## **Classification of Engineering Problems**

- Crandall (1956)
- Equilibrium Problems
  - Time-independent problem
  - equilibrium stress, steady state temperature, pressure
- Eigenvalue Problems
  - natural frequency (vibration)
  - buckling load (stability)
- Propagation Problems
  - Time-dependent problem
  - propagation of displacements, heat, wave
  - IVP, transient and unsteady-state phenomena

Linear (elastic) dynamic problems  $M\ddot{u} + C\dot{u} + Ku = f(t)$ Zero damping and external loads  $M\ddot{u} + Ku = 0$ Solution form:  $u(t) = \Phi e^{i\omega t}$ Generalized eigenvalue problem  $(\boldsymbol{K}-\omega^2\boldsymbol{M})\boldsymbol{\Phi}=0$ Non-trivial (eigen-)solutions :  $\left( \begin{array}{c} \omega_i \\ \text{eigenfrequency modeshape} \end{array} \right)$ 

#### **Optimization Problem**

$$\begin{array}{l}
\max_{\rho} \omega_{i}^{2} \\
\text{subject to} \quad \sum_{e=1}^{N} v_{e} \rho_{e} \leq V^{*} \\
0 < \rho_{\min} \leq \rho \leq 1 \\
\left( \mathbf{K} - \omega^{2} \mathbf{M} \right) \Phi = 0
\end{array}$$

design sensitivity of single eigenvalue

$$\frac{d\omega_i^2}{d\rho_e} = \phi_i^T \left(\frac{\partial \mathbf{K}}{\partial\rho_e} - \omega_i^2 \frac{\partial \mathbf{M}}{\partial\rho_e}\right) \phi_i$$
$$\phi_i^T \mathbf{M} \phi_i = 1$$

#### **Fundamental problems**

- Optimization problem is not well posed (less structure → higher eigenfrequency)
  - Solutions: impose mass equality constraint, solve problem as a reinforcement problem or include non-structural masses
- Spurious modes in low density regions
  - Solution: tailored interpolation functions
- Mode switching (non-smooth)
  - Solutions: include more modes or use bound formulation
- Multiple eigenvalues  $\rightarrow$  incorrect sensitivities
  - Solution: compute correct sensitivities !

#### **Spurious Modes**

**Research papers** 

Struct Multidisc Optim 20, 2-11 © Springer-Verlag 2000

#### Maximization of eigenvalues using topology optimization





Vehicle Structure Optimization

#### Topology Optimization: Extension - 5

#### Modified SIMP



Topology Optimization: Extension - 6

#### **Discontinuous Stiffness**



#### **RAMP** Interpolation

Struct Multidisc Optim 22, 116–124 © Springer-Verlag 2001

## An alternative interpolation scheme for minimum compliance topology optimization

#### M. Stolpe and K. Svanberg



Vehicle Structure Optimization

Topology Optimization: Extension - 8

## Mode Switching (1)

• Weighted average



Bound formulation

$$\max_{\rho} \beta$$
  
subject to  $\beta - \omega_i^2 \le 0$ ,  $i \in \{1, i_{\max}\}$ 
$$\sum_{e=1}^N v_e \rho_e \le V^*$$
$$0 < \rho_{\min} \le \rho \le 1$$
$$\left(\mathbf{K} - \omega^2 \mathbf{M}\right) \Phi = 0$$

Choose  $i_{max}$  large enough to ensure smoothness !

## Mode Switching (2)

Mean eigenvalue

$$\Lambda = \lambda_0 + \left( \sum_{i=1}^m w_i \left( \lambda_{n_i} - \lambda_{0_i} \right)^n / \sum_{i=1}^m w_i \right)^{\frac{1}{n}} \quad (\text{for } n = \pm 1, \pm 2, \dots; n \neq 0)$$

- $\lambda_{n_i}$ : chosen eigenvalues
- $w_i$ : given weighting coefficients
- $\lambda_0, \lambda_{0_i}$ : given shift parameters
- *m*: number of eigenvalues of interest

*n*: given power

$$\begin{cases} \text{case 1: } n = -1, \ m = 1, \ \lambda_0 = 0, \ \sum_{i=1}^m w_i = 1 \Longrightarrow \Lambda = \lambda_{n_1} \ \text{(lowest eigenvalue)} \\ \text{case 2: } n = -1, \ \sum_{i=1}^m w_i = 1 \Longrightarrow \Lambda = \lambda_0 + \frac{1}{\sum_{i=1}^m \frac{w_i}{\lambda_{n_i} - \lambda_{0_i}}} \to \tilde{\Lambda} = \frac{1}{\Lambda - \lambda_0} = \sum_{i=1}^m \frac{w_i}{\lambda_{n_i} - \lambda_{0_i}} \end{cases}$$

Maximize  $\Lambda$  (maximize the mean eigenvalue) Maximize  $\Lambda$  with  $\lambda_{0_i} = (2\pi\omega_{0_i})^2$  (away from the specified eigenvalue) Minimize  $\Lambda$  with  $\lambda_{0_i} = (2\pi\omega_{0_i})^2$  (approach to the specified eigenvalue)

#### Sensitivity Analysis (1)

$$\begin{aligned} \text{Maximize } \Lambda &\to \text{Minimize } f\left(x\right) = -\Lambda \\ \Lambda &= \lambda_0 + \left(\sum_{i=1}^m w_i \left(\lambda_{n_i} - \lambda_{0_i}\right)^n \middle/ \sum_{i=1}^m w_i\right)^{\frac{1}{n}} \to \left(\Lambda - \lambda_0\right)^n = \frac{\sum_{i=1}^m w_i \left(\lambda_{n_i} - \lambda_{0_i}\right)^n}{\sum_{i=1}^m w_i} \\ &\to \overline{f} = \left(\Lambda - \lambda_0\right)^n \sum_{i=1}^m w_i - \sum_{i=1}^m w_i \left(\lambda_{n_i} - \lambda_{0_i}\right)^n = 0 \\ &\to \frac{\partial \overline{f}}{\partial \rho} = n \left(\Lambda - \lambda_0\right)^{n-1} \frac{\partial \Lambda}{\partial \rho} \sum_{i=1}^m w_i - n \sum_{i=1}^m w_i \left(\lambda_{n_i} - \lambda_{0_i}\right)^{n-1} \frac{\partial \lambda_{n_i}}{\partial \rho} = 0 \\ &\to \frac{\partial \Lambda}{\partial \rho} = \frac{\left(\Lambda - \lambda_0\right)^{1-n}}{\sum_{i=1}^m w_i} \sum_{i=1}^m w_i \left(\lambda_{n_i} - \lambda_{0_i}\right)^{n-1} \frac{\partial \lambda_{n_i}}{\partial \rho} \\ &= \frac{n - 1}{\sum_{i=1}^m w_i} \frac{\partial \Lambda}{\partial \rho} = \frac{\left(\Lambda - \lambda_0\right)^2}{\sum_{i=1}^m w_i} \sum_{i=1}^m \frac{w_i}{\left(\lambda_{n_i} - \lambda_{0_i}\right)^2} \left(\frac{\partial \lambda_{n_i}}{\partial \rho}\right) \end{aligned}$$

#### Sensitivity Analysis (2)

$$\frac{\partial \overline{\Lambda}}{\partial \rho_{e}} = \frac{\left(\Lambda - \lambda_{0}\right)^{2}}{\sum_{i=1}^{m} w_{i}} \sum_{i=1}^{m} \frac{w_{i}}{\left(\lambda_{n_{i}} - \lambda_{0_{i}}\right)^{2}} \frac{\partial \lambda_{n_{i}}}{\partial \rho_{e}}$$

$$\frac{\left\{\frac{\partial \lambda_{n_{i}}}{\partial \rho_{e}} = \phi_{n_{i}}^{T} \left(\frac{\partial \mathbf{K}}{\partial \rho_{e}} - \lambda_{n_{i}} \frac{\partial \mathbf{M}}{\partial \rho_{e}}\right) \phi_{n_{i}} = \phi_{n_{i}}^{T} \left(\frac{\partial \mathbf{k}_{e}}{\partial \rho_{e}} - \lambda_{n_{i}} \frac{\partial \mathbf{m}_{e}}{\partial \rho_{e}}\right) \phi_{n_{i}}$$
where
$$\frac{\left\{\frac{\partial \mathbf{k}_{e}}{\partial \rho_{e}} = \int_{\Omega_{e}} \mathbf{B}_{e}^{T} \frac{\partial \mathbf{E}_{e}}{\partial \rho_{e}} \mathbf{B}_{e} d\Omega \right\}}{\left(\frac{\partial \mathbf{m}_{e}}{\partial \rho_{e}}\right)^{2} = \int_{\Omega_{e}} m_{0} N_{e}^{T} N_{e} d\Omega$$

$$\xrightarrow{\text{filtering}} \xrightarrow{\frac{\partial \overline{\Lambda}}{\partial \rho_{e}}} = \frac{1}{\rho_{e}} \sum_{f=1}^{N} H_{f} \rho_{e} \frac{\partial \overline{\Lambda}}{\partial \rho_{e}}$$

$$\frac{\partial \overline{g}}{\partial x_{e}} = \frac{v_{e}}{V_{0}}$$

#### Modified Optimality Criteria Method

$$\begin{cases} \underset{x}{\text{Minimize }} f\left(x\right) = \tilde{\Lambda} \\ \text{subject to } g\left(x\right) = \frac{V\left(x\right)}{V_{0}} - 1 = \frac{\sum_{e} v_{e} \rho_{e}}{V_{0}} - 1 \le 0 \\ 0 < \rho_{\min} \le \rho_{e} \le 1 \end{cases} \\ \rightarrow \frac{\partial f}{\partial \rho_{e}} + \lambda \frac{\partial g}{\partial \rho_{e}} = \alpha_{i}^{l} - \alpha_{i}^{u} \rightarrow D_{e} = \frac{1}{\lambda} \left( -\frac{\partial f}{\partial \rho_{e}} \middle/ \frac{\partial g}{\partial \rho_{e}} \right) = 1 \quad \text{for } 0 < \rho_{\min} \le \rho_{e} \le 1 \\ \rightarrow \rho_{e}^{k+1} = \left( D_{e}^{k} \right)^{\eta} \rho_{e}^{k} \quad \text{for } 0 < \rho_{\min} \le \rho_{e} \le 1 \\ D_{e}^{k} : \text{ positive } ? \\ \left( \frac{\partial f}{\partial \rho_{e}} - \mu \frac{\partial g}{\partial \rho_{e}} \right) + \lambda^{*} \frac{\partial g}{\partial \rho_{e}} = 0 \quad \text{for } \rho_{e}^{l} < \rho_{e} < \rho_{e}^{u} \quad \left( \lambda^{*} = \lambda + \mu \right) \\ \overline{D}_{e} = \frac{1}{\lambda^{*}} \left( \mu - \frac{\partial f}{\partial \rho_{e}} \middle/ \frac{\partial g}{\partial \rho_{e}} \right) = 1 \quad \text{for } \rho_{e}^{l} < \rho_{e} < \rho_{e}^{u} \quad \text{where } \mu \ge \max_{e} \left( \frac{\partial f}{\partial \rho_{e}} \middle/ \frac{\partial g}{\partial \rho_{e}} \right) \\ \rightarrow v_{e}^{k+1} = \left( \overline{D}_{e}^{k} \right)^{\eta} \rho_{e}^{k} \quad \text{for } 0 < \rho_{\min} \le v_{e} \le 1 \end{cases}$$

## Multiple Eigenvalues (1)

- Inherent property for 2- or 3-dimensional homogeneous and symmetric (eg square or cubic) structures
- Often an outcome of the optimization procedure



## Multiple Eigenvalues (2)





**b**  $\omega_{2a} = \omega_{1a}^{opt} = 174.7$ 



double eigenvalue:  $(\lambda = \omega^2, \phi_1, \phi_2)$ 

normalized mode shapes

$$\begin{cases} \boldsymbol{\phi}_1^T \boldsymbol{M} \boldsymbol{\phi}_1 = 1 \\ \boldsymbol{\phi}_2^T \boldsymbol{M} \boldsymbol{\phi}_2 = 1 \end{cases}$$

mode shapes are orthogonal

$$\boldsymbol{\phi}_1^T \boldsymbol{M} \boldsymbol{\phi}_2 = \boldsymbol{0}$$

## Sensitivity of Multiple Eigenvalue

$$\begin{pmatrix} \lambda, \phi_{1}, \phi_{2} \\ \phi_{1}^{T}M\phi_{2} = 0 \end{pmatrix} \rightarrow \begin{cases} \overline{\phi} = c_{1}\phi_{1} + c_{2}\phi_{2} \rightarrow \frac{d\lambda}{d\rho_{e}} = \overline{\phi}^{T} \left( \frac{dK}{d\rho_{e}} - \lambda \frac{dM}{d\rho_{e}} \right) \overline{\phi} \end{cases} \qquad \qquad \text{There is the state of the set of the set$$

Vehicle Structure Optimization

Topology Optimization: Extension - 16

Available online at www.sciencedirect.com

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## Mode Tracking (1)

- the order of eigenmodes may fluctuate
- critical to keep track of the target modes accurately during the optimization process
- MAC (modal assurance criterion)
  - used mostly in checking the correlation between experimental and numerical mode shapes
  - not explicitly used in the definition of the objective or the constraint function



PERGAMON

Computers and Structures 74 (2000) 375-383

www.elsevier.com/locate/compstruc

Computers & Structures

Mac-based mode-tracking in structural topology optimization

Tae Soo Kim, Yoon Young Kim\*

$$\mathrm{MAC}(\boldsymbol{\Phi}_{a},\boldsymbol{\Phi}_{b}) = \frac{\left|\boldsymbol{\Phi}_{a}^{T}\boldsymbol{\Phi}_{b}\right|^{2}}{\left(\boldsymbol{\Phi}_{a}^{T}\boldsymbol{\Phi}_{a}\right)\left(\boldsymbol{\Phi}_{b}^{T}\boldsymbol{\Phi}_{b}\right)}$$

 $\boldsymbol{\Phi}_{a}, \boldsymbol{\Phi}_{b}$ : two mode shape vectors of interest  $0 \leq MAC(\boldsymbol{\Phi}_{a}, \boldsymbol{\Phi}_{b}) \leq 1$  $MAC(\boldsymbol{\Phi}_{a}, \boldsymbol{\Phi}_{b}) = 1 \Rightarrow$  exactly the same mode shape

## Mode Tracking (2)



#### Flowchart



#### **Buckling Problem**



Structural Optimization 10, 71-78 © Springer-Verlag 1995

INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING Int. J. Numer. Meth. Engng 2002; 54:809–834 (DOI: 10.1002/nme.449)

#### Technical Papers

Generalized topology design of structures with a buckling load criterion

M.M. Neves, H. Rodrigues and J.M. Guedes IDMEC-Instituto Superior Técnico, Mechanical Engineering Department, Av. Rovisco Pais 1, P-1096 Lisboa Codex, Portugal

#### Topology optimization of periodic microstructures with a penalization of highly localized buckling modes

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#### Topology Optimization: Extension - 20

#### **Frequency Response Problems**

$$M\ddot{u} + C\dot{u} + Ku = f \xrightarrow{f = Fe^{j\omega t}} (K + j\omega C - \omega^2 M) U = F$$

- 직접법(Direct Frequency Response):  $U = (K + j\omega C \omega^2 M)^{-1} F$
- 모드중첩법(Modal Frequency Response):

$$\boldsymbol{u} = \sum_{i=1}^{n} \phi_{i} p_{i} \rightarrow \boldsymbol{M} \ddot{\boldsymbol{u}} + \boldsymbol{K} \boldsymbol{u} = \boldsymbol{f} \rightarrow \underbrace{\overline{\phi_{i}}^{T} \boldsymbol{M} \phi_{i}}_{m_{i}} \ddot{p}_{i} + \underbrace{\overline{\phi_{i}}^{T} \boldsymbol{K} \phi_{i}}_{k_{i}} p_{i} = \overline{\phi_{i}}^{T} \boldsymbol{f} \quad (i = 1, ..., n)$$

$$\xrightarrow{\boldsymbol{f} = \boldsymbol{F} e^{j\omega t}, \text{ orthogonality}}_{\boldsymbol{u} = U e^{j\omega t}, p_{i} = q_{i} e^{j\omega t}} \rightarrow \boldsymbol{q}_{i} = \frac{\overline{\phi_{i}}^{T} \boldsymbol{F}}{k_{i} - \omega^{2} m_{i}} = \frac{\overline{\phi_{i}}^{T} \boldsymbol{F}}{m_{i} \left(\omega_{i}^{2} - \omega^{2}\right)}$$

$$\leftrightarrow \boldsymbol{U} = \sum_{i=1}^{n} \phi_{i} \left(\frac{\boldsymbol{\phi_{i}}^{T} \boldsymbol{F}}{\omega_{i}^{2} - \omega^{2}}\right) = \sum_{i=1}^{n} \phi_{i} q_{i} = \Phi \boldsymbol{Q} \leftarrow \begin{cases} \omega_{i}^{2}: \text{ eigenvalue} \\ \phi_{i}: \text{ eigenvector} \end{cases}$$

#### Frequency Response Problems: Objective Function

Minimize 
$$\pi = \frac{1}{2} | \boldsymbol{U}^T \boldsymbol{D} \boldsymbol{U} |$$

- Dynamic mean compliance:  $D = 2(K + j\omega C \omega^2 M)$
- Internal energy of the structure: D = K
- Kinetic energy of the structure: D = M
- Norm of frequency responses: D = I
- Frequency response at specified DOF of the structure:

$$\boldsymbol{D} = diag\{w_i\} \text{ where } \begin{cases} w_i \neq 0 \text{ at specified DOF's} \\ w_i = 0 \text{ at the other DOF's} \end{cases}$$

• 특정 주파수 대역([ $\omega_a, \omega_b$ ])에서 고려

Minimize 
$$\Pi = \int_{\omega_a}^{\omega_b} \pi(\omega) d\omega = \sum_{i} w_i \pi(\omega_i)$$
  
( $w_i, \omega_i$ ): quadrature weights and point



Topology Optimization: Extension - 23











Topology Optimization: Extension - 26

## **Compliant Mechanism**

- Mechanisms which intentionally use the structural fle xibility as a mechanism function
- Kinematics (Flexibility) + Structure (Stiffness)
- Elastic Deformation  $\Rightarrow$  Mechanical Function
- Jointless Mechanism : Less Wear, Noise, etc.
- Manufacturing with NO Assembly
  - Micro Machine, MEMS application





## **Design of Flexible Structures**

• Kinematic function  $\rightarrow$  Flexibility



• Structural function  $\rightarrow$  Stiffness



#### **Optimization Problem**



where *w* is the weighting coefficient such that  $0 \le w \le 1$ 

#### **Example: Compliant Clamp**



Topology Optimization: Extension - 30

## **Example: Compliant Gripper**



To maximize the displacement in the direction of F2 at point P2 when the force is applied at point P1.



## **Comparison of Analogous Properties**

| Conorio Torm              | Discipline                    |                             |                               |  |
|---------------------------|-------------------------------|-----------------------------|-------------------------------|--|
| Generic Term              | Elasticity                    | Heat Conduction             | Magnetostatics                |  |
| Potential function        | {u}                           | u                           | {A}                           |  |
| "Strain"                  | {3}                           | -{∇u}                       | {B}                           |  |
| "Stress"                  | <b>{σ}</b>                    | {q}                         | {H}                           |  |
| Load Density              | {p}                           | {Q}                         | {J}                           |  |
| Boundary Load             | {t}                           | {q <sup>b</sup> }           | Нхп                           |  |
| Definition of Strain      | {ε} = [L] {u}                 | -{∇u}                       | $\{B\} = \nabla x \{A\}$      |  |
| Constitutive Relationship | {σ} = [D] {ε}                 | $\{q\} = -[K] \{\nabla u\}$ | $\{H\} = [\mu]^{-1} [B]$      |  |
| Potential Energy          | 1⁄2 {σ} <sup>T</sup> {ε}      |                             | ½ {H} <sup>⊺</sup> {B}        |  |
| Equilibrium               | $\Sigma \sigma_{ij,j} = -P_i$ | ∇ • {q} = Q                 | $\nabla \times \{H\} = \{J\}$ |  |
| Inertia Loading           | $\{p^i\} = -\rho\{u,_{tt}\}$  | $Q^s = -\rho cu,_t$         |                               |  |

R. H. MacNeal, Finite Elements: Their Design and Performance, Marcel Dekker, 1994

#### Heat Transfer Problem

Conduction and convection in steady state

$$(\text{strong form}) \begin{cases} \frac{\partial}{\partial x_i} \left( K_{ij} \frac{\partial T}{\partial x_j} \right) = f & \text{in } \Omega \\ T = g & \text{on } \Gamma_g \\ \left( -K_{ij} \frac{\partial T}{\partial x_j} \right) n_i = h_i & \text{on } \Gamma_h \\ \left( -K_{ij} \frac{\partial T}{\partial x_j} \right) n_i = k_0 (T - T_\infty) & \text{on } \Gamma_\infty \end{cases} \xrightarrow{\Gamma_g} \Gamma_g$$

$$\Rightarrow (\text{weak form}) \begin{cases} \int_{\Omega} K_{ij} \frac{\partial T}{\partial x_j} \frac{\partial T}{\partial x_i} d\Omega - \int_{\Omega} f \widetilde{T} d\Omega + \int_{\Gamma_h} h \widetilde{T} d\Gamma - \int_{\Gamma_u} k_0 (T_\infty - T) \widetilde{T} d\Gamma = 0 \\ \widetilde{T} = 0 & \text{on } \Gamma_g \end{cases}$$

#### **Optimization Problem**

temperature  $T \leftrightarrow$  displacement  $u_i$ thermal conductivity  $K_{ij} \leftrightarrow$  elasticity  $E_{ijkl}$ heat flux  $q_i \leftrightarrow$  stress  $\sigma_{ij}$ temperature gradient  $T_i \leftrightarrow$  strain  $\varepsilon_{ii}$ 

 $\begin{cases} \text{heat flux energy} \leftrightarrow \text{strain energy density} \\ \text{high conductivity structure} \leftrightarrow \text{stiff structure} \end{cases}$ 

 $\begin{array}{c|c} \underset{\substack{design\\ \text{subject to}\\ equilibrium\\ volume}}{\text{minimize}} & \text{heat flux energy} \Rightarrow \underset{\substack{design\\ \text{subject to}\\ volume}}{\text{minimize}} & \text{total potential energy} \end{array}$ 

$$\begin{array}{l} \underset{\substack{(a,b,\theta)\\\text{subject to}\\\int_{\Omega}\rho d\Omega \leq \Omega_{s}}{} & \text{minimize } F(T) \\ F(T) = \frac{1}{2} \int_{\Omega} K_{ij} \frac{\partial T}{\partial x_{i}} \frac{\partial T}{\partial x_{i}} d\Omega - \int_{\Omega} fT d\Omega + \int_{\Gamma_{h}} hT d\Gamma + \frac{1}{2} \int_{\Gamma_{\infty}} k_{0} T^{2} d\Gamma - \int_{\Gamma_{\infty}} k_{0} T_{\infty} T d\Gamma \end{array}$$

#### **Example: Heat Sources and Sinks**

- Heat source:  $f = 0.2 \text{ W/m}^2$
- Fixed temperature:  $T_1 = 0$  °C
- Material:  $k_1 = 75$  W/m C (white),  $k_2 = 2 k_1$  (black)
- Volume ratio  $(k_1: k_2) = 1:1$
- Measure: high conductivity, integral of  $|T T_1|$



#### **Example: Fixed Temperatures**

- Fixed temperature:  $T_1 = 100$  °C,  $T_2 = 0$  °C
- Material: aluminum ( $k = 236 \text{ W/m}^{\circ}\text{C}$ ) 30%
- Measure: high conductivity, amount of heat delivered from high to low temperature



#### **Example: Thermal Convection**

- Fixed temperature:  $T_1 = 100 \text{ °C}$ ,  $T_0 = 25 \text{ °C}$
- Boundary:  $h_0 = 5.677 \text{ W/m}^2 \text{ °C}$
- Material:  $k_1 = 75$  W/m°C (white),  $k_2 = 2 k_1$  (black)
- Measure: high conductivity, amount of heat transferred by convection along boundaries



#### **Example: Shell**

- Fixed temperature:  $T_{\rm H}$  = 100 °C,  $T_{\rm L}$  = 0 °C
- Material: aluminum (k = 236 W/m°C)

910

·820

.730

.640

. 550

- 460

.370

-290

.190

.100

\_010

• 30%, 50%, 70%



.910

.820

.730

.640

- 2250

- 460

.370

-200

.190

.100

\_010

#### **Electromagnetic Problem**

 J. Yoo and N. Kikuchi, Topology Optimization in Magnetic Fields Using the Homogenization Design Method, *Int. J. Numer. Meth. Engrg.* 48, pp.1463-1479, 2000



stress distribution



magnetic flux distribution

| field    | by FEM | stress/flux density                                | constitutive Eq.         |
|----------|--------|----------------------------------------------------|--------------------------|
| Elastic  | Ku = F | $\boldsymbol{\varepsilon} = \nabla \boldsymbol{u}$ | $\sigma = E \varepsilon$ |
| Magnetic | KA = J | $\boldsymbol{B} = \nabla \times \boldsymbol{A}$    | $H = \frac{1}{\mu}B$     |

#### **Optimization Problem**

Magnetic energy

$$W_m = \frac{1}{2} \int_{\Omega} \mathbf{B}^T \frac{1}{\mathbf{\mu}^h} \mathbf{B} d\Omega = \frac{1}{2} \int_{\Omega} \mathbf{J} \cdot \mathbf{A} d\Omega = \frac{1}{2} N I \varphi$$

where **B**: magnetic flux density,  $\mu^{h}$ : homogenized magnetic permeability

J: current density, A: vector potential, H: magnetic field strength

N: number of turns, I: current,  $\varphi$ : magnetic flux

 $(W_m)_e = \frac{1}{2} \int_{\Omega_e} \mathbf{B}_e^T \mathbf{M}_e^h \mathbf{B}_e d\Omega$  where  $\mathbf{M}_e^h$ : homogenized permeability matrix

• Maximize magnetic mean compliance = maximize magnetic flux  $f_{MMC} = NI\phi$ 

maximize  $f_{\text{MMC}} = NI\phi$ subject to  $\sum_{e=1}^{N} v_e - V \le 0$ 

• Sensitivity considering the saturation effect

$$\frac{\partial f_{MMC}}{\partial x} = \frac{1}{2} \int_{\Omega} \mathbf{B}^{\mathrm{T}} \left[ \frac{\partial}{\partial x} \frac{1}{\boldsymbol{\mu}(\mathbf{B})} + \frac{\partial}{\partial \mathbf{B}} \left( \frac{1}{\boldsymbol{\mu}(\mathbf{B})} \right) \frac{\partial \mathbf{B}}{\partial x} \right] \mathbf{B} d\Omega$$

Vehicle Structure Optimization

Topology Optimization: Extension - 40

#### Example: C-core





Linear case



Nonlinear case

#### Sound-Vibrations Coupled Problems

- Sound insulation using laminated structures composed of porous media
- To obtain the optimal layout of porous media



#### **Porous Materials**

- Two phases material
  - Solid layer and liquid layer
  - Coupled problems with respect to inertias
  - Viscous damping
- Numerical analysis based on Biot's model



## Porous media model



#### Example

To minimize sound pressure at specified frequency ranges



## **Optimal Configurations**

- Volume constraint is set to 25% of total volume
  - (a) From 55 Hz through 85 Hz

Porous media

- (b) From 85 Hz through 350 Hz



Air

#### Sound Pressure Response w.r.t Frequency



## Additive Manufacturing

- Produce geometrically complex components layer-by-layer
- Reduce the geometric complexity restrictions imposed on topology optimization
- Make near-full use of the freeform structural evolution of topology optimization

#### Chinese Journal of Aeronautics, (2021), 34(1): 91-110



#### A review of topology optimization for additive manufacturing: Status and challenges

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Structural and Multidisciplinary Optimization (2018) 57:2457–2483 https://doi.org/10.1007/s00158-018-1994-3

#### **REVIEW ARTICLE**



## Current and future trends in topology optimization for additive manufacturing

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Vehicle Structure Optimization

#### Topology Optimization: Extension - 48

## Aerospace bracket designed by topology optimization and manufactured by AM



## Additive Manufacturing: Issues

- Support structure design
  - Support slimming
  - Overhang-free topology optimization
- Porous infill design
  - Porous infill optimization
  - Lattice material optimization (meta-material optimization
- Material feature in AM
  - Material anisotropy
  - Microstructure control via topology optimization
- Multi-material and nonlinear topology optimization
  - Multi-material topology optimization
  - Nonlinear (multi-material) topology optimization
  - Topology optimization of structures with specific functionalities
- Robust design incorporating material and manufacturing uncertainties
  - Topology optimization under material uncertainty
  - Topology optimization under manufacturing uncertainty





(a) Topologically optimized support<sup>101</sup>



Innovative support structures. Fig. 17





(a) Non-self-support design

Fig. 18 3D printed topologically optimized industrial frame.<sup>109</sup>



(a) Prototype fabricated by SLM

(b) Solid structure

(c) Lattice structure

Fig. 8 A satellite bracket filled with lattice.



(a) Optimized beam infilled with hexagonal lattice<sup>68</sup>



(b) An engine bracket infilled with strut-based lattice<sup>64</sup>

(c) Bi-scale design of bridge-type structure<sup>27</sup>



C/SiC FGM 5 layer Pure SiC C-C 100 µm

(b) Backscattered electron image of C/SiC FGM<sup>149</sup>

(c) An optimized functionally graded microstructure<sup>151</sup>

Fig. 24 Illustrations of FGMs.

# (b) Self-support design

#### Topology Optimization: Extension - 51

20 mm

Some topology optimization examples for hierarchical structures. Fig. 9 Vehicle Structure Optimization

## Toyota's Lightweight Car Seat

- Challenge: create such a revolutionizing model, but manipulate and build such a large file
  - apply the 3D geometry at the slice level instead of at the STL level, and save all information about structures and textures as metadata
  - (STL) 250GB (metadata) 36MB
- Reduction: (volume) 72%, (weight)  $25 \rightarrow 7$ kg, (heat capacity)  $35.4 \rightarrow 14.5$  J/K



https://www.materialise.com/en/cases/materialise-slicing-technology-enables-toyota%E2%80%99s-lightweight-car-seat Vehicle Structure Optimization Topology Optimization: Extension - 52

#### REVIEW ARTICLE

#### **Topology optimization approaches**

A comparative review

Ole Sigmund · Kurt Maute

$$\begin{split} \min_{\rho} &: F = F(\mathbf{u}(\rho), \rho) = \int_{\Omega} f(\mathbf{u}(\rho), \rho) dV \\ s.t. &: G_0(\rho) = \int_{\Omega} \rho(\mathbf{x}) dV - V_0 \leq 0 \\ &: G_i(\mathbf{u}(\rho), \rho) \leq 0, \ j = 1, \dots, M \\ &: \rho(\mathbf{x}) = 0 \text{ or } 1, \ \forall \mathbf{x} \in \Omega \end{split}$$
$$\end{split}$$
$$\begin{split} \min_{\rho\rho} &: F(\mathbf{u}(\rho), \rho) = \sum_i \int_{\Omega_i} f(\mathbf{u}(\rho_i), \rho_i) dV \\ s.t. &: G_0(\rho) = \sum_i v_i \rho_i - V_0 \leq 0 \\ &: G_j(\mathbf{u}(\rho), \rho) \leq 0, \ j = 1, \dots, M \\ &: \rho_i = 0 \text{ or } 1, \ i = 1, \dots, N \end{split}$$
$$\end{split}$$
$$\end{split}$$
$$\begin{split} \min_{\rho\rho} &: F(\mathbf{u}(\rho), \rho) = \sum_i \int_{\Omega_i} f(\mathbf{u}(\rho_i), \rho_i) dV \\ s.t. &: G_0(\rho) = \sum_i v_i \rho_i - V_0 \leq 0 \\ &: G_j(\mathbf{u}(\rho), \rho) \leq 0, \ j = 1, \dots, M \\ &: \rho_i \leq 1, \ i = 1, \dots, N \end{cases}$$

- Topology optimization approaches
  - Density approach
  - Topological derivatives
  - Level set approach
  - Phase field approach
- Discrete approaches
  - Evolutionary approaches
- Lagrangian approaches and combined shape and topology optimization

#### Comparison of methods

- Use of filtering and smoothing operators
- Convergence and density vs phase field updates
- Local vs global regularization
- Continuous vs. discrete design variables
- Optimizers
- Boundary dependent loads and critical boundary conditions
- Body fitted meshes
- Provision of research codes to the community
- Benchmark problems
- Local constraints
- On the need for level set approaches
- Lagrangian methods

## Main Challenges

- Efficiency: large scale 3D problems
- General applicability: arbitrary physics problems
- Multiple constraints
- Complex boundary conditions
- Independence on starting guess
- Few tuning parameters
- Mesh-independent convergence
- Ease of use
- Alternatives to finite element analysis
  - Finite volume methods for compressible flow problems
  - Finite difference methods for nano-optical problems

#### WCSMO-13: Statistics

- The 13<sup>th</sup> World Congress of Structural and Multidisciplinary Optimization
- 20th~24th May, 2019 Beijing, China
- 553 presentations (475 oral + 78 poster) in total
  - China(273), Korea(48), Japan(44), USA(43), Germany(35)
- 13 topics
  - Topology Optimization(204), Structural Optimization(40), Robust and Reliability-Based Design Optimization(32), Design Optimization(24), Multidisciplinary Design Optimization(21)

## WCSMO-13: SOTA(State-of the Art) Discussion



Niels Aage Technical University of Denmark, Denmark

Multiphysics & Multiscale design: Achievements and Current Frontier



Wei Chen Northwestern University, USA

Machine Learning and Data Driven Techniques: Status and Opportunities



Hai Huang Beihang University, China

Promote the service of Topology Optimization for Layout Design of Engineering Structural Systems



Yoshihiro Kanno The University of Tokyo, Japan

Reliability- and Robustness-Based Design

imization: Extension - 57

#### Multiphysics & Multiscale design: Achievements and Current Frontier



Presentations: multiscales (60), multiphysics (39)

- Discussions
  - Fluid-Structure for large Reynolds number
  - CAD-CAE interaction
  - Benchmarking problem

#### Machine Learning and Data Driven Techniques: Status and Opportunities



#### Metamodeling

- Metamodeling will continue to play an important role in SMO with expensive physicsbased simulations, for managing information complexity (multiscale/multicomponent/multidiscipline), and for solving inverse problems.
- Choice of metamodeling techniques and sampling decisions need to be integrated and viewed as a resource allocation problem.

#### Machine learning/deep learning

- Machine learning/dimension reduction helps to gain knowledge and draw insights into physical relations associated with geometry.
- The use of deep learning needs to be justified.
- Physics-based machine learning and transfer learning are needed to make ML practically useful.
- Benefits of using ML-based TO over traditional TO methods need to be further studied (Nonlinear? Multiscale? Hybrid?). Community needs good benchmark problems.

Vehicle Structure Op .....

#### Machine Learning and Data Driven Techniques: Status and Opportunities

#### Pros

- ML models are fast to evaluate.
- They can capture highly nonlinear behavior.
- They can handle high dimensional inputs such as images.
- No explicit domain knowledge is required.

#### Cons

- ML models are difficult to train.
- They are in general only as good as the data.
- Large amount of high quality data are desired.
- They are hard to interpret.





Research on applying data-driven/machine learning methods in SMO is growing in recent years.

\*Up to 05/02/2019, SMO has 184 publications in 2019.

#### Promote the service of Topology Optimization for Layout Design of Engineering Structural Systems





Presentations: TO (200+ presentations, 38 sessions)

AM (60+), Explicit (30+), Multiscale(30+), Stress (20+), Large scale (20+), {IGA, ML, MMTO, Multiphysics, Shell, Mathematical/Numerical} (10+)

#### Discussions

- Stress problem
- Benchmarking problem

#### Reliability and Robustness Based Design



#### Presentations: 70 (East Asia 47)

- Discussions
  - Only a few group
  - Modeling uncertainty