EXERCISE 14.1 Here is the solution to go from plane strain to plane stress:

```
ClearAll[Em,n,Emstar,nstar]; Efac=Em*(1-n)/((1+n)*(1-2n));
Emat=Efac*{{1,n/(1-n),0},{n/(1-n),1,0},{0,0,(1-2*n)/(2*(1-n))}};
Print["Plane strain E matrix:", Emat//MatrixForm];
EEmat=Simplify[Emat/.{Em->Emstar,n->nstar}];
sol=Solve[{EEmat[[1,1]]==Em/(1-n^2),EEmat[[1,2]]==Em*n/(1-n^2)},
    {Emstar,nstar}]; sol=Simplify[sol];
Print["replacement rule:",sol];
Print["Check - this should be the plane stress E matrix:",
       Simplify[EEmat/.sol[[1]]]//MatrixForm];
```

Plane strain E matrix: $\begin{pmatrix} \frac{\text{Em}}{(1-2\nu)(1+\nu)} & \frac{\text{Em}\,\nu}{(1-2\nu)(1+\nu)} & 0 \\ \frac{\text{Em}\,\nu}{(1-2\nu)(1+\nu)} & \frac{\text{Em}}{(1-2\nu)(1+\nu)} & 0 \\ 0 & 0 & \frac{\text{Em}}{2(1+\nu)} \end{pmatrix}$

replacement rule: $\left\{\left\{\text{Emstar} \to \frac{\text{Em}\,(1+2\nu)}{(1+\nu)^2},\ \text{vstar} \to \frac{\nu}{1+\nu}\right\}\right\}$

Check - this should be the plane stress E matrix: $\begin{pmatrix} \frac{\text{Em}}{1-\nu^2} & \frac{\text{Em}\,\nu}{1-\nu^2} & 0 \\ \frac{\text{Em}\,\nu}{1-\nu^2} & \frac{\text{Em}}{1-\nu^2} & 0 \\ 0 & 0 & \frac{\text{Em}}{2+2\nu} \end{pmatrix}$

FIGURE E14.2. Solution for Exercise 14.1: plane strain to plane stress.

It gives $E^* = E\dfrac{(1+2\nu)}{(1+\nu)^2}$ and $\nu^* = \dfrac{\nu}{1+\nu}$.

Here is to go from plane stress to plane strain:

```
ClearAll[Em,n,Emstar,nstar]; Efac=Em/(1-n^2);
Emat=Simplify[Efac*{{1,n,0},{n,1,0},{0,0,(1-n)/2}}];
Print["Plane stress E matrix:", Emat//MatrixForm];
EEmat=Simplify[Emat/.{Em->Emstar,n->nstar}];
sol=Solve[{EEmat[[1,1]]==Em*(1-n)/((1+n)*(1-2n)),
          EEmat[[1,2]]==Em*n/((1+n)*(1-2n))},
          {Emstar,nstar}]; sol=Simplify[sol];
Print["replacement rule:",sol];
Print["Check - this should be the plane strain E matrix:",
       Simplify[EEmat/.sol[[1]]]//MatrixForm];
```

Plane stress E matrix: $\begin{pmatrix} \frac{\text{Em}}{1-n^2} & \frac{\text{Em}\,n}{1-n^2} & 0 \\ \frac{\text{Em}\,n}{1-n^2} & \frac{\text{Em}}{1-n^2} & 0 \\ 0 & 0 & \frac{\text{Em}}{2+2n} \end{pmatrix}$

replacement rule:99Emstar $\frac{\text{Em}}{1-n^2}$, nstar $-\frac{n}{-1+n}$ ==

Check - this should be the plane strain E matrix: $\begin{pmatrix} \frac{\text{Em}\,H\text{-}1+nL}{-1+n+2n^2} & -\frac{\text{Em}\,n}{-1+n+2n^2} & 0 \\ -\frac{\text{Em}\,n}{-1+n+2n^2} & \frac{\text{Em}\,H\text{-}1+nL}{-1+n+2n^2} & 0 \\ 0 & 0 & \frac{\text{Em}}{2+2n} \end{pmatrix}$

It gives $E^* = \dfrac{E}{1-\nu^2}$ and $\nu^* = \dfrac{\nu}{1-\nu}$. Credit is given for doing it either way.

EXERCISE 15.4

$$f_{x1} = \frac{1}{6} h\, L_{21}(2q_{x1} + q_{x2}), \quad f_{x2} = \frac{1}{6} h\, L_{21}(q_{x1} + 2q_{x2}),$$
$$f_{y1} = \frac{1}{6} h\, L_{21}(2q_{y1} + q_{y2}), \quad f_{y2} = \frac{1}{6} h\, L_{21}(q_{y1} + 2q_{y2}), \quad f_{x3} = f_{y3} = 0.$$

Here $L_{21} = \sqrt{x_{21}^2 + y_{21}^2}$ is the length of side 1-2.