

Introduction & ODE by COMSOL

Computational Design Laboratory
Department of Automotive Engineering
Hanyang University, Seoul, Korea



OUTLINE

- **Lecture Goals**
 - ✓ 다양한 Physics의 유한요소해석 소프트웨어인 COMSOL의 솔루션들을 소개하고, 이를 활용한 미분 방정식 해법을 도출하는 과정을 실습한다
- **Content**
 - ✓ Introduction
 - ✓ COMSOL desktop
 - ✓ Mathematic module
 - ✓ ODE examples
 - ✓ Assignment

- **Finite element analysis and simulation software package for various physics and engineering applications**
- **Founded by Savante Littmarck and Farhad Saeidi in 1986**
- **FEMLAB: Early version of COMSOL (before 2005)**



Dr. h.c. Svante Littmarck
CEO of the COMSOL Group



Mr. Farhad Saeidi
President of COMSOL AB

COMSOL BACKGROUND

Before the 1980's, **Germund Dahlquist** pioneered the use of personal computers to solving a system of partial differential equations using numerical operations.

In 1986, Two of his students at the time, **Svante Littmarck** and **Farhad Saeidi** began to work on such a software package outside of the work already required by their graduate program. Littmarck and Saeidi made the decision to forgo their studies and begin building a software company under the name COMSOL.

Dahlquist provided administrative advice and even went so far as to put the two budding entrepreneurs into contact with **MathWorks**, who had already built their empire around successful software package **MATLAB**.

COMSOL released their first software package on September 1st, 1998 with a **structural mechanics model that included CAD optimization, material models, thermal stresses, waves, and much more**.

On April 12, 2001 the Chemical Engineering Module was added and nine years later on **April 20, 2010, software integration with products such as Solidworks and MATLAB were introduced**.

COMSOL ran into a near crippling lawsuit from MathWorks for copyright infringement in 2006. Nonetheless, the company rebounded and continues to draw in large profits. Today, COMSOL has about 50,000 users and boasts NASA as its largest consumer.

[Reference-COMSOL Background – COMSOL Testosterone Transport Project](#)

MODULES

www.comsol.com

COMSOL Multiphysics®

ELECTRICAL	MECHANICAL	FLUID	CHEMICAL	MULTIPURPOSE	INTERFACING	
AC/DC Module	Heat Transfer Module	CFD Module	Chemical Reaction Engineering Module	Optimization Module®	LiveLink™ for MATLAB®	LiveLink™ for Excel®
RF Module	Structural Mechanics Module	Microfluidics Module	Batteries & Fuel Cells Module	Material Library	CAD Import Module	ECAD Import Module
MEMS Module	Nonlinear Structural Materials Module	Subsurface Flow Module	Electrodeposition Module	Particle Tracing Module	LiveLink™ for SolidWorks®	LiveLink™ for SpaceClaim®
Plasma Module	Geomechanics Module	Pipe Flow Module	Corrosion Module		LiveLink™ for Inventor®	LiveLink™ for AutoCAD®
	Fatigue Module				LiveLink™ for Creo™ Parametric	LiveLink™ for Pro/ENGINEER®
	Acoustics Module				LiveLink™ for Solid Edge®	File Import for CATIA®V5

AC/DC MODULE

ELECTRICAL

AC/DC
Module

RF
Module

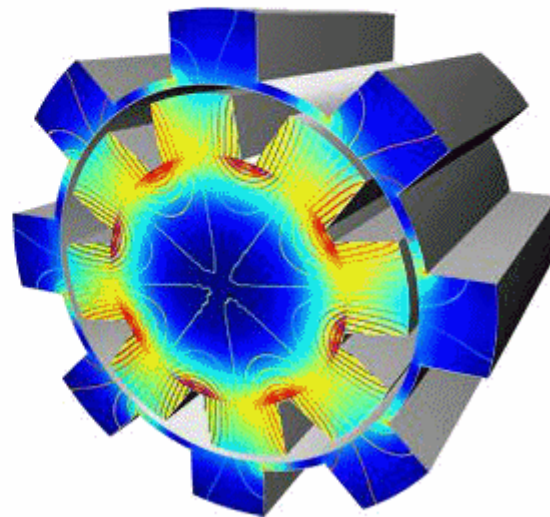
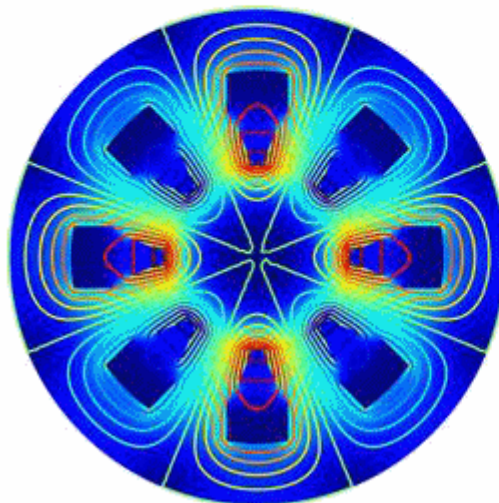
MEMS
Module

Plasma
Module

Dynamics of a Generator

This example shows how the circular motion of a rotor with permanent magnets in a generator results in an induced EMF in the stator winding. The generated voltage is calculated as a function of time during the rotation.

The plot on the left shows the magnetic flux density along with a contour plot of the magnetic potential. Note the brighter regions, which indicate the position of the permanent magnets in the rotor. The figure on the right shows the geometry and a simulation of the generator in 3D.

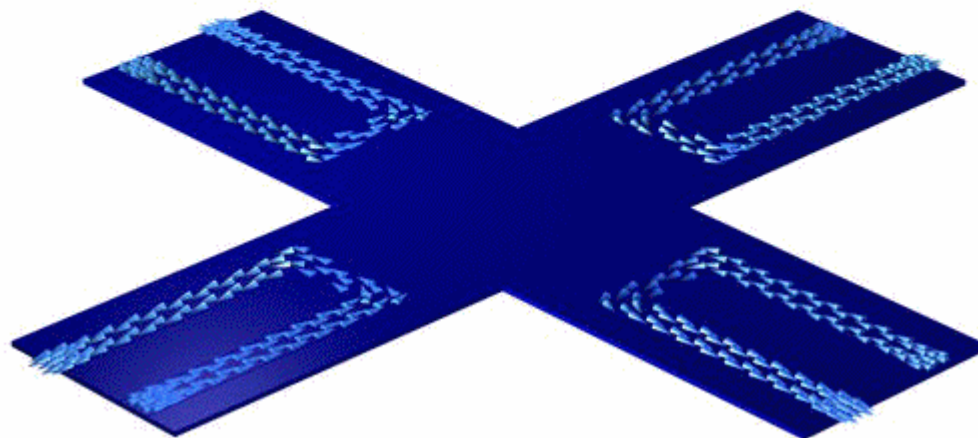


HEAT TRANSFER MODULE

Deformation of a Thermomechanical Microvalve

Thermomechanical microvalves are common flow control components in microfluidics systems. Here, an electric current generates movement by resistively heating the actuator structure, thereby causing mechanical stress and deformation.

In this example, a parametric study shows how an increasing voltage applied to each of the legs leads to temperature rise causing more and more deformation.



MECHANICAL

Heat Transfer
Module

Structural
Mechanics Module

Nonlinear Structural
Materials Module

Geomechanics
Module

Fatigue
Module

Acoustics
Module

CFD MODULE

FLUID

CFD
Module

Microfluidics
Module

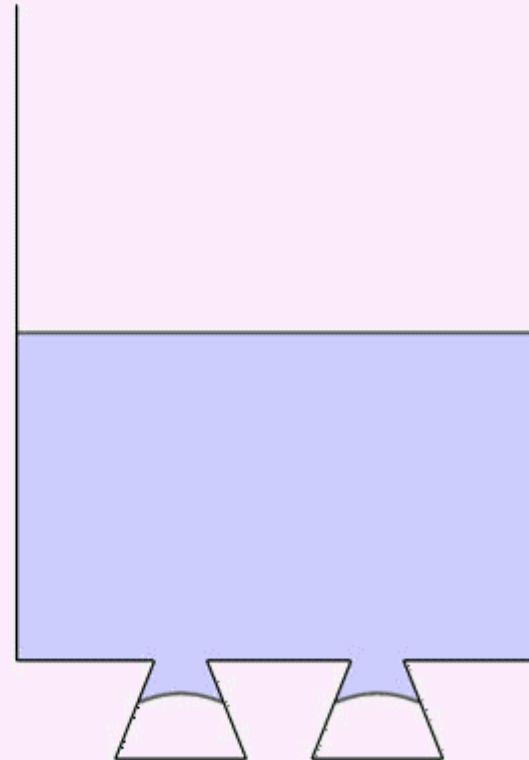
Subsurface Flow
Module

Pipe Flow
Module

Boiling Water

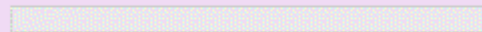
This model studies the film boiling of water. A heat flux above the Leidenfrost point is applied at the surface of two cavities. A layer of vapor is maintained at the hot surface - liquid interface where film-boiling results.

The animation shows the fluids volume fraction over time as a surface and contour plot.



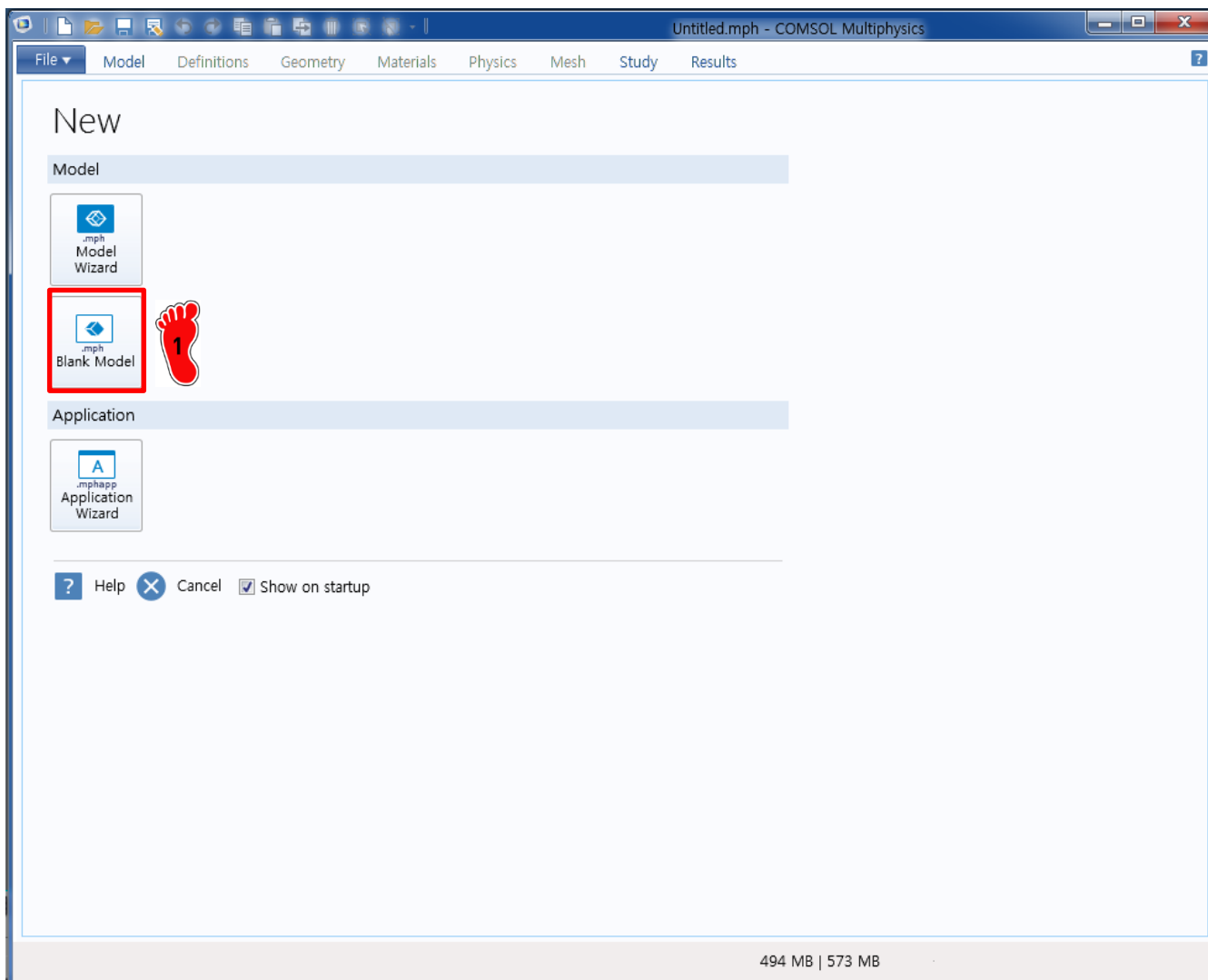
MULTIPHYSICS

COMSOL Multiphysics®



- **COMSOL desktop**
 - ✓ **Toolbars & Ribbon tabs**
 - ✓ **Windows**
 - ✓ **Physics**
 - ✓ **Study types**
 - ✓ **Setting flow**
 - ✓ **Setting result**

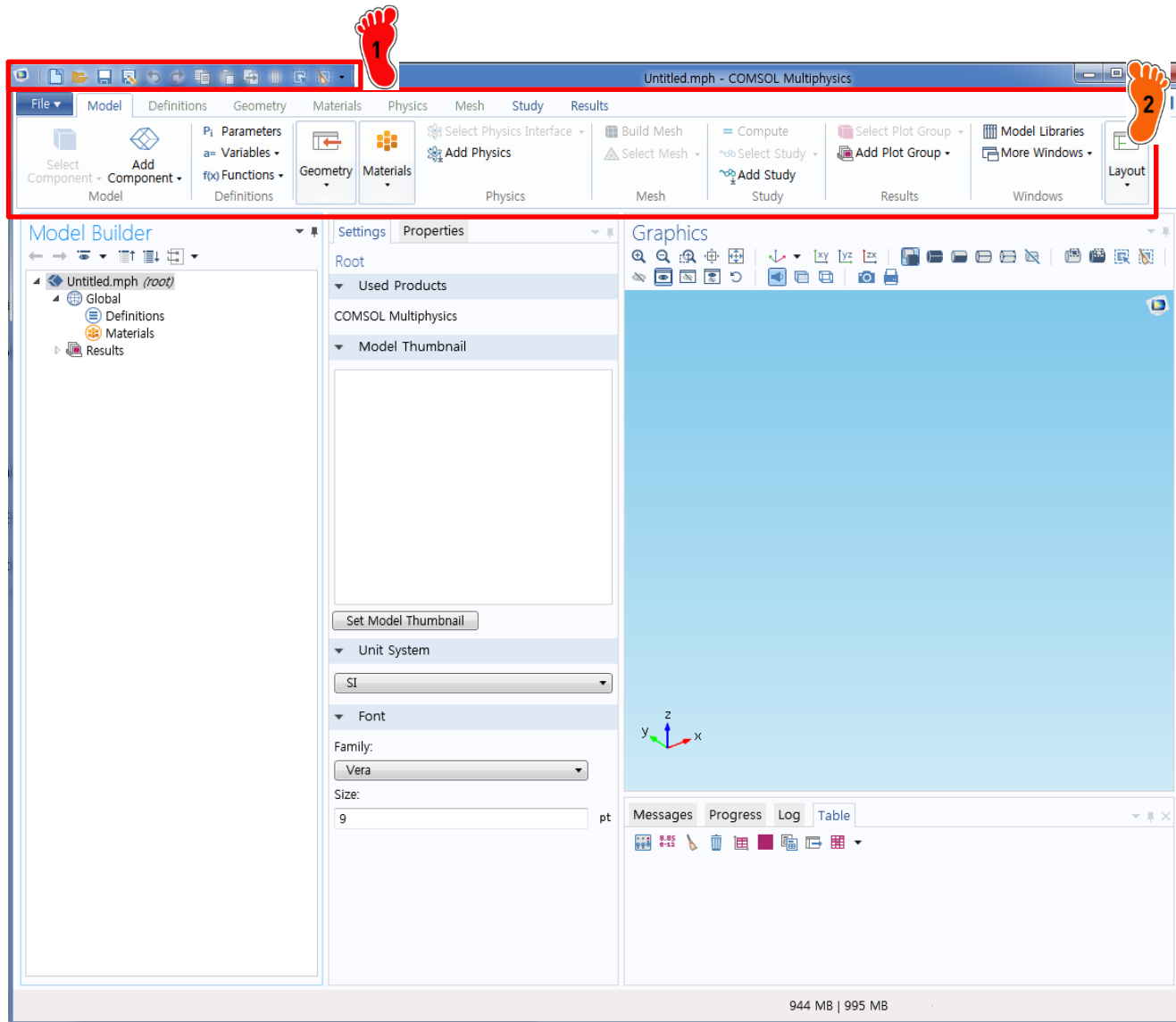
CREATING NEW MODEL



Blank Model

The Blank Model 옵션은 Physics 또는 Study 설정없이 COMSOL Desktop interface를 실행

TOOLBAR & RIBBON TABS

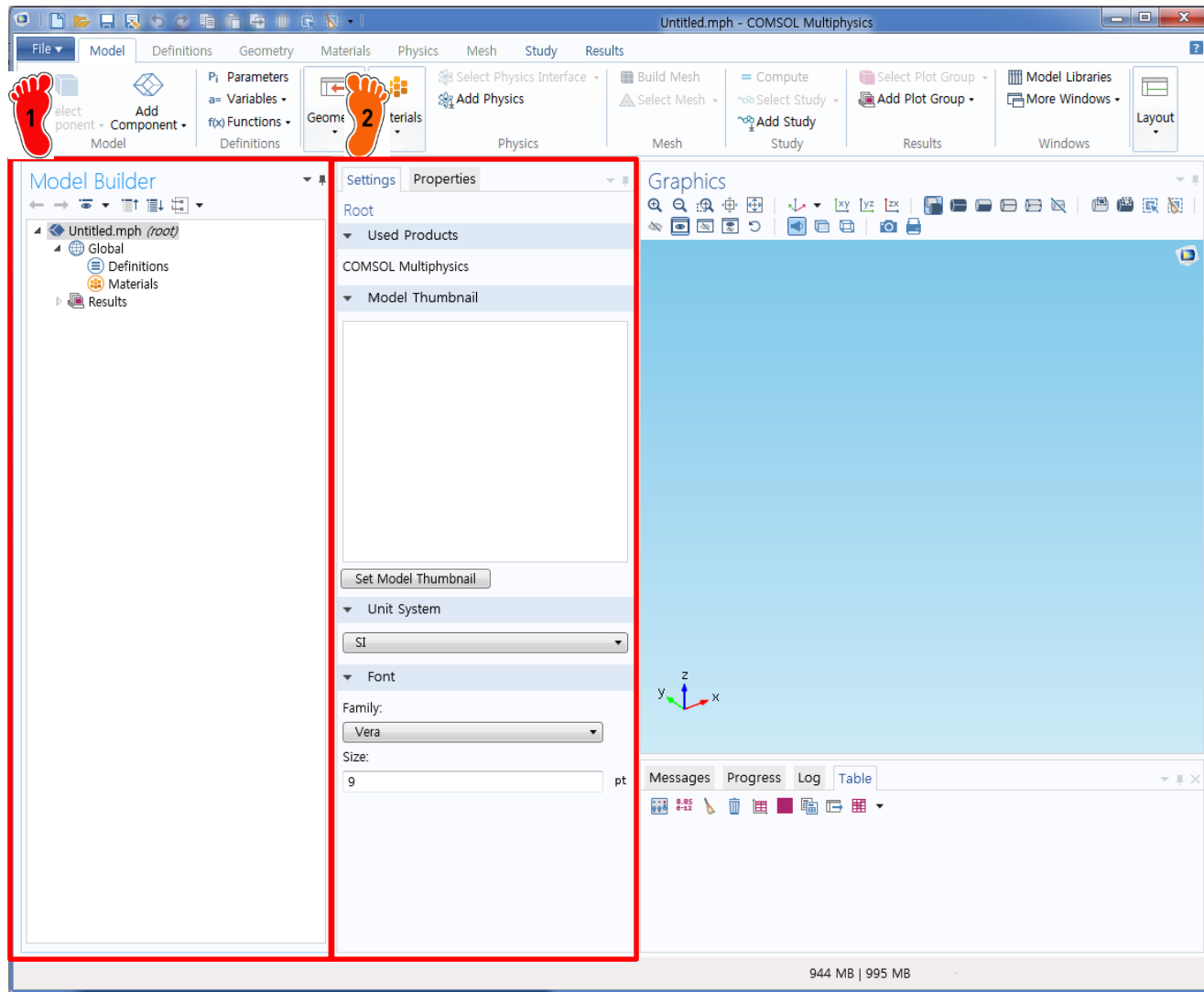


Quick Access Toolbar

Ribbon tabs

COMSOL Desktop 환경의
The ribbon tabs은 모델링
워크플로우를 반영하고
각 모델링에 사용할 수 있는
기능에 대한 개요 제공

WINDOWS



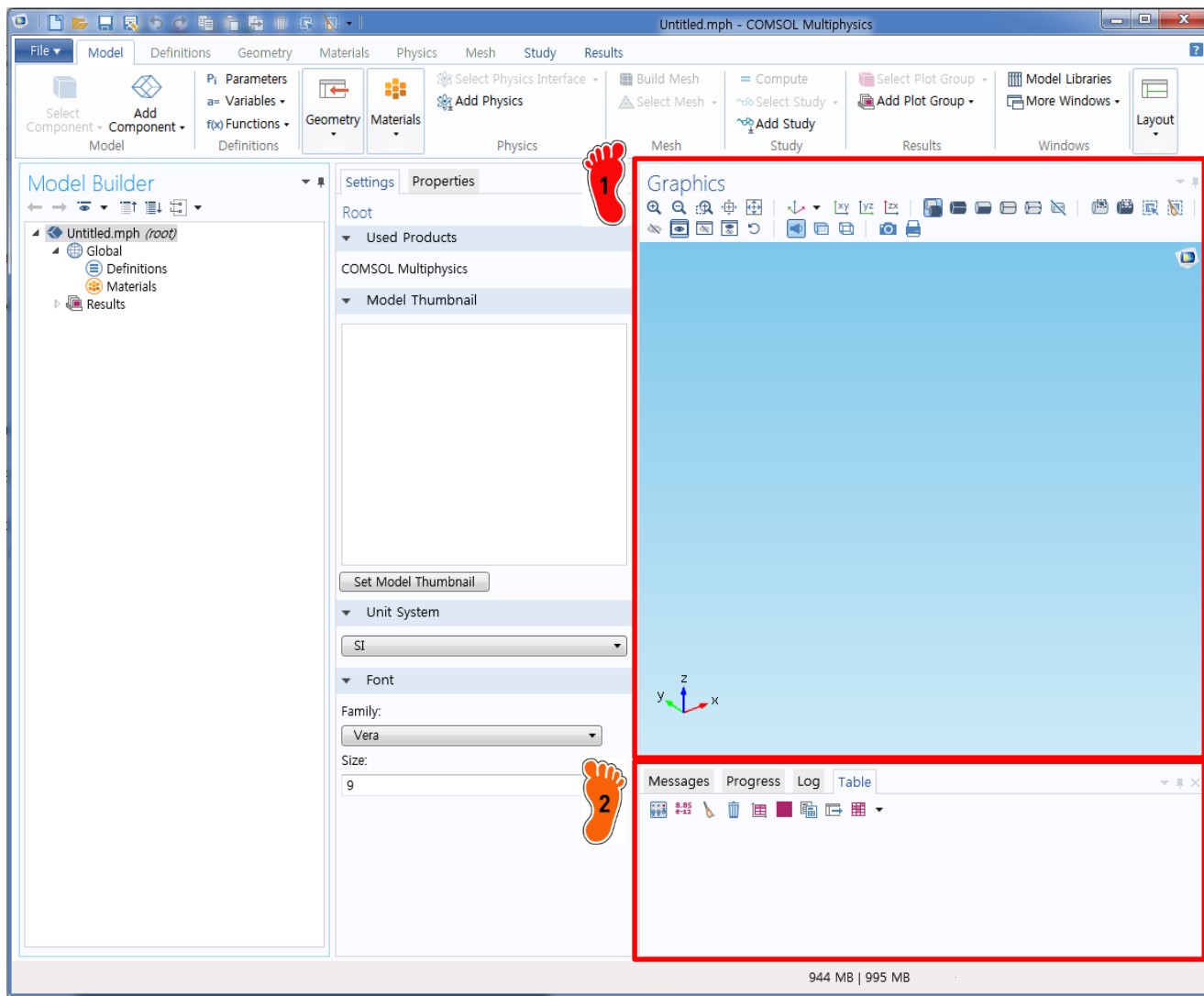
1 Model builder window

The Model Builder 는 사용자 정의하는 Model 과 Physics를 정의하는 tool

2 Setting window

Model builder의 세부 node 세팅을 편집

WINDOWS



Graphics window

Mesh와 해석 결과를 시각화합니다. 회전, 축소확대, 선택적 시각화 등 옵션 설정 가능



Information windows

정보 창에는 솔루션 시간, 솔루션 진행률, 메시 통계 및 솔버 로그, 결과 테이블과 같은 시뮬레이션 동안 중요한 모델 정보 표시

SETTING FLOW



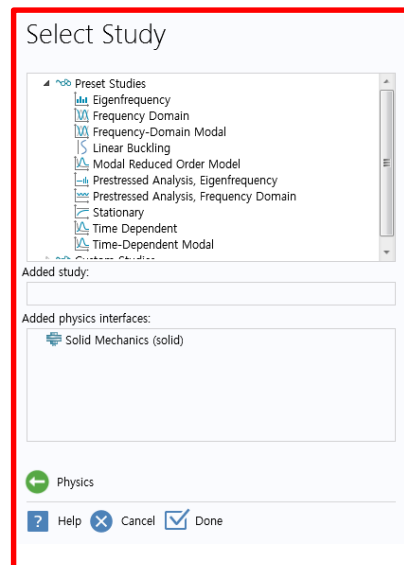
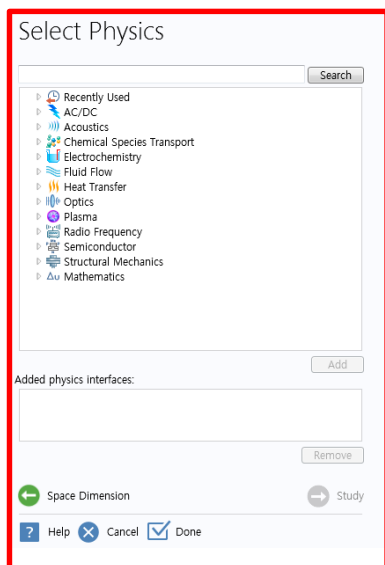
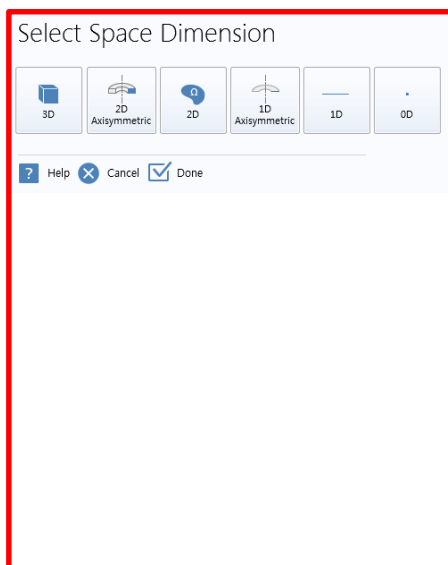
Model Wizard

The Model Wizard는 차원, Physics, Study type을 설정

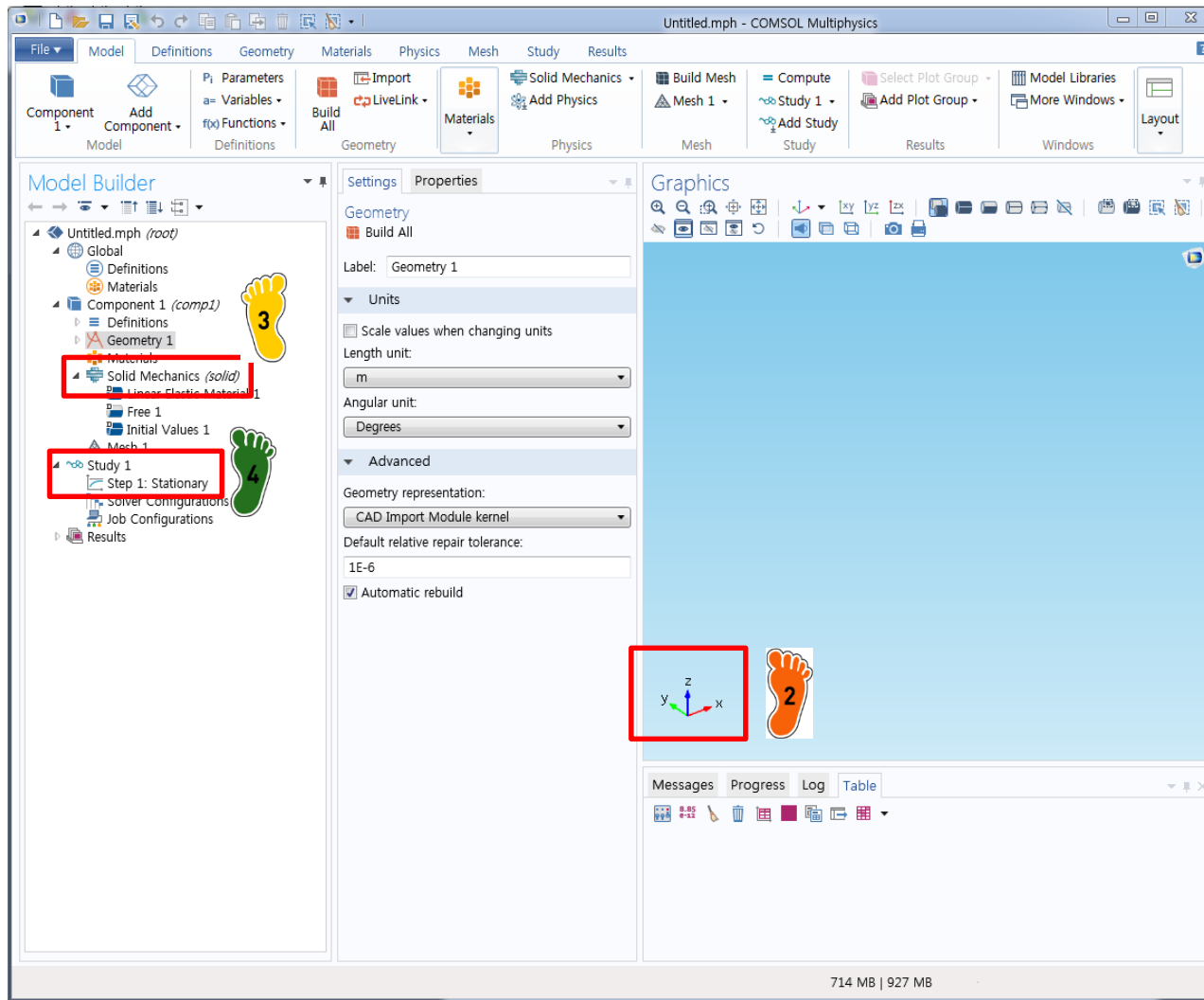
실제 모델링 전 진행 과정

1. Dimension 선택
2. 해석 물리현상(physics) 선택
3. 해석 조건(study) 선택

ex) 3D – Solid Mechanics (solid) – Stationary



SETTING RESULT



1 전 페이지에서 선택한 setting 결과

2 3차원

3 Solid mechanics

4 Stationary

- **Mathematics module**
 - ✓ **Coefficient form PDE**
 - ✓ **PDE Interfaces**

COEFFICIENT FORM PDE

Equation of coefficient form PDE

$$\left\{ \begin{array}{ll} e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c \nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + a u = f & \text{in } \Omega \\ \mathbf{n} \cdot (c \nabla u + \alpha u - \gamma) + q u = g - h^T \mu & \text{on } \partial \Omega \\ 0 = R & \text{on } \partial \Omega \end{array} \right.$$

where

- Ω is the computational domain—the union of all domains
- $\partial \Omega$ is the domain boundary
- $\mathbf{n}$ is the outward unit normal vector on $\partial \Omega$

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} - \nabla \cdot (c \nabla u + \alpha u - \gamma) + \beta \cdot \nabla u + a u = f$$

Diagram illustrating the physical interpretation of the coefficient form PDE terms:

- $e_a \frac{\partial^2 u}{\partial t^2}$: Mass
- $d_a \frac{\partial u}{\partial t}$: Damping/ Mass
- $\nabla \cdot (c \nabla u)$: Diffusion
- $\nabla \cdot (\alpha u)$: Convection
- $\nabla \cdot (-\gamma)$: Source
- $\beta \cdot \nabla u$: Convection
- $a u$: Source
- f : Source
- γ : Conservative Flux
- $h^T \mu$: Absorption

TABLE 16-1: CLASSICAL PDES IN COMPACT AND COMPONENT NOTATION

EQUATION	COMPACT NOTATION	COMPONENT NOTATION (2D)
Laplace's equation	$-\nabla \cdot (\nabla u) = 0$	$-\frac{\partial}{\partial x} \frac{\partial u}{\partial x} - \frac{\partial}{\partial y} \frac{\partial u}{\partial y} = 0$
Poisson's equation	$-\nabla \cdot (c \nabla u) = f$	$-\frac{\partial}{\partial x} \left(c \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left(c \frac{\partial u}{\partial y} \right) = f$
Helmholtz equation	$-\nabla \cdot (c \nabla u) + a u = f$	$-\frac{\partial}{\partial x} \left(c \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left(c \frac{\partial u}{\partial y} \right) + a u = f$
Heat equation	$d_a \frac{\partial u}{\partial t} - \nabla \cdot (c \nabla u) = f$	$d_a \frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left(c \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left(c \frac{\partial u}{\partial y} \right) = f$
Wave equation	$e_a \frac{\partial^2 u}{\partial t^2} - \nabla \cdot (c \nabla u) = f$	$e_a \frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x} \left(c \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left(c \frac{\partial u}{\partial y} \right) = f$
Convection- diffusion equation	$d_a \frac{\partial u}{\partial t} - \nabla \cdot (c \nabla u) + \beta \cdot \nabla u = f$	$d_a \frac{\partial u}{\partial t} - \frac{\partial}{\partial x} \left(c \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left(c \frac{\partial u}{\partial y} \right) + \beta_x \frac{\partial u}{\partial x} + \beta_y \frac{\partial u}{\partial y} = f$

PDE INTERFACES

The screenshot shows the COMSOL Multiphysics interface with the following steps highlighted:

- Model Wizard → 1D 클릭**: The 'Select Space Dimension' dialog box is open, and the '1D' option is selected.
- 모듈 선택 메뉴에서 Mathematics - PDE Interfaces 를 클릭**: The 'Select Physics' dialog box is open, and the 'PDE Interfaces' folder is expanded, showing options like 'Coefficient Form PDE (c)', 'General Form PDE (g)', 'Wave Form PDE (wahw)', and 'Weak Form PDE (w)'.
- Add 클릭**: The 'Add' button is clicked to add the selected PDE interface to the 'Added physics interfaces' list.
- Done 클릭**: The 'Done' button is clicked to complete the selection process.

The 'Review Physics Interface' panel on the right shows the 'Coefficient Form PDE (c)' interface with the following settings:

- Dependent Variables**: Field name: ; Number of dependent variables: ; Dependent variables:
- Units**: Dependent variable quantity: ; Source term quantity: ; Unit:

1 Model Wizard → 1D 클릭

2 모듈 선택 메뉴에서
Mathematics - PDE
Interfaces 를 클릭

PDE Interfaces 는 PDE 를
풀기 위한 physics 모음

5가지 physics 선택 가능

Coefficient form PDE (선택)
General Form PDE
Wave Form PDE
Weak Form PDE
Lower Dimensions

3 Add 클릭

4 Done 클릭

PDE INTERFACES

Model Builder tree structure:

- Untitled.mph (root)
 - Global Definitions
 - Materials
 - Component 1 (comp1)
 - Definitions
 - Geometry 1
 - Materials
 - Coefficient Form PDE (c)**
 - Coefficient Form PDE 1** (selected)
 - Zero Flux 1
 - Initial Values 1
 - Mesh 1
 - Results

Settings for Coefficient Form PDE:

- Label: Coefficient Form PDE 1
- Domain Selection: All domains
- Equation:

$$e_s \frac{\partial^2 u}{\partial t^2} + d_s \frac{\partial u}{\partial t} + \nabla \cdot (-c \nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + a u = f$$

$$\nabla = \frac{\partial}{\partial x}$$
- Diffusion Coefficient:
 - c: 1
- Absorption Coefficient:
 - a: 0 $1/m^2$
- Source Term:
 - f: 1 $1/m^2$
- Mass Coefficient:
 - e_a: 0 s^2/m^2

Graphics window: COMSOL 5.2.0.166. Finalized geometry is empty. Finalized geometry is empty.

1 Model builder 창 tree 에서 Coefficient form PDE 선택

2 Equation 탭 메뉴를 확장시키면 독립변수 t, x 와 종속 변수 u 에 대한 수식이 나옴

3 수식 안에 있는 계수들의 값을 입력할 수 있는 창

- **ODE examples**

- ✓ **Example 25.5**
- ✓ **Example 28.2: predator-prey model**
- ✓ **Example 25.14**
- ✓ **Example 27.10**
- ✓ **Previous case study I**
- ✓ **Previous case study II**

EXAMPLE 25.5



ODE 함수 예제



ordinary differential equation

$$y' = 4e^{0.8t} - 0.5y$$

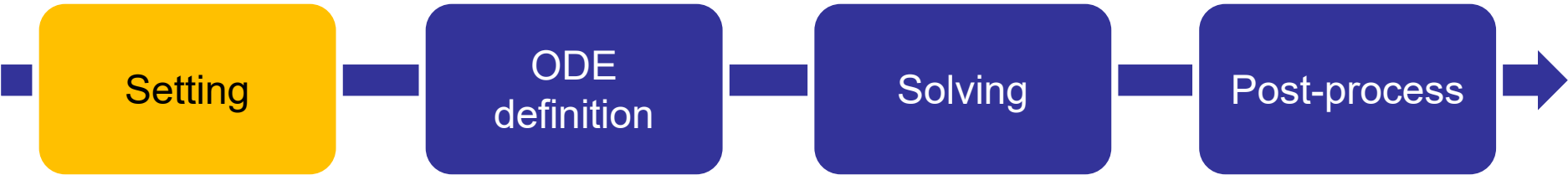
initial condition

$$t = 0, y = 2$$

analytic solution

$$y = \frac{4}{1.3} (e^{0.8t} - e^{-0.5t}) + 2e^{-0.5t}$$

ANALYSIS FLOW



Setting

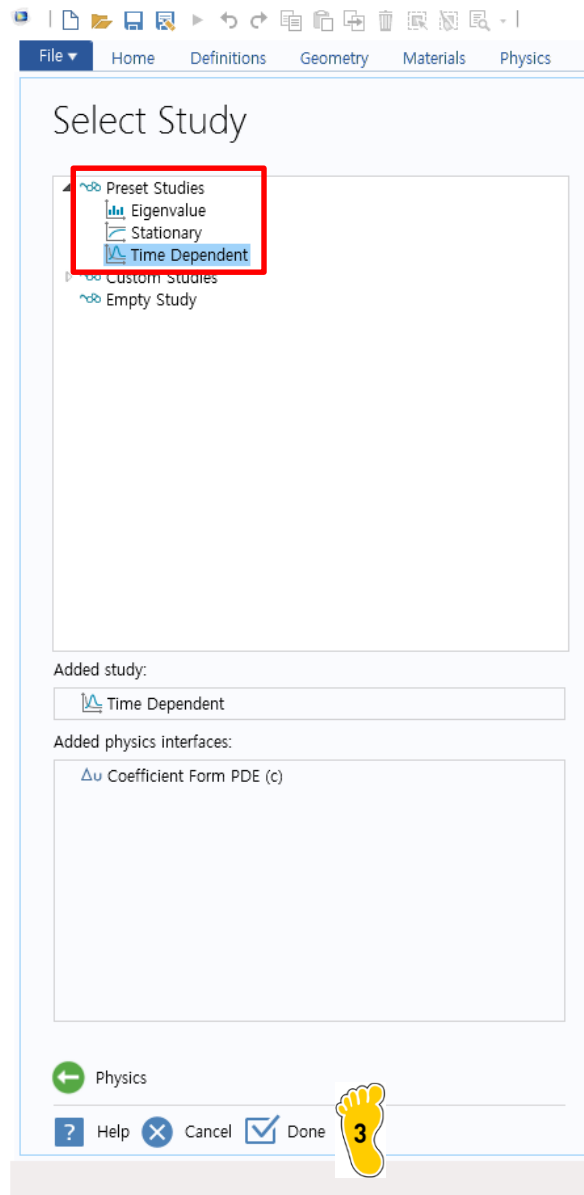
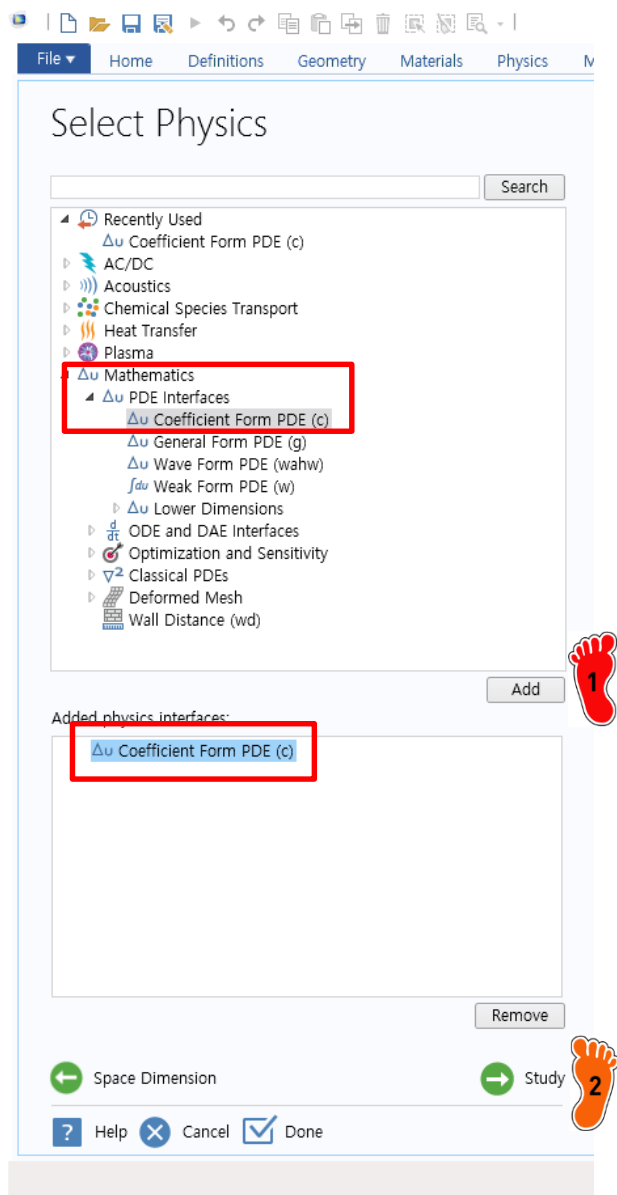
ODE
definition

Solving

Post-process

- ✓ Dimension selection
- ✓ Physics selection
- ✓ Study type selection

SETTING



1 Model Wizard → 1D
→ Mathematics - PDE
Interfaces – Coefficient
form PDE 선택
→ Add 클릭

2 Study 클릭

3 Time Dependent 선택 후
Done 클릭

ANALYSIS FLOW

```
graph LR; A[Setting] --> B[ODE definition]; B --> C[Solving]; C --> D[Post-process]; D --> E[ ]; style B fill:#FFD700
```

Setting

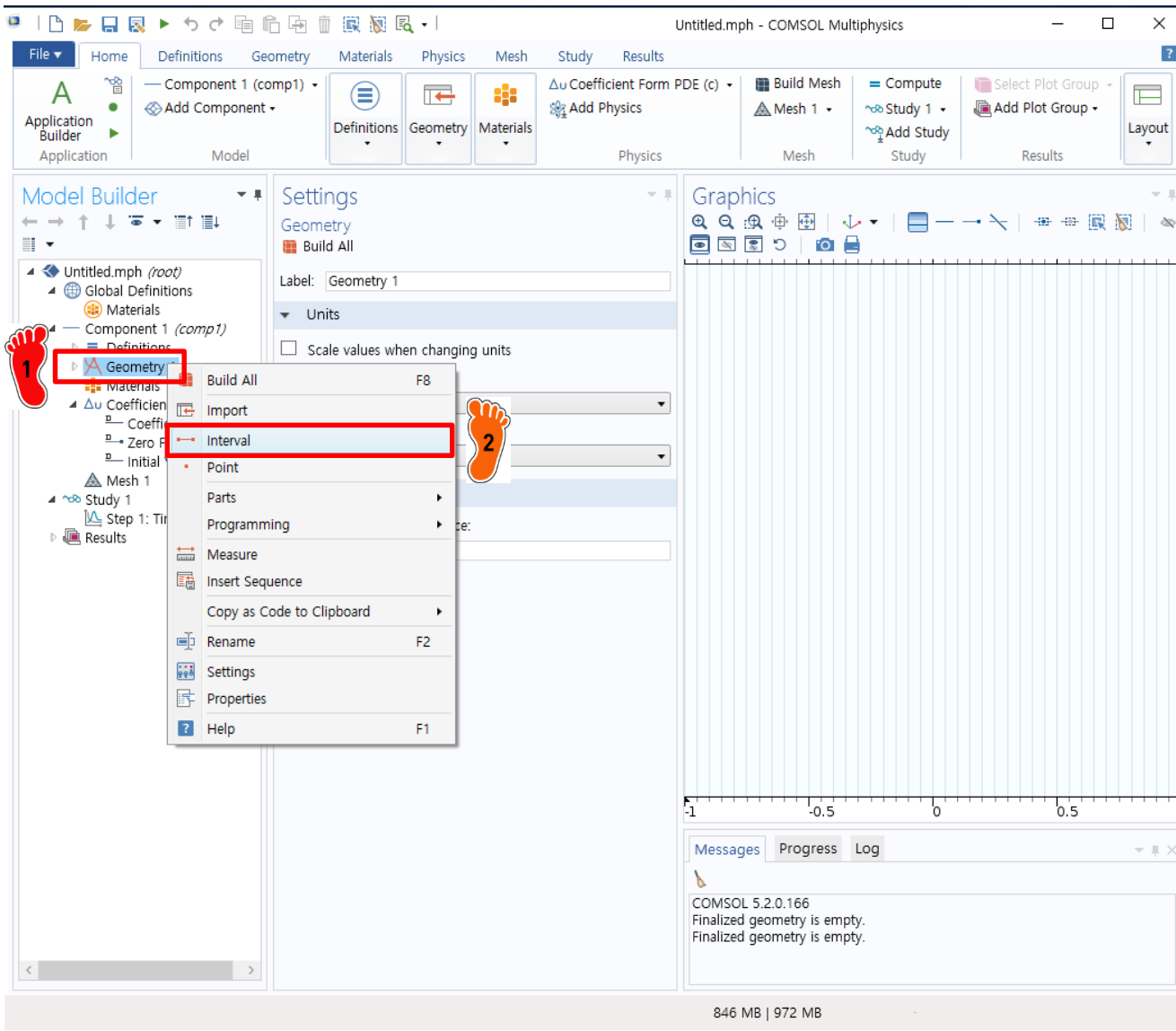
ODE
definition

Solving

Post-process

- ✓ Geometry creation
- ✓ Coefficient input
- ✓ Initial value input

GEOMETRY CREATION



Geometry 1 메뉴를 마우스 우클릭



Interval 클릭

GEOMETRY CREATION

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results

Application Builder Model

Model Builder

- Untitled.mph (root)
 - Global Definitions
 - Materials
 - Component 1 (comp1)
 - Definitions
 - Geometry 1
 - Interval 1 (11)
 - Form Union (fin)
 - Materials
 - Coefficient Form PDE (c)
 - Coefficient Form PDE
 - Zero Flux 1
 - Initial Values 1
 - Mesh 1
 - Study 1
 - Step 1: Time Dependent
 - Results

Settings

Interval

Build Selected Build All Objects

Label: Interval 1

Interval

Number of intervals: One

Left endpoint: 0 m

Right endpoint: 1 m

Selections of Resulting Entities

Contribute to: None New

☐ Resulting objects selection

Show in physics: Domain selection

Graphics

0 0.2 0.4 0.6 0.8 1

Messages Progress Log

COMSOL 5.2.0.166
Finalized geometry is empty.
Finalized geometry is empty.

845 MB | 973 MB

1 Left endpoint 에 "0"
Right endpoint 에 "1"
값을 입력

2 Build selected 클릭

COEFFICIENT INPUT

1 Coefficient Form PDE (c)

2 계수 값 입력 (나머지 0)

Diffusion Coefficient

c 0 1

Absorption Coefficient

a 0 $1/m^2$

Source Term

f $4 \cdot \exp(0.8 \cdot t) - 0.5 \cdot u$ $1/m^2$

Mass Coefficient

e_a 0 s^2/m^2

Damping or Mass Coefficient

d_a 1 s/m^2

Conservative Flux Convection Coefficient

α 0 $1/m$

Convection Coefficient

β 0 $1/m$

Conservative Flux Source

γ 0 $1/m$

Messages

Progress Log

COMSOL 5.2.0.166
Finalized geometry is empty.
Finalized geometry is empty.
Finalized geometry has 1 domain and 2 boundaries.



Coefficient Form PDE 1
선택



계수 값 입력 (나머지 0)

$$d_a = 1$$

$$f = 4e^{0.8t} - 0.5u$$



$$e \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c \nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + a u = f \leftrightarrow \frac{\partial u}{\partial t} = 4e^{0.8t} - 0.5u$$

INITIAL VALUE INPUT

The screenshot displays the COMSOL Multiphysics interface for a new model. The top toolbar includes icons for File, Home, Definitions, Geometry, Materials, Physics, Mesh, Study, and Results. The left sidebar shows the Model Builder tree with the following structure:

- Untitled.mph (root)
 - Global Definitions
 - Materials
 - Component 1 (comp1)
 - Definitions
 - Geometry 1
 - Interval 1 (f1)
 - Form Union (fin)
 - Materials
 - Coefficient Form PDE (c)
 - Coefficient Form PDE 1
 - Zero Flux 1
 - Initial Values 1** (highlighted with a red box and a red footprint icon labeled '1')
 - Mesh 1
 - Study 1
 - Step 1: Time Dependent
 - Results

The central Settings pane is set to the 'Initial Values' tab. It shows the following configuration:

- Label: Initial Values 1
- Domain Selection: Selection: All domains, Active: 1
- Override and Contribution: Initial Values
- Initial value for u: u (input field contains '2', highlighted with a red box and a red footprint icon labeled '2')
- Initial time derivative of u: $\frac{\partial u}{\partial t}$ (input field contains '0')

The right sidebar shows the Graphics pane with a plot area and a Messages pane at the bottom. The Messages pane displays the following text:

```
COMSOL 5.2.0.166
Finalized geometry is empty.
Finalized geometry is empty.
Finalized geometry has 1 domain and 2 boundaries.
```

The status bar at the bottom indicates 849 MB | 928 MB.



Initial Values 1 메뉴 클릭



u 창에 "2" 입력



ANALYSIS FLOW

```
graph LR; A[Setting] --> B[ODE definition]; B --> C[Solving]; C --> D[Post-process]; D --> E[ ]; style C fill:#ffcc00
```

Setting

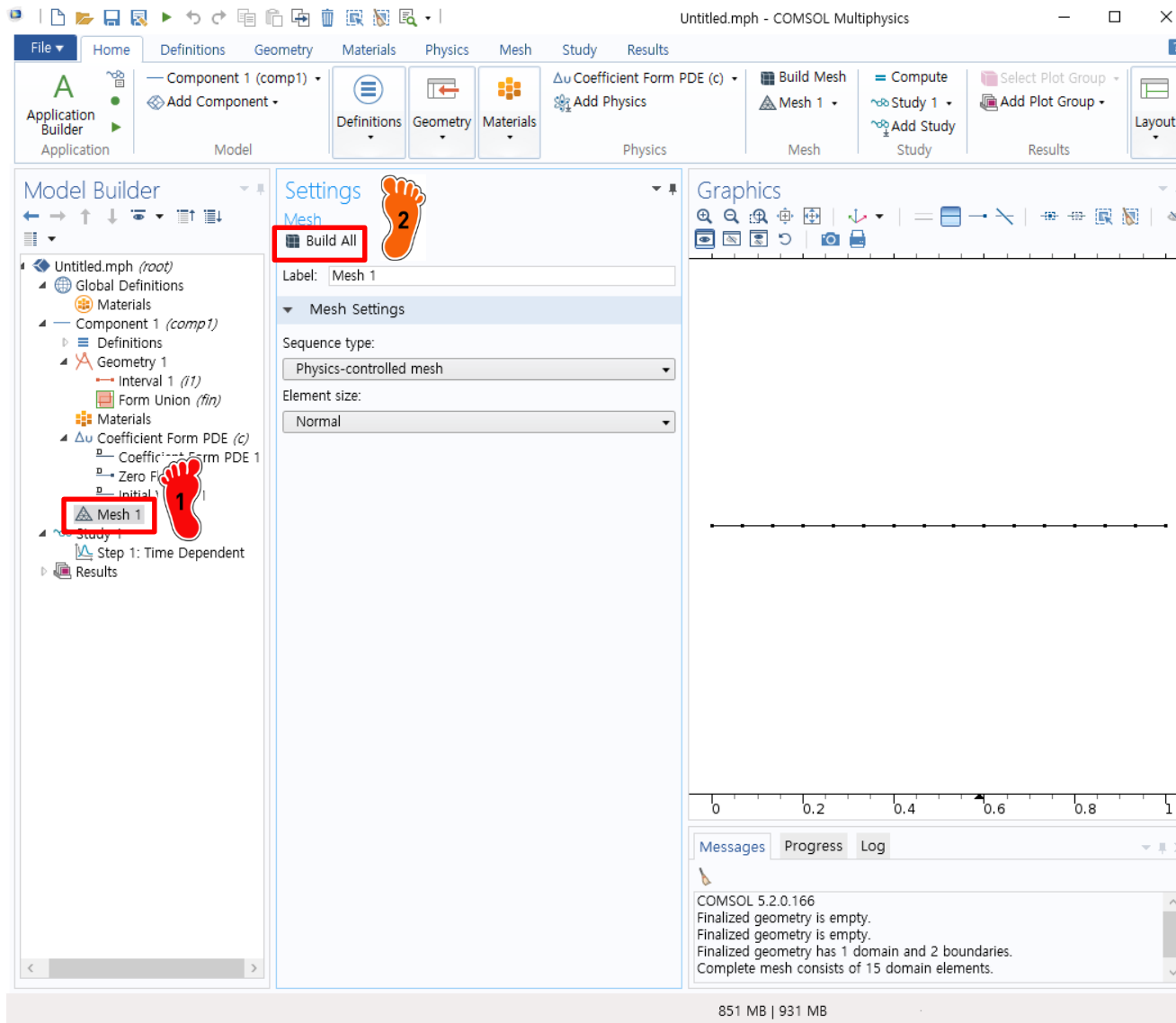
ODE
definition

Solving

Post-process

- ✓ Mesh creation
- ✓ Time range input
- ✓ Compute

MESH CREATION



1 Mesh 1 메뉴 클릭

2 Build All 클릭

TIME RANGE INPUT

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results

Application Builder Model

Model Builder

- Untitled.mph (root)
 - Global Definitions
 - Materials
 - Component 1 (comp1)
 - Definitions
 - Geometry 1
 - Interval 1 (it)
 - Form Union (fin)
 - Materials
 - Coefficient Form PDE (c)
 - Coefficient Form PDE 1
 - Zero Flux 1
 - Initial Values 1
 - Mesh 1
 - Study 1
 - Step 1: Time Dependent
 - Results

Settings

Time Dependent

Compute

Label: Time Dependent

Study Settings

Time unit: s

Times: range(0,1,4) s

Relative tolerance: ☐ 0.01

Results While Solving

Physics and Variables Selection

Modify physics tree and variables for study step

Physics interface Solve for Discretization

Coefficient Form PDE (c) ☒ Physics settings: ▾

Values of Dependent Variables

Mesh Selection

Study Extensions

Graphics

0 0.2 0.4 0.6 0.8 1

Messages Progress Log

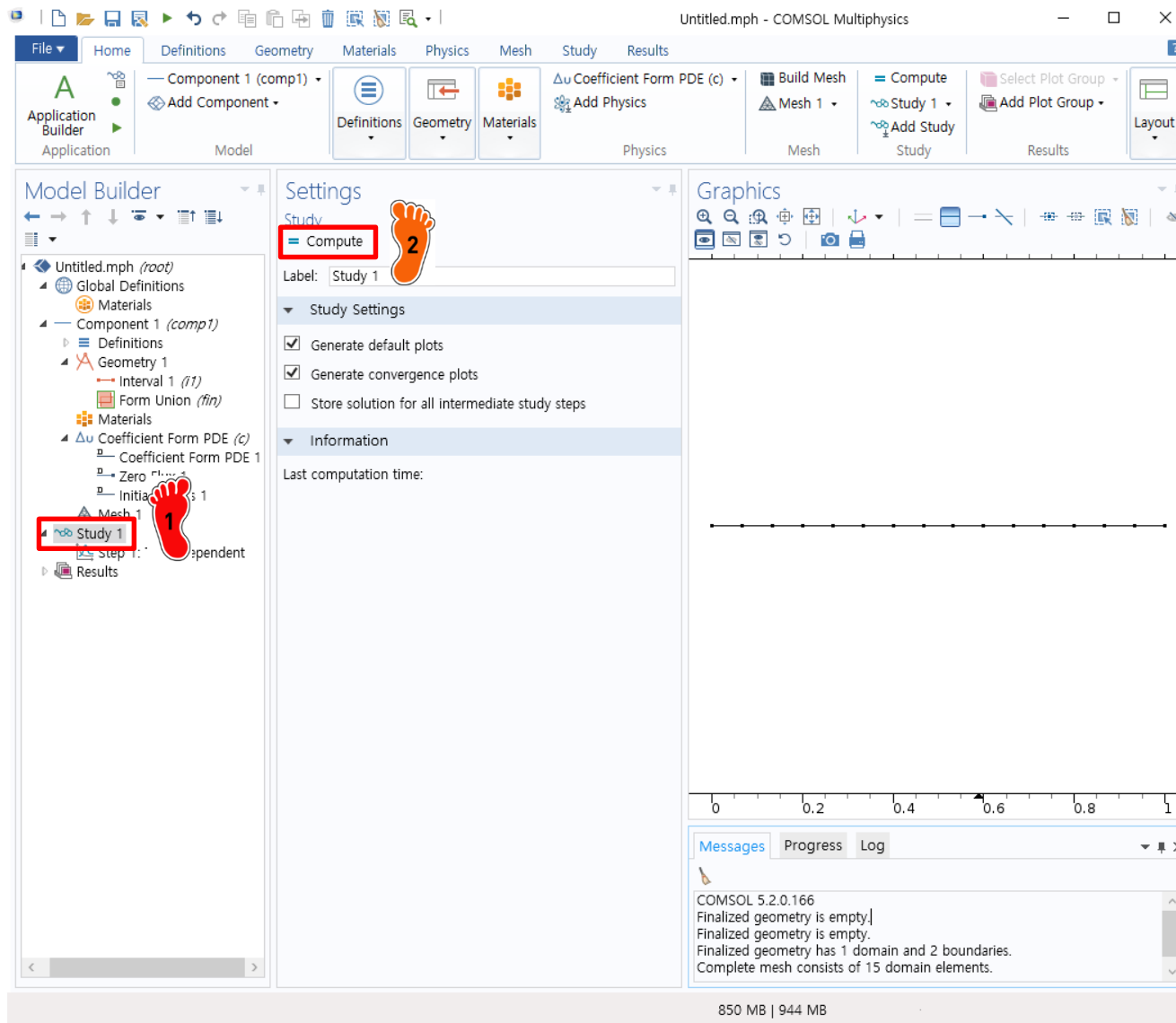
COMSOL 5.2.0.166
 Finalized geometry is empty.
 Finalized geometry is empty.
 Finalized geometry has 1 domain and 2 boundaries.
 Complete mesh consists of 15 domain elements.

843 MB | 936 MB

1 Study 1 → Step 1: Time Dependent 메뉴 클릭

2 Times: 창에 "range(0,1,4)" 입력

COMPUTE



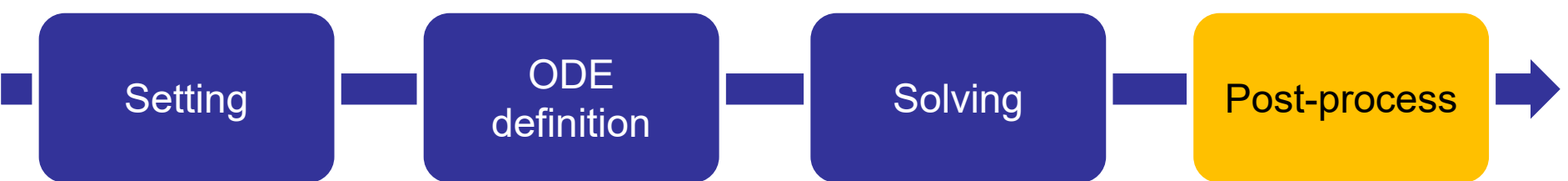
Study 1 메뉴 선택



Compute 클릭

850 MB | 944 MB

ANALYSIS FLOW



```
graph LR; A[Setting] --> B[ODE definition]; B --> C[Solving]; C --> D[Post-process]; D --> E[ ]; style D fill:#ffcc00; style E fill:none,stroke:none;
```

Setting

ODE
definition

Solving

Post-process

✓ Result plot

RESULT PLOT

The screenshot displays the COMSOL Multiphysics interface for 'Untitled.mph - COMSOL Multiphysics'. The 'Results' tab is active, and the '1D Plot Group 1' context menu is open. The menu options include Plot, Plot In, Line Graph, Point Graph, Global, Table Graph, Histogram, Ray, Nyquist Plot, Far Field, Octave Band Plot, Mesh, Add Image to Export, Copy as Code to Clipboard, Duplicate, Delete, Disable, Rename, Settings, and Properties. The 'Point Graph' option is highlighted with a red box and a red footprint icon labeled '2'. The '1D Plot Group 1' option in the left sidebar is also highlighted with a red box and a red footprint icon labeled '1'. The main plot area shows a 'Line Graph: Dependent variable u (1)' with a y-axis ranging from 0 to 75 and an x-axis ranging from 0 to 1. The plot displays a horizontal line at y=15. The bottom status bar indicates '887 MB | 958 MB'.

1D Plot Group 1 우클릭

Point Graph 클릭

RESULT PLOT

Untitled.mph - COMSOL Multiphysics

1D Plot Group 1

Color Expression

Filter

Attributes

1D Image

Animation

Export

Model Builder

Settings: Point Graph 1

Label: Point Graph 1

Data

Data set: From parent

Selection

Selection: Manual

1

Active

y-Axis Data

Expression: u

Unit: 1

Description: Dependent variable u

Title

x-Axis Data

Parameter: Time

Unit: s

Graphics

Convergence Plot 1

Messages

Progress

Log

Finalized geometry is empty.
Finalized geometry has 1 domain and 2 boundaries.
Complete mesh consists of 15 domain elements.
Number of degrees of freedom solved for: 31 (plus 2 internal DOFs).
Solution time (Study 1): 1 s.

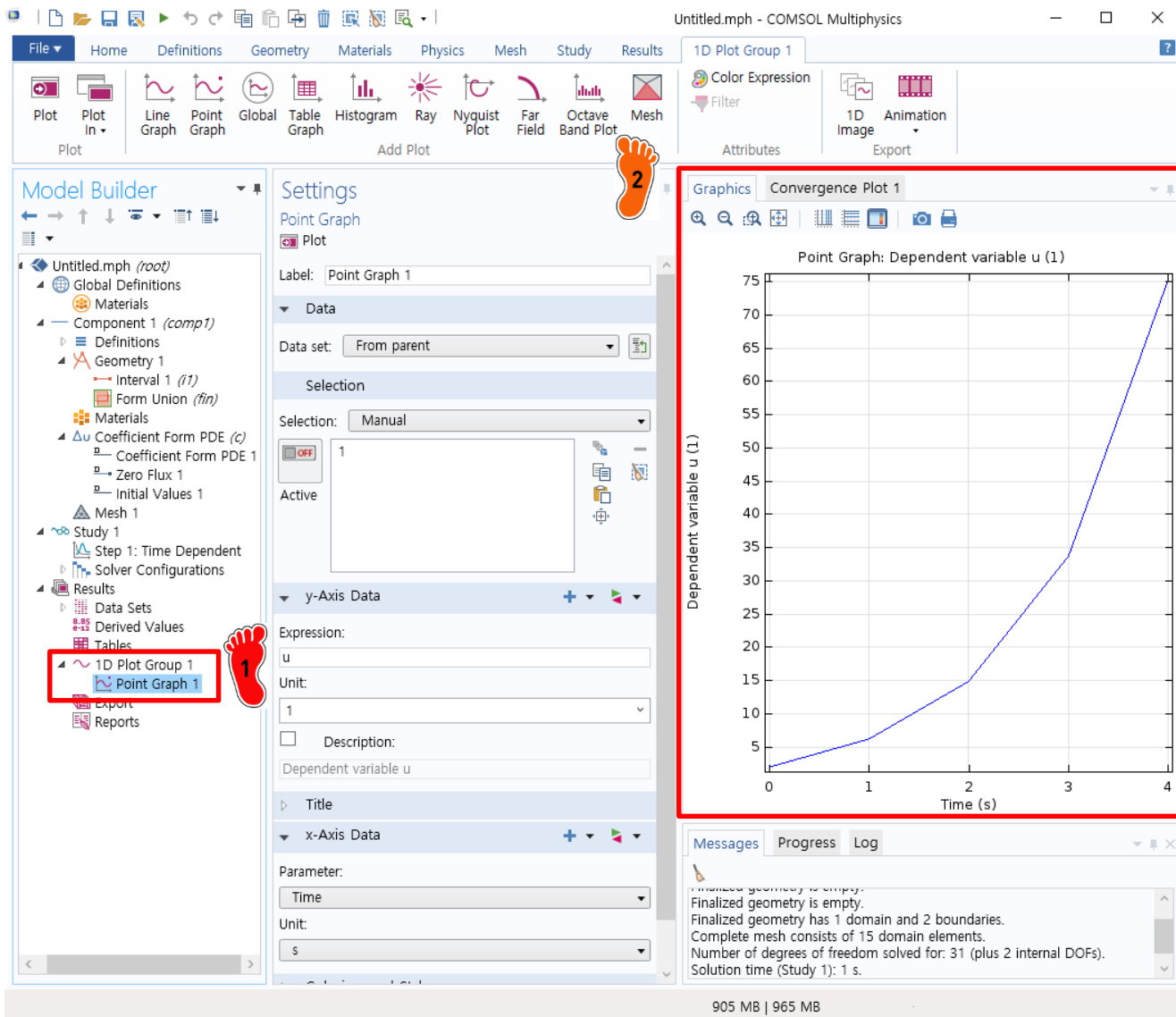
902 MB | 980 MB

1 왼쪽 point 클릭

2 Selection에 "1" 확인

3 Plot 클릭

RESULT PLOT



1 기존 Line Graph 1 삭제
(우클릭 → Delete 또는
선택 후 "Del" 키)

2 결과 그래프 확인

- **ODE examples**
 - ✓ **Example 25.5**
 - ✓ **Example 28.2: predator-prey model**
 - ✓ **Example 25.14**
 - ✓ **Example 27.10**
 - ✓ **Previous case study I**
 - ✓ **Previous case study II**

EXAMPLE 28.2

[predator – prey model]

nonlinear ordinary differential equations

$$\begin{cases} \frac{dy_1}{dt} = ay_1 - by_1y_2 \\ \frac{dy_2}{dt} = -cy_2 + dy_1y_2 \end{cases}$$

$$a = 1.2, b = 0.6, c = 0.8, d = 0.3$$

initial condition

$$t = 0, y_1 = 2, y_2 = 1$$



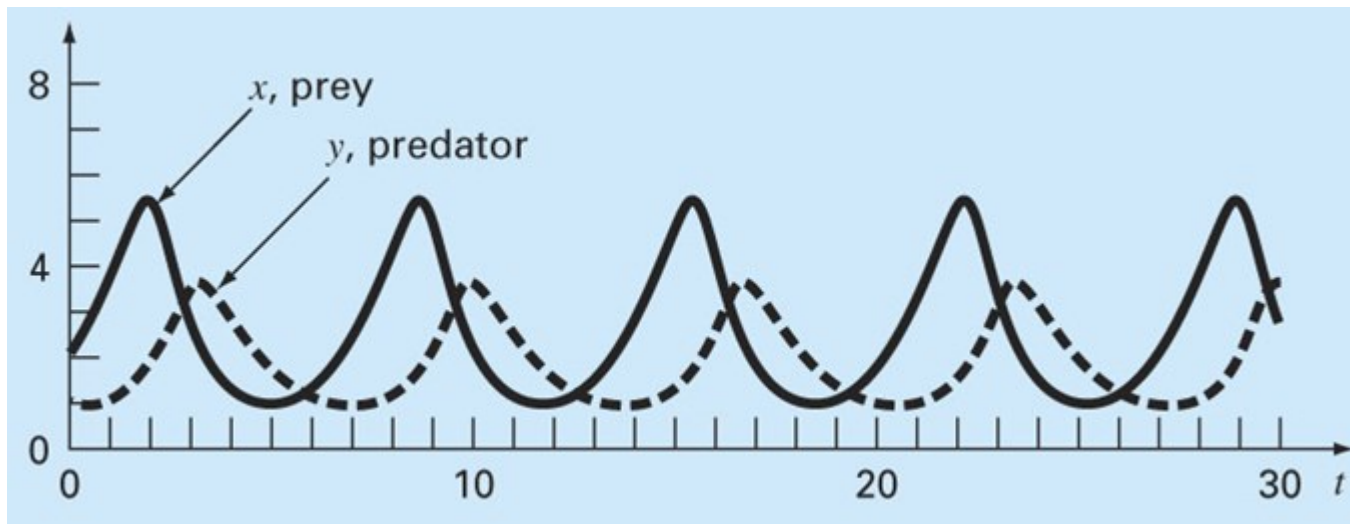
Predator-prey model developed by the Italian mathematician Vito Volterra and the American biologist Alfred J. Lotka.

먹이사슬에 관한 미분방정식

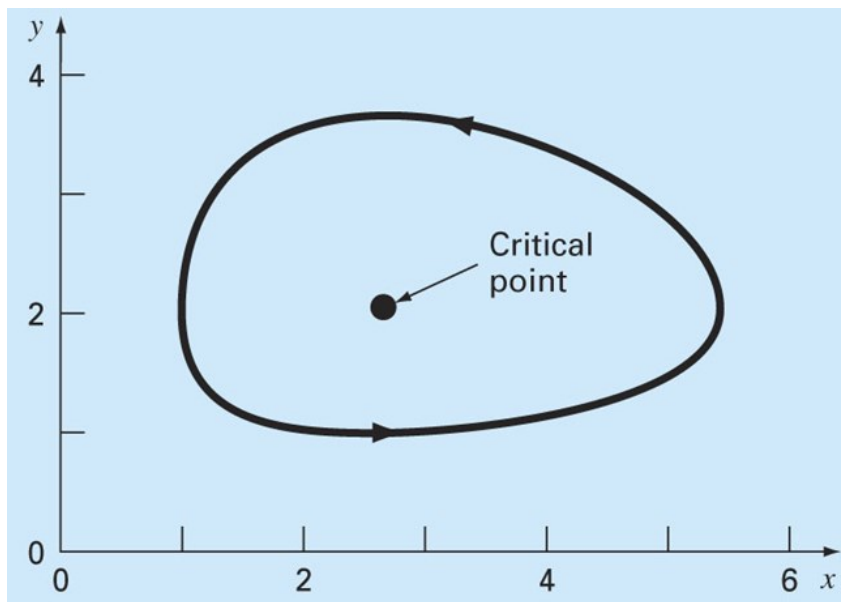
a = the prey growth rate
 c = the predator death rate

$b=d$ = the rate characterizing the effect of the predator-prey interaction on prey death and predator growth

EXAMPLE 28.2



Parameters: change of magnitudes of peaks, lags, period



[state-space representation]

$$\frac{dx}{dt} = \frac{dy}{dt} = 0 \rightarrow (x, y) = (0, 0) \text{ and } \left(\frac{c}{d}, \frac{a}{b}\right)$$

$\left(\frac{0.8}{0.3}, \frac{1.2}{0.6}\right)$ populations will remain constant

SETTING

09_25p5.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results 1D Plot Group 1

Select Physics

Search

- Recently Used
- AC/DC
- Acoustics
- Chemical Species Transport
- Heat Transfer
- Plasma
- Mathematics
 - PDE Interfaces**
 - Coefficient Form PDE (c)**
 - General Form PDE (g)
 - Wave Form PDE (waww)
 - Weak Form PDE (w)
 - Lower Dimensions
 - ODE and DAE Interfaces
 - Optimization and Sensitivity
 - Classical PDEs
 - Deformed Mesh
 - Wall Distance (wd)

Add

Added physics interfaces:

- Coefficient Form PDE (c)

Remove

Space Dimension

Study

Help Cancel Done

Review Physics Interface

Coefficient Form PDE (c)

Dependent Variables

Field name: u

Number of dependent variables: 2

Dependent variables: u1, u2

Units

Dependent variable quantity: Dimensionless (1)

Source term quantity: None

Unit: m⁻²

1D → Coefficient form PDE
선택 → Add 클릭

Number of dependent
variables: "2" 입력

Study 클릭
→ Time Dependent 선택
→ Done 클릭

COEFFICIENT INPUT

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results

Application Builder Model

Component 1 (comp1) Add Component

Definitions Geometry Materials

Physics Mesh Study Results

Build Mesh Mesh 1 Compute Study 1 Add Study Add Plot Group Select Plot Group Add Plot Group Layout

Model Builder

Untitled.mph (root)

Global Definitions

Materials

Component 1 (comp1)

Definitions

Geometry 1

Interval 1 (f1)

Form Union (fin)

Materials

Δu Coefficient Form PDE (c)

Coefficient Form PDE 1

Zero Flux 1

Initial Values 1

Mesh 1

Study 1

Step 1: Time Dependence

Results

Settings

Coefficient Form PDE

c

0 1 0 1

0 1 0 1

Absorption Coefficient

a

0 1/m² 0 1/m²

0 1/m² 0 1/m²

Source Term

f

1.2*u1-0.6*u1*u2 1/m²

-0.8*u2+0.3*u1*u2 1/m²

Mass Coefficient

e_a

0 s²/m² 0 s²/m²

0 s²/m² 0 s²/m²

Damping or Mass Coefficient

d_a

1 s/m² 0 s/m²

0 s/m² 1 s/m²

Conservative Flux Convection Coefficient

α

0 1/m 0 1/m

0 1/m 0 1/m

Convection Coefficient

β

0 1/m 0 1/m

0 1/m 0 1/m

Conservative Flux Source

γ

0 1/m

0 1/m

Graphics

Convergence Plot 1

0 0.2 0.4 0.6 0.8 1

Messages Progress Log

Number of degrees of freedom: 100000 (approx. 2.5 million elements).

Solution time (Study 1): 1 s.

Saved file: 09_25p2.mph

Saved file: 09_25p5.mph

Finalized geometry has 1 domain and 2 boundaries.

892 MB | 929 MB

1 전 예제와 동일한 방식으로 geometry 생성

2 Coefficient Form PDE 1에 계수 입력

종속변수가 2개 이므로
벡터 또는 행렬로 입력 됨
(나머지 0)

$$d_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$f = \begin{bmatrix} 1.2u_1 - 0.6u_1u_2 \\ -0.8u_2 + 0.3u_1u_2 \end{bmatrix}$$

INITIAL VALUE INPUT



Initial Values 1 클릭 후
u1: 2, u2: 1 입력

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results

Application Builder Model

Component 1 (comp1) Add Component

Definitions Geometry Materials

Physics

Mesh

Study

Results

Model Builder

Untitled.mph (root)

Global Definitions

Materials

Component 1 (comp1)

Definitions

Geometry 1

Interval 1 (int)

Form Union (fin)

Materials

Δu Coefficient Form PDE (c)

Coefficient Form PDE 1

Zero Flux 1

Initial Values 1

Mesh 1

Study 1

Step 1: Time Dependent

Results

Settings

Initial Values

Label: Initial Values 1

Domain Selection

Selection: All domains

1

Active

Override and Contribution

Initial Values

Initial value for u1:

u1 2 1

Initial value for u2:

u2 1 1

Initial time derivative of u1:

$\frac{\partial u1}{\partial t}$ 0 1/s

Initial time derivative of u2:

$\frac{\partial u2}{\partial t}$ 0 1/s

Graphics

Convergence Plot 1

Messages

Progress

Log

Number of degrees of freedom solved for: 21 (plus 2 internal DOFs).

Solution time (Study 1): 1 s.

Saved file: 09_25p2.mph

Saved file: 09_25p5.mph

Finalized geometry has 1 domain and 2 boundaries.

909 MB | 941 MB

TIME RANGE INPUT

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results

Application Builder Model

Component 1 (comp1) Add Component

Definitions Geometry Materials

Physics

Build Mesh Mesh 1 Compute Study 1 Add Study

Select Plot Group Add Plot Group Layout

Model Builder

Untitled.mph (root)

Global Definitions

Materials

Component 1 (comp1)

Definitions

Geometry 1

Interval 1 (i1)

Form Union (fin)

Materials

Coefficient Form PDE (c)

Coefficient Form PDE 1

Zero Flux 1

Initial Values 1

Mesh 1

Study 1

Step 1: Time Dependent

Results

Settings

Time Dependent

Compute

Label: Time Dependent

Study Settings

Time unit: s

Times: range(0,0.1,20) s

Relative tolerance: 0.01

Results While Solving

Physics and Variables Selection

Modify physics tree and variables for study step

Physics interface Solve f Discretization

Coefficient Form PDE (c) [x] Physics settings

Values of Dependent Variables

Mesh Selection

Study Extensions

Graphics

Convergence Plot 1

0 0.2 0.4 0.6 0.8 1

Messages Progress Log

Number of degrees of freedom selected for: 1 (plus 2 internal DOFs)

Solution time (Study 1): 1 s.

Saved file: 09_25p2.mph

Saved file: 09_25p5.mph

Finalized geometry has 1 domain and 2 boundaries.

894 MB | 919 MB

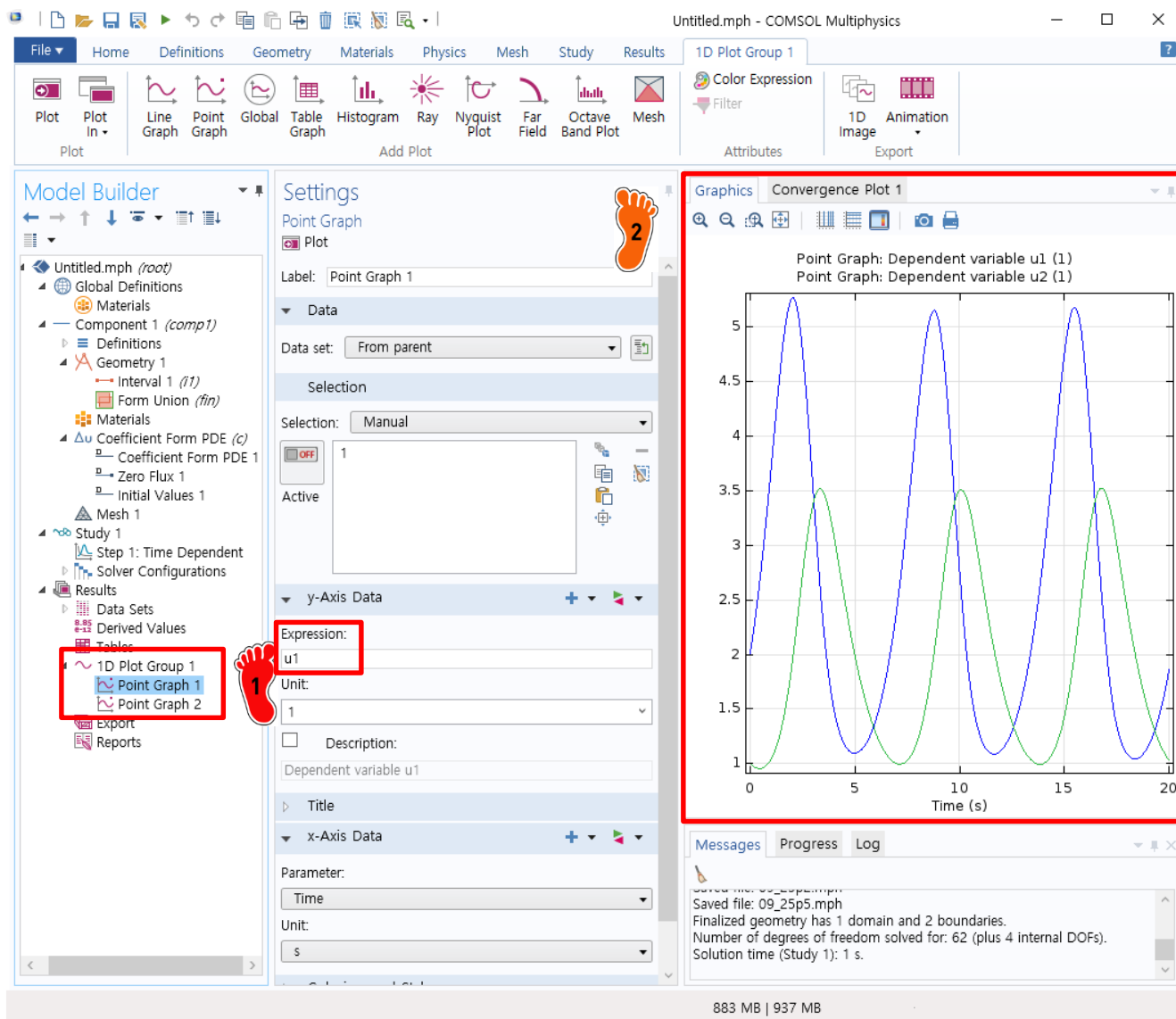


Step 1: Time Dependent 메뉴 선택 후

Times: range(0,0.1,20) 입력

이 후 Study 선택 후
Compute 클릭

RESULT PLOT



1 두 개의 Point Graph 생성
(Expression에 u1, u2 입력)

2 결과 그래프 확인

883 MB | 937 MB

- **ODE examples**
 - ✓ **Example 25.5**
 - ✓ **Example 28.2: predator-prey model**
 - ✓ **Example 25.14**
 - ✓ **Example 27.10**
 - ✓ **Previous case study I**
 - ✓ **Previous case study II**

EXAMPLE 25.14

ordinary differential equation

$$\frac{dy}{dt} = 10e^{-(t-2)^2 / [2(0.075)^2]} - 0.6y$$

initial condition

$$t = 0, y = 0.5$$



Coefficient Input

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c \nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + a u = f$$

▼	Absorption Coefficient
a	0.6 1/m ²
▼	Source Term
f	10*exp(-(t-2)^2)/(2*(0.075)^2) 1/m ²
▼	Damping or Mass Coefficient
d _a	1 s/m ²

Initial Value Input

▼	Initial Values
Initial value for u:	
u	0.5

Time Range Input

▼	Study Settings
Time unit:	s
Times:	range(0,1,4)

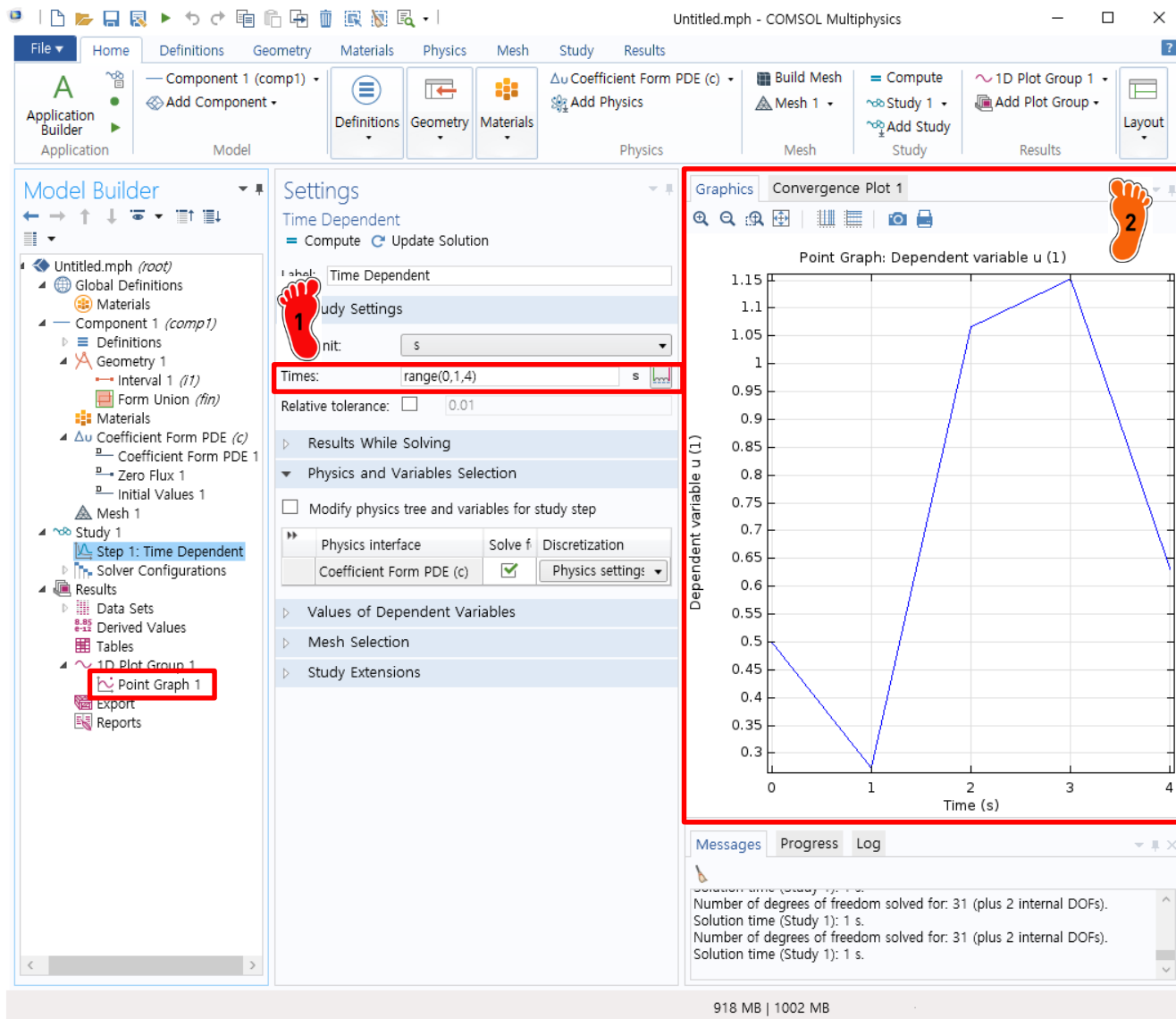


Adaptive ODE solver 를 보여주기 위한 미분방정식



모델링 입력 항목

RANGE INPUT



1 range(0,1,4) 로 입력할 경우 step size 가 상대적으로 큼

2 결과 그래프에서 데이터는 step size 1 로 결정이 됨

ADAPTIVE OPTION SELECTION

The screenshot shows the COMSOL Multiphysics interface for a 1D plot group. The **Model Builder** on the left shows the hierarchy: **Study 1** > **Step 1: Time Dependent** > **Solver Configurations** > **Solution 1 (sol1)** > **Time-Dependent Solver 1**. The **Settings** pane for **Time-Dependent Solver 1** is open, showing the **General** tab. The **Times to store** dropdown is set to **Steps taken by solver**. The **Output** section has **Store reaction forces**, **Store time derivatives**, and **Store solution on disk** checked. The **Advanced** section is expanded, showing **Log** settings. The **Point Graph: Dependent variable u (1)** is displayed, showing a plot of **Dependent variable u (1)** versus **Time (s)**. The plot shows a curve that starts at 0.5, decreases to a minimum of about 0.2 at 1.5s, and then increases to a peak of about 1.8 at 2.0s. The **Coloring and Style** pane is open, showing **Line style** set to **Solid**, **Color** set to **Cycle**, and **Width** set to **1**. The **Line markers** section is also open, showing **Marker** set to **Circle** and **Positioning** set to **In data points**.

1 Time-Dependent Solver 1 메뉴 선택

2 Times to store: Steps taken by solver 변경

3 step size 가 adaptive 하게 변경이 됨

Point Graph 1

- **ODE examples**

- ✓ **Example 25.5**
- ✓ **Example 28.2: predator-prey model**
- ✓ **Example 25.14**
- ✓ **Example 27.10**
- ✓ **Previous case study I**
- ✓ **Previous case study II**

EXAMPLE 27.10



[Van der Pol equation]

$$\frac{d^2 y_1}{dt^2} - \mu(1 - y_1^2) \frac{dy_1}{dt} + y_1 = 0$$

initial condition

$$t = 0, y_1 = 1, \frac{dy_1}{dt} = 1$$

convert process →

$$\begin{cases} \frac{dy_1}{dt} = y_2 \\ \frac{dy_2}{dt} = \mu(1 - y_1^2)y_2 - y_1 \end{cases}$$



Dependent Variables

Dependent Variables	
Field name:	u
Number of dependent variables:	2
Dependent variables:	u1 u2

Coefficient Input

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c \nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + a u = f$$

Source Term	
f	u2 1/m ²
	μ(1-u1^2)*u2-u1 1/m ²

Damping or Mass Coefficient			
d _a	1 s/m ²	0 s/m ²	
	0 s/m ²	1 s/m ²	



stiff 한 정도가 mu 값에 따라서 변하는 van der Pol equation.



모델링 입력 항목

Initial Value Input

Initial Values	
Initial value for u1:	u1 1
Initial value for u2:	u2 1

Time Range Input

Study Settings	
Time unit:	s
Times:	range(0,0.1,20)

TIME DEPENDENT OPTION

The screenshot displays the COMSOL Multiphysics interface for a time-dependent study. The 'Study' tab is active, and the 'Time Dependent' option is selected in the 'Study Settings' section. A red box highlights the 'Results While Solving' checkbox, which is checked. The 'Plot' checkbox is also checked. The 'Time unit' is set to 's' (seconds), and the 'Times' range is 'range(0,0.1,20)'. The 'Relative tolerance' is set to '0.01'. The 'Plot group' is '1D Plot Group 1', and the 'Update at' is 'Times stored in output'. The 'Physics and Variables Selection' section shows the 'Coefficient Form PDE (c)' interface selected. The 'Values of Dependent Variables' section is expanded, showing 'u1' and 'u2'. The 'Mesh Selection' and 'Study Extensions' sections are also visible. The 'Graphics' window shows a 'Convergence Plot 1' with two data series: 'Point Graph: Dependent variable u1 (1)' (blue circles) and 'Point Graph: Dependent variable u2 (1)' (green circles). The plot shows oscillatory behavior over time. The 'Messages' window at the bottom displays the solution progress and statistics.

1.11 GB | 1.17 GB



Step 1: Time Dependent
 → Result While Solving
 → Plot 체크 후 Compute

ADAPTIVE OPTION SELECTION

The screenshot displays the COMSOL Multiphysics interface for a time-dependent study. The left sidebar shows the Model Builder tree with the following structure:

- Untitled.mph (root)
 - Global Definitions
 - Materials
 - Component 1 (comp1)
 - Definitions
 - Geometry 1
 - Interval 1 (t1)
 - Form Union (fin)
 - Materials
 - Coefficient Form PDE (c)
 - Coefficient Form PDE 1
 - Zero Flux 1
 - Initial Values 1
 - Mesh 1
 - Study 1
 - Step 1: Time Dependent
 - Solver Configurations
 - Solution 1 (sol1)
 - Compile Equation:
 - Time-Dependent Solver 1 (highlighted with a red box and a red footprint icon labeled '1')

The Settings window for the Time-Dependent Solver 1 is shown on the right. The 'General' tab is active, with the following settings:

- Label: Time-Dependent Solver 1
- Defined by study step: Step 1: Time Dependent
- Time unit: s
- Time: range(0,0.1,20)
- Relative tolerance: 0.01
- Times to store: Steps taken by solver (highlighted with a red box and a red footprint icon labeled '2')
- Store reaction forces: ☒
- Use lumping when computing fluxes: ☐
- Store time derivatives: ☒
- Store solution on disk: ☒
- Store solution before and after events: ☐
- Advanced:
 - Log:
 - Keep warnings in stored log: ☐

The bottom right shows the Messages window with the following output:

```

Time-Dependent Solver 1 in Study 1/Solution 1
Time-dependent solver (BDF)
Number of degrees of freedom solved for: 62 (
Nonsymmetric matrix found.
Scales for dependent variables:
Dependent variable u1 (comp1.u1): 1
Dependent variable u2 (comp1.u2): 1
  
```

The Graphics window displays a Convergence Plot 1, showing the Point Graph: Dependent variable u1 (1) and Point Graph: Dependent variable u2 (1). The plot shows two oscillating curves, one in blue and one in green, over a time range from 0 to 20 seconds. The y-axis ranges from -2.5 to 2.5.

1 Time-Dependent Solver 1
메뉴 선택 → Steps taken
by solver 선택

2 Print step size 비교

STIFF COEFFICIENT INPUT



mu 값을 1000으로 변경

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results

Application Builder Model

Component 1 (comp1) Add Component

Definitions Geometry Materials

Physics

Build Mesh Mesh 1

Compute Study 1 Add Study

1D Plot Group 1 Add Plot Group

Results Layout

Model Builder

Untitled.mph (root)

Global Definitions

Materials

Component 1 (comp1)

Definitions

Geometry 1

Interval 1 (f1)

Form Union (fin)

Materials

Coefficient Form PDE (c)

Coefficient Form PDE 1

Zero Flux 1

Initial Values 1

Mesh 1

Study 1

Step 1: Time Dependent

Solver Configurations

Solution 1 (sol1)

Compile Equations: Time

Dependent Variables 1

Time-Dependent

Results

Data Sets

Derived Values

Tables

1D Plot Group 1

Point Graph 1

Point Graph 2

Export

Reports

Settings

Coefficient Form PDE

Equation

Show equation assuming:

Study 1, Time Dependent

$$e_a \frac{\partial^2 \mathbf{u}}{\partial t^2} + d_a \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (-c \nabla \mathbf{u} - \alpha \mathbf{u} + \gamma) + \beta \cdot \nabla \mathbf{u} + \alpha \mathbf{u} = \mathbf{f}$$

$$\mathbf{u} = [u_1, u_2]^T$$

$$\nabla = \frac{\partial}{\partial x}$$

Diffusion Coefficient

c	0	1	0	1
	0	1	0	1

Absorption Coefficient

a	0	1/m ²	0	1/m ²
	0	1/m ²	0	1/m ²

Source Term

f	u2	1/m ²
	(1-u1^2)*u2-u1	1/m ²

Mass Coefficient

e_a	0	s ² /m ²	0	s ² /m ²
	0	s ² /m ²	0	s ² /m ²

Damping or Mass Coefficient

d_a	1	s/m ²	0	s/m ²
	0	s/m ²	1	s/m ²

Conservative Flux Convection Coefficient

Graphics

Convergence Plot 1

Messages Progress Log

Number of degrees of freedom solved for: 62 (plus 4 internal DOFs).
Solution time (Study 1): 6 s.
Number of degrees of freedom solved for: 62 (plus 4 internal DOFs).
Solution time (Solution 1 (sol1)): 5 s.

1.08 GB | 1.16 GB

TIME RANGE INPUT

Untitled.mph - COMSOL Multiphysics

File Home Definitions Geometry Materials Physics Mesh Study Results

Application Builder Model

Component 1 (comp1) Add Component

Definitions Geometry Materials

Physics

Build Mesh Mesh 1

Compute Study 1 Add Study

1D Plot Group 1 Add Plot Group

Layout

Model Builder

Untitled.mph (root)

Global Definitions

Materials

Component 1 (comp1)

Definitions

Geometry 1

Interval 1 (t1)

Form Union (fin)

Materials

Coefficient Form PC

Coefficient Form PDE 1

Zero Flux 1

Initial Values 1

Mesh 1

Study 1

Step 1: Time Dependent

Solver Configurations

Solution 1 (sol1)

Results

Data Sets

Derived Values

Tables

1D Plot Group 1

Point Graph 1

Point Graph 2

Export

Reports

Settings

Time Dependent

Compute Update Solution

Label: Time Dependent

Study Settings

Time unit: s

Times: range(0,0.1,2000) s

Relative tolerance: 0.01

Results While Solving

Plot

Plot group: 1D Plot Group 1

Update at: Times stored in output

Probes: All

Update at: Times stored in output

Physics and Variables Selection

Modify physics tree and variables for study step

Physics interface Solve f Discretization

Coefficient Form PDE (c) ☒ Physics settings

Values of Dependent Variables

Mesh Selection

Study Extensions

Graphics

Convergence Plot 1

Point Graph: Dependent variable u1 (1)

Point Graph: Dependent variable u2 (1)

Time (s)

Messages Progress Log

Number of degrees of freedom solved for: 62 (plus 4 internal DOFs).

Solution time (Solution 1 (sol1)): 1 s.

Number of degrees of freedom solved for: 62 (plus 4 internal DOFs).

Solution time (Study 1): 16 s.

Number of degrees of freedom solved for: 62 (plus 4 internal DOFs).

Solution time (Study 1): 4 s.

1.16 GB | 1.21 GB



range(0,0.1,2000)으로 변경
→ Compute

PLOT GROUP DISABLE

Untitled.mph - COMSOL Multiphysics

1D Plot Group 1

Color Expression

Filter

Attributes

1D Image

Animation

Export

Model Builder

Settings

Point Graph

Label: Point Graph 2

Data

Data set: From parent

Plot

Plot In

Color Expression

Add Plot Data to Export

Copy as Code to Clipboard

Move Up

Ctrl+Up

Copy

Duplicate

Delete

Del

Disable

F3

Rename

F2

Settings

Properties

Help

F1

Point Graph 2

Convergence Plot 1

Point Graph: Dependent variable u1 (1)

Point Graph: Dependent variable u2 (1)

Time (s)

Messages

Progress

Log

Number of degrees of freedom solved for: 62 (plus 4 internal DOFs).

Solution time (Study 1): 14 s.

Number of degrees of freedom solved for: 62 (plus 4 internal DOFs).

Solution time (Solution 1 (sol1)): 1 s.

Number of degrees of freedom solved for: 62 (plus 4 internal DOFs).

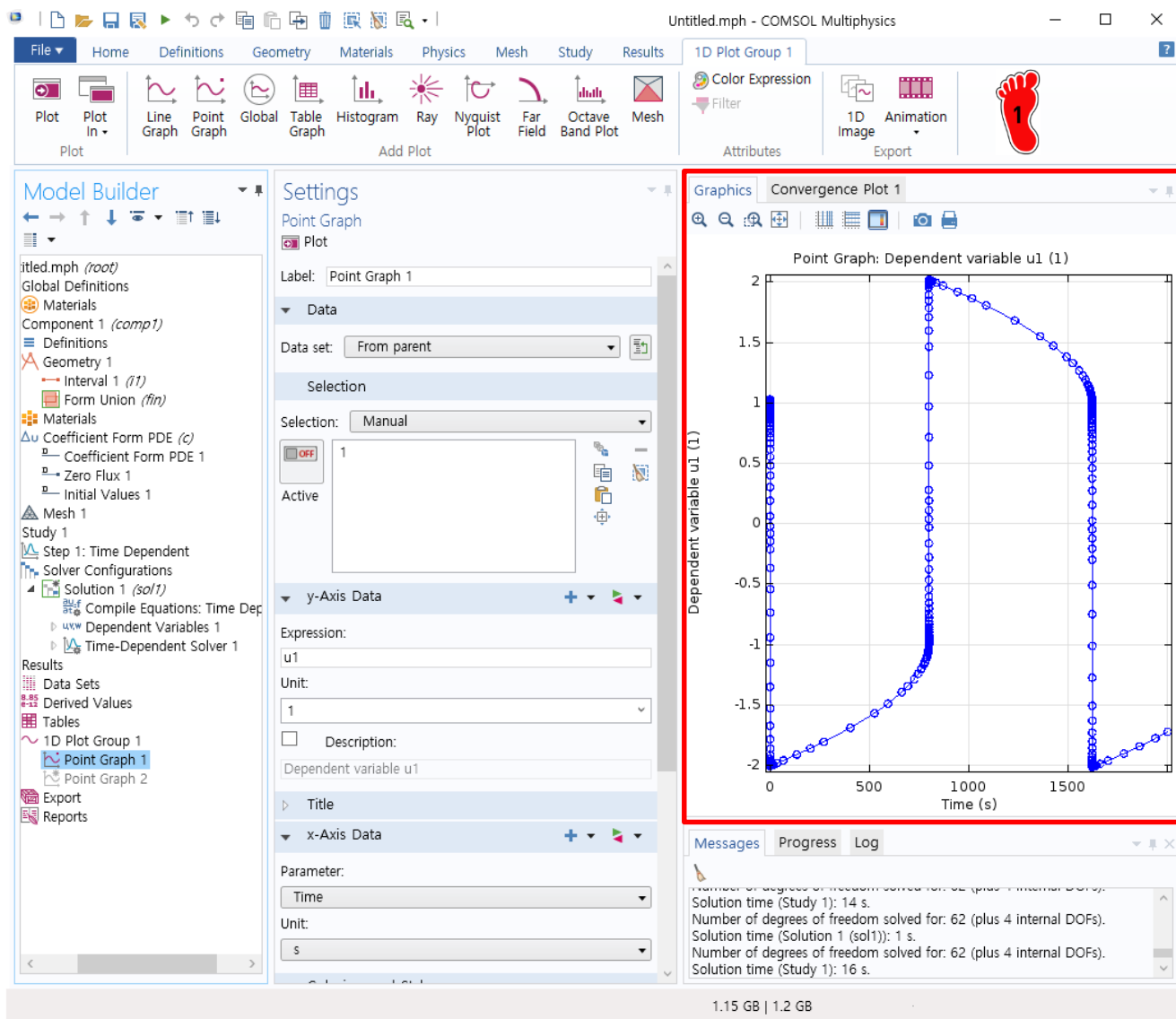
Solution time (Study 1): 16 s.

1.15 GB | 1.2 GB

1 Point Group 2 우클릭

2 Disable 클릭
(u2의 경우 stiff한 구간에서 값이 급격하게 증가하기 때문에(결과 그래프 참조) u1 결과를 동시에 확인 어려움)

RESULT PLOT



결과 확인

stiff 한 시스템인 경우 시간이 소요되는 부분은 기울기가 급격히 변화하는 부분임을 확인

Times to store: Steps taken by solver
☒ Store reaction Specified values
☐ Use lumping when computing fluxes Steps taken by solver

옵션에 따른 결과 확인

- **ODE examples**
 - ✓ **Example 25.5**
 - ✓ **Example 28.2: predator-prey model**
 - ✓ **Example 25.14**
 - ✓ **Example 27.10**
 - ✓ **Previous case study I**
 - ✓ **Previous case study II**

CASE STUDY I

Background. Electric circuits where the current is time-variable rather than constant are common. A transient current is established in the right-hand loop of the circuit shown in Fig. 28.11 when the switch is suddenly closed.

Equations that describe the transient behavior of the circuit in Fig. 28.11 are based on Kirchhoff's law, which states that the algebraic sum of the voltage drops around a closed loop is zero (recall Sec. 8.3). Thus,

$$L \frac{di}{dt} + Ri + \frac{q}{C} - E(t) = 0 \quad (28.9)$$

where $L(di/dt)$ = voltage drop across the inductor, L = inductance (H), R = resistance (Ω), q = charge on the capacitor (C), C = capacitance (F), $E(t)$ = time-variable voltage source (V), and

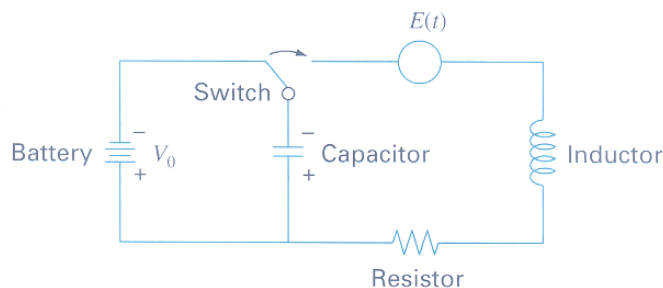
$$i = \frac{dq}{dt} \quad (28.10)$$

Equations (28.9) and (28.10) are a pair of first-order linear differential equations that can be solved analytically. For example, if $E(t) = E_0 \sin \omega t$ and $R = 0$,

$$q(t) = \frac{-E_0}{L(p^2 - \omega^2)} \frac{\omega}{p} \sin pt + \frac{E_0}{L(p^2 - \omega^2)} \sin \omega t \quad (28.11)$$

FIGURE 28.11

An electric circuit where the current varies with time.



$$E = E_0 \sin(\omega t)$$

$$L = 1 \text{ H}$$

$$E_0 = 1 \text{ V}$$

$$C = 0.25 \text{ C}$$

$$\omega^2 = 3.5 \text{ rad/s}^2$$

$$R = 0$$

$$i(0) = 0$$

$$q(0) = 0$$

$$t = (0, 100) \Delta t = 0.1$$

$$q = y_1$$

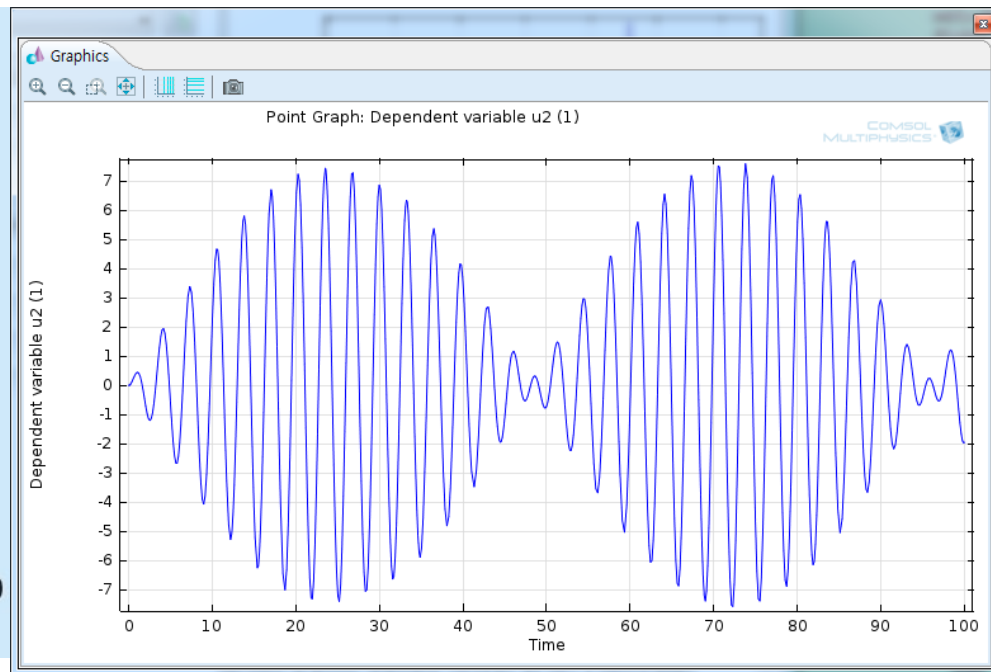
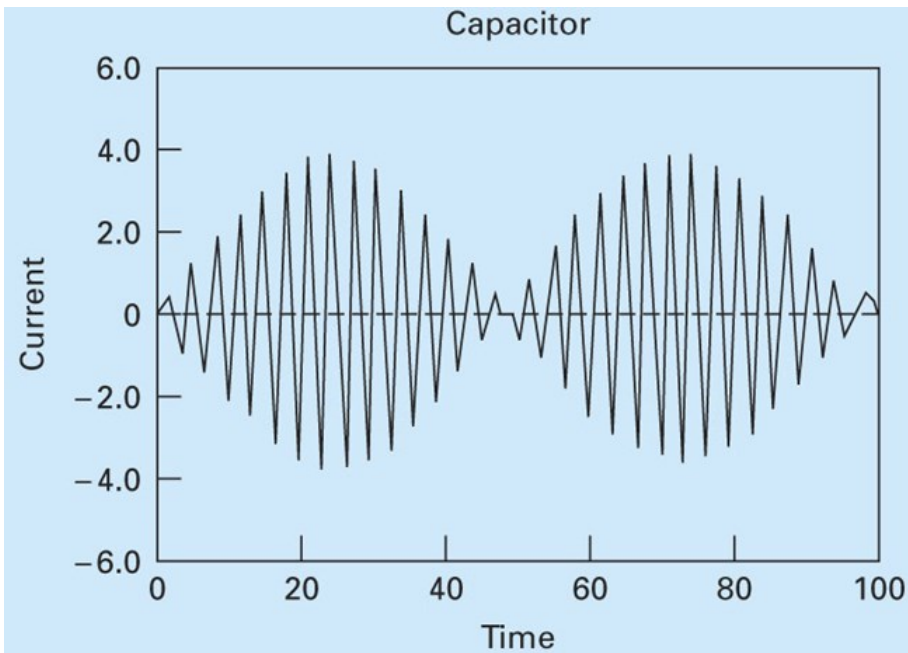
$$i = \frac{dq}{dt} = \frac{dy_1}{dt} = y_2$$

$$L \frac{di}{dt} + Ri + \frac{q}{C} - E(t) = 0$$

$$\rightarrow \frac{dy_2}{dt} = \frac{1}{L} \left(E(t) - Ri - \frac{y_1}{C} \right)$$

$$\begin{cases} \frac{dy_1}{dt} = y_2 \\ \frac{dy_2}{dt} = \frac{1}{L} \left(E(t) - Ri - \frac{y_1}{C} \right) \end{cases}$$

CASE STUDY I: RESULT



- **ODE examples**

- ✓ **Example 25.5**
- ✓ **Example 28.2: predator-prey model**
- ✓ **Example 25.14**
- ✓ **Example 27.10**
- ✓ **Previous case study I**
- ✓ **Previous case study II**

CASE STUDY II

Background. Mechanical engineers (as well as all other engineers) are frequently faced with problems concerning the periodic motion of free bodies. The engineering approach to such problems ultimately requires that the position and velocity of the body be known as a function of time. These functions of time invariably are the solution of ordinary differential equations. The differential equations are usually based on Newton's laws of motion.

As an example, consider the simple pendulum shown previously in Fig. PT7.1. The particle of weight W is suspended on a weightless rod of length l . The only forces acting on the particle are its weight and the tension R in the rod. The position of the particle at any time is completely specified in terms of the angle θ and l .

The free-body diagram in Fig. 28.16 shows the forces on the particle and the acceleration. It is convenient to apply Newton's laws of motion in the x direction tangent to the path of the particle:

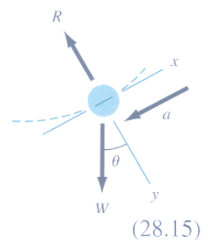
$$\Sigma F = -W \sin \theta = \frac{W}{g} a$$

where g = the gravitational constant (32.2 ft/s²) and a = the acceleration in the x direction. The angular acceleration of the particle (α) becomes

$$\alpha = \frac{a}{l}$$

FIGURE 28.16

A free-body diagram of the swinging pendulum showing the forces on the particle and the acceleration.



(28.15)

Therefore, in polar coordinates ($\alpha = d^2\theta/dt^2$),

$$-W \sin \theta = \frac{Wl}{g} \alpha = \frac{Wl}{g} \frac{d^2\theta}{dt^2}$$

or

$$\frac{d^2\theta}{dt^2} + \frac{g}{l} \sin \theta = 0$$

This apparently simple equation is a second-order nonlinear differential equation. In general, such equations are difficult or impossible to solve analytically. You have two choices regarding further progress. First, the differential equation might be reduced to a form that can be solved analytically (recall Sec. PT7.1.1), or second, a numerical approximation technique can be used to solve the differential equation directly. We will examine both of these alternatives in this example.

$$\sin \theta \approx \theta$$

$$g = 9.81 \text{ m/s}^2$$

$$\theta_0 = \pi / 4$$

$$l = 0.6096 \text{ m}$$

$$\theta = y_1$$

$$\frac{d\theta}{dt} = \frac{dy_1}{dt} = y_2$$

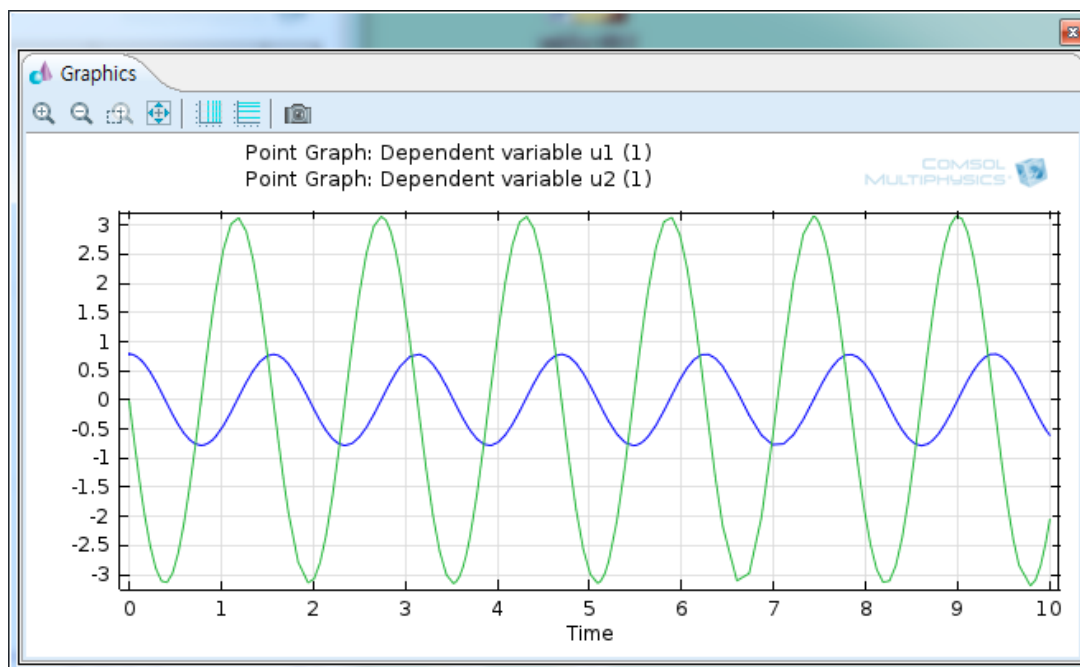
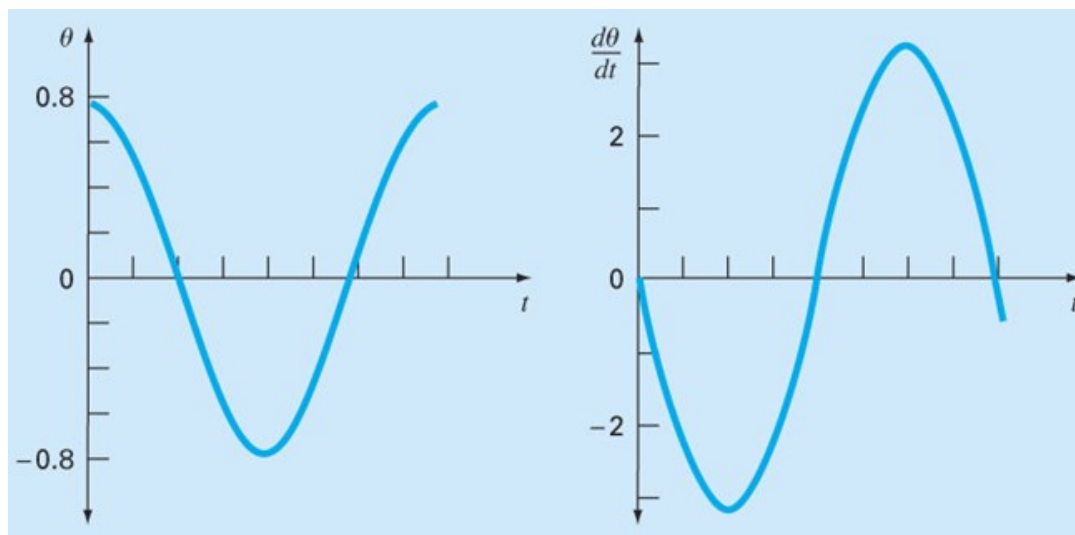
$$\frac{d^2\theta}{dt^2} + \frac{g}{l} \sin \theta = 0$$

$$\rightarrow \frac{dy_2}{dt} = -\frac{g}{l} \sin y_1$$

$$\rightarrow \frac{dy_2}{dt} = -\frac{g}{l} y_1$$

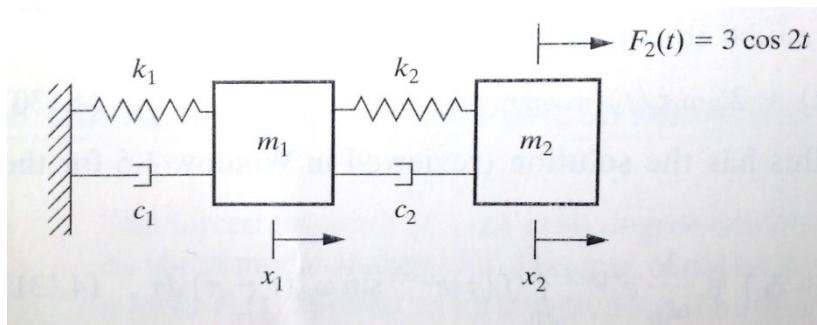
$$\begin{cases} \frac{dy_1}{dt} = y_2 \\ \frac{dy_2}{dt} = -\frac{g}{l} y_1 \end{cases}$$

CASE STUDY II: RESULT



ASSIGNMENT(3RD WEEK: ODE)

2-DOF system



$$m_1 = 9 \text{ kg}, m_2 = 1 \text{ kg}$$

$$k_1 = 24 \text{ N/m}, k_2 = 3 \text{ N/m}$$

$$c_1 = 2.4 \text{ Ns/m}, c_2 = 0.3 \text{ Ns/m}$$

$$F_2(t) = 3 \cos 2t$$

※ Ref. : Daniel J. Inman, "Engineering Vibration",
Prentice Hall International, Inc., pp 296-298, 2001

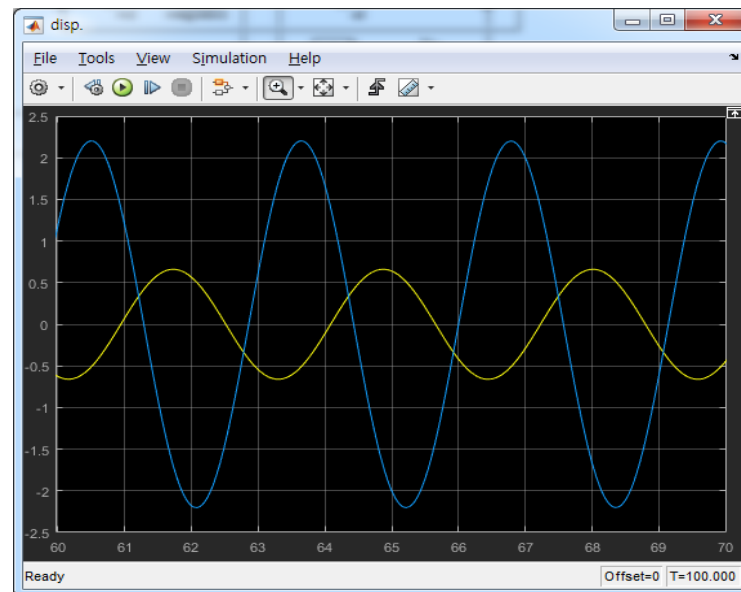
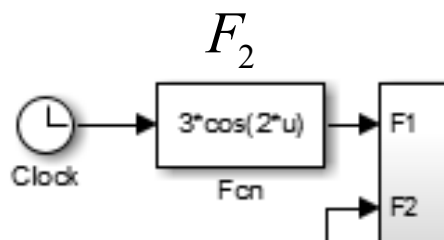
$$m_1 \ddot{x}_1 = -k_2(x_1 - x_2) - c_2(\dot{x}_1 - \dot{x}_2) - k_1 x_1 - c_1 \dot{x}_1$$

$$m_2 \ddot{x}_2 = k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2) + F(t)$$

※ Analytic Solution (Steady State)

$$x_1(t) = 0.2451 \cos(2t - 0.1974) - 0.6249 \sin 2t$$

$$x_2(t) = 0.7354 \cos(2t - 0.1974) + 1.8749 \sin 2t$$



ASSIGNMENT(3RD WEEK: ODE)

