Introduction & ODE by COMSOL

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OUTLINE

- **Lecture Goals**
  - ✓ 다양한 Physics의 유한요소해석 소프트웨어인 COMSOL의 솔루션들을 소개하고, 이를 활용한 미분 방정식 해법을 도출하는 과정을 실습한다
- **Content**
  - ✓ Introduction
  - ✓ COMSOL desktop
  - ✓ Mathematic module
  - ✓ ODE examples
  - ✓ Assignment

COMSOL MULTIPHYSICS

- Finite element analysis and simulation software package for various physics and engineering applications
- Founded by Savante Littmarck and Farhad Saeidi in 1986
- FEMLAB: Early version of COMSOL (before 2005)

Dr. h.c. Svante Littmarck
CEO of the COMSOL Group

Mr. Farhad Saeidi
President of COMSOL AB

COMSOL BACKGROUND

Before the 1980's, **Germund Dahlquist** pioneered the use of personal computers to solving a system of partial differential equations using numerical operations.

In 1986, Two of his students at the time, **Svante Littmarck** and **Farhad Saeidi** began to work on such a software package outside of the work already required by their graduate program. Littmarck and Saeidi made the decision to forgo their studies and begin building a software company under the name COMSOL.

Dahlquist provided administrative advice and even went so far as to put the two budding entrepreneurs into contact with **MathWorks**, who had already built their empire around successful software package **MATLAB**.

COMSOL released their first software package on September 1st, 1998 with a **structural mechanics model that included CAD optimization, material models, thermal stresses, waves, and much more**.

On April 12, 2001 the Chemical Engineering Module was added and nine years later on **April 20, 2010, software integration with products such as Solidworks and MATLAB were introduced.**

COMSOL ran into a near crippling lawsuit from MathWorks for copyright infringement in 2006. Nonetheless, the company rebounded and continues to draw in large profits. Today, COMSOL has about 50,000 users and boasts NASA as its largest consumer.

Reference-COMSOL Background – COMSOL Testosterone Transport Project

MODULES

www.comsol.com

COMSOL Multiphysics®

ELECTRICAL	MECHANICAL	FLUID	CHEMICAL	MULTIPURPOSE	INTERFACING	
AC/DC Module	Heat Transfer Module	CFD Module	Chemical Reaction Engineering Module	Optimization Module®	LiveLink™ for MATLAB®	LiveLink™ for Excel®
RF Module	Structural Mechanics Module	Microfluidics Module	Batteries & Fuel Cells Module	Material Library	CAD Import Module	ECAD Import Module
MEMS Module	Nonlinear Structural Materials Module	Subsurface Flow Module	Electrodeposition Module	Particle Tracing Module	LiveLink™ for SolidWorks®	LiveLink™ for SpaceClaim®
Plasma Module	Geomechanics Module	Pipe Flow Module	Corrosion Module		LiveLink™ for Inventor®	LiveLink™ for AutoCAD®
	Fatigue Module				LiveLink™ for Creo™ Parametric	LiveLink™ for Pro/ENGINEER®
	Acoustics Module				LiveLink™ for Solid Edge®	File Import for CATIA®V5

AC/DC MODULE

ELECTRICAL

AC/DC Module

RF Module

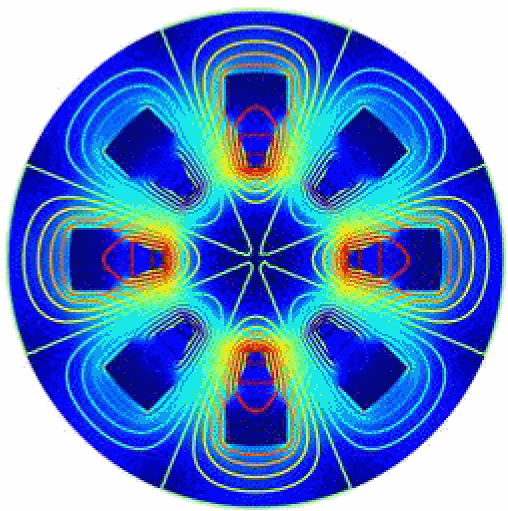
MEMS Module

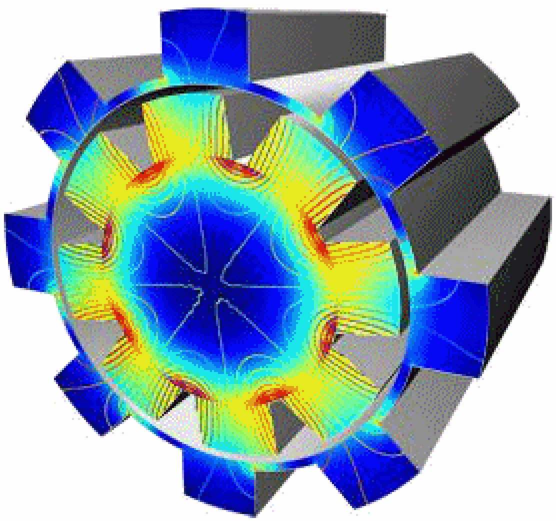
Plasma Module

Dynamics of a Generator

This example shows how the circular motion of a rotor with permanent magnets in a generator results in an induced EMF in the stator winding. The generated voltage is calculated as a function of time during the rotation.

The plot on the left shows the magnetic flux density along with a contour plot of the magnetic potential. Note the brighter regions, which indicate the position of the permanent magnets in the rotor. The figure on the right shows the geometry and a simulation of the generator in 3D.



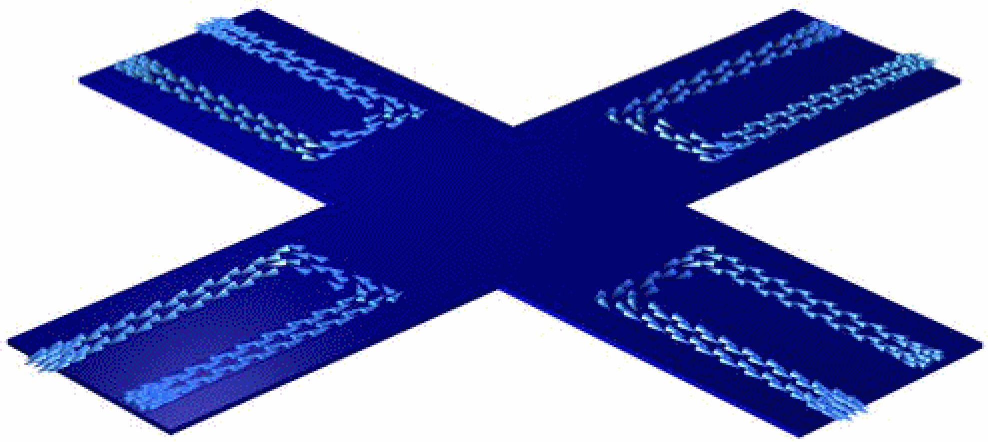


HEAT TRANSFER MODULE

Deformation of a Thermomechanical Microvalve

Thermomechanical microvalves are common flow control components in microfluidics systems. Here, an electric current generates movement by resistively heating the actuator structure, thereby causing mechanical stress and deformation.

In this example, a parametric study shows how an increasing voltage applied to each of the legs leads to temperature rise causing more and more deformation.



FLUID

CFD Module

Microfluidics Module

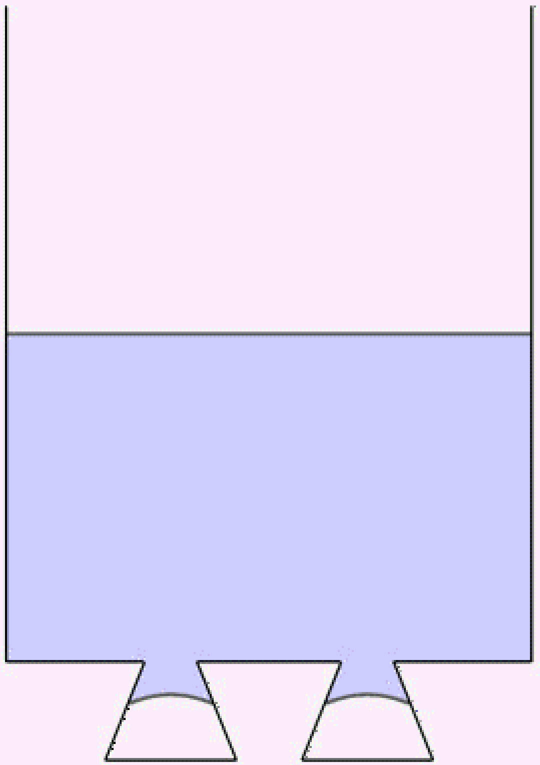
Subsurface Flow Module

Pipe Flow Module

Boiling Water

This model studies the film boiling of water. A heat flux above the Leidenfrost point is applied at the surface of two cavities. A layer of vapor is maintained at the hot surface - liquid interface where film-boiling results.

The animation shows the fluids volume fraction over time as a surface and contour plot.

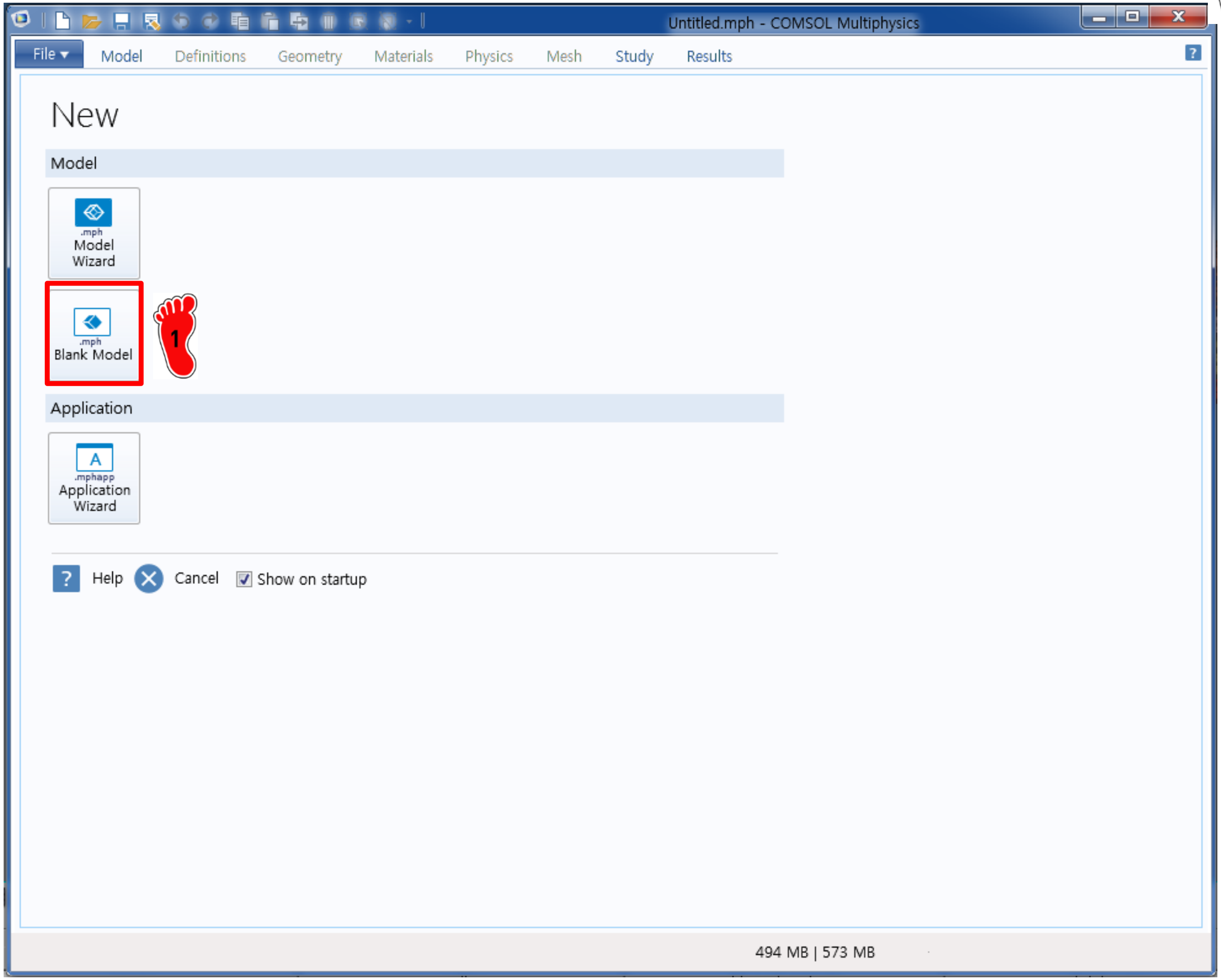


COMSOL Multiphysics®

- # COMSOL desktop

  - ✓ Toolbars & Ribbon tabs
  - ✓ Windows
  - ✓ Physics
  - ✓ Study types
  - ✓ Setting flow
  - ✓ Setting result

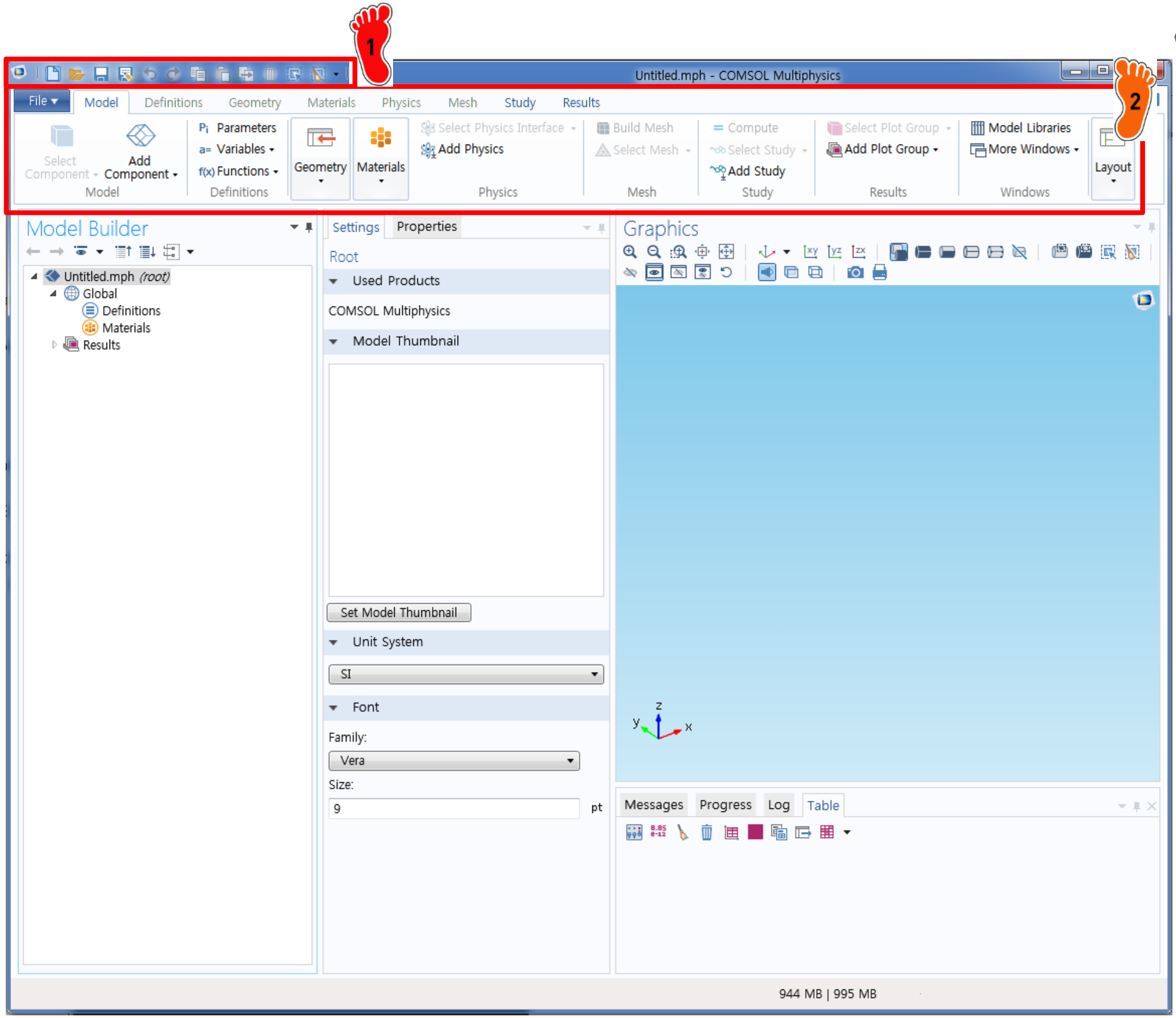
CREATING NEW MODEL



1 Blank Model

The Black Model 옵션은 Physics 또는 Study 설정없이 COMSOL Desktop interface를 실행

TOOLBAR & RIBBON TABS

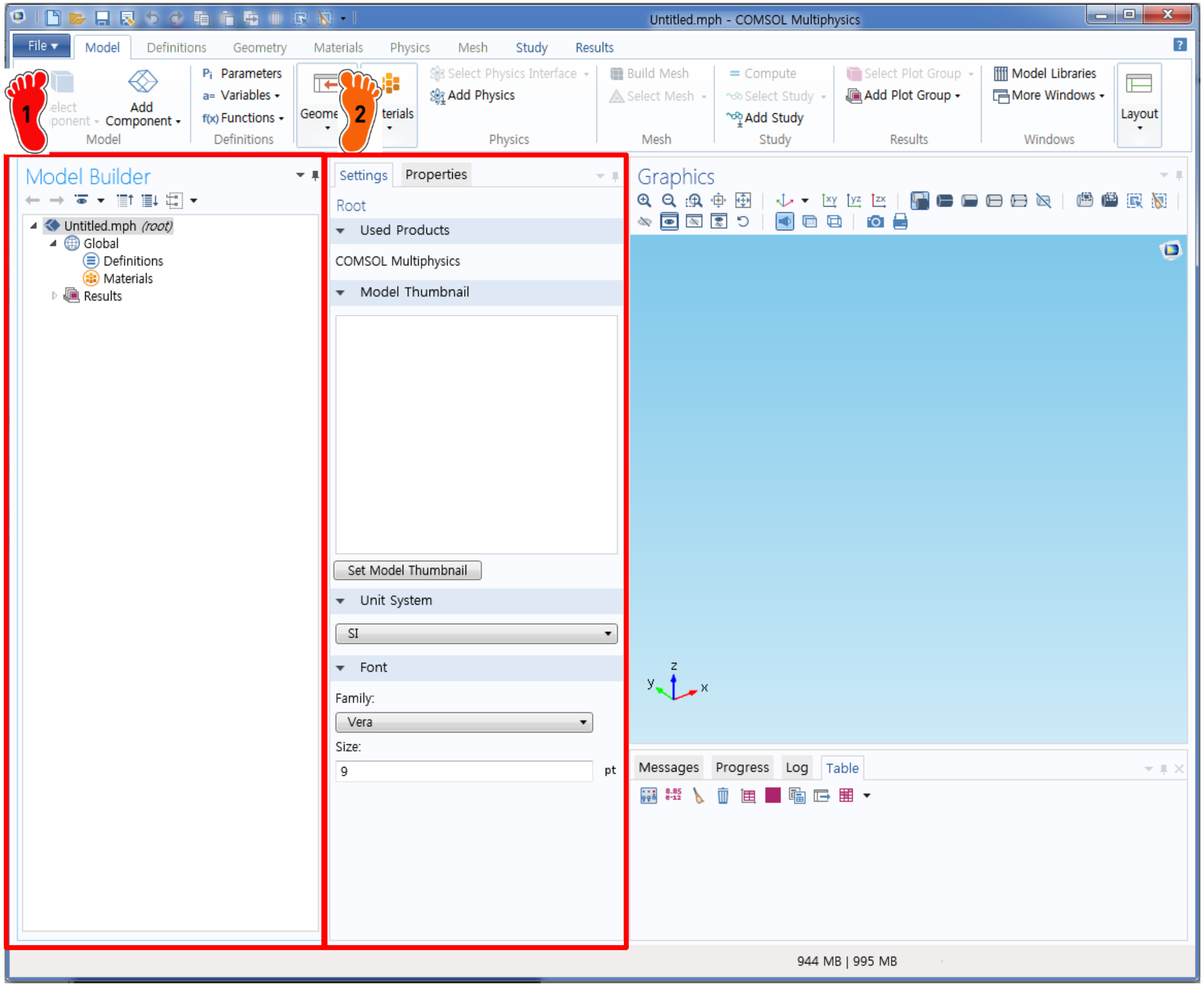


1 Quick Access Toolbar

2 Ribbon tabs

COMSOL Desktop 환경의
The ribbon tabs은 모델링
워크플로우를 반영하고
각 모델링에 사용할 수 있는
기능에 대한 개요 제공

WINDOWS



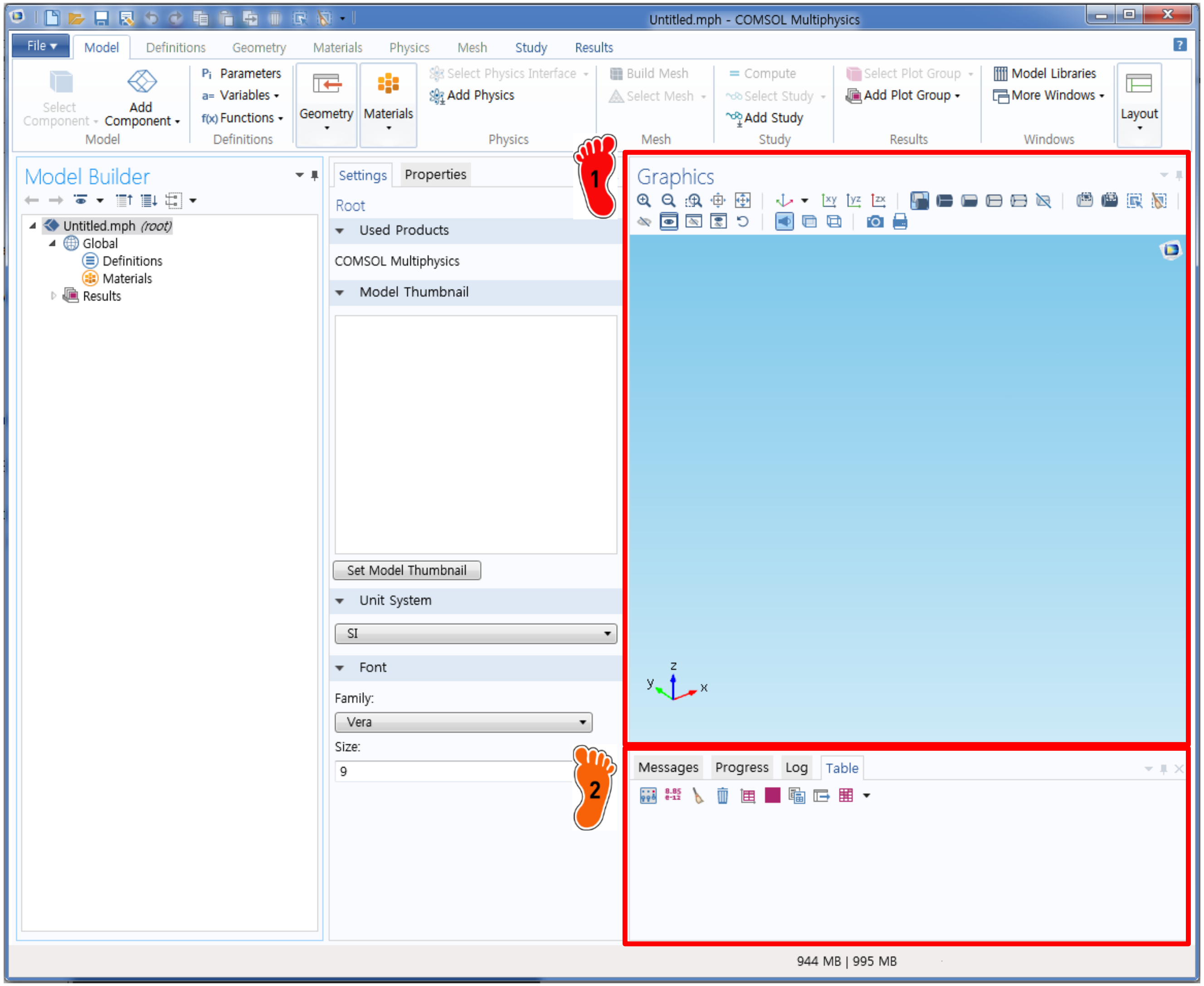
1 Model builder window

The Model Builder 는 사용자가 정의하는 Model 과 Physics를 정의하는 tool

2 Setting window

Model builder의 세부 node 세팅을 편집

WINDOWS



1 Graphics window

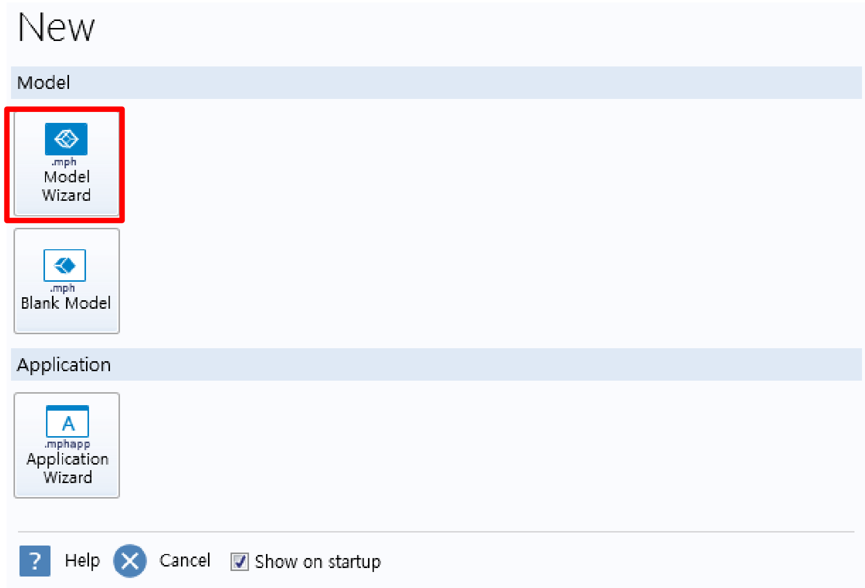
Mesh와 해석 결과를 시각화합니다. 회전, 축소확대, 선택적 시각화 등 옵션 설정 가능

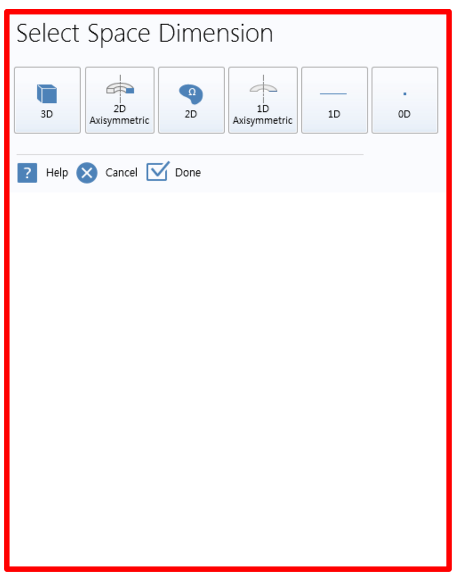
Information windows

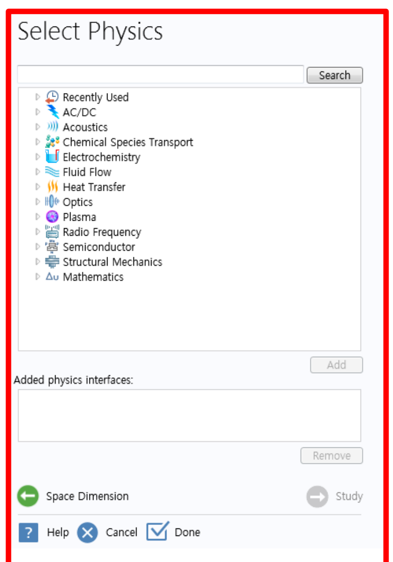
정보 창에는 솔루션 시간, 솔루션 진행률, 메시 통계 및 솔버 로그, 결과 테이블과 같은 시뮬레이션 동안 중요한 모델 정보 표시

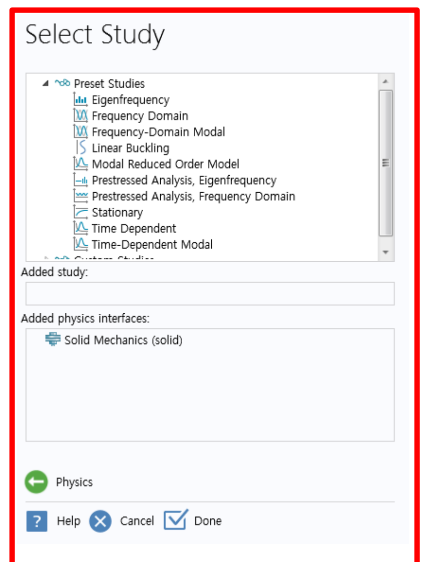
2

SETTING FLOW









1 Model Wizard

The Model Wizard는 차원, Physics, Study type을 설정

실제 모델링 전 진행 과정

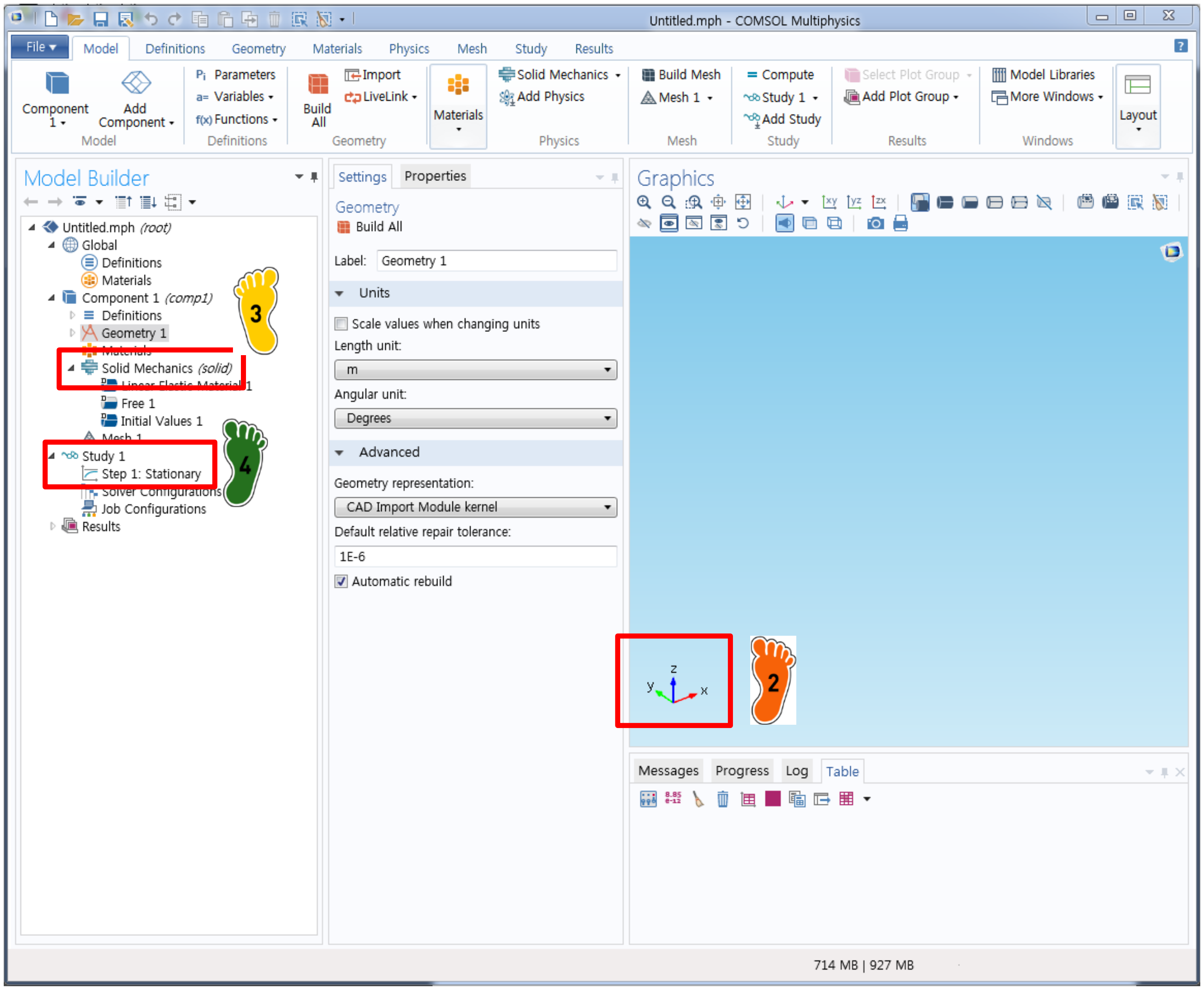
1. Dimension 선택

2. 해석 물리현상(physics) 선택

3. 해석 조건(study) 선택

ex) 3D – Solid Mechanics (solid) – Stationary

SETTING RESULT



1. 전 페이지에서 선택한 setting 결과
2. 3차원
3. Solid mechanics
4. Stationary

- **Mathematics module**
  - ✓ **Coefficient form PDE**
  - ✓ **PDE Interfaces**

COEFFICIENT FORM PDE

Equation of coefficient form PDE

$$\begin{cases} e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + au = f & \text{in}\,\Omega \\ \mathbf{n} \cdot (c\nabla u + \alpha u - \gamma) + qu = g - h^T \mu & \text{on}\,\partial\Omega \\ 0 = R & \text{on}\,\partial\Omega \end{cases}$$

where

- Ω is the computational domain—the union of all domains
- $\partial\Omega$ is the domain boundary
- $\mathbf{n}$ is the outward unit normal vector on $\partial\Omega$

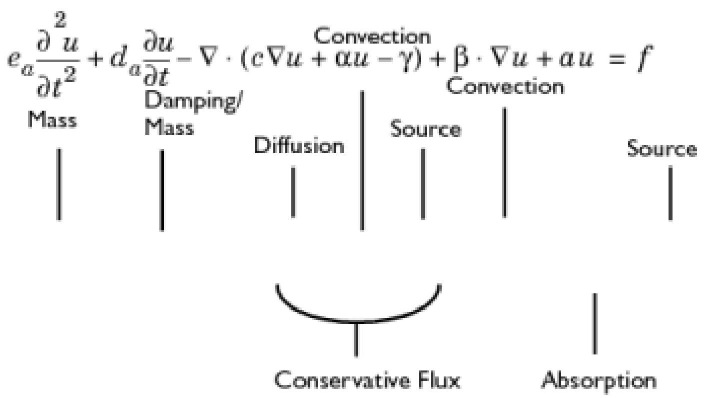
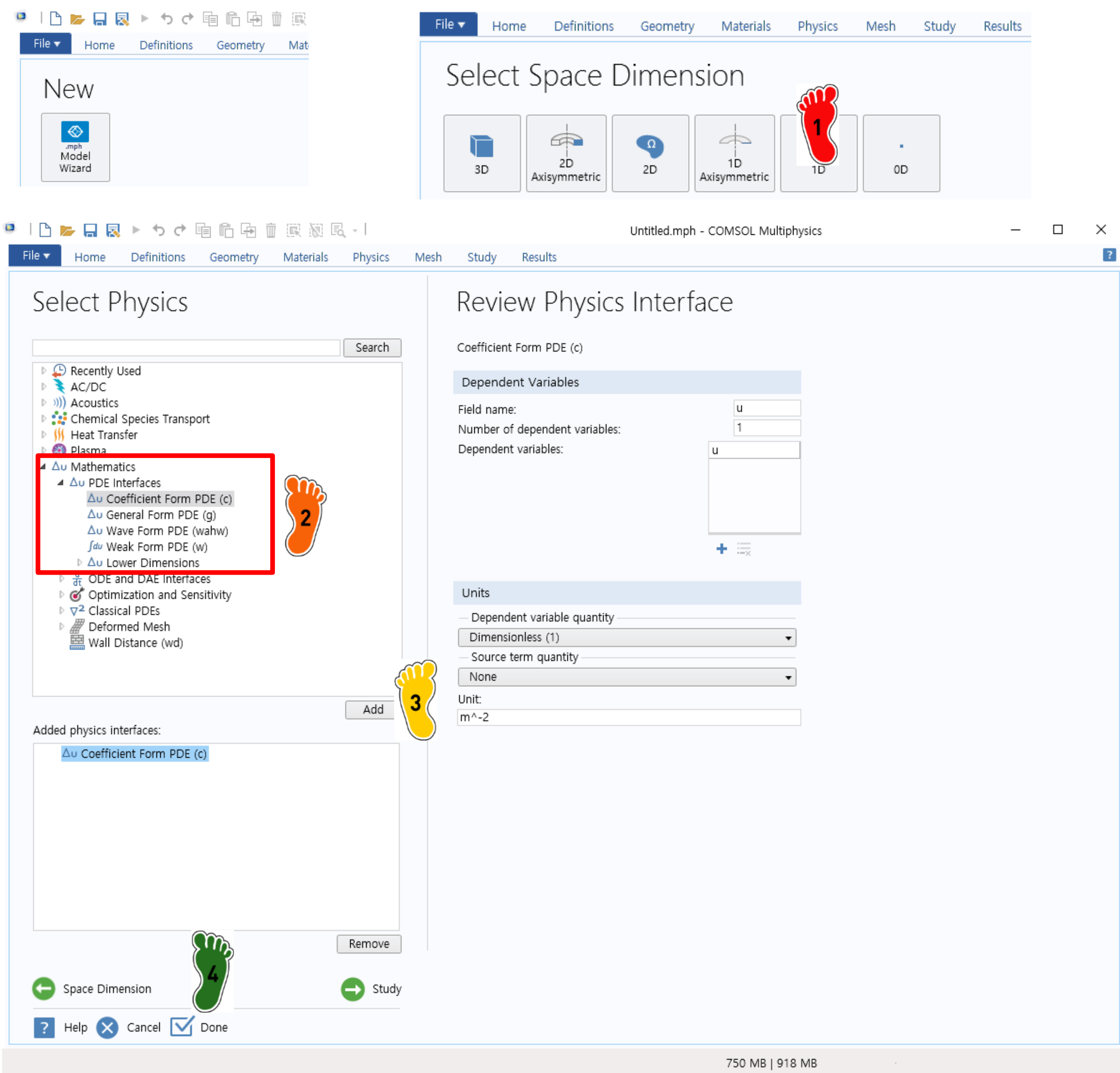


TABLE 16-1: CLASSICAL PDES IN COMPACT AND COMPONENT NOTATION

EQUATION	COMPACT NOTATION	COMPONENT NOTATION (2D)
Laplace's equation	$-\nabla \cdot (\nabla u) = 0$	$-\frac{\partial}{\partial x}\frac{\partial u}{\partial x} - \frac{\partial}{\partial y}\frac{\partial u}{\partial y} = 0$
Poisson's equation	$-\nabla \cdot (c\nabla u) = f$	$-\frac{\partial}{\partial x}\left(c\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(c\frac{\partial u}{\partial y}\right) = f$
Helmholtz equation	$-\nabla \cdot (c\nabla u) + au = f$	$-\frac{\partial}{\partial x}\left(c\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(c\frac{\partial u}{\partial y}\right) + au = f$
Heat equation	$d_a\frac{\partial u}{\partial t} - \nabla \cdot (c\nabla u) = f$	$d_a\frac{\partial u}{\partial t} - \frac{\partial}{\partial x}\left(c\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(c\frac{\partial u}{\partial y}\right) = f$
Wave equation	$e_a\frac{\partial^2 u}{\partial t^2} - \nabla \cdot (c\nabla u) = f$	$e_a\frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial x}\left(c\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(c\frac{\partial u}{\partial y}\right) = f$
Convection- diffusion equation	$d_a\frac{\partial u}{\partial t} - \nabla \cdot (c\nabla u) + \beta \cdot \nabla u = f$	$d_a\frac{\partial u}{\partial t} - \frac{\partial}{\partial x}\left(c\frac{\partial u}{\partial x}\right) - \frac{\partial}{\partial y}\left(c\frac{\partial u}{\partial y}\right) + \beta_x\frac{\partial u}{\partial x} + \beta_y\frac{\partial u}{\partial y} = f$

PDE INTERFACES



1 Model Wizard → 1D 클릭

2 모듈 선택 메뉴에서 Mathematics - PDE Interfaces 를 클릭

PDE Interfaces 는 PDE 를 풀기 위한 physics 모음

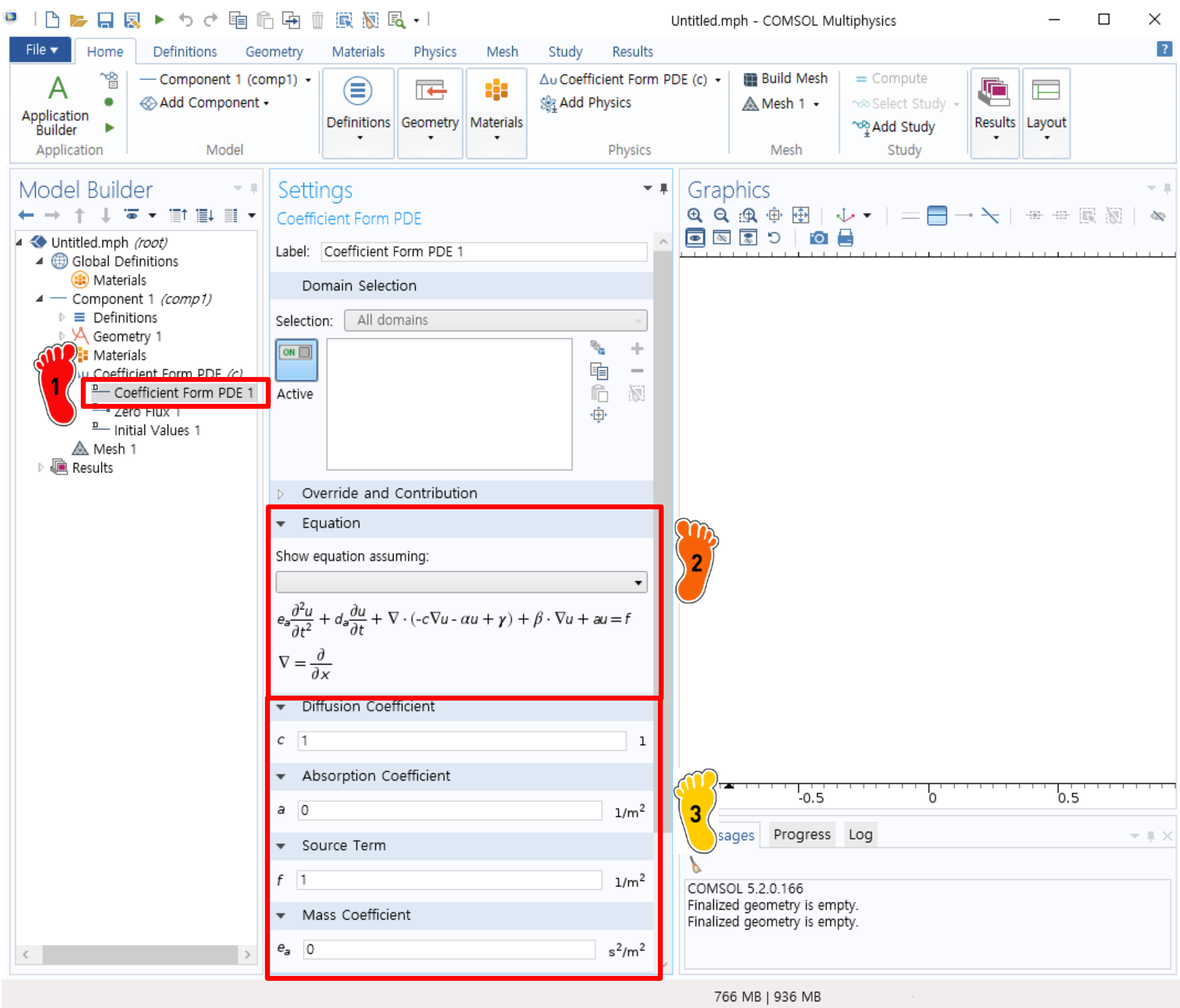
5가지 physics 선택 가능

Coefficient form PDE (선택)
General Form PDE
Wave Form PDE
Weak Form PDE
Lower Dimensions

3 Add 클릭

4 Done 클릭

PDE INTERFACES



1. Model builder 창 tree 에서 Coefficient form PDE 선택

2. Equation 탭 메뉴를 확장시키면 독립변수 t, x 와 종속변수 u 에 대한 수식이 나옴

3. 수식 안에 있는 계수들의 값을 입력할 수 있는 창

- **ODE examples**
  - ✓ Example 25.5
  - ✓ Example 28.2: predator-prey model
  - ✓ Example 25.14
  - ✓ Example 27.10
  - ✓ Previous case study I
  - ✓ Previous case study II

EXAMPLE 25.5

ODE 함수 예제

1

ordinary differential equation

$$y' = 4e^{0.8t} - 0.5y$$

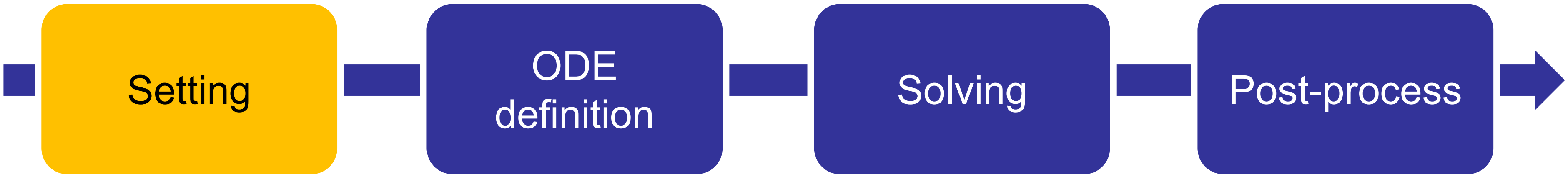
initial condition

$$t = 0, y = 2$$

analytic solution

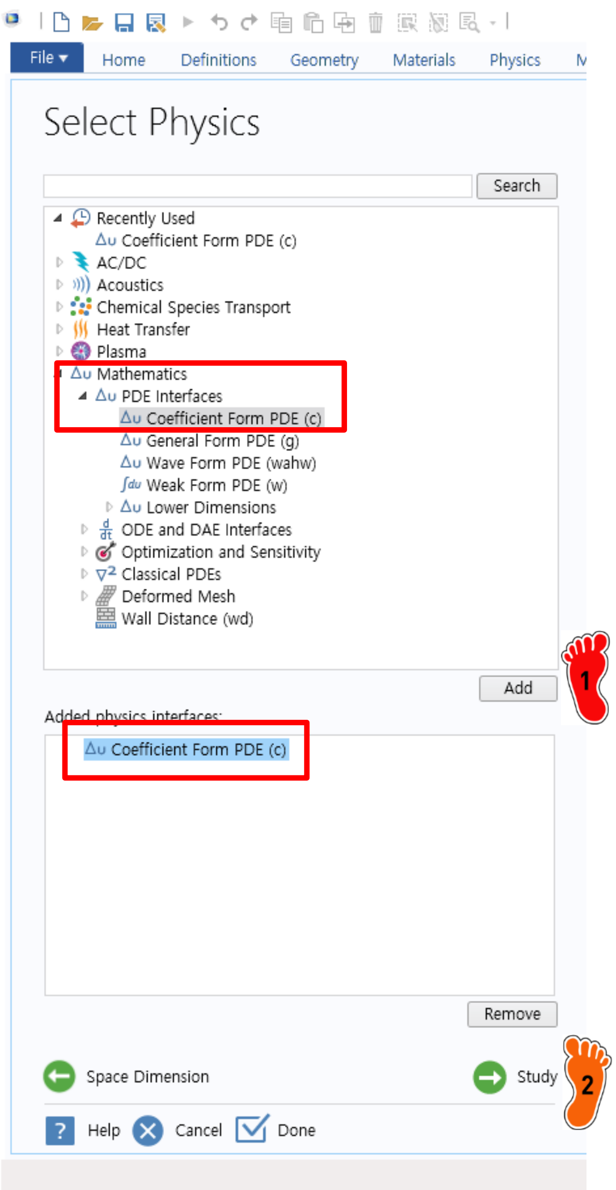
$$y = \frac{4}{1.3}\left(e^{0.8t} - e^{-0.5t}\right) + 2e^{-0.5t}$$

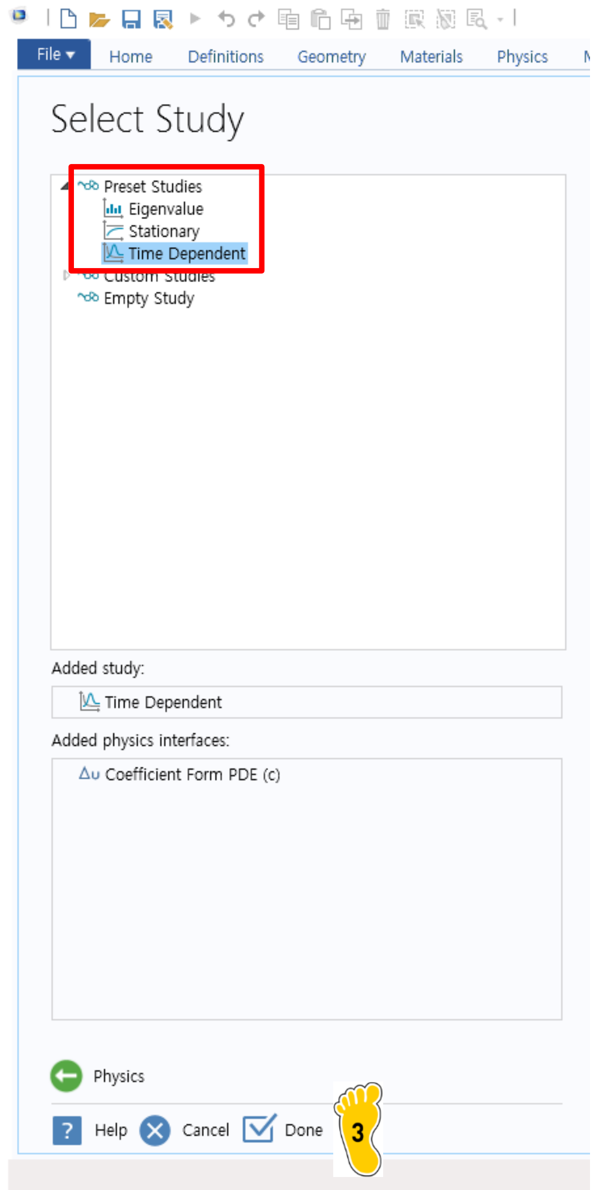
ANALYSIS FLOW



- ✓ Dimension selection
- ✓ Physics selection
- ✓ Study type selection

SETTING



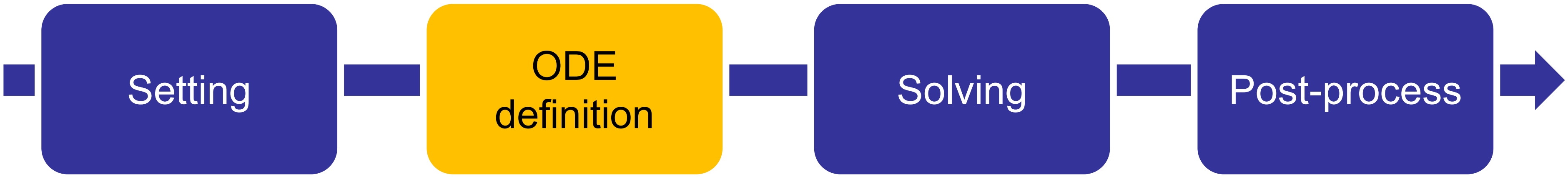


1 Model Wizard → 1D → Mathematics - PDE Interfaces – Coefficient form PDE 선택 → Add 클릭

2 Study 클릭

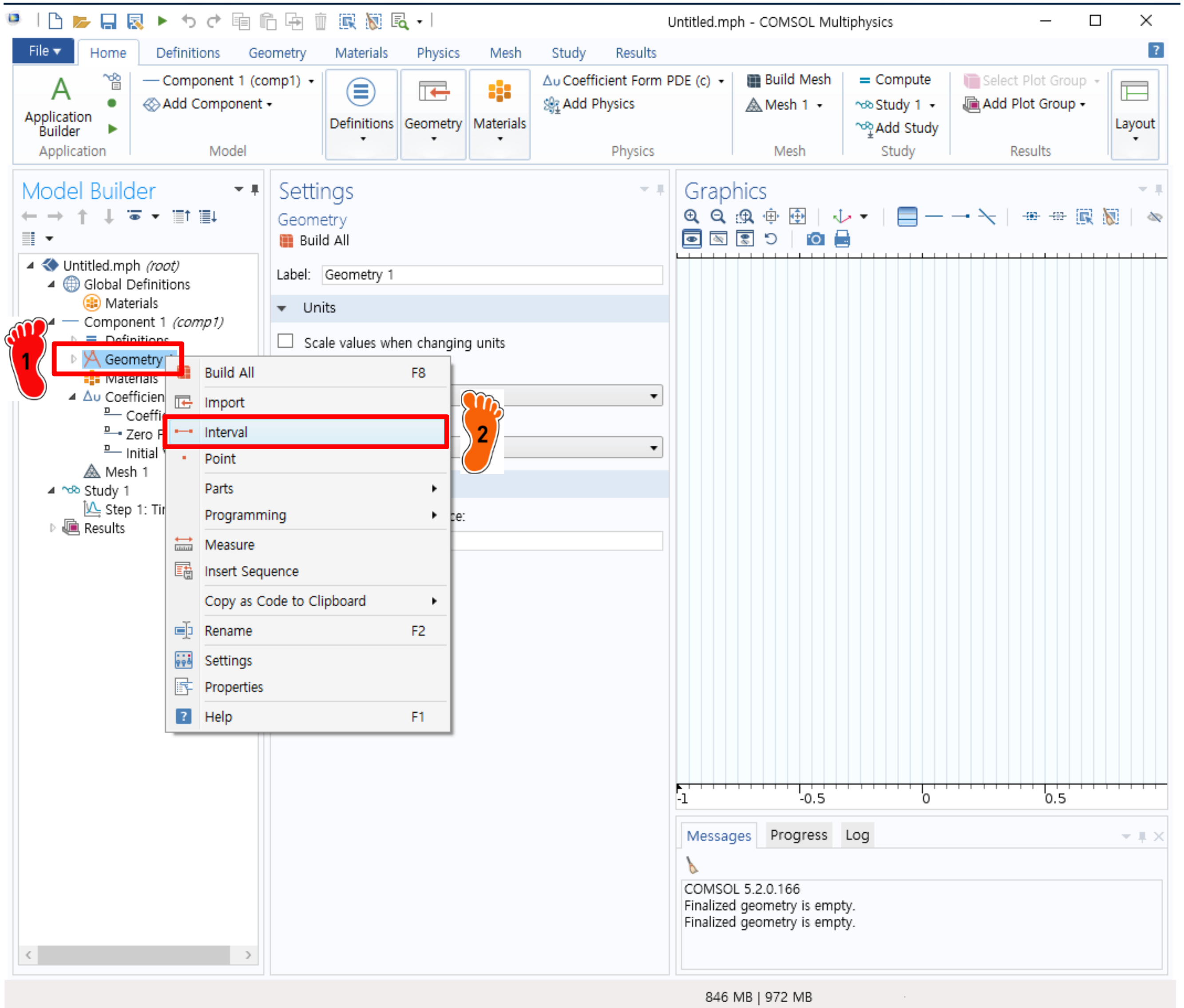
3 Time Dependent 선택 후 Done 클릭

ANALYSIS FLOW



- ✓ Geometry creation
- ✓ Coefficient input
- ✓ Initial value input

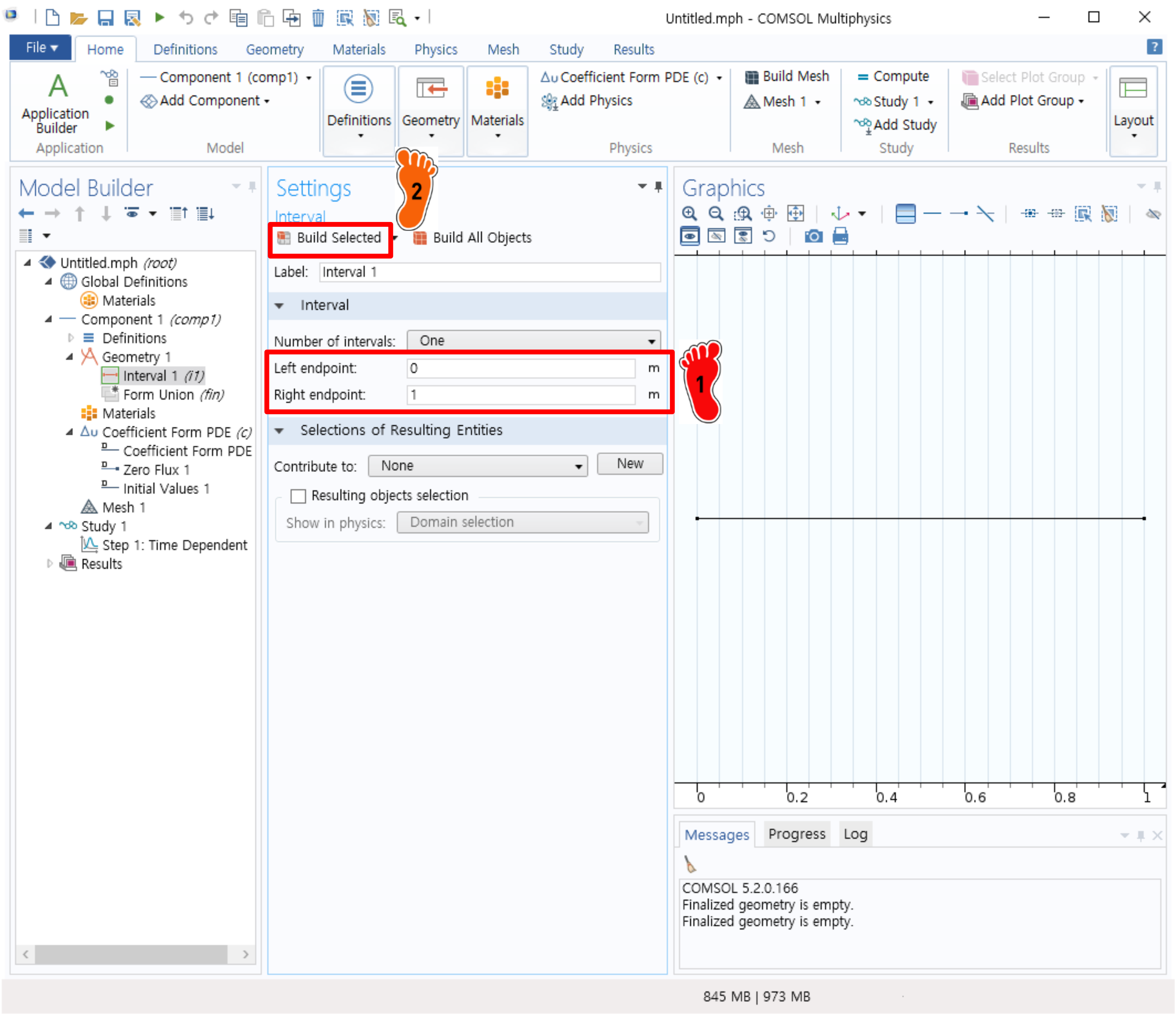
GEOMETRY CREATION



1 Geometry 1 메뉴를 마우스 우클릭

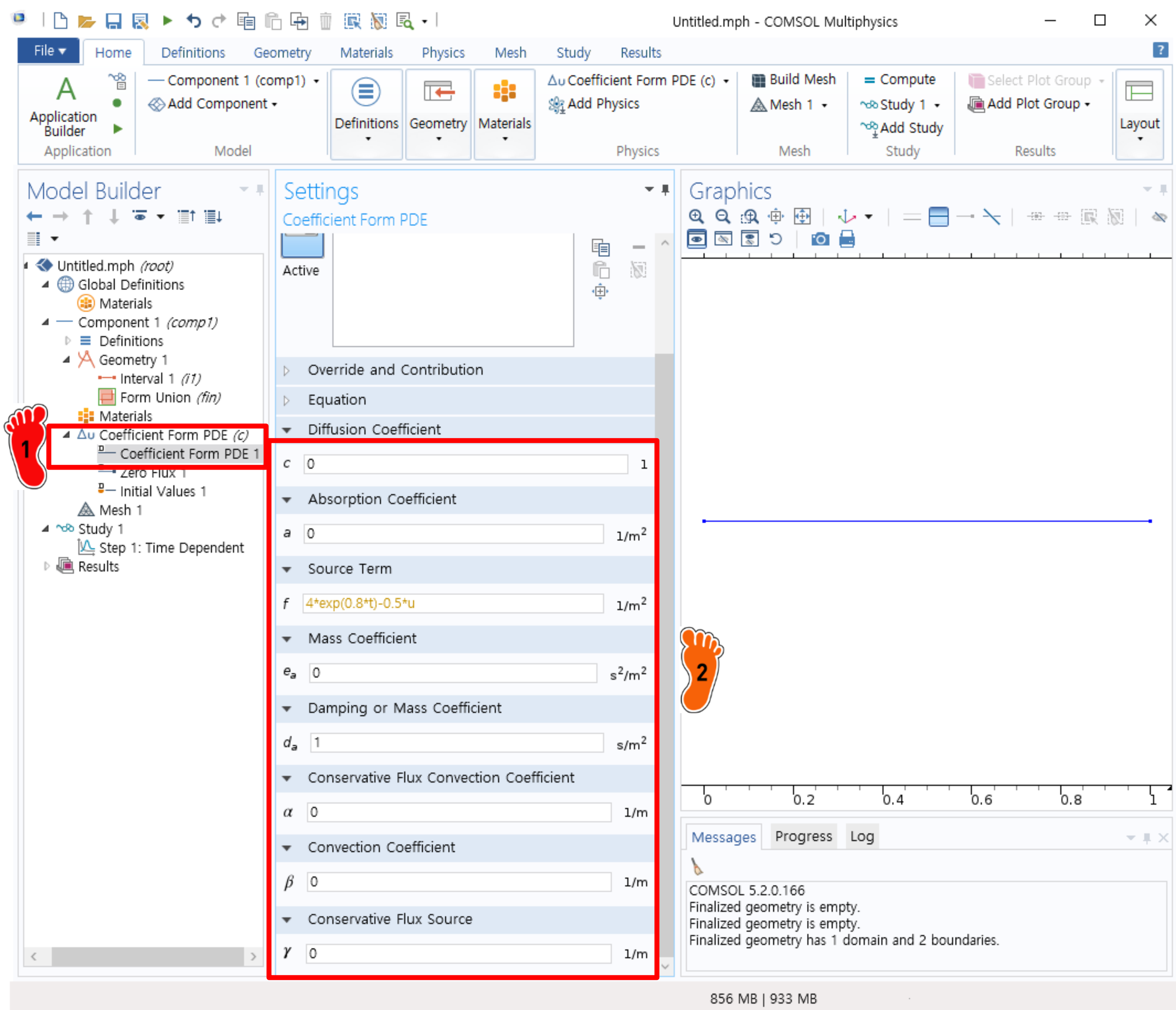
2 Interval 클릭

GEOMETRY CREATION



1. Left endpoint 에 "0"
   Right endpoint 에 "1"
   값을 입력
2. Build selected 클릭

COEFFICIENT INPUT



1. Coefficient Form PDE 1 선택

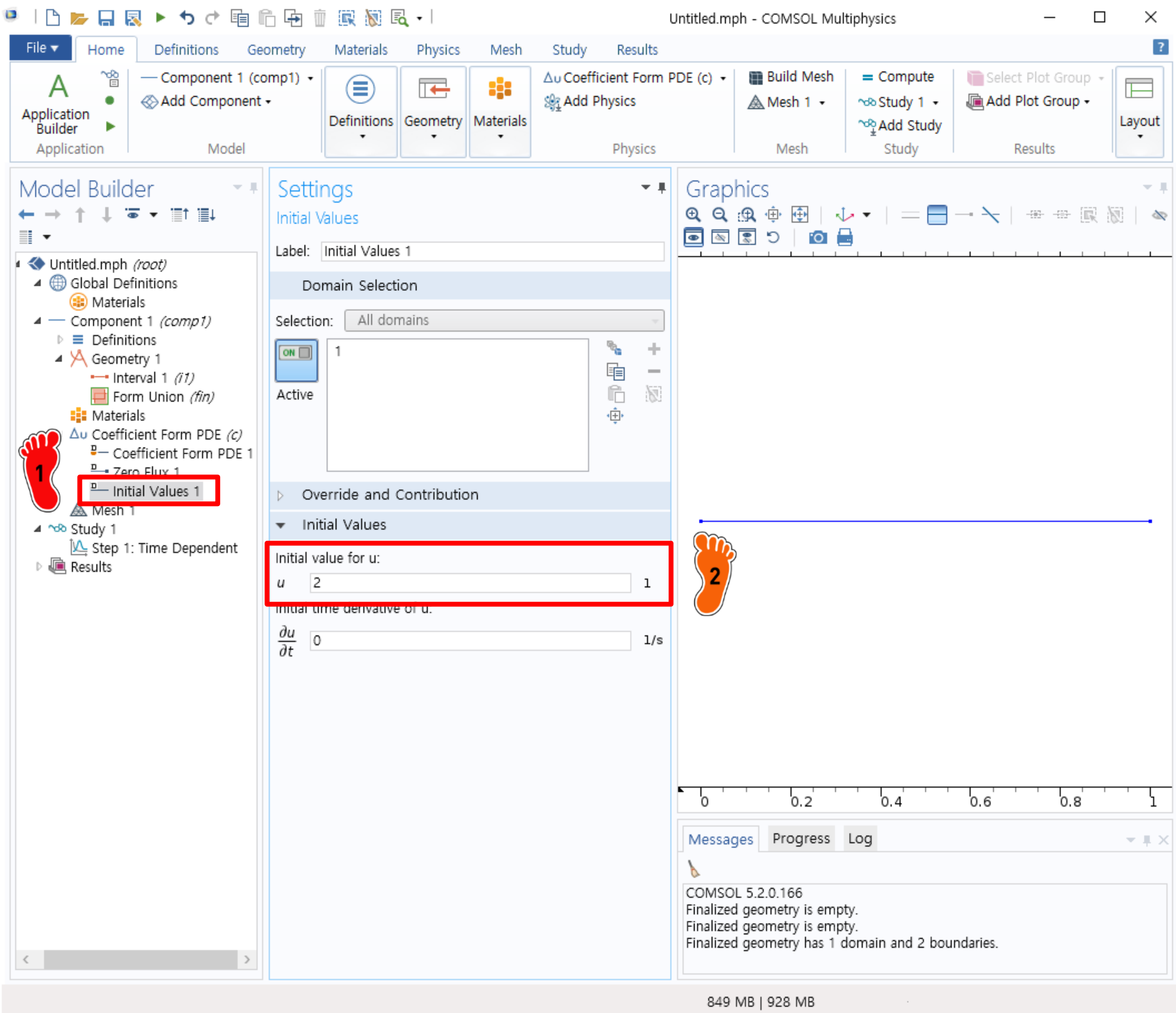
2. 계수 값 입력 (나머지 0)

$$d_a = 1$$

$$f = 4e^{0.8t} - 0.5u$$

$$e\frac{\partial^2 u}{\partial t^2} + d_a\frac{\partial u}{\partial t} + \nabla\cdot\left(-c\nabla u - \alpha u + \gamma\right) + \beta\cdot\nabla u + au = f \leftrightarrow \frac{\partial u}{\partial t} = 4e^{0.8t} - 0.5u$$

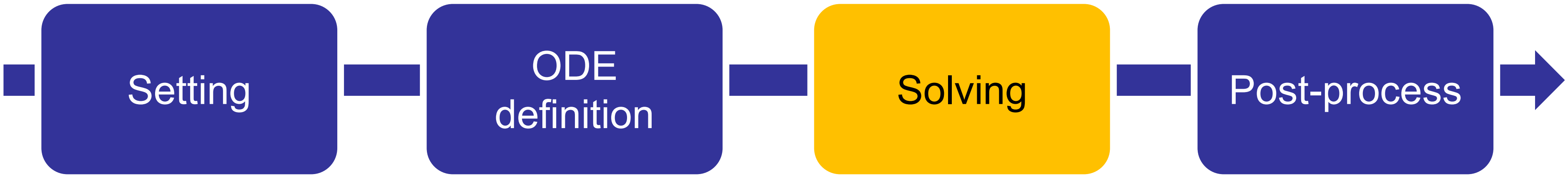
INITIAL VALUE INPUT



1 Initial Values 1 메뉴 클릭

2 u 창에 “2” 입력

ANALYSIS FLOW

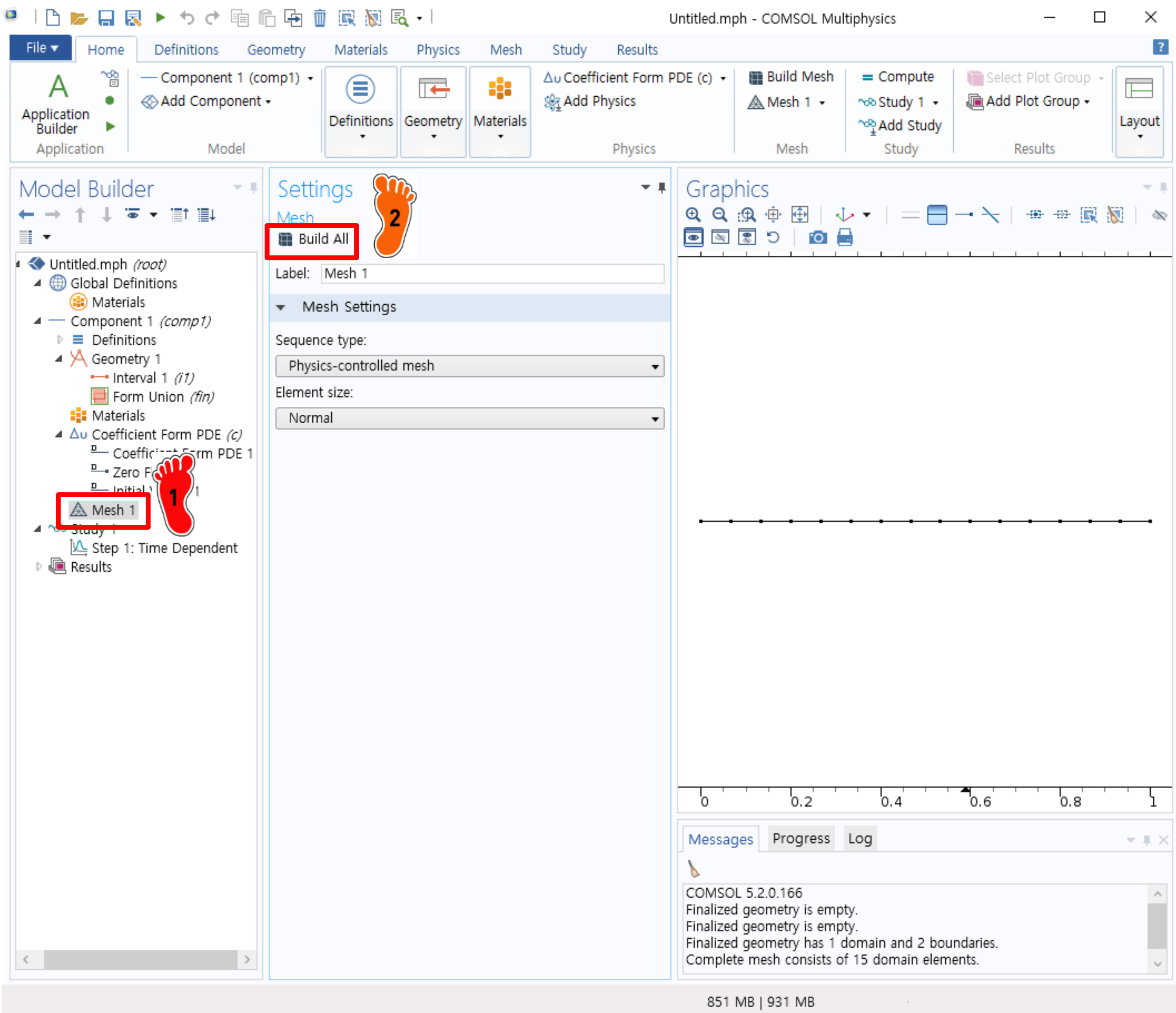


- ✓ Mesh creation
- ✓ Time range input
- ✓ Compute

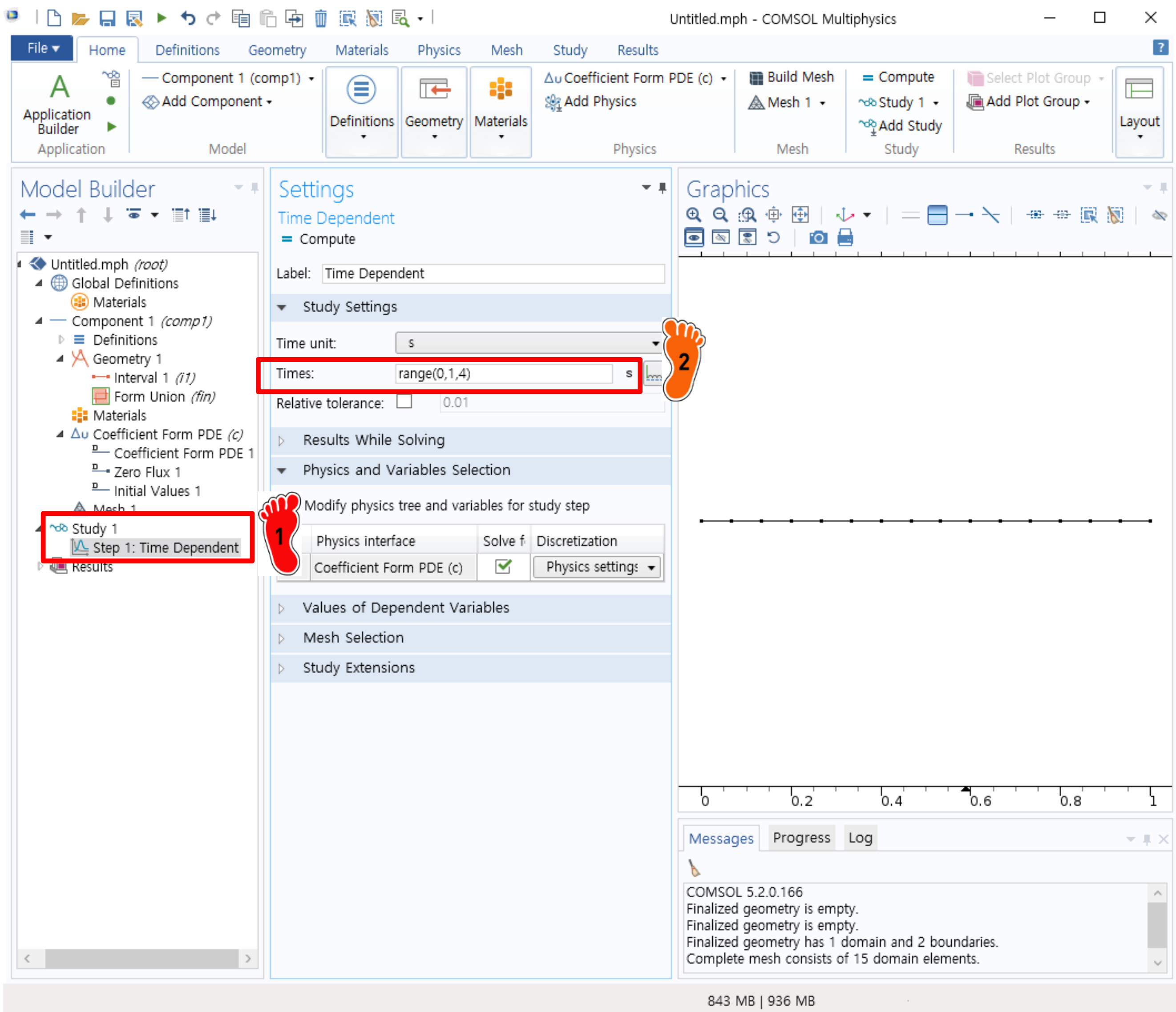
MESH CREATION

1 Mesh 1 메뉴 클릭

2 Build All 클릭

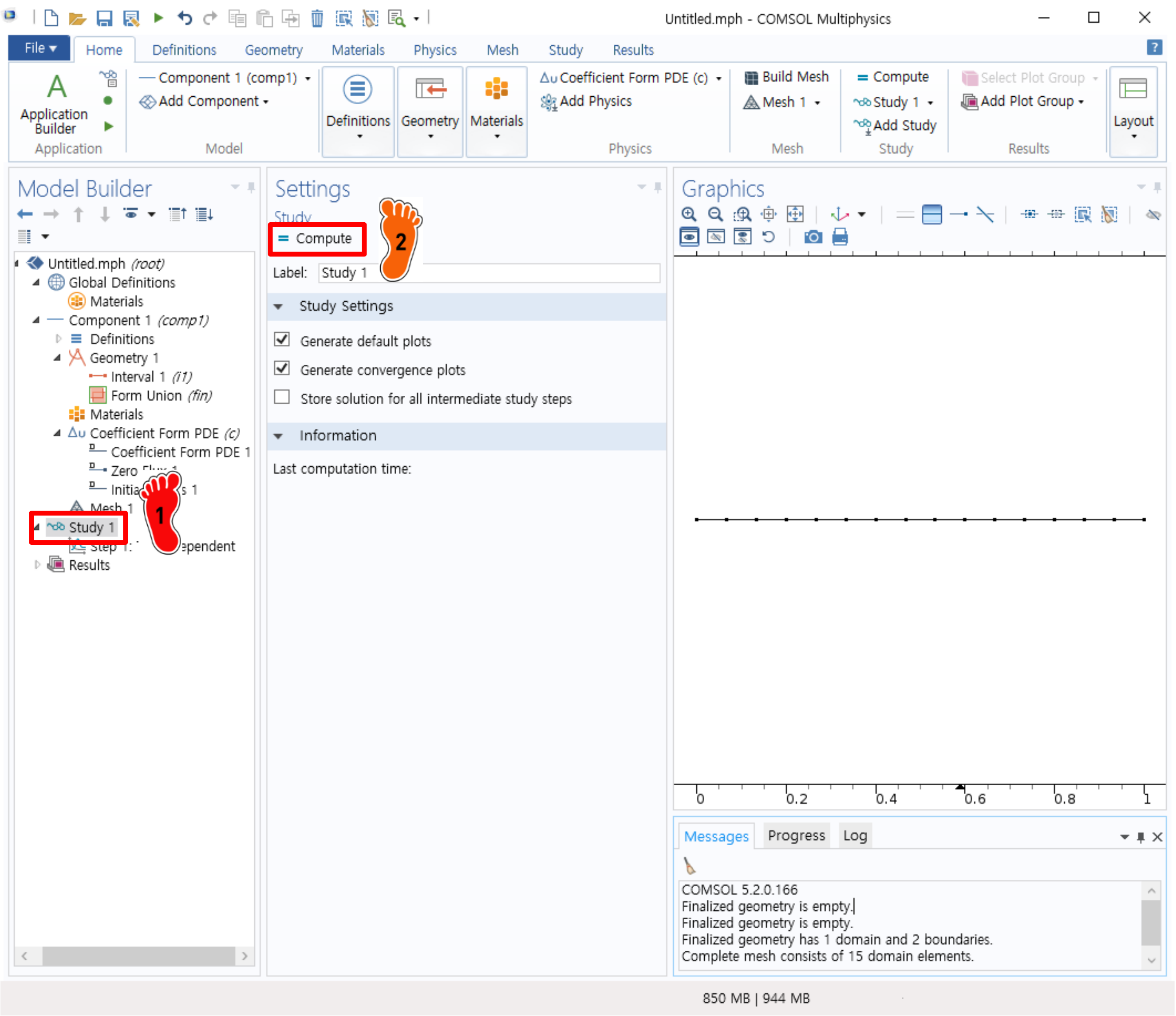


TIME RANGE INPUT



1. Study 1 → Step 1: Time Dependent 메뉴 클릭
2. Times: 창에 “range(0,1,4)” 입력

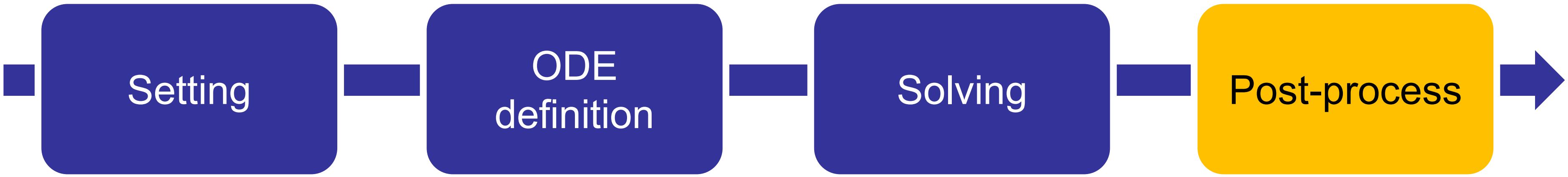
COMPUTE



1 Study 1 메뉴 선택

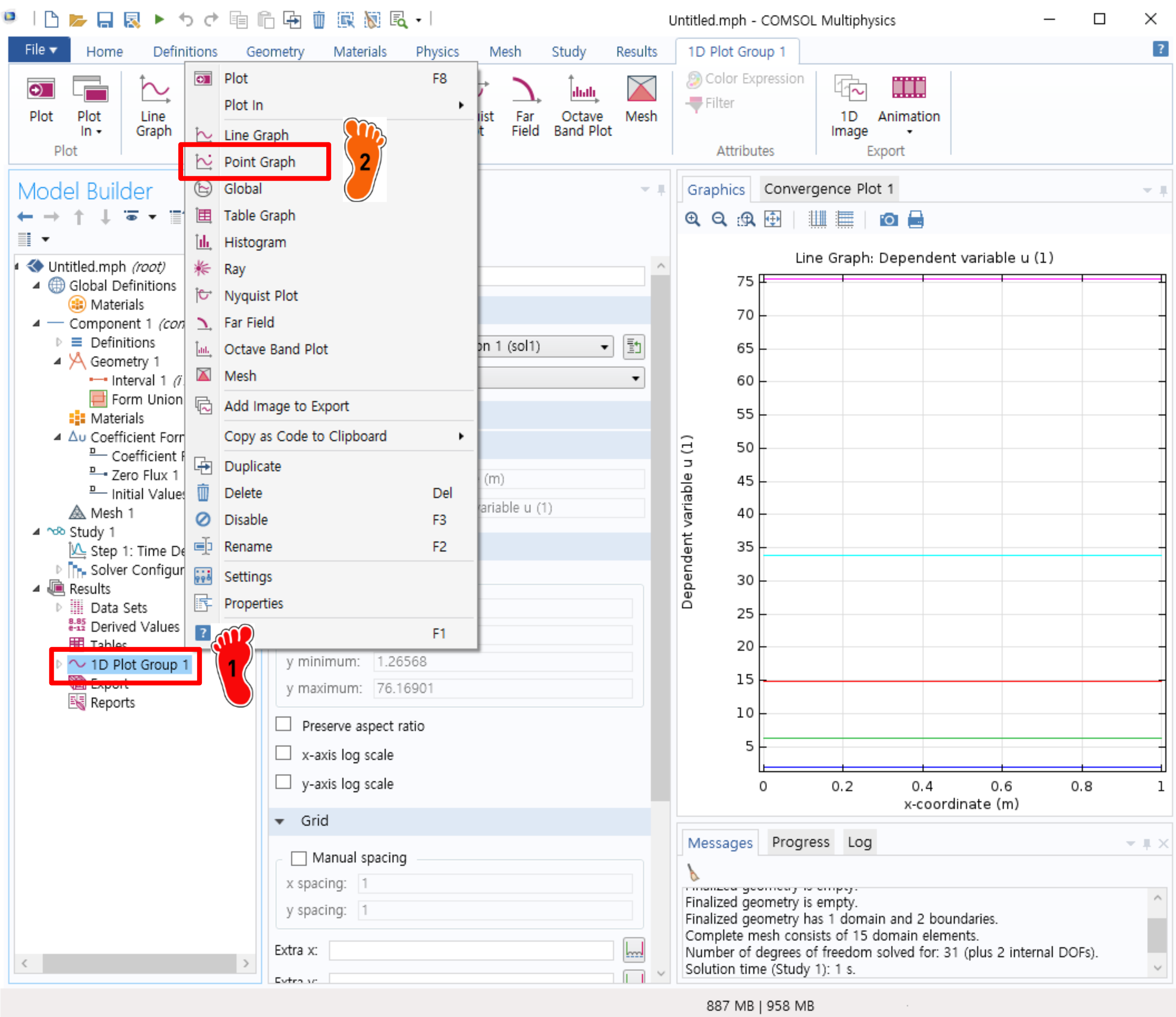
2 Compute 클릭

ANALYSIS FLOW



✓ Result plot

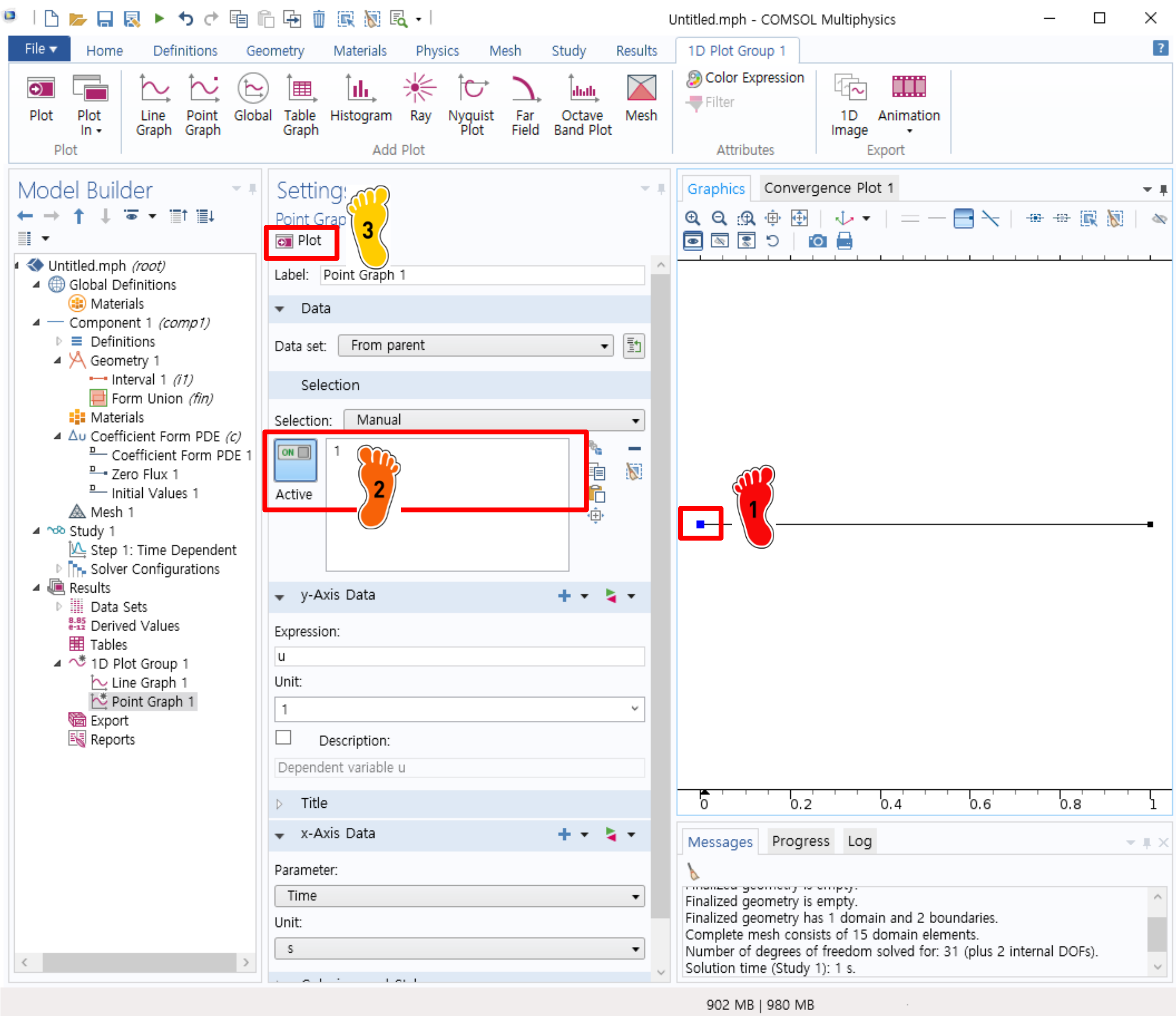
RESULT PLOT



1 1D Plot Group 1 우클릭

2 Point Graph 클릭

RESULT PLOT

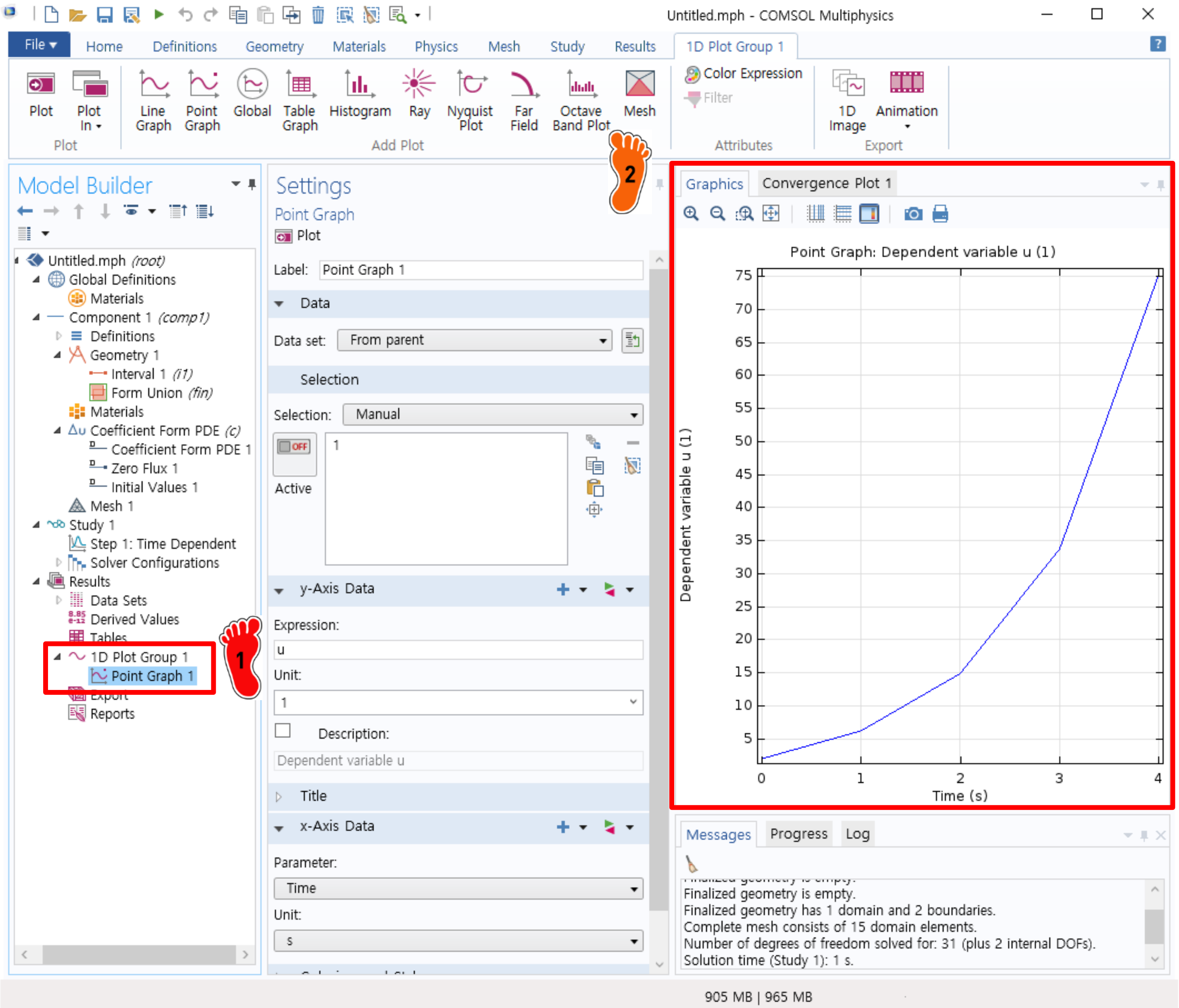


1 왼쪽 point 클릭

2 Selection에 "1" 확인

3 Plot 클릭

RESULT PLOT



1 기존 Line Graph 1 삭제 (우클릭 → Delete 또는 선택 후 "Del" 키)

2 결과 그래프 확인

- **ODE examples**

  - ✓ Example 25.5
  - ✓ Example 28.2: predator-prey model
  - ✓ Example 25.14
  - ✓ Example 27.10
  - ✓ Previous case study I
  - ✓ Previous case study II

EXAMPLE 28.2

[predator – prey model]

nonlinear ordinary differential equations

$$\begin{cases} \dfrac{dy_1}{dt} = ay_1 - by_1y_2 \\ \dfrac{dy_2}{dt} = -cy_2 + dy_1y_2 \end{cases}$$

$a = 1.2, b = 0.6, c = 0.8, d = 0.3$

initial condition

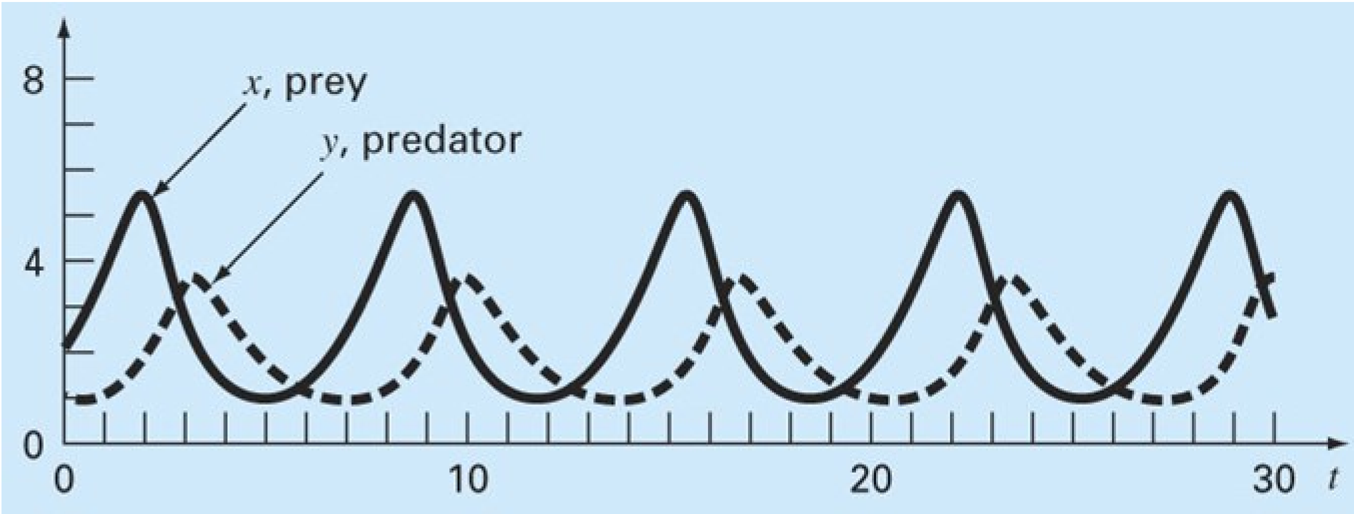
$t = 0, y_1 = 2, y_2 = 1$

1 Predator-prey model developed by the Italian mathematician Vito Volterra and the American biologist Alfred J. Lotka.

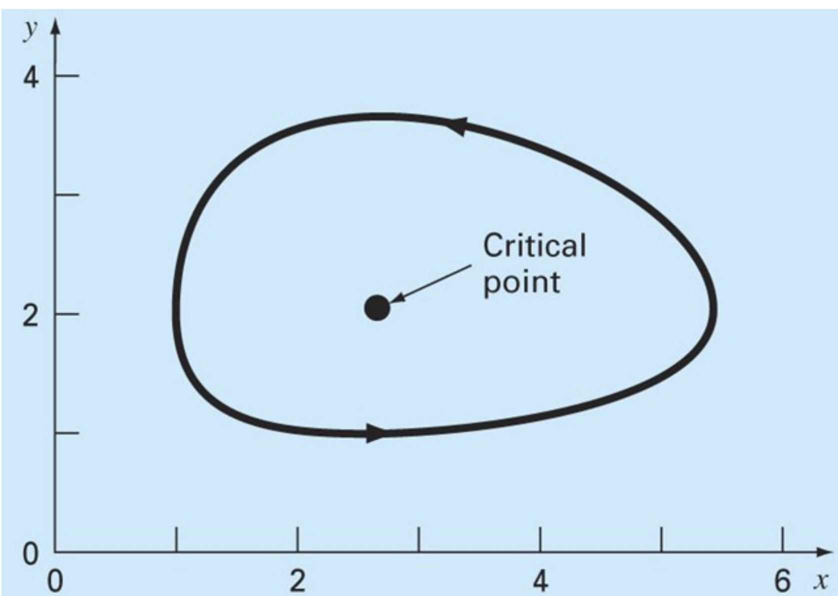
먹이사슬에 관한 미분방정식

a = the prey growth rate
c = the predator death rate
b=d= the rate characterizing the effect of the predator-prey interaction on prey death and predator growth

EXAMPLE 28.2



Parameters: change of magnitudes of peaks, lags, period

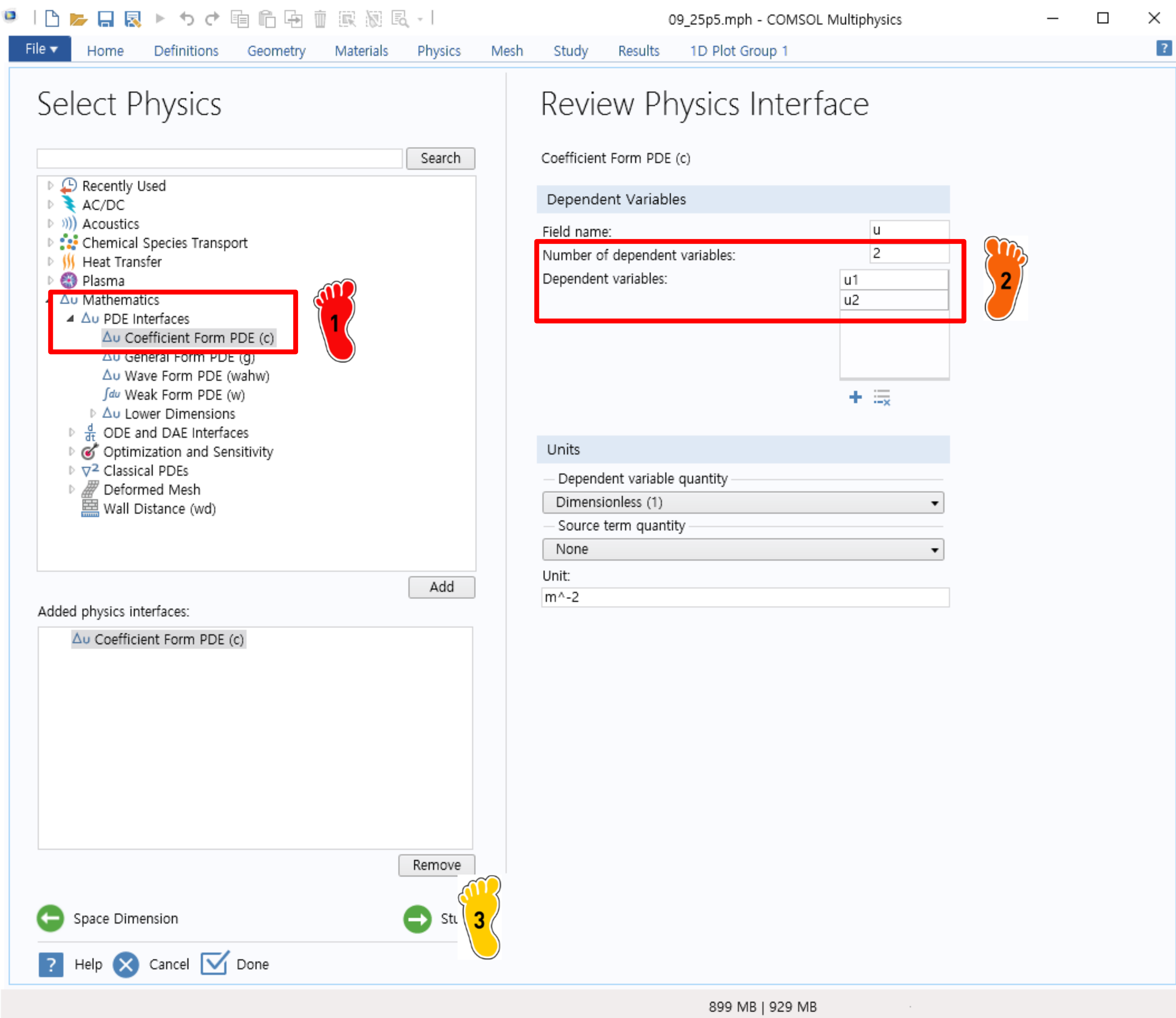


[state-space representation]

$$\frac{dx}{dt} = \frac{dy}{dt} = 0 \rightarrow (x, y) = (0,0) \text{ and } \left(\frac{c}{d}, \frac{a}{b}\right)$$

$\left(\frac{0.8}{0.3}, \frac{1.2}{0.6}\right)$ populations will remain constant

SETTING

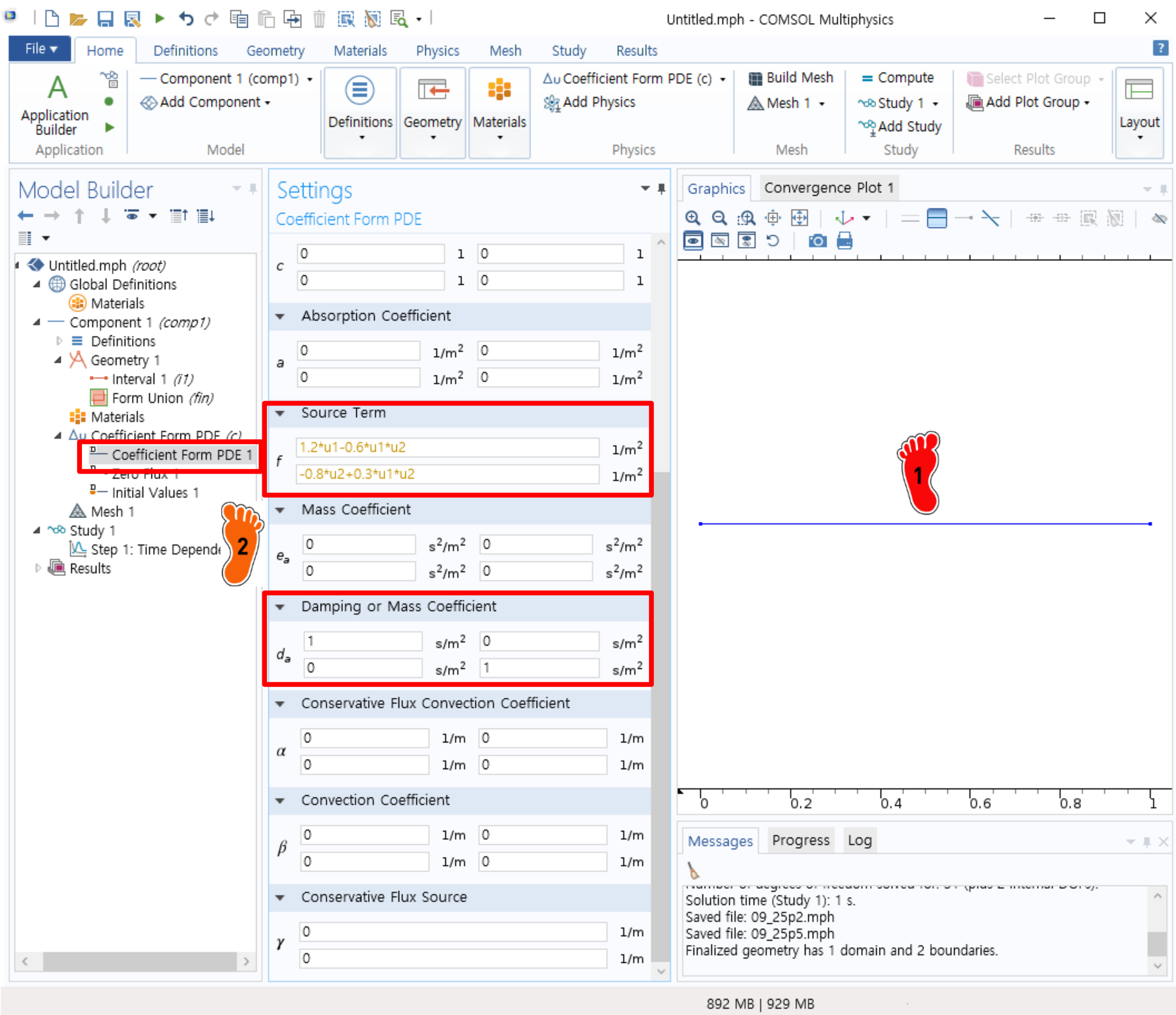


1D → Coefficient form PDE 선택 → Add 클릭

Number of dependent variables: "2" 입력

Study 클릭
→ Time Dependent 선택
→ Done 클릭

COEFFICIENT INPUT



1 전 예제와 동일한 방식으로 geometry 생성

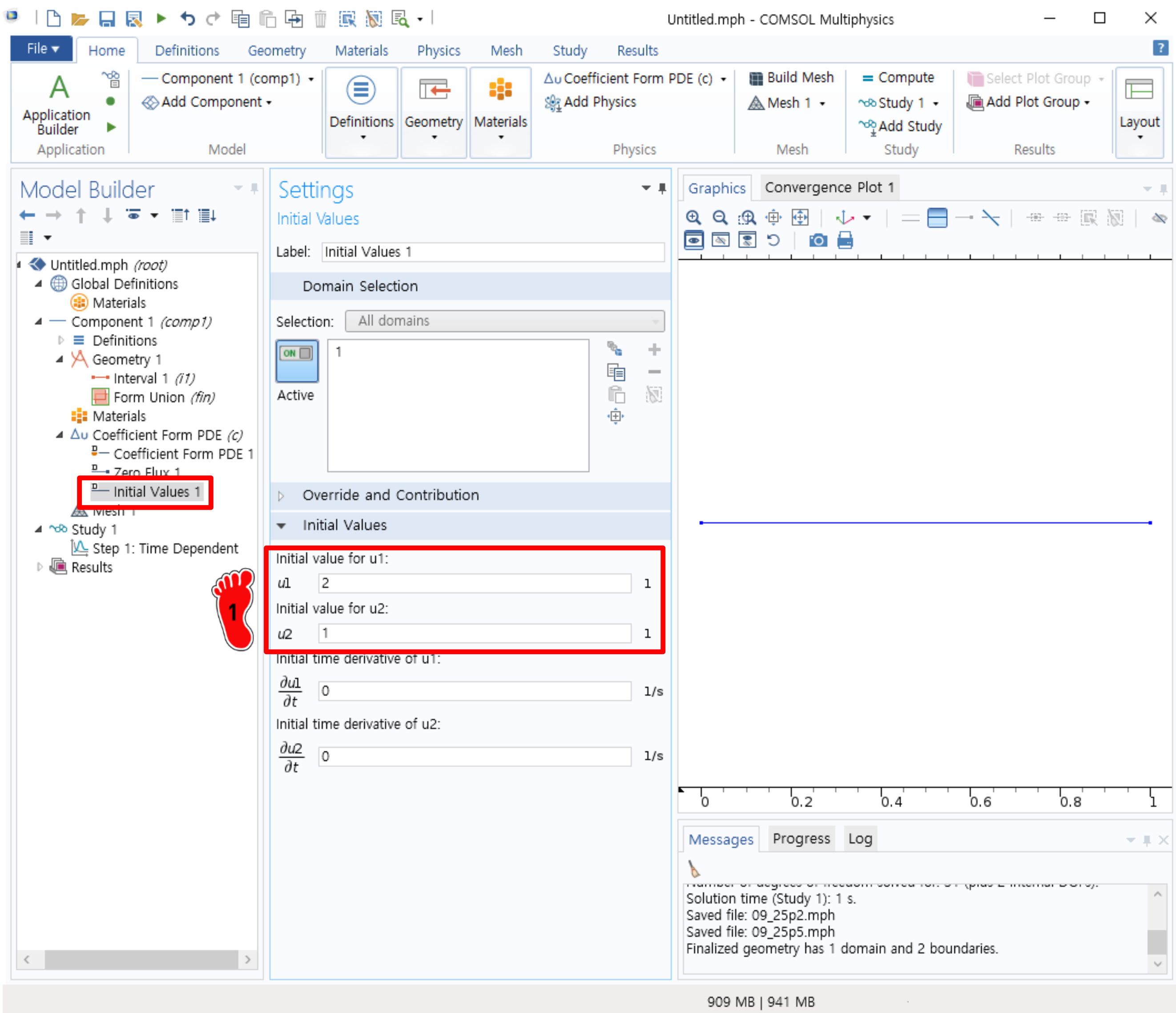
2 Coefficient Form PDE 1에 계수 입력

종속변수가 2개 이므로 벡터 또는 행렬로 입력 됨 (나머지 0)

$$d_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

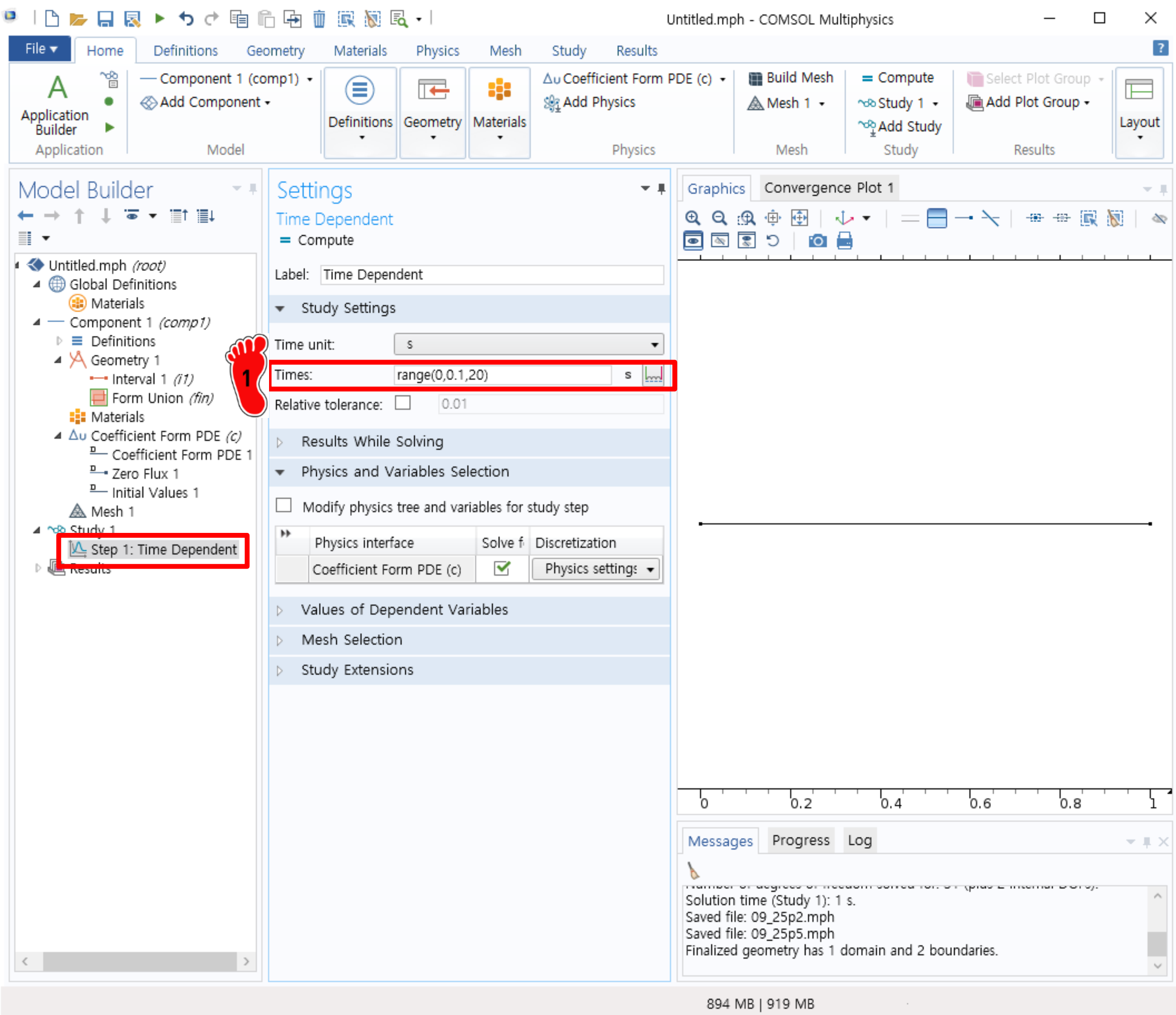
$$f = \begin{bmatrix} 1.2u_1 - 0.6u_1u_2 \\ -0.8u_2 + 0.3u_1u_2 \end{bmatrix}$$

INITIAL VALUE INPUT



1 Initial Values 1 클릭 후
u1: 2, u2: 1 입력

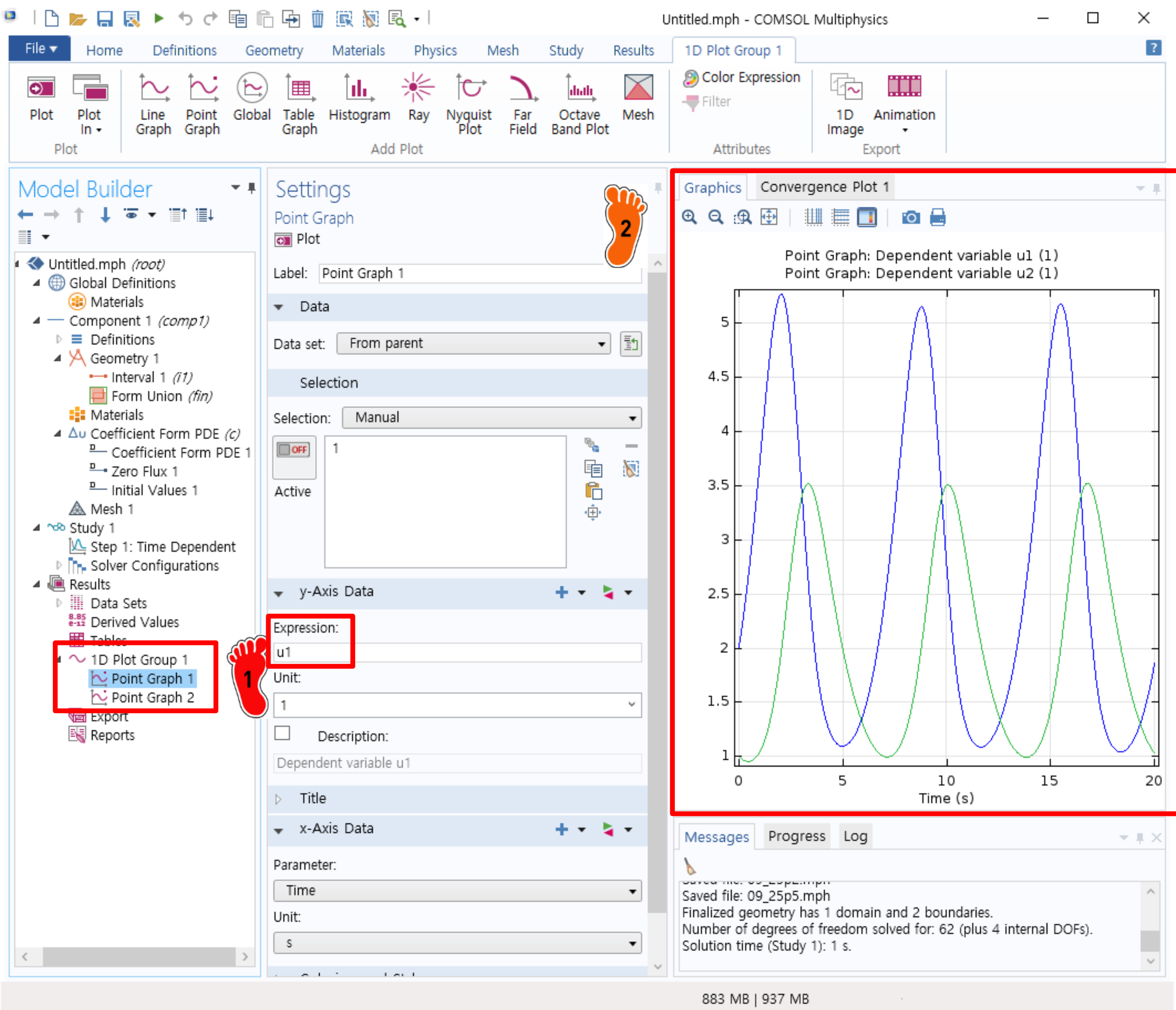
TIME RANGE INPUT



Step 1: Time Dependent 메뉴 선택 후
Times: range(0,0.1,20) 입력

이 후 Study 선택 후
Compute 클릭

RESULT PLOT



1 두 개의 Point Graph 생성 (Expression에 u1, u2 입력)

2 결과 그래프 확인

- **ODE examples**
  - ✓ Example 25.5
  - ✓ Example 28.2: predator-prey model
  - ✓ Example 25.14
  - ✓ Example 27.10
  - ✓ Previous case study I
  - ✓ Previous case study II

EXAMPLE 25.14

ordinary differential equation

$$\frac{dy}{dt} = 10e^{-(t-2)^2/\left[2(0.075)^2\right]} - 0.6y$$

initial condition

$$t = 0,\ y = 0.5$$

1 Adaptive ODE solver 를 보여주기 위한 미분방정식

2 모델링 입력 항목

Coefficient Input

$$e_a\frac{\partial^2 u}{\partial t^2} + d_a\frac{\partial u}{\partial t} + \nabla\cdot(-c\nabla u - \alpha u + \gamma) + \beta\cdot\nabla u + au = f$$

Absorption Coefficient

a 0.6 1/m^2

Source Term

f 10*exp(-((t-2)^2)/(2*(0.075)^2)) 1/m^2

Damping or Mass Coefficient

d_a 1 s/m^2

Initial Value Input

Initial Values

Initial value for u:

u 0.5

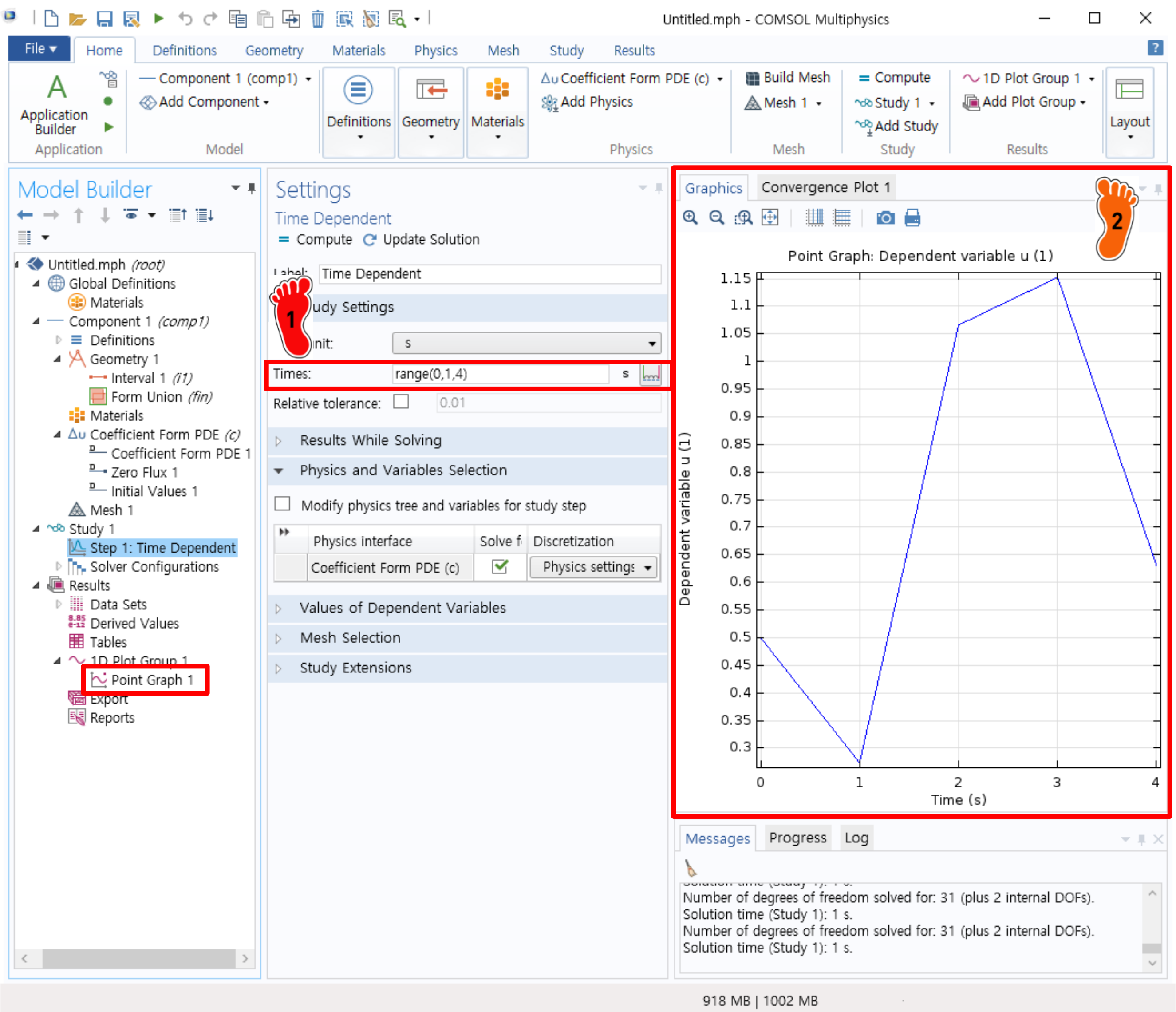
Time Range Input

Study Settings

Time unit: s

Times: range(0,1,4)

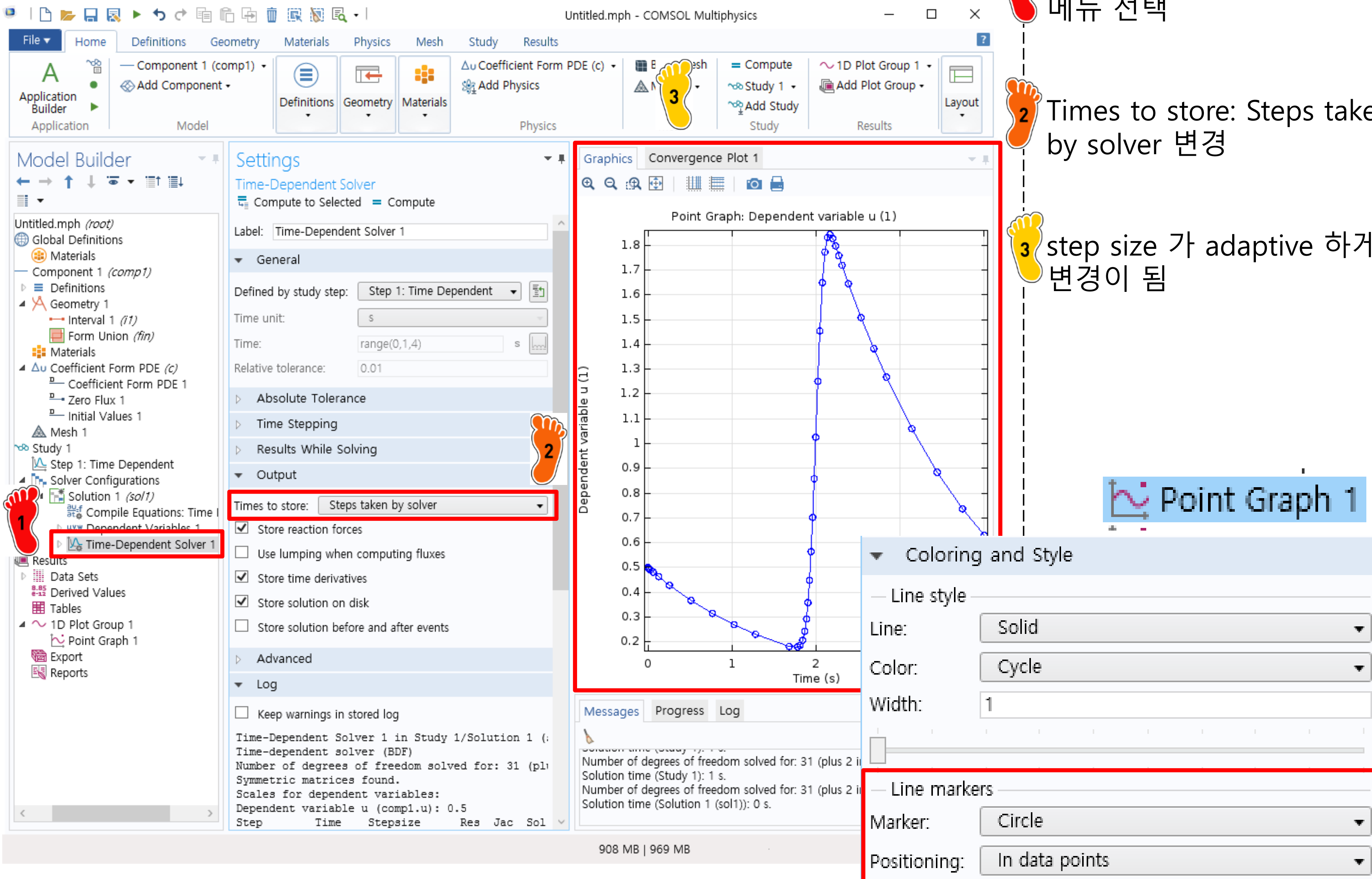
RANGE INPUT



1 range(0,1,4) 로 입력할 경우 step size 가 상대적으로 큼

2 결과 그래프에서 데이터는 step size 1 로 결정이 됨

ADAPTIVE OPTION SELECTION



1. Time-Dependent Solver 1 메뉴 선택
2. Times to store: Steps taken by solver 변경
3. step size 가 adaptive 하게 변경이 됨

- **ODE examples**
  - ✓ Example 25.5
  - ✓ Example 28.2: predator-prey model
  - ✓ Example 25.14
  - ✓ <u>Example 27.10</u>
  - ✓ Previous case study I
  - ✓ Previous case study II

EXAMPLE 27.10

[Van der Pol equation]

$$\frac{d^2 y_1}{dt^2} - \mu\left(1 - y_1^2\right)\frac{dy_1}{dt} + y_1 = 0$$

initial condition

$$t = 0,\ y_1 = 1,\ \frac{dy_1}{dt} = 1$$

convert process →

$$\begin{cases} \dfrac{dy_1}{dt} = y_2 \\ \dfrac{dy_2}{dt} = \mu\left(1 - y_1^2\right) y_2 - y_1 \end{cases}$$

1. stiff 한 정도가 mu 값에 따라서 변하는 van der Pol equation.
2. 모델링 입력 항목

Dependent Variables

Dependent Variables

Field name:	u
Number of dependent variables:	2
Dependent variables:	u1
	u2

Coefficient Input

$$e_a\frac{\partial^2 u}{\partial t^2} + d_a\frac{\partial u}{\partial t} + \nabla\cdot(-c\nabla u - \alpha u + \gamma) + \beta\cdot\nabla u + au = f$$

Source Term

f	u2	1/m²
	(μ) *(1-u1^2)*u2-u1	1/m²

Damping or Mass Coefficient

d_a	1	s/m²	0	s/m²
	0	s/m²	1	s/m²

Initial Value Input

Initial Values

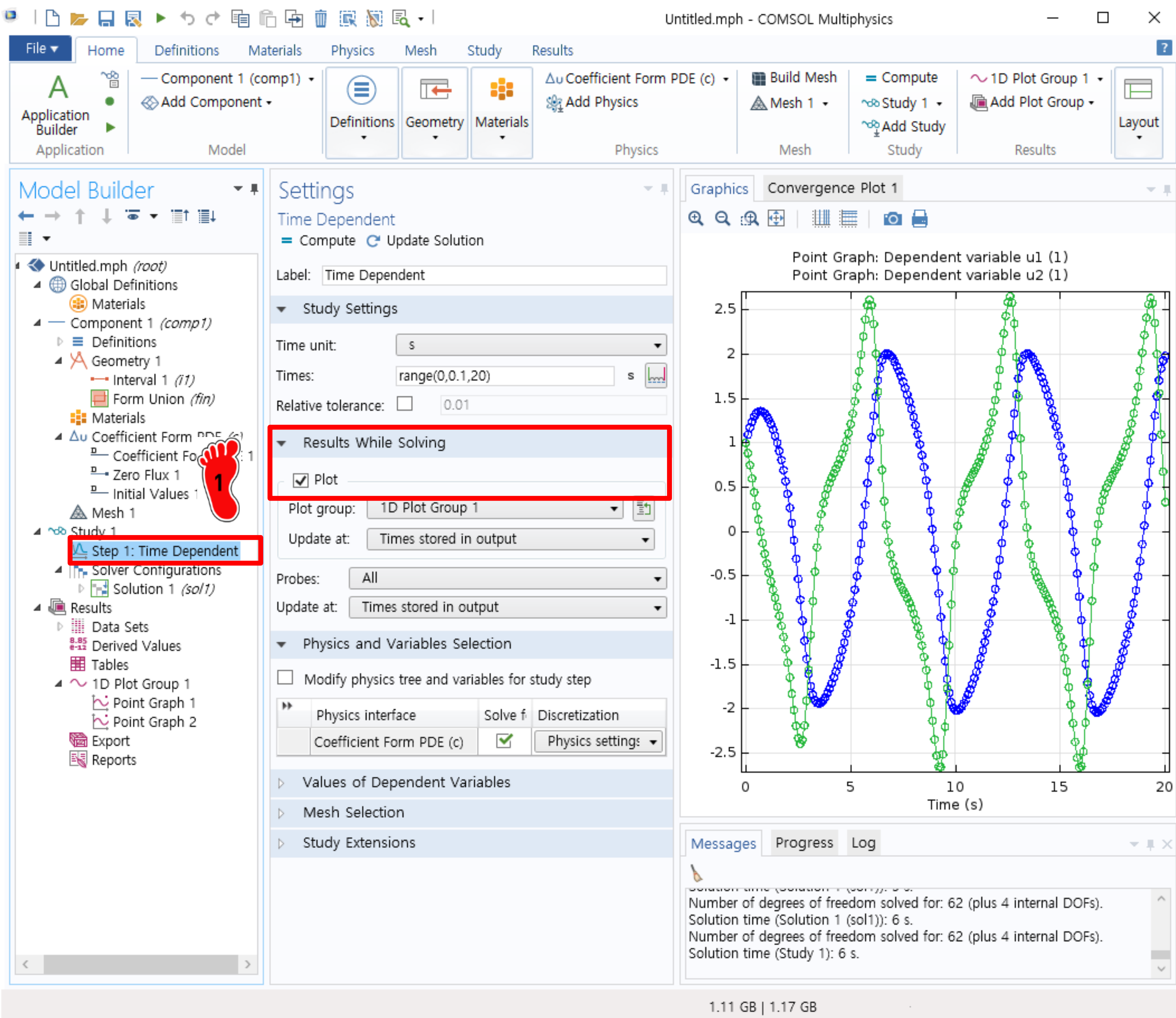
Initial value for u1:	
$u1$	1
Initial value for u2:	
$u2$	1

Time Range Input

Study Settings

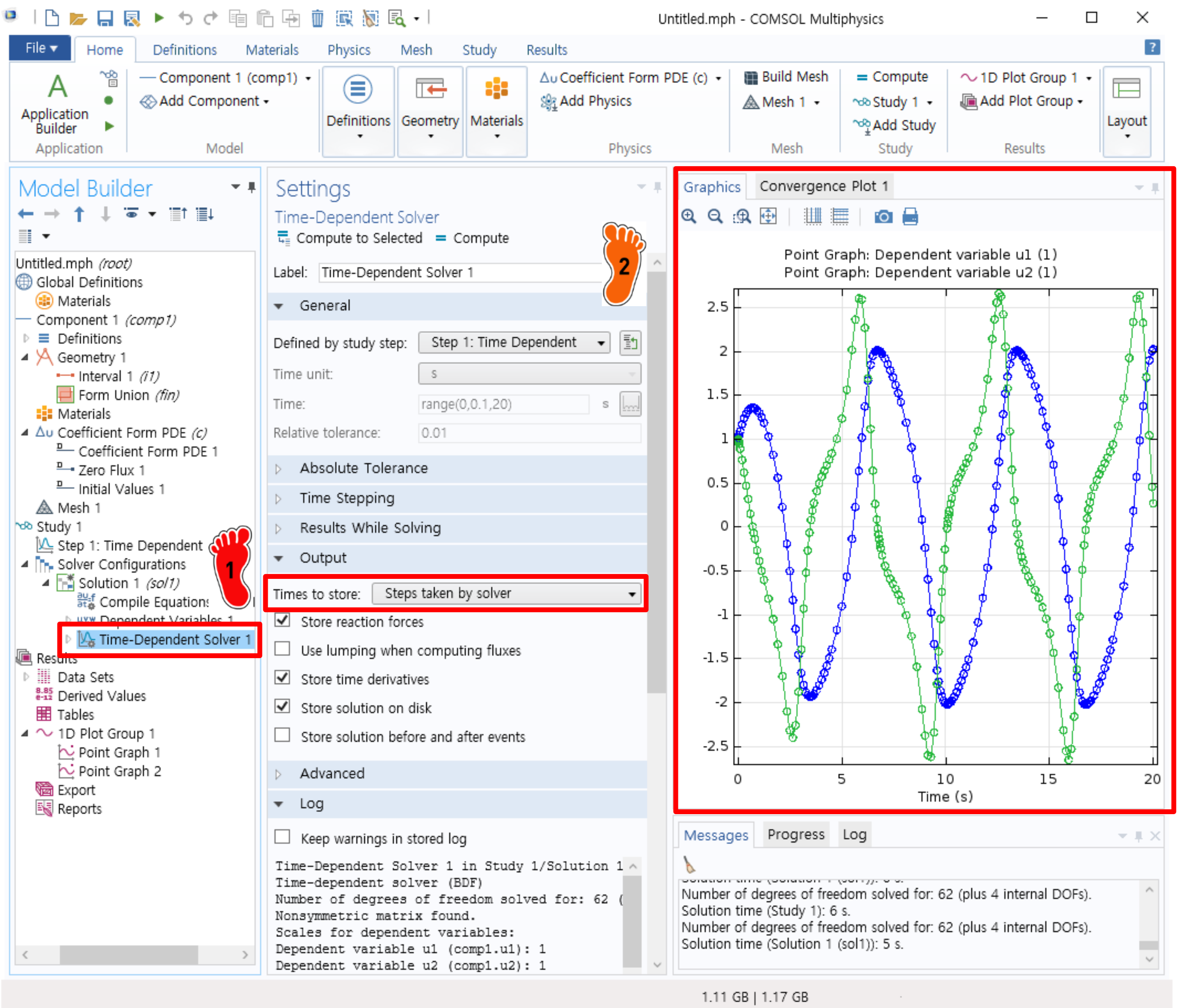
Time unit:	s
Times:	range(0,0.1,20)

TIME DEPENDENT OPTION



Step 1: Time Dependent
→ Result While Solving
→ Plot 체크 후 Compute

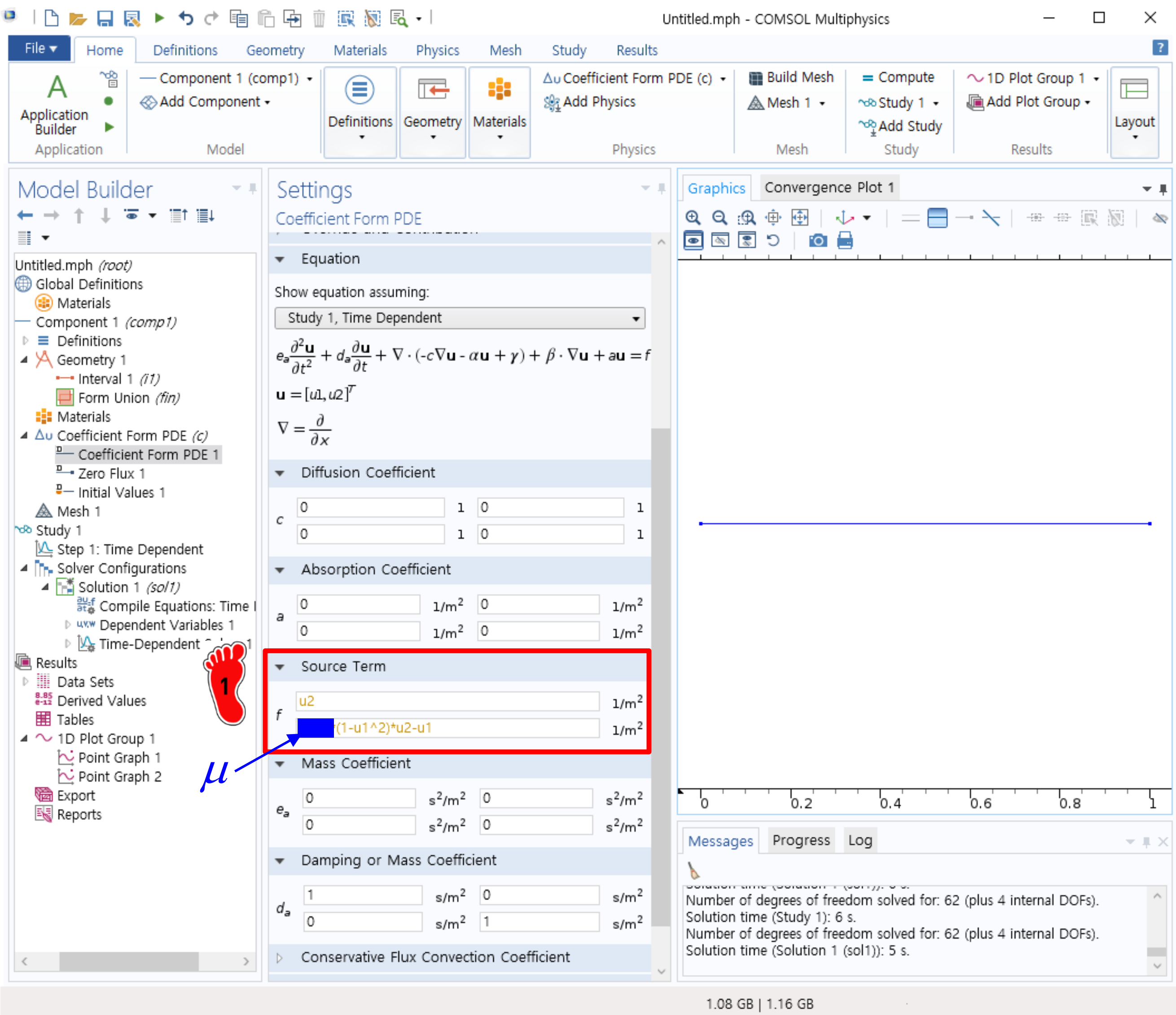
ADAPTIVE OPTION SELECTION



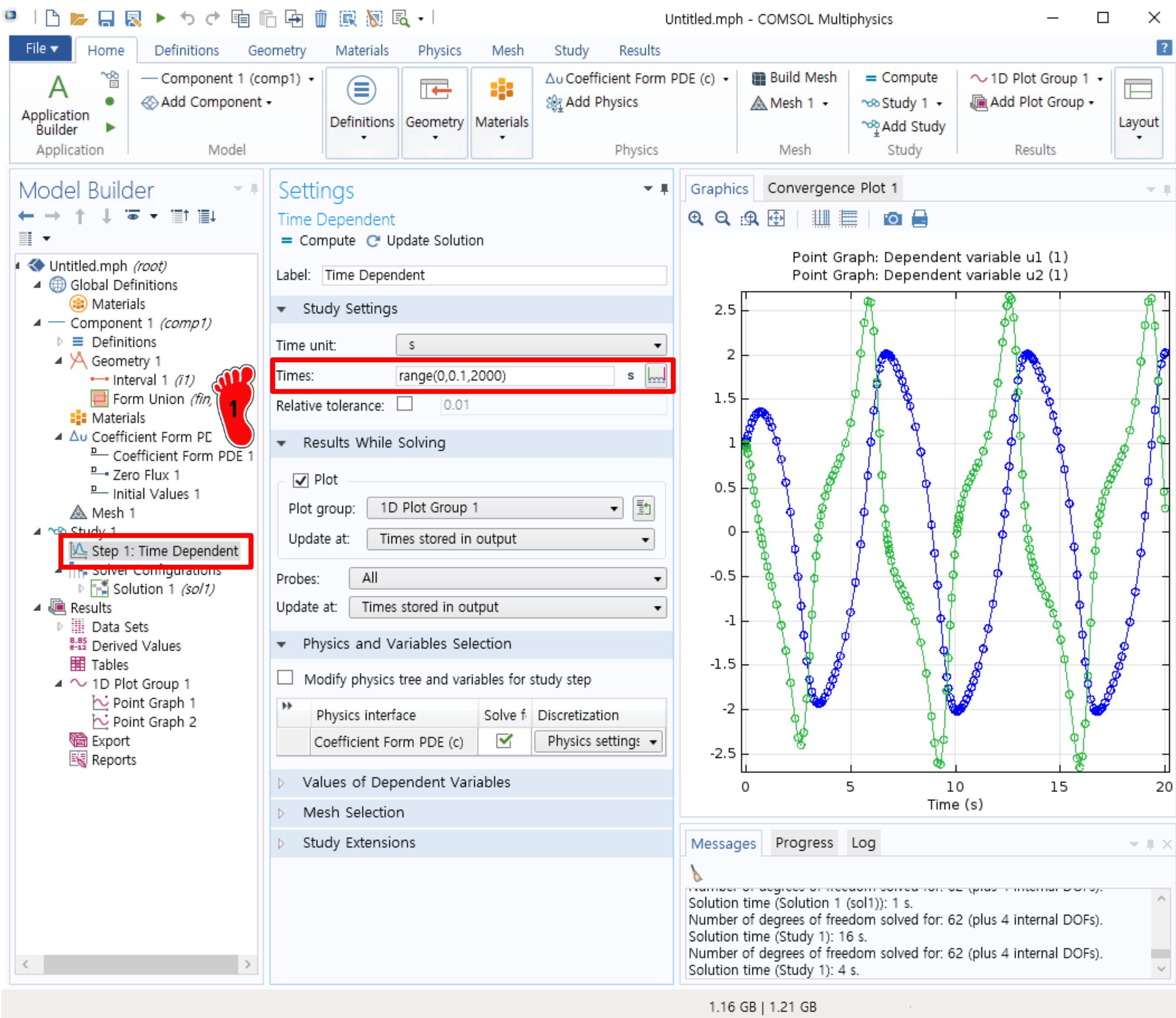
1. Time-Dependent Solver 1 메뉴 선택 → Steps taken by solver 선택
2. Print step size 비교

STIFF COEFFICIENT INPUT

1 mu 값을 1000으로 변경

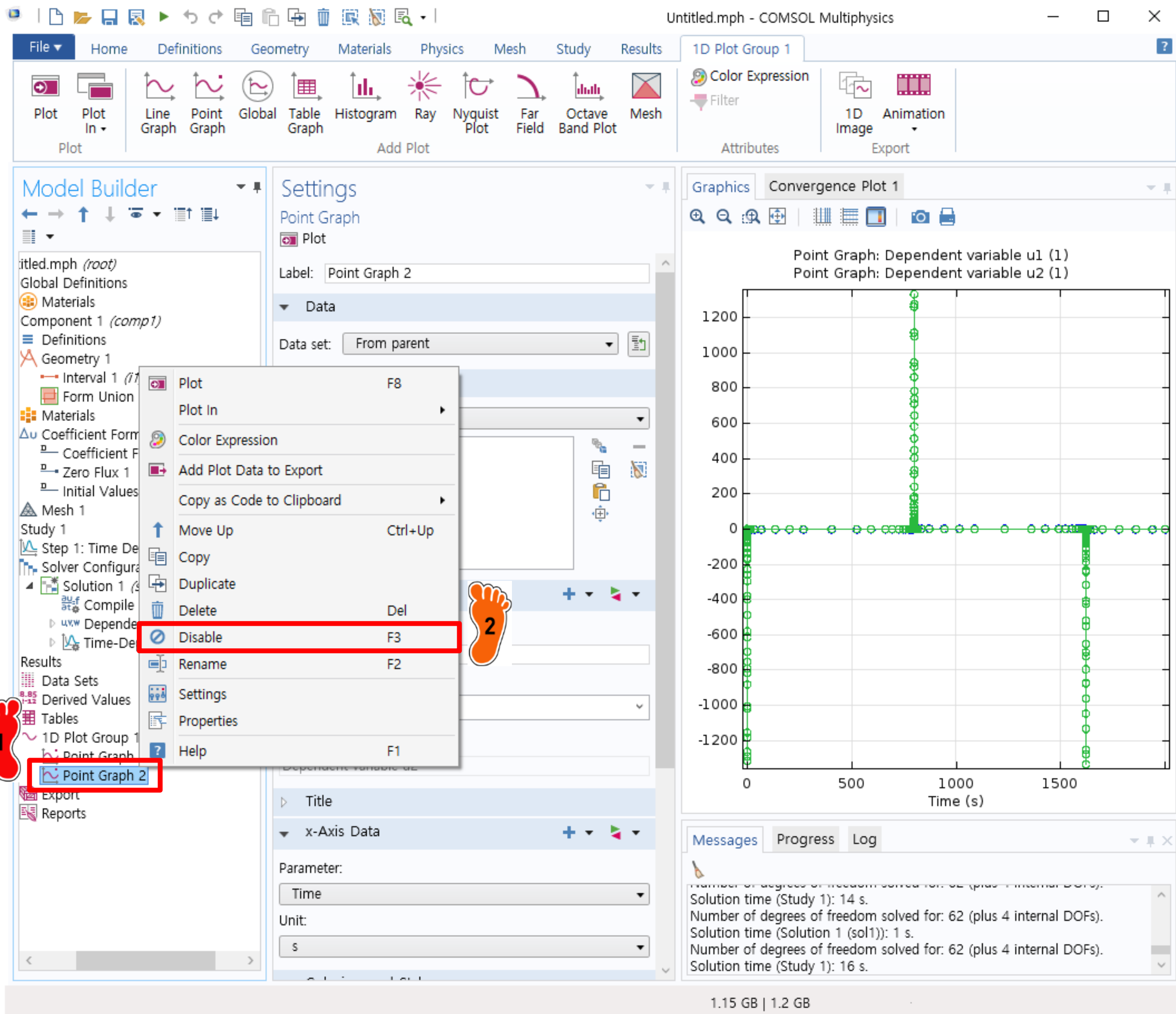


TIME RANGE INPUT



1 range(0,0.1,2000)으로 변경 → Compute

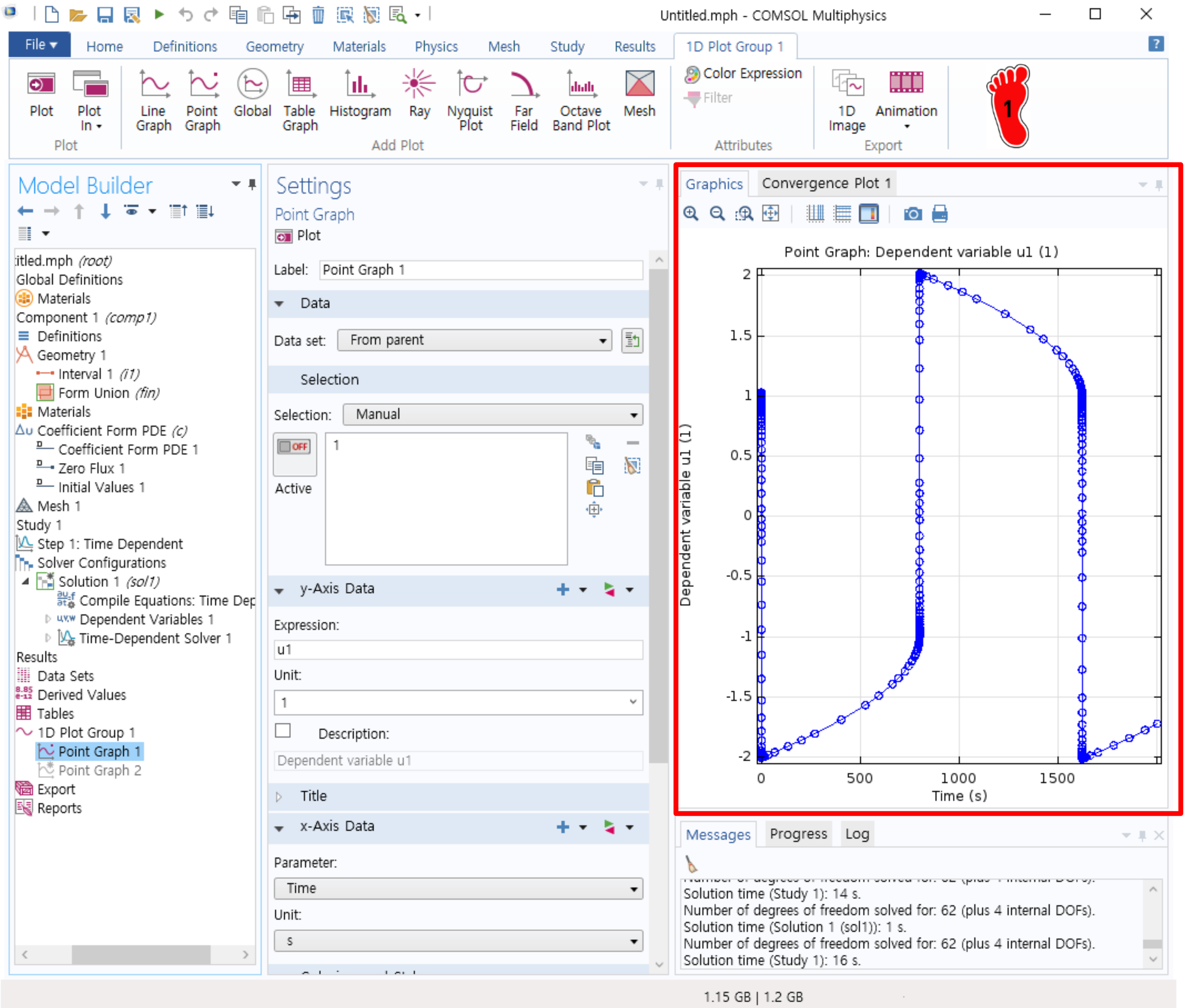
PLOT GROUP DISABLE



1 Point Group 2 우클릭

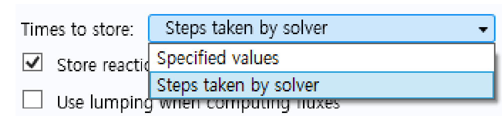
2 Disable 클릭
(u2의 경우 stiff한 구간에서 값이 급격하게 증가하기 때문에(결과 그래프 참조) u1 결과를 동시에 확인 어려움)

RESULT PLOT



결과 확인

stiff 한 시스템인 경우 시간이 소요되는 부분은 기울기가 급격히 변화하는 부분임을 확인



옵션에 따른 결과 확인

- **ODE examples**
  - ✓ Example 25.5
  - ✓ Example 28.2: predator-prey model
  - ✓ Example 25.14
  - ✓ Example 27.10
  - ✓ Previous case study I
  - ✓ Previous case study II

CASE STUDY I

Background. Electric circuits where the current is time-variable rather than constant are common. A transient current is established in the right-hand loop of the circuit shown in Fig. 28.11 when the switch is suddenly closed.

Equations that describe the transient behavior of the circuit in Fig. 28.11 are based on Kirchhoff's law, which states that the algebraic sum of the voltage drops around a closed loop is zero (recall Sec. 8.3). Thus,

$$L\frac{di}{dt} + Ri + \frac{q}{C} - E(t) = 0 \tag{28.9}$$

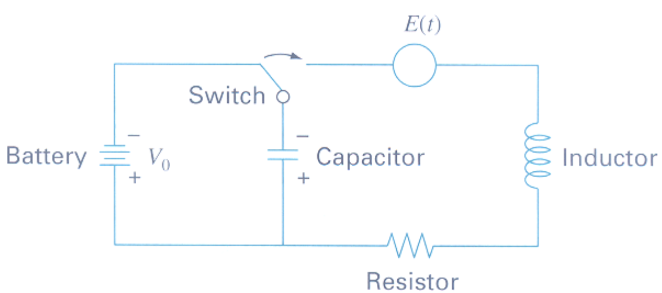
where $L(di/dt)$ = voltage drop across the inductor, L = inductance (H), R = resistance (Ω), q = charge on the capacitor (C), C = capacitance (F), $E(t)$ = time-variable voltage source (V), and

$$i = \frac{dq}{dt} \tag{28.10}$$

Equations (28.9) and (28.10) are a pair of first-order linear differential equations that can be solved analytically. For example, if $E(t) = E_0 \sin \omega t$ and $R = 0$,

$$q(t) = \frac{-E_0}{L(p^2-\omega^2)}\frac{\omega}{p}\sin pt + \frac{E_0}{L(p^2-\omega^2)}\sin \omega t \tag{28.11}$$

FIGURE 28.11
An electric circuit where the current varies with time.



$$E = E_0 \sin(\omega t)$$

$$L = 1 \text{ H}$$

$$E_0 = 1 \text{ V}$$

$$C = 0.25 \text{ C}$$

$$\omega^2 = 3.5 \text{ rad/s}^2$$

$$R = 0$$

$$i(0) = 0$$

$$q(0) = 0$$

$$t = (0,100)\ \Delta t = 0.1$$

$$q = y_1$$

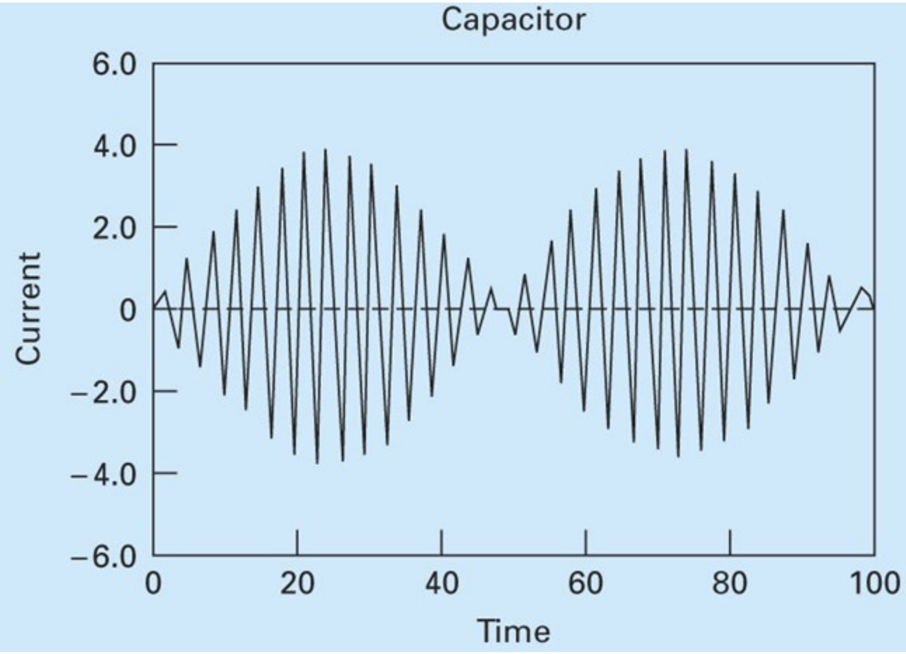
$$i = \frac{dq}{dt} = \frac{dy_1}{dt} = y_2$$

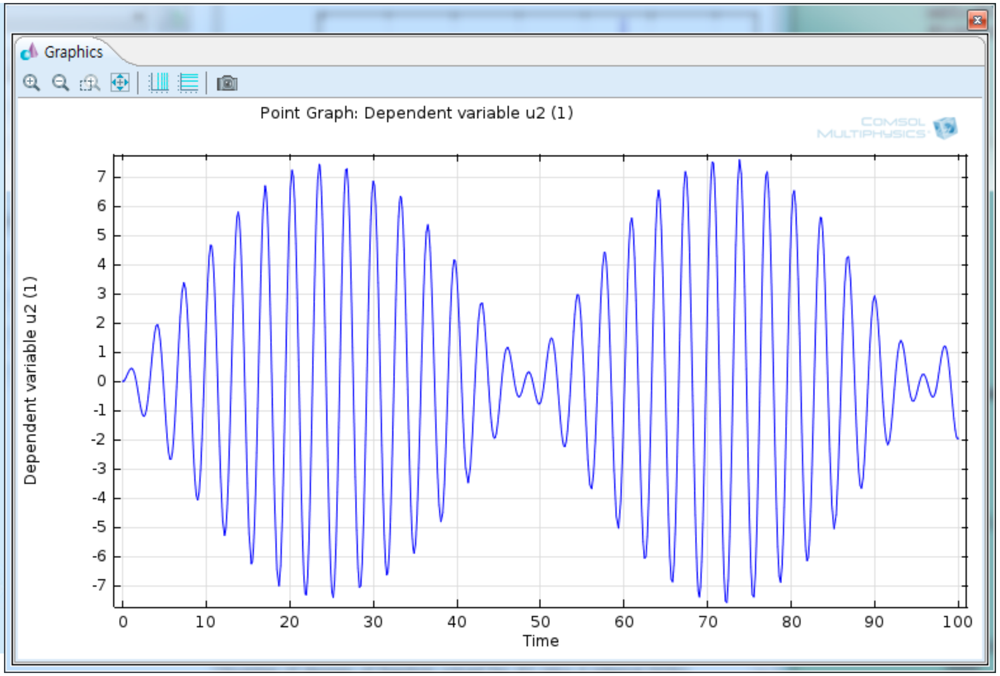
$$L\frac{di}{dt} + Ri + \frac{q}{C} - E(t) = 0$$

$$\rightarrow \frac{dy_2}{dt} = \frac{1}{L}\left(E(t) - Ri - \frac{y_1}{C}\right)$$

$$\begin{cases} \dfrac{dy_1}{dt} = y_2 \\ \dfrac{dy_2}{dt} = \dfrac{1}{L}\left(E(t) - Ri - \dfrac{y_1}{C}\right) \end{cases}$$

CASE STUDY I: RESULT





- **ODE examples**
  - ✓ Example 25.5
  - ✓ Example 28.2: predator-prey model
  - ✓ Example 25.14
  - ✓ Example 27.10
  - ✓ Previous case study I
  - ✓ Previous case study II

CASE STUDY II

Background. Mechanical engineers (as well as all other engineers) are frequently faced with problems concerning the periodic motion of free bodies. The engineering approach to such problems ultimately requires that the position and velocity of the body be known as a function of time. These functions of time invariably are the solution of ordinary differential equations. The differential equations are usually based on Newton's laws of motion.

As an example, consider the simple pendulum shown previously in Fig. PT7.1. The particle of weight W is suspended on a weightless rod of length l. The only forces acting on the particle are its weight and the tension R in the rod. The position of the particle at any time is completely specified in terms of the angle θ and l.

The free-body diagram in Fig. 28.16 shows the forces on the particle and the acceleration. It is convenient to apply Newton's laws of motion in the x direction tangent to the path of the particle:

$$\Sigma F = -W \sin\theta = \frac{W}{g} a$$

where g = the gravitational constant (32.2 ft/s^2) and a = the acceleration in the x direction. The angular acceleration of the particle (α) becomes

$$\alpha = \frac{a}{l}$$

Therefore, in polar coordinates ($\alpha = d^2\theta/dt^2$),

$$-W \sin\theta = \frac{Wl}{g}\alpha = \frac{Wl}{g}\frac{d^2\theta}{dt^2}$$

or

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0 \qquad (28.15)$$

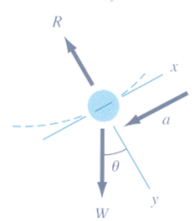


FIGURE 28.16
A free-body diagram of the swinging pendulum showing the forces on the particle and the acceleration.

This apparently simple equation is a second-order nonlinear differential equation. In general, such equations are difficult or impossible to solve analytically. You have two choices regarding further progress. First, the differential equation might be reduced to a form that can be solved analytically (recall Sec. PT7.1.1), or second, a numerical approximation technique can be used to solve the differential equation directly. We will examine both of these alternatives in this example.

$$\sin\theta \approx \theta$$

$$g = 9.81 \text{ m/s}^2$$

$$\theta_0 = \pi / 4$$

$$l = 0.6096 \text{ m}$$

$$\theta = y_1$$

$$\frac{d\theta}{dt} = \frac{dy_1}{dt} = y_2$$

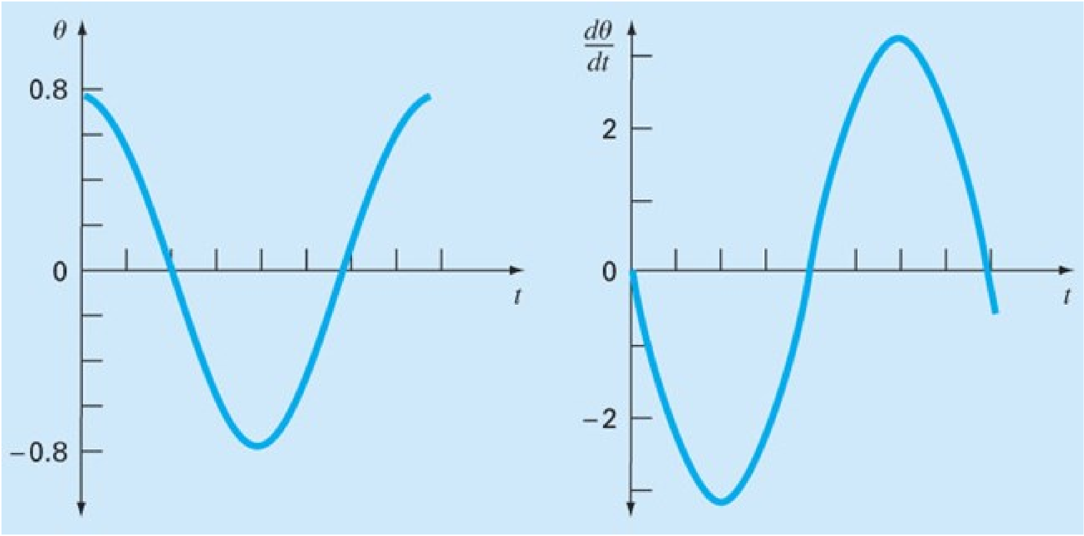
$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

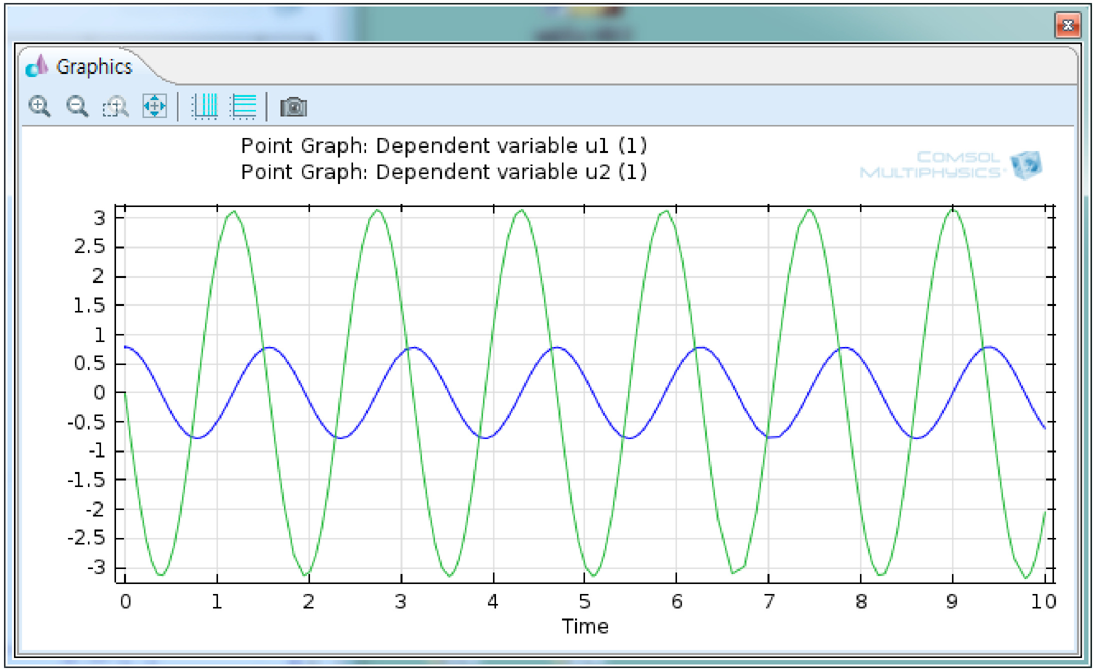
$$\rightarrow \frac{dy_2}{dt} = -\frac{g}{l}\sin y_1$$

$$\rightarrow \frac{dy_2}{dt} = -\frac{g}{l} y_1$$

$$\begin{cases} \dfrac{dy_1}{dt} = y_2 \\ \dfrac{dy_2}{dt} = -\dfrac{g}{l} y_1 \end{cases}$$

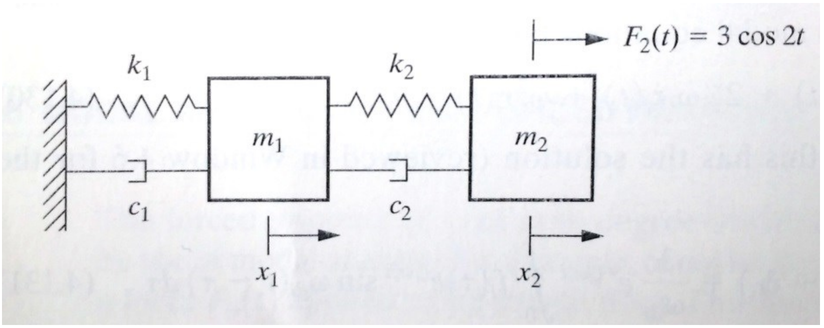
CASE STUDY II: RESULT





ASSIGNMENT(3RD WEEK: ODE)

2-DOF system



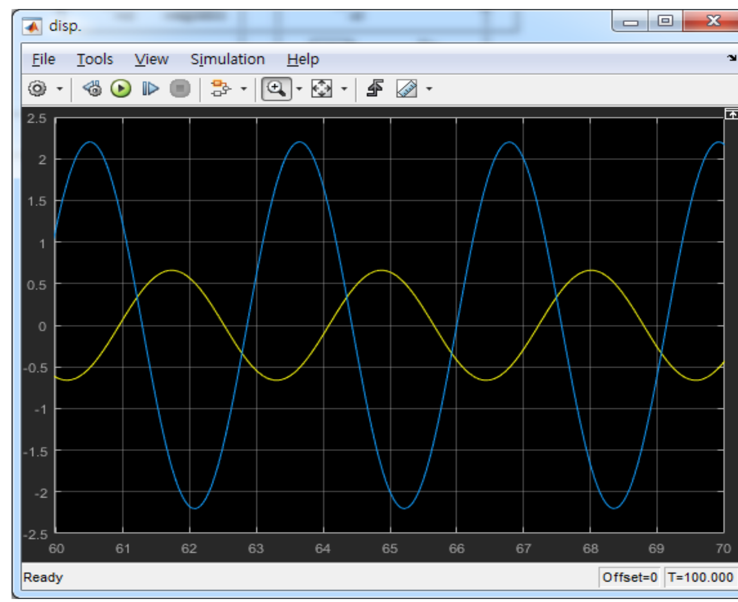
$$m_1 = 9\,\text{kg}, m_2 = 1\,\text{kg}$$
$$k_1 = 24\,\text{N/m}, k_2 = 3\,\text{N/m}$$
$$c_1 = 2.4\,\text{Ns/m}, c_2 = 0.3\,\text{Ns/m}$$
$$F_2(t) = 3\cos 2t$$

※ Ref. : Daniel J. Inman, "Engineering Vibration",
Prentice Hall International, Inc., pp 296-298, 2001

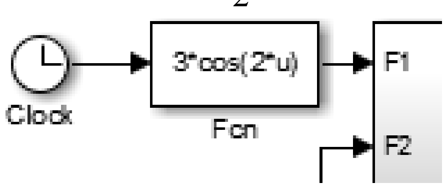
$$\mathbf{m_1\ddot{x}_1 = -k_2(x_1 - x_2) - c_2(\dot{x}_1 - \dot{x}_2) - k_1x_1 - c_1\dot{x}_1}$$
$$\mathbf{m_2\ddot{x}_2 = k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2) + F(t)}$$

※ Analytic Solution (Steady State)

$$x_1(t) = 0.2451\cos(2t - 0.1974) - 0.6249\sin 2t$$
$$x_2(t) = 0.7354\cos(2t - 0.1974) + 1.8749\sin 2t$$



F_2



ASSIGNMENT(3RD WEEK: ODE)