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  - Suppressing rigid body motions
  - Taking advantages of symmetry and anti-symmetry

FEM (Finite Element Method)

- Distinguished from MSA(Matrix Structural Analysis)
  - Ability to go beyond structures
  - Increasing importance of continuum models in two and three space dimension
  - Key role of variational mathematics
- Terminology
  - Degree of freedom (DOF), generalized coordinates
  - State (primary) variables: displacements in mechanics
  - Conjugate (dual) variables: forces in mechanics
  - Stiffness matrix
  - Master stiffness equations

$$\mathbf{K}\mathbf{u} = \mathbf{f} = \mathbf{f}_M + \mathbf{f}_I$$

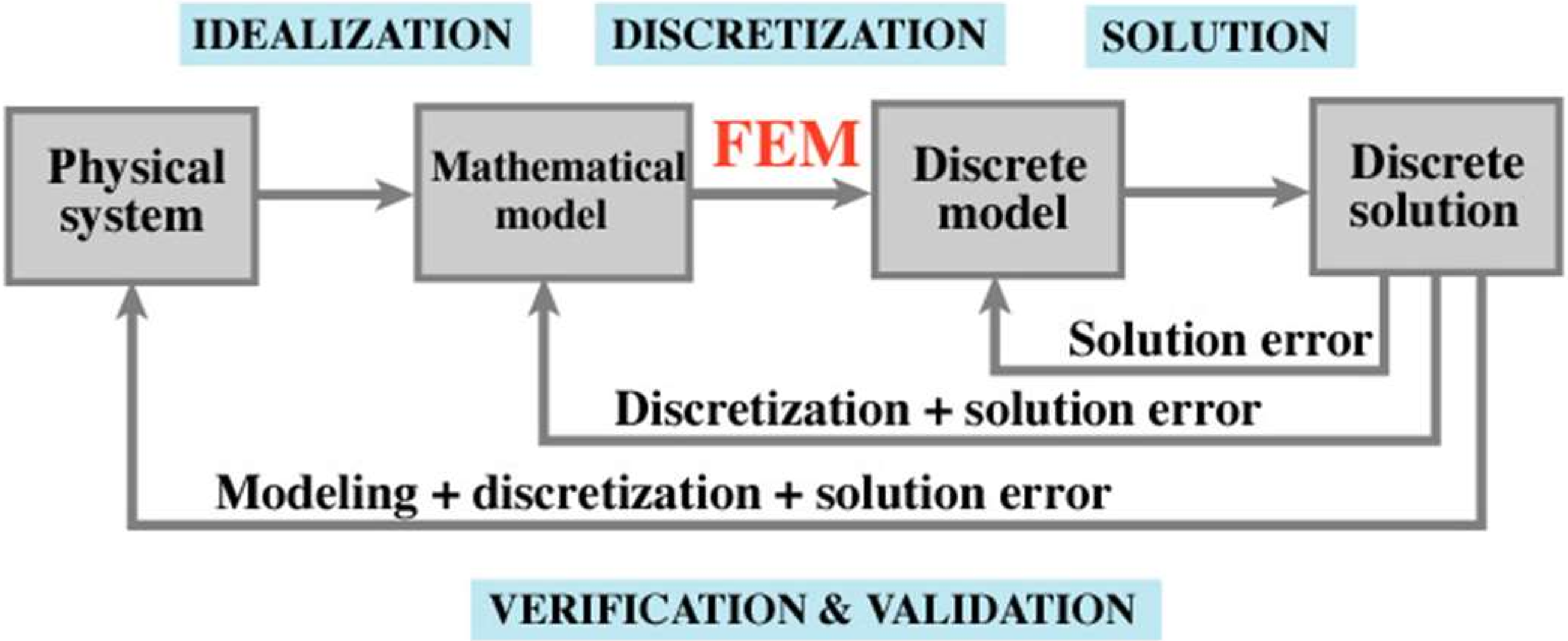
$\mathbf{f}_M$: vector of mechanical forces

$\mathbf{f}_I$: initial node force vector

Physical Significance

Application Problem	State Vector (u)	Forcing Vector (f)
Structures and solid mechanics	Displacement	Mechanical force
Heat conduction	Temperature	Heat flux
Acoustic fluid	Displacement potential	Particle velocity
Potential flows	Pressure	Particle velocity
General flows	Velocity	Fluxes
Electrostatics	Electric potential	Charge density
Magnetostatics	Magnetic potential	Magnetic intensity

Model Based Simulation



Idealization

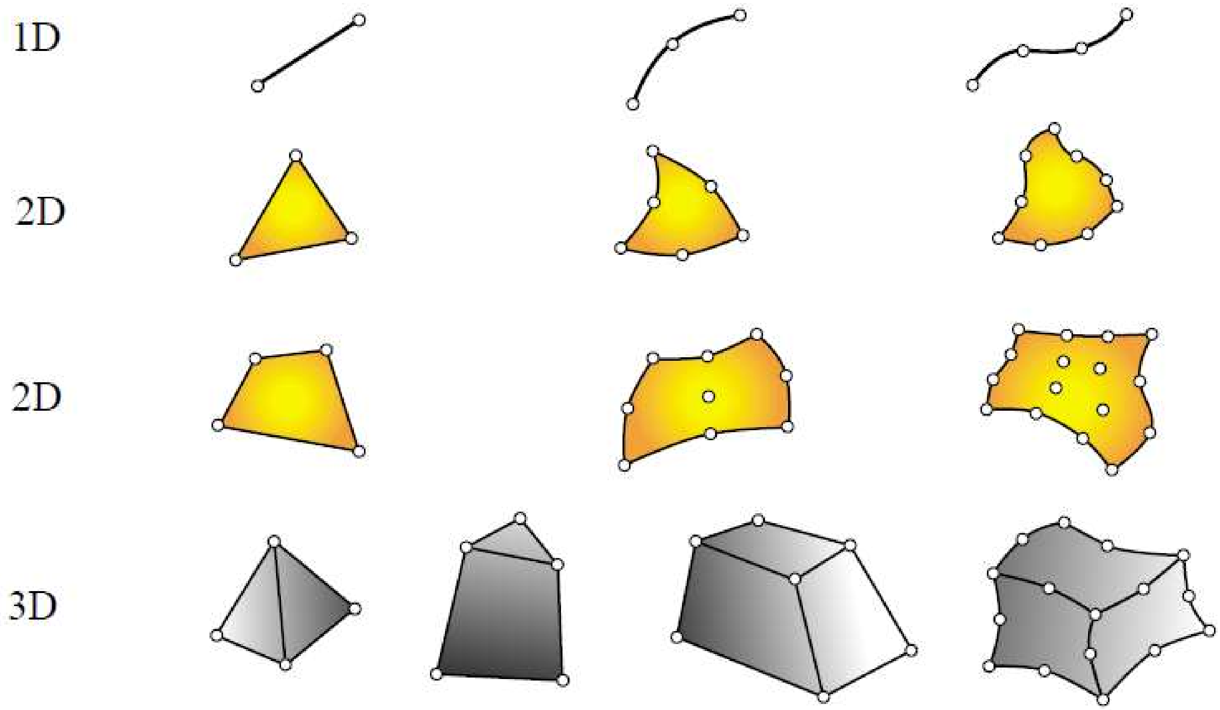
- Most important step in engineering practice
  - By human: physical system → mathematical model
- Models
  - Symbolic device built to simulate and predict aspects of behavior of a system
- Mathematical models
  - Abstraction of the physical reality
  - Eg. flat plate structure subject to transverse loading
    - Very thin / thin / moderately thin / very thick
  - Complexity control: filter out physical details that are not relevant to the design and analysis process
- Implicit (“black box”) vs. explicit modeling

Discretization

- Analytical or Numerical?
  - Closed form solutions: regular geometries, simple BCs
  - Discretization: space / time (finite number of DOF)
- Error sources and approximation
  - Modeling error: need validation (experimental results)
  - Discretization error ← approximation theory
    - Discrepancy between the computed solution of discrete model and the exact solution of mathematical model
  - Solution error
- Other discretization methods
  - BEM(Boundary Element Method)
    - Problems involving infinite domains
  - FDM(Finite Difference Method), FVM(Finite Volume Method)
    - Fluid mechanics, electromagnetics, CFD (moderate to high Re)

Element Attributes (1)

- Element dimensionality: 1D, 2D, 3D
  - Zero dimension: lumped spring, point mass
- Element nodes: geometry/connection



Element Geometry by Node Locations

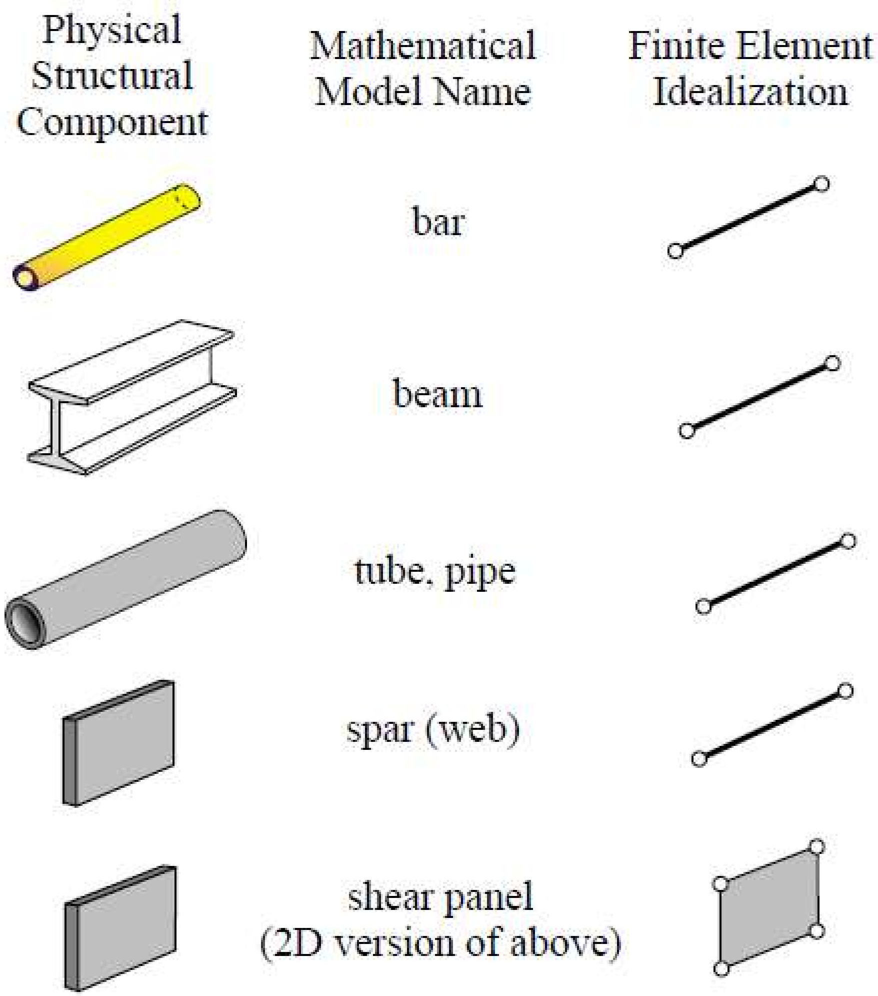
Element Attributes (2)

- Element geometry
  - 1D: straight line, curved segments
  - 2D: triangular or quadrilateral shape
  - 3D: tetrahedra, pentahedra(wedge, prism), hexahedra(brick)
- Element degrees of freedom
- Nodal forces
- Element constitutive properties
  - Linear elastic: elastic modulus E, thermal expansion coeff. α
- Element fabrication properties
  - Cross sectional properties of bar, beam, shaft
  - Thickness: plate, shell

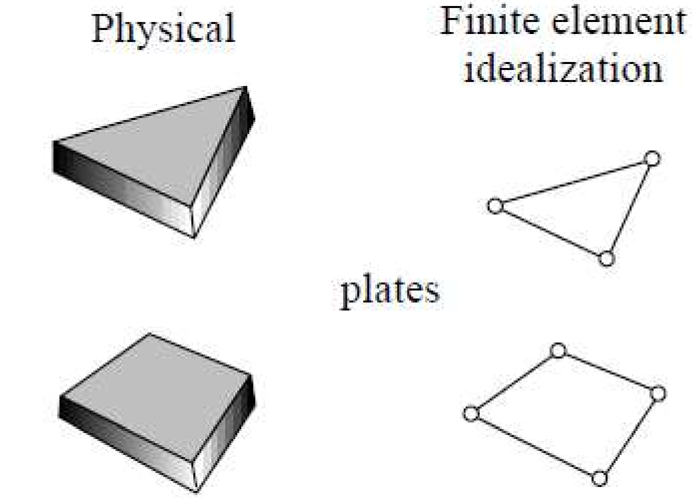
Classification of Mechanical Elements

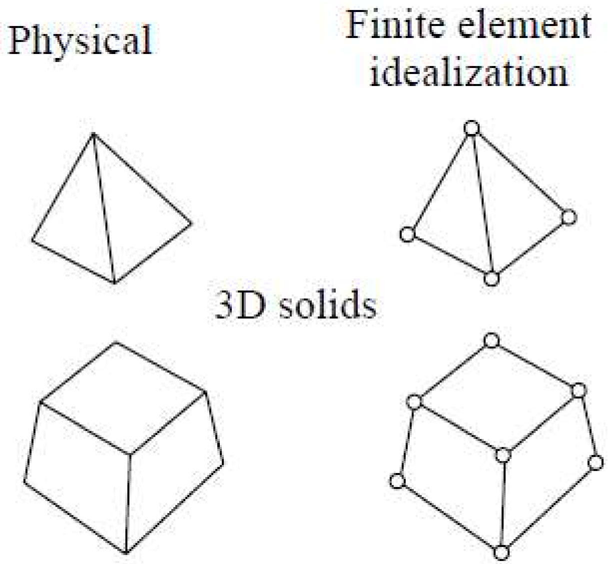
- Primitive structural elements
- Continuum elements
- Special elements
- Macroelements
- Substructures

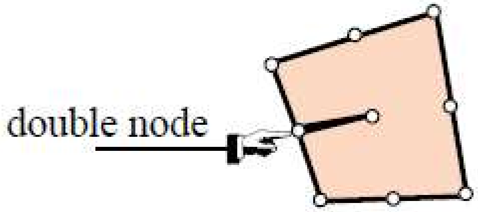
Primitive Structural Elements



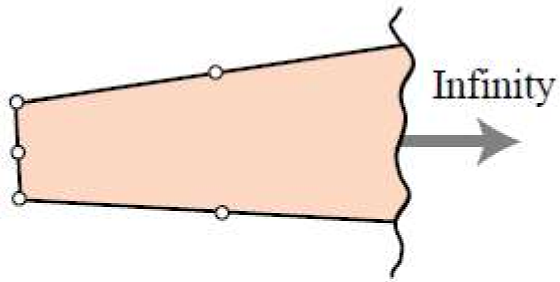
Continuum Element / Special Element



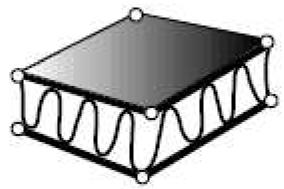




Crack element

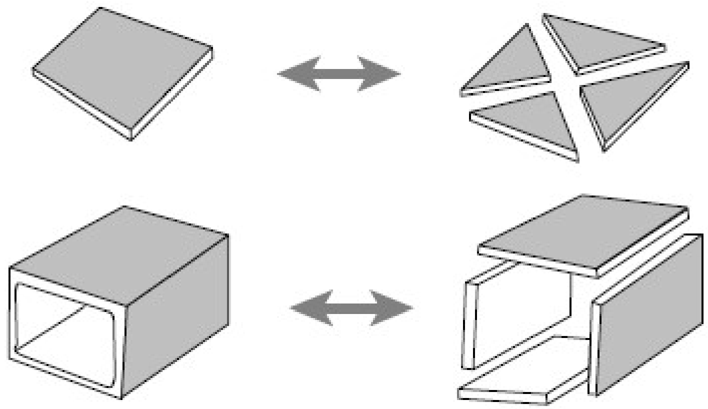


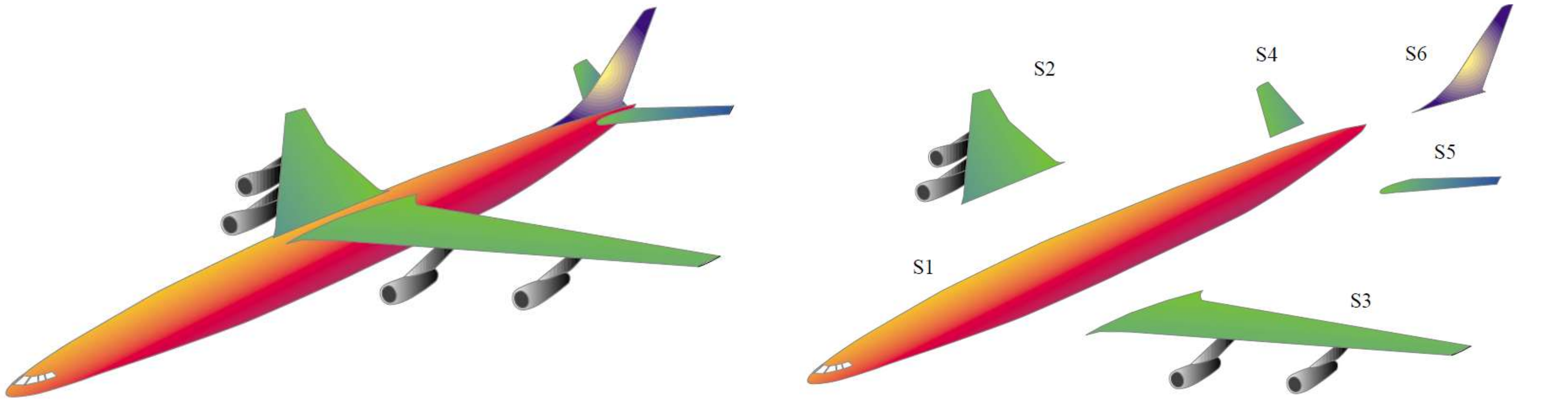
Infinite element



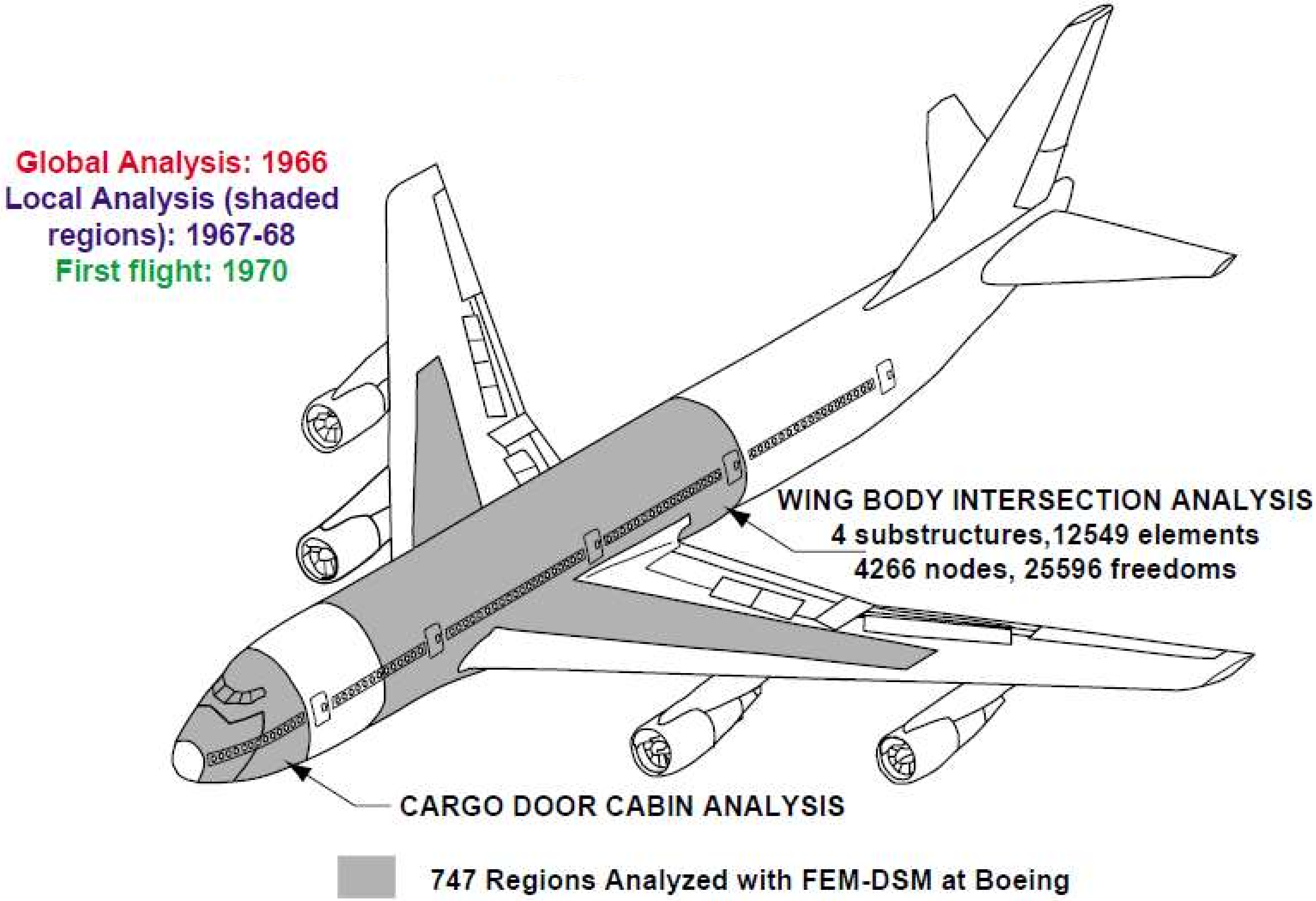
Honeycomb panel

Macroelements / Substructures





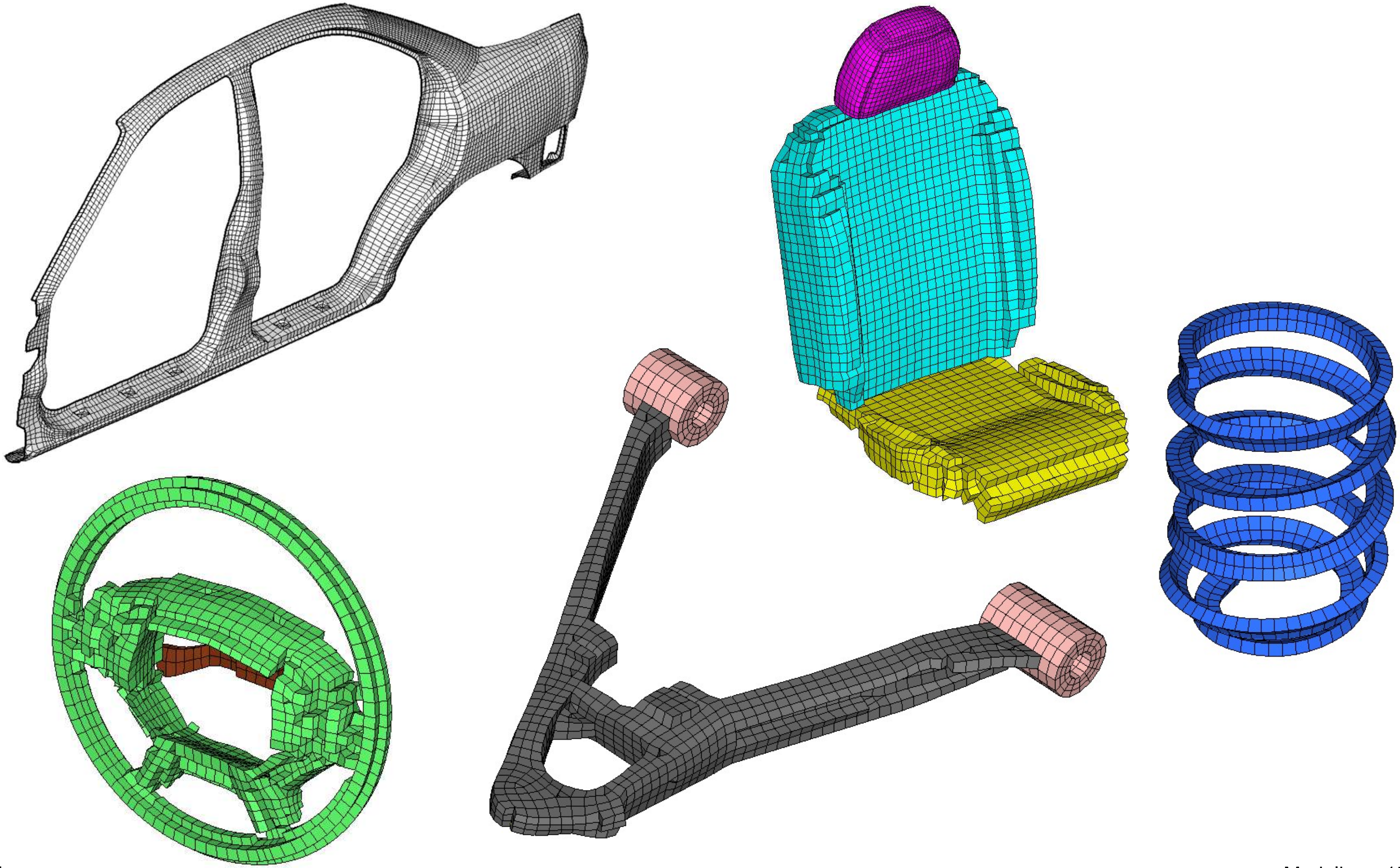
FEM Substructuring: Boeing 747



Boundary Conditions

- The most difficult topic for FEM program users
- The devil hides in the boundary
- Two types: essential vs. natural
  – If a BC involves one or more DOF in a direct way, it is essential and goes to the Left Hand Side (LHS) of Ku=f, such as displacements or rotations
  – Otherwise, it is natural and goes to the Right Hand Side (RHS) of Ku=f

Examples of Structural Models



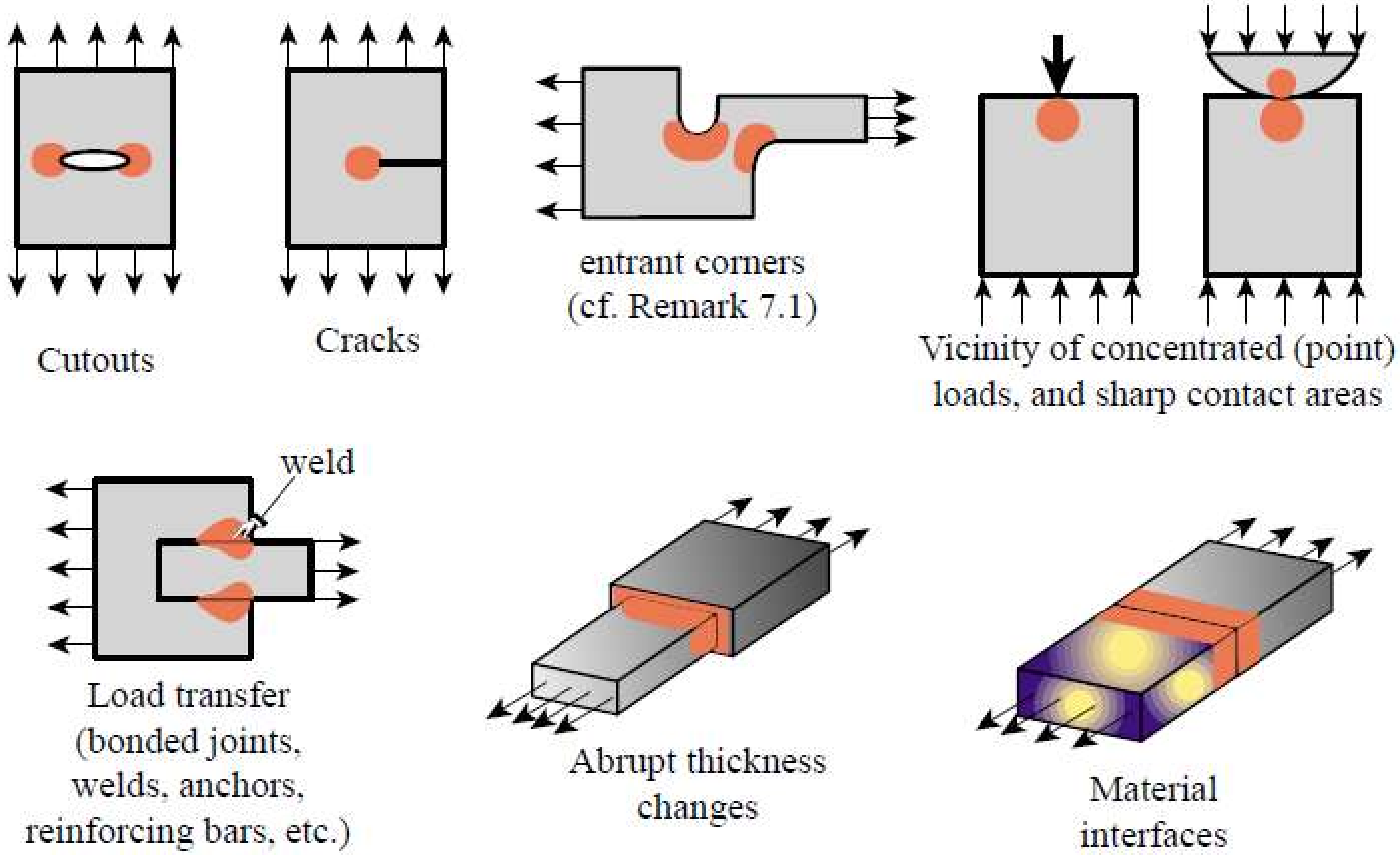
General FEM Modeling Rules

- Keep It Simple
- Use the simplest elements that will do the job
- Never, never, never use complicated or special elements unless you are absolutely sure of what you are doing
- Use the coarsest mesh that will capture the dominant behavior of the physical model, particularly in design situations
  - In product design situations, several FEM models of increasing refinement will be set up as design evolves
  - Ergo, do not overkill at the beginning

Guidelines on Element Layout

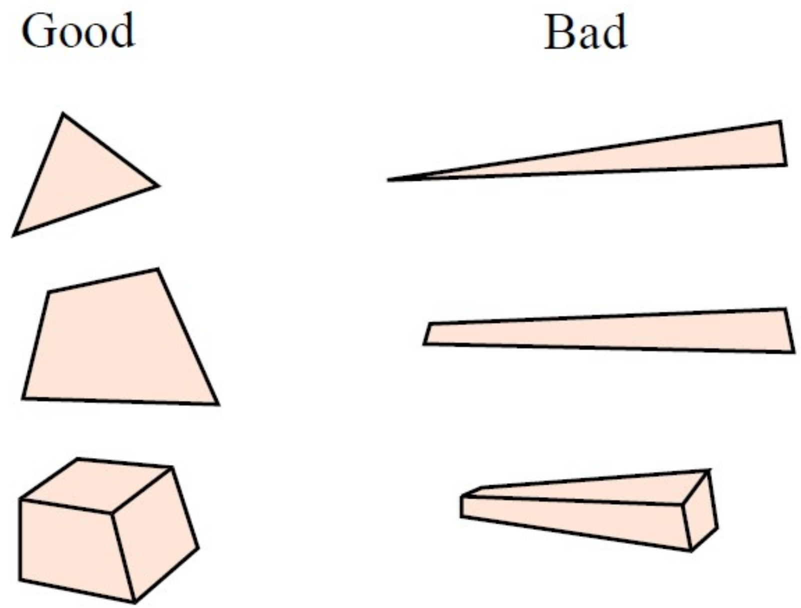
- Mesh Refinement
  - Near entrant corners or sharply curved edges
  - In the vicinity of concentrated (point) loads, concentrated reactions, cracks and cutouts
  - In the interior of structures with abrupt changes in thickness, material properties or cross sectional areas
- Element Aspect Ratios
- Physical Interfaces
- Preferred Shapes
  - Other things being equal, prefer
    - in 2D: Quadrilaterals over Triangles
    - in 3D: Bricks over Wedges, Wedges over Tetrahedra

Where Finer Meshes Should be Used

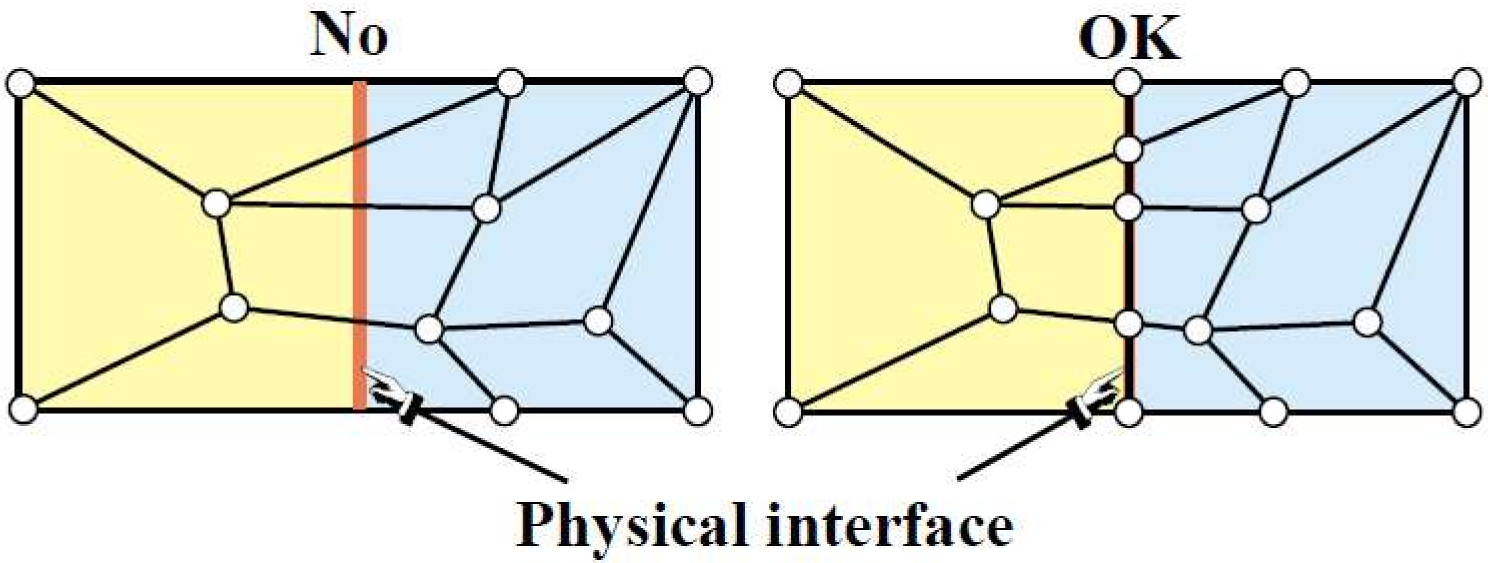


Avoid 2D/3D Elements of Bad Aspect Ratio

- “thin” structures modeled as continuous bodies
  - Elongated or “skinny” element
- Aspect ratio
  - Ratio between its largest and smallest dimension
  - > 3: caution
  - > 10: alarm



Elements Must Not Cross Interfaces

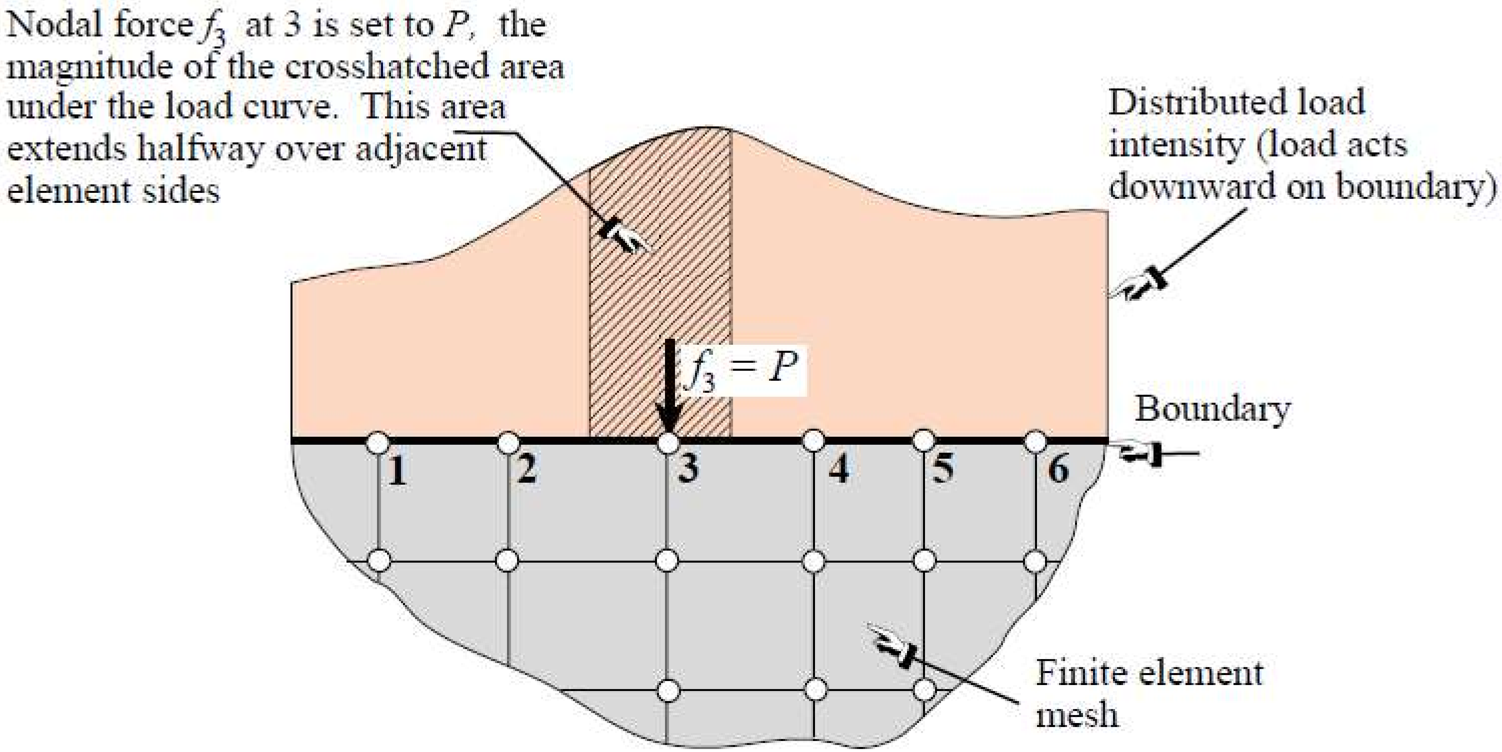


Direct Lumping of Distributed Loads

- Distributed loads vs. concentrated (point) loads

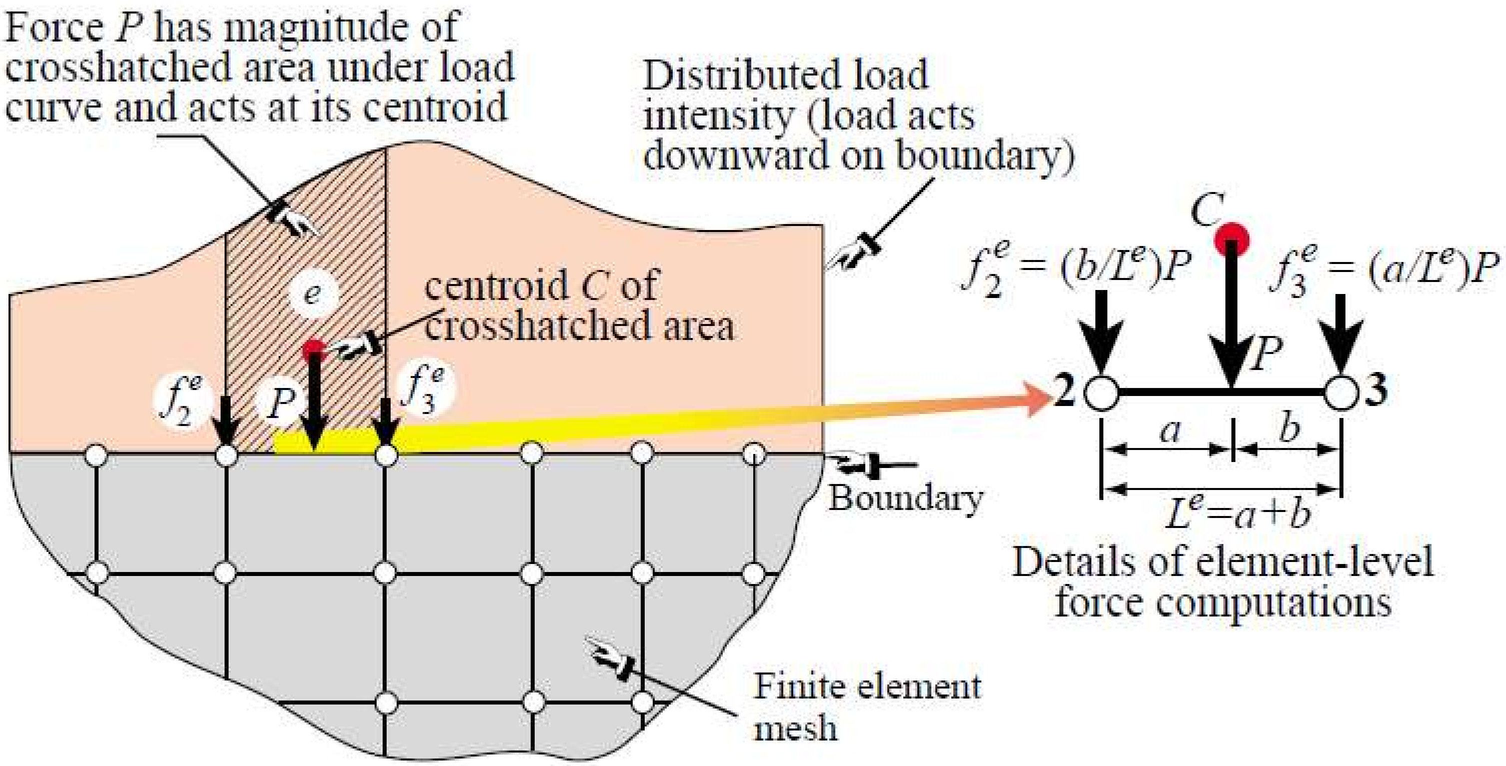
- Distributed surface loads (surface tractions)
  - Wind or water pressure, snow weight on roofs, lift in airplanes, live loads on bridges
- Volume loads (body forces)
  - Own weight(gravity), inertial/centrifugal/thermal/prestress/ electromagnetic effects
- Line loads
  - Integration of surface loads along one transverse direction
  - Integration of volume loads along two transverse directions
- Distributed loads → consistent nodal forces
  - Same external work

Node by Node (NbN) Lumping



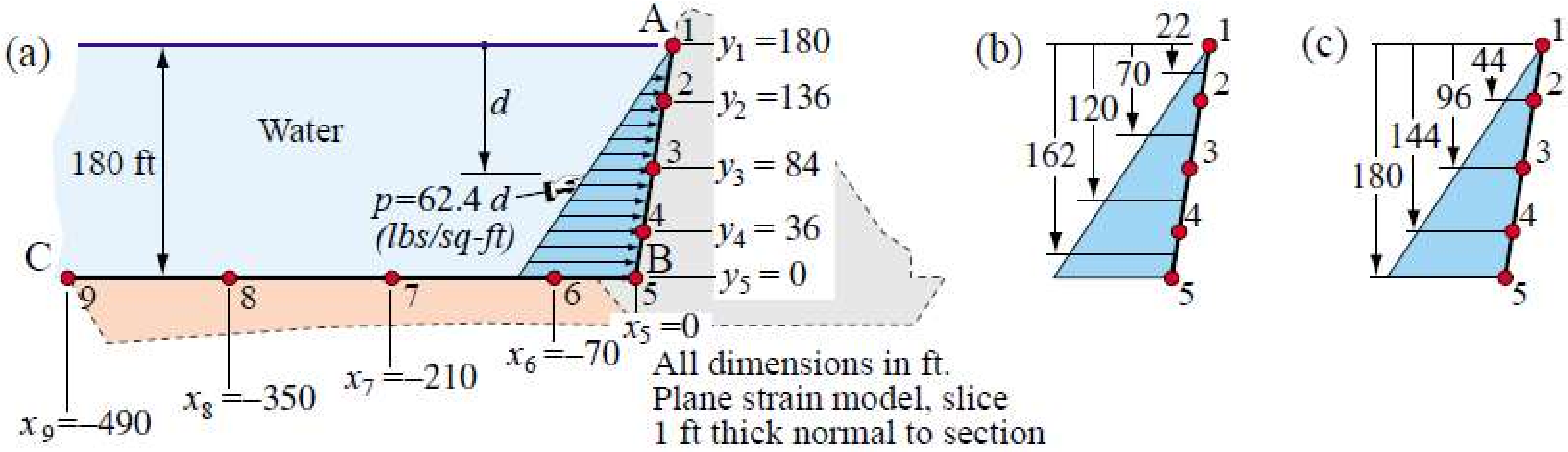
- Tributary (or contributing) region method
  - Not requiring the computation of centroid ☺
  - Rapidly varying applied forces or act only over portions of the tributary regions ☹

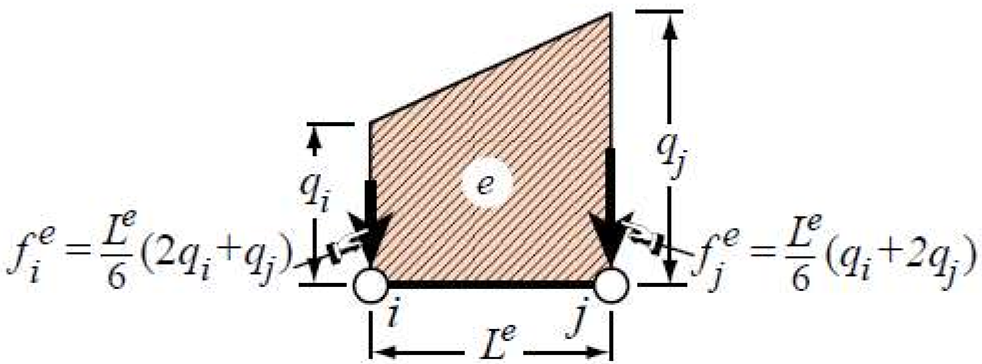
Element by Element (EbE) Lumping



- Resultant load at the centroid → element nodes

Example: Norfork Dam

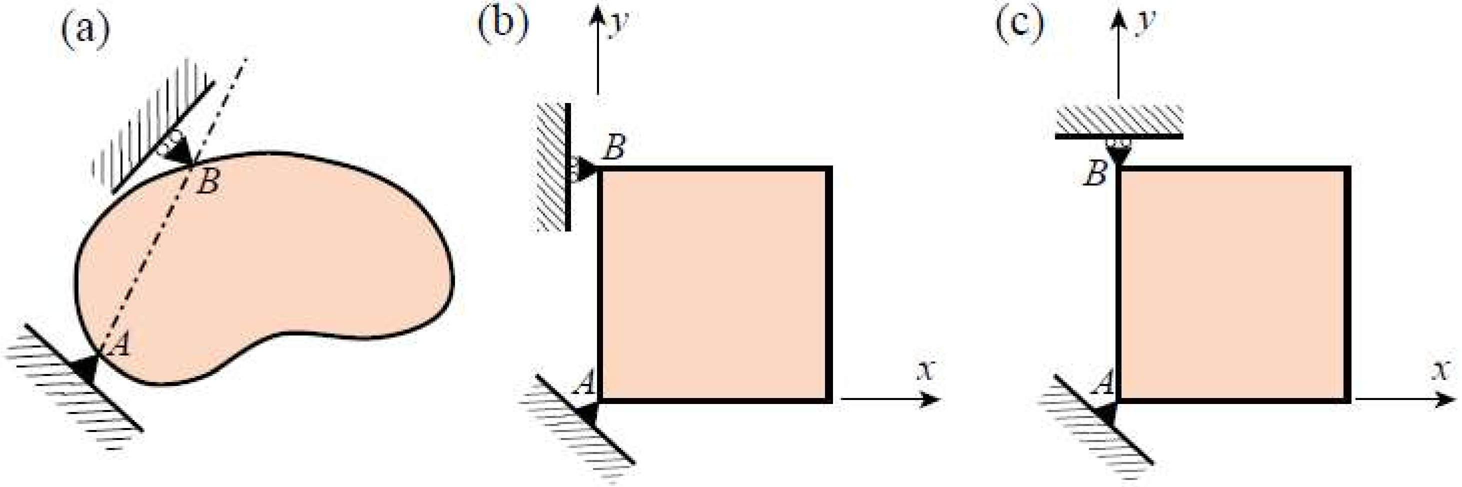


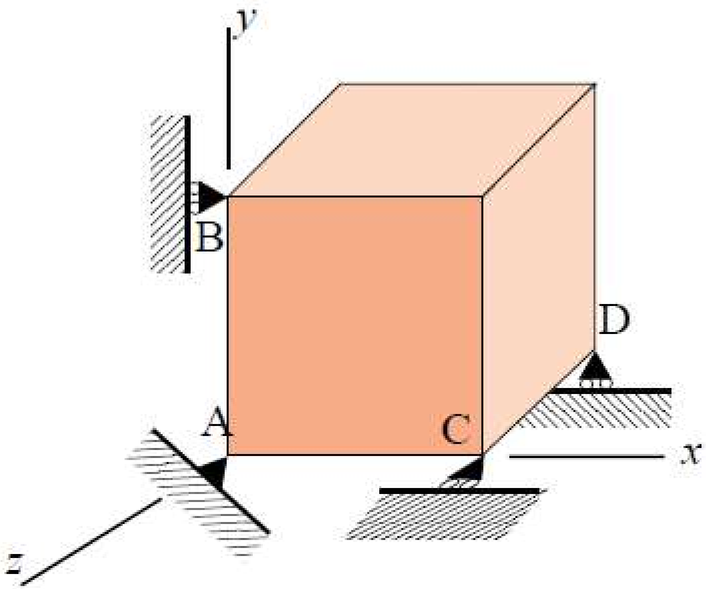


Boundary Conditions

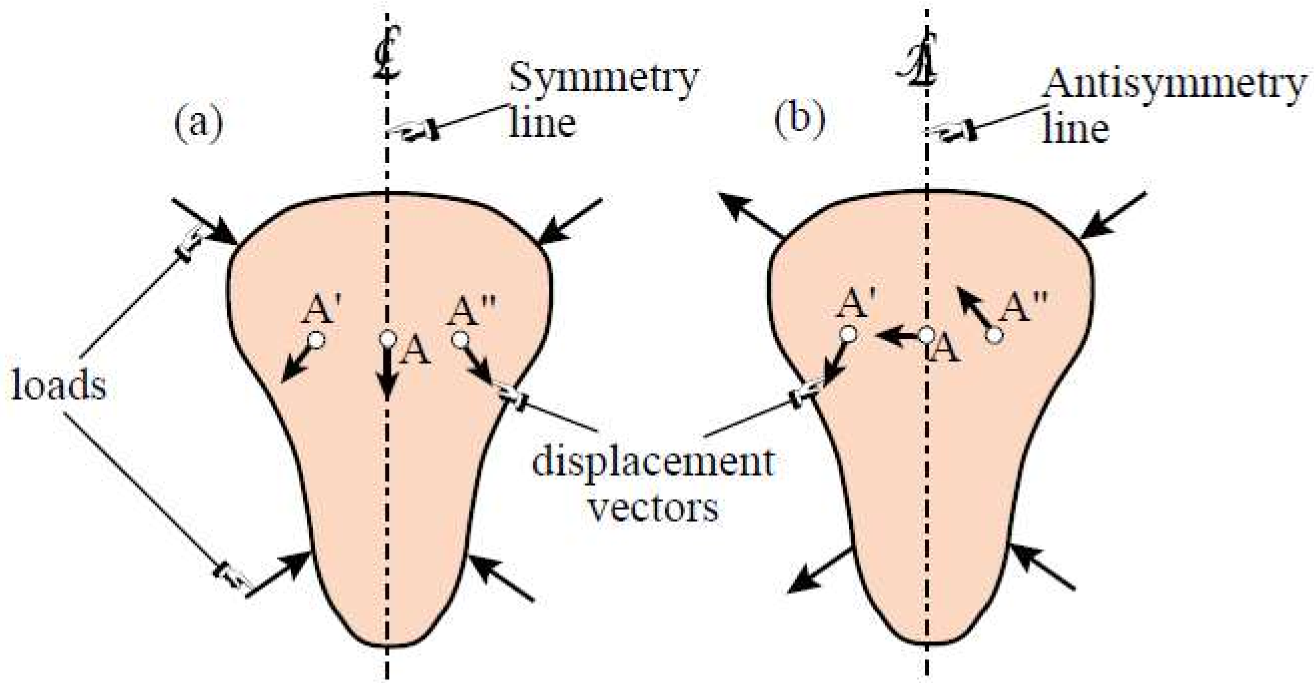
- Essential vs. Natural
  - Directly involve the nodal freedom (displacements or rotations) or not
- Simplest essential BCs
  - Support conditions
    - Restrain structures against relative rigid body motions
    - Attach to earth ground or ground structure
    - Minimum number of constraints: 3 for 2D, 6 for 3D
  - Symmetry/Antisymmetry conditions
    - Recognition by visualization of displacement field or imaging certain rotational/reflection motions
    - Symmetry line(plane): 180° rotation → original problem
    - Antisymmetry line(plane): 180° rotation → original problem but all applied loads are reversed

Minimum Support Conditions to Suppress Rigid Body Motions in 2D/3D

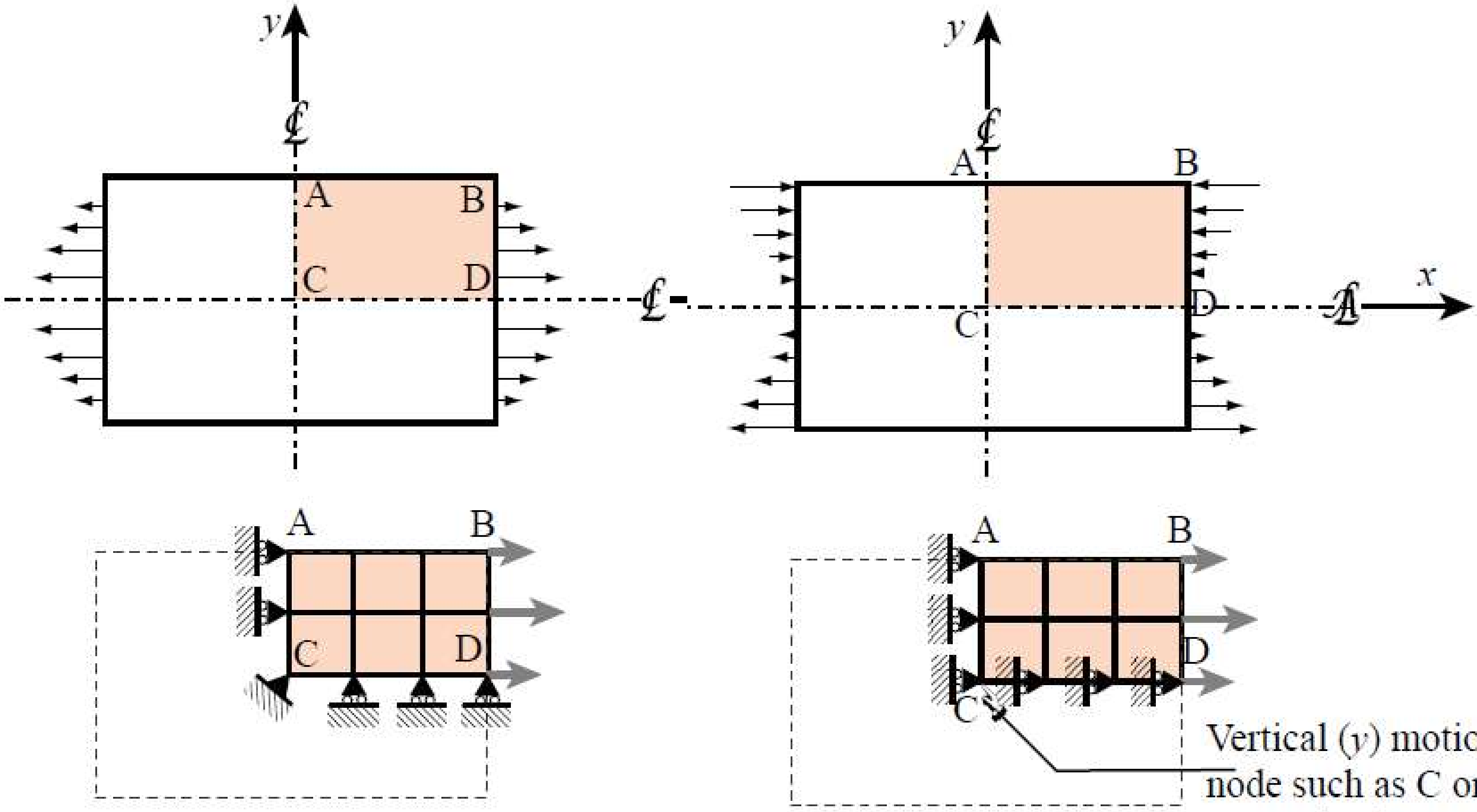




Visualizing Symmetry and Antisymmetry Conditions in 2D

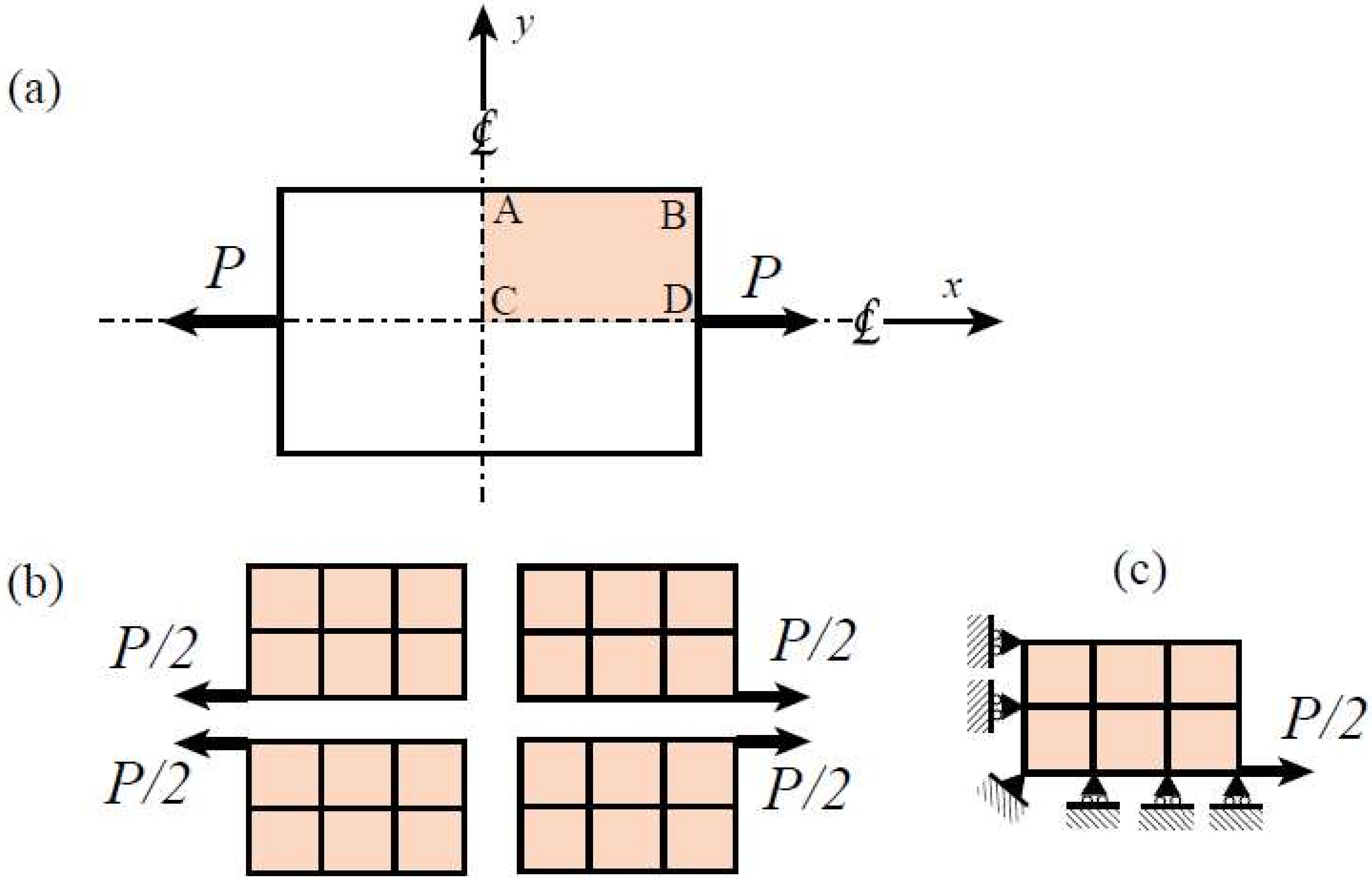


Example of Symmetry/Antisymmetry BCs



Vertical (y) motion of one node such as C or D may be constrained to suppress y-RBM

"Breaking Up" Point Loads at Symmetry BCs



"Breaking Up" Point Loads at Antisymmetry BCs