Ch.4 Design for Body Bending

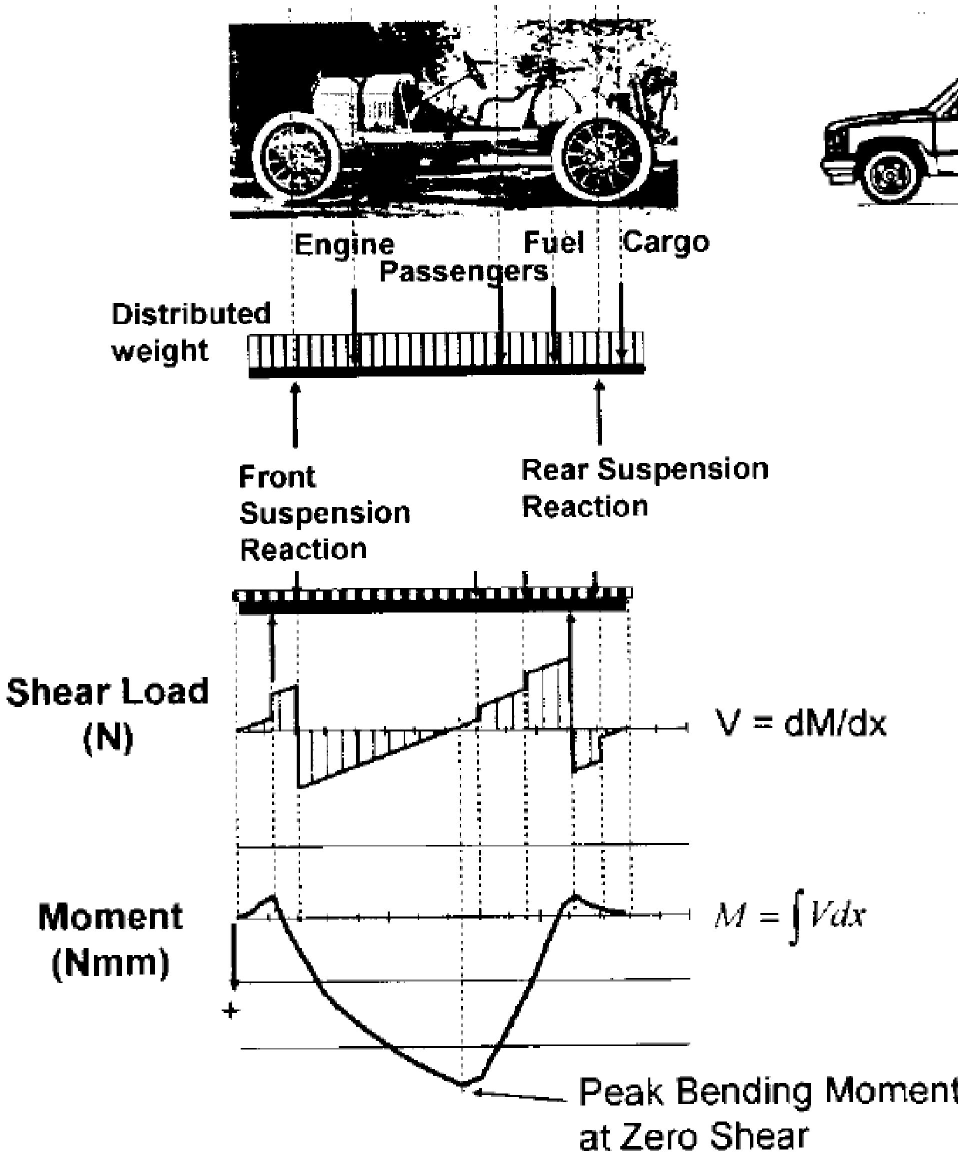
- Consider the overall body structure supported and loaded similar to a single beam
- Symmetrical supporting points and loads applied to the vehicle center line

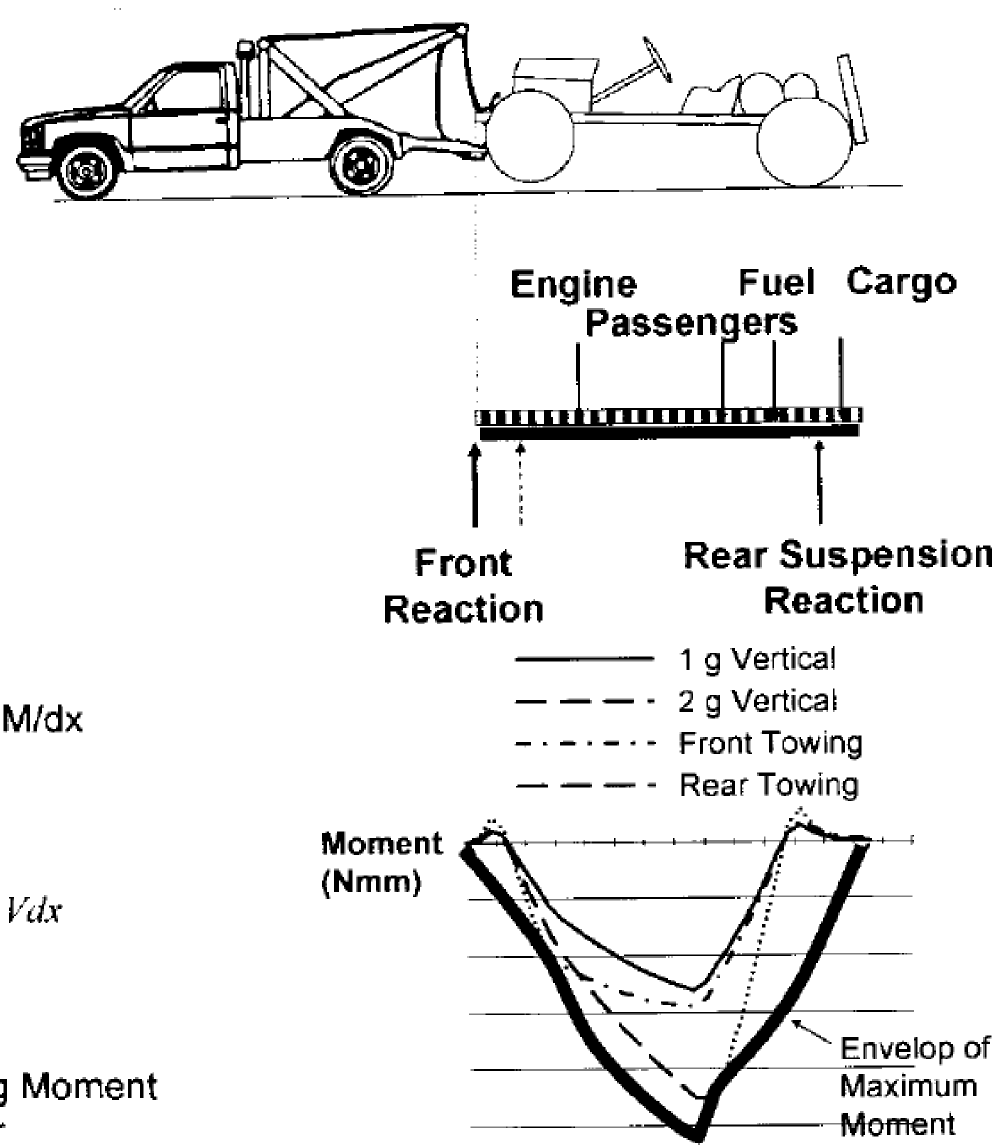
- Body bending requirements
  - For midsize vehicle: K = 7000 N/mm, F = 6680 N
- Internal loads during global bending: load path analysis
- Analysis of body bending stiffness
- Principles of good joint design

4.1 Body Bending Strength Requirement

- Most basic structure requirement
  - Locate and retain the vehicle subsystems in correct positions
  - Support powertrain, occupants, suspension, etc.
- Envelop of maximum bending moments: NO failure
  - Static weight loading
  - Dynamic loading: inertia loads of the subsystem mass (x 2g)
  - Front/Rear jacking or towing: one support point is moved to an end of the vehicle

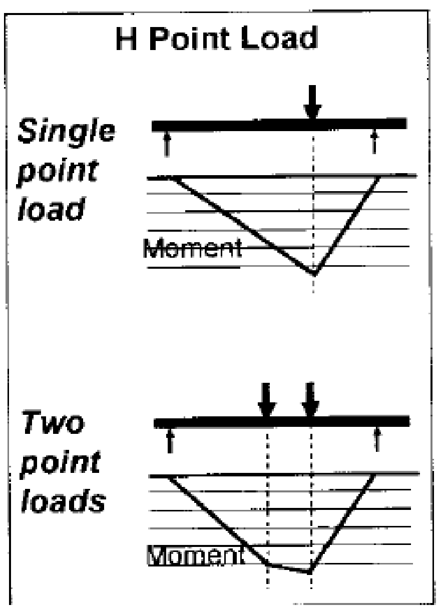
Body Loaded by Subsystem Weight

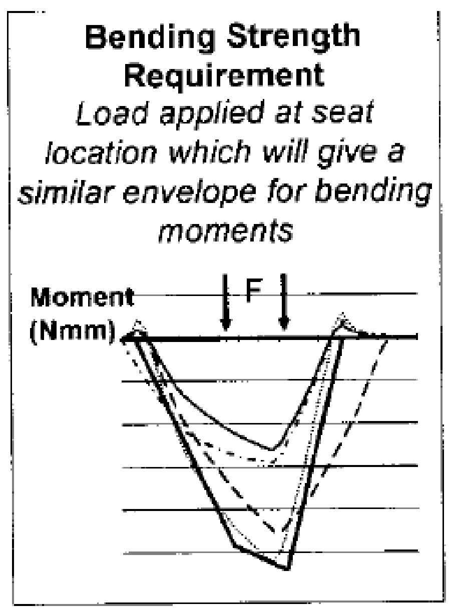


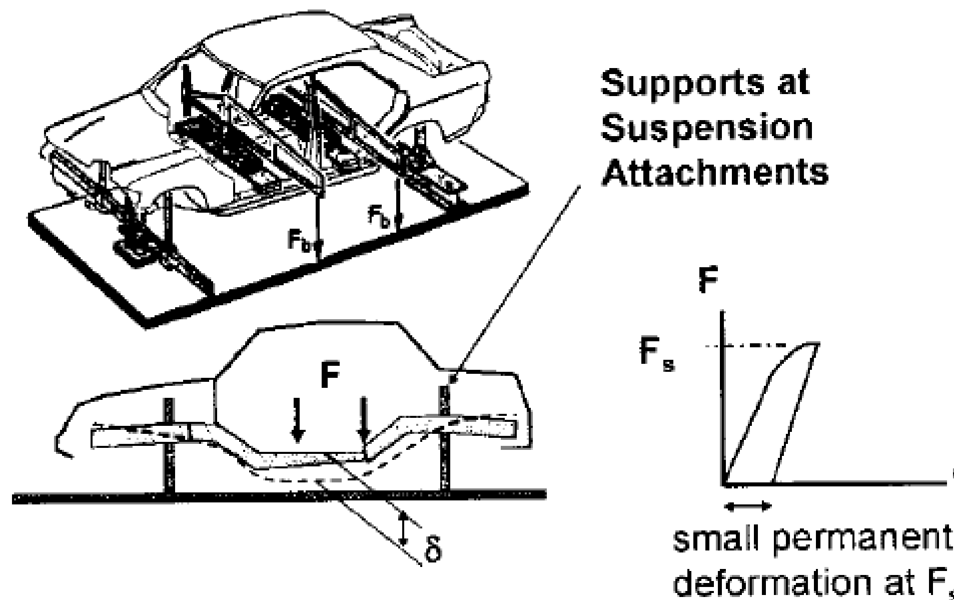


H point Bending Test

- Consider body supported at the suspension points and loaded by just one or two loads at the seating position (H point)
- Superimpose the diagram over bending moments for the vehicle
- Vary the magnitude of the H point load
- F_s: bending strength for the body



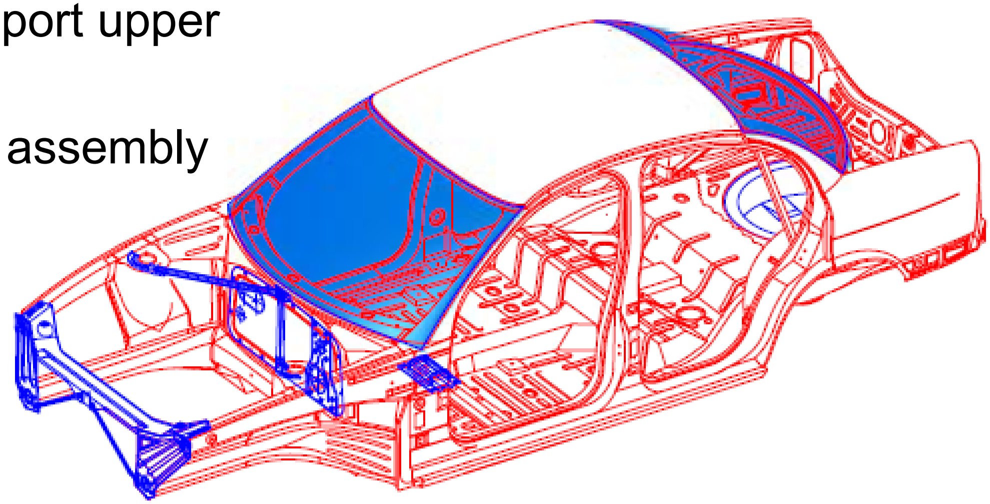




Test: Configuration

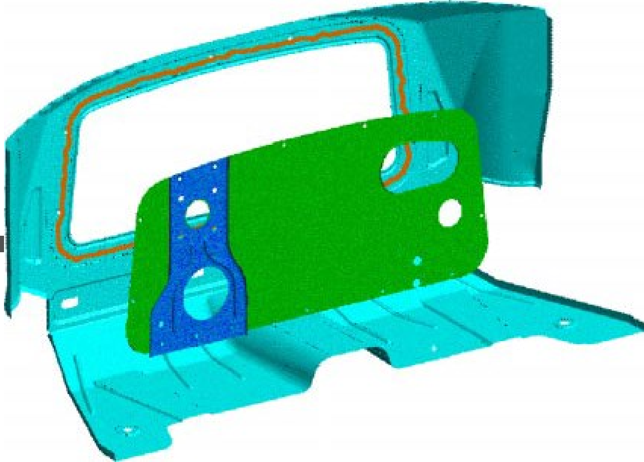
- Welded body structure
- Bonded windshield and back light (aluminum panels)
- Bonded and bolted panel dash insert
- Bonded panel spare tire tub
- Bolted reinforcement panel dash brake booster
- Bolted braces radiator
- Bolted reinforcement radiator rail closeout RH/LH
- Bolted reinforcement radiator support upper
- Bolted tunnel bridge lower/upper
- Bolted brace cowl to shock tower assembly

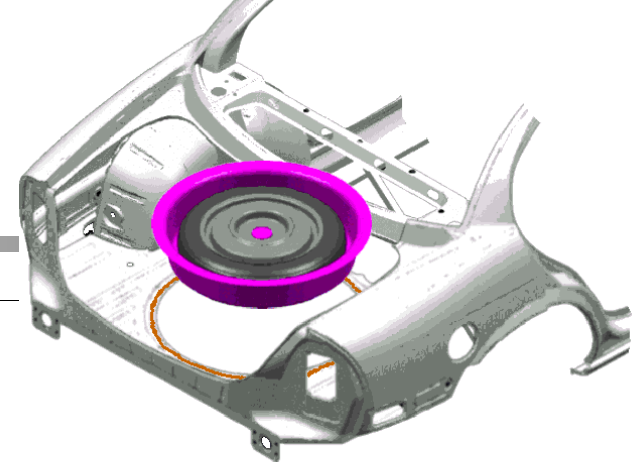
– Holding
  - Front: at panel skirt RH/LH
  - Rear: at plate rear spring upper

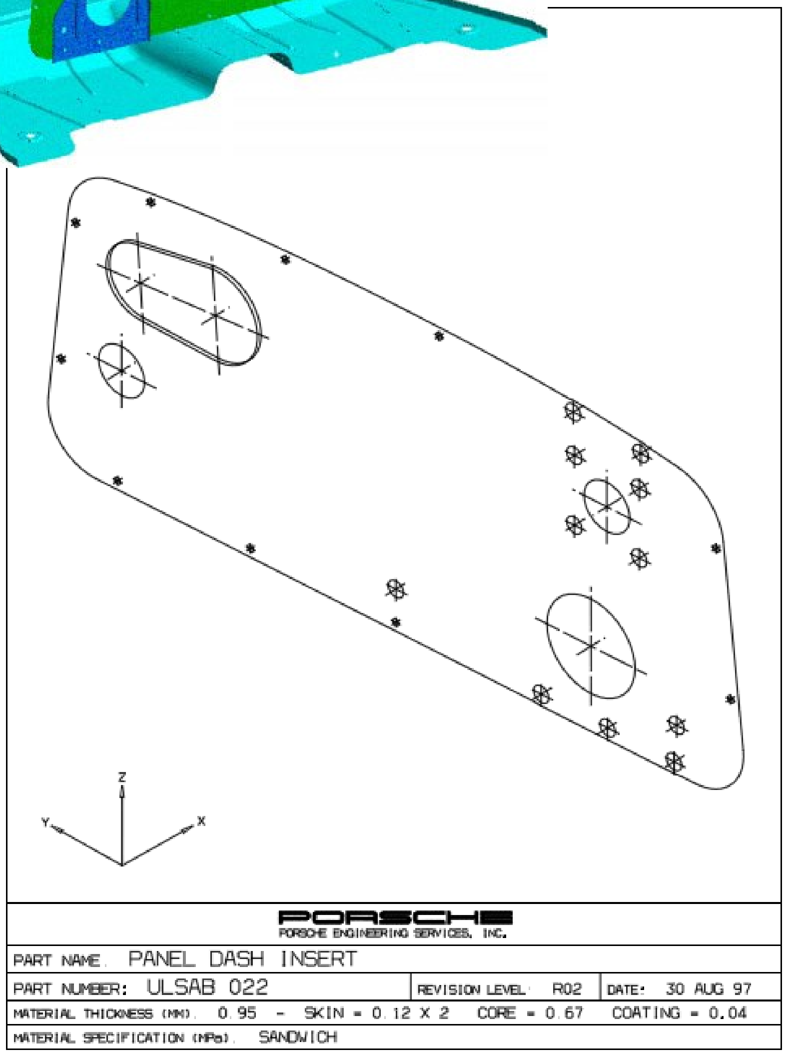


– Measurement
  - 12 stadia rods along the front rails, rockers, rear rails

Parts (1)



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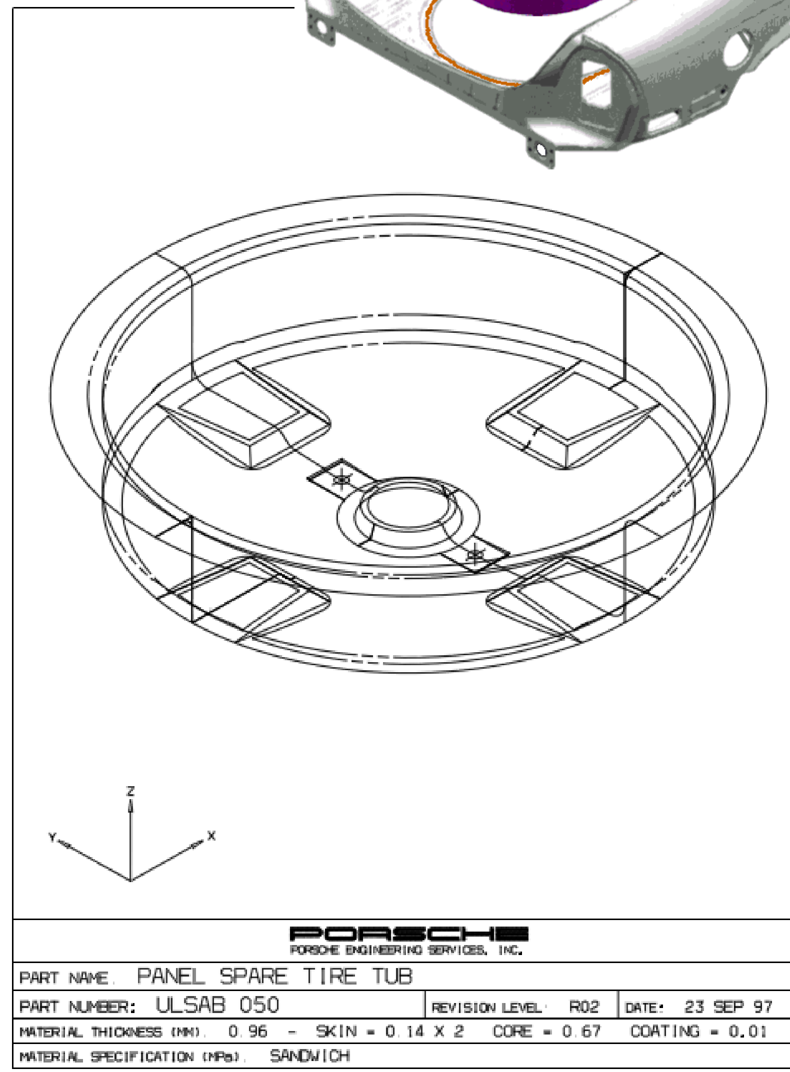
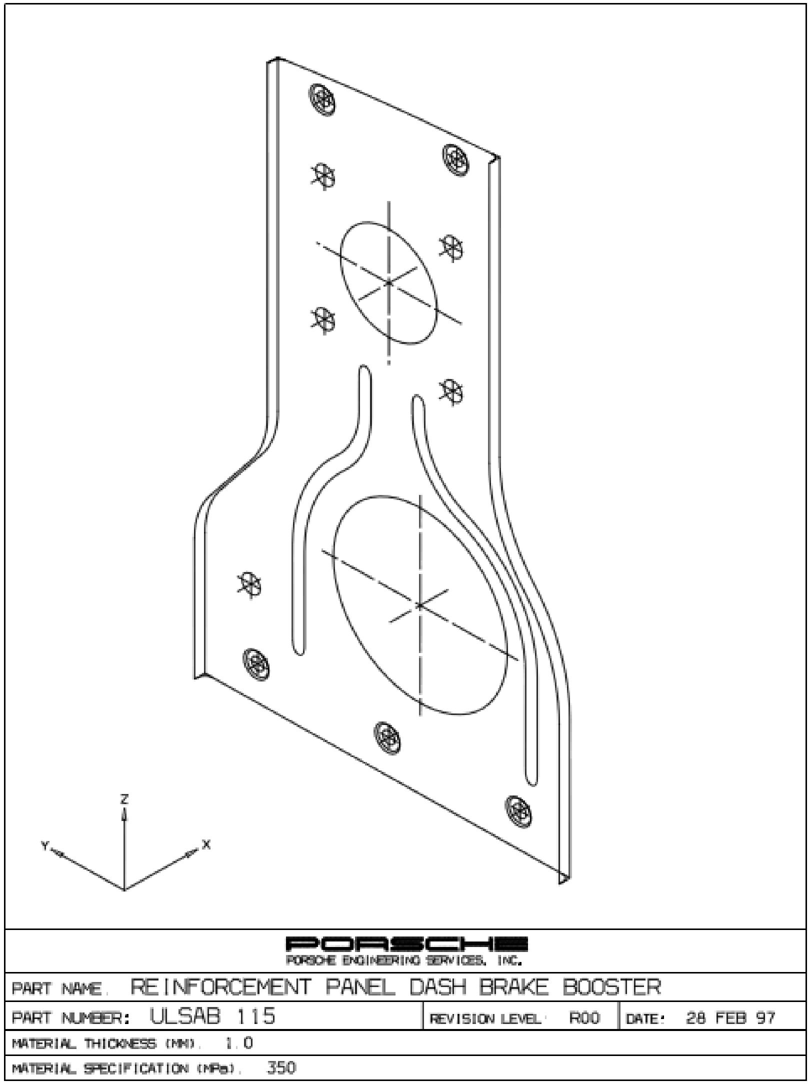
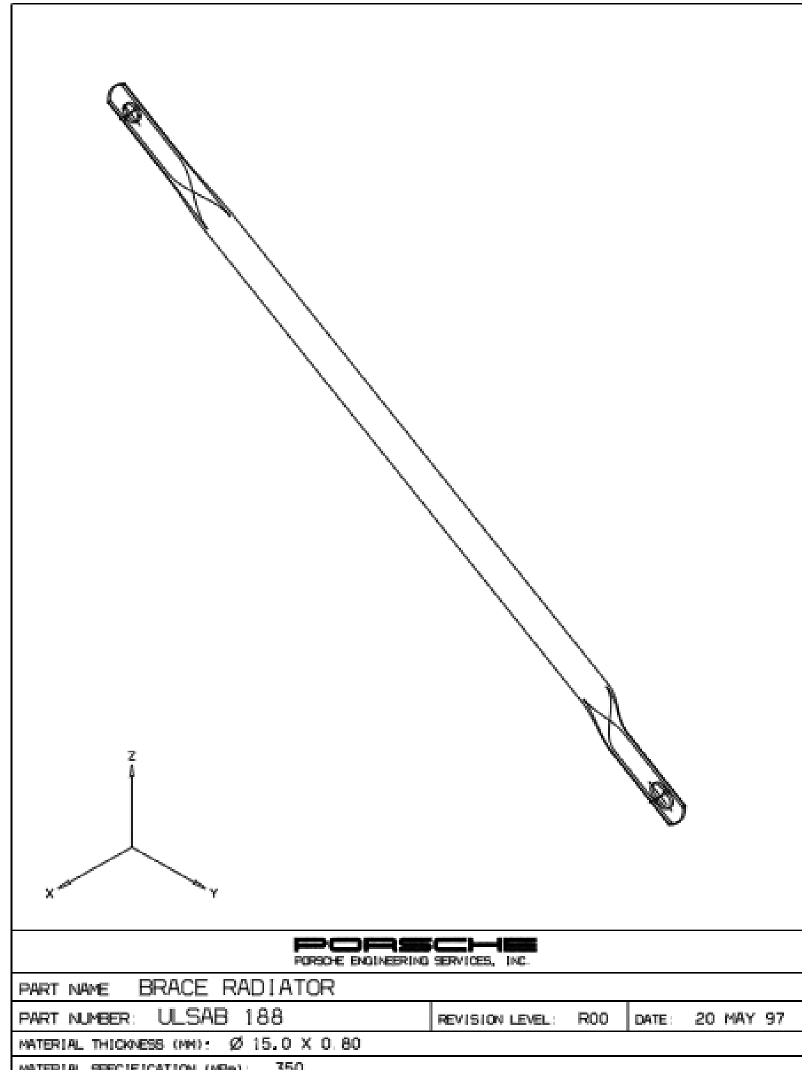
Y
X
PORSCHE
PORSCHE ENGINEERING SERVICES, INC.
PART NAME: PANEL DASH INSERT
PART NUMBER: ULSAB 022
REVISION LEVEL: R02
DATE: 30 AUG 97
MATERIAL THICKNESS (MM): 0.95 - SKIN = 0.12 X 2 CORE = 0.67 COATING = 0.04
MATERIAL SPECIFICATION (MPa): SANDWICH


Z
Y
X
PORSCHE
PORSCHE ENGINEERING SERVICES, INC.
PART NAME: PANEL SPARE TIRE TUB
PART NUMBER: ULSAB 050
REVISION LEVEL: R02
DATE: 23 SEP 97
MATERIAL THICKNESS (MM): 0.96 - SKIN = 0.14 X 2 CORE = 0.67 COATING = 0.01
MATERIAL SPECIFICATION (MPa): SANDWICH

Parts (2)

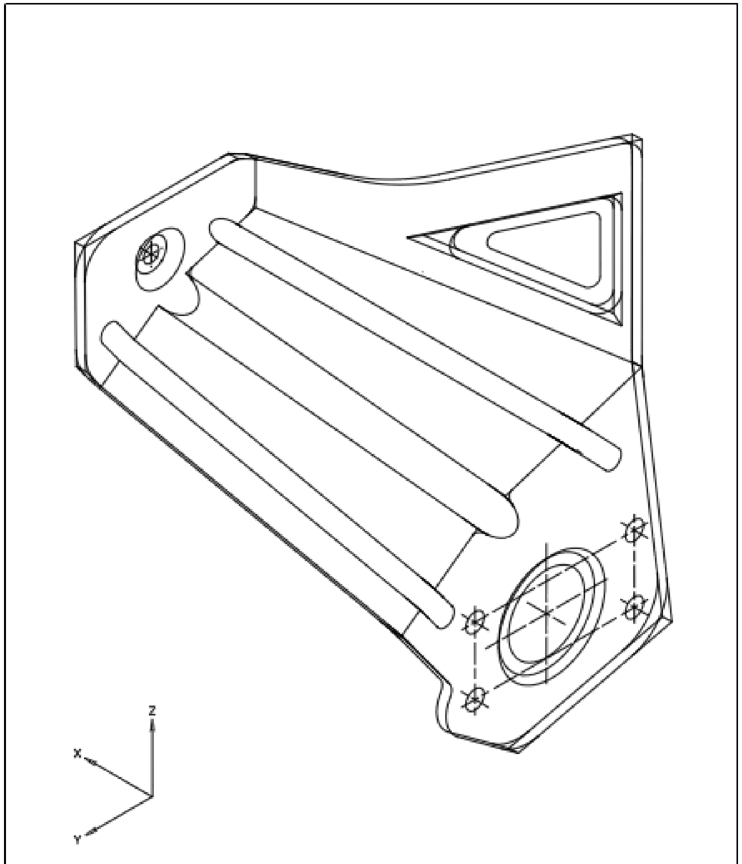


Z
Y
X
PORSCHE
PORSCHE ENGINEERING SERVICES, INC.
PART NAME: REINFORCEMENT PANEL DASH BRAKE BOOSTER
PART NUMBER: ULSAB 115
REVISION LEVEL: R00
DATE: 28 FEB 97
MATERIAL THICKNESS (MM): 1.0
MATERIAL SPECIFICATION (MPa): 350

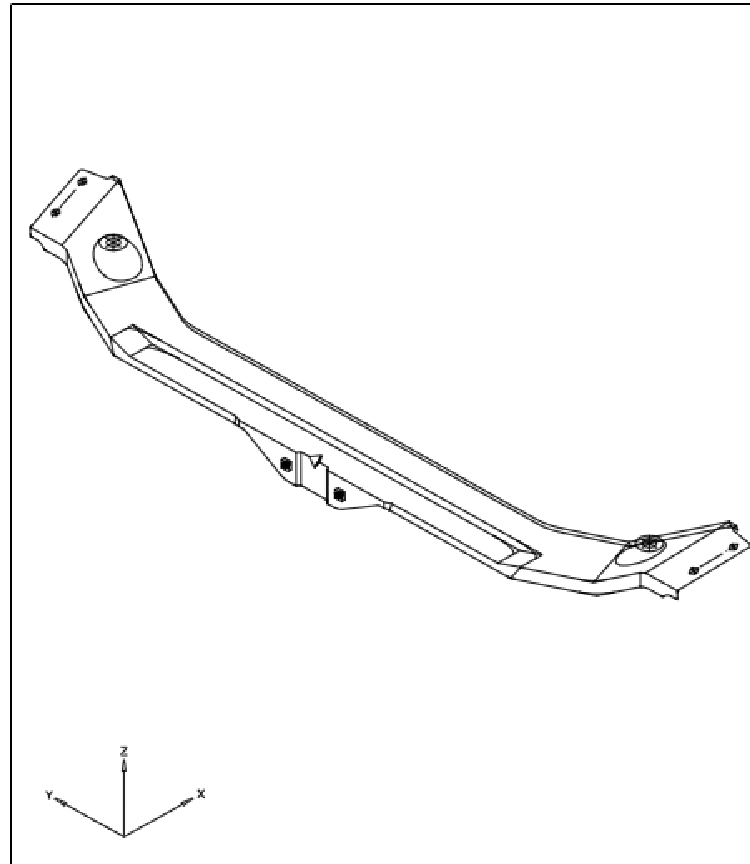


Z
X
Y
PORSCHE
PORSCHE ENGINEERING SERVICES, INC.
PART NAME: BRACE RADIATOR
PART NUMBER: ULSAB 188
REVISION LEVEL: R00
DATE: 20 MAY 97
MATERIAL THICKNESS (MM): Ø 15.0 X 0.80
MATERIAL SPECIFICATION (MPa): 350

Parts (3)

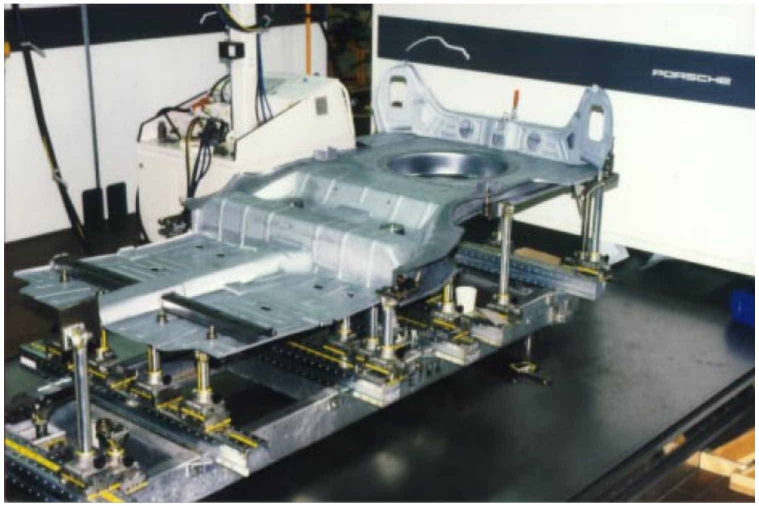


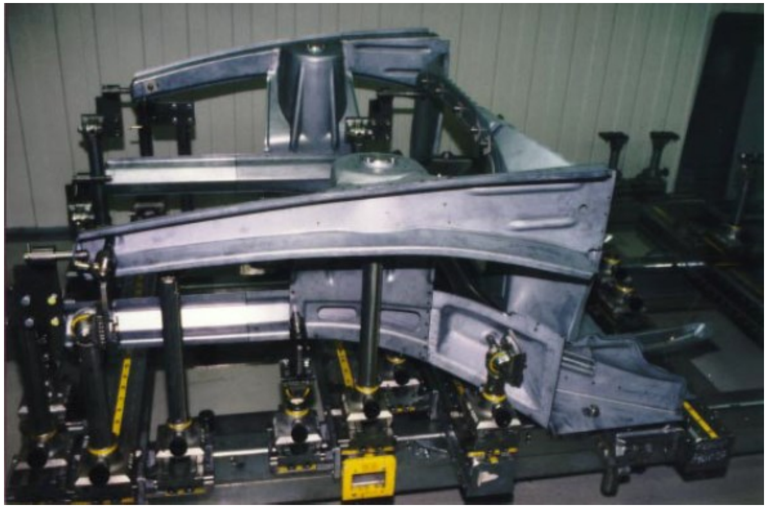
PORSCHE ENGINEERING SERVICES, INC.			
PART NAME: REINFORCEMENT RADIATOR RAIL CLOSEOUT			
PART NUMBER: ULSAB 094 / 095	REVISION LEVEL: R02	DATE: 01 DEC 97	
MATERIAL THICKNESS (MM): 1.00			
MATERIAL SPECIFICATION (MPa): 350			



PORSCHE ENGINEERING SERVICES, INC.			
PART NAME ASSY REINFORCEMENT RADIATOR SUPPORT UPPER			
PART NUMBER: ULSAB 001	REVISION LEVEL: R02	DATE: 30 AUG 97	
MATERIAL THICKNESS (MM): 1.00			
MATERIAL SPECIFICATION (MPa): 350			

Underbody





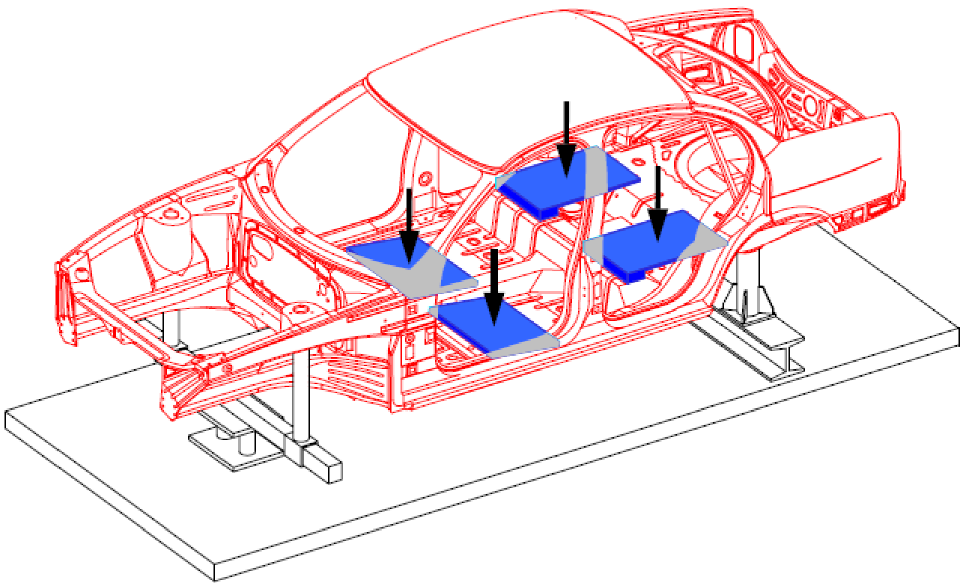
Rear floor

Frond End

Body Side Outer/Inner + Underbody

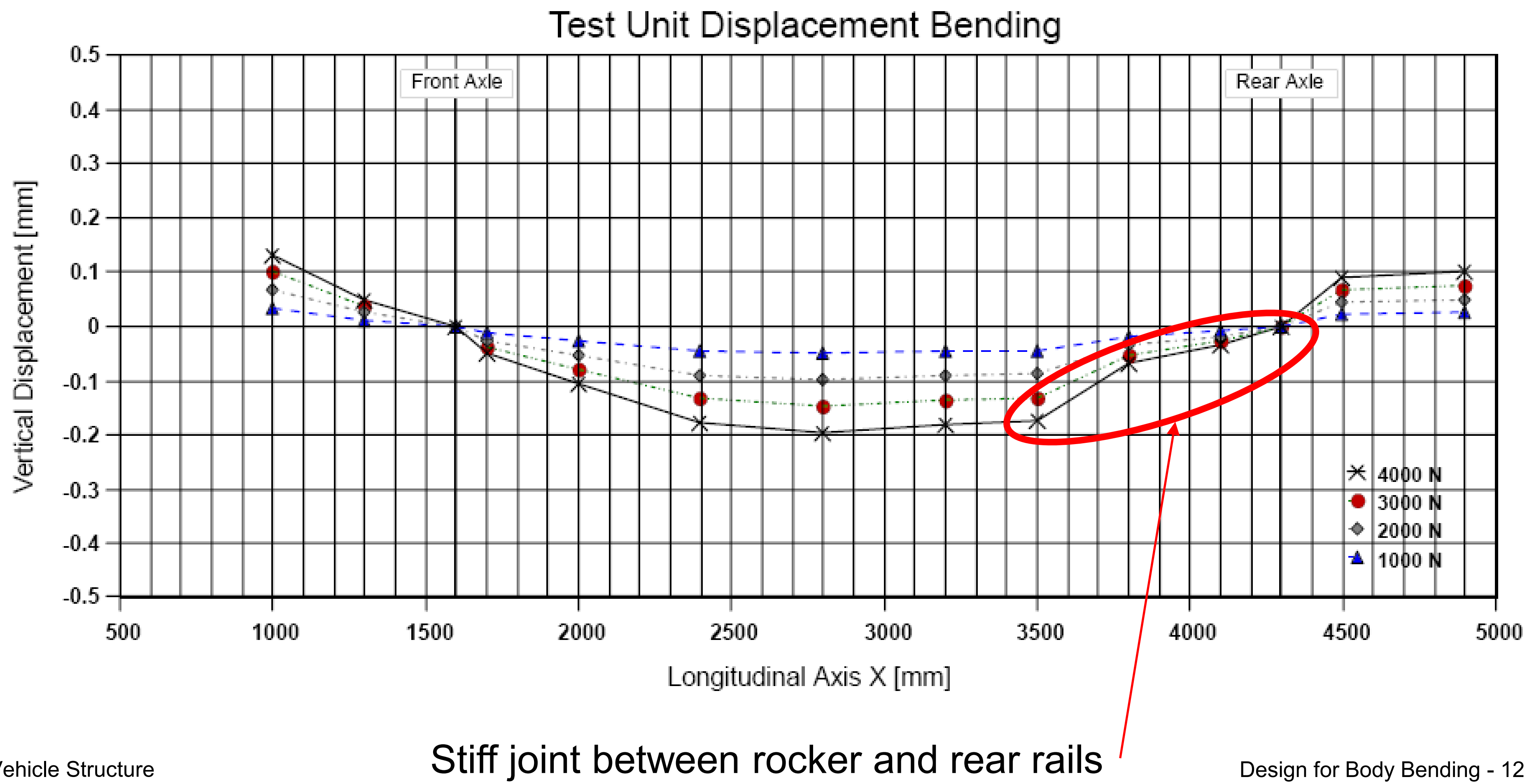
Test: Static Bending

– Constraint: four fixing points
– Load: center of the front seats and center of the two outer rear seats, from F = 1000N (4X250N) to 4000N (4X1000N)



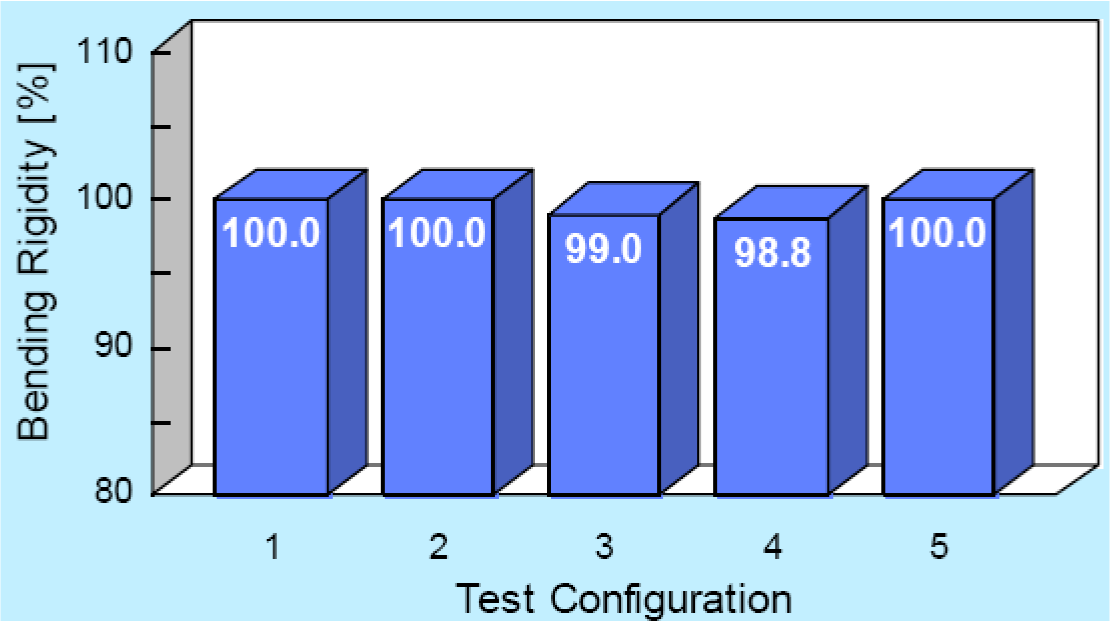
Test: Results for Static Bending

- With glass: 20,460 N/mm
- Without glass: 17,150 N/mm



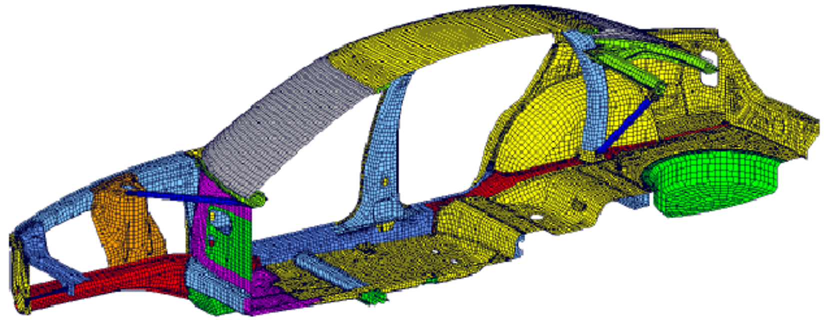
Static Bending: impact of bonded and/or bolted parts

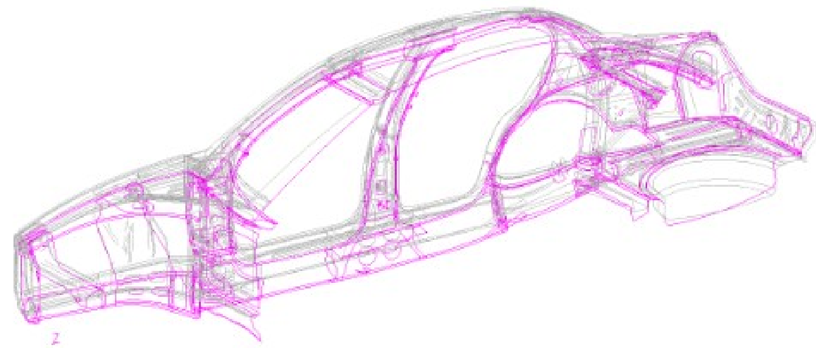
– 1. full configuration
– 2. as 1, but without braces radiator
– 3. as 2, but without radiator support upper
– 4. as 3, but without bolted brace cowl to shock tower assembly
– 5. as 4, but without tunnel bridge

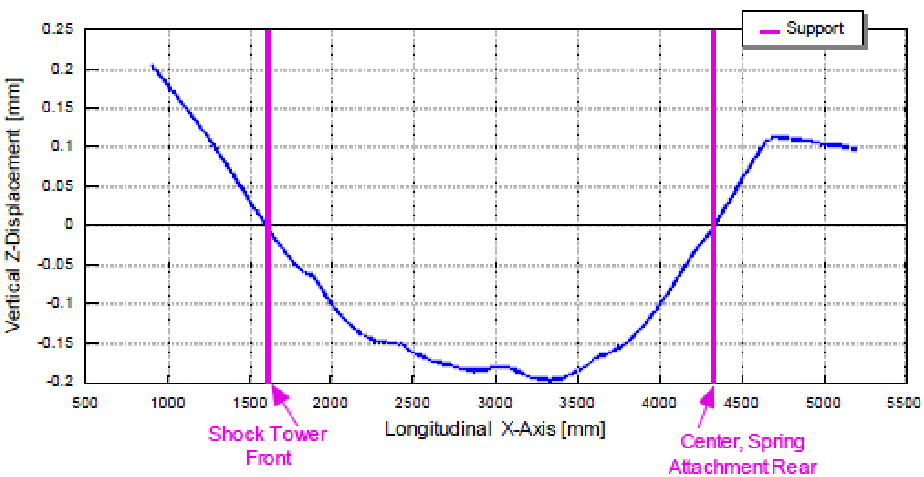


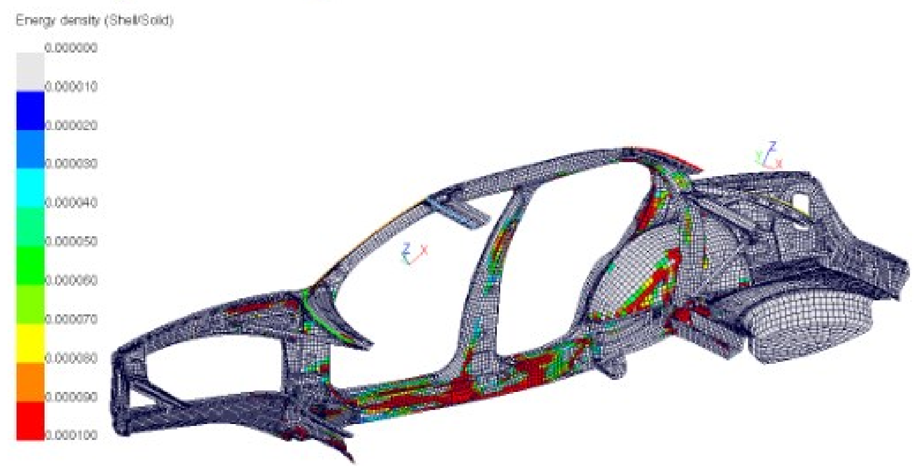
CAE Analysis

- Half model
  - 54,521 shell elements
  - 53,460 nodes
- Static bending stiffness
  - 20,540 N/mm



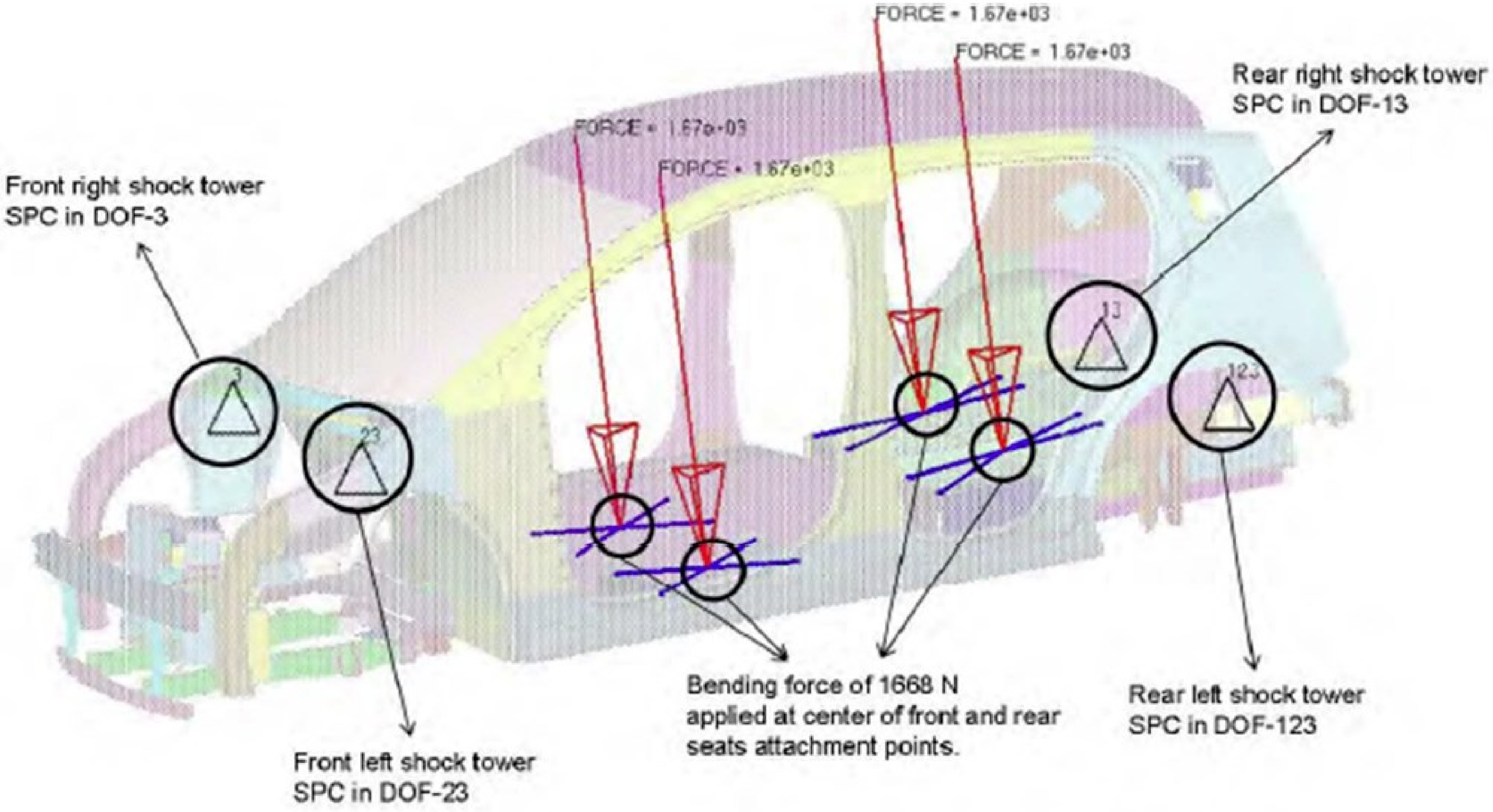






Bending Stiffness

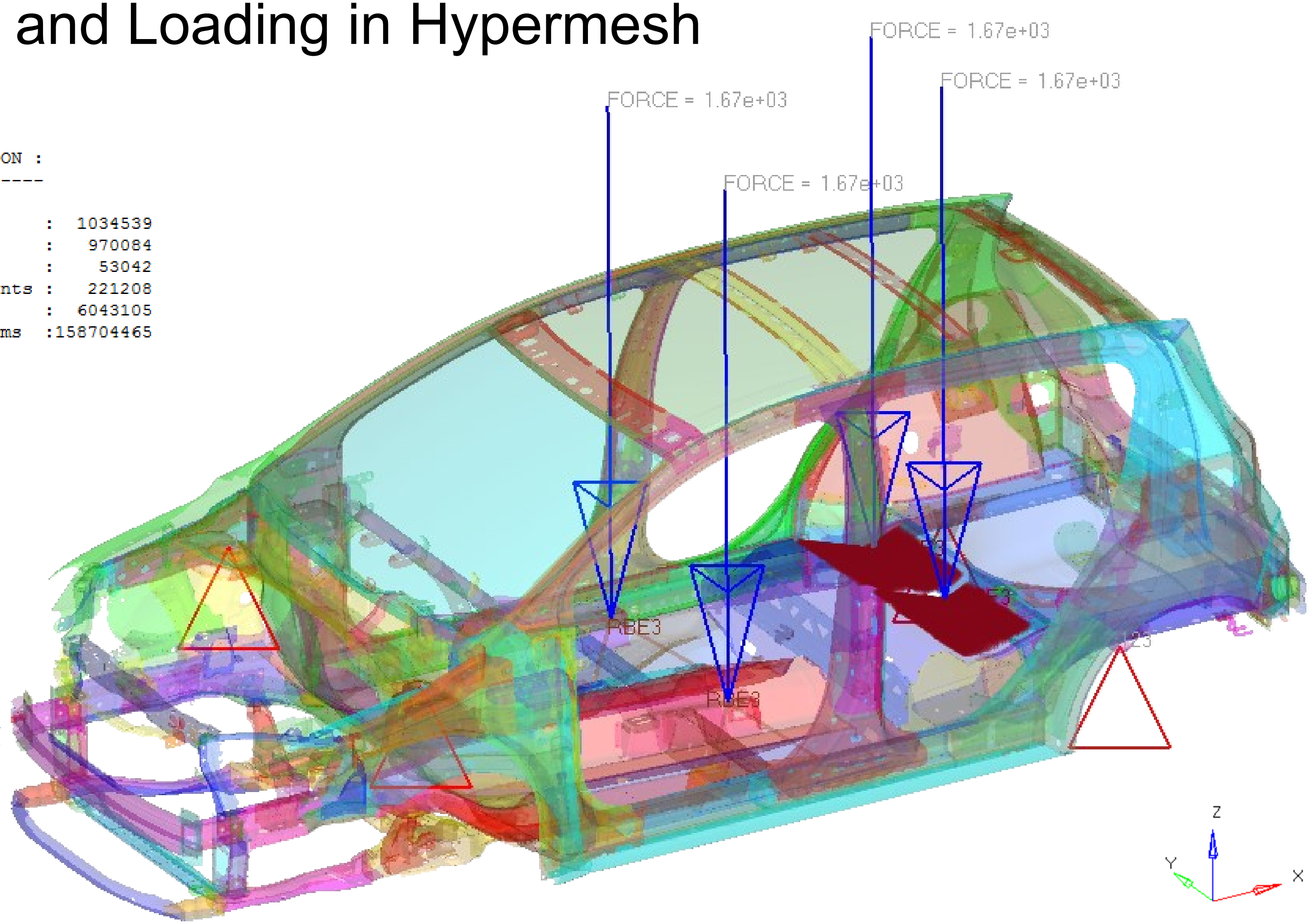
- Constraints and Loading in FSV Report



Bending Stiffness

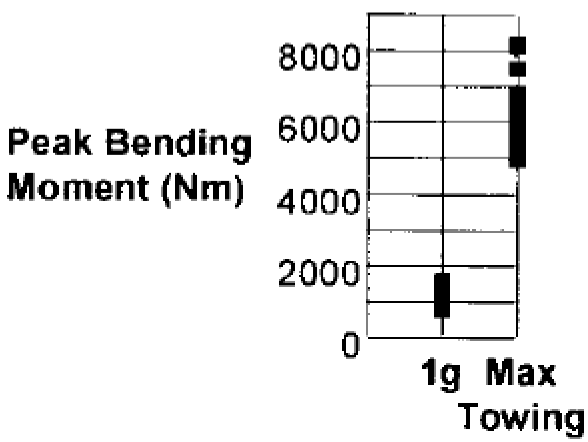
- Constraints and Loading in Hypermesh

```
FINITE ELEMENT MODEL DATA INFORMATION :
---------------------------------------

  Total # of Grids (Structural)          :  1034539
  Total # of Elements                    :   970084
  Total # of Rigid Elements              :    53042
  Total # of Rigid Element Constraints   :   221208
  Total # of Degrees of Freedom          :  6043105
  Total # of Non-zero Stiffness Terms    :158704465


  Element Type Information
  ------------------------

   CHEXA    Elements    :     12119
   CPENTA   Elements    :        98
   CQUAD4   Elements    :    900965
   CTRIA3   Elements    :     55437
   PLOTEL   Elements    :      1465

  Load and Boundary Information
  -----------------------------

   FORCE Sets         :       2
   SPC   Sets         :       2

  Material and Property Information
  ---------------------------------

   PSOLID Cards       :      12
   PSHELL Cards       :     420
   MAT1   Cards       :      34
```

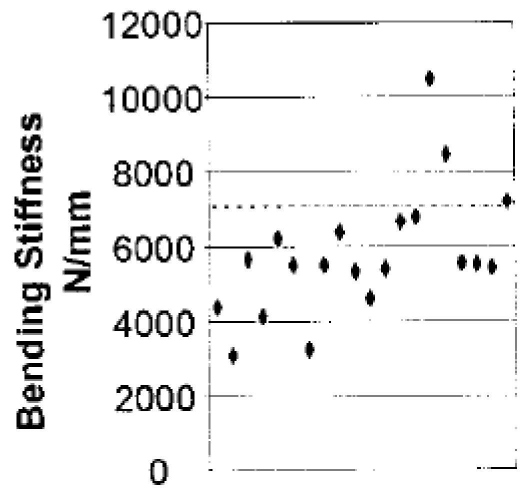


Benchmark

  – Maximum bending moments for a sampling of 20 vehicles
- Peak bending moments depend on
  – Placement of the subsystem mass
  – Longitudinal dimensions of the vehicle: wheelbase



Reference Vehicles- Honda Accord, Lexus LS400. Nisson 300ZX, Lumina, Century, Infinity Q45, Transport, Grand Prix, Toyota Camry. (AISI Data)

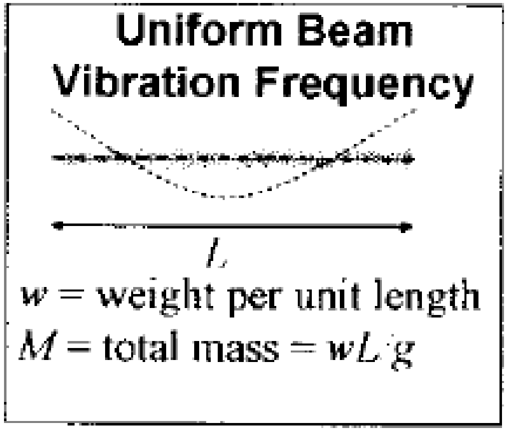


Reference Vehicles- Honda Accord, Lexus LS400. Nisson 300ZX, Lumina, Century, Infinity Q45, Transport, Grand Prix, Toyota Camry. (AISI Data)

4.2 Body Bending Stiffness Requirement

- Bending stiffness: slope of the load-deflection curve in the linear region from H point bending test
- Feeling of solidness: subjective → body vibration resonance
- Benchmarking (body bending resonant frequency)
  - body shell > full vehicle
- Desirable range for vehicle bending frequency: 22~25 Hz
- Uniform beam
    - ω_n: bending resonant frequency (rad/sec)
    - w: weight per unit length
    - M: total mass
    - L: beam length

$$\omega_n = (2\pi f_n) = \frac{22.4}{L^2}\sqrt{\frac{EIg}{w}} \xrightarrow{M=\frac{wL}{g}} \omega_n = 22.4L^{-(3/2)}\sqrt{\frac{EI}{M}}$$



Lateral Vibration of Beam

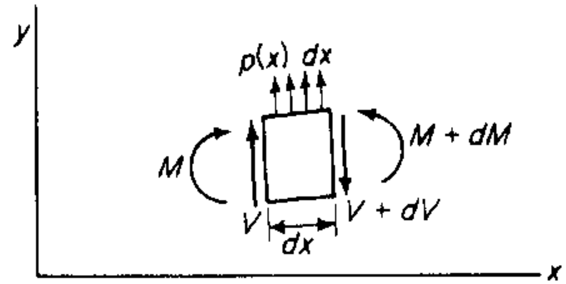
- Beam vibrating about its static equilibrium position under its own weight
  - Load per unit length = inertia load due to its mass and acceleration
  - Assuming harmonic motion

$$\left.\begin{aligned} \frac{d^2M}{dx^2} &= \frac{dV}{dx} = p(x) \\ M &= EI\frac{d^2y}{dx^2} \end{aligned}\right\} \to \frac{d^2}{dx^2}\left(EI\frac{d^2y}{dx^2}\right) = p(x) = \rho\omega^2 y \xrightarrow{EI=const} EI\frac{d^4y}{dx^4} - \rho\omega^2 y = 0$$

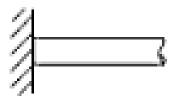
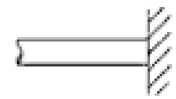
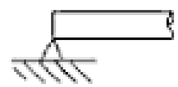
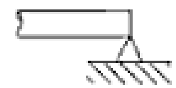
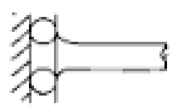
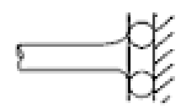
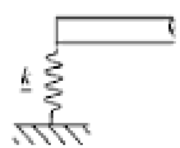
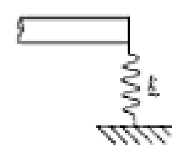
$$\xrightarrow{\beta^4=\rho\frac{\omega^2}{EI}} \frac{d^4y}{dx^4} - \beta^4 y = 0 \to y = e^{ax} \to \begin{Bmatrix} a = \pm\beta \\ a = \pm i\beta \end{Bmatrix} \to \begin{Bmatrix} e^{\pm\beta x} = \cosh\beta x \pm \sinh\beta x \\ e^{\pm i\beta x} = \cos\beta x \pm i\sin\beta x \end{Bmatrix}$$

$$\to \begin{cases} y = C_1 e^{\beta x} + C_2 e^{-\beta x} + C_3 e^{i\beta x} + C_4 e^{-i\beta x} \\ y = C_1\cos\beta x + C_2\sin\beta x + C_3\cosh\beta x + C_4\sinh\beta x \end{cases}$$

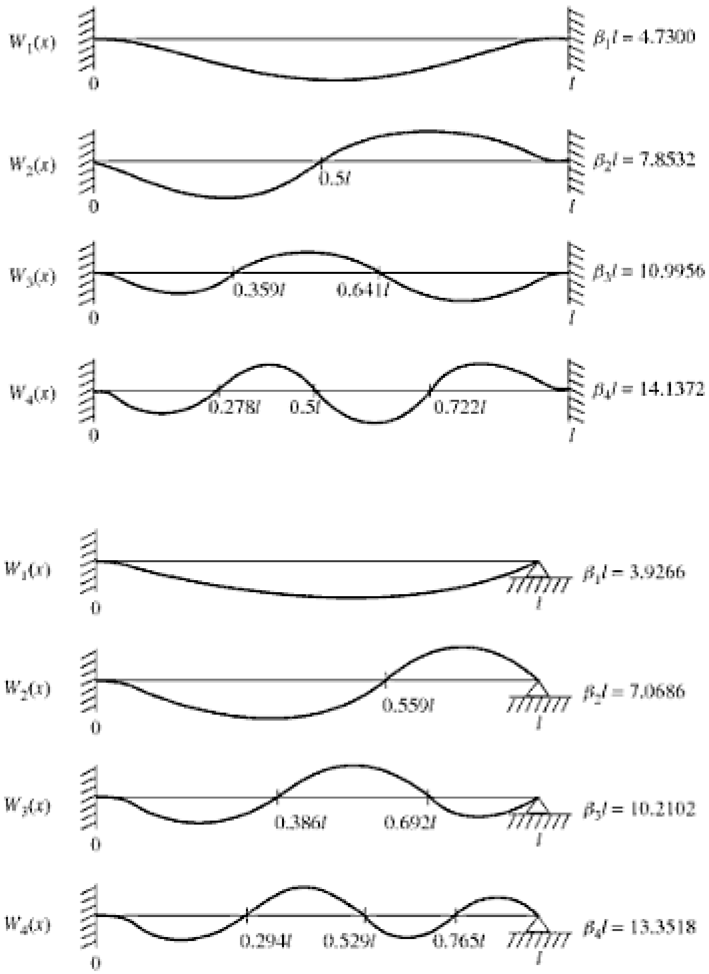
$$\omega_n = \beta_n^{\,2}\sqrt{\frac{EI}{\rho}} = (\beta_n l)^2\sqrt{\frac{EI}{\rho l^4}}$$

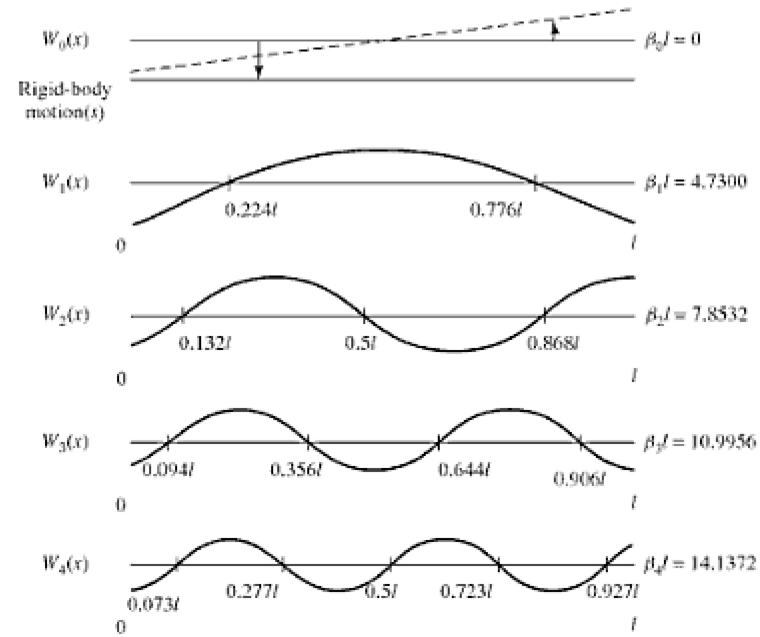


Boundary Conditions

Boundary condition	At left end ($x = 0$)		At right end ($x = l$)	
1. Free end (bending moment = 0, shear force = 0)	 $x = 0$	$EI\frac{\partial^2 w}{\partial x^2}(0, t) = 0$ $\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(0,t)} = 0$	 $x = l$	$EI\frac{\partial^2 w}{\partial x^2}(l, t) = 0$ $\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(l,t)} = 0$
2. Fixed end (deflection = 0, slope = 0)	 $x = 0$	$w(0, t) = 0$ $\frac{\partial w}{\partial x}(0, t) = 0$	 $x = l$	$w(l, t) = 0$ $\frac{\partial w}{\partial x}(l, t) = 0$
3. Simply supported end (deflection = 0, bending moment = 0)	 $x = 0$	$w(0, t) = 0$ $EI\frac{\partial^2 w}{\partial x^2}(0, t) = 0$	 $x = l$	$w(l, t) = 0$ $EI\frac{\partial^2 w}{\partial x^2}(l, t) = 0$
4. Sliding end (slope = 0, shear force = 0)	 $x = 0$	$\frac{\partial w}{\partial x}(0, t) = 0$ $\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(0,t)} = 0$	 $x = l$	$\frac{\partial w}{\partial x}(l, t) = 0$ $\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(l,t)} = 0$
5. End spring (spring constant = $\underset{\sim}{k}$)	 $x = 0$	$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert_{(0,t)} =$ $-\underset{\sim}{k}w(0, t)$ $EI\frac{\partial^2 w}{\partial x^2}(0, t) = 0$	 $x = l$	$\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\Big\vert(l, t) =$ $\underset{\sim}{k}w(l, t)$ $EI\frac{\partial^2 w}{\partial x^2}(l, t) = 0$

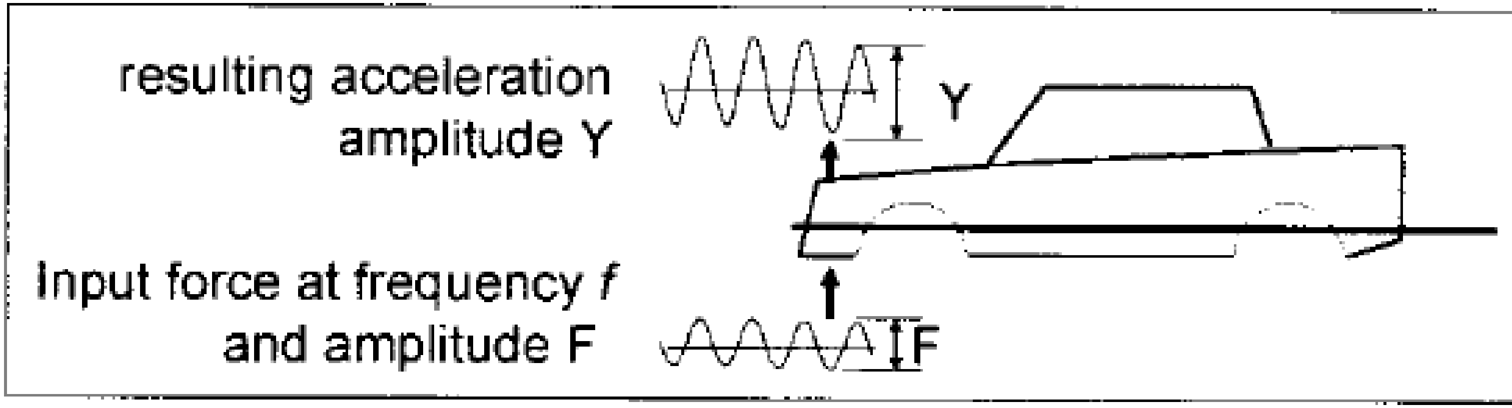
Natural Frequencies and Mode Shape



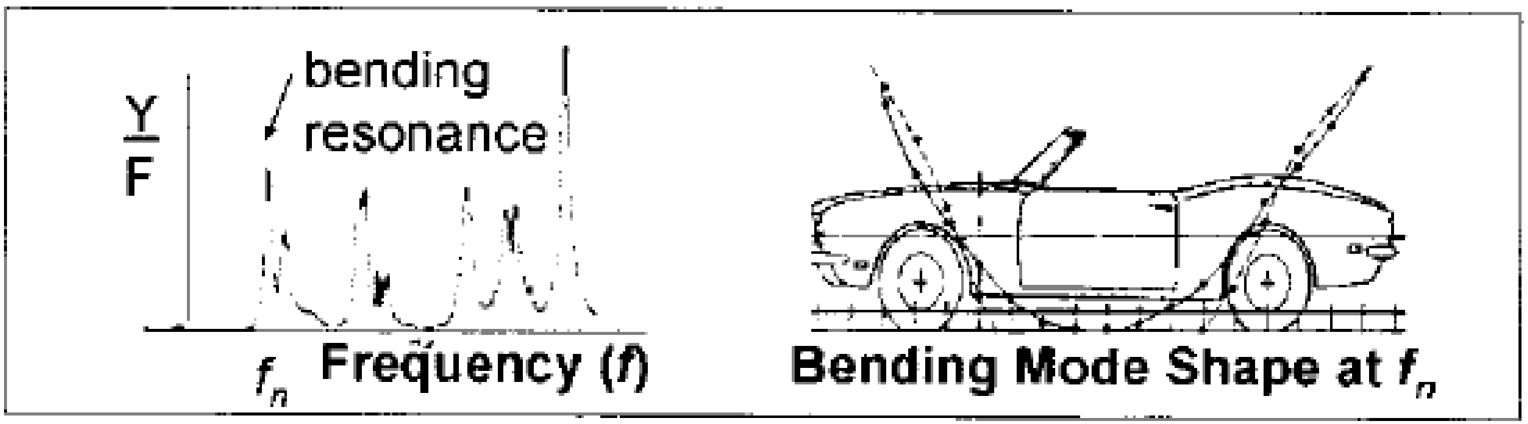


Beam Configuration	$(\beta_1 l)^2$ Fundamental	$(\beta_2 l)^2$ Second Mode	$(\beta_3 l)^2$ Third Mode
Simply supported	9.87	39.5	88.9
Cantilever	3.52	22.0	61.7
Free-free	22.4	61.7	121.0
Clamped-clamped	22.4	61.7	121.0
Clamped-hinged	15.4	50.0	104.0
Hinged-free	0	15.4	50.0

Body Vibration Test and Behavior

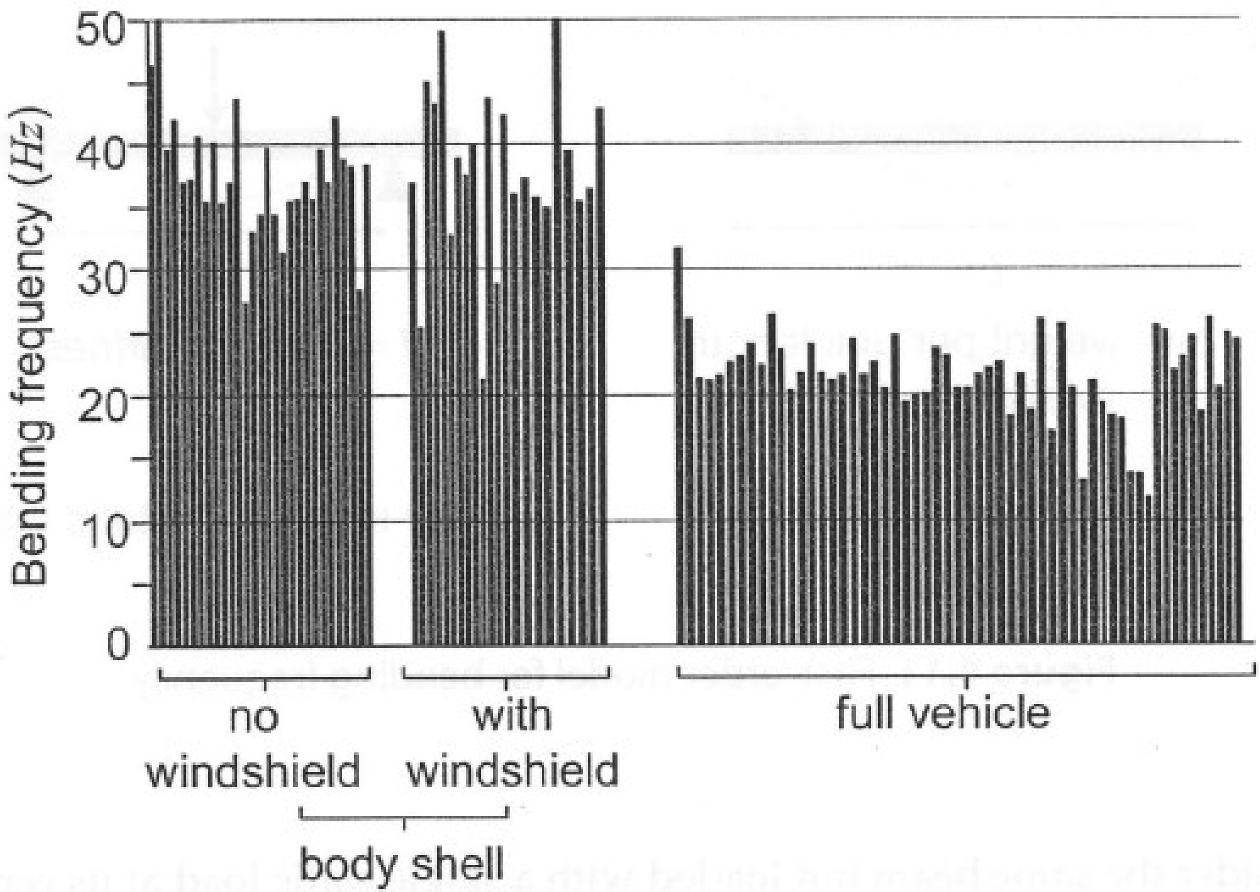


(a) Vibration Test Set up



(b) Typical Frequency Response and Mode Shape

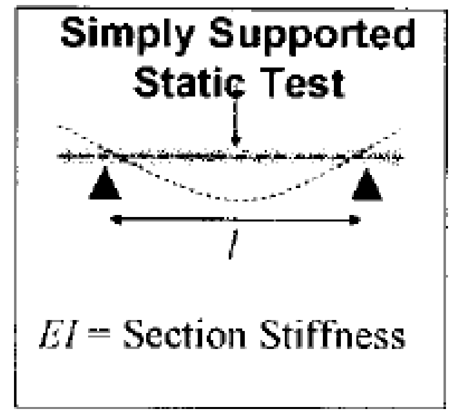
Bending Resonant Frequency Benchmark



Body Bending Stiffness Requirement

- Simply supported static test loaded at its center
  - l: wheelbase
  - M: rigidly mounded mass
  - K: required bending stiffness

$$K = \frac{48EI}{l^3} \rightarrow EI = \frac{Kl^3}{48}$$



$$\omega_n = 22.4L^{-(3/2)}\sqrt{\frac{Kl^3}{48M}} = 3.2332\left(\frac{l}{L}\right)^{(3/2)}\sqrt{\frac{K}{M}} \rightarrow K = 0.096\omega_n{}^2 M\left(\frac{L}{l}\right)^3$$

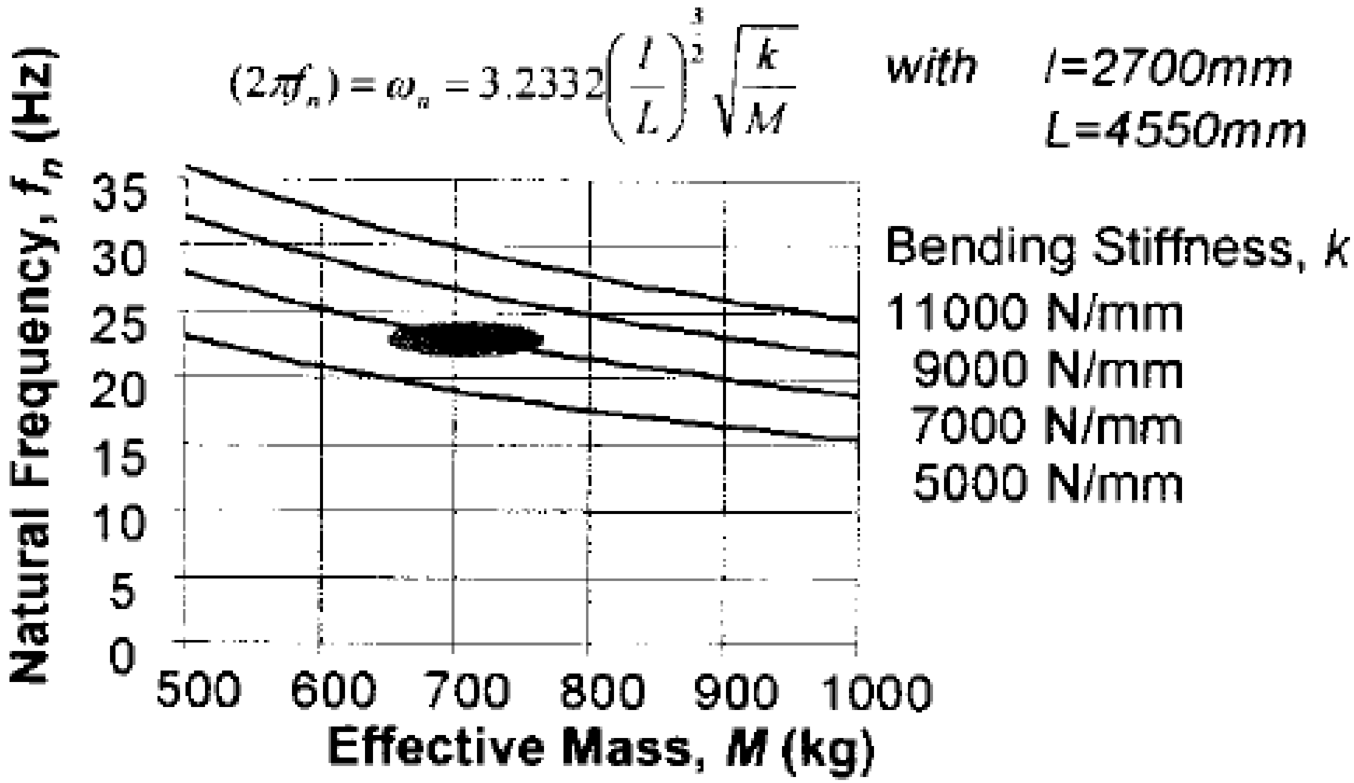
- Typical midsize vehicle

$$l = 2700mm,\ L = 4550mm,\ M \approx \left(0.4 \sim 0.6\right)\times\left(\text{curb mass} = 1446kg\right)$$

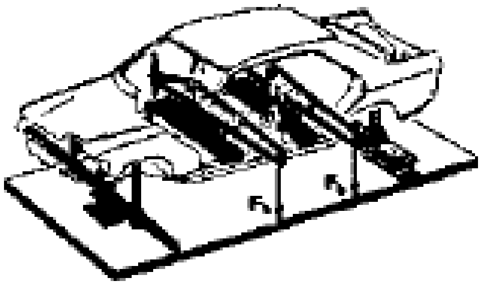
$$578 \le M \le 868,\ f_n = 22 \sim 25Hz \rightarrow K = 7000N/mm$$

- High bending stiffness required to achieve the same ω_n
  - Higher mass loading (optioned luxury car), long overall length (four door sedans vs. two seat sport coupes)
  - Reduce relative deformations which cause squeaks and rattles

First Order Estimation of Bending Stiffness



Typical Bending Requirements: Midsize Vehicle

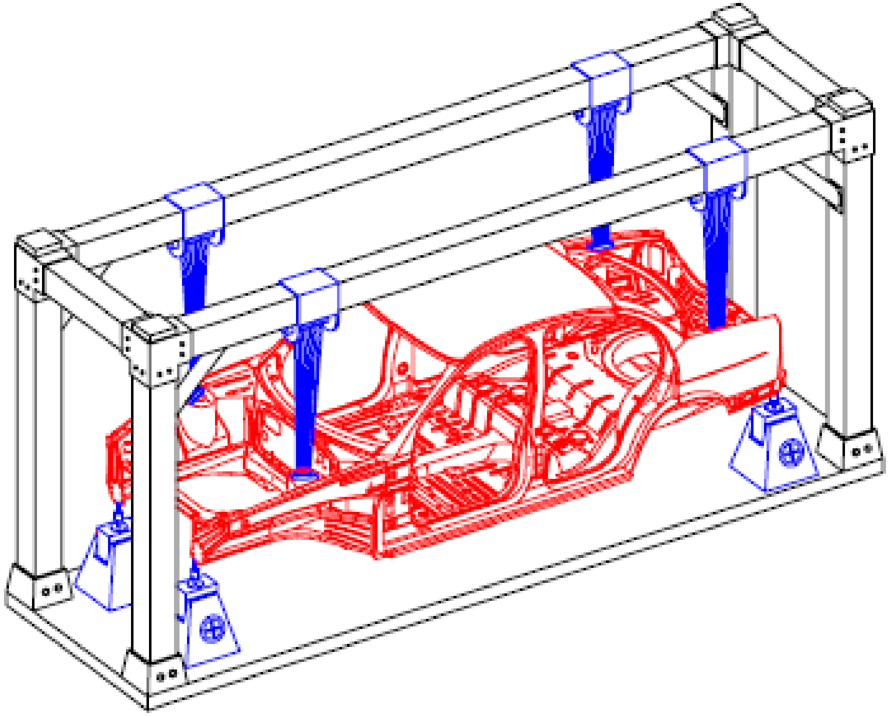


Restraints at
Suspension
Attachments

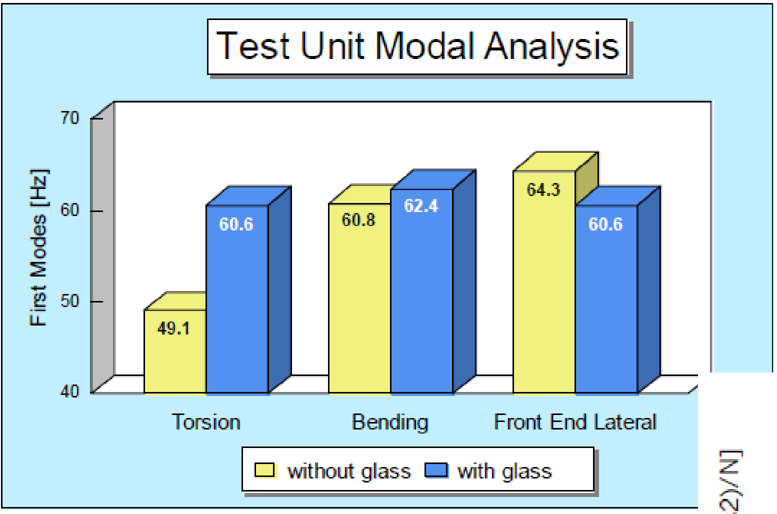
Bending Stiffness	Bending Strength
Nominal Value Stiffness = 7000 N/mm	Nominal Value F= 6680 N no permanent deformation

Test: Modal Analysis

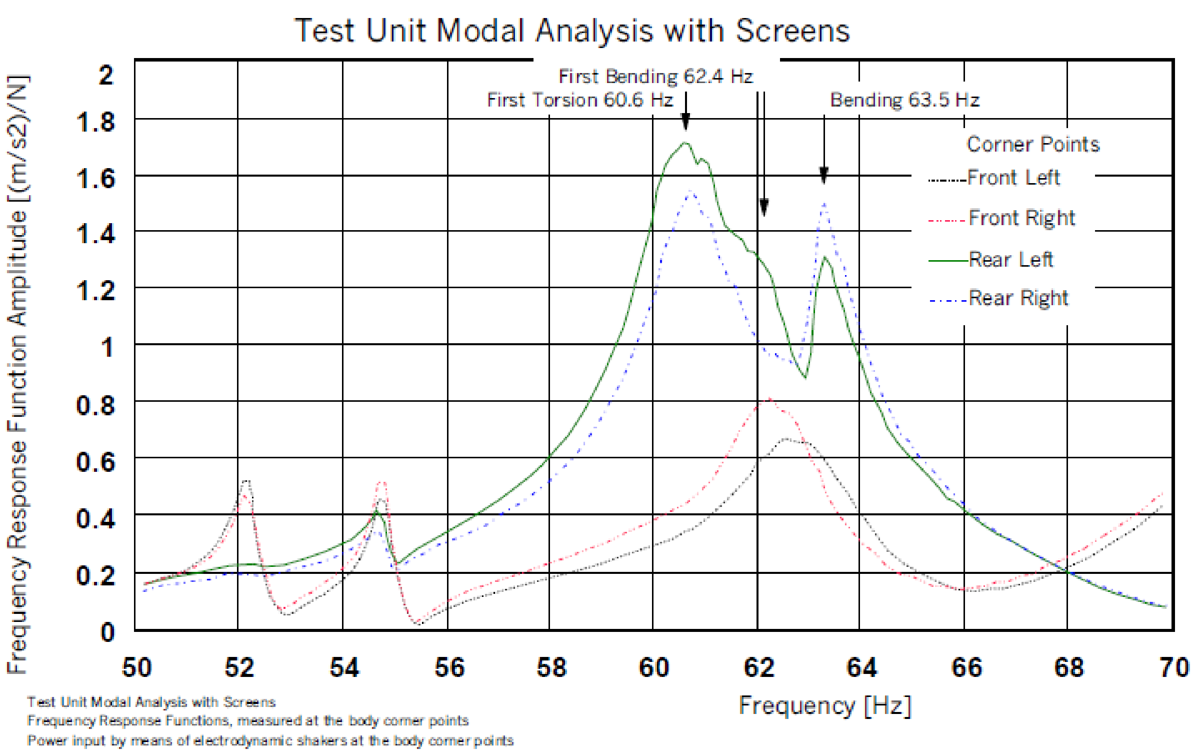
- resonance frequencies of the specific structure and the corresponding mode shapes (how the structure vibrates)
  - Input: applied energy (electrodynamic shakers at corner points)
  - Output: acceleration at different points



Test: Results for Modal Analysis

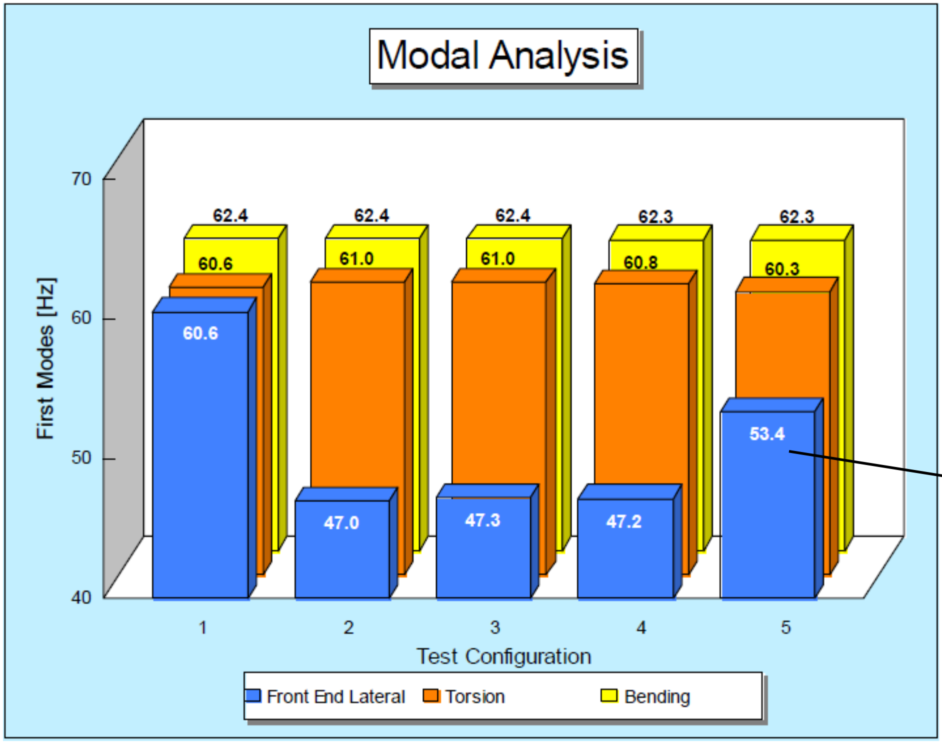


Second bending mode at 63.5 Hz occurs mainly in the rear; whereas the first bending mode occurs in the front and rear of the structure



Modal Analysis: impact of bonded and/or bolted parts

- 1. full configuration
- 2. as 1, but without bolted brace cowl to shock tower assembly
- 3. as 2, but without braces radiator
- 4. as 3, but without tunnel bridge
- 5. as 4, but without radiator support upper



influence of the mass of assembly radiator support

ULSAB Testing Results Overview vs. CAE Results

Testing	Testing DH #2	Testing Test Unit	CAE Final Version	CAE Test Unit	Benchmark Average	Targets
Static Rigidity						
Torsion (Nm/deg)	20,800	21,620	20,350	19,020	11,531	≥ 13,000
Bending (N/mm)	18,100	20,460	20,540	20,410	11,902	≥ 12,200
Modal Analysis						
Torsion (Hz)	60.1	60.6	61.4	61.1	38*	≥ 40
Bending (Hz)	63.9	62.4	61.8	64.1	38*	≥ 40
Front End Lateral (Hz)	64.9	60.6	60.3	58.5	38*	≥ 40

FSV: Body Structure Performance

- Static Stiffness

Analysis Type	Target	FSV Model Results
Torsion stiffness (KN-m/deg)	20.0	19.604
Bending stiffness (N/mm)	12.0	15.552
Global Modes	**Target**	**Frequency (Hz)**
Torsion	>40 Hz (both modes), separated by 3 Hz	54.8
Vertical bending		60.6

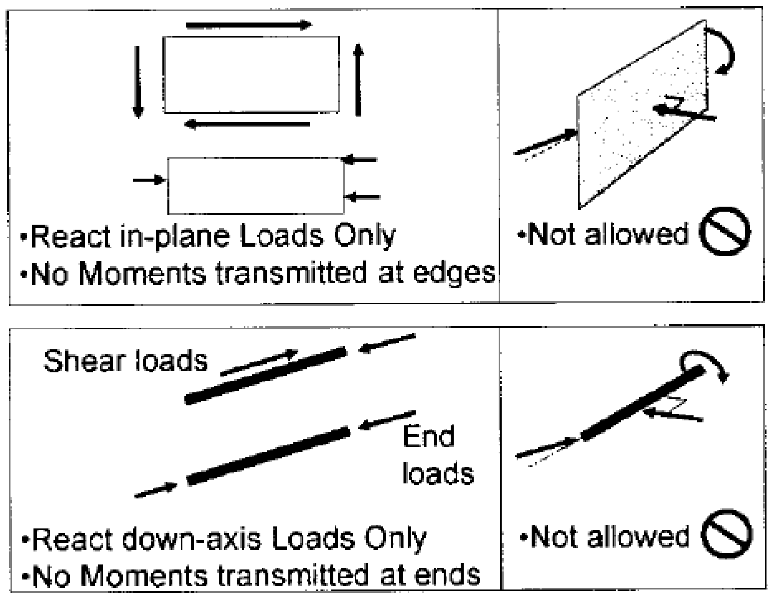
- Durability

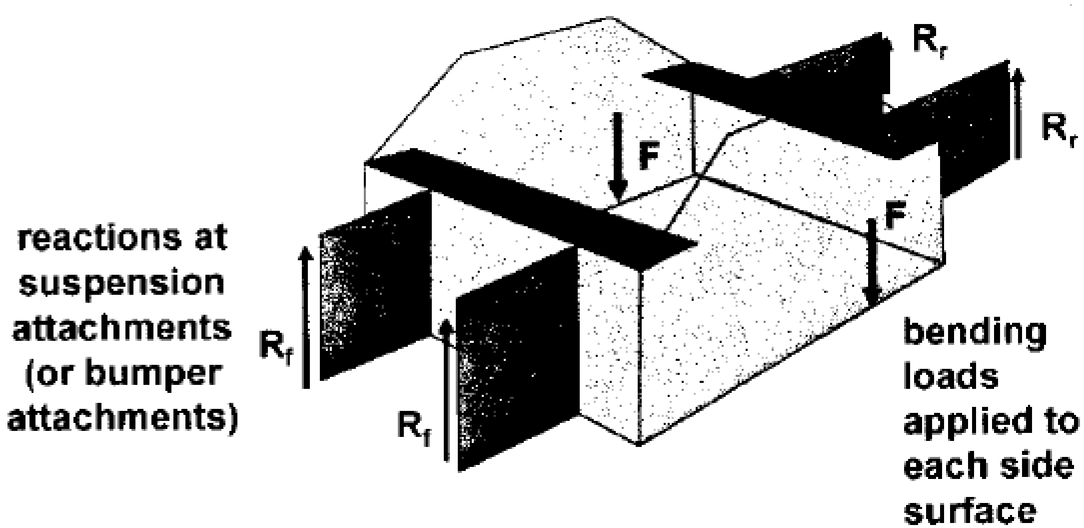
Analysis Type	Target life cycles	Predicted life cycles (FSV Model)
3 g pot hole	200,000	927,100
0.7 g cornering	100,000	1,676,000
0.8 g forward braking	100,000	274,700 (engine cradle life), 17,340,000(body life)

4.3 Load Path Analysis: Global Bending

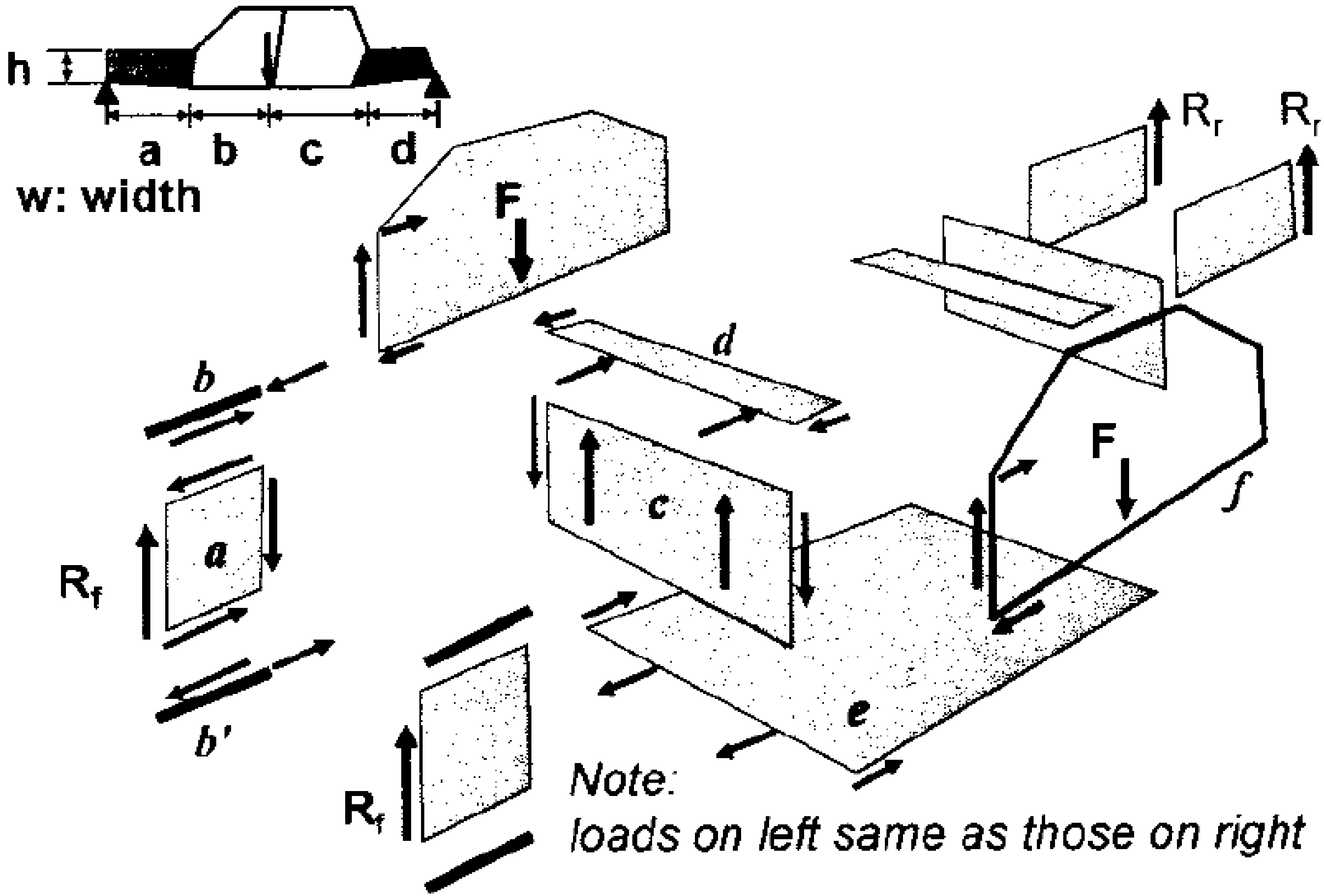
- Understand how global body requirements flow down to loads on structural elements
- Idealize the body as a set of structural surface and bar
  - Structural surface: flat element loaded in shear along edges
  - Bar element: linear element only reacts loads along it's axis either end loads or shearing loads along the length
- Apply loads in the global bending strength test
- Find loads on each individual structural subsystem
- Find internal element loads using static equilibrium
- Find appropriate sections to react these loads

Structural Surface and Bar Body Model



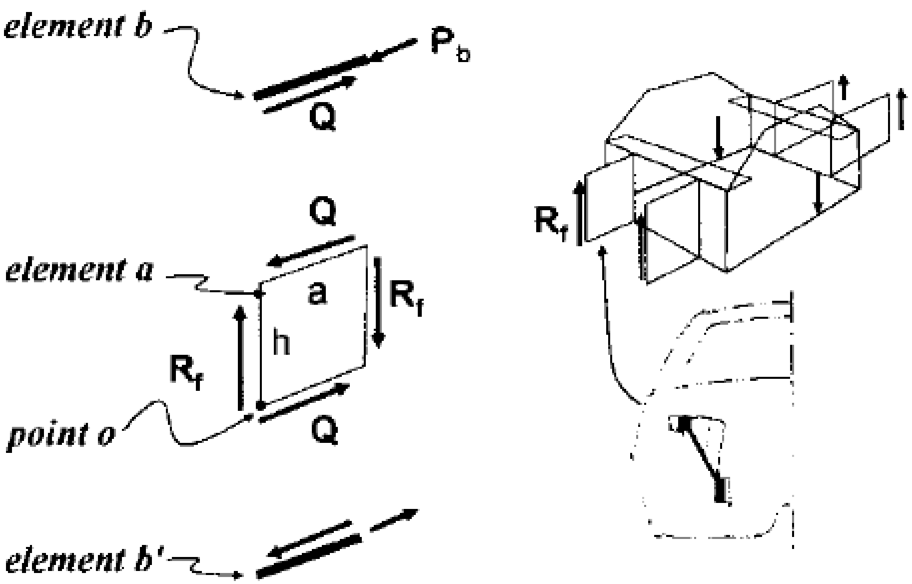


Internal Loads on Structural Surface Model

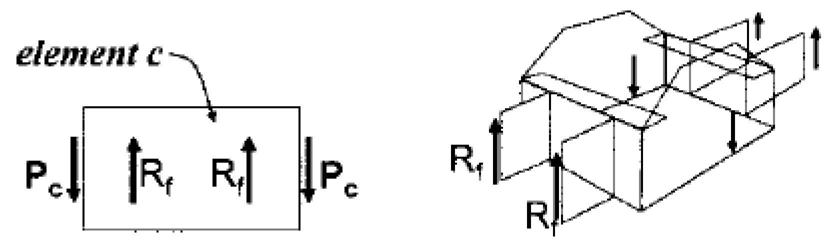


Internal Loads (1)

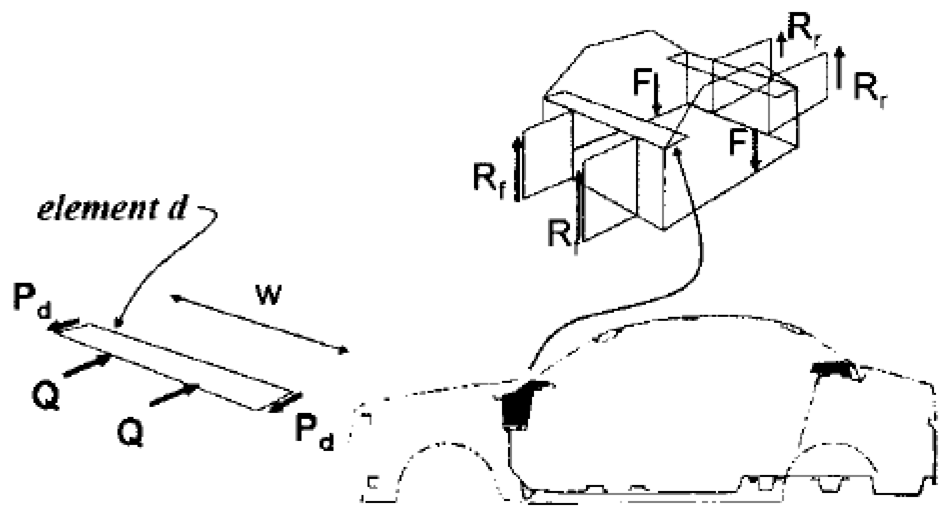
- Motor compartment panel



- Structural surface model

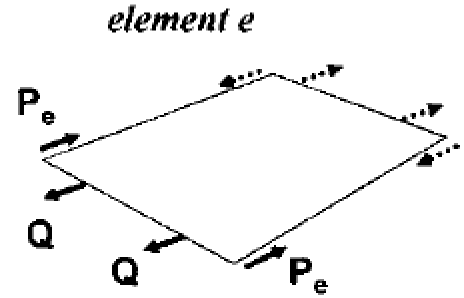


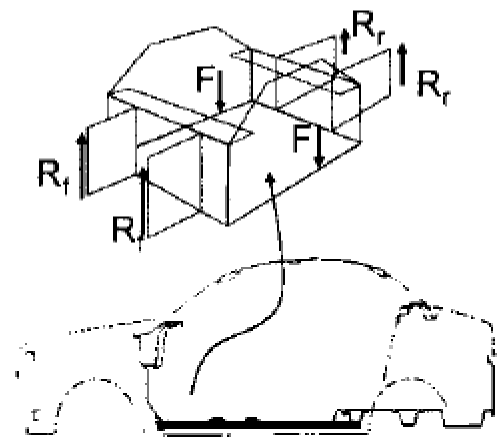
- Cowl or package shelf



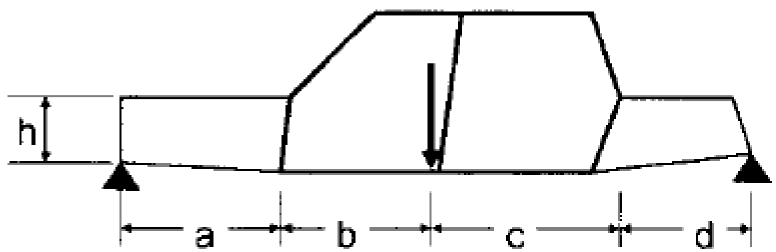
Internal Loads (2)

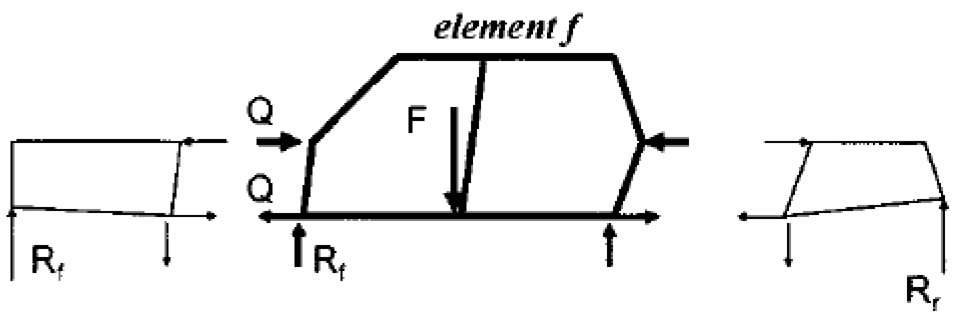
- Floor pan





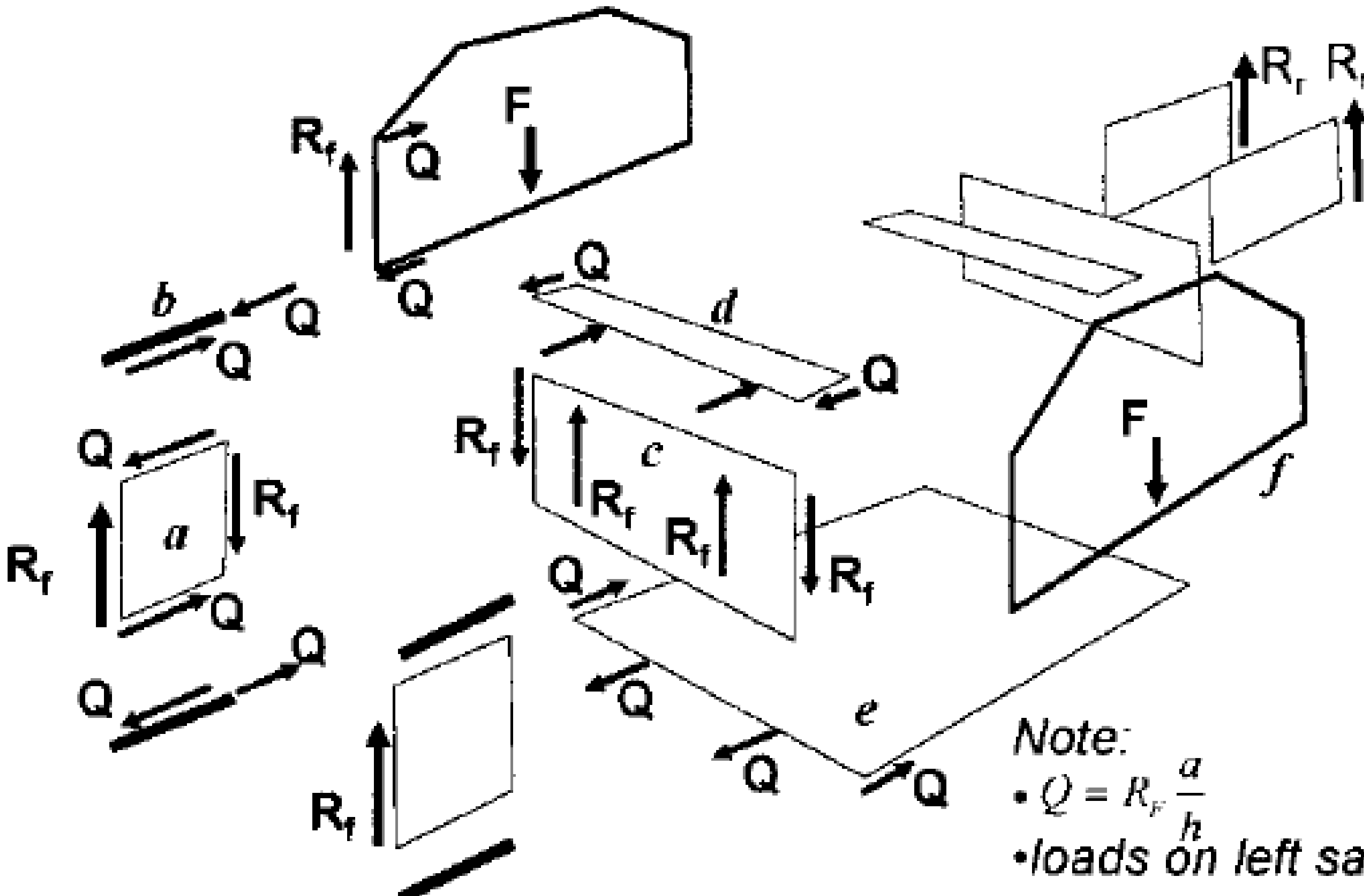
- Side frame





$R_f = F(c+d)/(a+b+c+d)$ $R_r = F(a+b)/(a+b+c+d)$

Internal Loads on Structural Surface Model

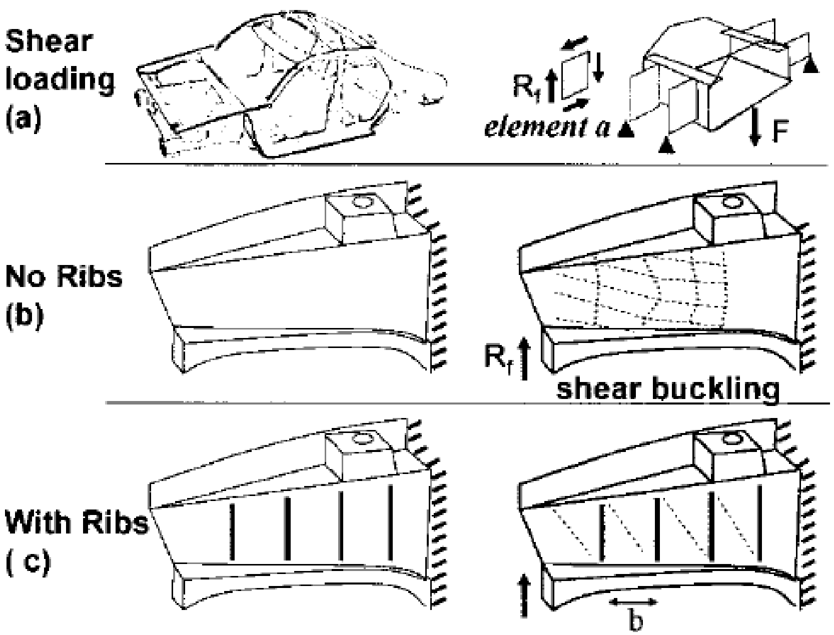


Note:

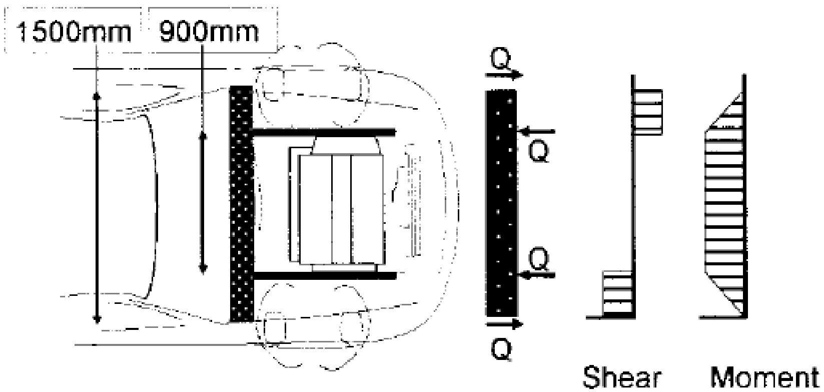
- $Q = R_F \frac{a}{h}$
- loads on left same as those on right

Structural Subsystem

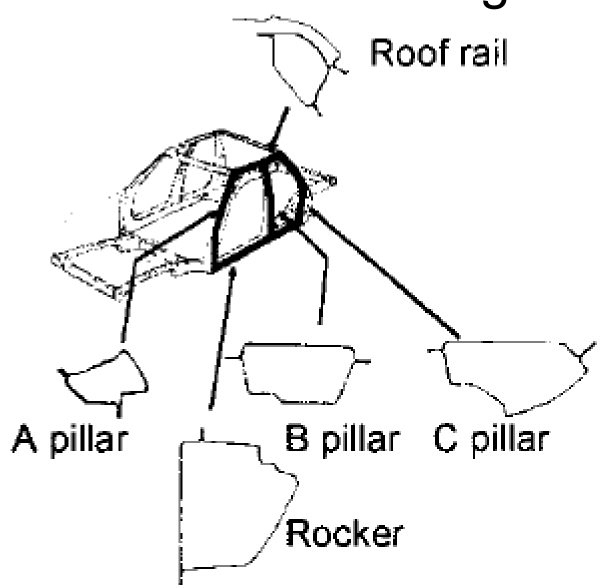
- Motor compartment side panel
  - Plate buckling



- Cowl: bending moment

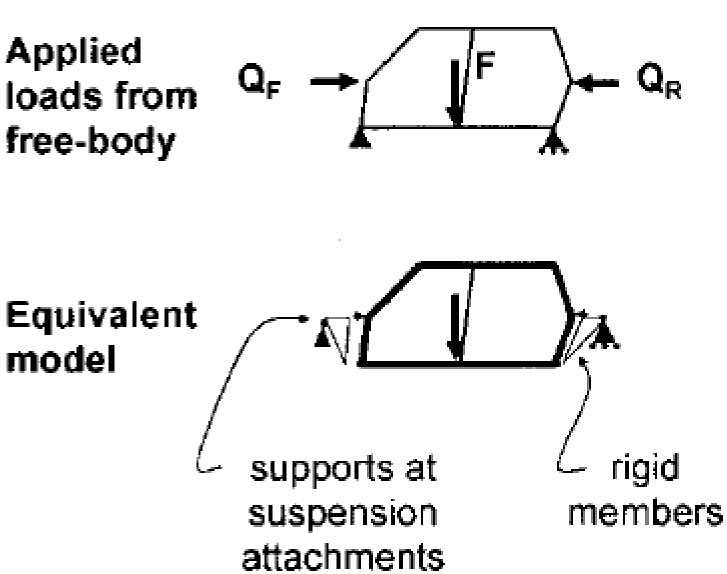


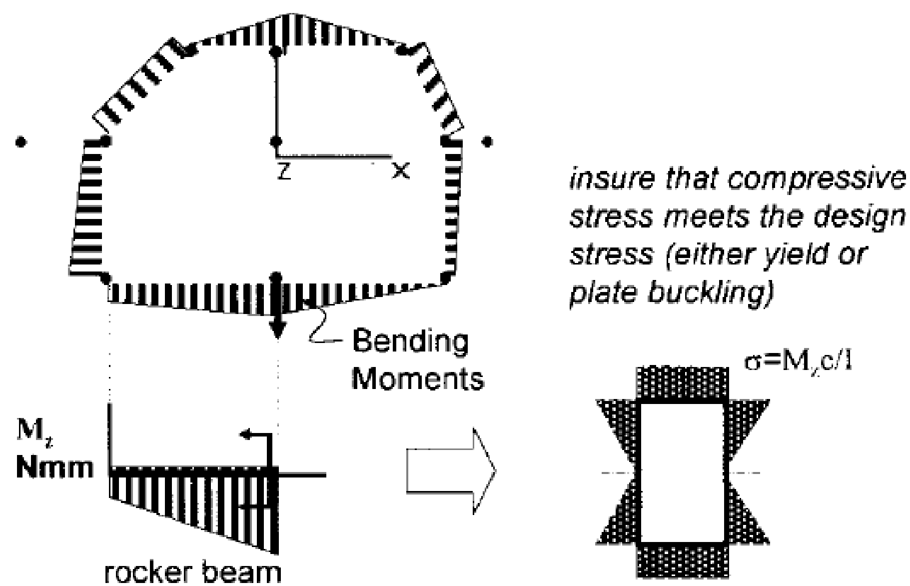
- Side frame: bending moment



Side Frame Beam Model

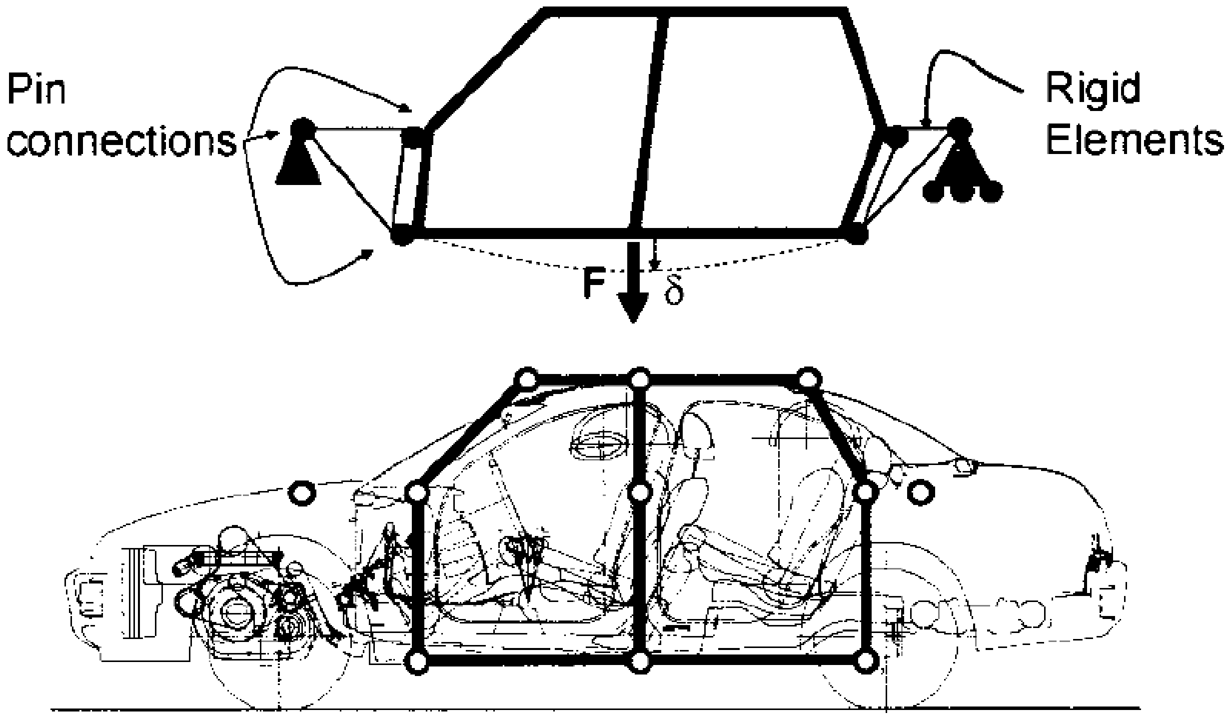
– What moments are applied to each beam by these loads so that we may design the beam section
  - Statically indeterminate
  - Relative stiffness of each beam



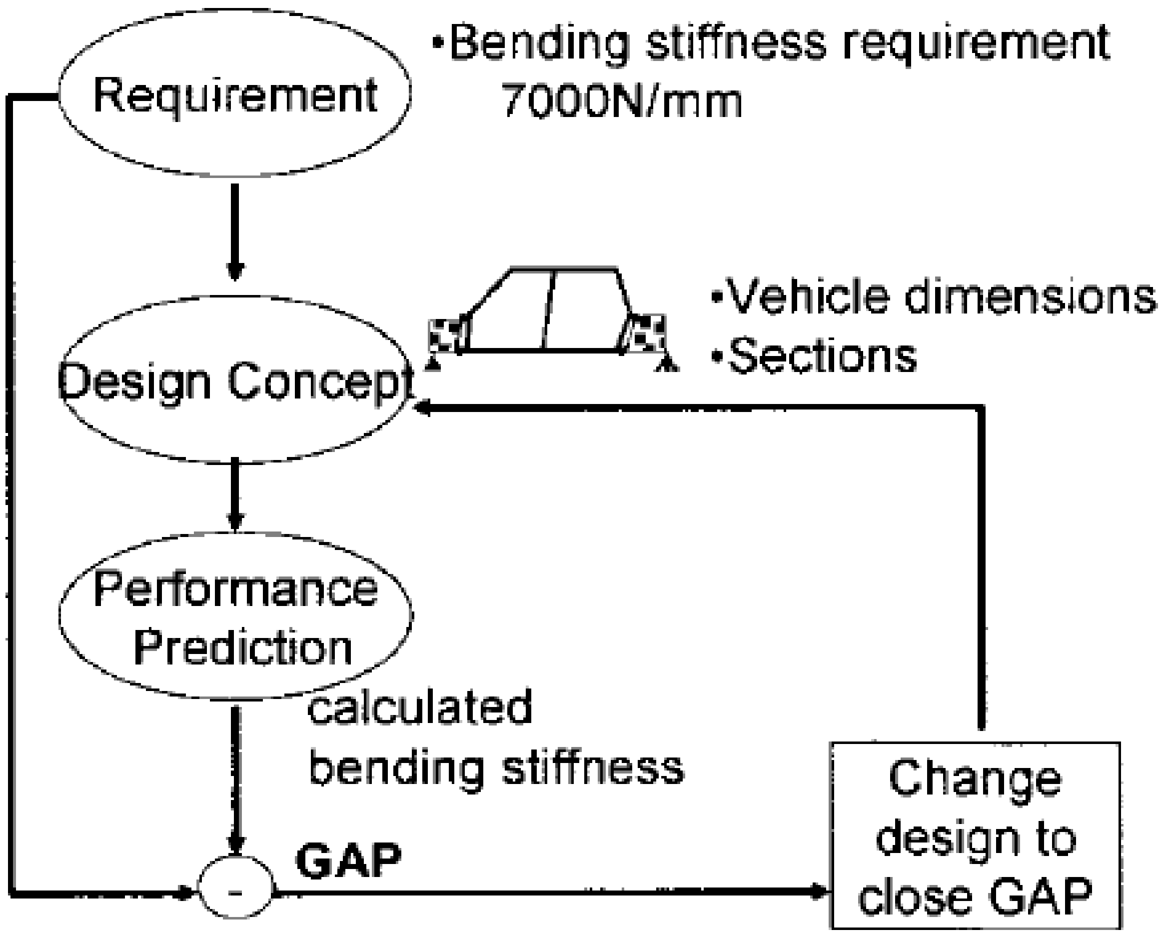


4.4 Analysis of Bending Stiffness

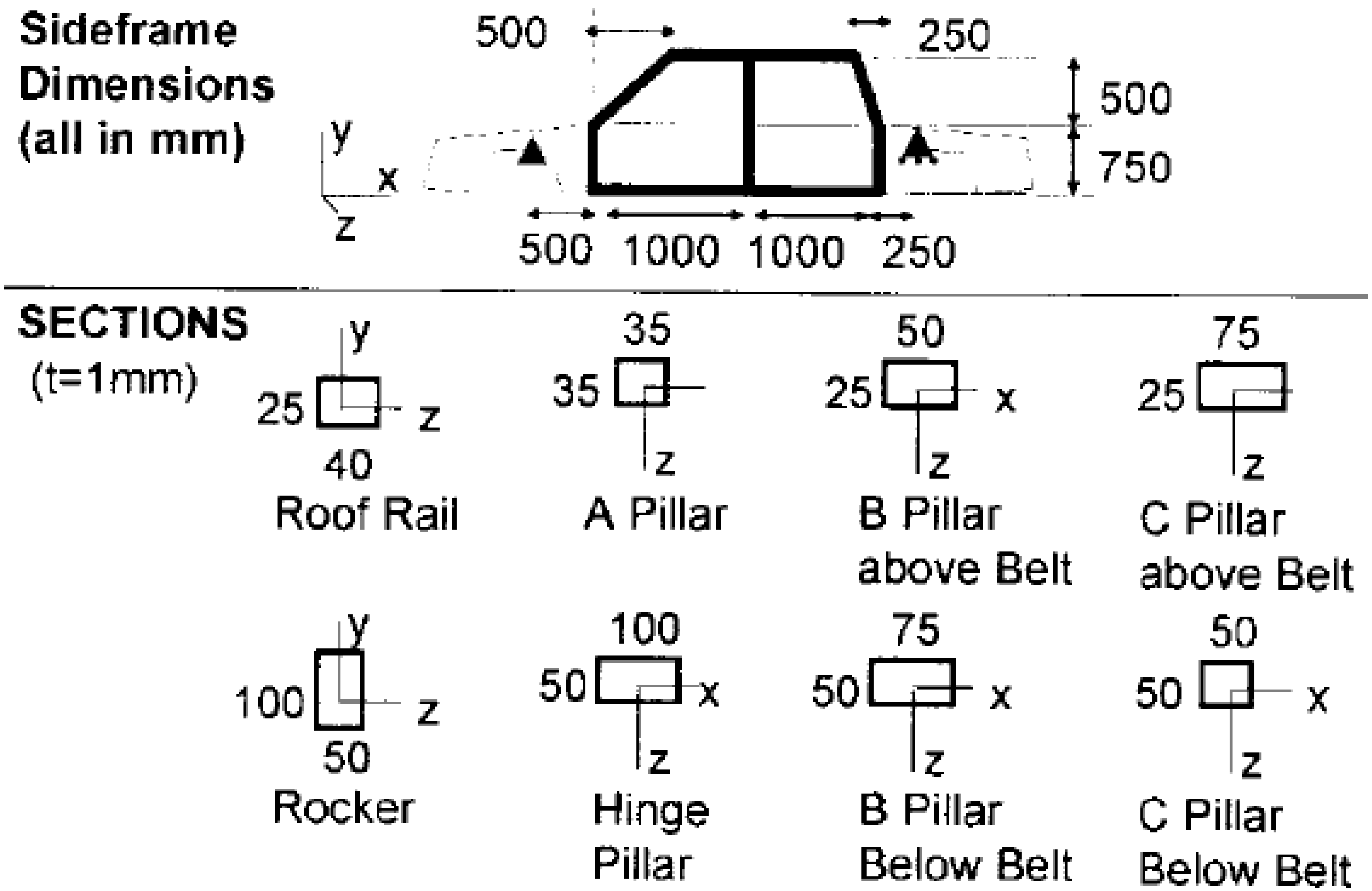
- Focus on side frame due to its dominant effect
  - Basic beam finite element model
  - Bending stiffness: ratio of applied load to deflection at the node of load application



System Design Procedure

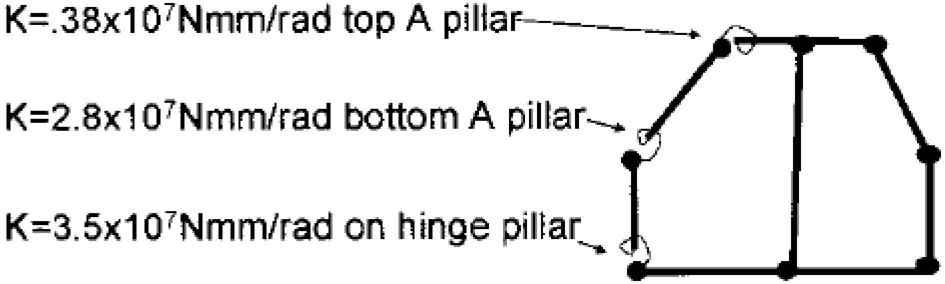


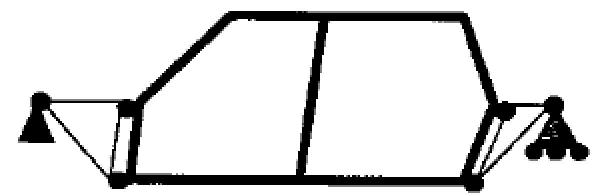
Example: Side Frame Model

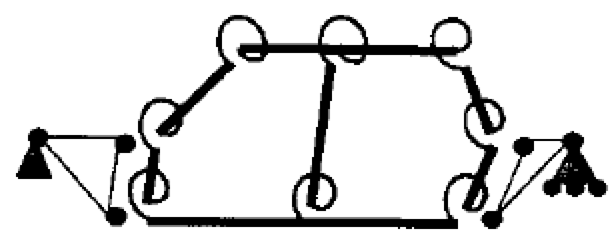


FEA Results

- F = 6680 N $\rightarrow \delta$ = 6.4 mm
  - K = 1044 N/mm per side = 2088 N/mm bending stiffness
  - 30% of 7000 N/mm target
  - Twice the actual stiffness: too stiff ?
- Modified model with flexible joints



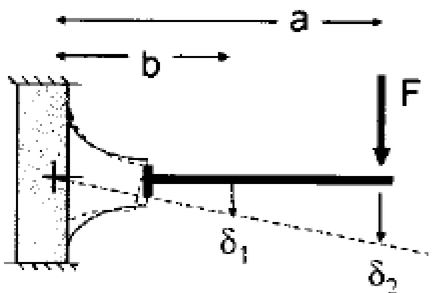




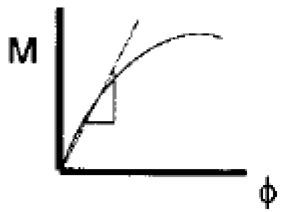
- F = 6680 N $\rightarrow \delta$ = 7.7 mm
  - K = 1735 N/mm bending stiffness

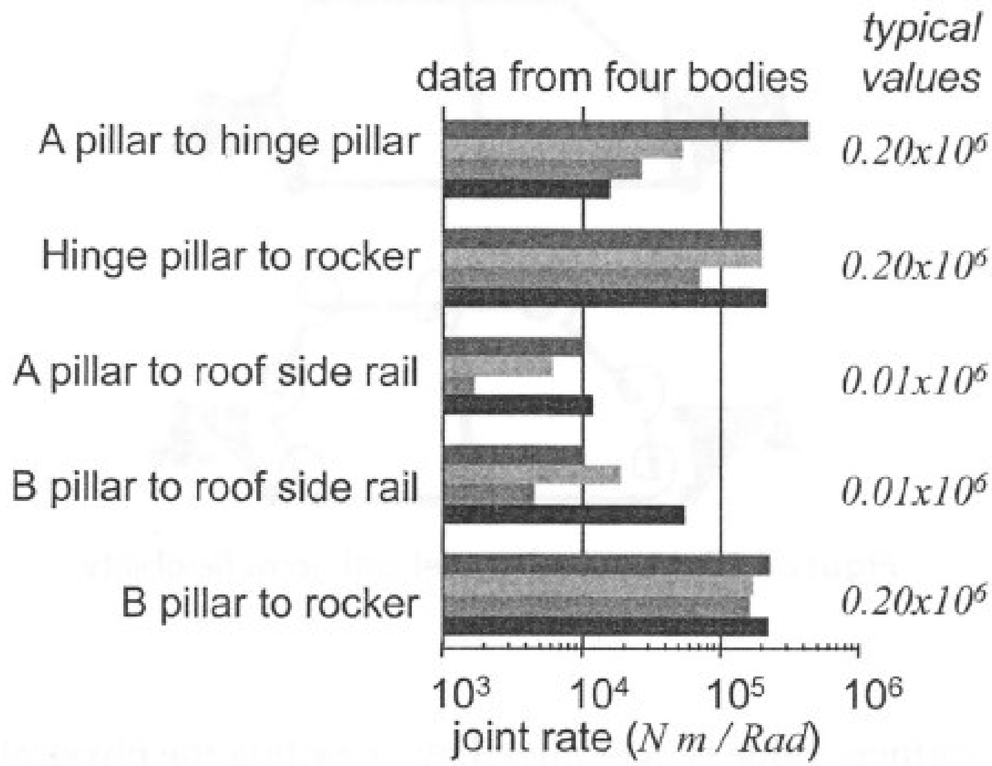
Joint Flexibility

- Two or more thin walled beams are joined → considerable local deformation
- Rigid joints → flexible joints: rotational stiffness

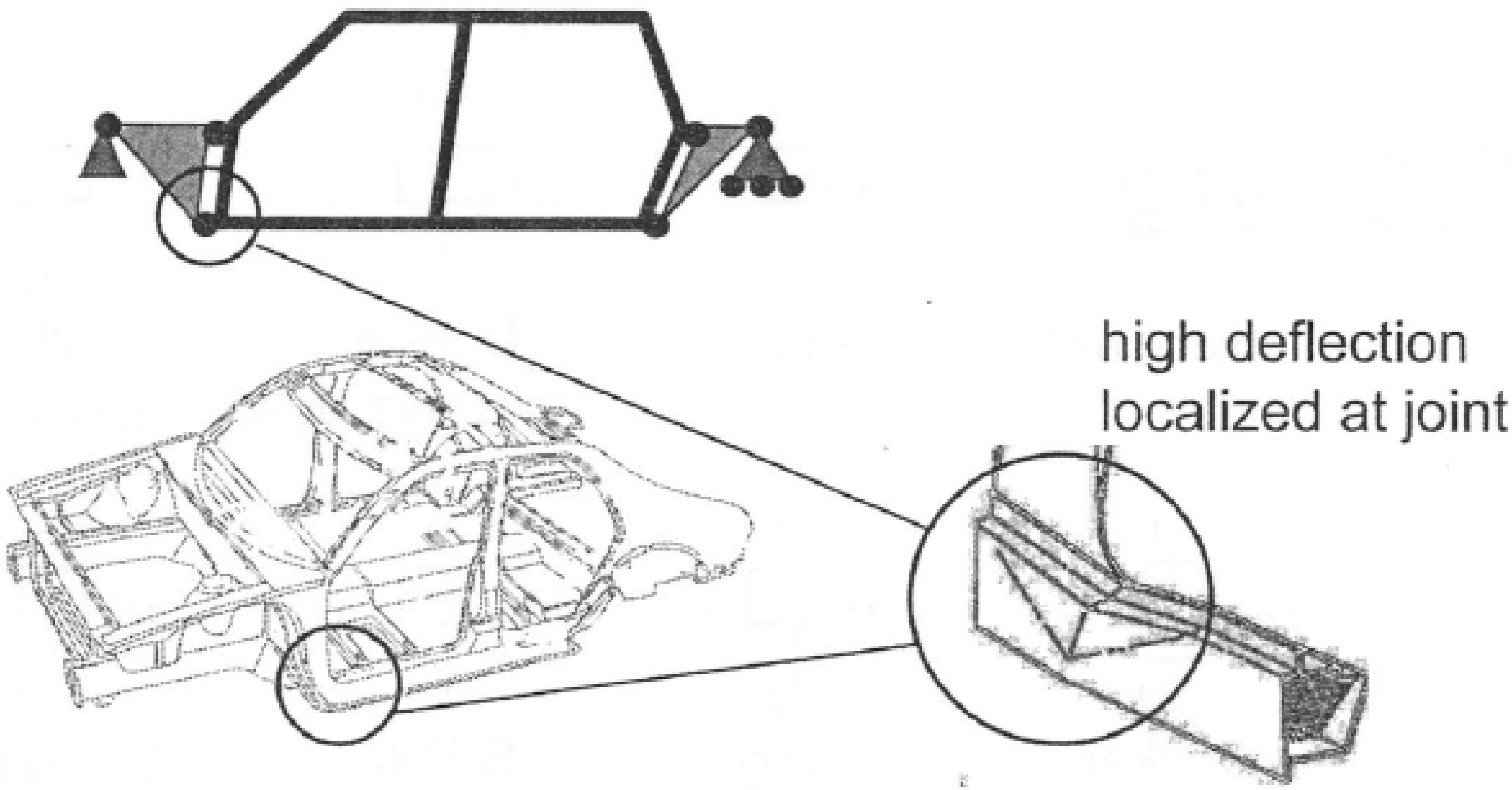


$\phi = (\delta_2 - \delta_1) / (a - b)$
$M = a F$
K = slope of M vs. ϕ plot:





Local Deformation at Joint

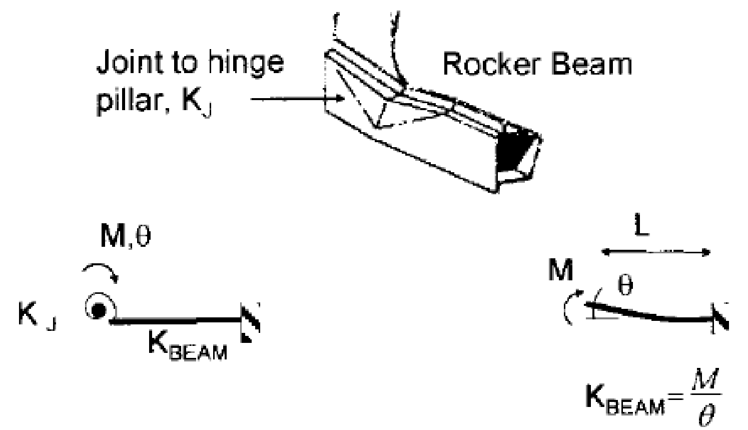


Joint Stiffness

- Is 0.2×10^6 Nm/rad very stiff or very flexible?
- Joint efficiency
  - Ratio of the combined of the beam with joint to the stiffness of the beam alone (assuming a rigid joint)
  - $K_{joint} >> K_{beam}$: $f \approx 1$
  - Very low efficiency: joint deformation dominant

$$\left.\begin{array}{l} f = \dfrac{K_{\text{beam+joint}}}{K_{\text{beam}}} \\[2ex] K_{\text{beam+joint}} = \dfrac{K_{\text{joint}} K_{\text{beam}}}{K_{\text{joint}} + K_{\text{beam}}} \end{array}\right\} \rightarrow f =$$

$$\xrightarrow{K_{\text{beam}} = \frac{M}{\theta} = \frac{2EI}{L}} f =$$



Beam & Joint system stiffness

Beam stiffness alone

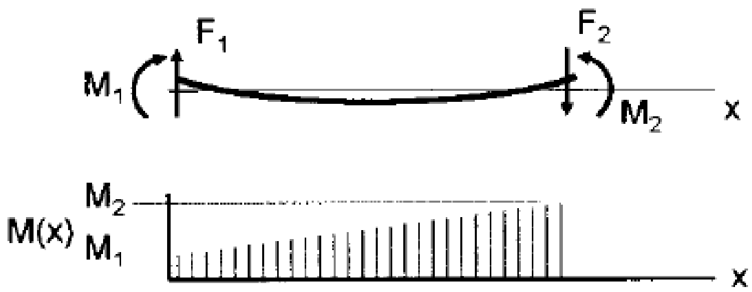
Example: Hinge Pillar-to-Rocker Joint

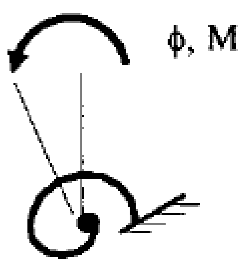
- $I = 4.15\text{x}10^6\ mm^4$ ($\leftarrow$ h=100, w=50, t=1)
- L = 1000 mm
- $K_{joint} = 0.2\text{x}10^6$ Nm/rad $= 0.2\text{x}10^9$ Nmm/rad
- $E = 207000\ N/mm^2$
- Joint efficiency ? 0.537

$$f = \frac{1}{1 + \frac{2EI}{LK_{\text{joint}}}}$$

Strain Energy and Stiffness

- If initial guess does not meet the required bending stiffness, which beams to adjust first and is it the minimum mass solution?
  – Increase the performance of the structural element with the highest fraction of strain energy
- Strain energy





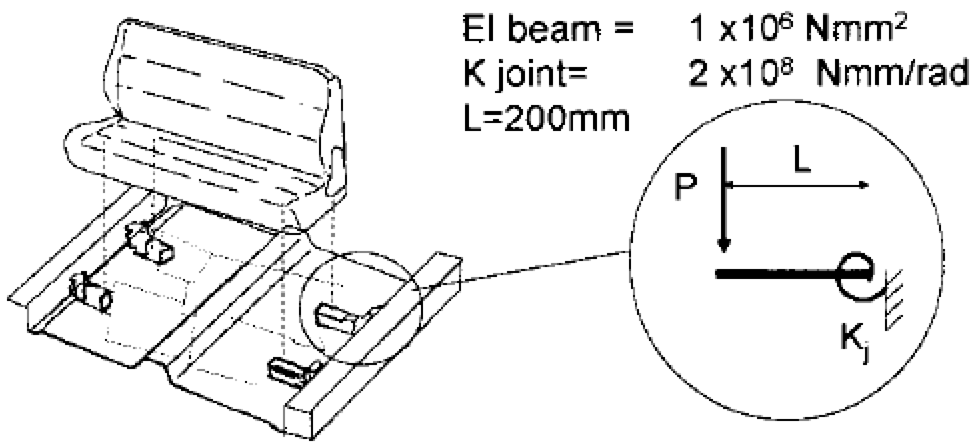
$$e_{\text{joint}} =$$

$$M = M_1 + \frac{M_2 - M_1}{L} x$$

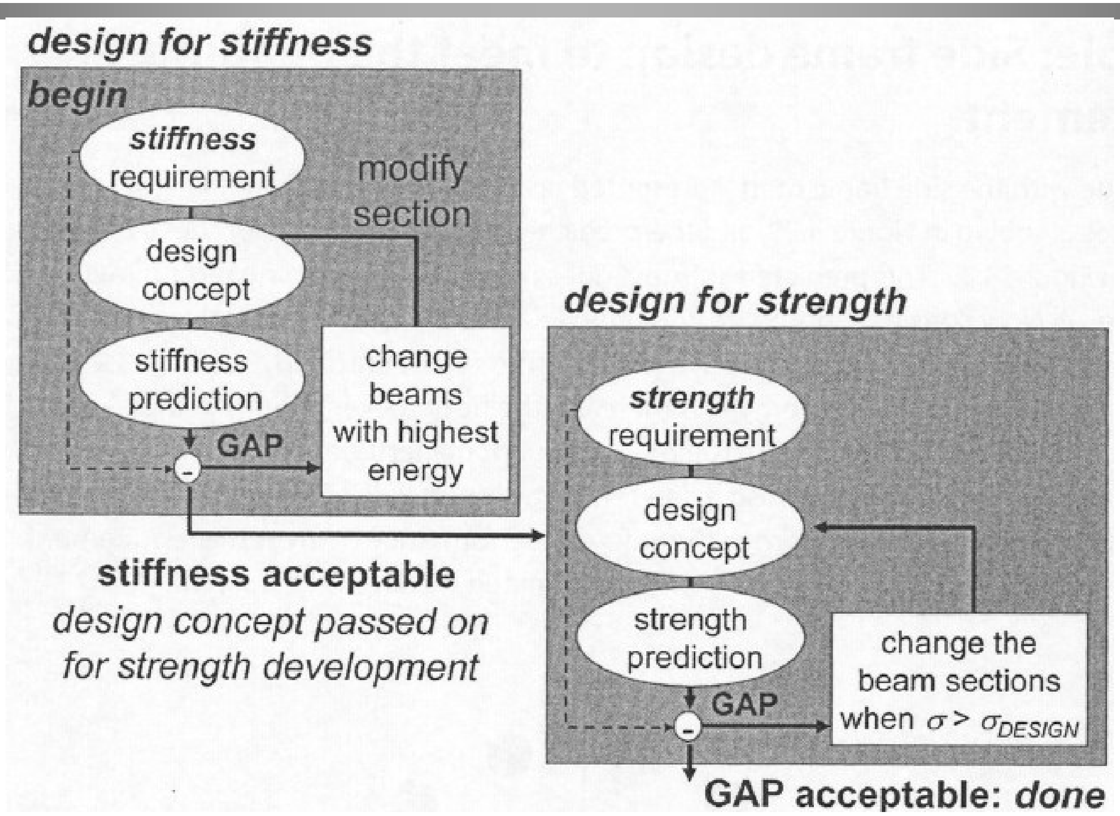
$$e_{\text{beam}} =$$

Example: Seat Mount System

– Assume that the current system design does not meet the (vertical) stiffness requirement, and we wish to know which element to change (beam or joint) to increase the stiffness of the system.

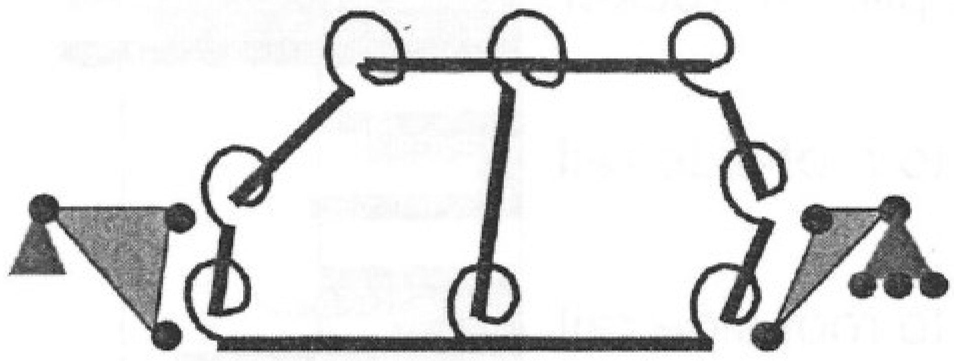


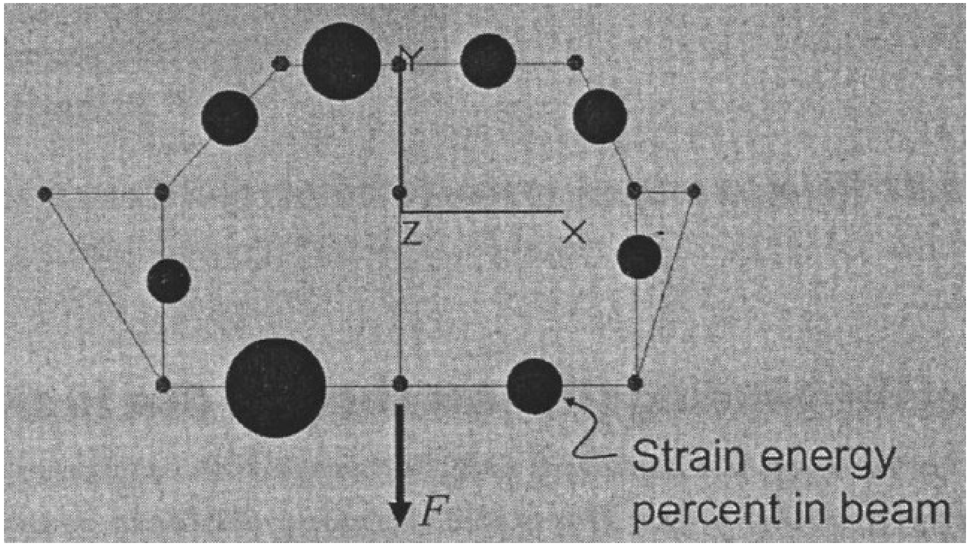
Iterative Process



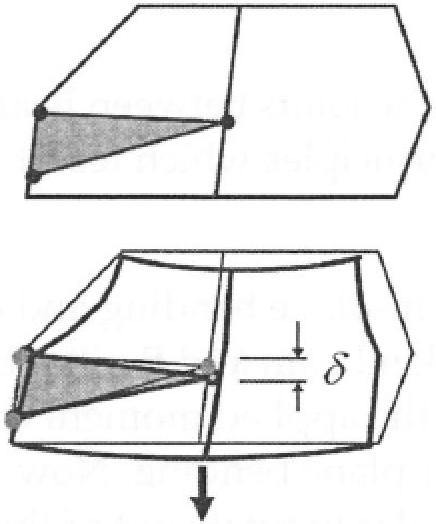
- If stress is greater than the design stress
  - Increase the buckling design stress by inhibiting elastic plate buckling
  - Choose a material with increased yield
  - Reduce the stress by increasing the section properties

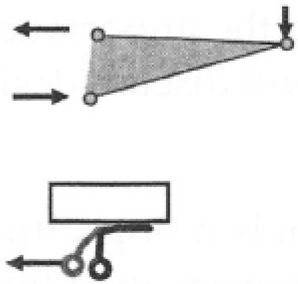
Side Frame Model





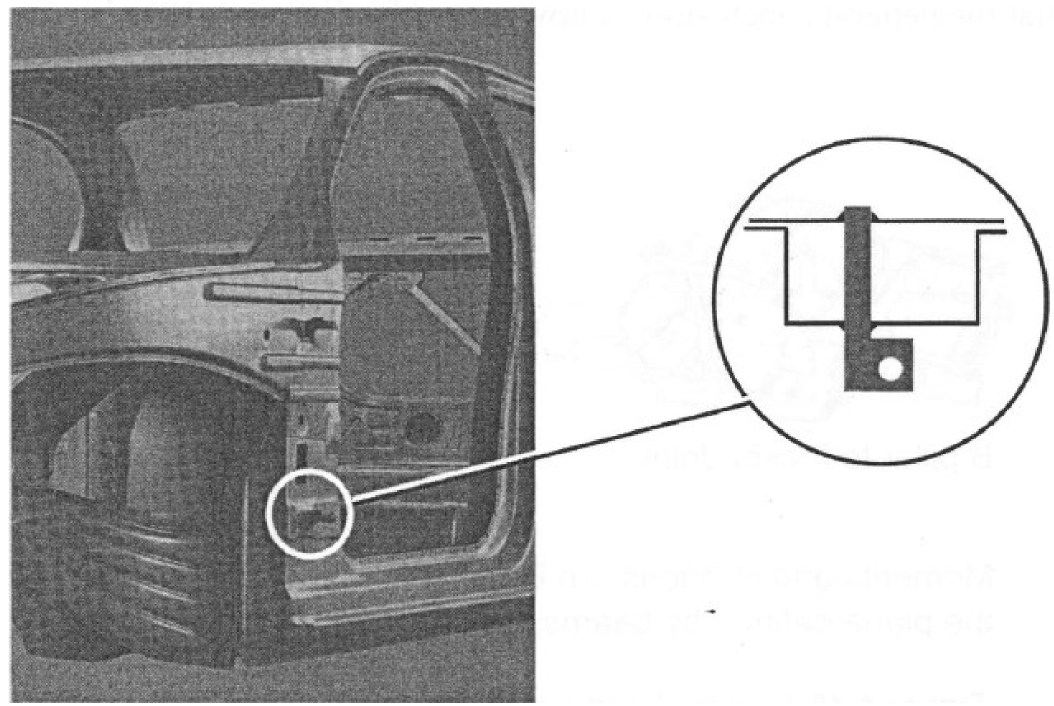
Effect of Door on Body Bending Stiffness





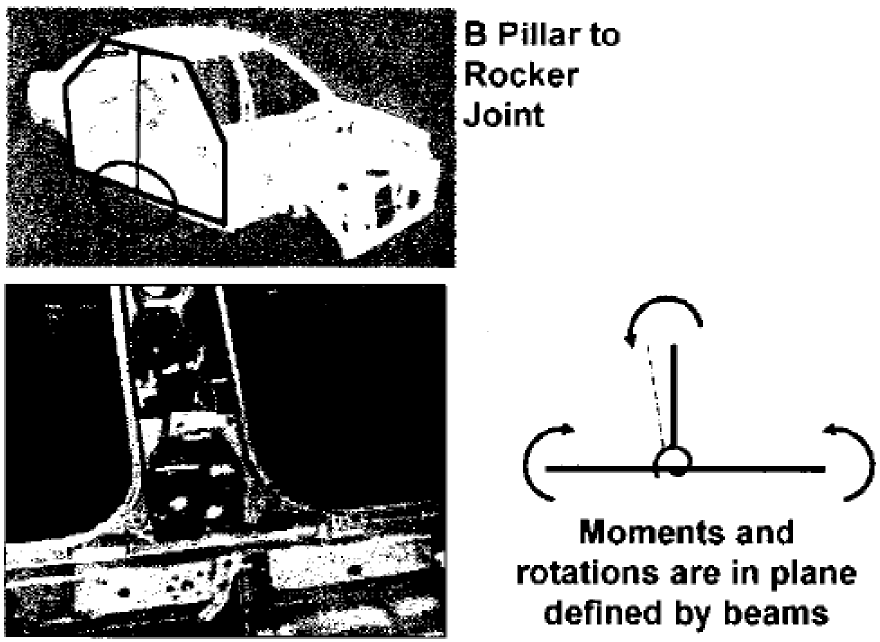
fore-aft hinge stiffness

Typical hinge stiffness is low compared to the door stiffness. Therefore door is considered as rigid.

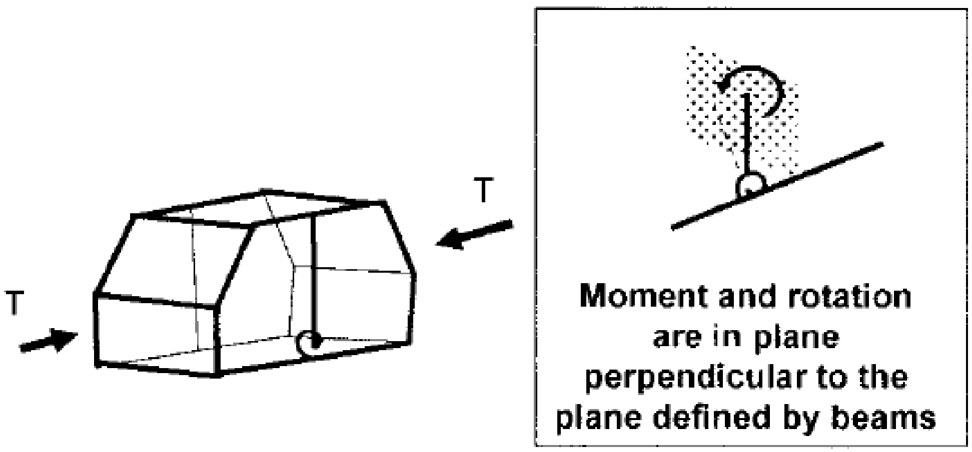


4.5 Principles of Good Joint Design

- Two cases for joint bending stiffness
  - In-plane bending: global body bending
  - out-of-plane bending: global body torsion



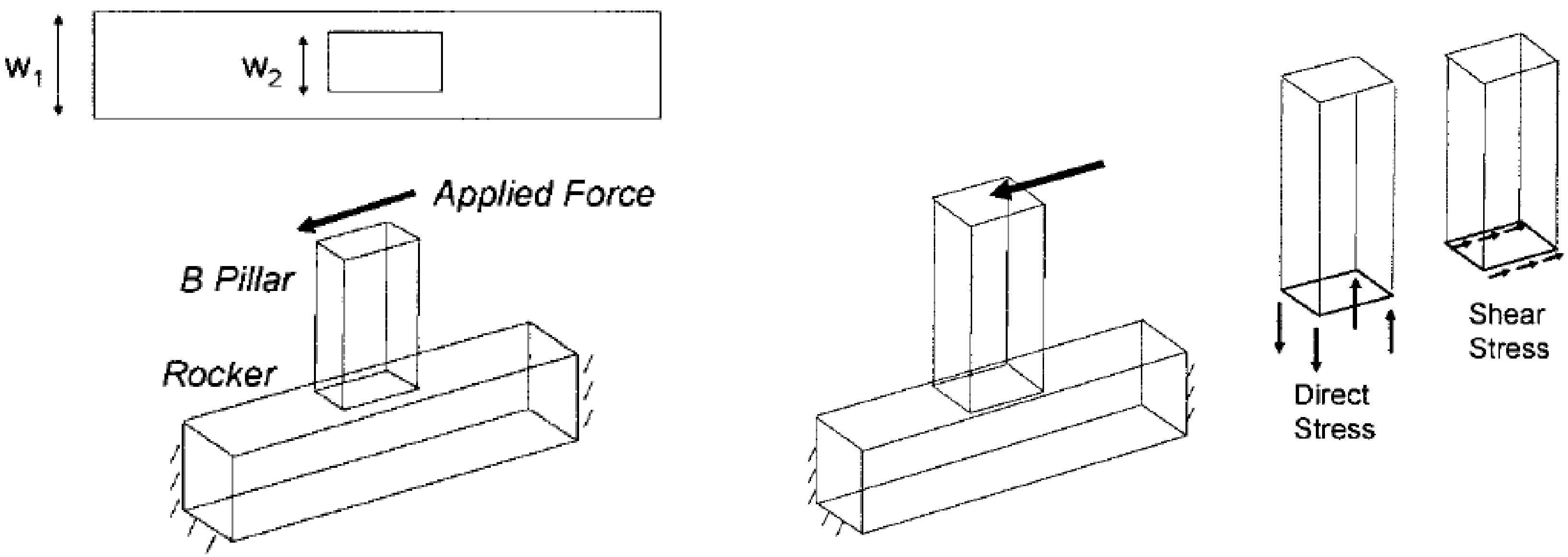
In-Plane Bending



Out-of-Plane Bending

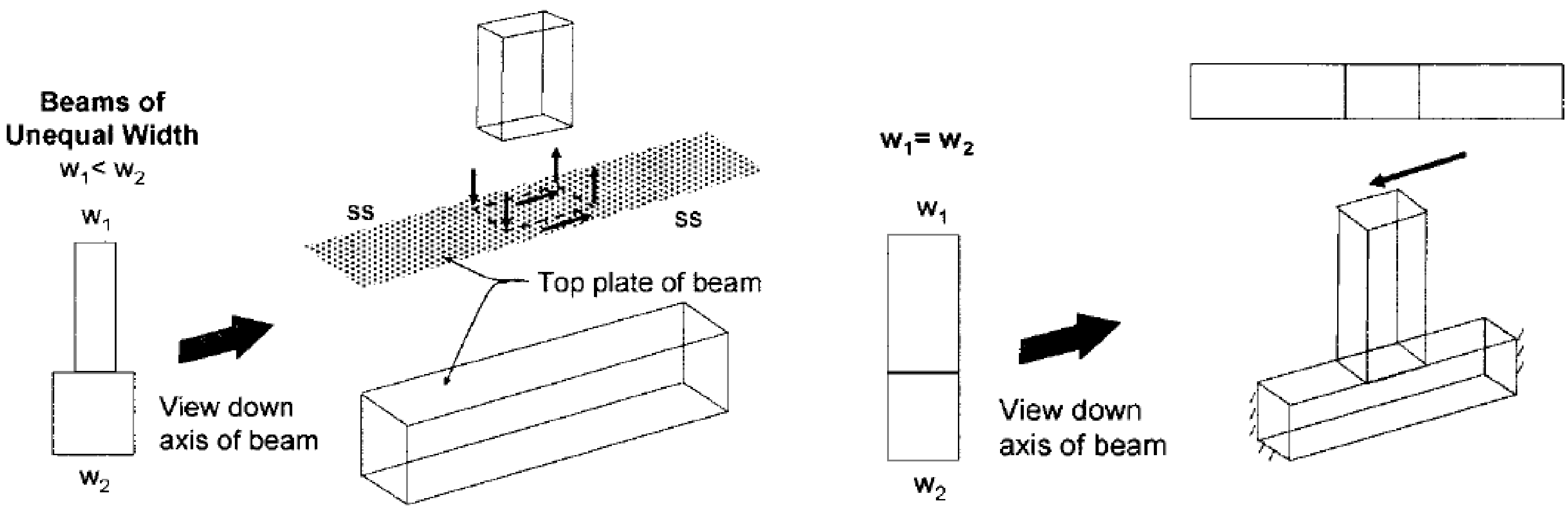
Simplified Joint of Thin-Walled Sections

- Under in-plane bending
  - Thin walled section: relatively high stress at the corners of the section than the center of the walls
  - Assuming that all of the stress is taken by the corners of the section



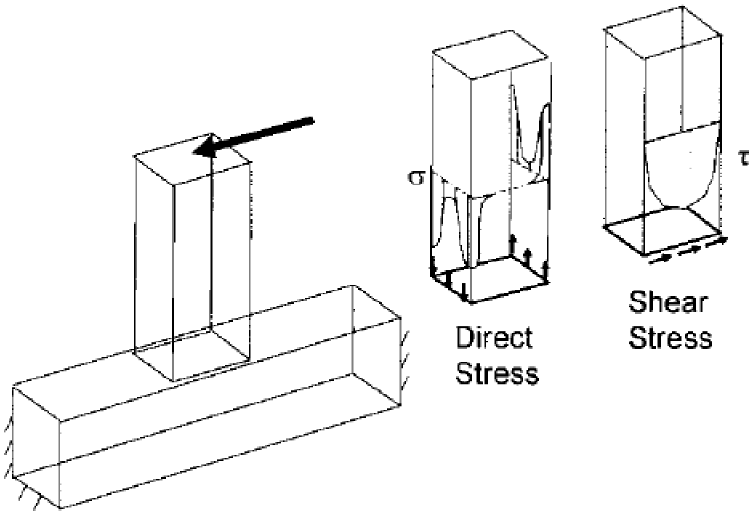
Guideline for Stiff Joint Construction

- For high joint stiffness, the shear walls of the connected beam should be aligned at the joint and flow smoothly from one beam to another

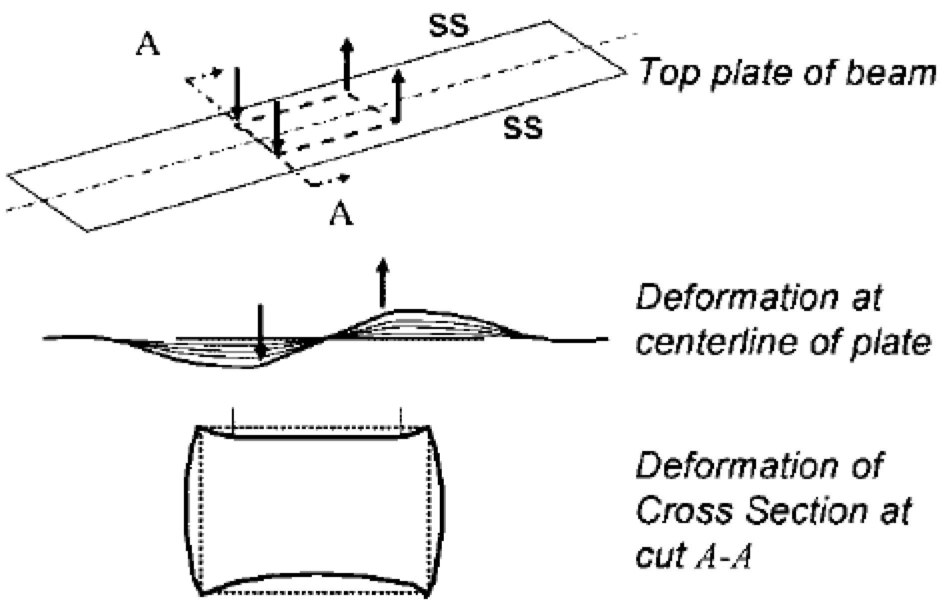


Joint Bending Stiffness

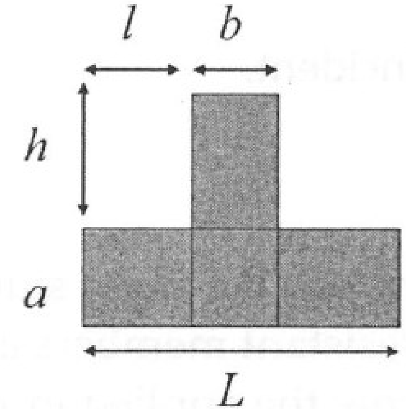
- Stress distribution

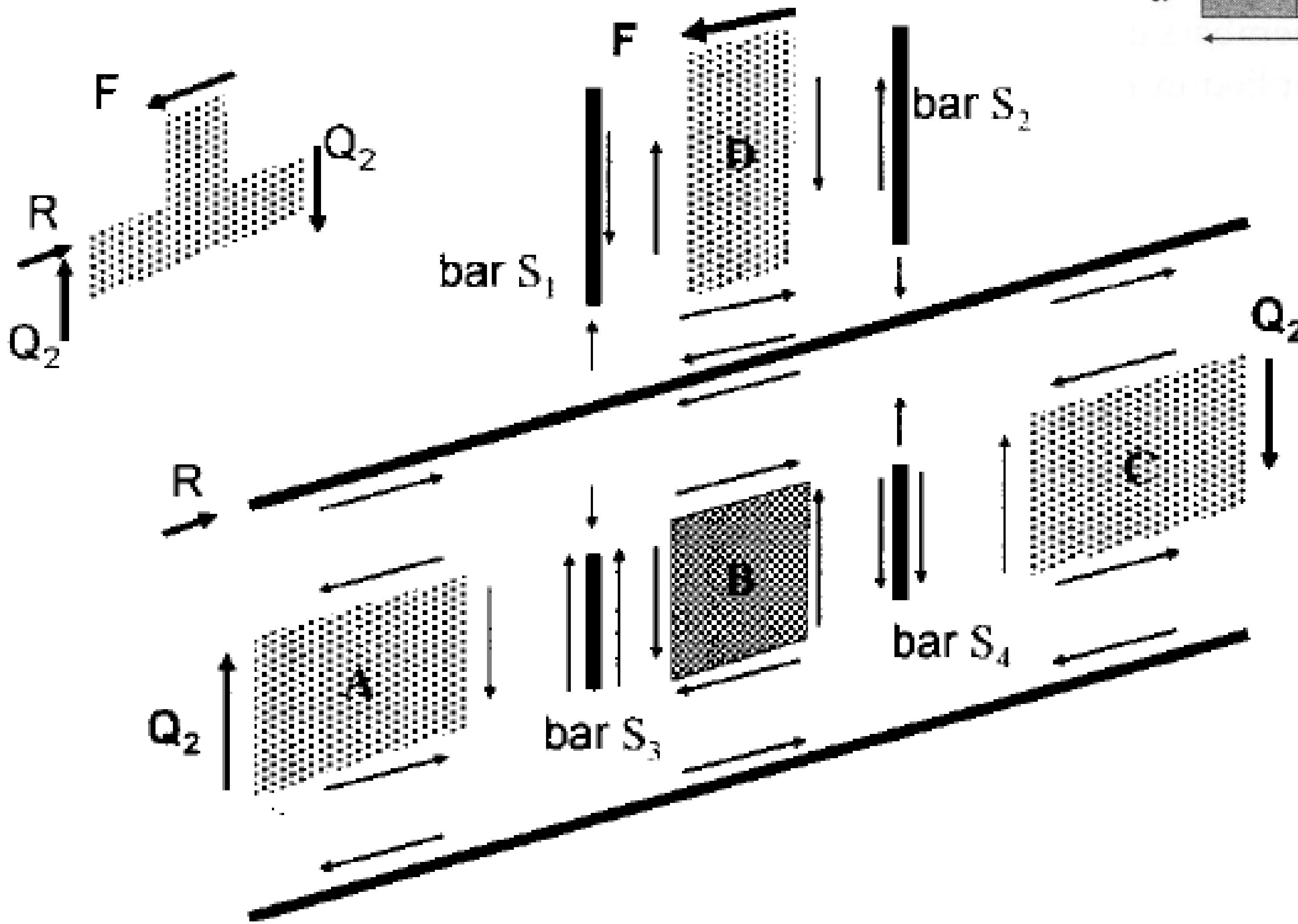


- Deformation



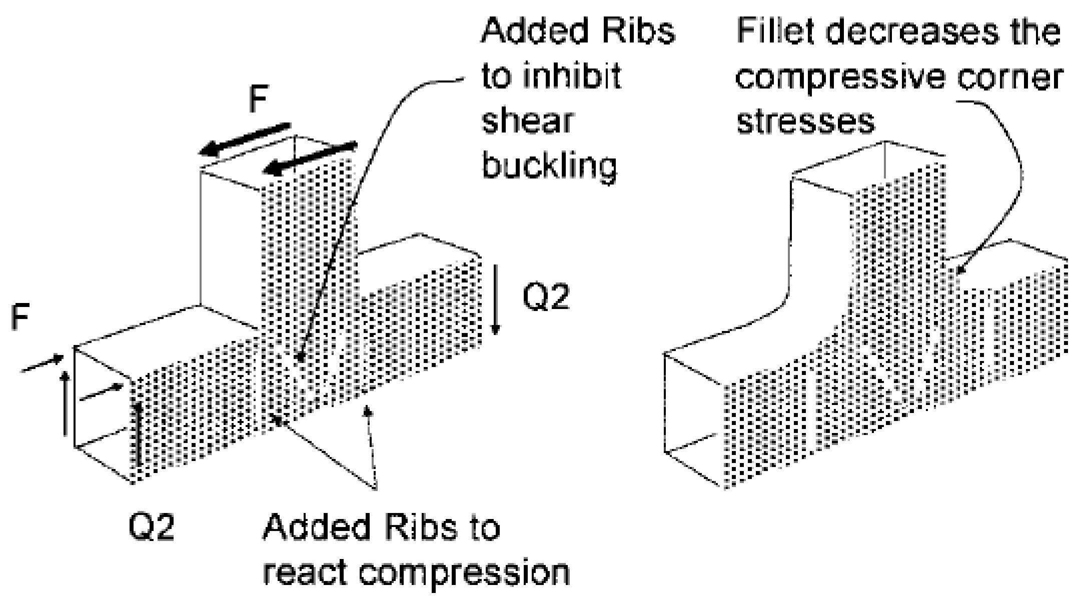
In-Plane Bending





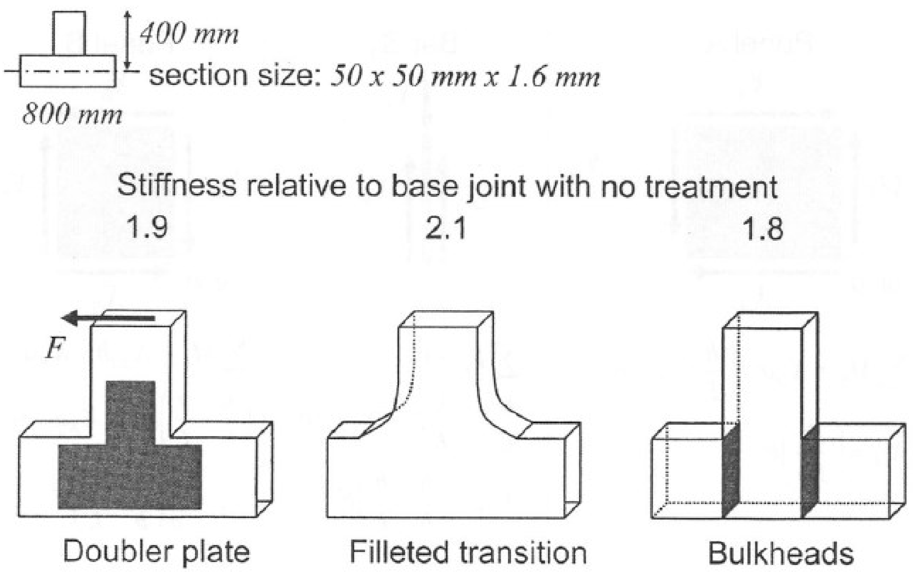
Modification to Joint Shear Wall

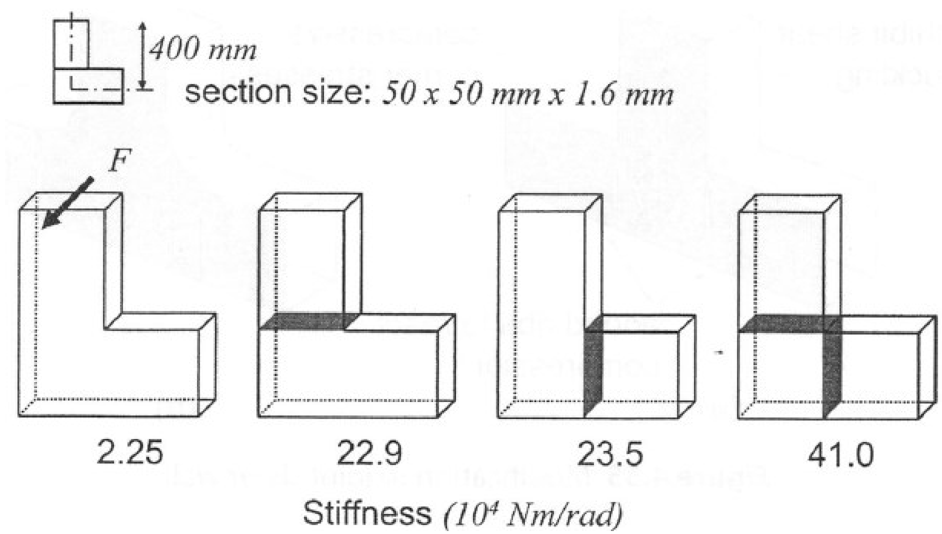
- Concerns: high compression, buckling
  - Rib pattern
    - V rib pattern: increase the shear buckling stress of the panel
    - Vertical rib: provide a path for the compressive load
  - Increase of the span: provide a filleted transition



Experimental Data

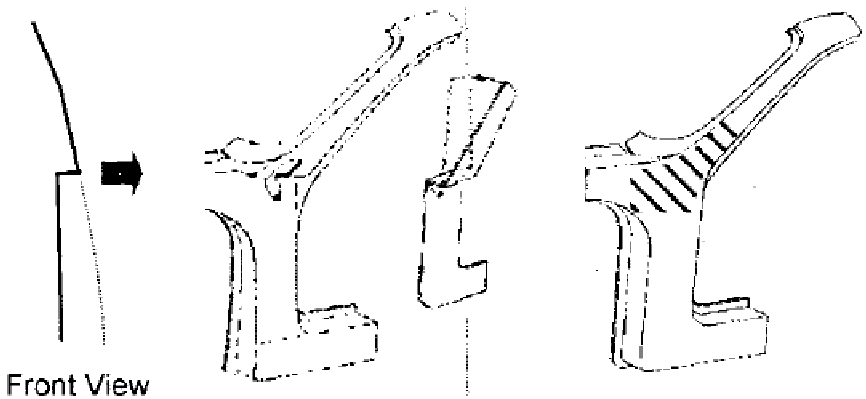
- Joint rigidity in-plane bending
  - Mass penalty
  - Additional load path
- Effect of added bulkhead on out-of-plane joint rigidity





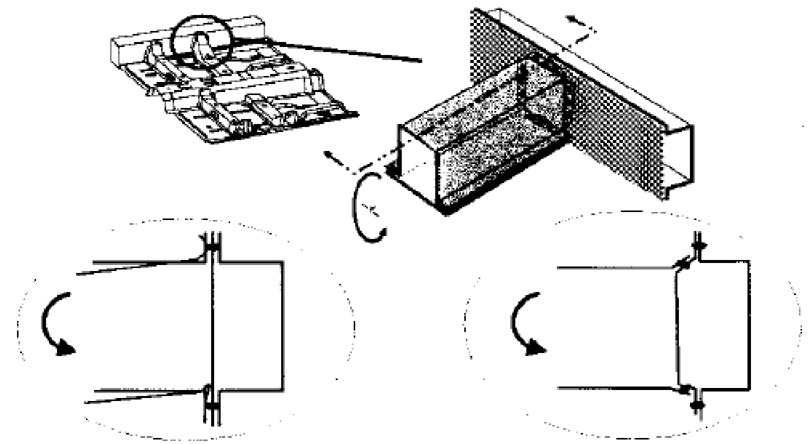
Example: Body Joint Design (1)

- A pillar to Hinge pillar joint
- Floor Cross Member to Rocker joint



Shear elements not aligned in front view with resulting distortion at joint

Shear elements aligned (joint much stiffer)

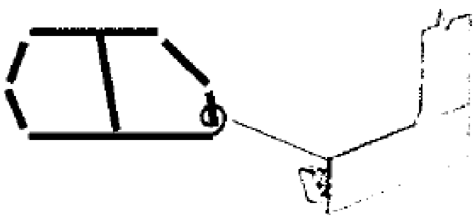


Flange distortion and weld peel (Poor)

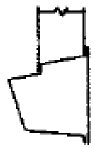
Loads transferred in shear (Better)

Example: Body Joint Design (2)

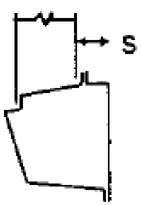
- Hinge pillar to Rocker joint



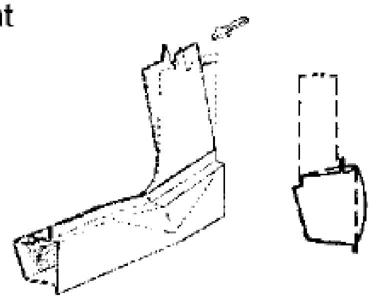
(a) Hinge Pillar to Rocker Joint



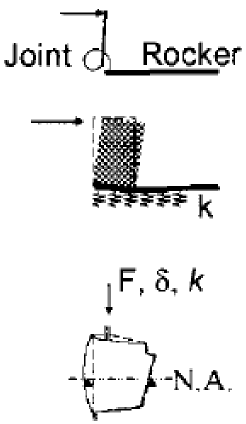
Ideal joint with shear planes in alignment



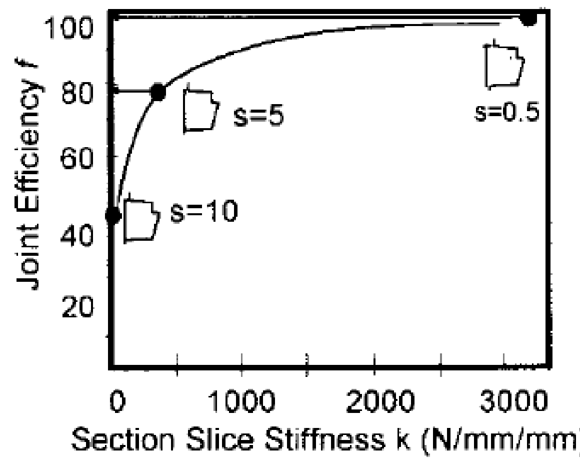
(b) offset, s, between beam shear walls



(c) Deflected shape when offset, s, present



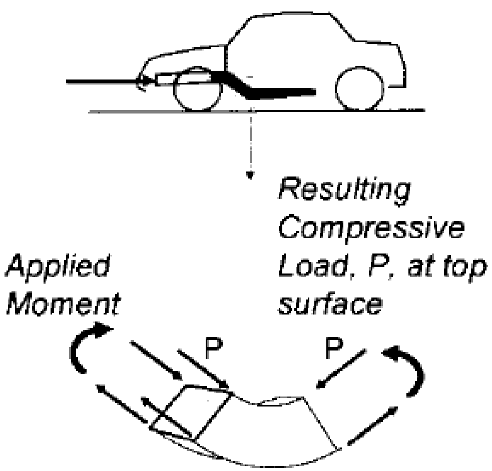
Local distortion of rocker behaving as a joint



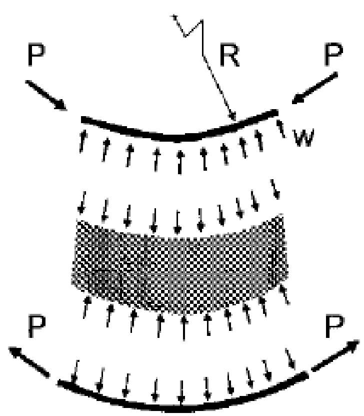
Joint efficiency as a function of section stiffness

Joint Behavior at Geometric Transitions

- Large localized deflection
  - Beam with two relatively straight portions and a central curved portion
- Treat the transition area as a joint stiffness
  - Increase the radius of curvature
  - Include radial ribs which react the compressive web loads
  - Smooth and gradual transition

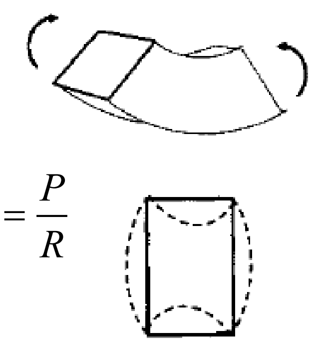


Curved Section

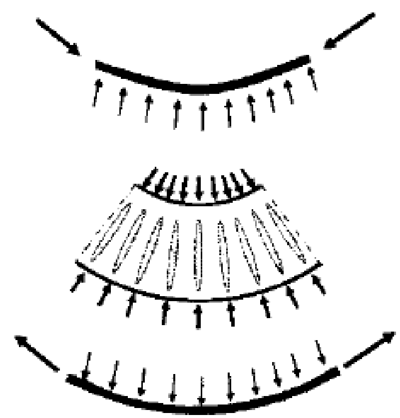


Internal loads

$$w = \frac{P}{R}$$



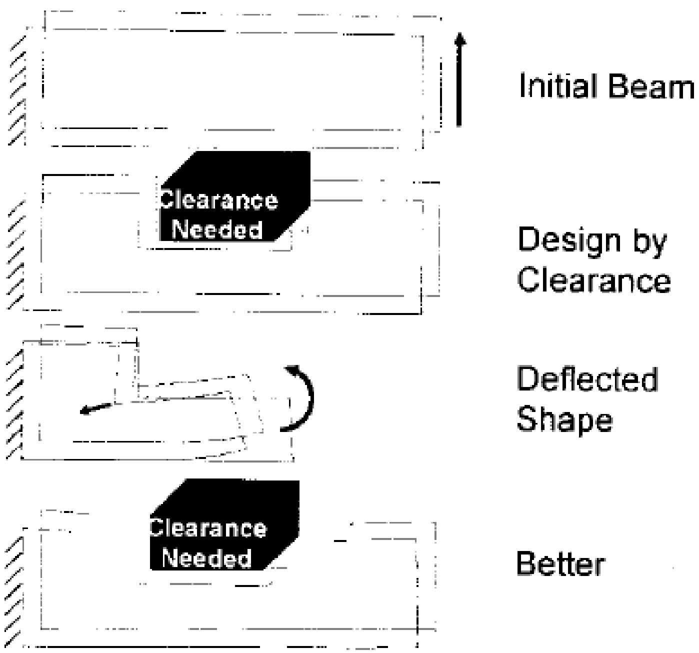
Distortion of Curved Section



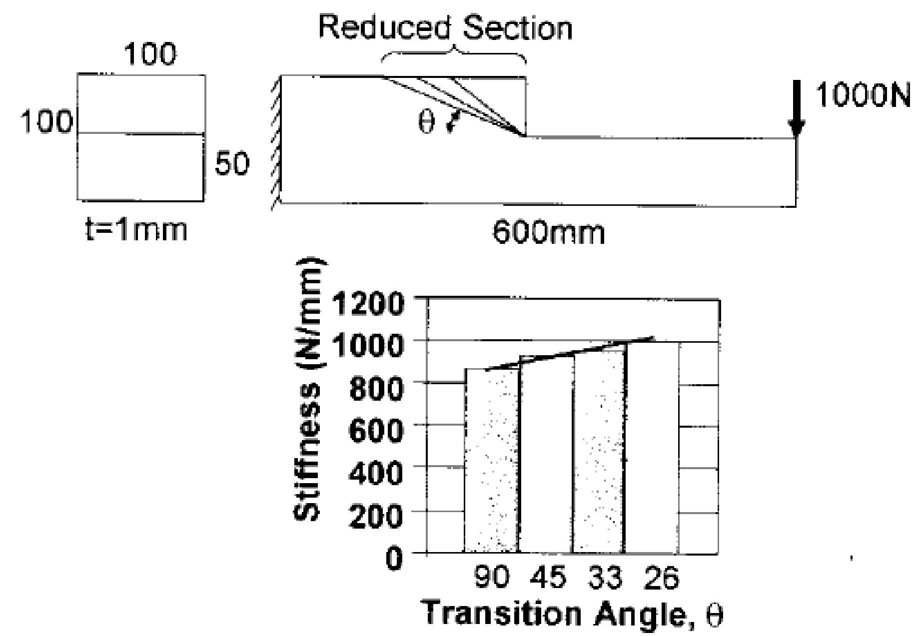
Radial Ribs to React Compression

Geometric Transition in a Straight Beam

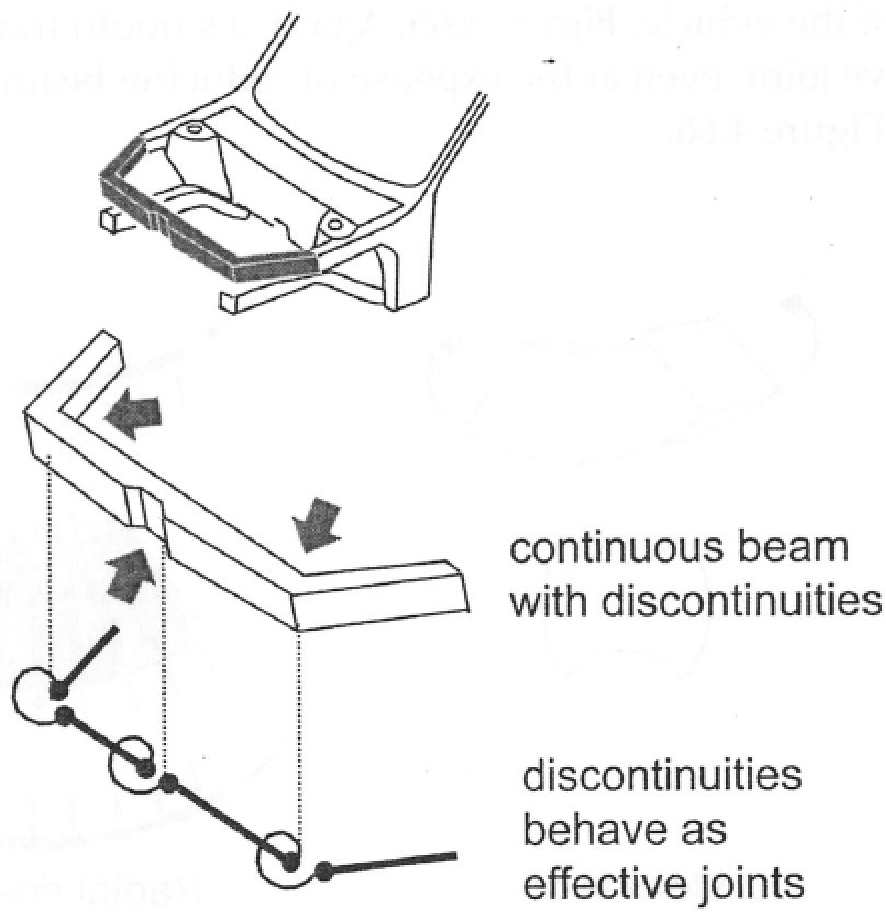
- Clearing some component of the vehicle
- Smooth transition: stiffer effective joint



Section Transition as a Joint



Section Transition


continuous beam
with discontinuities
discontinuities
behave as
effective joints