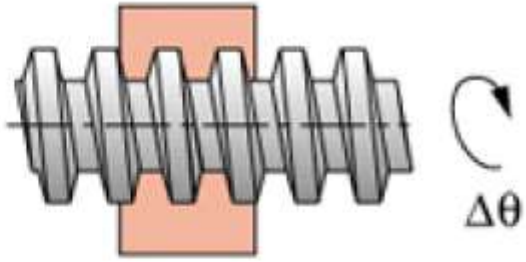
- Joint: connection between two or more links (at their nodes), which allows some motion between the links



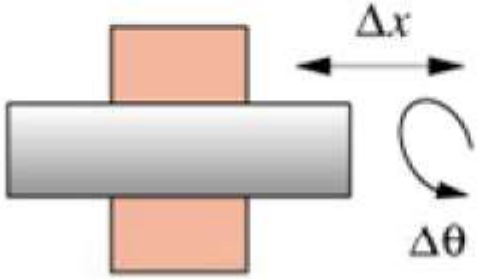
Revolute (R) joint—1 *DOF*



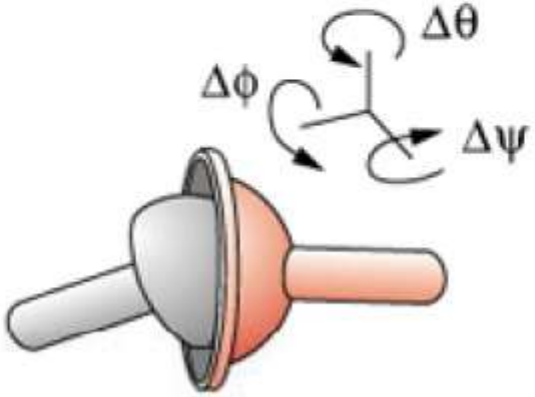
Prismatic (P) joint—1 *DOF*



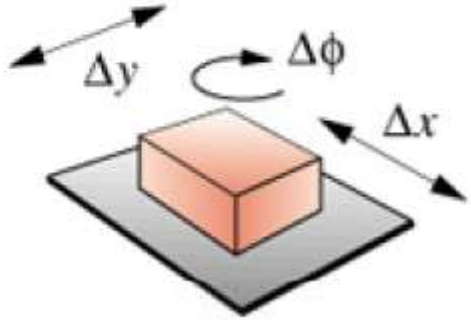
Helical (H) joint—1 *DOF*



Cylindric (C) joint—2 *DOF*



Spherical (S) joint—3 *DOF*



Planar (F) joint—3 *DOF*

Table 1. Freedom of general kinematic pair.

Translation/Rotation	0	1	2	3
0	Fixed	Revolute	Universal	Spherical
1	Translational	Cylindrical		
2		Planar		

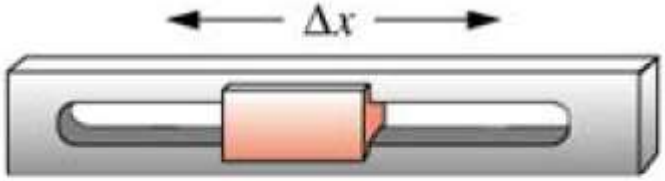
Joint (1)

- Joints are classified into several ways:
  - Number of DOF allowed at a joint
  - By type of contact
  - By type of physical closure (Force or Form closed)
  - Number of links (order of joint)

- Full joints: 1 DOF

Rotating full pin (R) joint (form closed)

pin (or revolute) joint

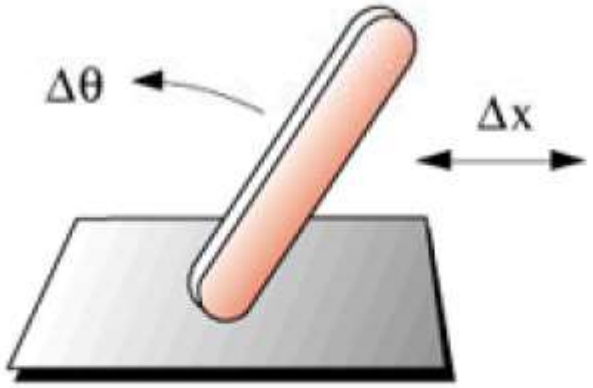


Translating full slider (P) joint (form closed)

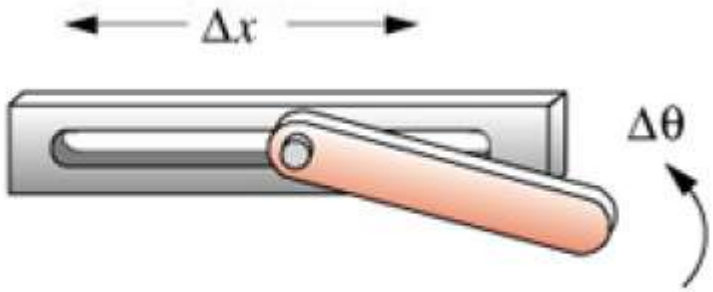
sliding (or prismatic) joint

Joint (2)

- Half (or RP, Roll-Slide) joints: 2 DOF



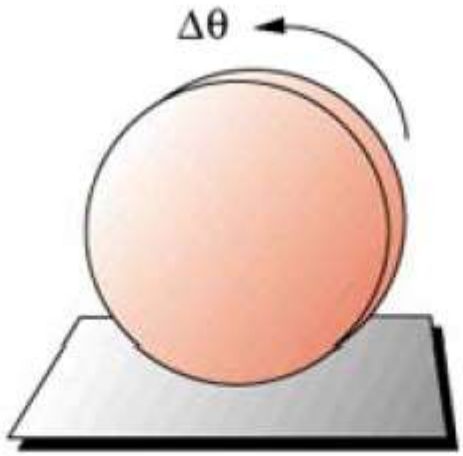
Link against plane (force closed)



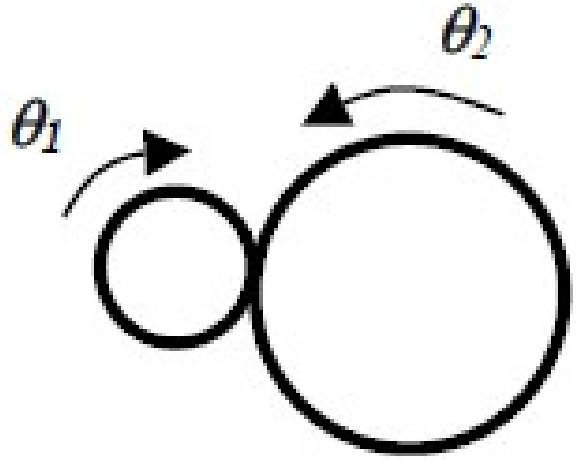
Pin in slot (form closed)

Joint (3)

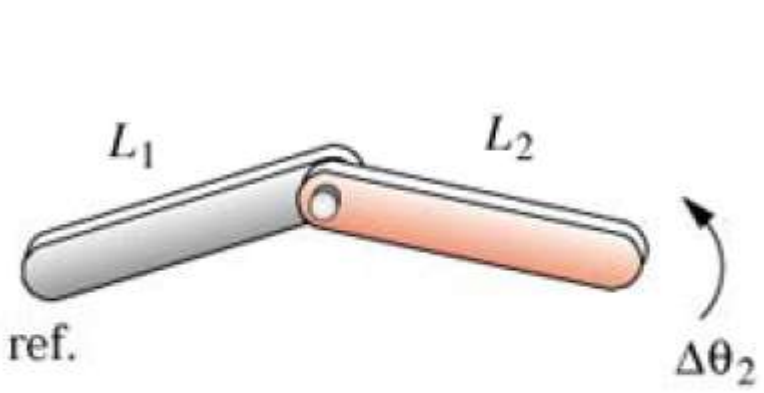
- Rolling joint: 1 or 2 DOF (may roll, slide, or roll-slide depending on friction)
  - Pure-roll (R), pure-slide (P), roll-slide (RP)



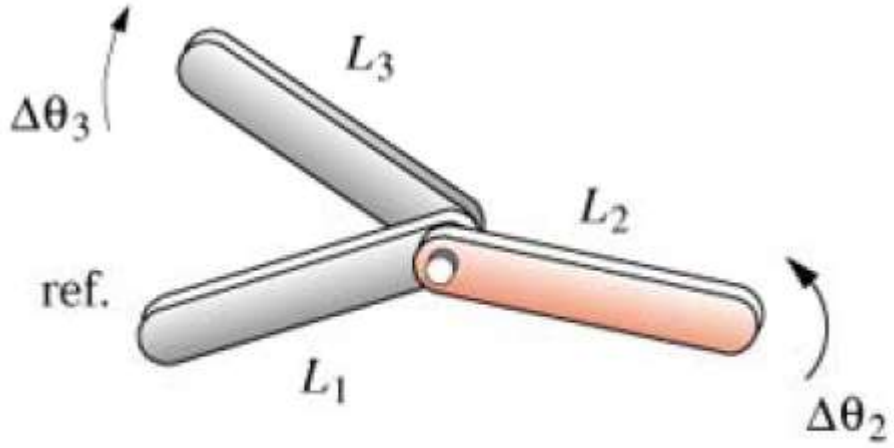
May roll, slide, or roll-slide, depending on friction



- Order = (number of links joined) – 1



First order pin joint - one *DOF*
(two links joined)

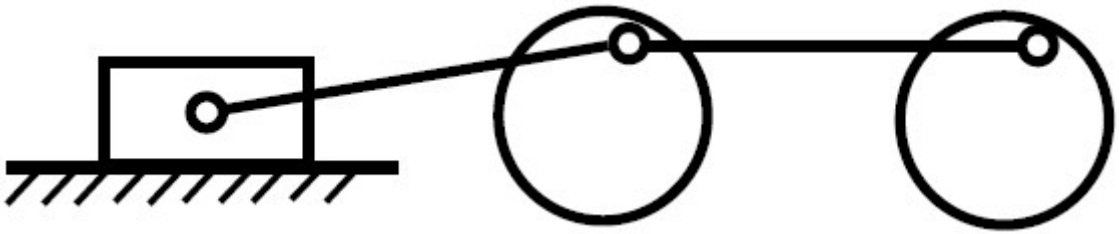


Second order pin joint - two *DOF*
(three links joined)

- Links are combined using joints to form kinematic chain or just linkage
- Linkage with at least one link fixed → mechanism

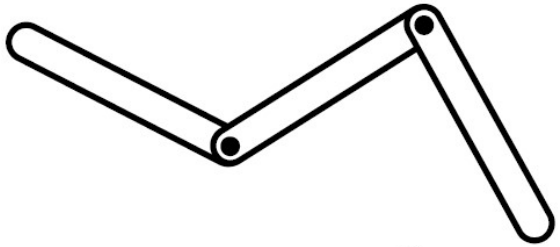
Planar Motion

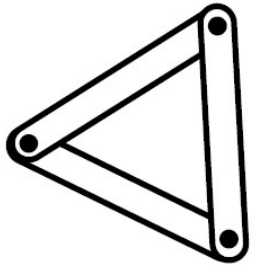
- Rectilinear translation
  - Points in the body move in parallel straight lines (piston)
- Curvilinear translation
  - Points in the body move along identical curves
  - Link does not rotate w.r.t. the ground (link connecting two disks)
- Rotation
  - Points in the body rotate about a single point, which is usually fixed to the ground (disks)
- General planar motion
  - General combination of rotation and translation (connecting rod joining piston and disk)

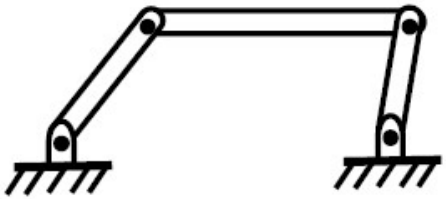


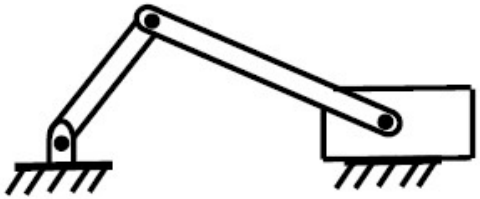
Linkage Classification by Number of Links

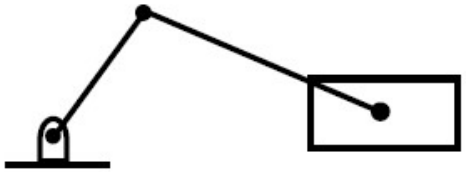
- Two links with one joint (dyad)
- Three links (triad)
- Four links (four-bar mechanism)
  - Joined by 4 pin-joints
  - Slider joint replaces one of the pin joints











- Kinematic chain
  - An assemblage of links and joints to provide a controlled output motion in response to a supplied input motion
- Crank
  - A link which makes a complete revolution and is pivoted to ground
- Rocker
  - A link which has an oscillatory rotation and is pivoted to ground
- Coupler (aka Connecting Rod)
  - Has a complex motion and is not pivoted to ground
- Ground
  - Any link that is fixed w.r.t. the reference frame
  - Note: reference frame may be in motion

Drawing Kinematic Diagrams

Binary link

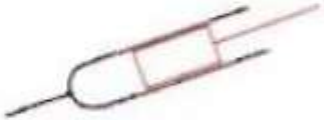
Ternary link

Quartenary link

Grounded half joint

Moving rotating joint

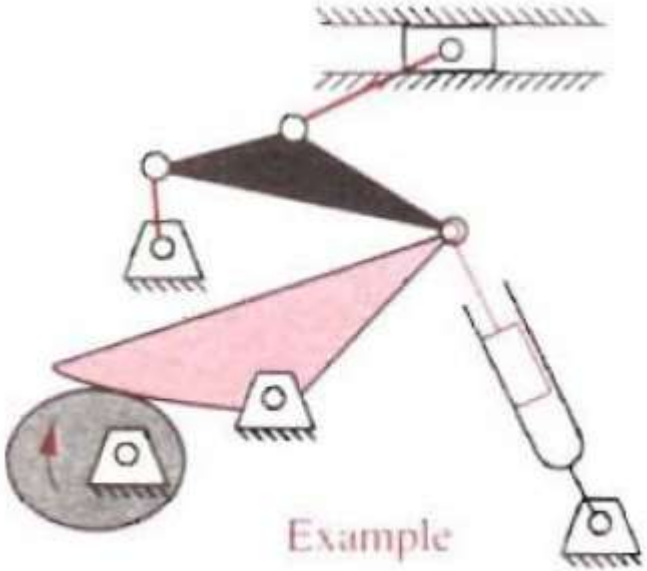
Grounded rotating joint



Moving translating joint

Grounded translating joint

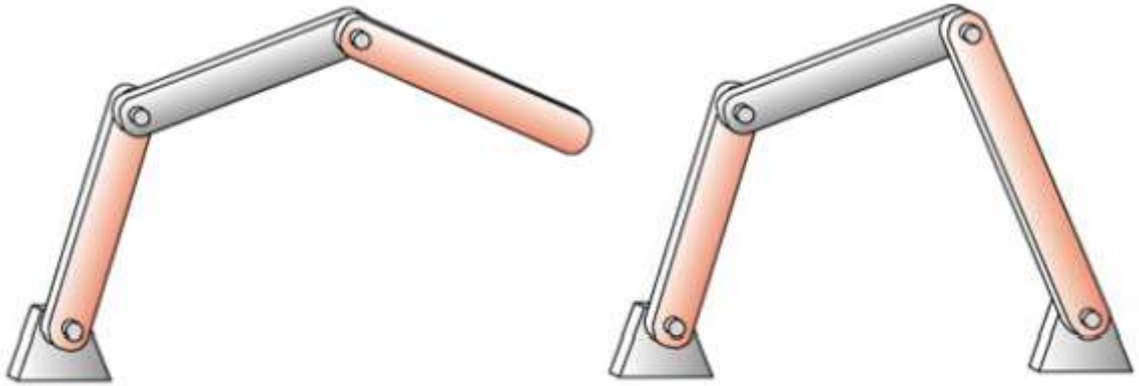
Moving half joint



Example

Determining DOF (1)

- Kinematic chains may be either OPEN or CLOSED



(*a*) Open mechanism chain (*b*) Closed mechanism chain

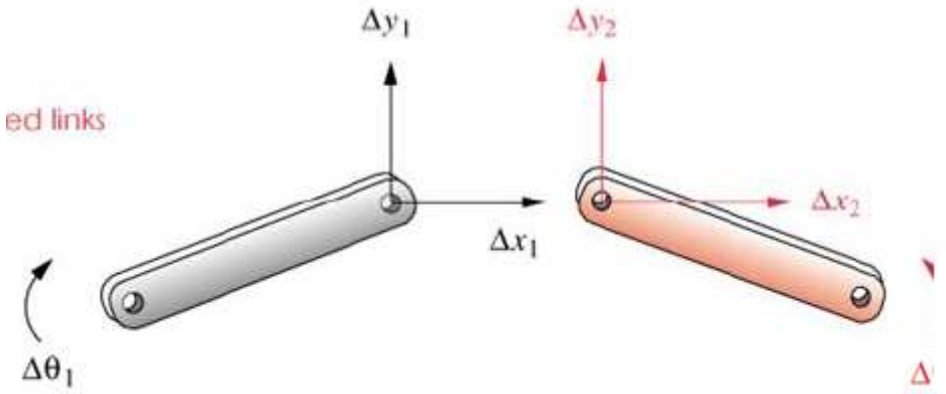
- Dyad
  - An open kinematic chain of two binary links and one joint

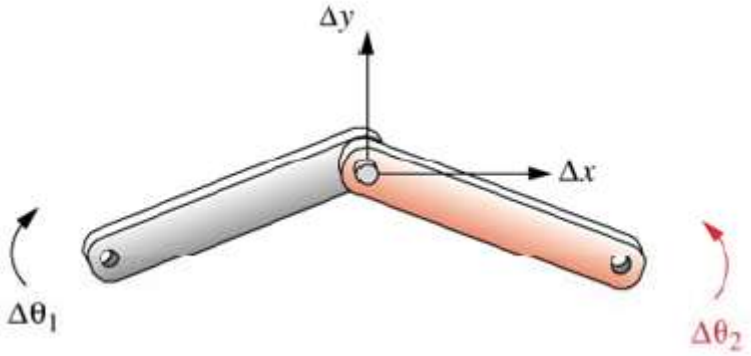
Determining DOF (2)

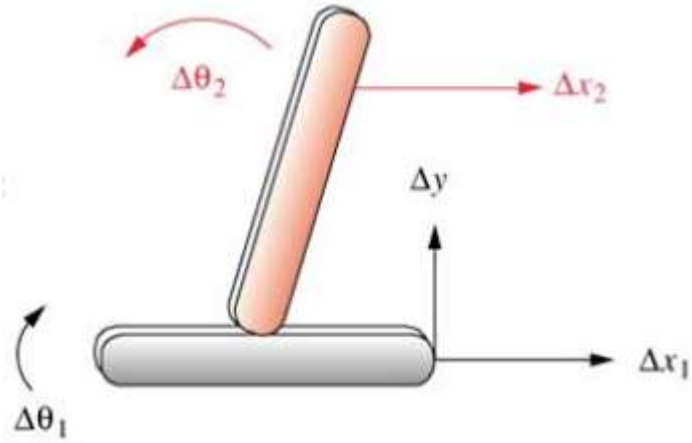
- To determine the overall DOF
  - Account for number of links and joints and their interactions
- Any free link on a plane has 3 DOF: Δx, Δy, $\Delta \theta$
- Gruebler's Equation
  - DOF = 3L - 2J - 3G
  - L: number of links
  - J: number of joints
  - G: number of ground
  - As there is always only one ground plane, G=1 always
  - DOF = 3(L – 1) - 2J
  - Note: J=1/2 for half joints because a half joint only removes one DOF

Determining DOF (3)

- Kutzbach's modification
  - DOF = $3(L - 1) - 2J_1 - J_2$
  - L: number of links
  - J_1: number of full joints
  - J_2 : number of half joints

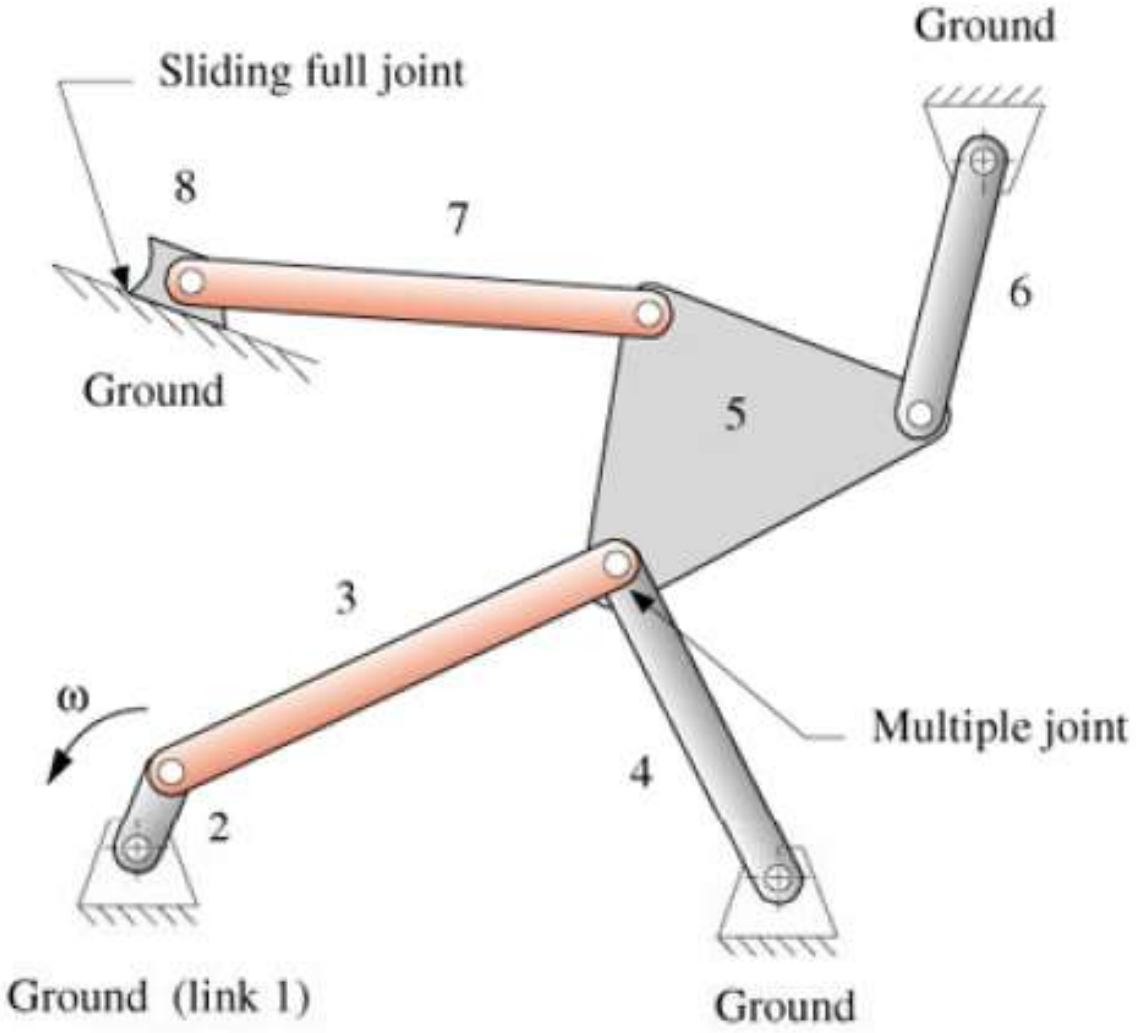




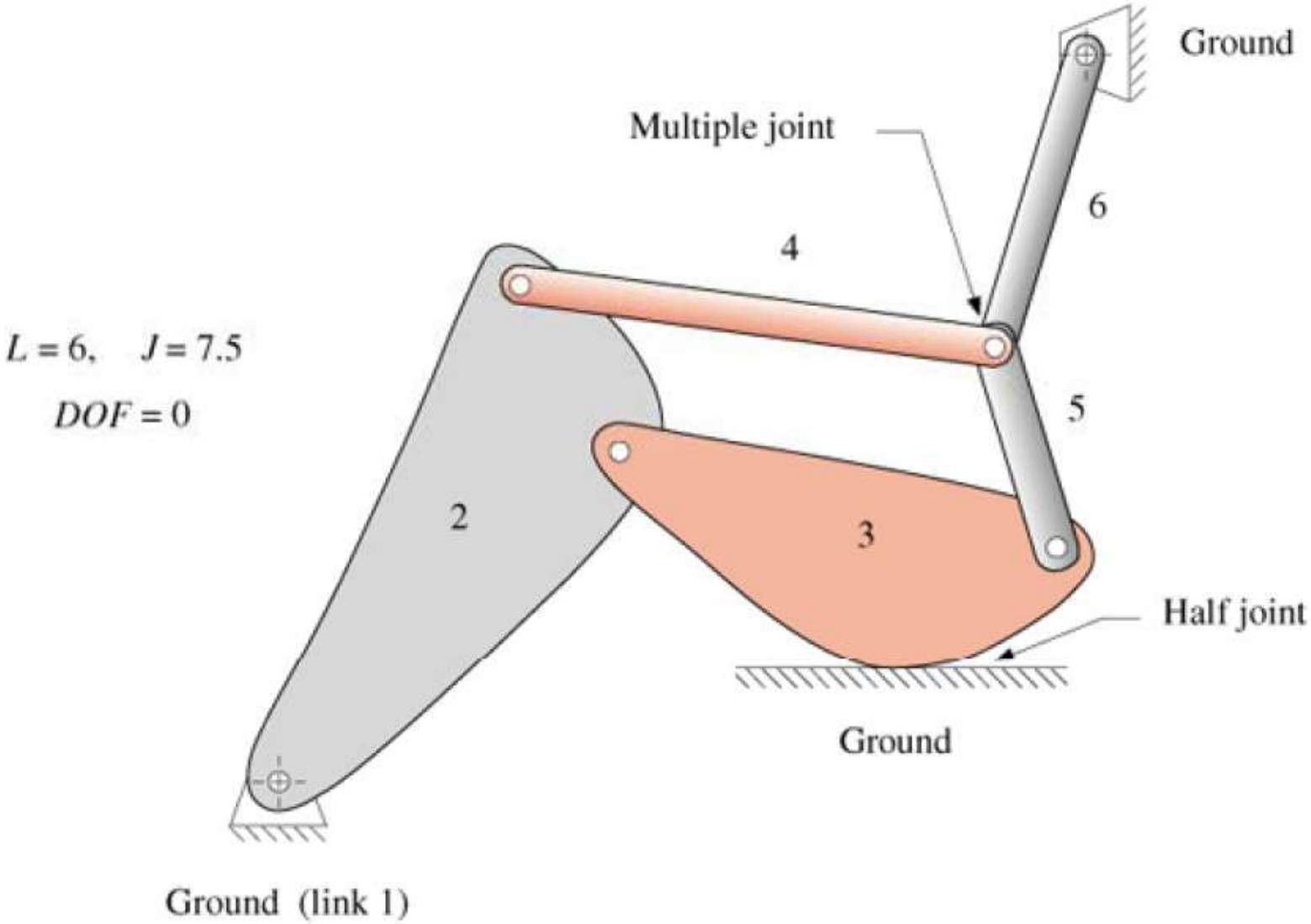


Determining DOF (4)

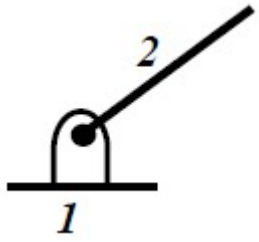
Name (Symbol)	*DOF*	Cont-ains
Revolute (R)	1	R
Prismatic (P)	1	P
Helical (H)	1	RP
Cylindric (C)	2	RP
Spherical (S)	3	RRR
Planar (F)	3	RPP

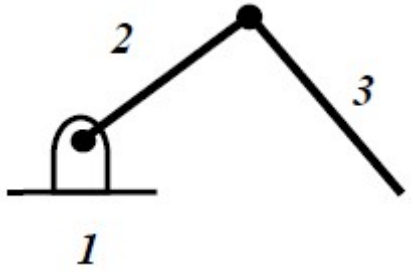


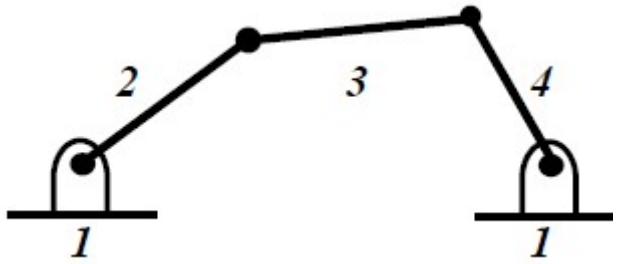
Determining DOF (5)

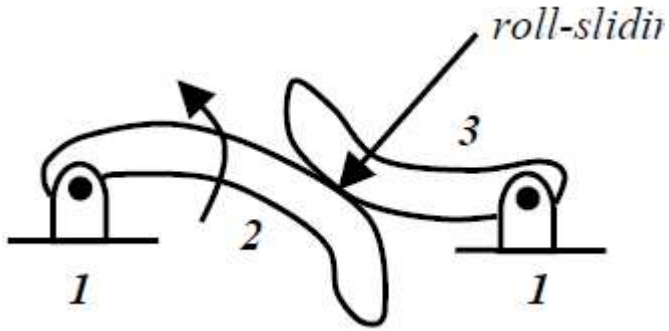


Calculation of Mobility

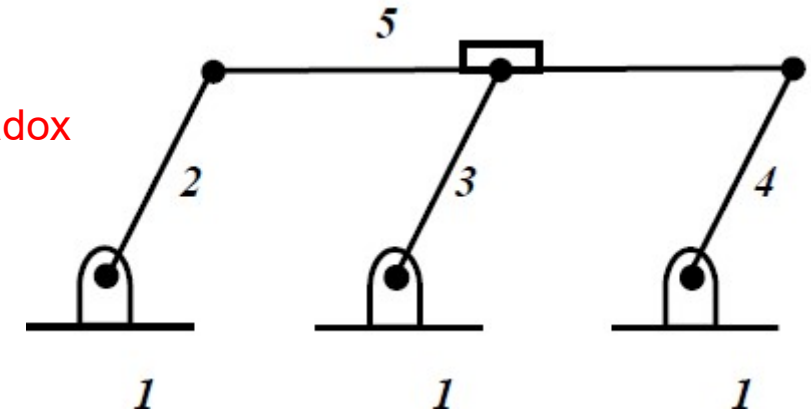


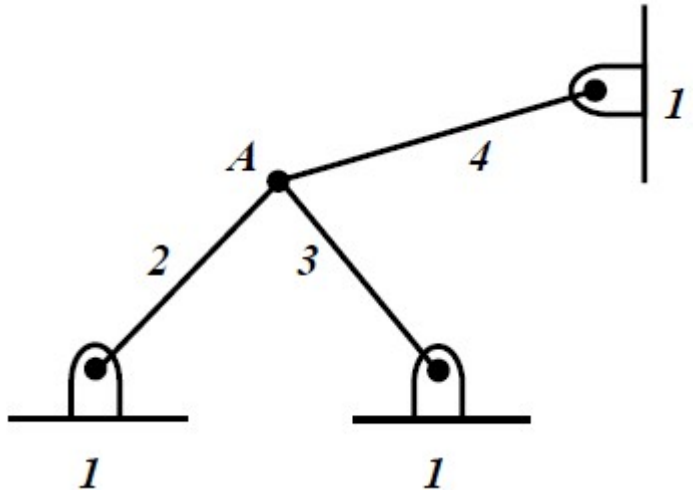






paradox

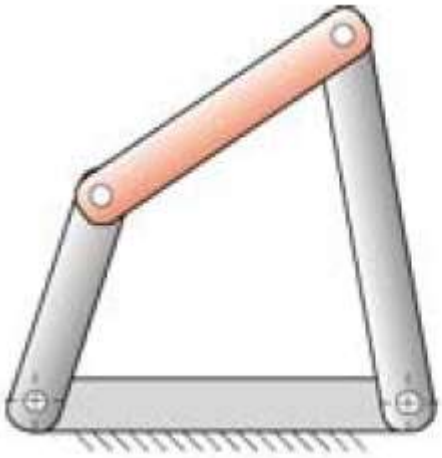


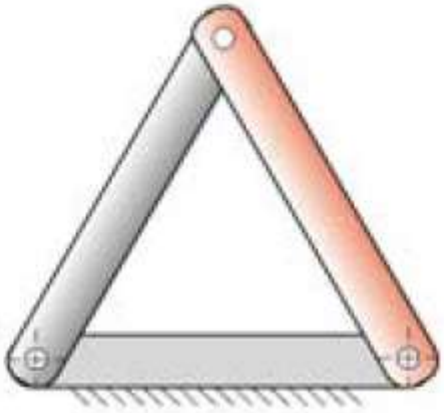


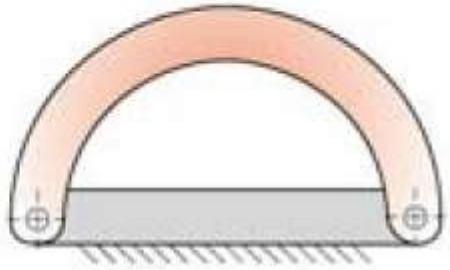
Statically indeterminate structure

Mechanisms and Structures

- Mechanism: DOF > 0 → possible motion
- Structure: DOF = 0 → no motion possible
- Preloaded structure: DOF < 0 → no motion and possible stresses

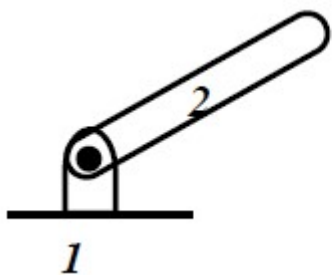


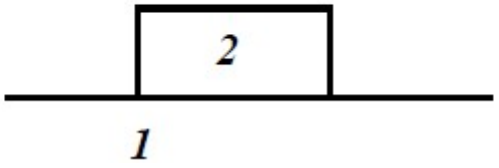


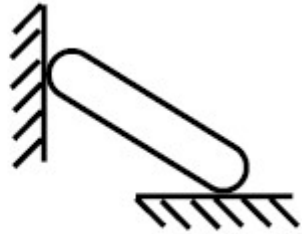


Simple Mechanisms (1)

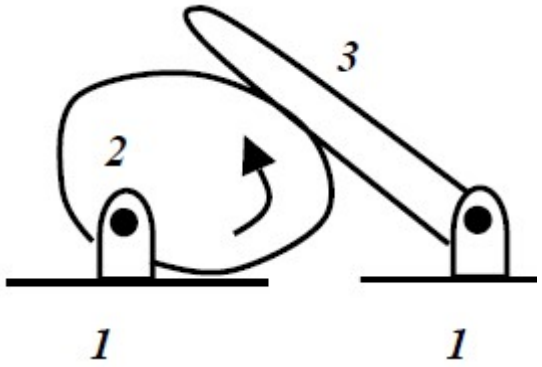
- 2 links: [3*(2-1)-2*1=1] by adding a single pin or slider joint

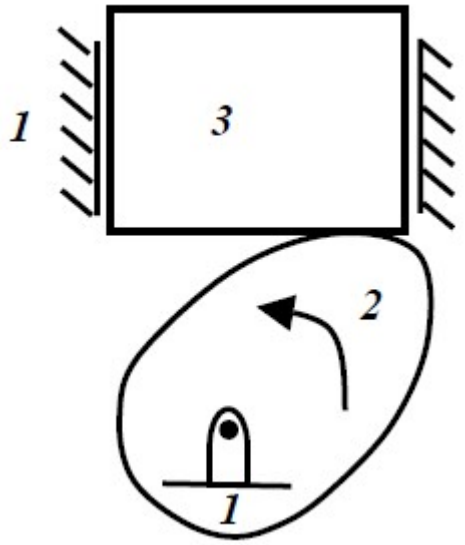






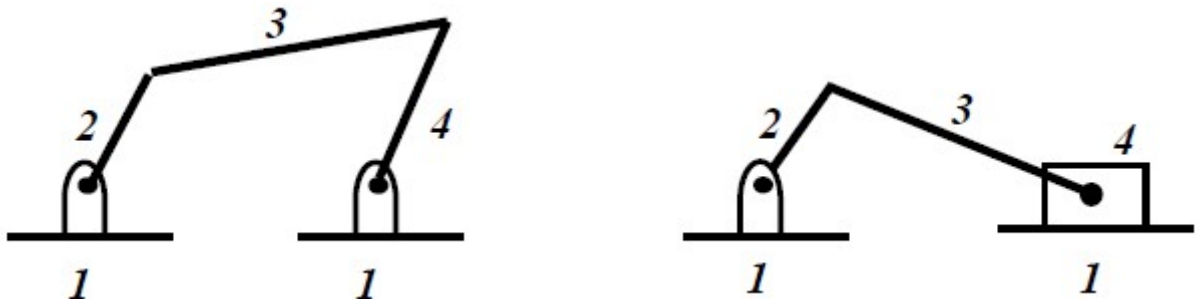
- 3 links: [3*(3-1)-2*2-1=1] by adding two pins and a roll-slider joint





Simple Mechanisms (2)

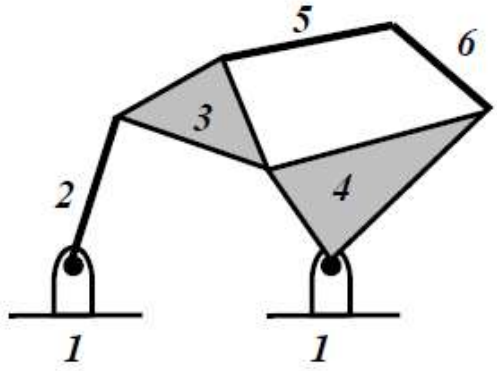
- 4 bars + 4 pins: 4-bar mechanism [3*(4-1)-2*4=1]



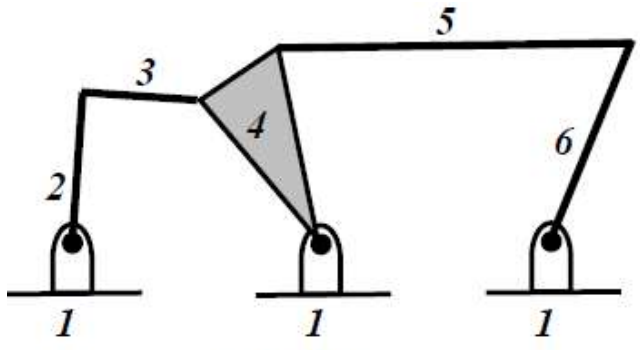
  - https://www.youtube.com/watch?v=KBFFwgCCP0U
- 5 links + pin or slider joints: not possible
- 6-bar linkage: [3*(6-1)-2*7=1]
  - 6 binary links? 6 pin joints (x)
  - 2 ternary (adjacent) + 4 binary links : Watt linkage
  - 2 ternary (separate) + 4 binary links : Stephenson linkage

Simple Mechanisms (3)

- Watt six-bar mechanism

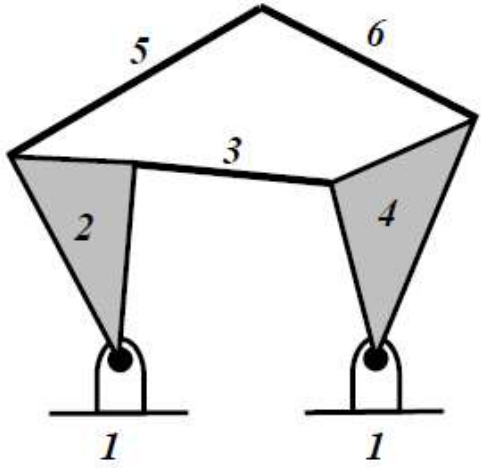


Watt I

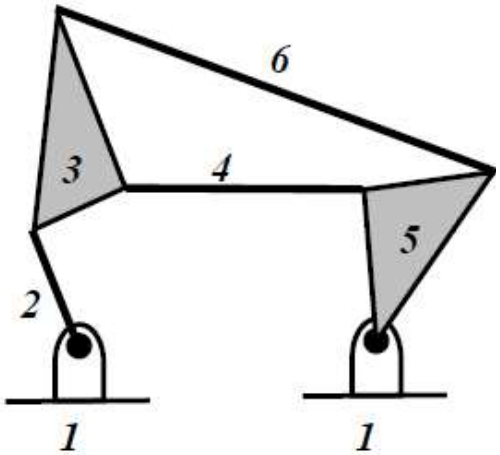


Watt II

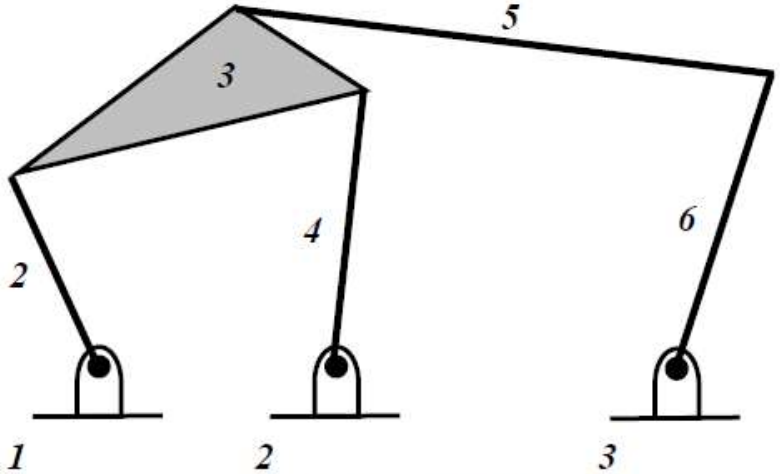
- Stephenson six-bar mechanism



Stephenson I



Stephenson II



Stephenson III

Number Synthesis

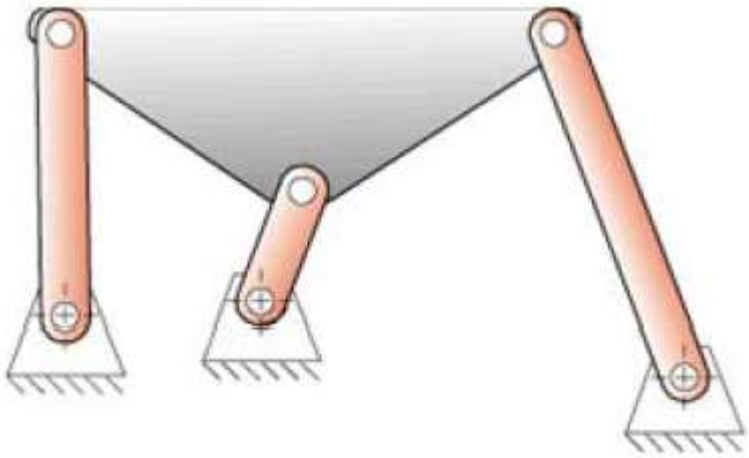
- The determination of the number and order of links and joints necessary to produce motion of a particular DOF
  - Gives all possible combinations of linkage combinations from which to choose
- Order: number of nodes per link
  - Binary, ternary, quaternary
- Example: drive all possible link combinations for one DOF for sets ≤ 8 links and link orders ≤ 6
  - Assume only full rotating joints

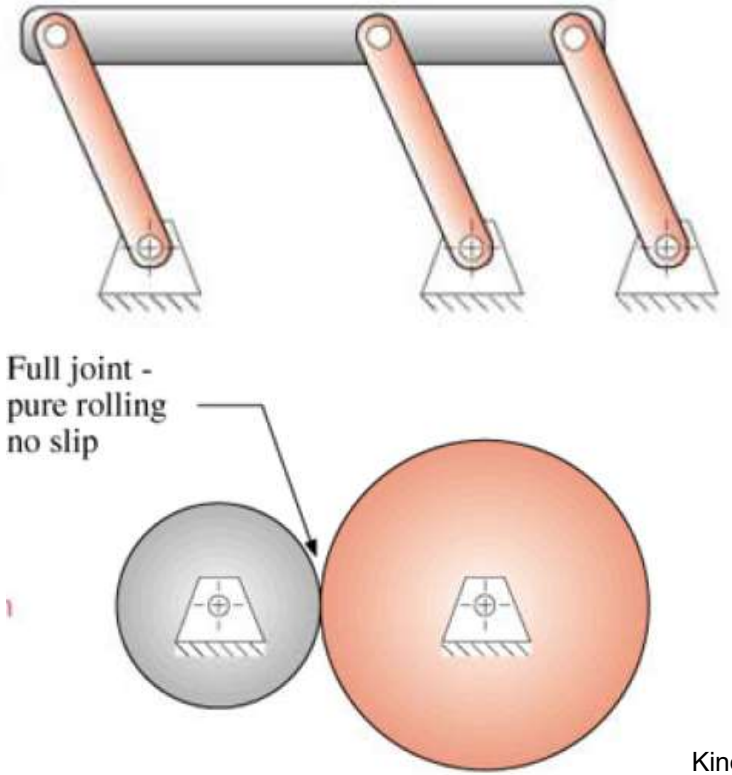
TABLE 2-2 1-*DOF* Planar Mechanisms with Revolute Joints and Up to 8 Links

Total Links	Link Sets				
	Binary	Ternary	Quaternary	Pentagonal	Hexagonal
4	4	0	0	0	0
6	4	2	0	0	0
6	5	0	1	0	0
8	7	0	0	0	1
8	4	4	0	0	0
8	5	2	1	0	0
8	6	0	2	0	0
8	6	1	0	1	0

Paradoxes

- Gruebler Equation ignores link sizes and shapes
  - It can give misleading conclusions for unique geometric configurations
  - Example: E-quintet

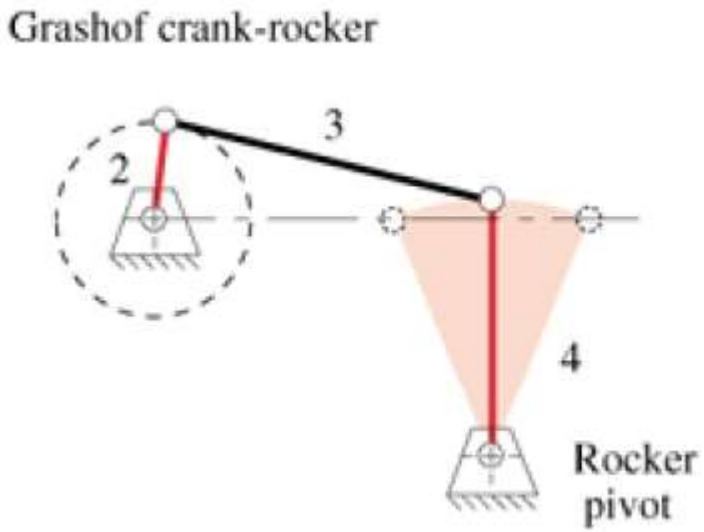


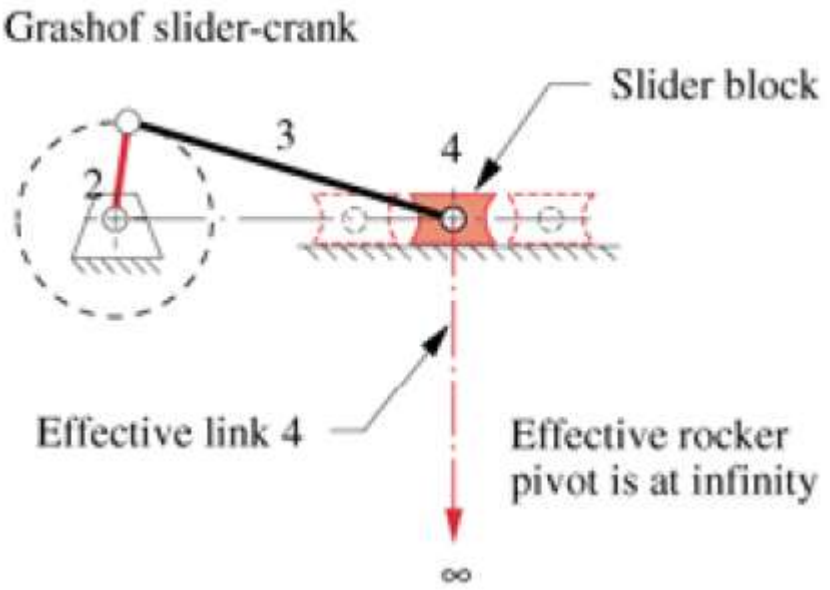


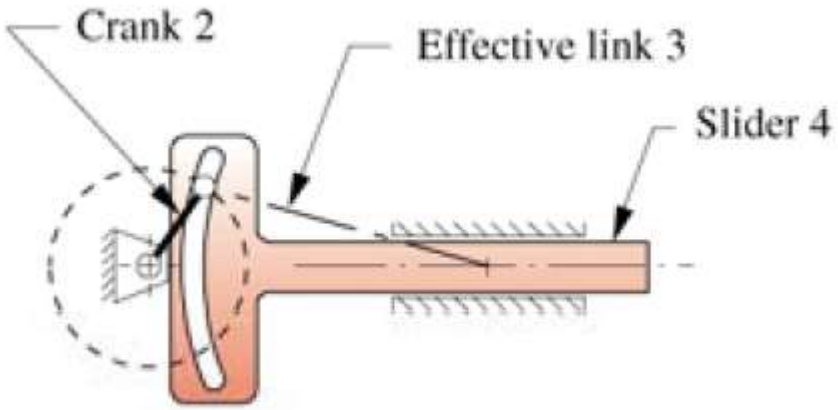
Linkage Transformation (1)

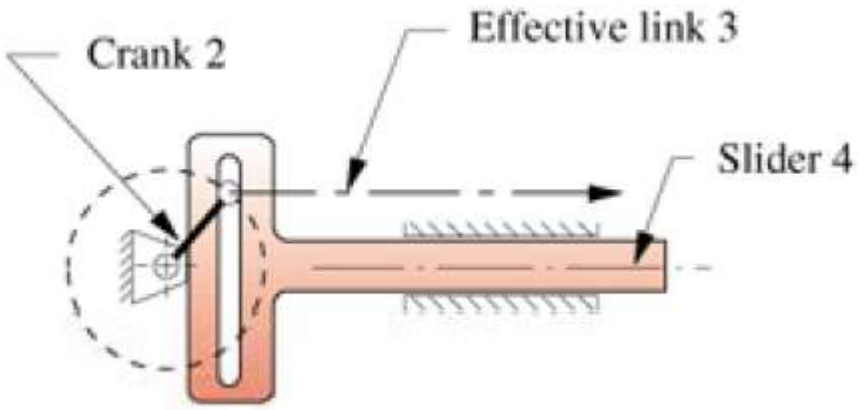
- [1] Any full rotating joint can be replaced by a sliding full joint with no change in DOF of the mechanism
- [2] Any full joint can be replaced by a half joint, but this will increase DOF by 1
- [3] Removal of a link will reduce DOF by 1
- [4] The combination of [2] and [3] will keep the original DOF unchanged
- [5] Any ternary or higher order link can be partially 'shrunk' to a lower order link by coalescing nodes
  - Creation of a multiple joint, but will not change DOF
- [6] Complete shrinkage of a link is equivalent to its removal
  - Creation of a multiple joint and DOF will be reduced

Linkage Transformation (2)

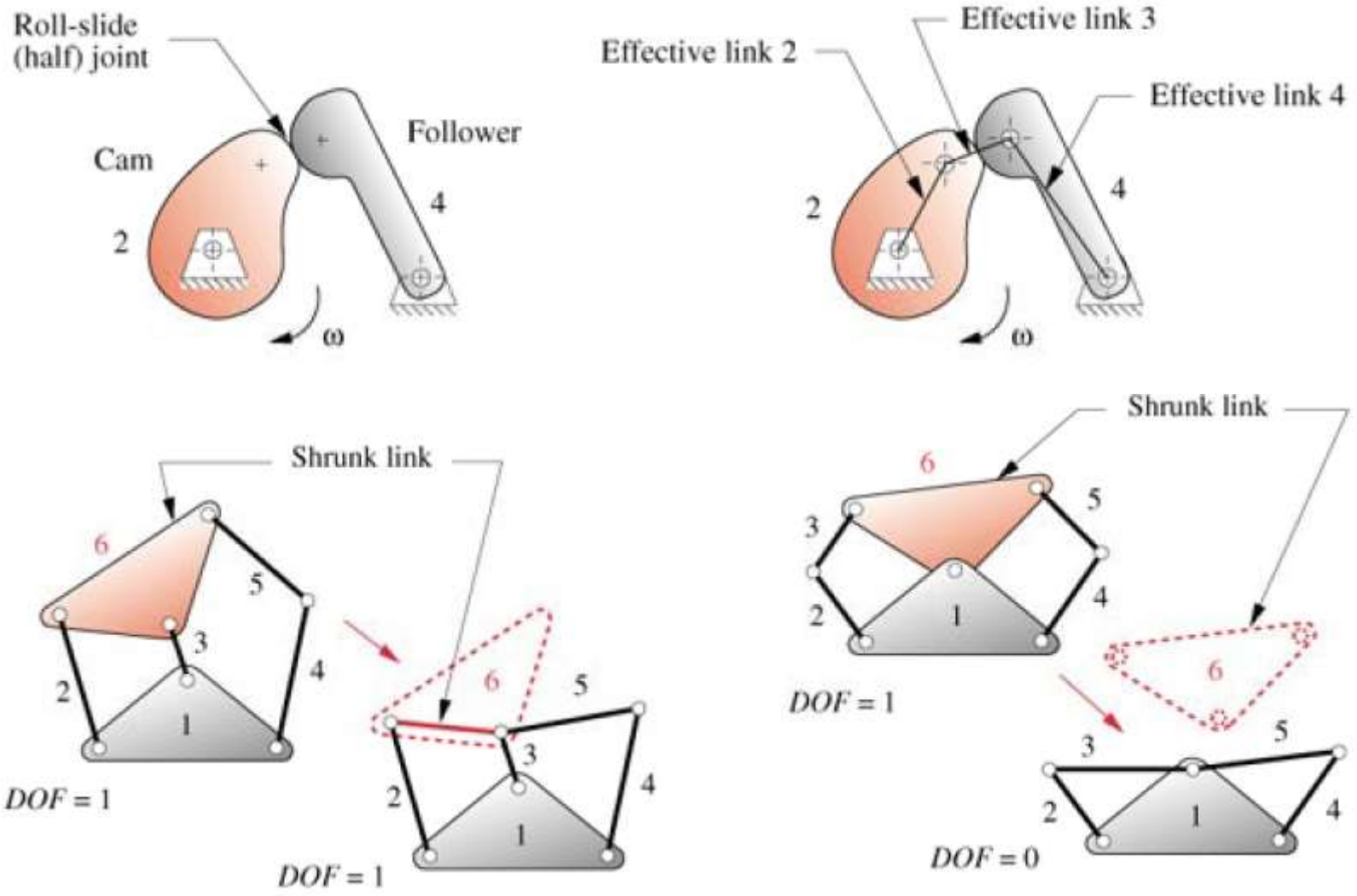






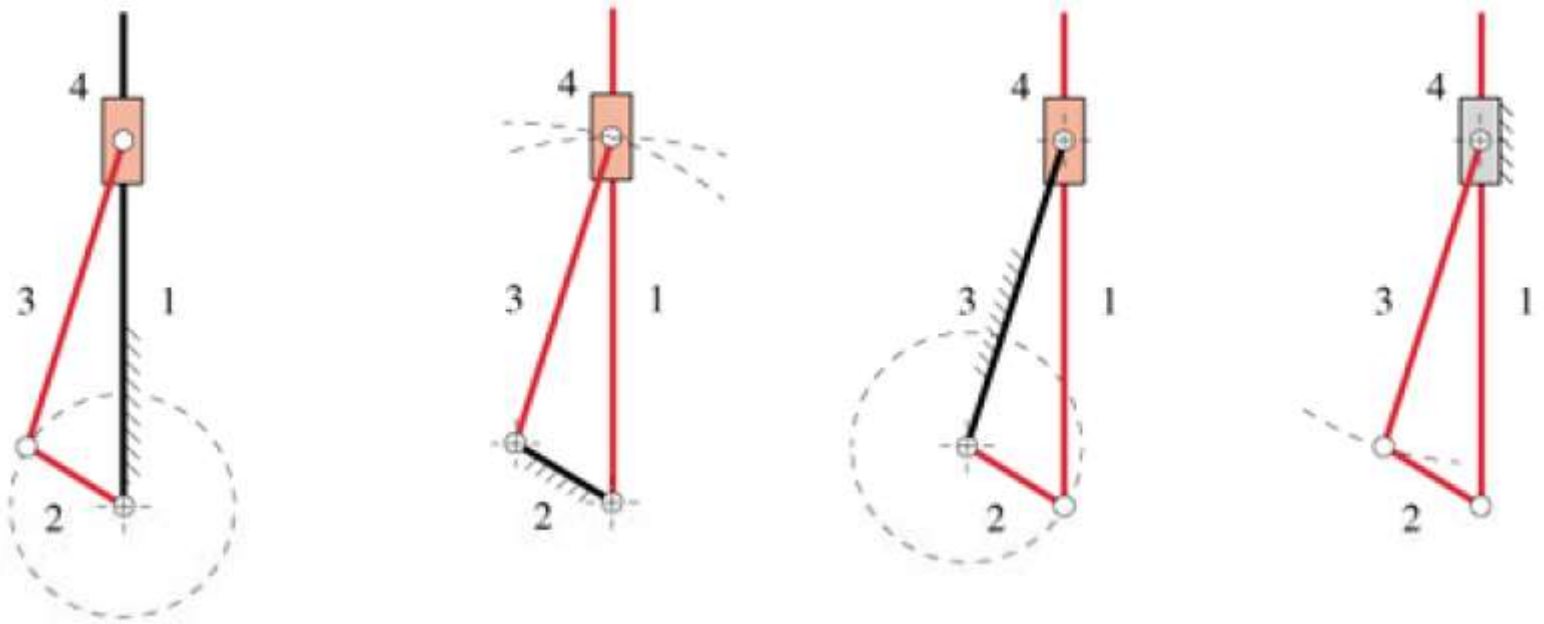


Linkage Transformation (3)

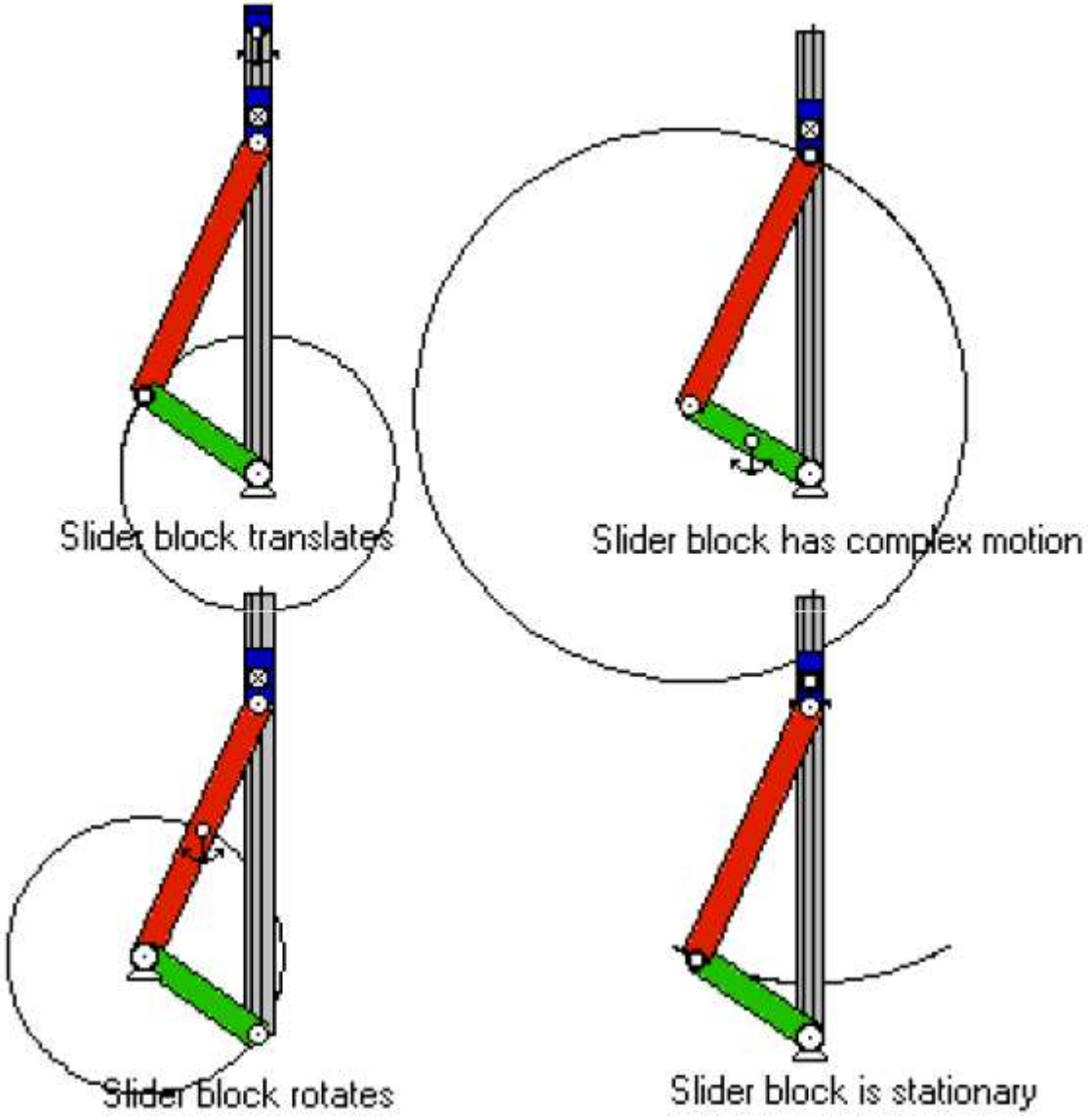


Inversion (1)

- An inversion is created by grounding a different link in the kinematic chain → there are as many inversions of a given linkage as there are links



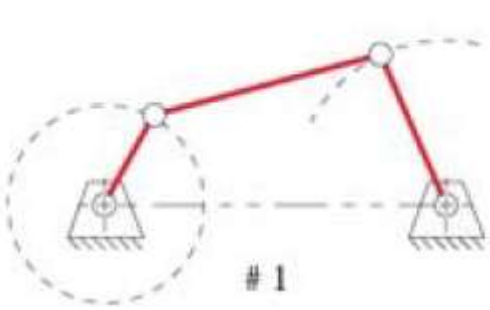
Inversion (2)

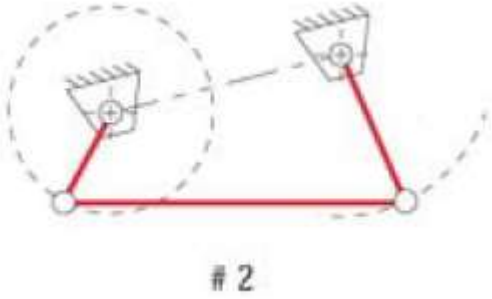

Slider block translates
Slider block has complex motion
Slider block rotates
Slider block is stationary

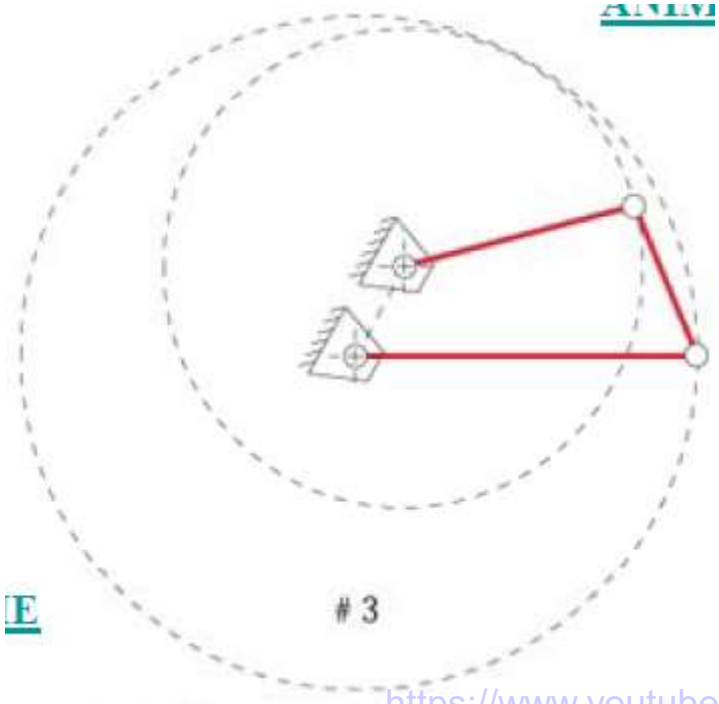
The Grashof Condition

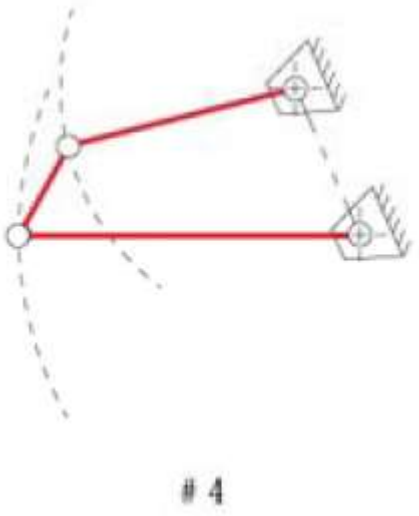
- A Fourbar linkage is the simplest possible pin jointed mechanism for a 1 DOF controlled motion
- Grashof Condition is a simple relation which predicts the behavior of fourbar linkage inversions
  - S : length of shortest link
  - L : length of longest link
  - P : length of one remaining link
  - Q : length of other remaining link
  - If $S + L \leq P + Q$, then the linkage is Grashof → At least one link will be capable of making full revolution w.r.t. the ground

All Inversions of the Grashof Fourbar Linkage







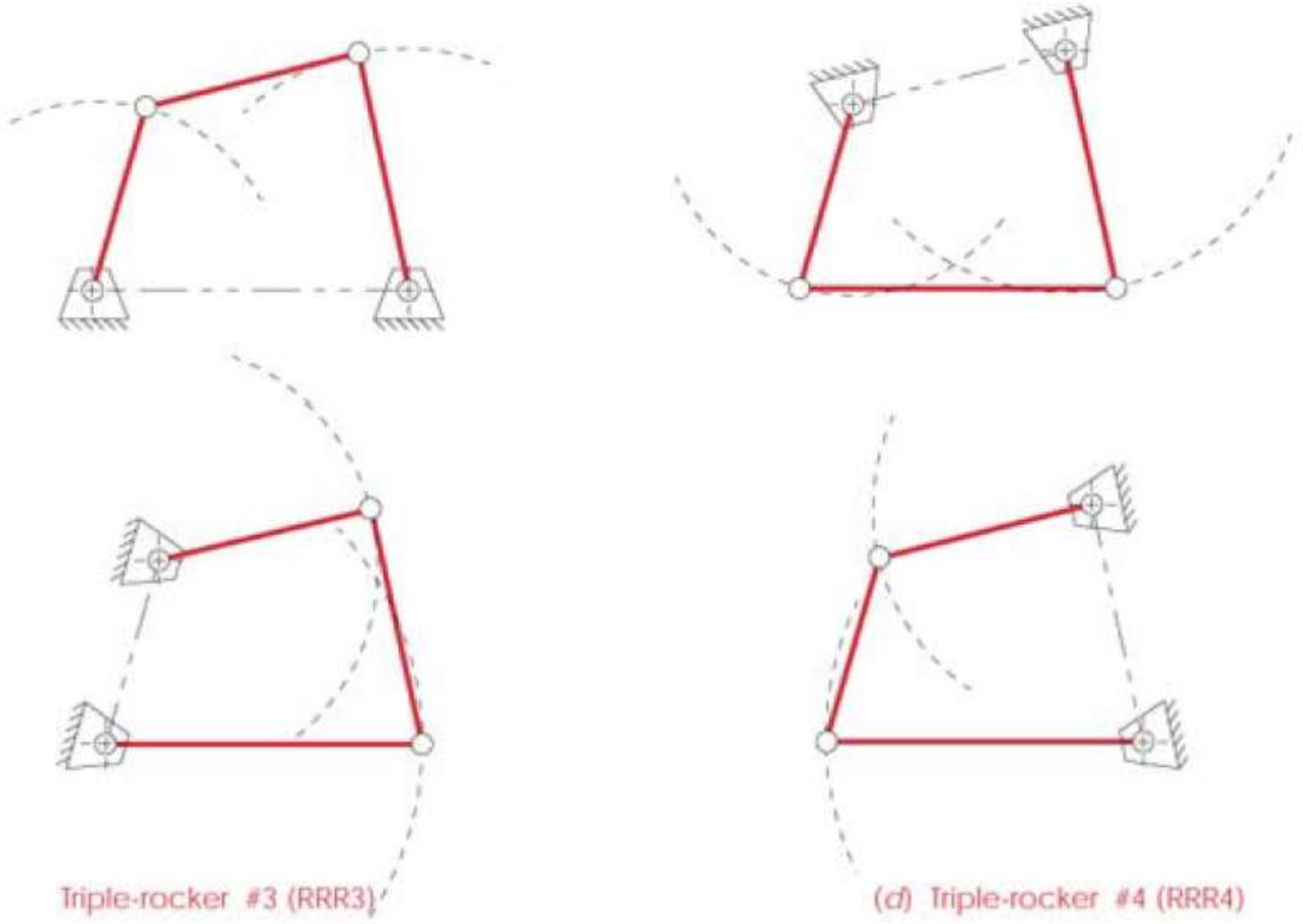


https://www.youtube.com/watch?v=qvD9QKDPRv8

Grashof

- Crank Rocker
  - https://www.youtube.com/watch?v=q0dd4SuUjKk
- Double Rocker
  - https://www.youtube.com/watch?v=mFnwPuUVB0k
- Rotating Coupler
  - https://www.youtube.com/watch?v=1_Yg_KAR7LI
- Special Case 1: Delta/Kite
  - https://www.youtube.com/watch?v=7nXKmvL3mOo
- Special Case 2: Parallel/Antiparallel
  - https://www.youtube.com/watch?v=gY0s0oiEnZI

All Inversions of the non-Grashof Fourbar Linkage

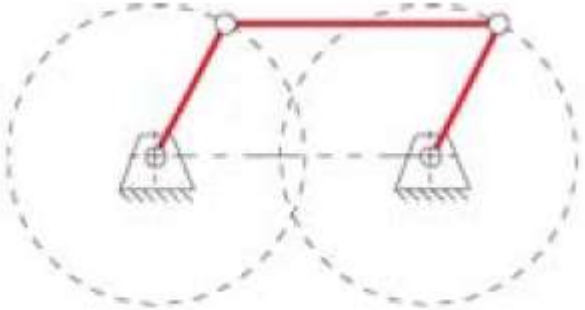


https://www.youtube.com/watch?v=CiEkyUHOUOI

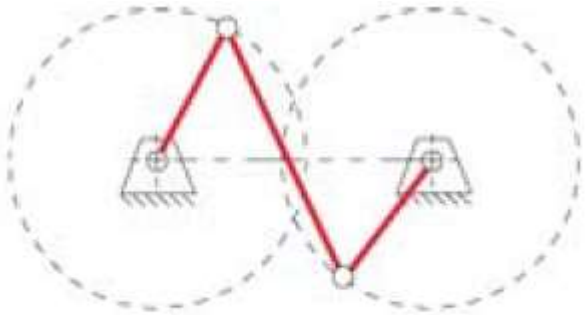
Non-Grashof

- Double Rocker: Outward/Outward
  - https://www.youtube.com/watch?v=mK51eFIUMAg
- Double Rocker: Inward/Inward
  - https://www.youtube.com/watch?v=AbtCFGzZzHM
- Double Rocker: Outward/Inward
  - https://www.youtube.com/watch?v=mL2HMsEjq_U

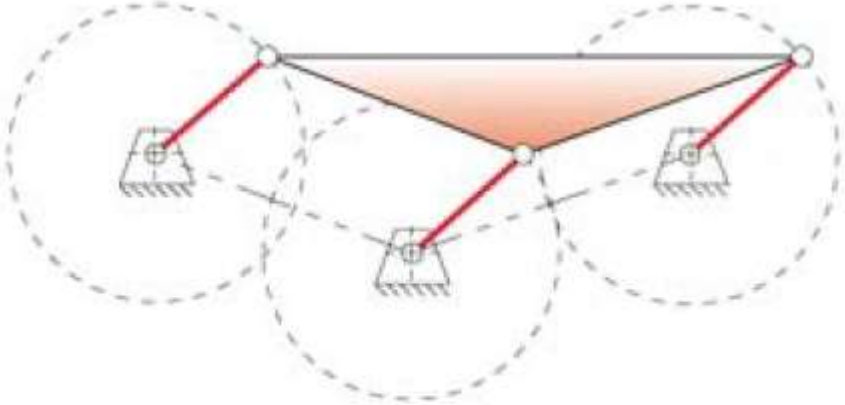
Special-case of Grashof Linkage (S+L=P+Q)



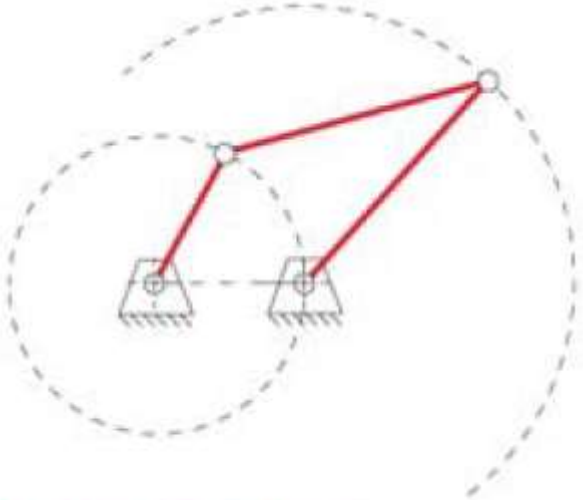
(a) Parallelogram form



(b) Antiparallelogram form



(c) Double-parallelogram linkage gives parallel motion (pure curvilinear translation) to coupler and also carries through the change points



(d) Deltoid or kite form