Automotive Body Structural Elements (1)

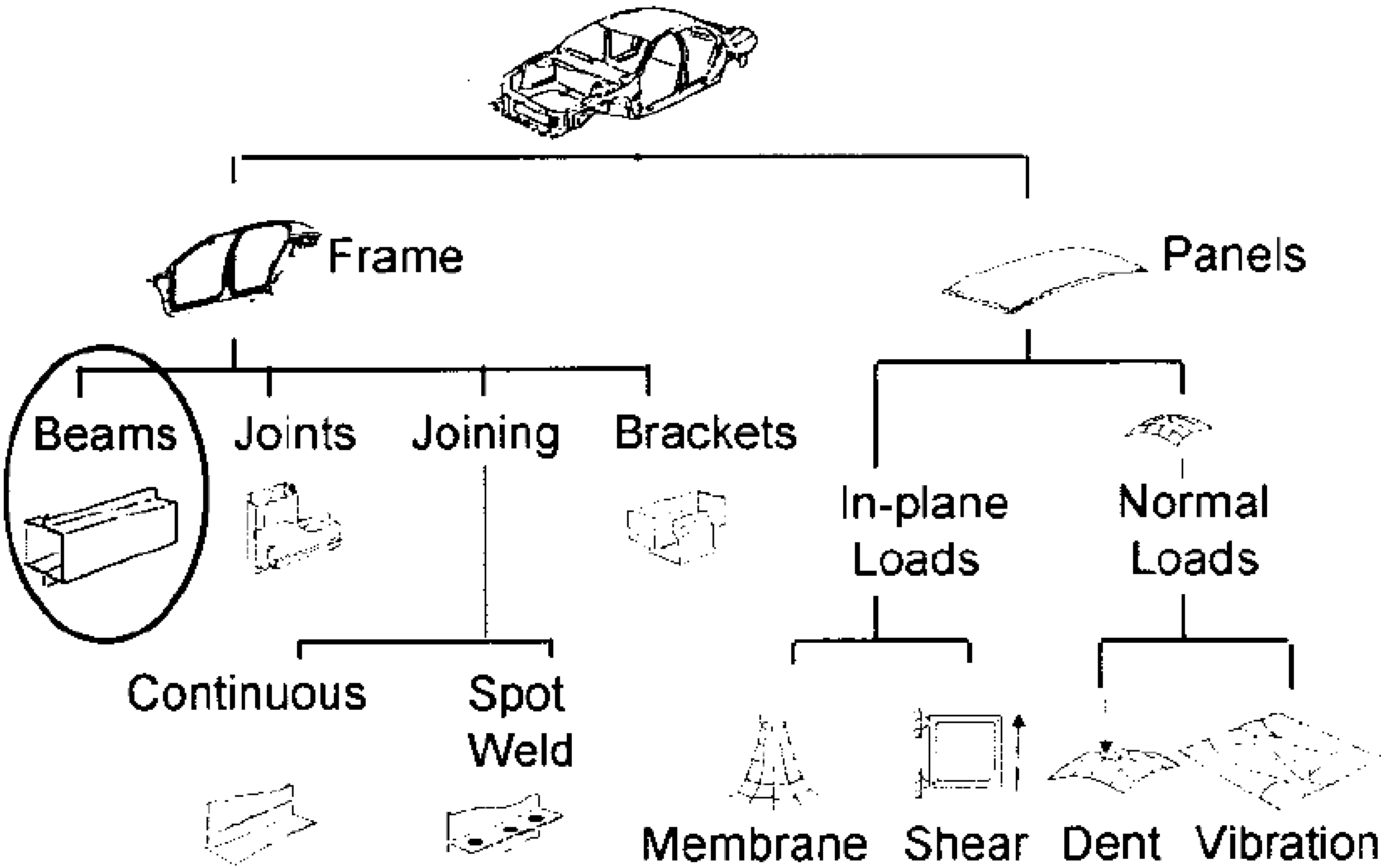
- Section design tools
  - How automotive structural elements respond to loading?
  - How they deflect? How they fail?
  - Predict stiffness and strength given the section geometry, the material and the bending moment, torque or applied force

- Classical beam behavior
- Design of automotive beam sections
  - Bending of non-symmetric beams
  - Point loading of thin walled sections

Automotive Body Structural Elements (2)

- Torsion of thin wall members
  - Torsion of member with closed/open section
  - Warping of open sections
  - Effect of spot welds on structural performance
  - Longitudinal stiffness of a shear loaded weld flange
- Thin wall beam section design
- Buckling of thin wall members
  - Plate buckling
  - Effective width
  - Techniques to inhibit buckling
- Panels: plates and membranes
  - Curved panel with normal loading
  - In-plane loading of panels
  - Membrane shaped panels

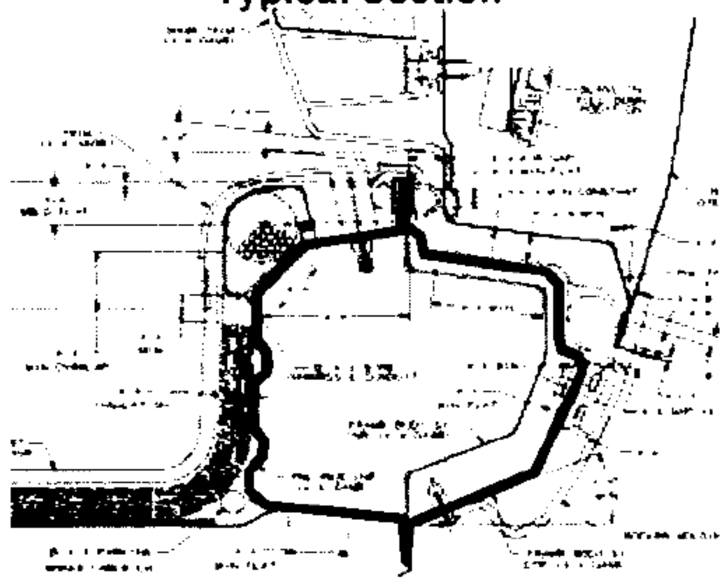
Structural Elements Classification



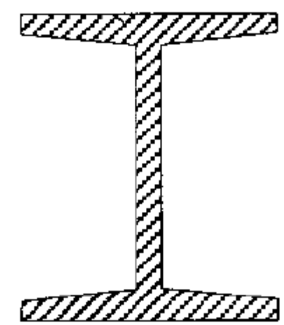
Beam Sections

- Thin walled structural elements
  - Relatively large width to thickness ratio
  - Non-symmetrical sections
  - Fabrication of several formed pieces spot welded

Automotive Rocker
Typical Section



Civil Engineering
Typical Section



3.1 Classical Beam Behavior

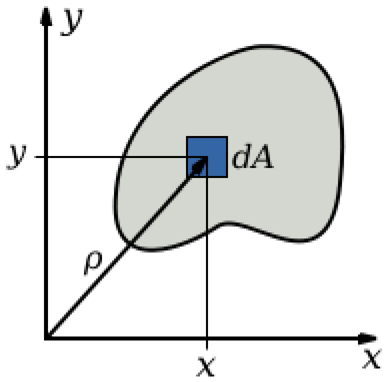
- Long straight beam with an I beam section
- Assumptions
  - Section is symmetric
  - Applied forces are down the axis of symmetry for the section
  - Section will not change shape upon loading
  - Deformation will be in the plane and in the direction of the applied load
  - Internal stresses vary in direct proportion with the strain
  - Failure: yielding of the outmost fiber
- Static equilibrium at a beam section: $M(x) = \int_0^x V dx$
- Stress over a beam section: $\sigma = -\frac{Mz}{I}$ where $I = \int_{\text{section}} z^2 dA$
- Beam deflection: $y = f(x),\ y'' = \frac{M(x)}{EI}$

Moment of Inertia

- Mass moment of inertia (관성모멘트)

$$I = kmr^2 = \sum_{i=1}^{n} m_i r_i^2 = \int r^2 dm = \iiint_V r^2 \rho(r) dV \rightarrow I = I_{cm} + md^2$$

- Area moment of inertia
  - Second moment of area (단면이차모멘트): bending
  - Polar moment of inertia (극관성모멘트): torsion
  - Product of inertia: unsymmetric geometry



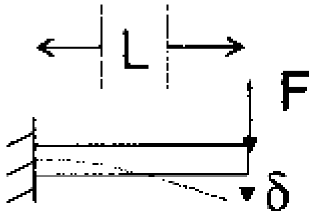
$$I_{xx} = \int_A y^2 dA \rightarrow I_{xx} = I_{xx\_c} + \bar{x}^2 A \text{ where } \bar{x}A = \int_A x dA$$

$$I_{yy} = \int_A x^2 dA$$

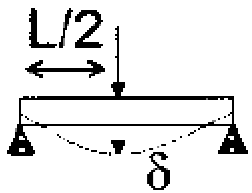
$$J(= I_z) = \int_A \rho^2 dA = \int_A (x^2 + y^2) dA = \int_A x^2 dA + \int_A y^2 dA = I_{xx} + I_{yy}$$

$$I_{xy} = \int_A xy dA$$

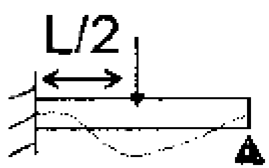
Beam Stiffness Equations



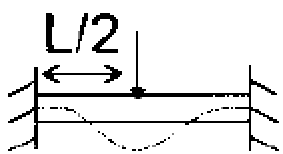
$$K = \frac{F}{\delta} = \frac{3EI}{L^3}$$



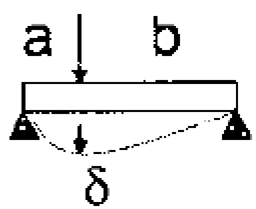
$$K = \frac{48EI}{L^3}$$



$$K = \frac{109.7EI}{L^3}$$



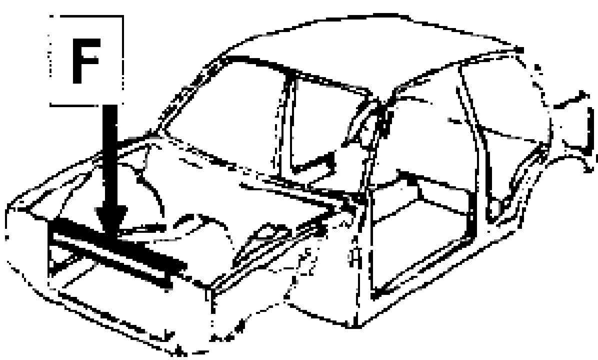
$$K = \frac{192EI}{L^3}$$

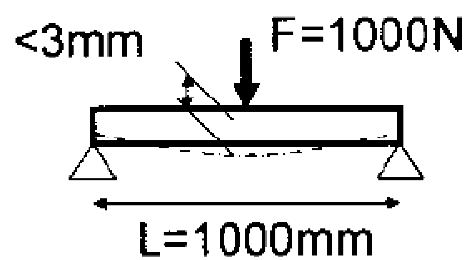


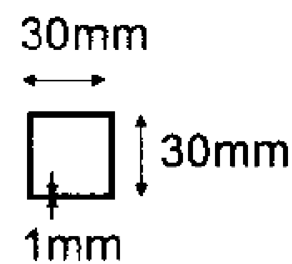
$$K = \frac{3EIL}{a^2b^2}$$

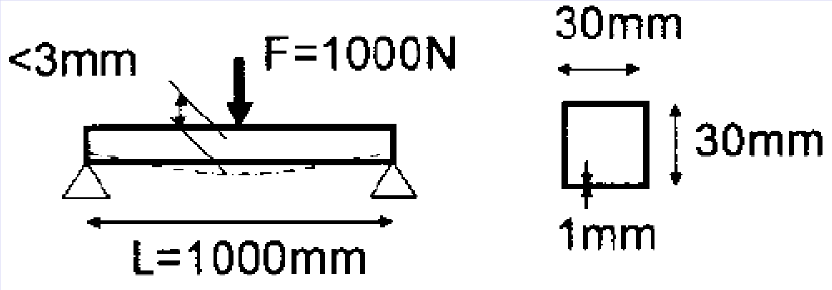
Example: Cross Member Beam

– Front motor compartment cross member holds the hood latch
– Under use, aerodynamic loading places a vertical load of 1000 N at the center of this beam
– Design requirements: section size ?
  • No yielding (σ_y = 210 N/mm^2) in the cross member
  • Maximum linear deflection at the hood latch of 3 mm

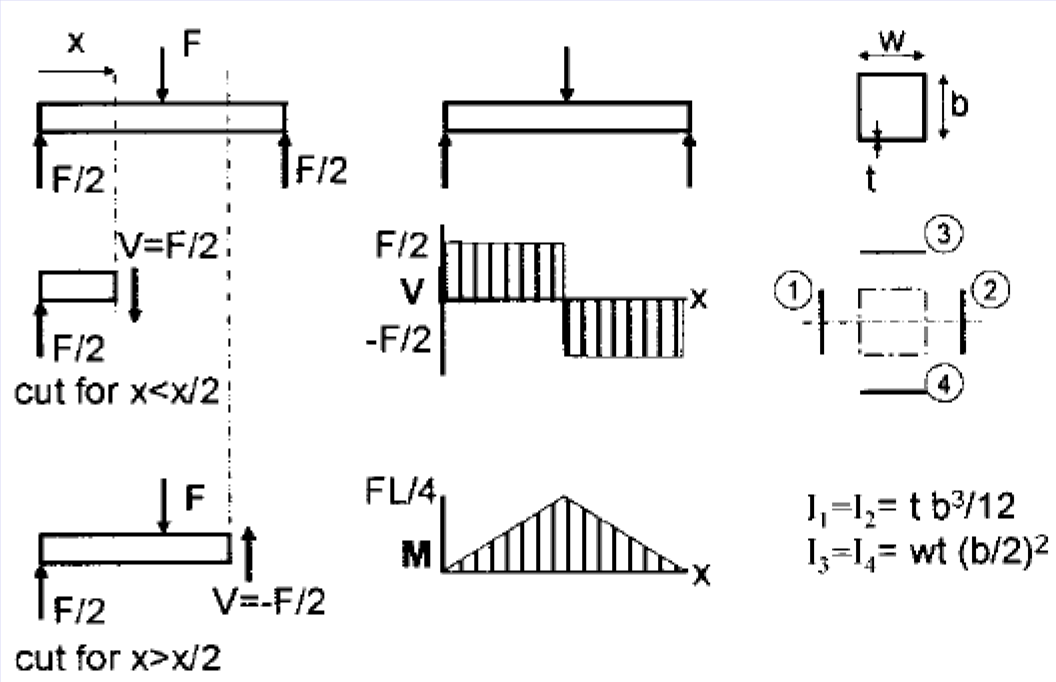








<3mm
F=1000N
L=1000mm
30mm
30mm
1mm



x
F
F/2
F/2
V=F/2
F/2
cut for x<x/2
F
F/2
V=-F/2
cut for x>x/2
F/2
V
-F/2
x
FL/4
M
x
w
b
t
1
2
3
4
$I_1=I_2= t\ b^3/12$
$I_3=I_4= wt\ (b/2)^2$

$$I = 2\left[\frac{1(30)^3}{12} + \left(\frac{30(1)^3}{12}\right) + 30(1)\left(\frac{30}{2}\right)^2\right] \approx 18000mm^4$$

$$\sigma = -\frac{Mz}{I_{zz}} = -\frac{(500N \times 500mm)(\mp 15mm)}{18000mm^4} = \pm 208N/mm^2 < \sigma_Y$$

$$k = \frac{48EI}{L^3} = \frac{F}{\delta} \rightarrow \delta = \frac{(1000mm)^3(1000N)}{48\left(207\times 10^3 N/mm^2\right)\left(18000mm^4\right)} = 5.6mm > 3mm$$

3.2 Design of Automotive Beam Sections

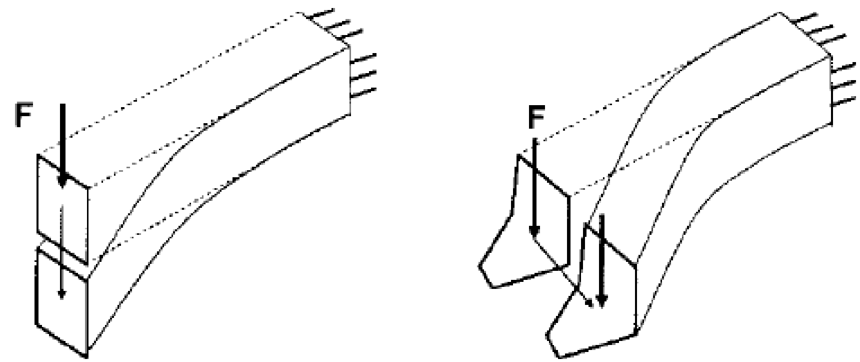
- Characteristics of automotive beams
  - Non-symmetrical nature of automotive beams
  - Local distortion of the section at the point of loading
  - Twisting of thin walled members
  - Effect of spot welds on structural performance

→ Stiffness reduction, How to design?

Bending of Non-Symmetric Beams

- Deflection
  - Resolve the load into components along each principle axis
  - Solve for the resulting deflection for each of these components
    - Moment of inertia is taken about the axis perpendicular to the load
    - Each of these deflections will be along the respective principle axis
  - Take the vector sum of the two deflections
- Stress
  - Resolve the moment into components along each principle axis
  - Solve for the resulting stress for each of these components
    - Dimension z is the distance to the point of interest from the axis which is colinear with the moment vector
  - Take the algebraic sum of two stresses for the resultant stress

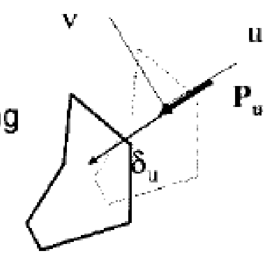
Non-Symmetric Beams

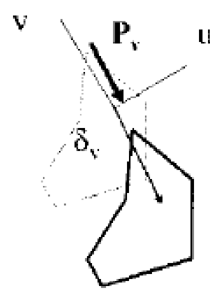


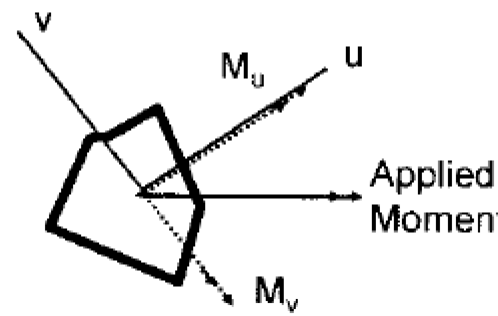
(a)
Symmetrical Beam

(b)
Non-Symmetrical Beam

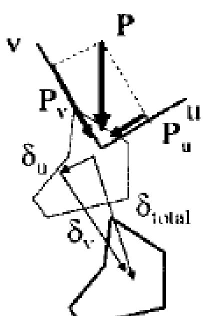
Deflections along principle axes







Vector sum of deflections along principle axes

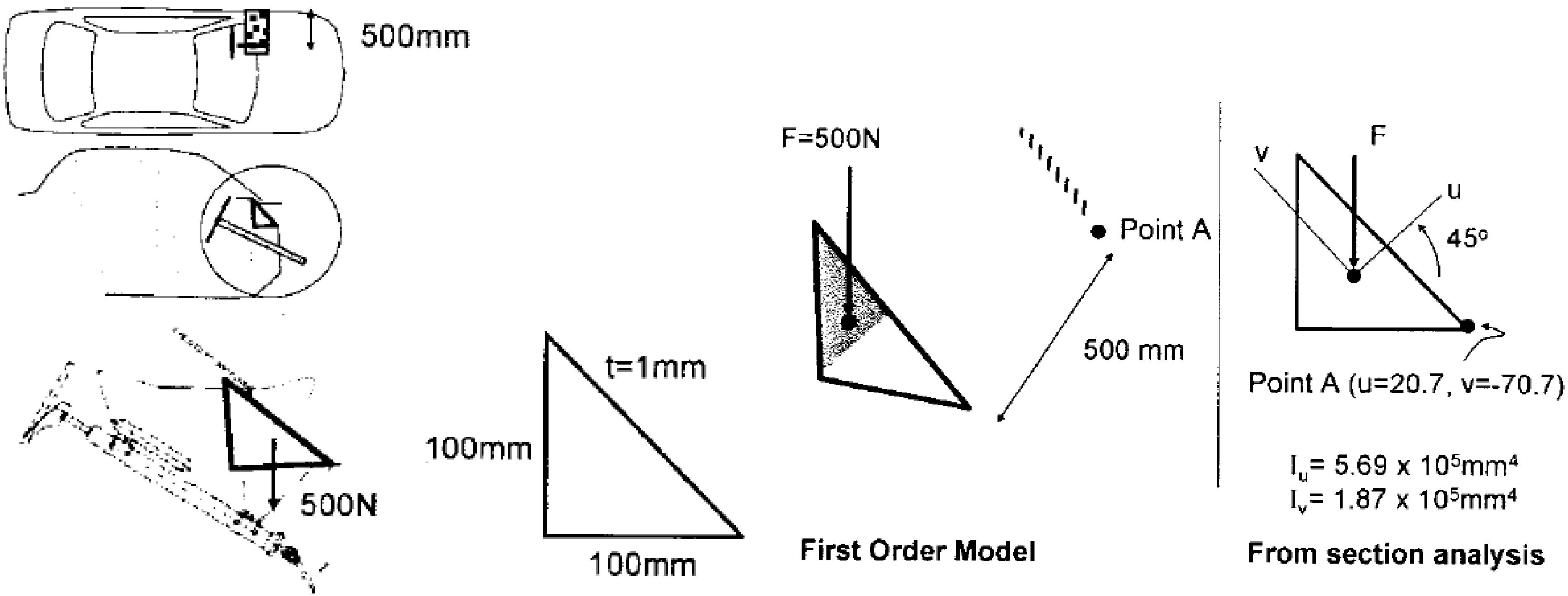


$$\sigma_{Mv} = -\frac{uM_v}{I_v}, \qquad \sigma_{Mu} = -\frac{vM_u}{I_u}$$

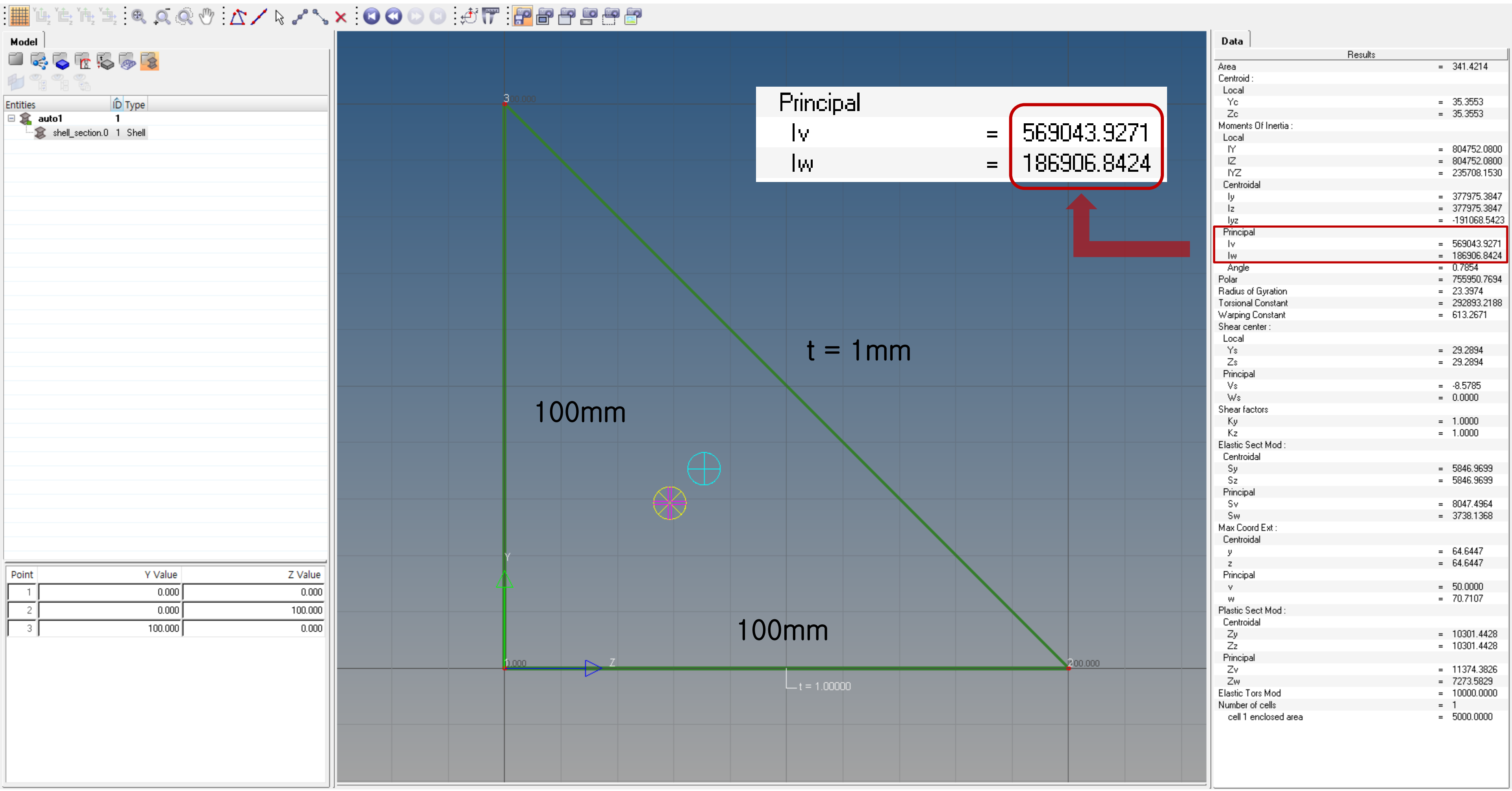
$$\sigma(u, v) = \sigma_{Mv} + \sigma_{Mu}$$

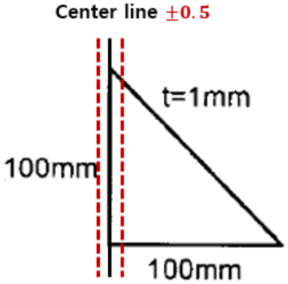
Example: Steering Column Mounting Beam

- Determine the tip deflection.
- Determine the stress at a specific point A where the beam joins the restraining structure.
- $E = 207 \times 10^3$ N/mm^2

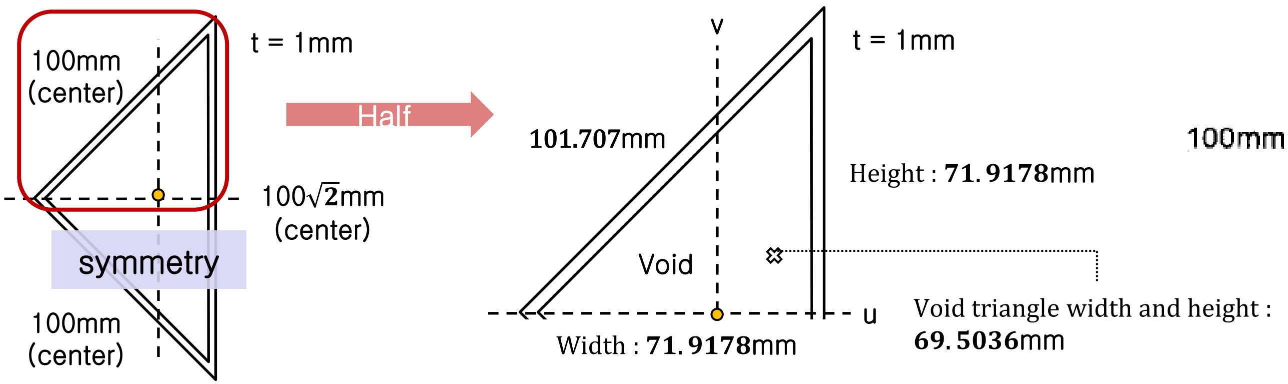


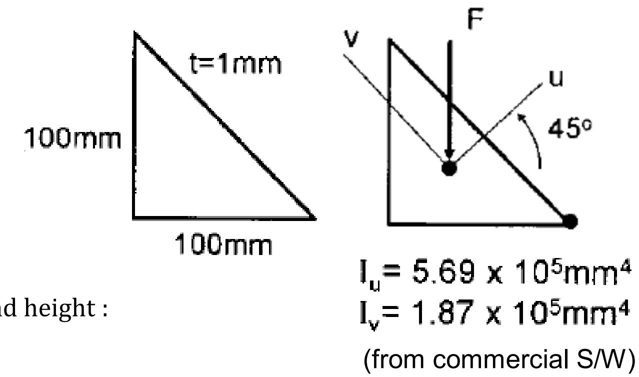
Area moment of inertia: HyperMesh





Area moment of inertia: Analysis (1)





(1) $I_{u,half} = I_{uc,half} + Ad^2 = \dfrac{(71.91)(71.91)^3}{36} + \left(\dfrac{(71.91)(71.91)}{2}\right)\left(\dfrac{71.91}{3}\right)^2 = 2229278\,mm^4$

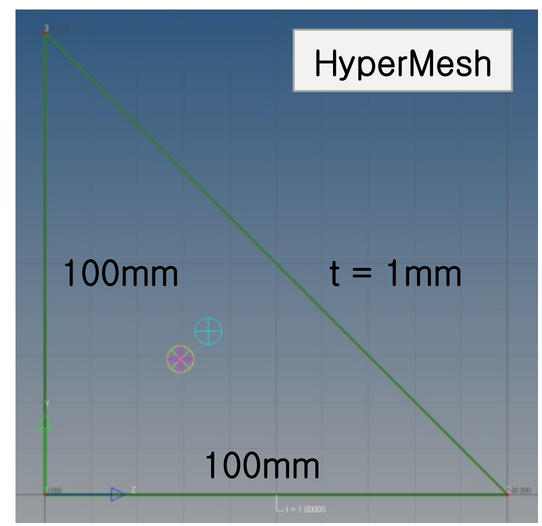
$I_{u,void} = I_{uc,void} + Ad^2 = \dfrac{(69.50)(69.50)^3}{36} + \left(\dfrac{(69.50)(69.50)}{2}\right)\left(\dfrac{69.50}{3}\right)^2 = 1944679\,mm^4$

$I_u = 2(I_{u,half} - I_{u,void}) = 569198 = 5.69 \times 10^5\,mm^4$ (error rate : 0.02%)

(2) $I_{v,half} = I_{vc,half} + Ad^2 = \dfrac{(71.91)^3(71.91)}{36} + \left(\dfrac{(71.91)(71.91)}{2}\right)(0)^2 = 743092\,mm^4$

$I_{v,void} = I_{vc,void} + Ad^2 = \dfrac{(69.50)^3(69.50)}{36} + \left(\dfrac{(69.50)(69.50)}{2}\right)(0.19)^2 = 648318\,mm^4$

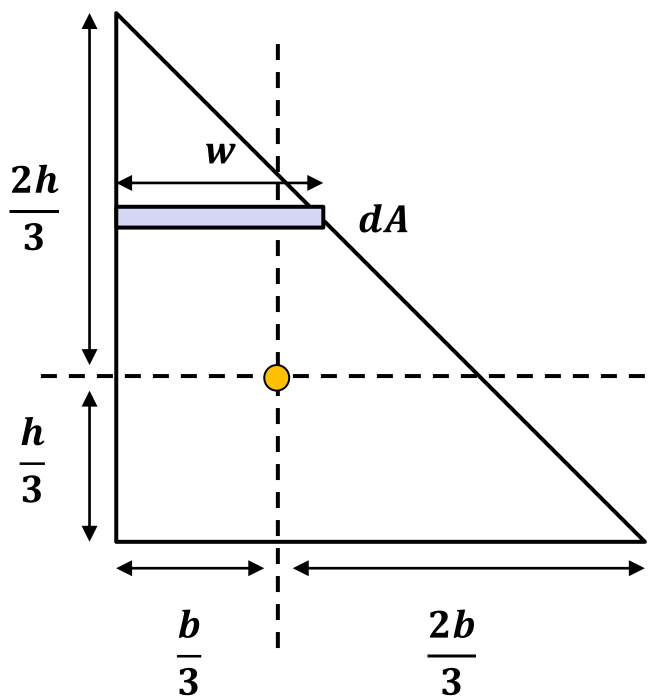
$I_v = 2(I_{v,half} - I_{v,void}) = 189548 = 1.89 \times 10^5\,mm^4$ (error rate : 1.41%)



Principal		
Iv	=	569043.9271
Iw	=	186906.8424

Area moment of inertia: Analysis (2)

- Moment of inertia of a triangle (centroidal axes)



$$dA = wdy,\ w = \frac{2b}{3} - \frac{b}{h}y$$

$$I_{x'} = \int y^2 dA$$

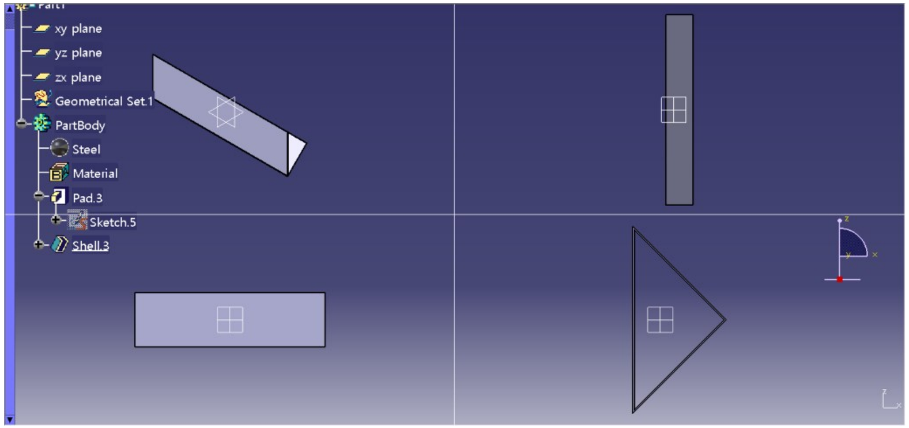
$$= \int_{h/3}^{2h/3} y^2 w dy = \int_{-h/3}^{2h/3} y^2 \left(\frac{2b}{3} - \frac{b}{h}y \right) dy$$

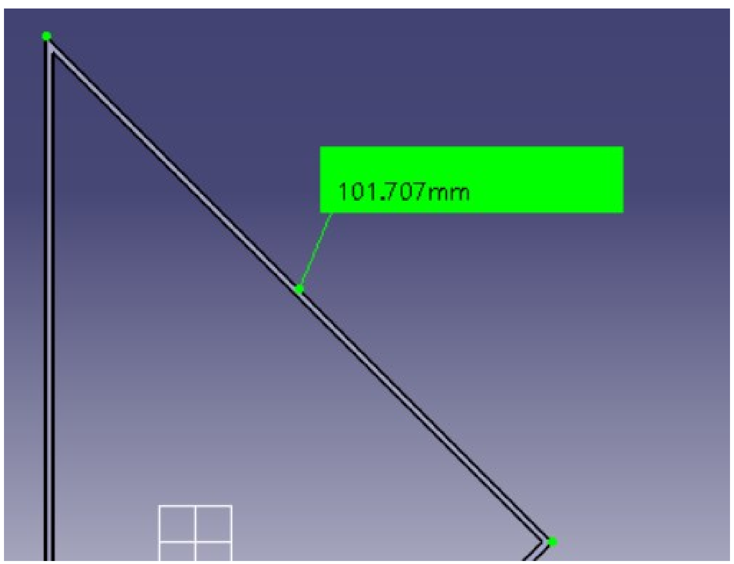
$$= \int_{-h/3}^{2h/3} \frac{2b}{3} y^2 - \frac{b}{h} y^3 dy = \left[\frac{2b}{3}\frac{y^3}{3} - \frac{b}{h}\frac{y^4}{4} \right]_{-h/3}^{2h/3}$$

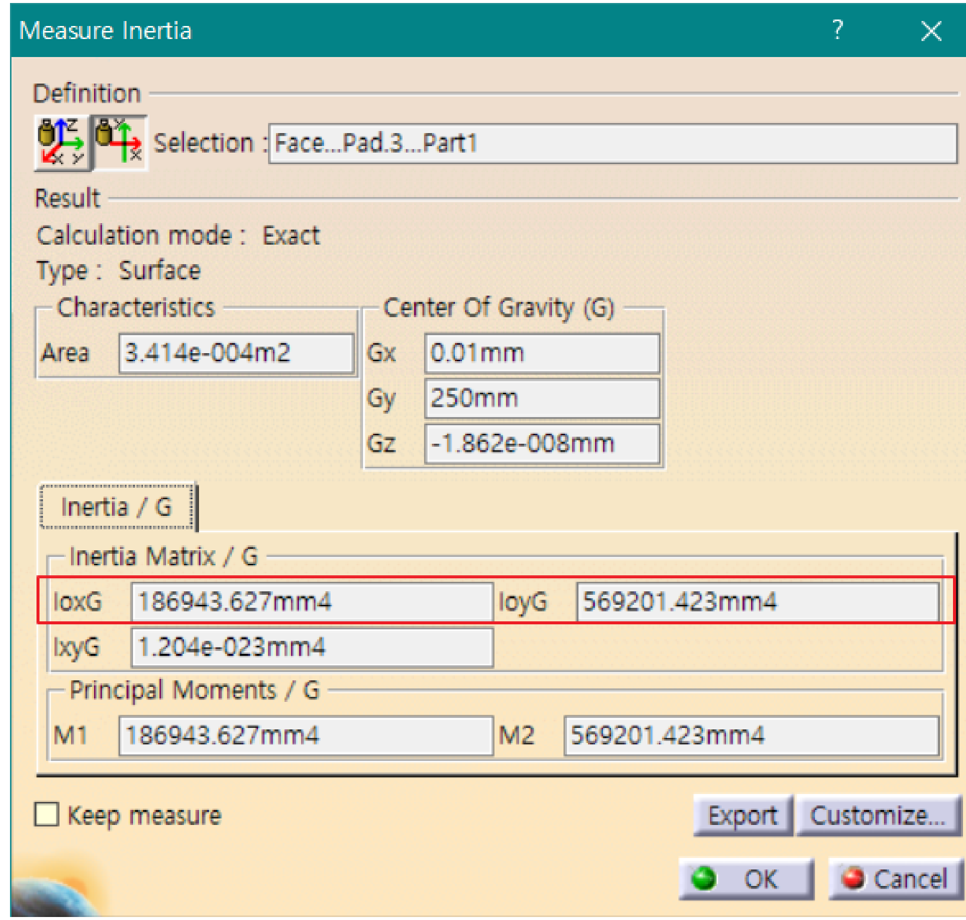
$$= \frac{2b}{9}\left(\frac{2h}{3}\right)^3 - \frac{b}{4h}\left(\frac{2h}{3}\right)^4 - \frac{2b}{9}\left(-\frac{h}{3}\right)^3 + \frac{b}{4h}\left(-\frac{h}{3}\right)^4$$

$$= \frac{bh^3}{36}$$

Area moment of inertia: CATIA







$$(1)\quad \delta_u = \frac{F_u l^3}{3EI_v} = \frac{(500N\sin 45°)(500mm)^3}{3\left(207\times10^3 N/mm^2\right)\left(1.87\times10^5 mm^4\right)} = 0.38mm$$

$$\delta_v = \frac{F_v l^3}{3EI_u} = \frac{(500N\cos 45°)(500mm)^3}{3\left(207\times10^3 N/mm^2\right)\left(5.69\times10^5 mm^4\right)} = 0.125mm$$

$$\delta = \sqrt{\left(\delta_u\right)^2 + \left(\delta_v\right)^2} = 0.4mm$$

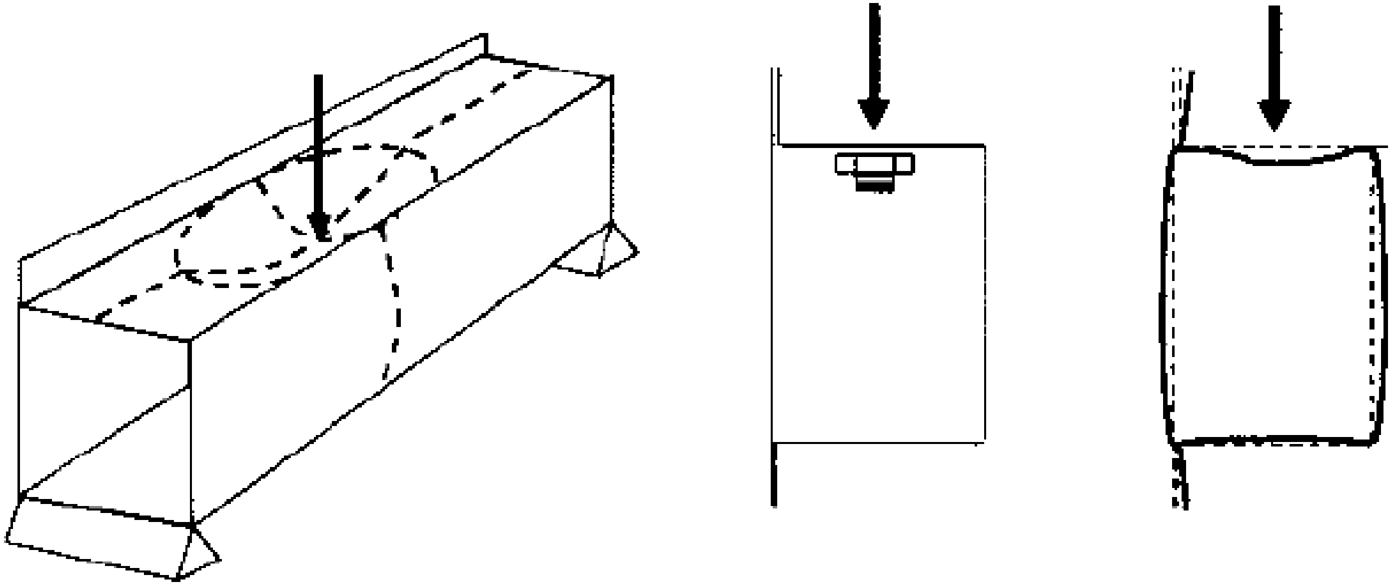
$$(2)\quad \sigma_u = -\frac{M_u v}{I_u} = -\frac{(-500N\cos 45°\times 500mm)\left(-70.7mm\right)}{5.69\times10^5 mm^4} = -22N/mm^2$$

$$\sigma_v = -\frac{M_v u}{I_v} = -\frac{(-500N\sin 45°\times 500mm)\left(20.7mm\right)}{1.87\times10^5 mm^4} = +19.57N/mm^2$$

$$\sigma_A = \sigma_u + \sigma_v = -2.43N/mm^2$$

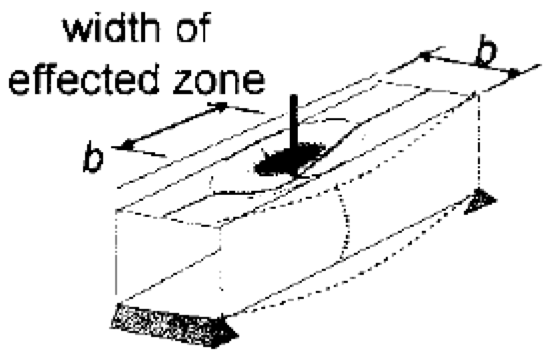
Point Loading of Thin Walled Sections

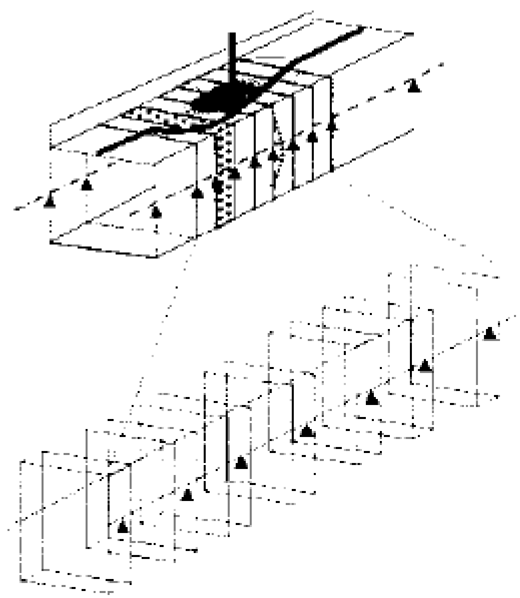
- Undesirable distortion in the vicinity of the load
  - Reduce apparent beam stiffness
  - Increase local stress

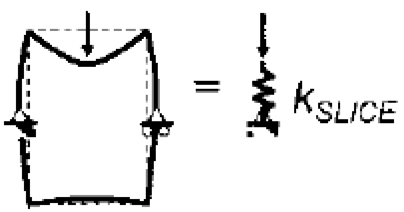


Prediction of Local Distortion

– Physical behavior: both beam deformation and local deformation
– Beam deformation eliminated by supporting beam along neutral axis leaving only local deformation: Local behavior isolated supporting beam along neutral axis
– Beam divided into slices of unit width over effective zone
– Slice characterized by a framework with stiffness k_{slice}

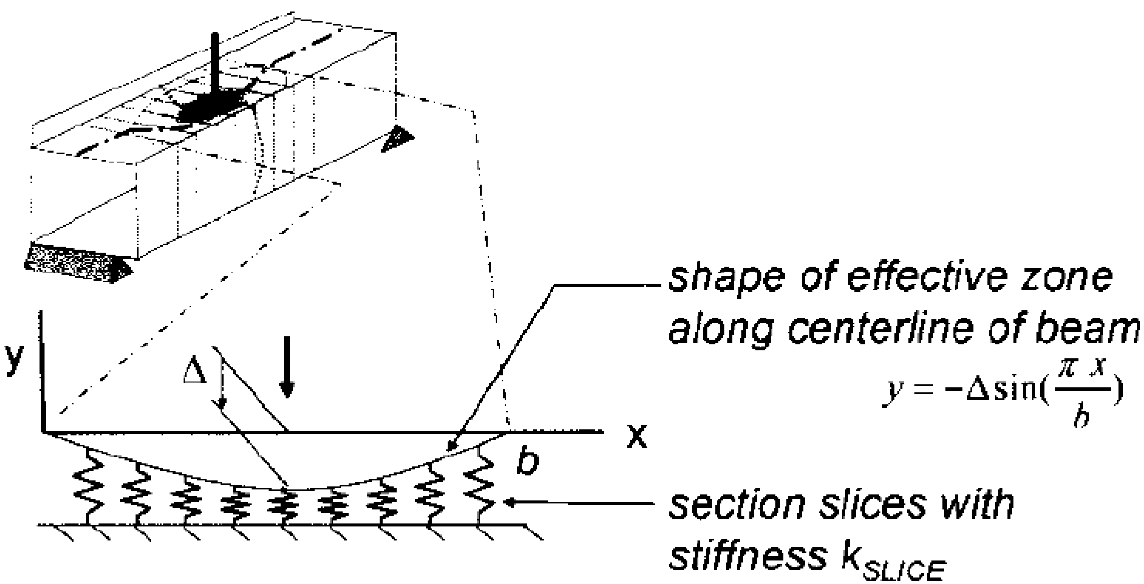


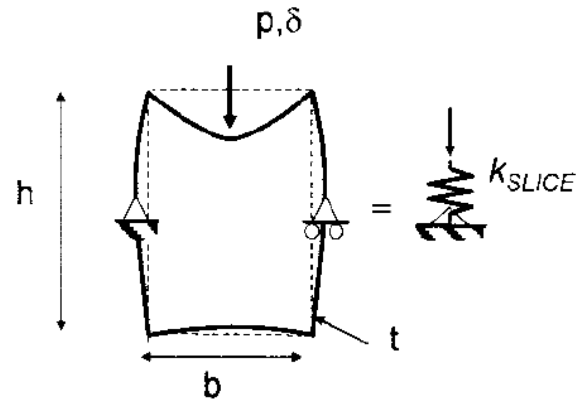




Idealized Beam Analysis

- (Energy stored by local stiffness at point of load application) = (Energy stored by distortion of all section slices)
- $K_{local} = F/\Delta$?



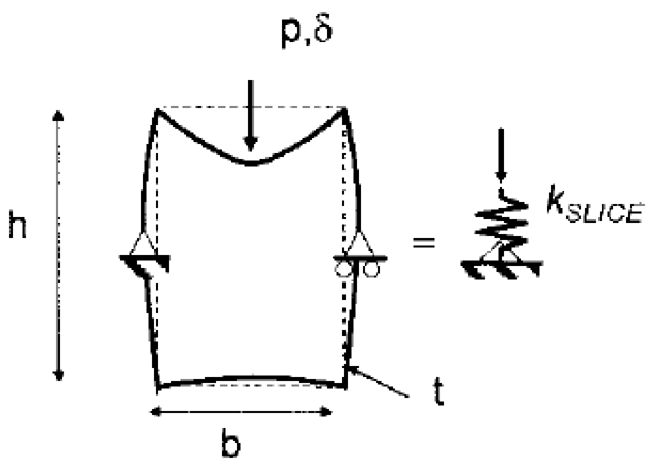


Rectangular Section Under Point Load

$$\begin{cases} work = \frac{1}{2} F\Delta \\ de = \frac{1}{2}\left(k_{slice} dx\right) y^2 \rightarrow energy = \frac{1}{2}\int_0^b k_{slice} y^2 dx = \frac{1}{2}\int_0^b k_{slice}\left(-\Delta \sin\frac{\pi x}{b}\right)^2 dx = \frac{1}{2} k_{slice}\Delta^2 \frac{b}{2} \end{cases}$$

$$\frac{1}{2} F\Delta = \frac{1}{2} k_{slice}\Delta^2 \frac{b}{2} \rightarrow \frac{F}{\Delta} = \frac{1}{2} k_{slice} b \rightarrow K_{local} = \frac{1}{2} k_{slice} b$$

$$k_{slice} = \frac{16Et^3\left(h+b\right)}{b^3\left(4h+b\right)} \rightarrow K_{local} = \frac{8Et^3\left(h+b\right)}{b^2\left(4h+b\right)}$$



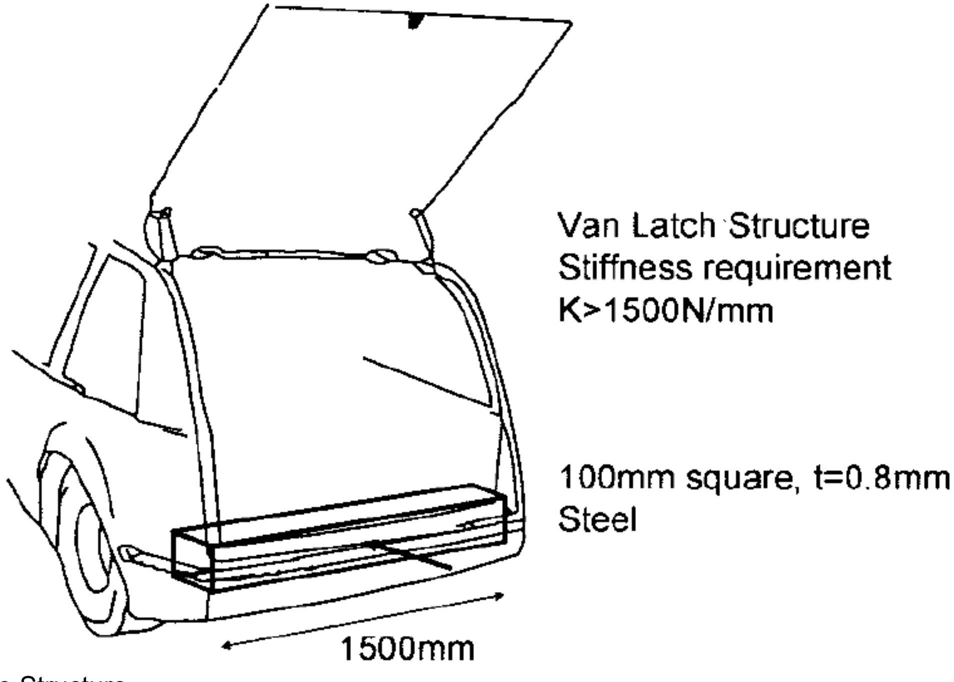
Slice of beam of unit length

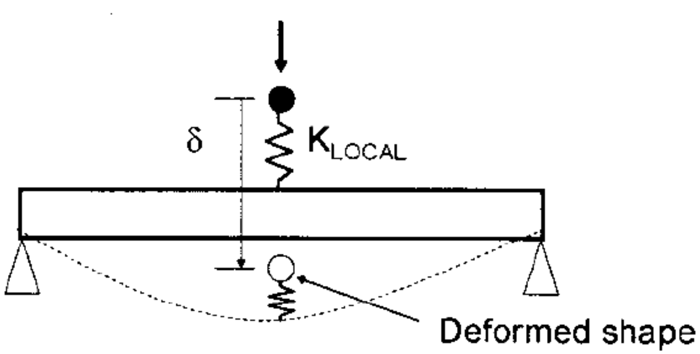
$$k_{SLICE} = 16E\left(\frac{t}{b}\right)^3\left(\frac{\frac{h}{b}+1}{4\frac{h}{b}+1}\right)$$

Stiffness of slice

Example: Van Cross Member

- Two springs in series
  - Idealized beam stiffness
  - Stiffness of the local distortion of the section





$$K_{system} = \frac{K_{ideal} K_{local}}{K_{ideal} + K_{local}}$$

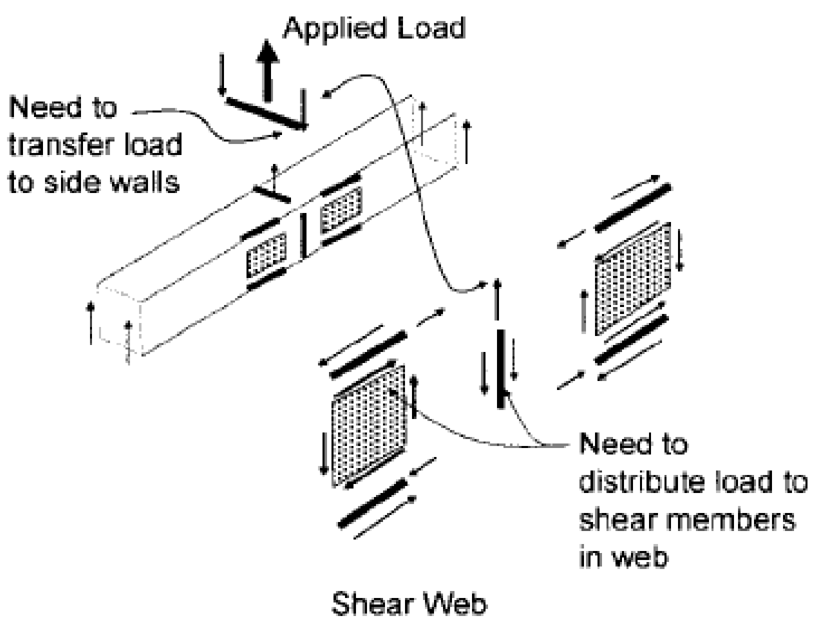
$$K_{ideal} = \frac{48EI}{L^3} = \frac{48(207000)}{1500^3} = 1569\, N/mm$$

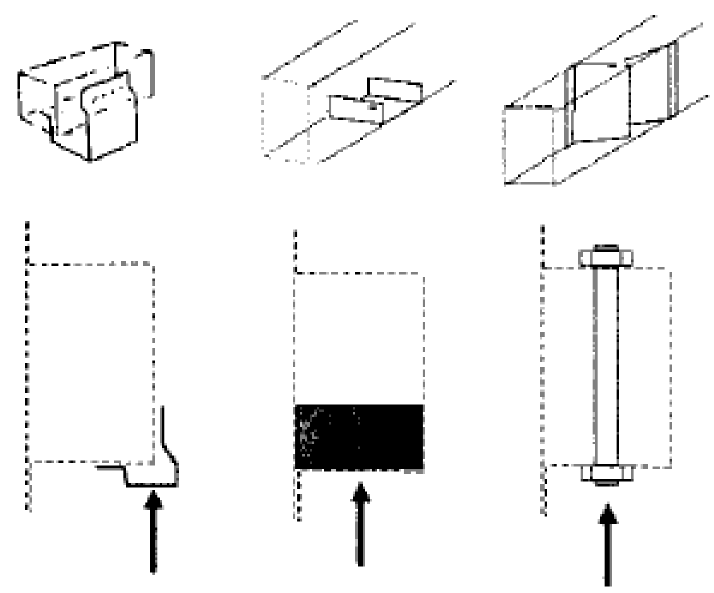
$$K_{local} = \frac{8Et^3(h+b)}{b^2(4h+b)} = \frac{8(207000)(0.8)^3(100+100)}{100^2(4\times100+100)} = 33.9\, N/mm$$

$$K_{system} = \frac{K_{ideal}K_{local}}{K_{ideal}+K_{local}} = 33.2\, N/mm \rightarrow 2\% \text{ of } K_{ideal}$$

Strategy to Reduce Local Distortion

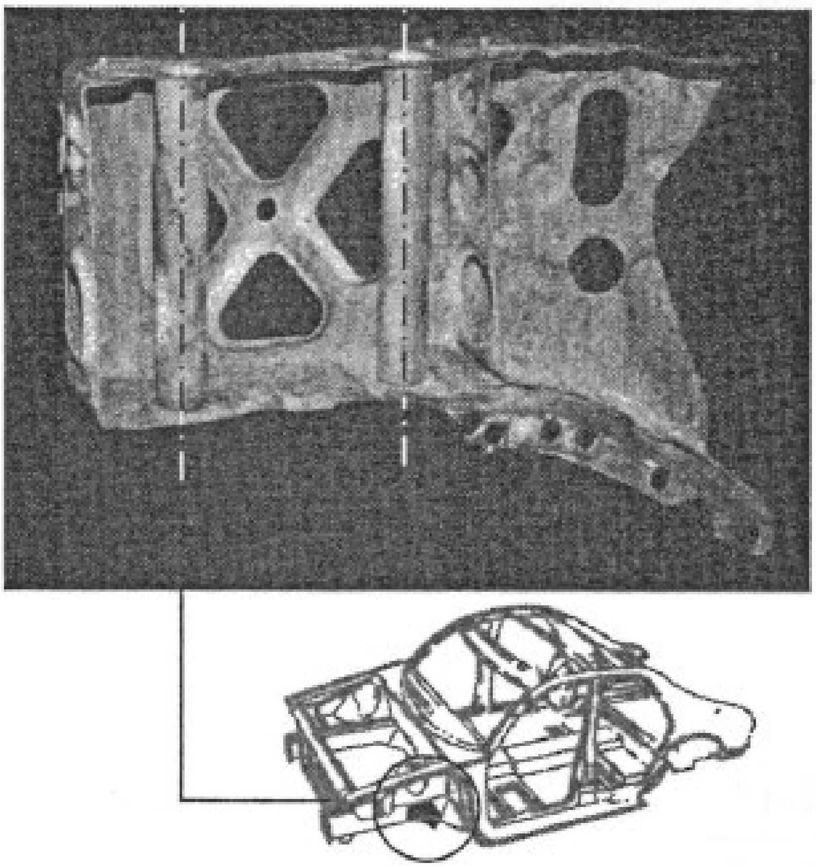
- Point load must load the shear web of the section directly
  - Moving the load point to align with the web
  - Adding stiff structural element to the section which reacts the load to the webs (local reinforcement)
  - Using through-section attachment with bulkhead to transfer the load to the web



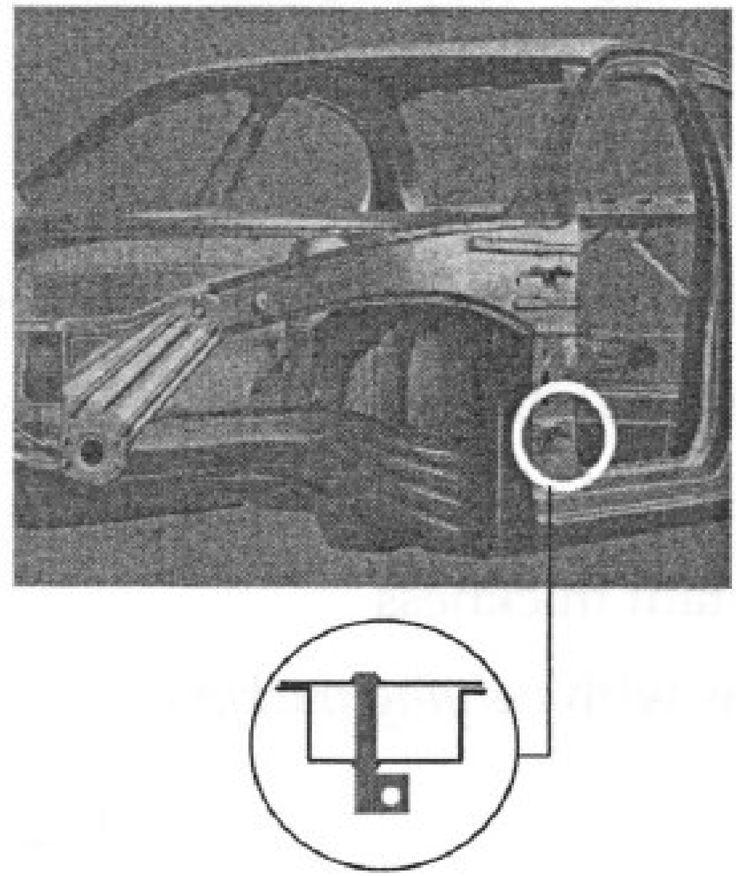


Example Sections Reacting a Point Load

Through-section engine mount attachment

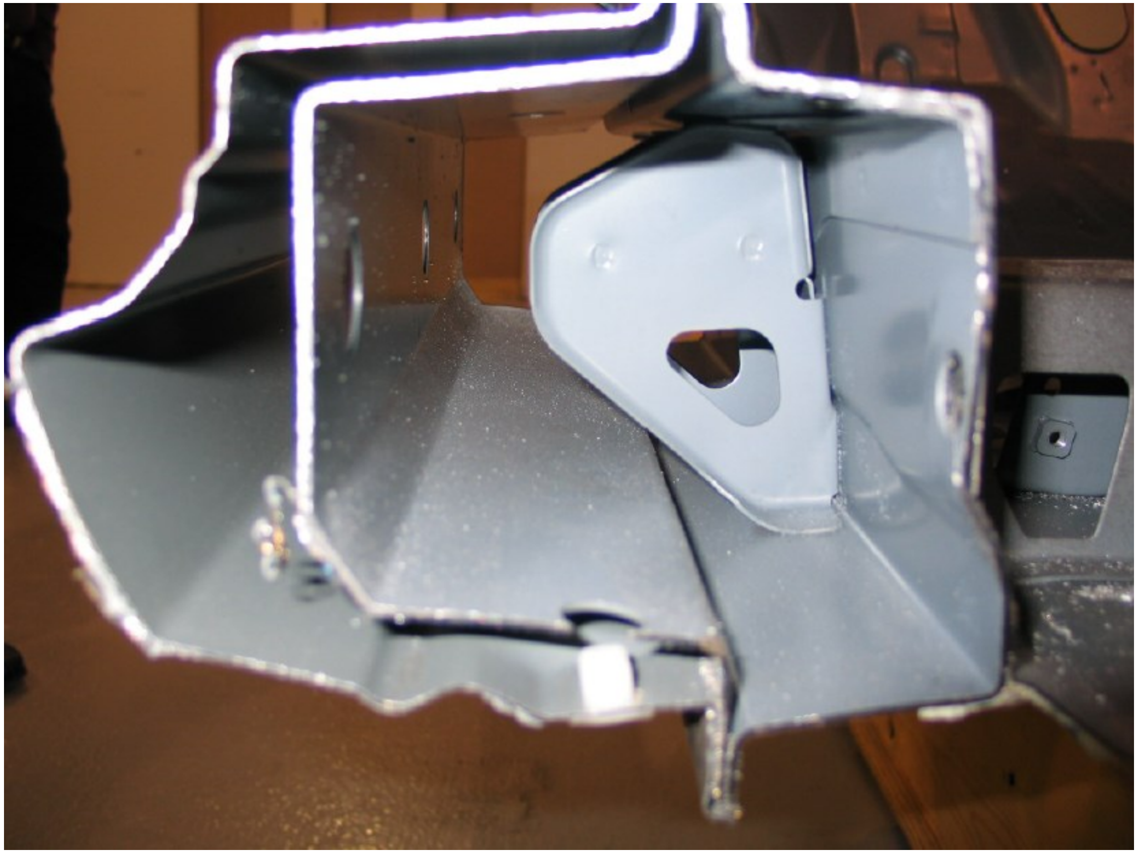


Through-section door hinge attachment



2003 Toyota Camry SE

- Local Stiffeners Inside Rocker To B-Pillar Joint
  - Bulkheads are used for local buckling prevention & FMVSS 214 Side Impact



3.3 Torsion of Thin Wall Members

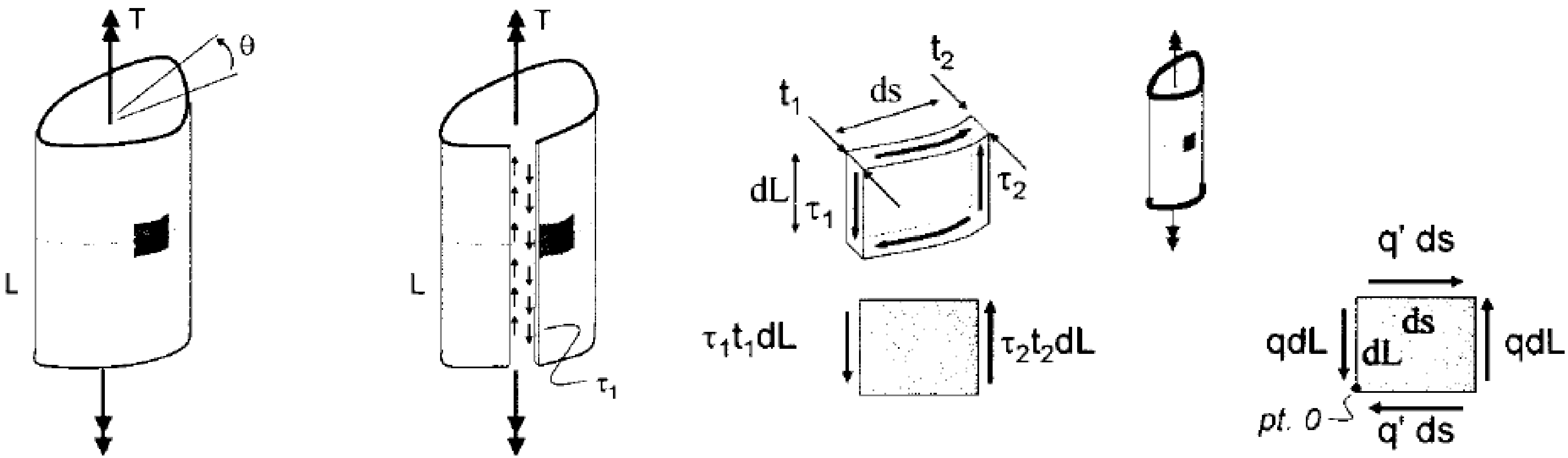
– For solid circular bar $\theta = \dfrac{TL}{GJ}, \quad \tau = \dfrac{Tr}{J}$

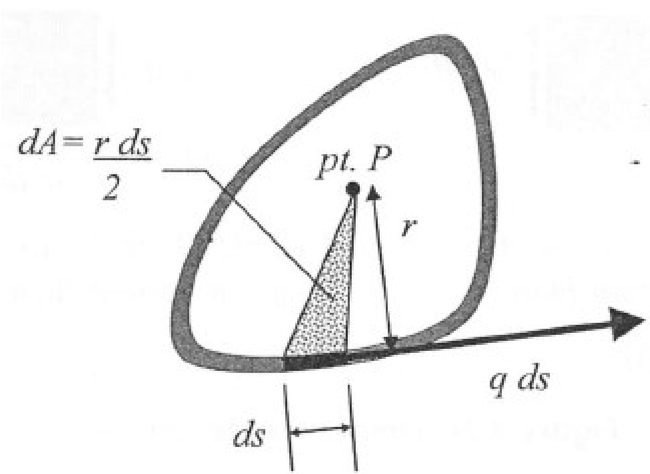
- Torsion of members with closed / open section

	closed section	open section
Angle of rotation	$\theta = \dfrac{TL}{GJ_{eff}}$	
Shear stress	$\tau = \dfrac{T}{2At}$	$\tau = \dfrac{Tt}{J_{eff}}$
Constant thickness	$J_{eff} = \dfrac{4A^2 t}{S}$	$J_{eff} = \dfrac{1}{3} t^3 S$
Non-uniform thickness	$J_{eff} = 4A^2 \Big/ \sum_i \dfrac{S_i}{t_i}$	$J_{eff} = \dfrac{1}{3} \sum_i t_i^{\,3} S_i$

- Warping of open sections under torsion
  – Warping constant

Torsion of Members with Closed Section (1)





$\tau_1 t_1 = \tau_2 t_2 \rightarrow q = \tau t$: shear flow (shearing force per unit length)

$dT = rdF = rqdS$

τ?

θ?

Torsion of Members with Closed Section (2)

$$T = \oint rq\,ds = q\oint r\,ds = q\oint r\frac{2dA}{r} = 2q\oint dA = 2qA \rightarrow q = \frac{T}{2A} = \tau t \rightarrow \tau = \frac{T}{2At}$$

$$\frac{1}{2}T\theta = \int \frac{1}{2}\tau\gamma\, dV = \int \frac{\tau^2}{2G}dV = \iint \frac{\tau^2}{2G}t\,ds\,dL$$

$$= \iint \frac{1}{2G}\left(\frac{T}{2At}\right)^2 t\,dL\,ds = \frac{1}{2G}\frac{T^2}{4A^2}\iint \frac{1}{t}dL\,ds = \frac{1}{2G}\frac{T^2 L}{4A^2}\oint \frac{ds}{t}$$

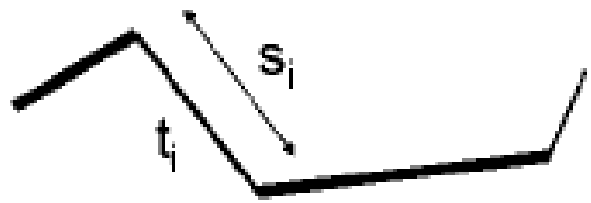
$$\theta = \frac{TL}{4GA^2}\oint \frac{ds}{t} = \frac{TL}{G\left(\dfrac{4A^2}{\oint \frac{ds}{t}}\right)} = \frac{TL}{GJ_{eff}}$$

Torsion of Members with Open Section

Uniform Thickness

$J_{eff} = {}^1/_3 s t^3$

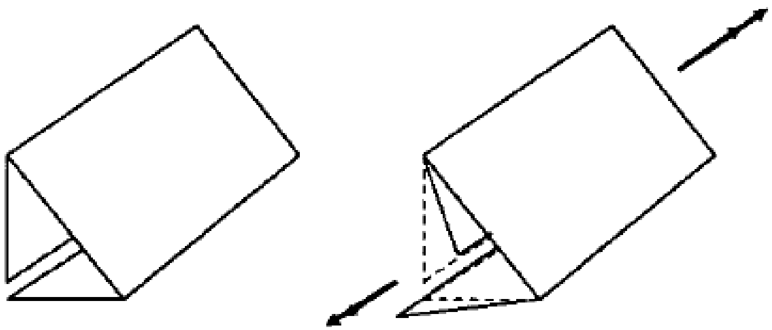
Variable Thickness

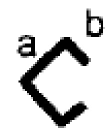


$J_{eff} = {}^1/_3 \Sigma_i s_i t_i^3$

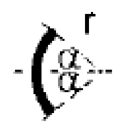
Warping of Open Sections under Torsion

- Warping in the longitudinal direction
  - Rigidly hold an end of an open tube and prevent warping, stiffness of the tube ↑

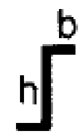




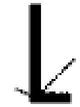
$$C_w = \frac{ta^4b^3}{6}\left(\frac{4a+3b}{2a^3-(a-b)^3}\right)$$



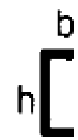
$$C_w = \frac{2tr^5}{3}\left(\alpha^3 - 6\frac{(\sin\alpha - \alpha\cos\alpha)^2}{\alpha - \sin\alpha\cos\alpha}\right)$$



$$C_w = \frac{th^2b^3}{12}\left(\frac{2h+b}{h+2b}\right)$$



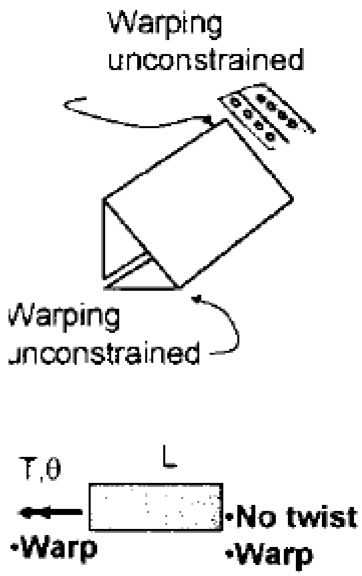
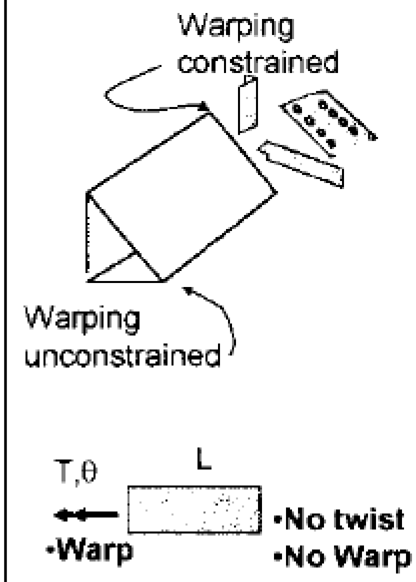
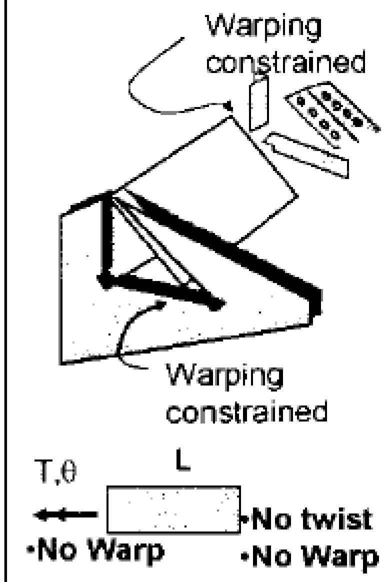
$$C_w = 0$$



$$C_w = \frac{th^2b^3}{12}\left(\frac{2h+3b}{h+6b}\right)$$

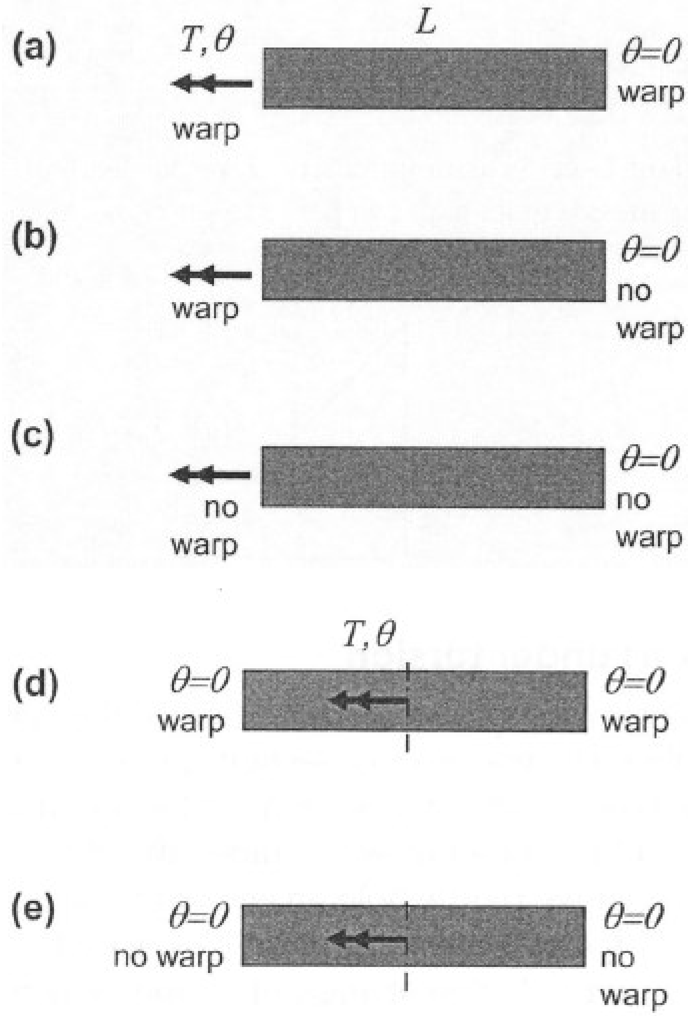
- Warping constant C_w
  - Depends on the geometry of the section
  - $C_w = 0$: section remains planar
  - Large C_w : greater out of plane deformation

Constrained Warping

		
$\theta = \frac{TL}{GJ}$	$\theta = \frac{TL}{GJ}\left(1 - \frac{\tanh kL}{kL}\right)$	$\theta = \frac{TL}{GJ}\left(1 - \frac{\tanh kL/2}{kL/2}\right)$

$$k = \sqrt{\frac{JG}{C_w E}}$$

Formulae for Twist of Warping Tubes



(a) $$\theta = \frac{TL}{JG}$$

(b) $$\theta = \frac{TL}{GJ}\left(1 - \frac{\tanh kL}{kL}\right)$$

(c) $$\theta = \frac{TL}{GJ}\left(1 - \frac{\tanh \frac{kL}{2}}{\frac{kL}{2}}\right)$$

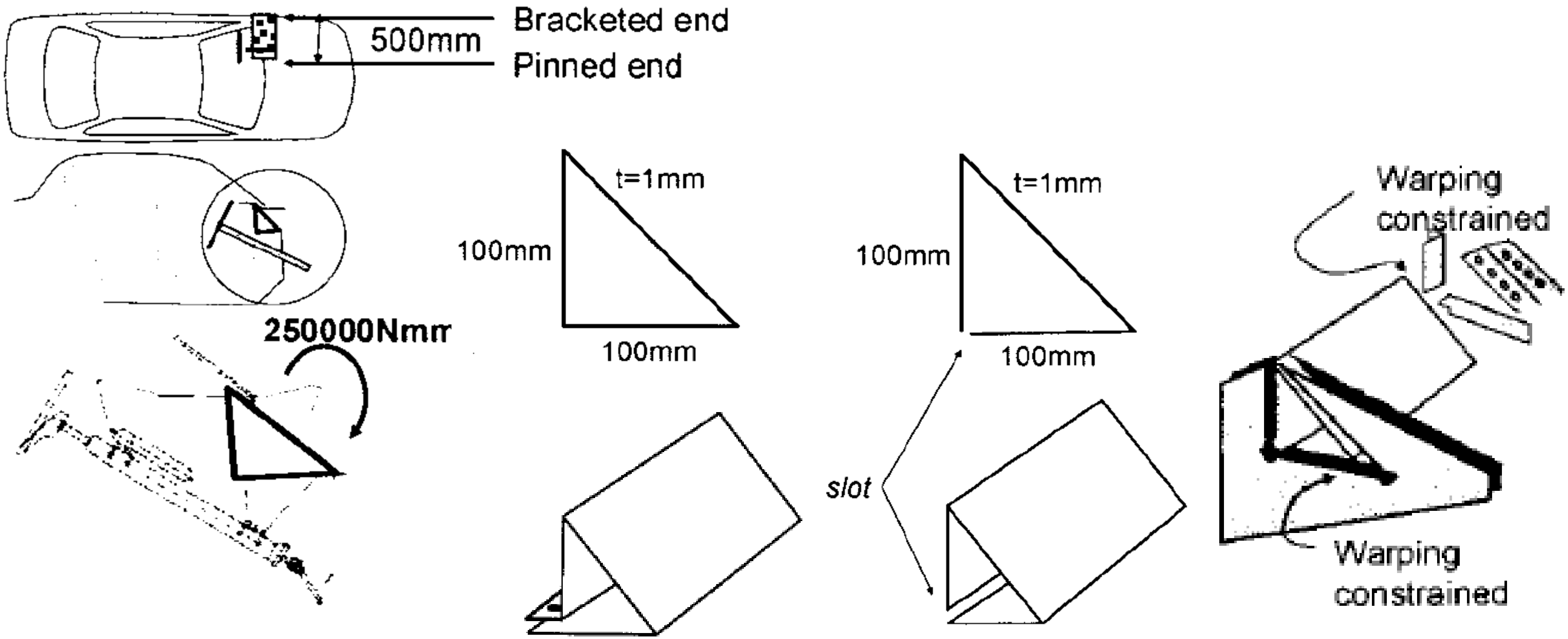
(d) $$\theta = \frac{TL}{4GJ}\left(1 - \frac{\tanh \frac{kL}{2}}{\frac{kL}{2}}\right)$$

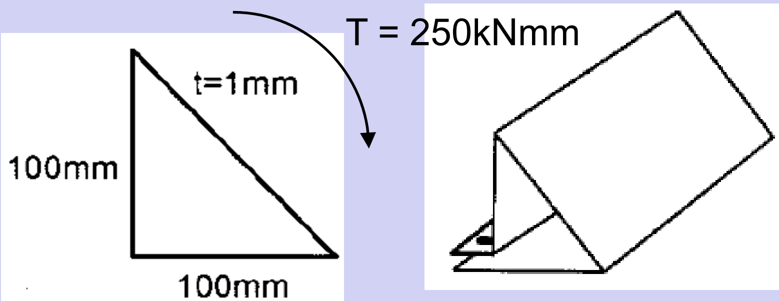
(e) $$\theta = \frac{TL}{4GJ}\left(1 - \frac{\tanh \frac{kL}{4}}{\frac{kL}{4}}\right)$$

$$k = \sqrt{\frac{JG}{C_w E}}$$

Example: Steering Column Mounting Beam

section	closed	open	No warping
Thin-wall torsion constant (mm^4)			
Angle of rotation (rad/degree)			
Shear stress (N/mm^2)			

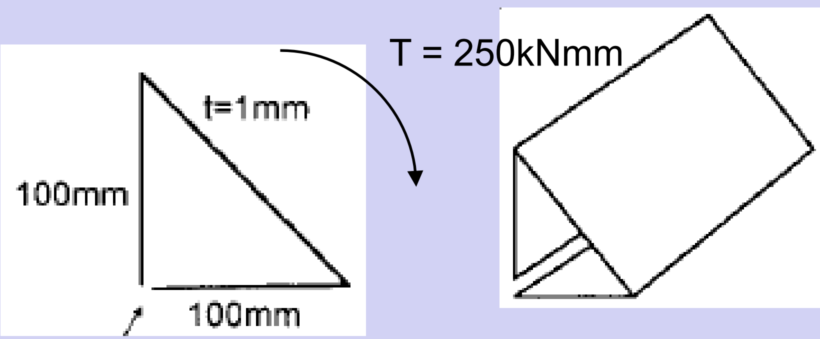




$$J_{eff} = \frac{4A^2}{\oint \frac{dS}{t}} = \frac{4A^2 t}{S} = \frac{4\left(\frac{1}{2}100^2\,mm^2\right)^2(1)}{\left(100+100+100\sqrt{2}\right)} = 2.93\times10^5\,mm^4$$

$$\theta = \frac{Tl}{GJ_{eff}} = \frac{(25\times10^4\,Nmm)(500mm)}{\left(78\times10^3\,N/mm^2\right)\left(2.93\times10^5\,mm^4\right)} = 5.47\times10^{-3}\,rad\,(0.312°)$$

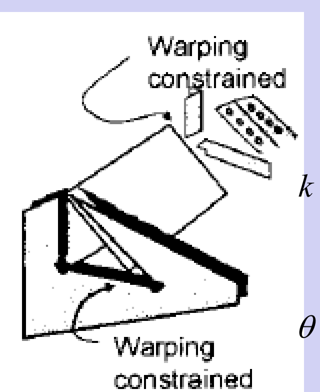
$$\tau = \frac{T}{2At} = \frac{25\times10^4\,Nmm}{2\left(\frac{1}{2}100^2\,mm^2\right)(1mm)} = 25N/mm^2$$



$$J_{eff} = \frac{1}{3}t^3 S = \frac{1}{3}(1mm)^3\left(100+100+100\sqrt{2}\right)mm = 113.8mm^4$$

$$\theta = \frac{Tl}{GJ_{eff}} = \frac{(25\times10^4\,Nmm)(500mm)}{\left(78\times10^3\,N/mm^2\right)\left(113.8mm^4\right)} = 14.1rad\,(803°?)$$

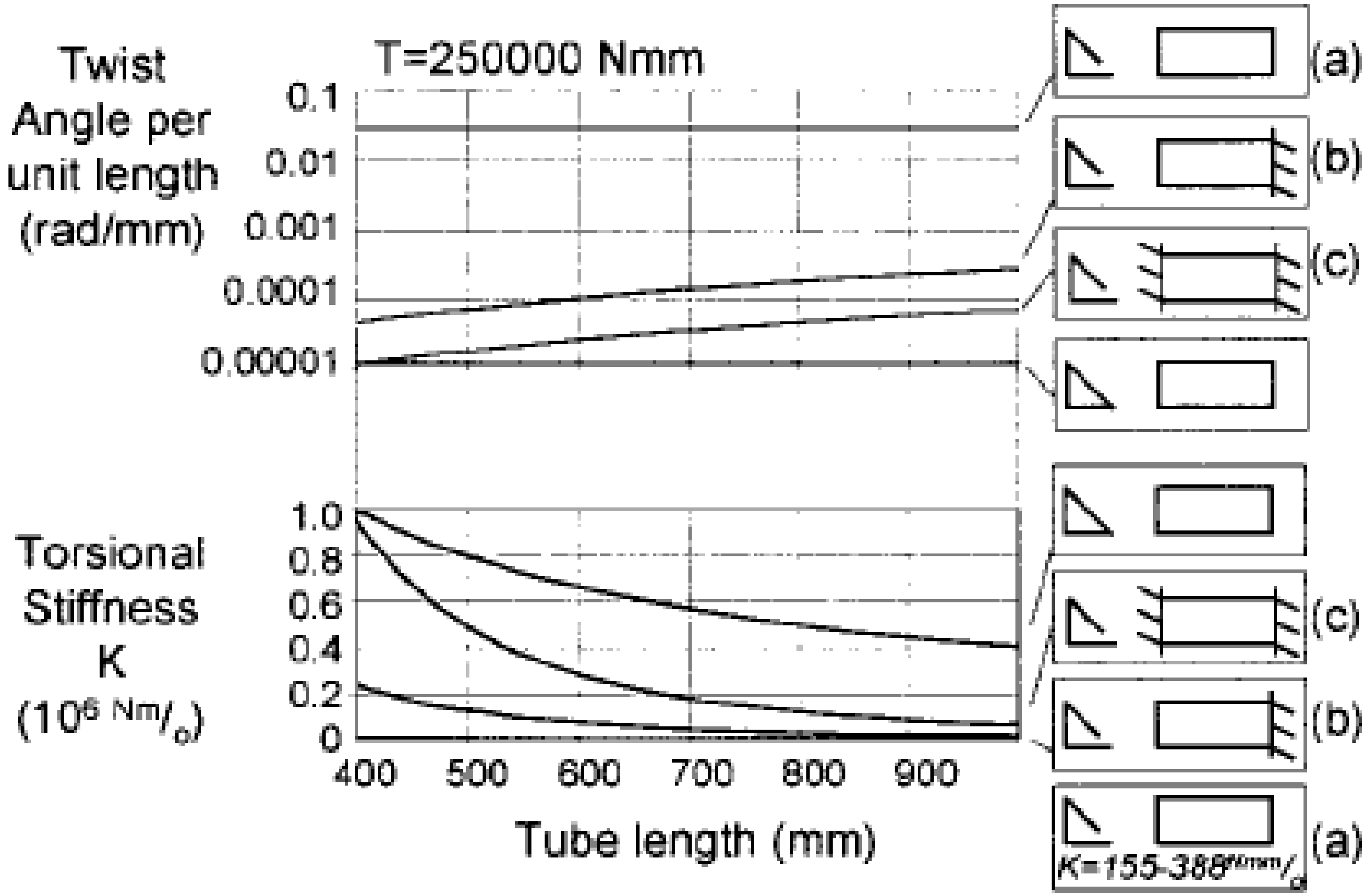
$$\tau = \frac{Tt}{J_{eff}} = \frac{25\times10^4\,Nmm(1mm)}{113.8mm^4} = 2197N/mm^2$$



$$k = \sqrt{\frac{JG}{C_w E}} = \sqrt{\frac{\left(113.8mm^4\right)\left(78\times10^3\,N/mm^2\right)}{\left(1.4\times10^9\,mm^6\right)\left(207\times10^3\,N/mm^2\right)}} = 1.75\times10^{-4}/mm$$

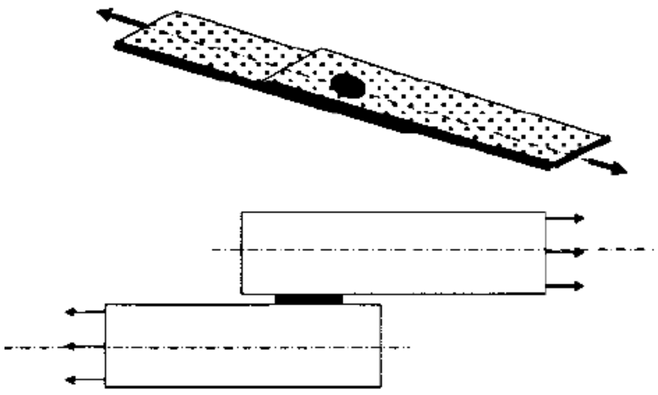
$$\theta = \frac{TL}{GJ}\left(1-\frac{\tanh kL/2}{kL/2}\right) = \frac{(25\times10^4\,Nmm)(500mm)}{\left(78\times10^3\,N/mm^2\right)\left(113.8mm^4\right)}\left[1-\frac{\tanh\left\{\left(1.75\times10^{-4}/mm\right)(500mm)/2\right\}}{\left(1.75\times10^{-4}/mm\right)(500mm)/2}\right] = 8.98\times10^{-3}\,rad\,(0.515°)$$

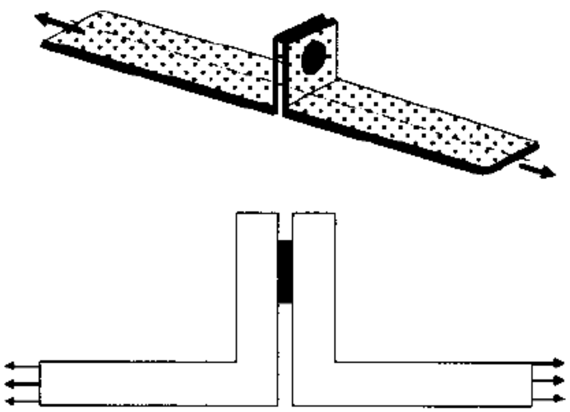
Effect of Beam Length on Angle of Rotation



Effect of Spot Welds on Structural Performance

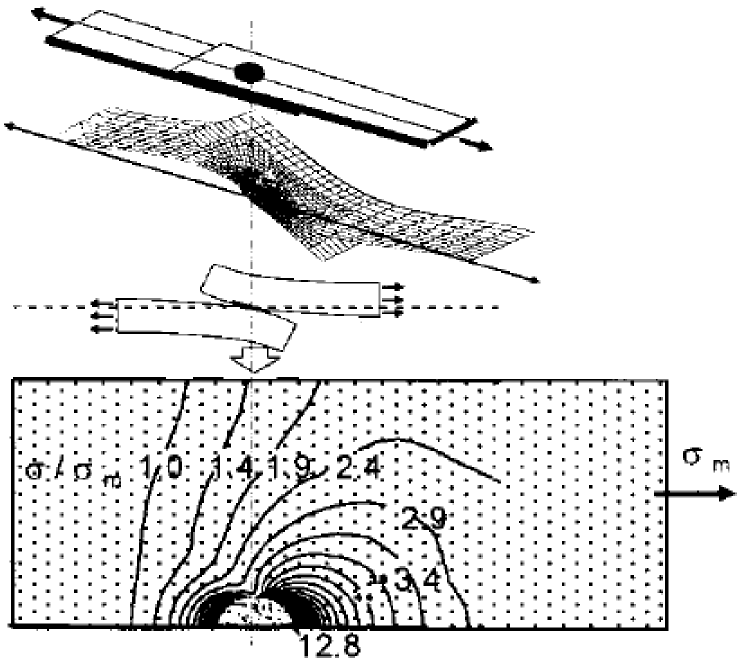
- Body sections
  - Fabrication of several formed element using spot welds
- Addition of shear flexibility in the section during torsion of fabricated sections
  - Tools to predict the degree of shear flexibility
  - Strategies to minimize the flexibility
- Shear vs. Peel loading

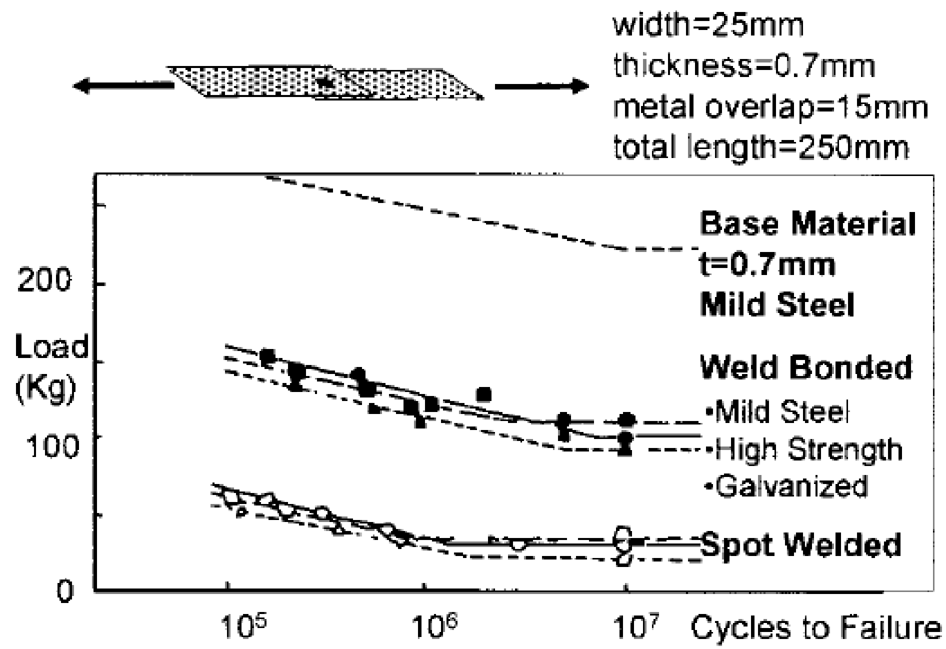




Shear Loading

- Create a moment at the weld
- Reduce fatigue limit by a factor of seven
  - Adhesive: more evenly distributed stress → fatigue performance

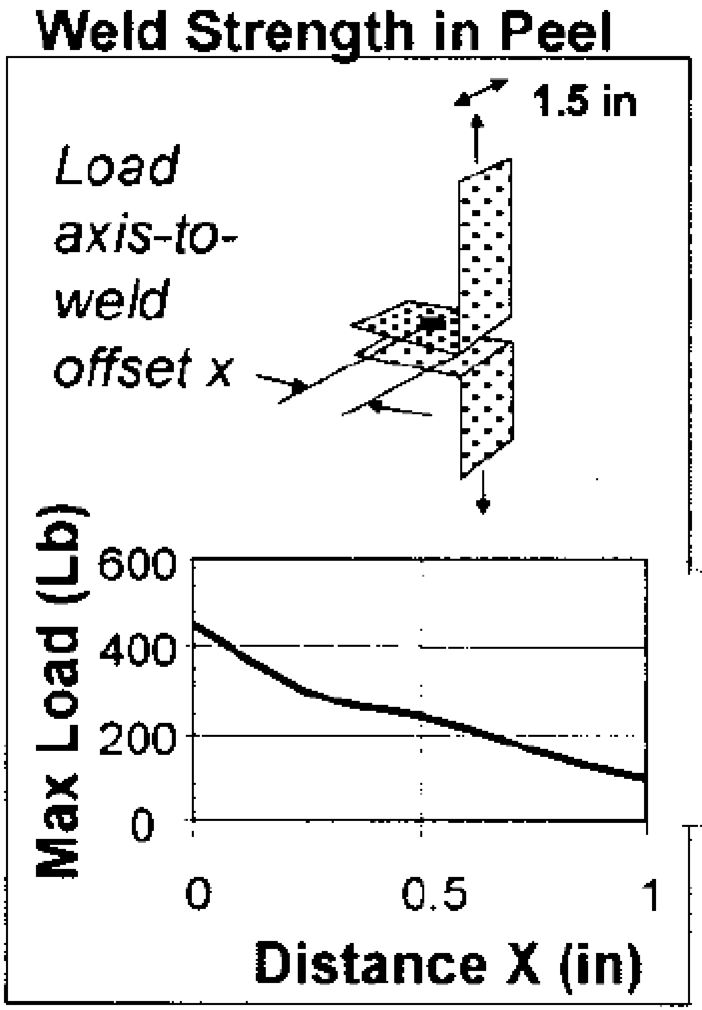


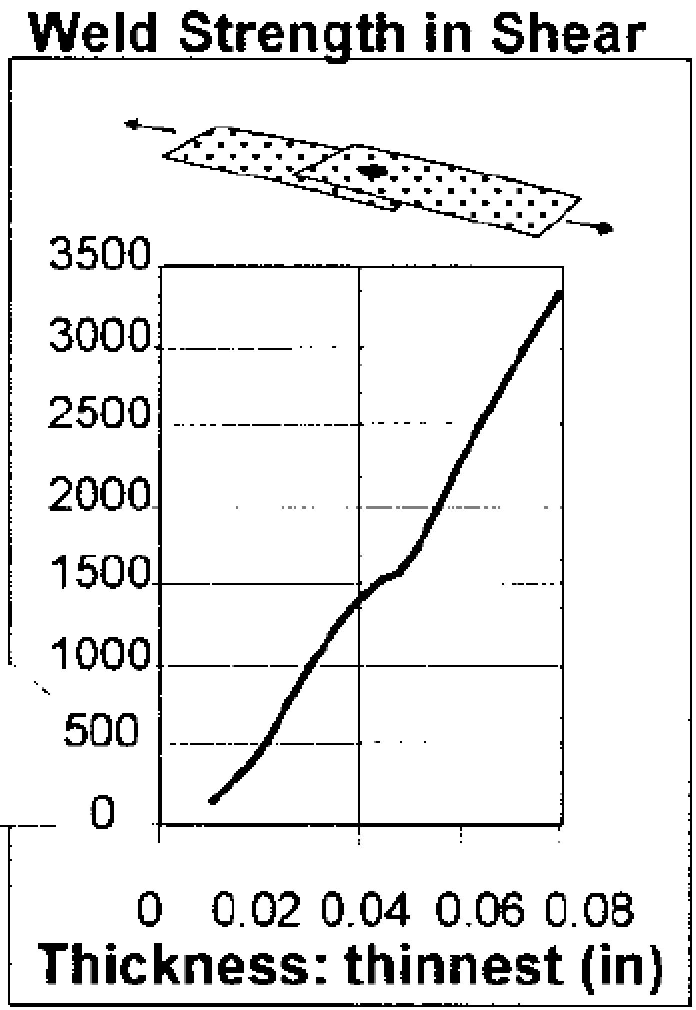


Peel Loading

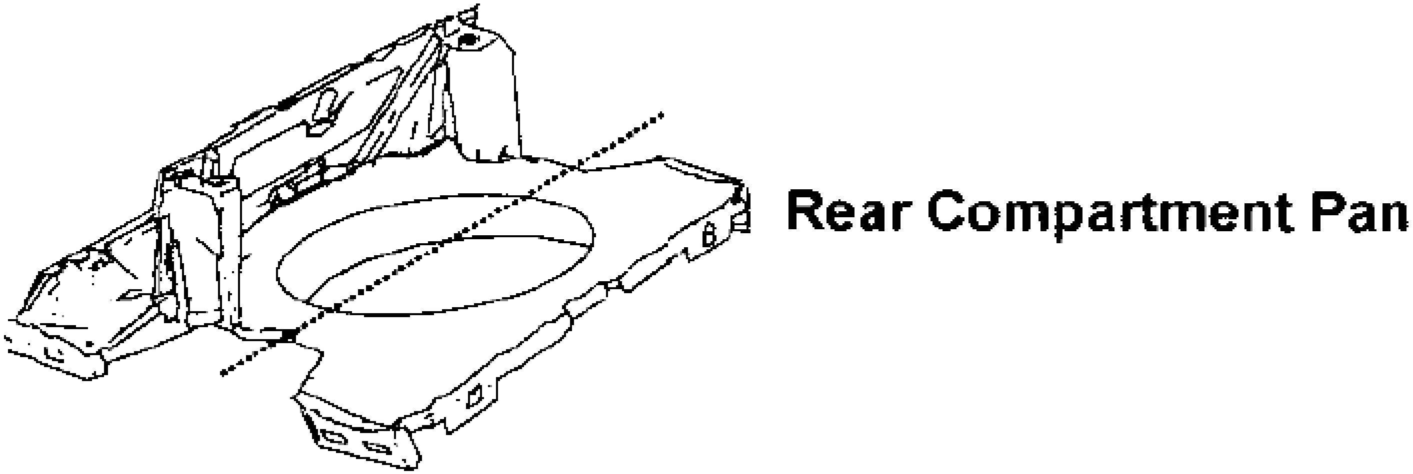
- Increase the detrimental offset
- Effect of increasing the loading offset beyond the sheet thickness
- Design practice
  - Assumption: tensile load within the plane of the thin wall material
  - Minimize the offset of this tensile load from the weld
  - Use part geometry to put welds into shear loading rather than peel loading

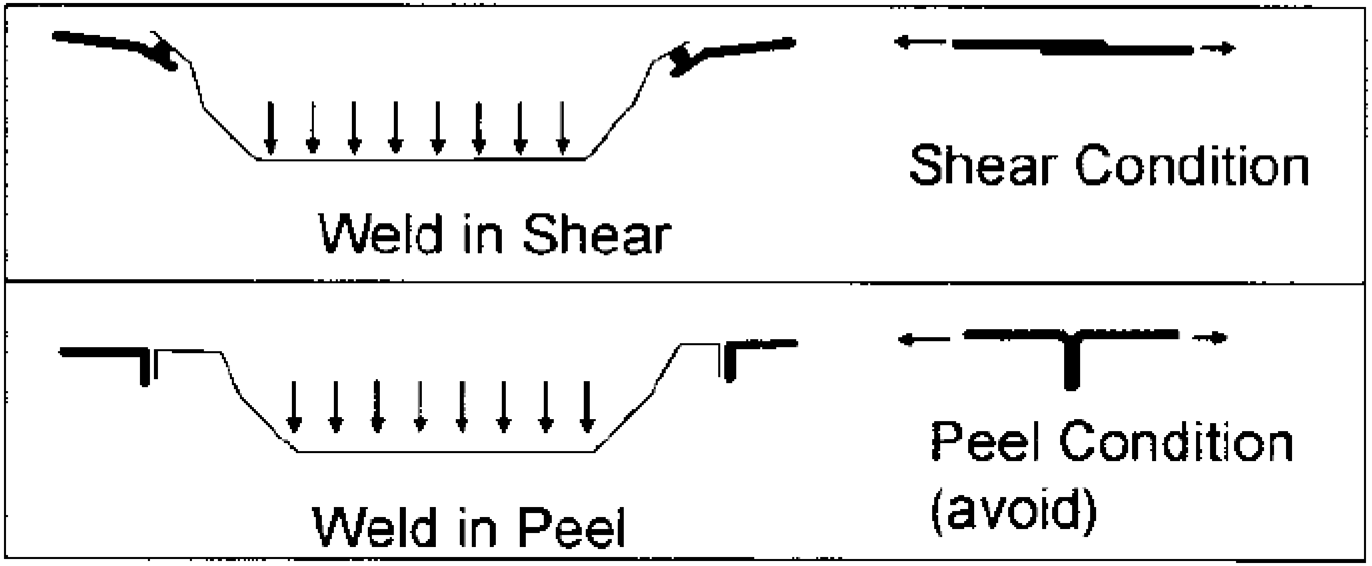
Offset Effect on Spot Welded Joint Strength





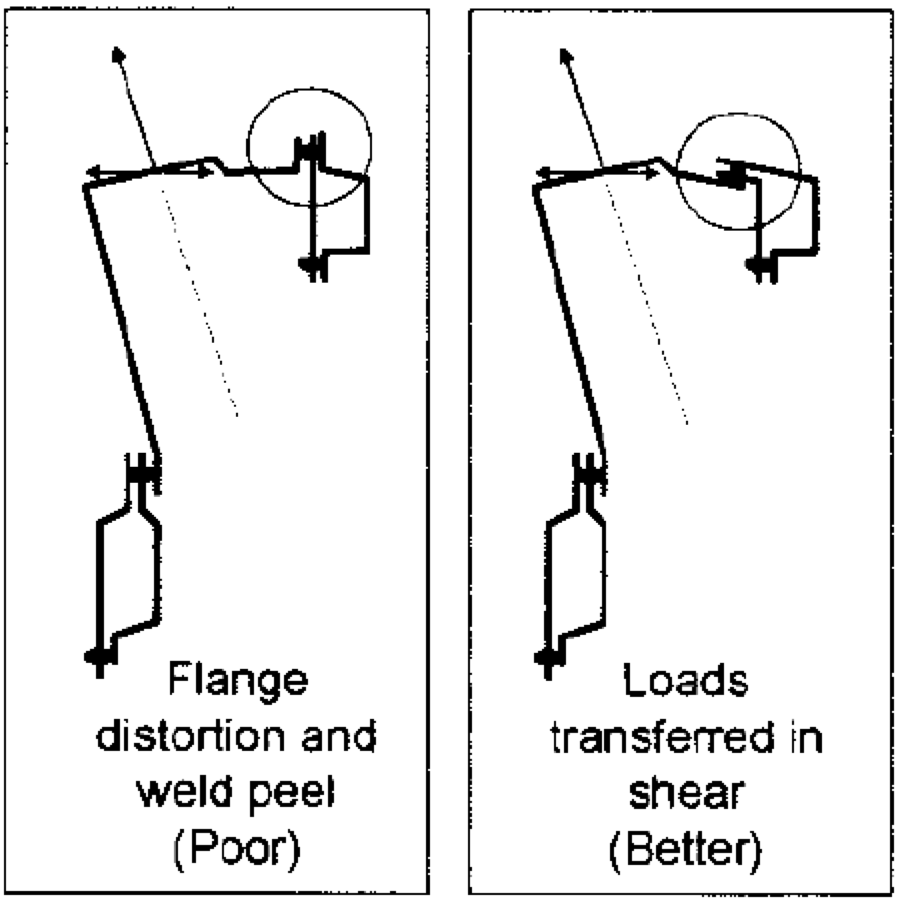
Examples of Joints in Shear (1)



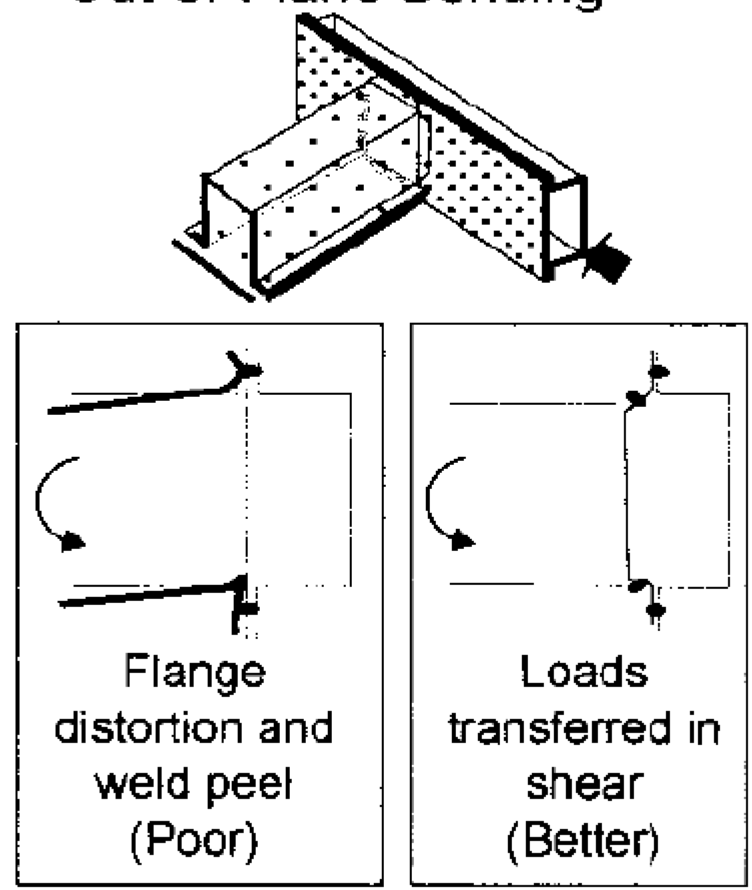


Examples of Joints in Shear (2)

Front Shock Tower Attachment

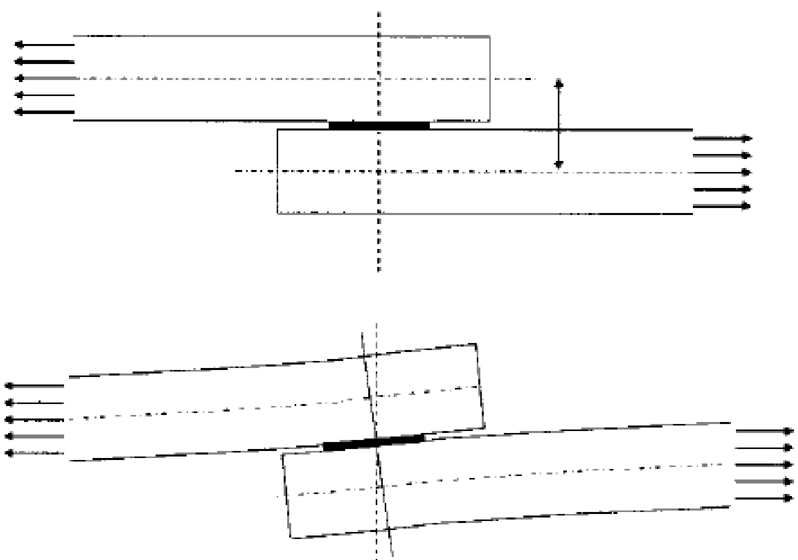


Seat Cross Member to Rocker Joint construction Out-of-Plane Bending

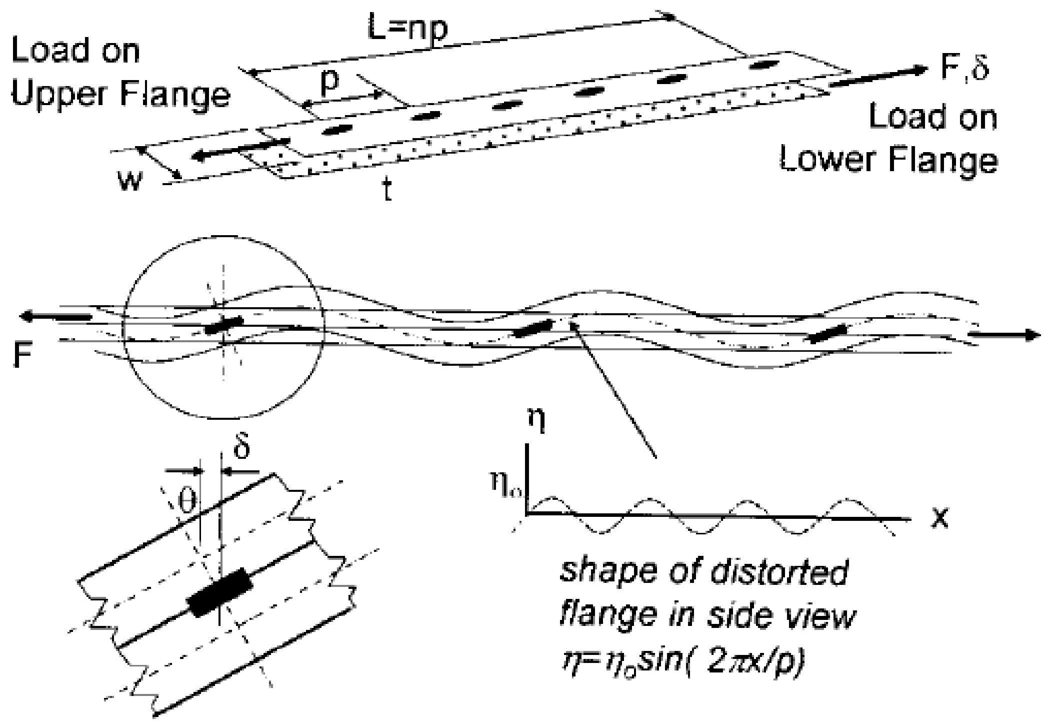


Longitudinal Stiffness of a Shear Loaded Weld Flange

- Local deformation → reduce the apparent stiffness of a section
- Distortion under a shear load: rotation with the center at the interface of the weld



Distortion at Spot Weld



Longitudinal Deflection

- Deflected shape of the flange η at each weld
- (work done by an external elastic shearing force through distance δ) = (bending strain energy in the distorted flange)
  - Deflection $\propto$ square of the weld pitch

$$\left.\begin{aligned} work &= \frac{1}{2}F\delta \\ energy &= \frac{1}{2}\int_V \sigma\varepsilon\, dV = \frac{1}{2}\int_V \frac{\sigma^2}{E}\, dV \\ &\xrightarrow[M=EIy'']{\sigma=\frac{My}{I}} \int_0^L \frac{1}{2}EI\left(\eta''\right)^2 dx \end{aligned}\right\} \rightarrow \delta = \frac{3p^2}{2E\pi^2 wt}q$$

Longitudinal Deflection

$$\eta = \eta_0 \sin\frac{2\pi x}{p} \rightarrow \frac{d\eta}{dx} = \eta_0 \frac{2\pi}{p}\cos\frac{2\pi x}{p}$$

$$\left.\begin{array}{l} \left.\dfrac{d\eta}{dx}\right|_{x=0,p,2p} = \eta_0 \dfrac{2\pi}{p} = \theta \\ \delta = \dfrac{t}{2}\theta \rightarrow \theta = \dfrac{2\delta}{t} \end{array}\right\} \rightarrow \eta_0 = \frac{p\delta}{\pi t}$$

$$\begin{cases} \dfrac{1}{2}F\delta = \dfrac{1}{2}(qL)\delta = \dfrac{1}{2}(qnp)\delta \\ \displaystyle\int_0^L \frac{1}{2}EI(\eta'')dx = \frac{1}{2}EI\int_0^{np}\left[-\eta_0\left(\frac{2\pi}{p}\right)^2 \sin\frac{2\pi x}{p}\right]^2 dx = \frac{1}{2}EI\eta_0{}^2\left(\frac{2\pi}{p}\right)^4 \int_0^L \sin^2\frac{2\pi x}{p}dx \\ \xrightarrow{\sin^2\theta = \frac{1-\cos 2\theta}{2}} \dfrac{1}{2}EI\eta_0{}^2\left(\dfrac{2\pi}{p}\right)^4 \dfrac{np}{2} \end{cases}$$

$$\rightarrow \frac{1}{2}(qnp)\delta = \frac{4EI\pi^2}{t^2}\frac{n\delta^2}{p} \rightarrow q = \frac{8EI\pi^2}{(pt)^2}\delta \xrightarrow{I=\frac{wt^3}{12}} q = \frac{2E\pi^2 wt}{3p^2}\delta \rightarrow \delta = \frac{3p^2}{2E\pi^2 wt}q$$

Tube Closed by a Single Spot Weld Flange

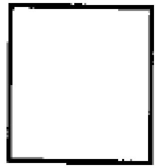
- Reduced stiffness in a twisted section by torque T
- (external energy) = (shear strain energy in tube wall) + (strain energy in distorted flange)
  - Estimate of the reduced stiffness in a twisted section when a single spot welded flange is present

$$(\text{stiffness of closed tube w/o weld flange}) = \frac{GJ}{L} = \frac{G\left(\frac{4A^2t}{S}\right)}{L} = \frac{4GA^2t}{LS}$$

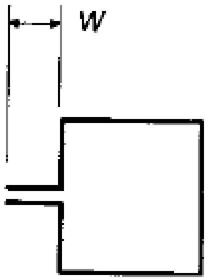
$$\begin{cases} work = \frac{1}{2}T\theta \\ energy = \frac{SL}{2Gt}q^2 + \frac{1}{2}F\delta \end{cases}$$

$$\rightarrow \frac{T}{\theta} = \frac{(\text{stiffness of closed tube w/o weld flange})}{\left[1 + \frac{3}{4\pi^2(1+\nu)}\frac{p^2}{wS}\right]}$$

Ideal closed tube



closed tube fabricated with a single weld flange



S: Perimeter without flange considered

$$\left(\text{stiffness of closed tube w/o weld flange}\right)=\frac{GJ}{L}=\frac{G\left(\frac{4A^2t}{S}\right)}{L}=\frac{4GA^2t}{LS}$$

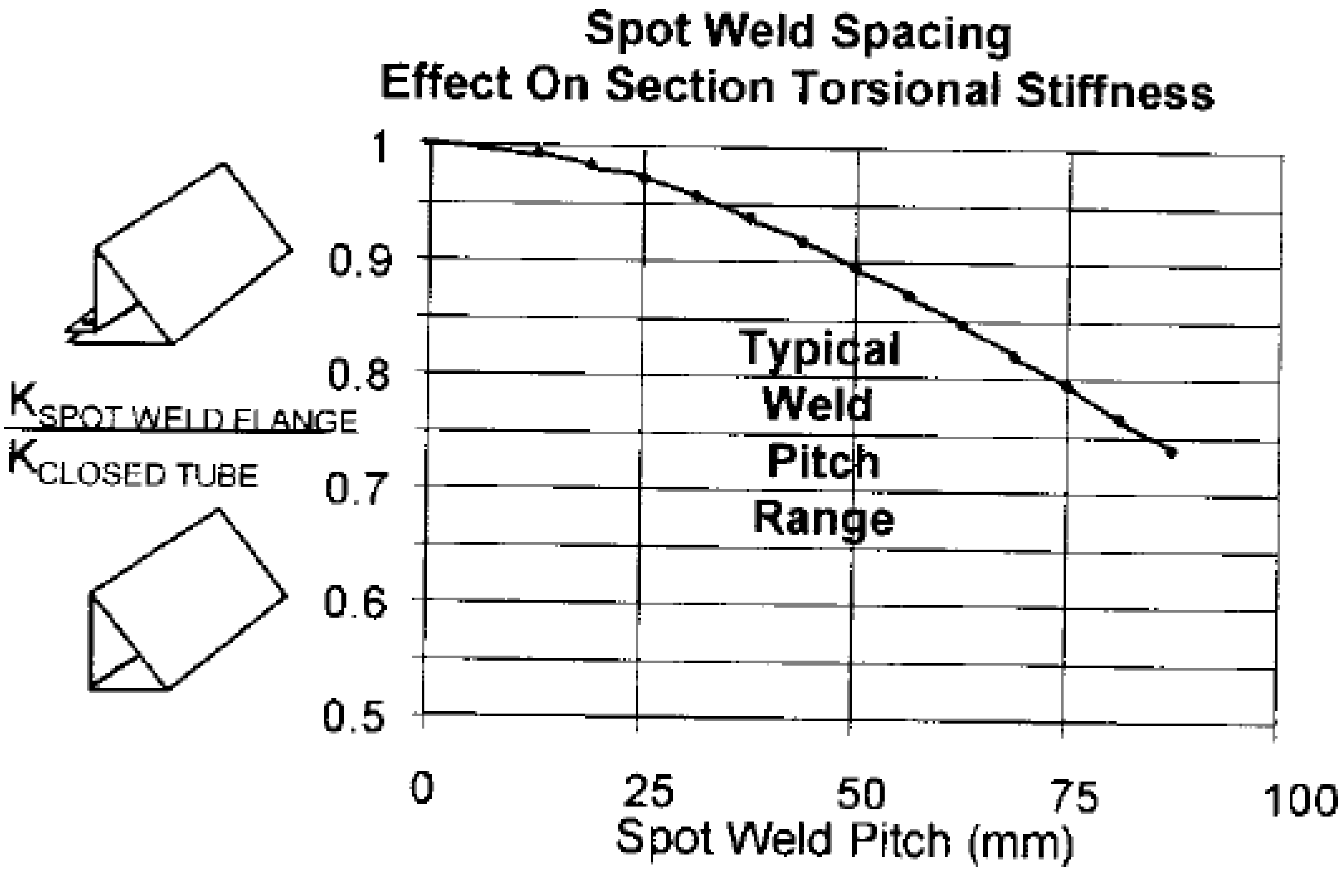
$$\begin{cases} work=\frac{1}{2}T\theta \\ energy=\int\frac{\tau^2}{2G}dV+\frac{1}{2}F\delta=\frac{(q/t)^2}{2G}tSL+\frac{1}{2}(qL)\delta=\frac{SL}{2Gt}q^2+\frac{1}{2}(qL)\frac{3p^2}{2E\pi^2wt}q \\ =\frac{SL}{\frac{E}{1+\nu}t}q^2+\frac{3p^2q^2L}{4E\pi^2wt}=\frac{(1+\nu)SLq^2}{Et}\left[1+\frac{3p^2}{4E\pi^2(1+\nu)wS}\right] \end{cases}$$

$$\frac{1}{2}T\theta=\frac{(1+\nu)SLq^2}{Et}\left[1+\frac{3p^2}{4E\pi^2(1+\nu)wS}\right]\rightarrow\theta=\frac{1}{T}\frac{SLq^2}{\frac{E}{2(1+\nu)}t}\left[1+\frac{3p^2}{4E\pi^2(1+\nu)wS}\right]$$

$$\frac{T}{\theta}=T^2\frac{Gt}{SLq^2}\frac{1}{\left[1+\frac{3p^2}{4E\pi^2(1+\nu)wS}\right]}\xrightarrow{T=2qA}\frac{T}{\theta}=\underbrace{\frac{4GA^2t}{SL}}_{\substack{\text{stiffness of}\\\text{closed tube}\\\text{w/o weld flange}}}\frac{1}{\left[1+\frac{3p^2}{4E\pi^2(1+\nu)wS}\right]}$$

$$\rightarrow\frac{T}{\theta}=\frac{\left(\text{stiffness of closed tube w/o weld flange}\right)}{\left[1+\frac{3}{4\pi^2(1+\nu)}\frac{p^2}{wS}\right]}$$

Spot Weld Spacing Effect



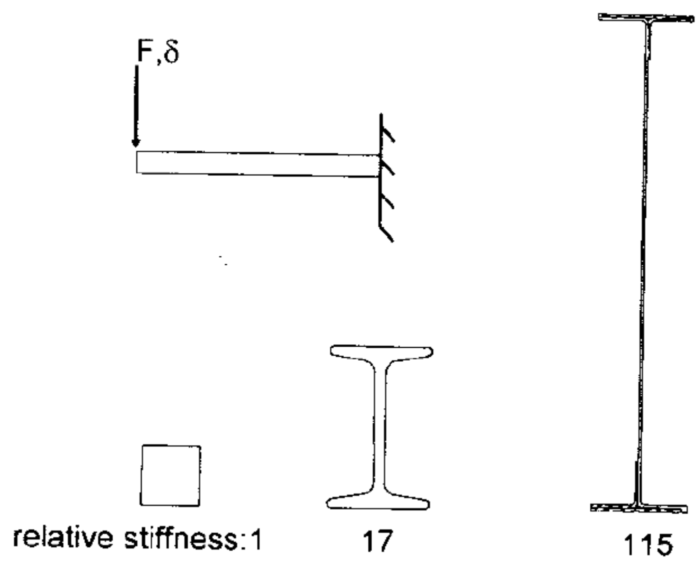
w (weld flange) = 8 mm, t = 1 mm

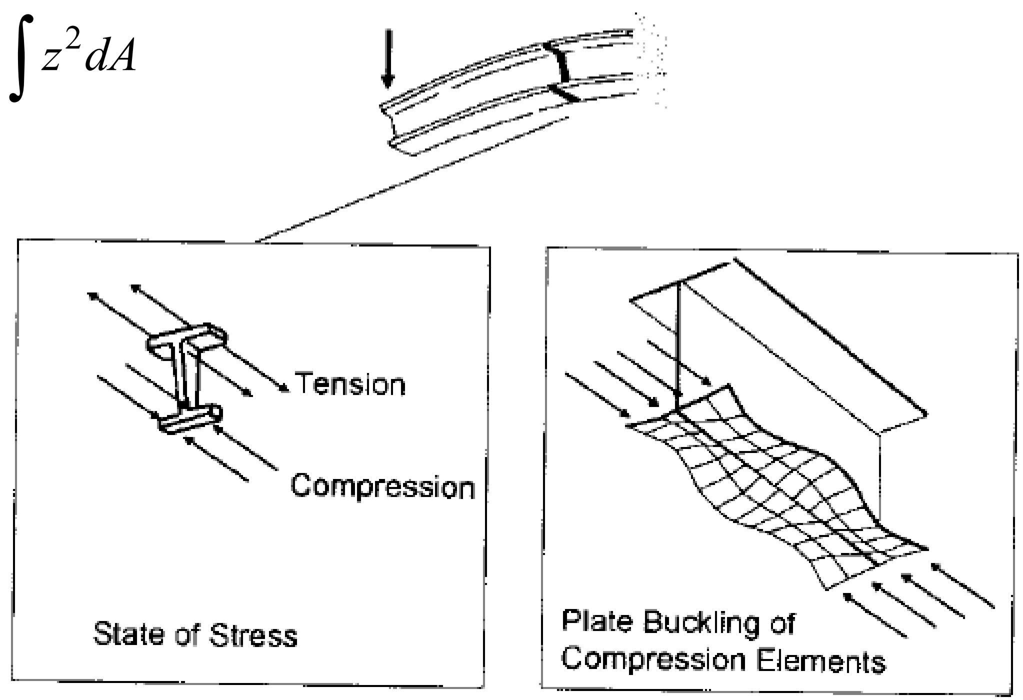
→ 40 mm < p < 60 mm

3.4 Thin Wall Beam Section Design

- Why are automotive sections so often thin walled?
  - Steel cantilever beam with a tip load
  - Cross section area is fixed: maximize strength and stiffness
  - Existence of new failure mode

$$k = \frac{3EI}{L^3} \quad \text{and} \quad F_{\max} = \frac{I\sigma_{design}}{Lc} \quad \text{where} \quad I = \int z^2 dA$$

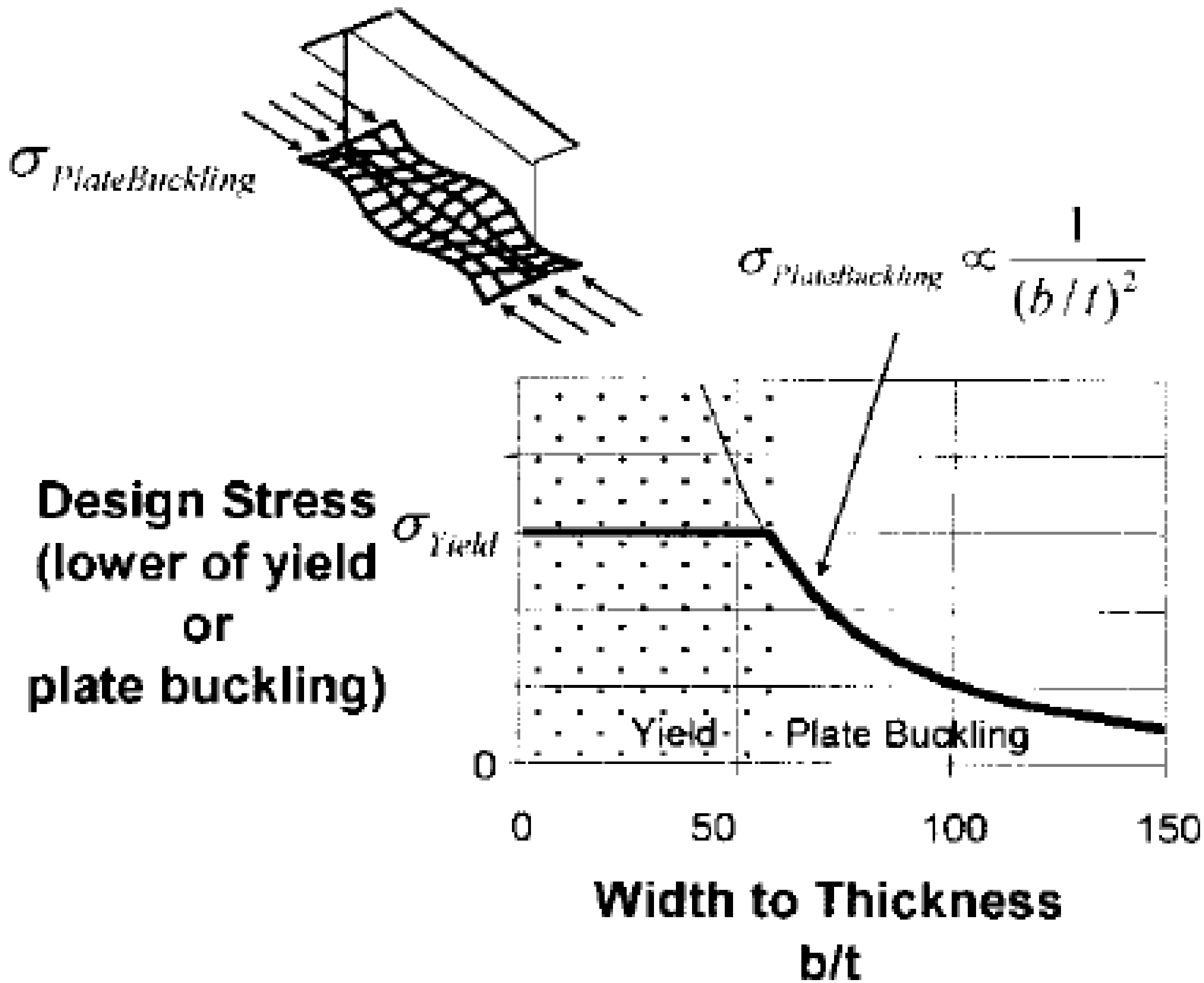




Elastic Plate Buckling

- General behavior of a compressively loaded plate
  - Bifurcate into the buckled shape if the plate is sufficiently thin
  - Compressive stress: plate width to thickness ratio
- Section design: trade-off
  - Thick walled section: higher strength but lower stiffness performance
  - Thin walled section: higher stiffness but lower strength performance due to plate buckling
  - Selection of the best section proportion: relationship of strength requirement to stiffness requirement

Plate Buckling Stress



Example: Rocker Sizing in Convertible

- Determine b/t to minimize rocker mass while meeting requirements

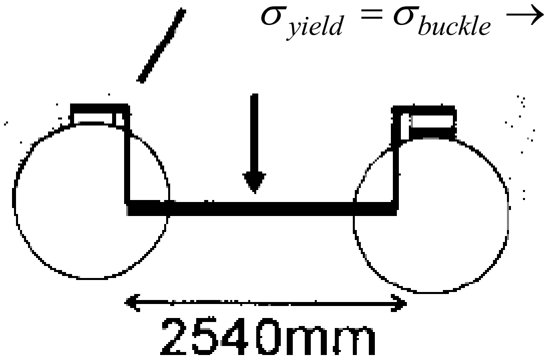
$$k = \frac{48EI}{L^3} \quad \text{and} \quad F_{\max} = \frac{4I\sigma_{design}}{Lc}$$

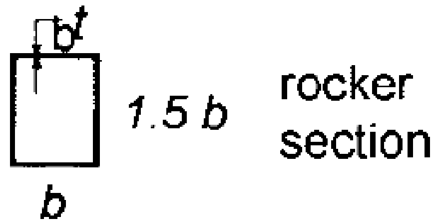
$$\sigma_{design} = \begin{cases} \sigma_{yield} = 207\ N/mm \\ \sigma_{buckle} = \dfrac{748355}{(b/t)^2}\ N/mm \end{cases}$$

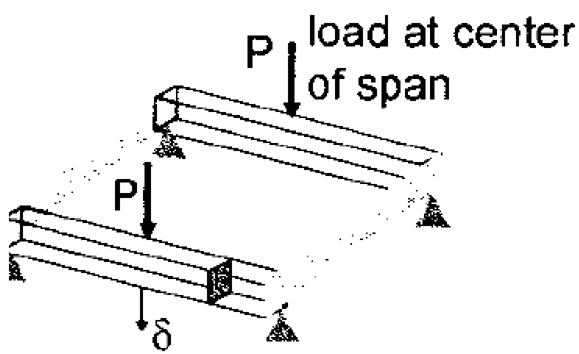
$$\sigma_{yield} = \sigma_{buckle} \rightarrow (b/t) \approx 60$$

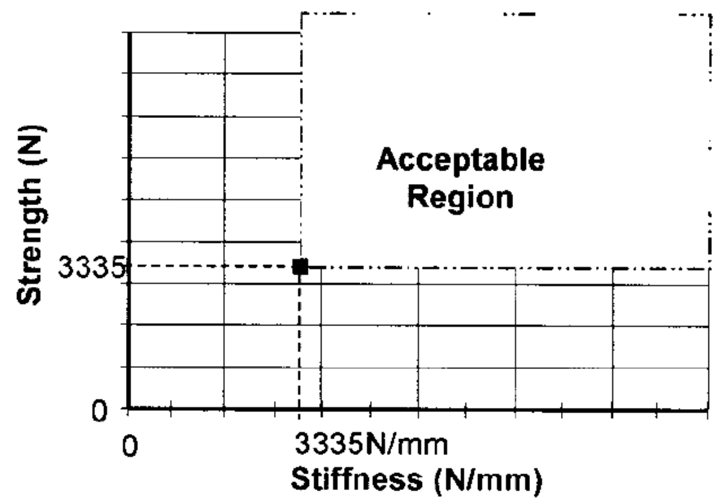
$$\left. \begin{aligned} t &= \frac{b}{(b/t)} \\ A &= 2(bt + 1.5bt) = 5bt \end{aligned} \right\} \rightarrow \begin{cases} t = \sqrt{\dfrac{A}{5(b/t)}} \\ b = \sqrt{\dfrac{A}{5}\left(\dfrac{b}{t}\right)} \end{cases}$$

$$\text{fix } A \rightarrow \text{change } (b/t) \rightarrow (b,t) \rightarrow I \rightarrow k, F_{\max}$$

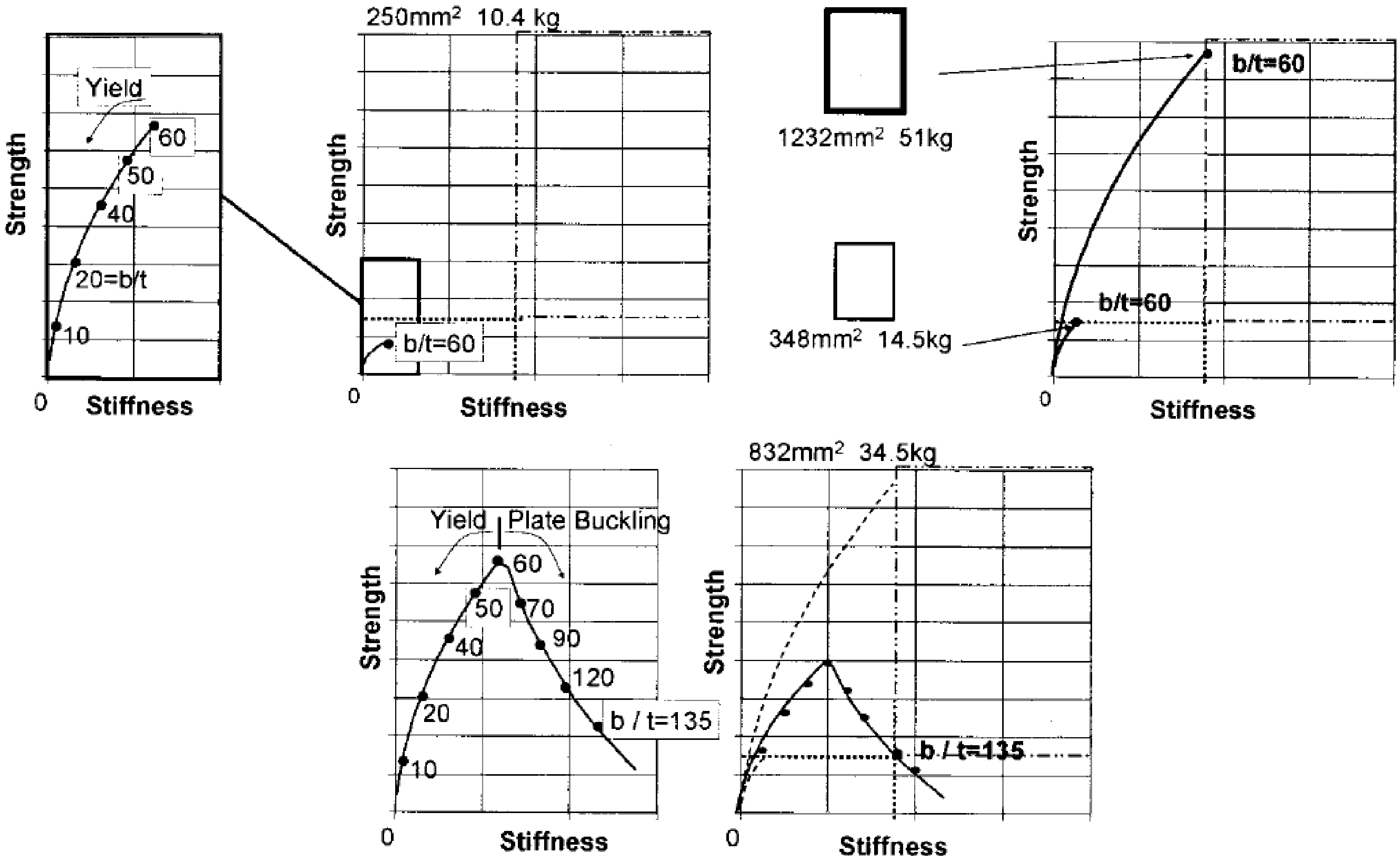




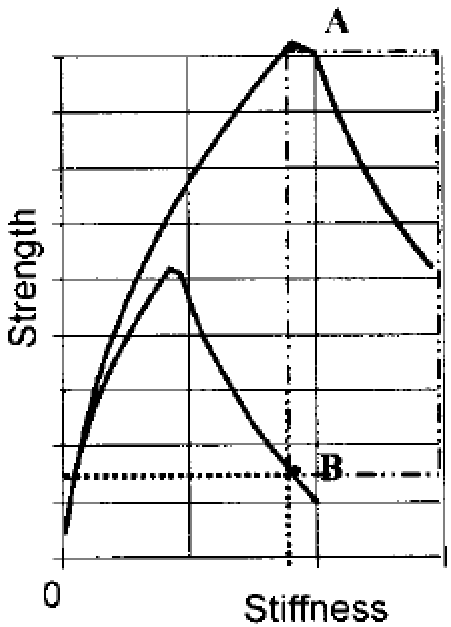




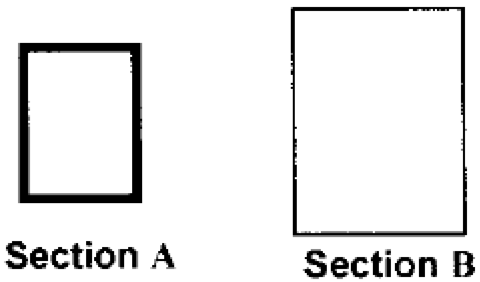
Thin Walled Section Performance



Mass Savings of Thin Walled Section

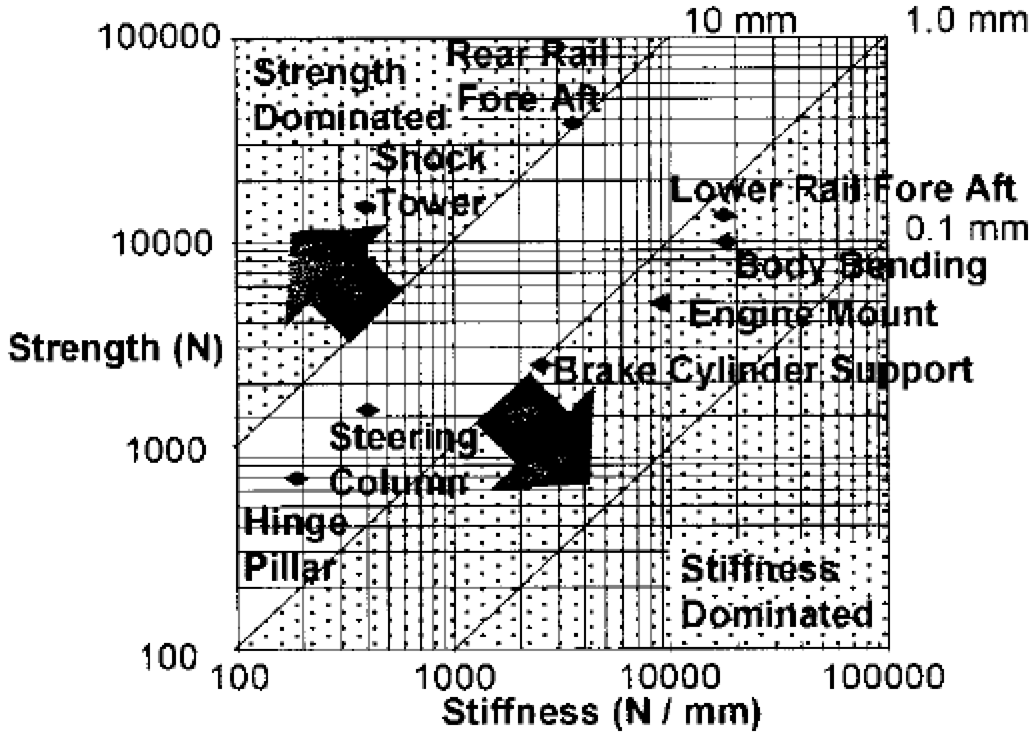


	b/t	failure mode	mass
A	60	yield	51.0kg
B	35	plate buckling	34.5kg



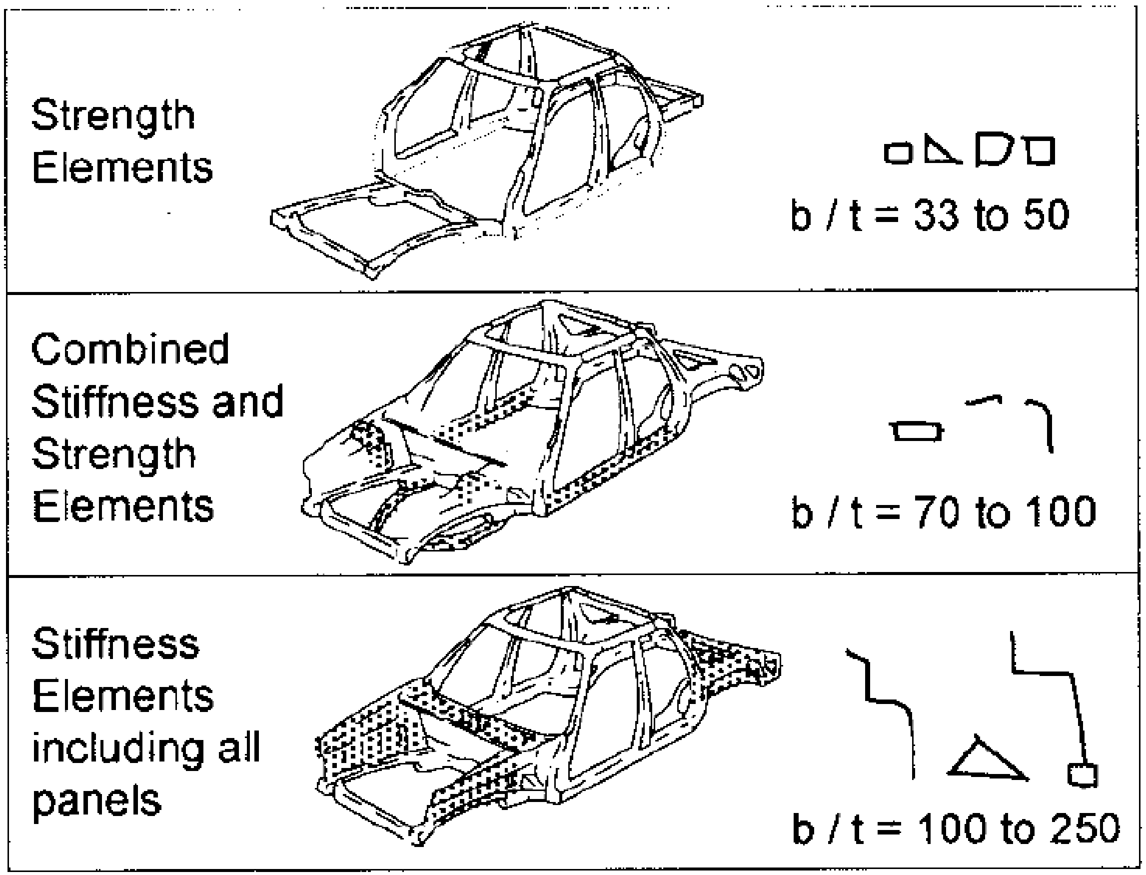
Dominant Structural Requirement

- Many structural elements in automobile body design are dominated by stiffness requirements
  - Thin wall sections: mass effective
  - Failure mode of buckling



Section Proportion

- Mass effective means to design for stiffness performance
  - Thin wall sections



Reacting loads in a crash: roof crush, side impact, maintaining cabin integrity

Major subsystem attachment: suspension, powertrain

Overall stiffness of the body

3.5 Buckling of Thin Walled Members

- Significant difference between automotive sections and others: failure mode by plate buckling
  - Plate buckling stress in section elements
  - Strength of a buckled section
- Plate buckling
- Identifying plate boundary conditions in practice
- Post buckling behavior of plates
- Effective width
- Thin walled section failure criteria
- Techniques to inhibit buckling

Plate Buckling (1)

- Static equilibrium of the element under loads
- Compatibility of deformations within the plate
- Material stress-strain relationship

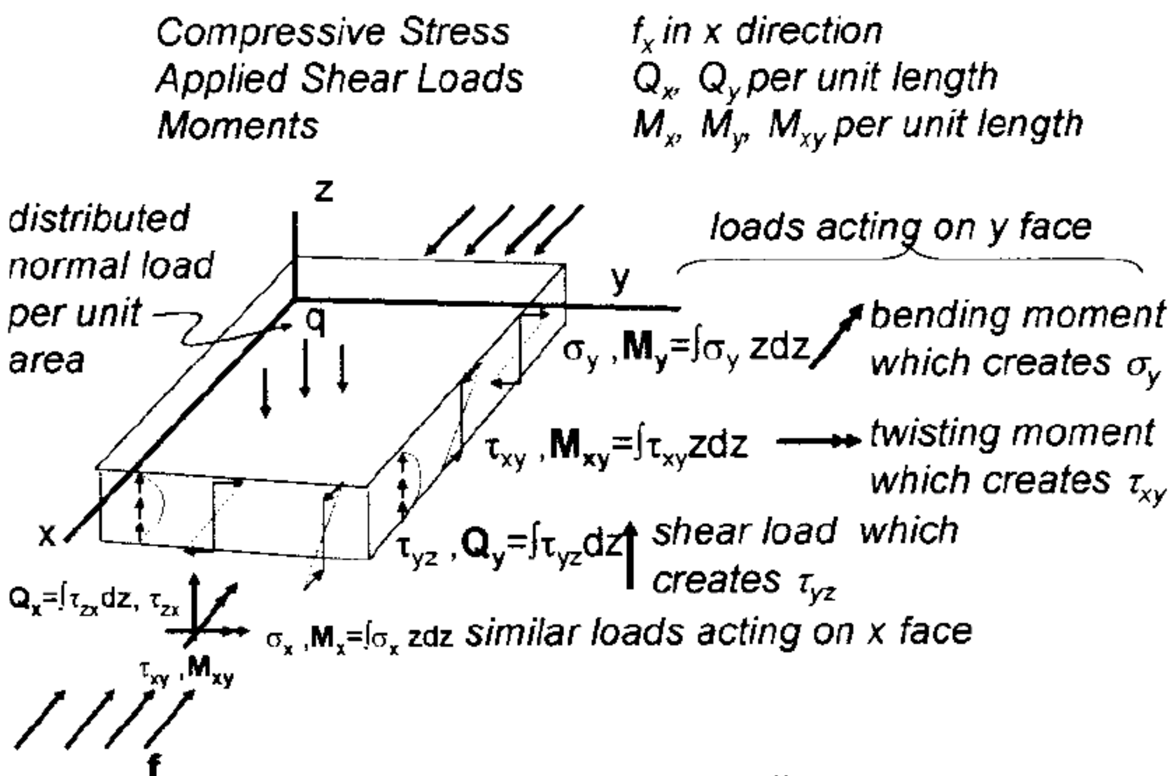


Plate Element Loading

bending moments

$M_x, M_y \rightarrow \sigma_x, \sigma_y$

twisting moment

$M_{xy} \rightarrow \tau_{xy}$

shear loads

$Q_x, Q_y \rightarrow \tau_{zx}, \tau_{yz}$

normal load, compressive stress

q, f_x

plate bending stiffness

$$D = \frac{Et^3}{12\left(1-\nu^2\right)}$$

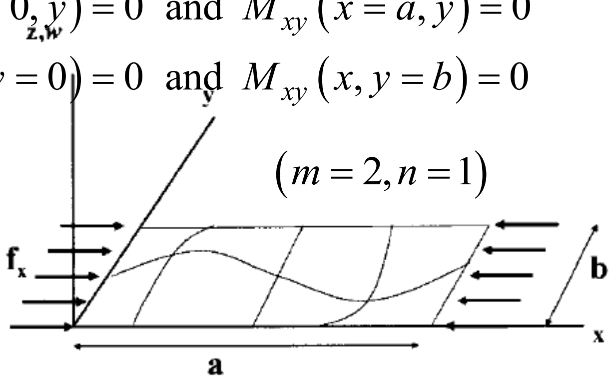
Plate Buckling (2)

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}+\frac{f_x t}{D}\frac{\partial^2 w}{\partial x^2}+\frac{q}{D}=0$$

$$\begin{cases} M_x = -D\left(\dfrac{\partial^2 w}{\partial x^2}+\nu\dfrac{\partial^2 w}{\partial y^2}\right) \\ M_y = -D\left(\nu\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}\right) \\ M_{xy} = -D(1-\nu)\dfrac{\partial^2 w}{\partial x \partial y} \end{cases}$$

simply supported plate

$$\begin{cases} M_x(x=0,y)=0 \text{ and } M_x(x=a,y)=0 \\ M_y(x,y=0)=0 \text{ and } M_y(x,y=b)=0 \\ M_{xy}(x=0,y)=0 \text{ and } M_{xy}(x=a,y)=0 \\ M_{xy}(x,y=0)=0 \text{ and } M_{xy}(x,y=b)=0 \end{cases}$$



$(m=2, n=1)$

reasonable guess at the deflected shape

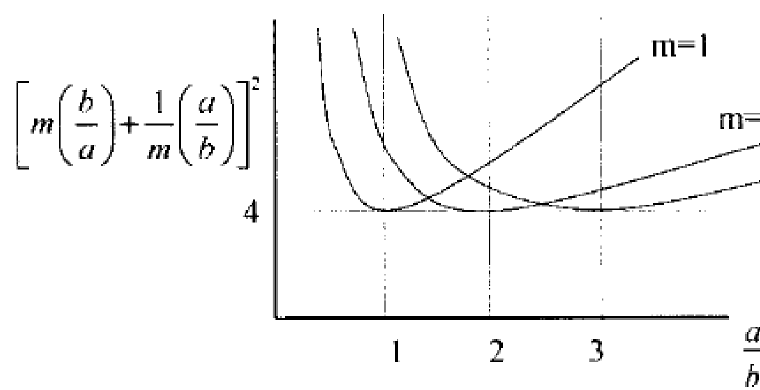
$$w(x,y)=A_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \quad \text{where } m,n=1,2,\ldots$$

$$\left[\pi^4\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^2-\frac{f_x t}{D}\frac{m^2\pi^2}{a^2}\right]A_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)=0$$

$$\rightarrow f_x=\frac{D\pi^2}{tb^2}\left[m\left(\frac{b}{a}\right)+\frac{n^2}{m}\left(\frac{a}{b}\right)\right]^2$$

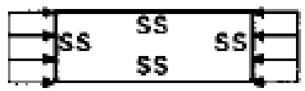
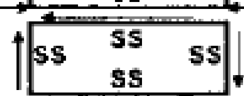
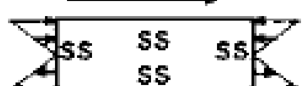
Note that lowest Buckling load occurs when n=1 or:

$$\sigma_{cr}=\frac{D\pi^2}{tb^2}\left[m\left(\frac{b}{a}\right)+\frac{1}{m}\left(\frac{a}{b}\right)\right]^2$$

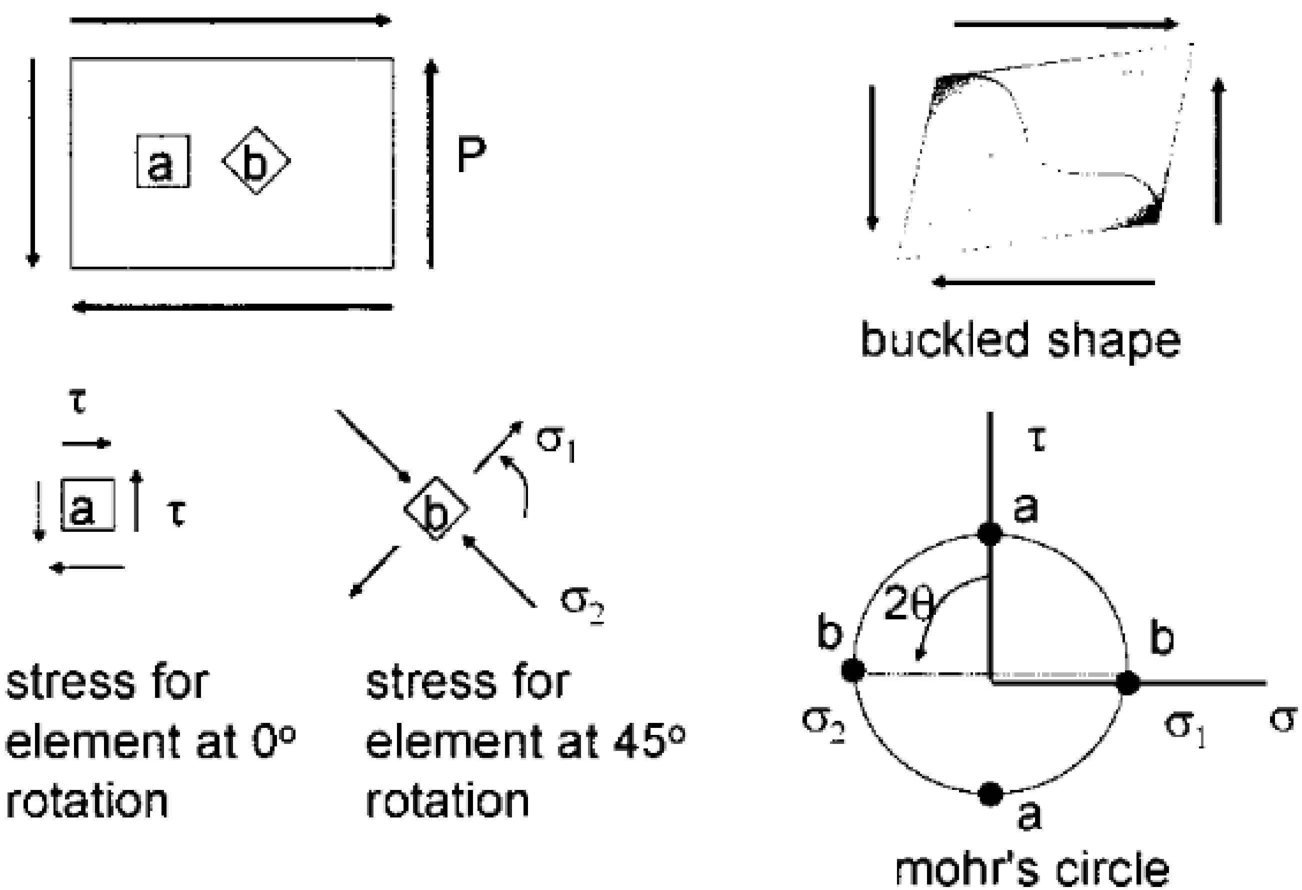


Buckling Constant for Various B.C.

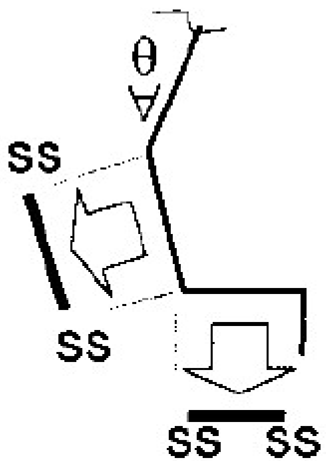
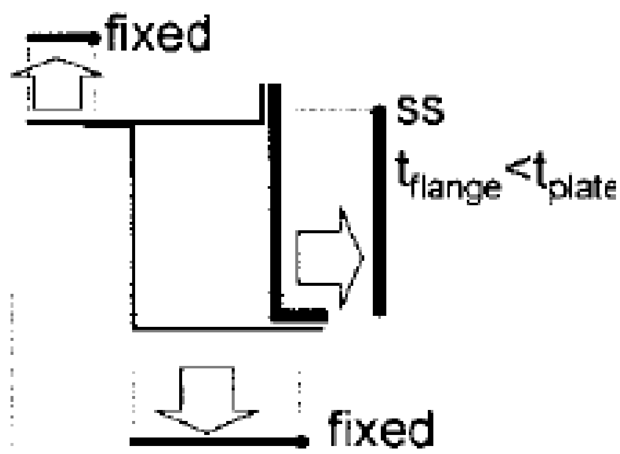
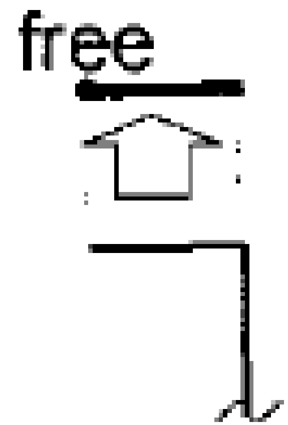
$$\sigma_{cr} = \frac{D\pi^2}{tb^2}k = \frac{E\pi^2}{12\left(1-\nu^2\right)\left(b/t\right)^2}k$$

Case	Boundary Condition	Loading	k
(a)		Compression	4.0
(b)		Compression	6.97
(c)		Compression	0.425
(d)		Compression	1.277
(e)		Compression	5.42
(f)		Shear	5.34
(g)		Shear	8.98
(h)		Bending	23.9
(i)		Bending	41.8

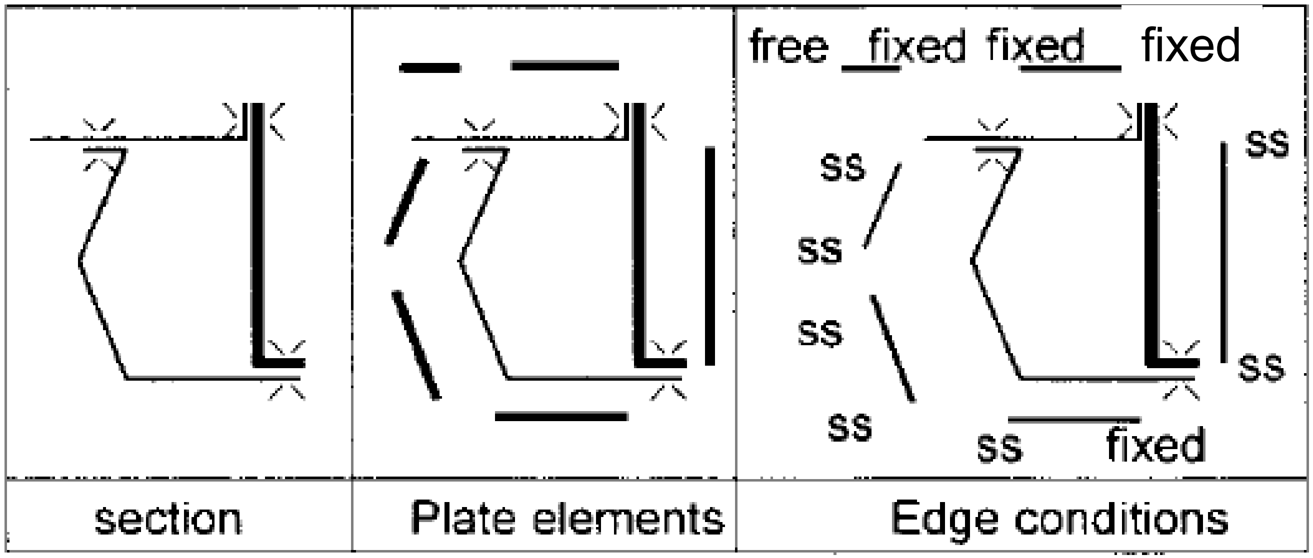
Compressive Stress in a Shear Panel



Identifying Plate Boundary Conditions

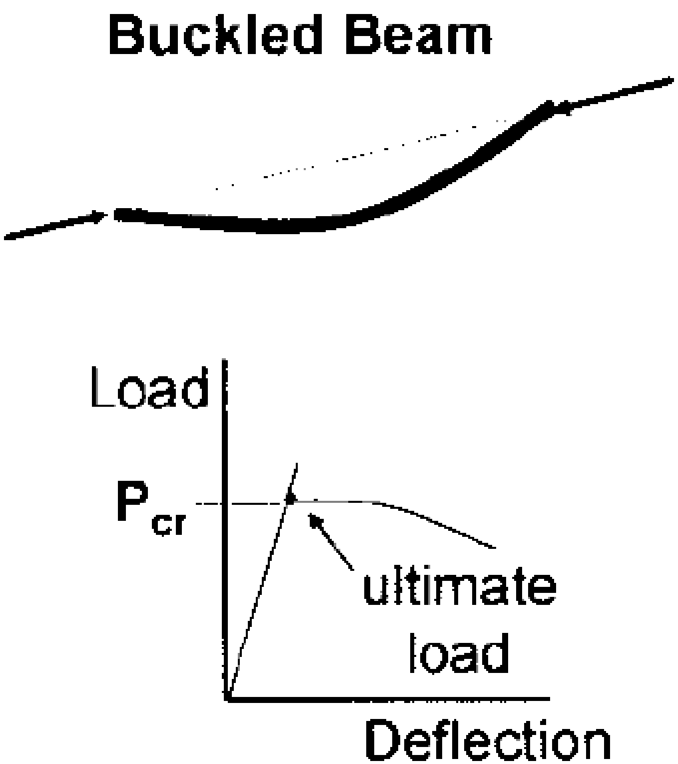
	Simply Supported	Fixed	Free
Deflection	N	N	Y
Rotation	Y	N	Y
			
	Each bent corner with angle > 40°	Support by a flat flange where $t_{flange} \geq t_{plate}$	Unconnected edge
Plate size	Corner to corner	From center line of weld	From edge

Example of Plate Edge Conditions

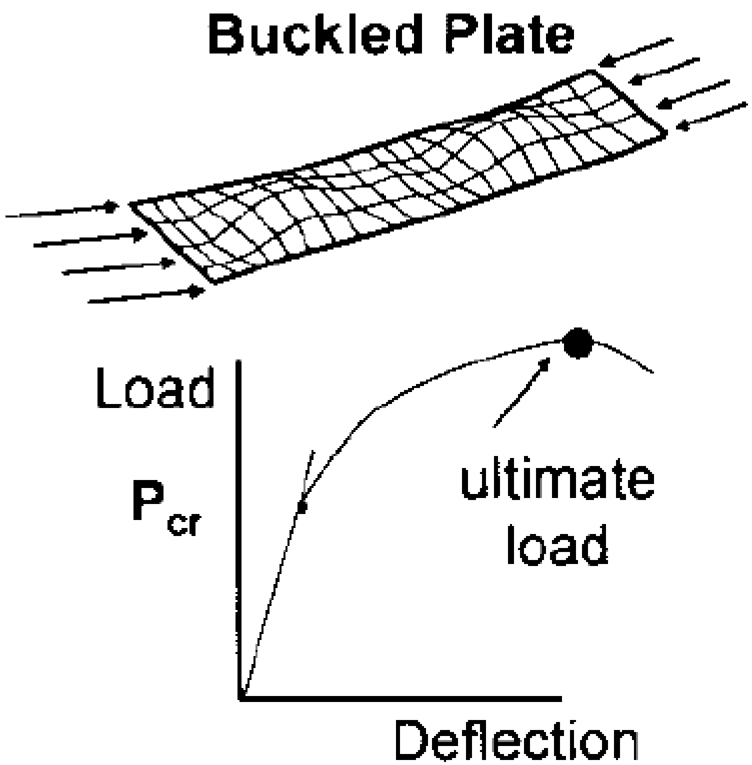


Post Buckling Behavior

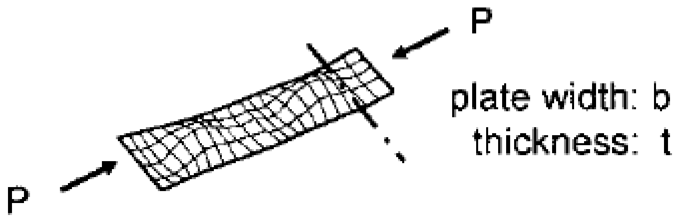
- Beam
  - Once buckled, a beam loses the ability to carry increased load



- Plate
  - Even after buckling, a plate can carry increased load



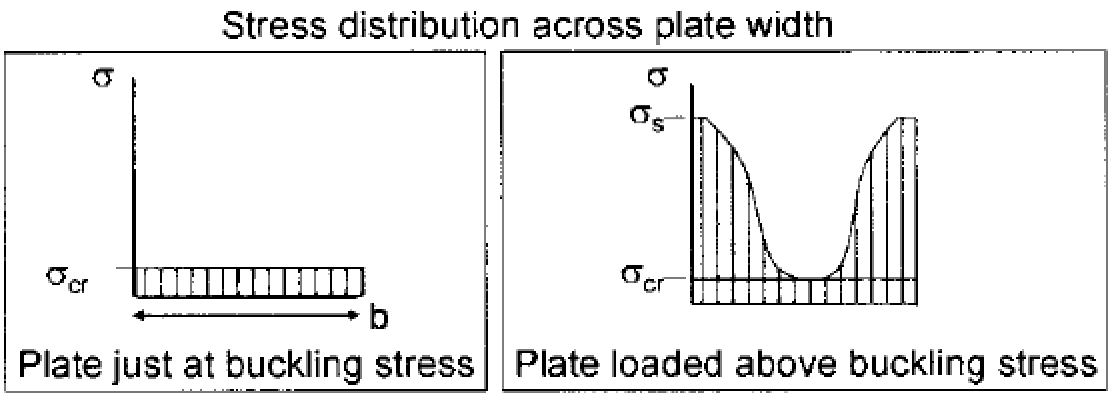
Effective Width (1)

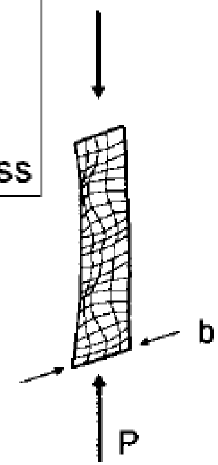


σ_{cr} : critical plate buckling stress

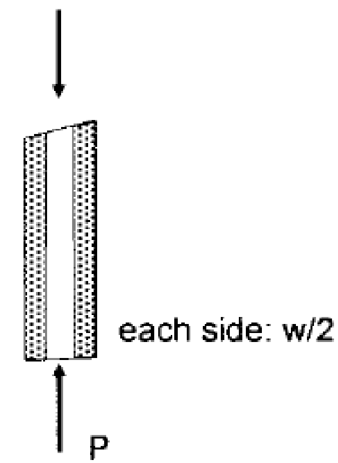
σ_S : maximum stress in the plate

$(b, \sigma) \leftrightarrow (w, \sigma_S)$





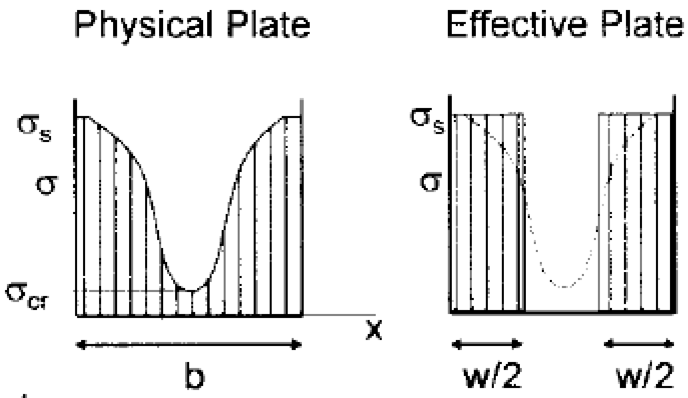
Physical Buckled plate with maximum stress σ_s thickness t and width b



Effective Unbuckled plate with uniform stress: σ_s thickness t and width w

Effective Width (2)

– Width of an imaginary effective plate which has a uniform stress of σ_s across it



Compressive load reacted by plate: $P = \int_0^b \sigma \; t \; dx$ $\qquad P = \sigma_s w t$

$$\begin{cases} P_{effective} = \sigma_s \left(wt \right) \\ P_{real} = \int_0^b \sigma \left(tdx \right) \\ \quad = \int_0^b \left[\left(\dfrac{\sigma_s + \sigma_{cr}}{2} \right) + \left(\dfrac{\sigma_s - \sigma_{cr}}{2} \right) \cos \dfrac{2\pi x}{b} \right] \left(tdx \right) \\ \quad = tb \left(\dfrac{\sigma_s + \sigma_{cr}}{2} \right) \end{cases}$$

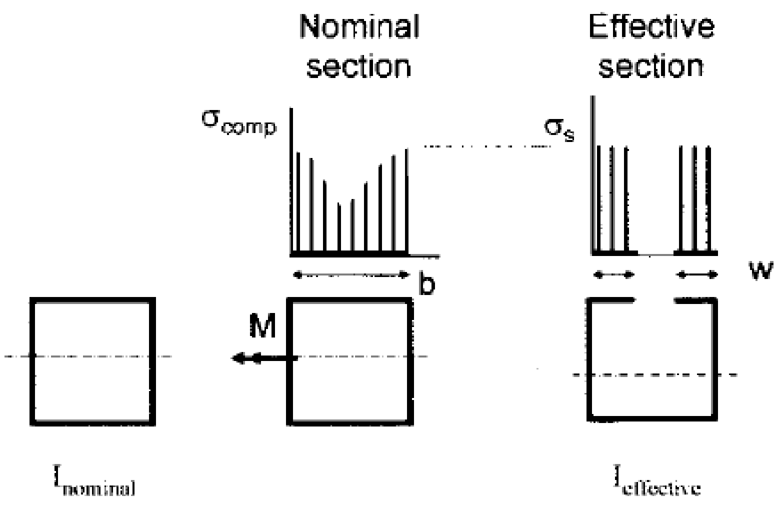
$$\rightarrow P_{effective} = P_{real} \rightarrow w = \frac{1}{2} \left(1 + \frac{\sigma_{cr}}{\sigma_s} \right) b$$

alternative empirical relationships for effective width

$$w = \begin{cases} 0.894 b \sqrt{\dfrac{\sigma_{cr}}{\sigma_s}} \\ b \sqrt{\dfrac{\sigma_{cr}}{\sigma_s}} \left(1 - 0.22 \sqrt{\dfrac{\sigma_{cr}}{\sigma_s}} \right) \end{cases}$$

Example

- Consider the 100 mm square thin wall steel beam of thickness 0.86 mm is loaded in compression. We determined the critical buckling stress for each side plate to be 55.35 N/mm^2 and the resulting compressive load to cause plate buckling to be 19040 N. What load will cause a maximum stress of 111 N/mm^2 in each plate?
- A 100 mm square thin walled steel beam of thickness 0.86 mm is loaded by a bending moment in the +x direction using the right hand rule. Under this moment, the maximum compressive stress in the top plate is 111 N/mm^2. What is the effective moment of inertia for the section under this moment loading?



$$\left.\begin{aligned}\sigma_s &= 111\,N/mm^2 \\ \sigma_{cr} &= 55.35\,N/mm^2\end{aligned}\right\} \rightarrow w = \frac{1}{2}\left(1+\frac{\sigma_{cr}}{\sigma_s}\right)b = \frac{1}{2}\left(1+\frac{55.35}{111}\right)100 = 75mm$$

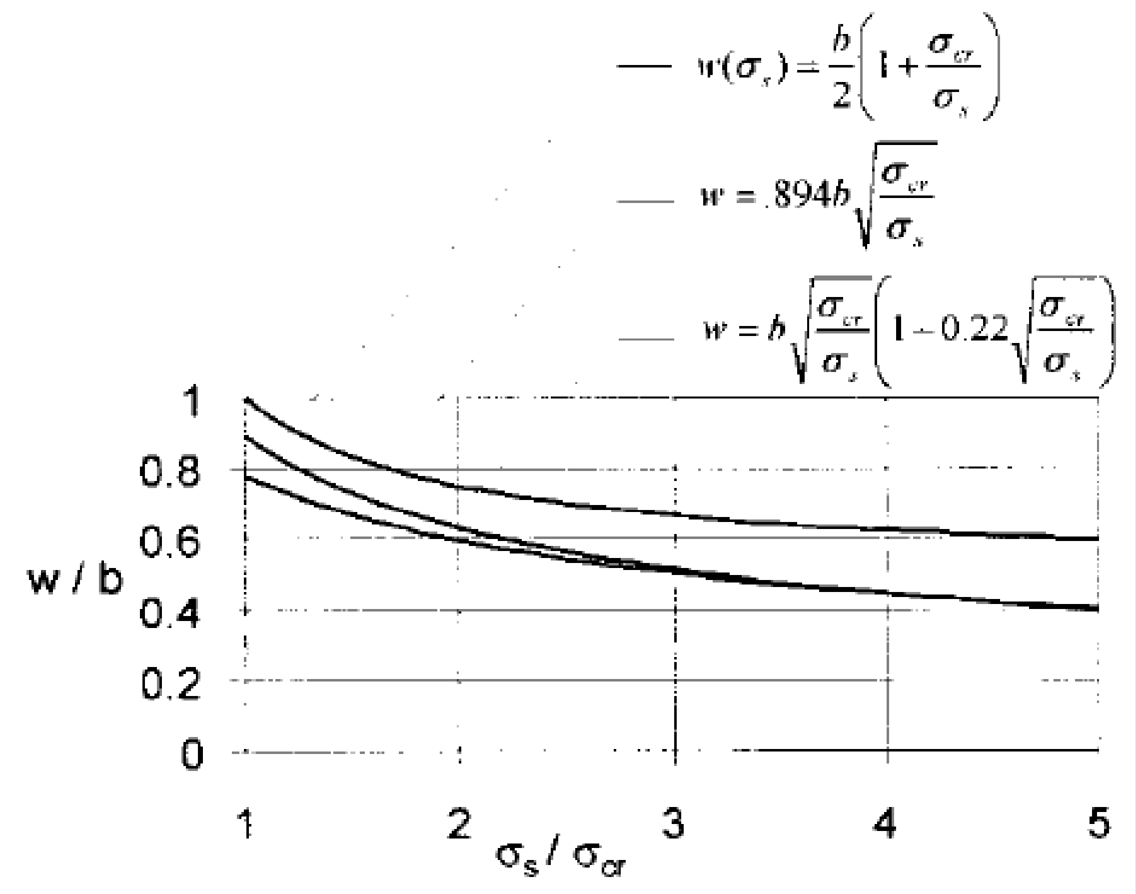
$$P_{effective} = \sigma_s\left(wt\right) = 111\left(75\times 0.86\right) = 7160\,N/side$$

$$I_{\text{nominal}} = 5.73\times 10^5\,mm^4 \rightarrow I_{\text{effective}} = 5.16\times 10^5\,\left(10\%\downarrow\right)$$

check the side walls: $k = 23.9$

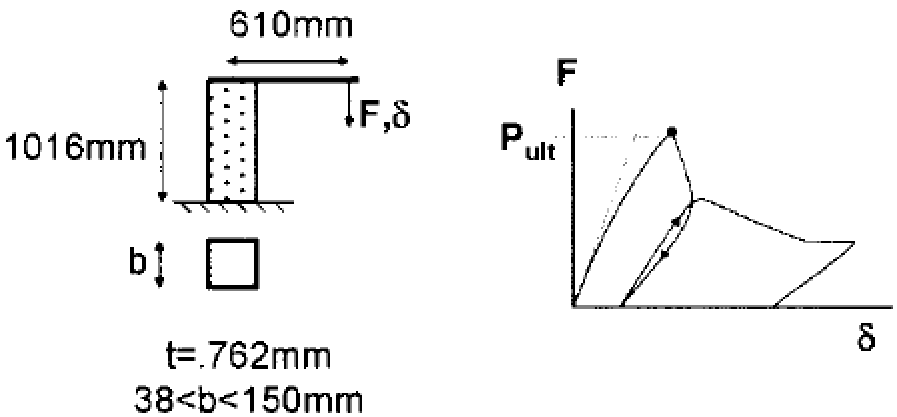
$$\rightarrow \sigma_{cr} = \frac{E\pi^2}{12\left(1-\nu^2\right)\left(b/t\right)^2}k = 331\,N/mm^2\,\left(OK!\right)$$

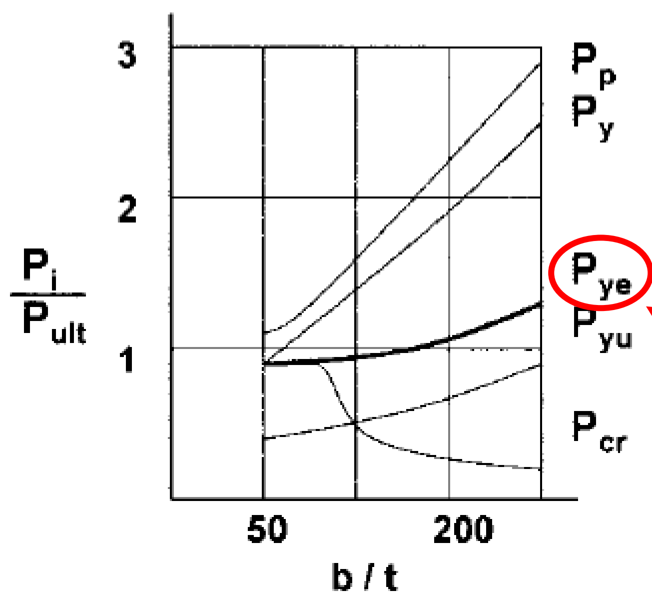
Comparison of Effective Width

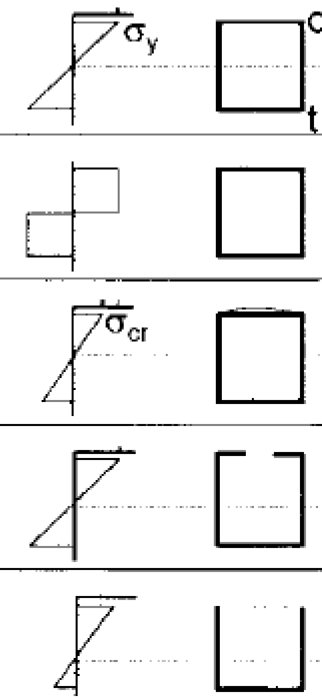


Thin Walled Section Failure Criteria

– Ultimate failure load for the thin walled plate (P_{ult})







Bending strength- Initial yield	P_y
Fully Plastic State	P_p
Onset of Plate Buckling of Compressive Element	P_{cr}
Yield of Effective Section	P_{ye}
U Section Yield	P_{yu}

Best predictor of ultimate load

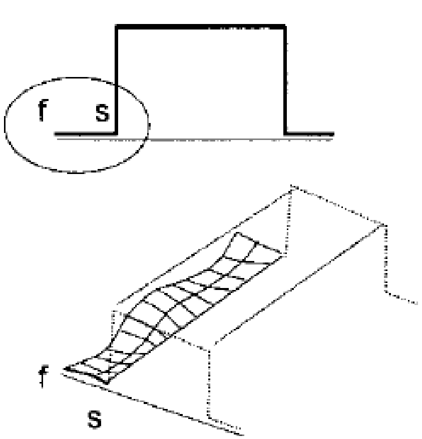
Techniques to Inhibit Buckling (1)

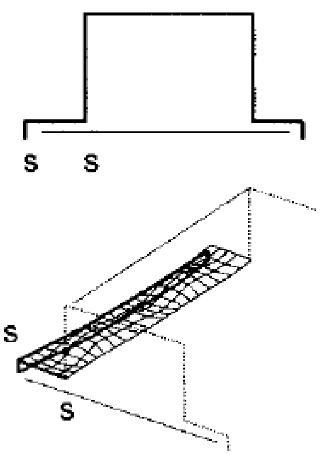
$$\sigma_{cr} = k \underbrace{\frac{E\pi^2}{12\left(1-\nu^2\right)}}\underbrace{\frac{1}{\left(b/t\right)^2}}$$

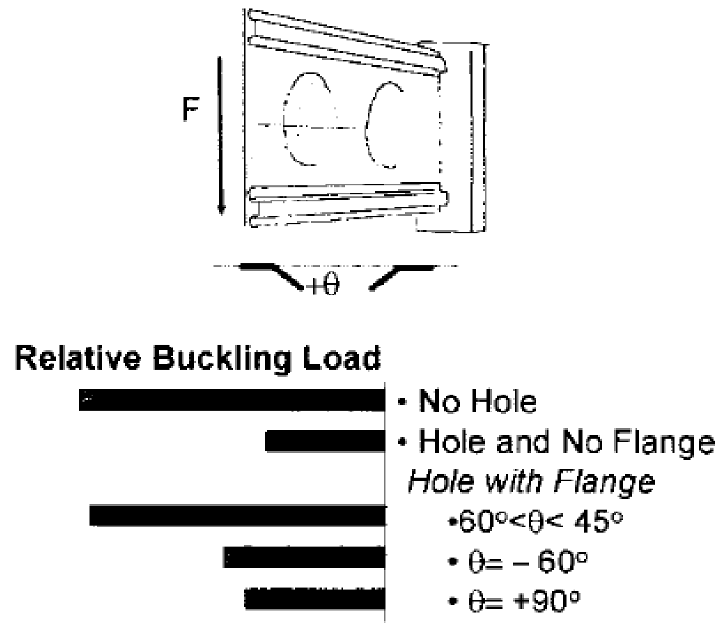
- Increase the critical plate buckling stress
  - Boundary conditions: flange curls, flanged holes
  - Normal stiffness of the plate: material, curved elements, foam filling
  - Width-to-thickness ratio: reducing width with beads and added edges (while maintaining moment of inertia)

Techniques to Inhibit Buckling (2)

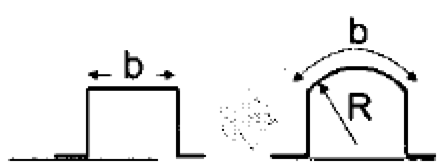
– Boundary conditions: flange curls, flanged holes





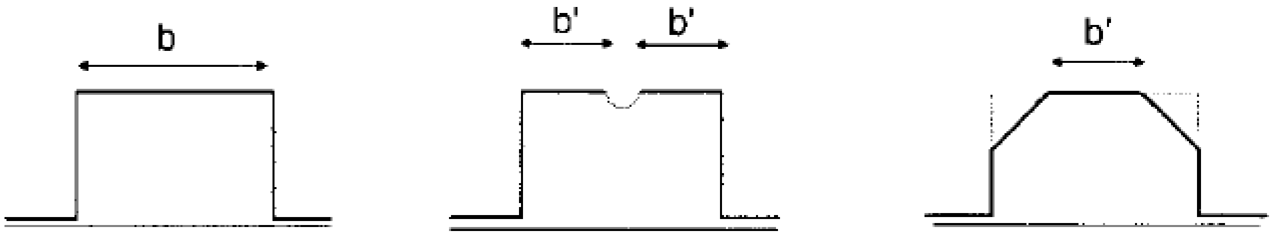


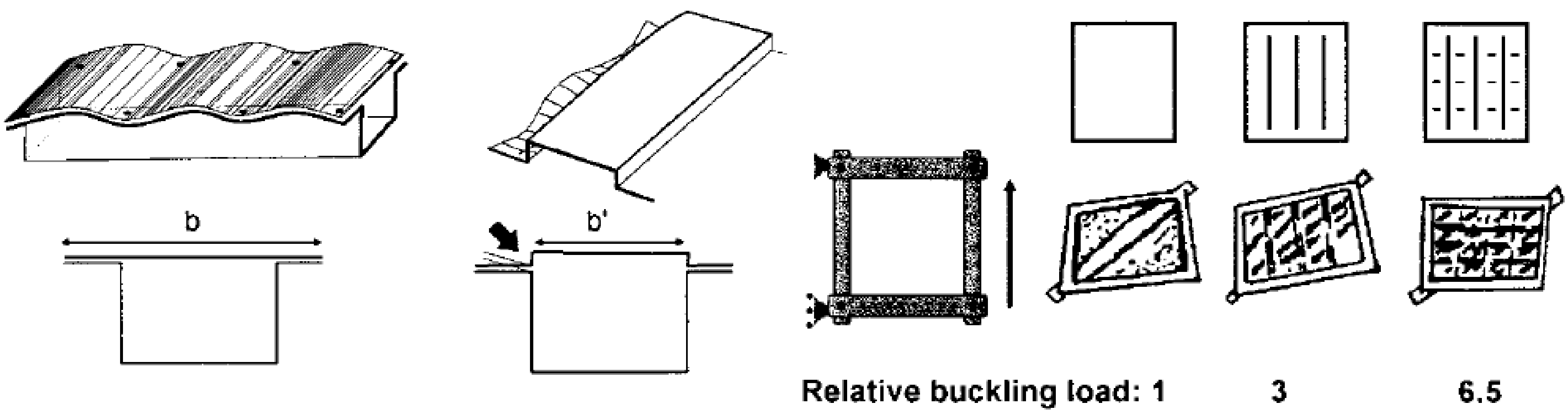
– Normal stiffness of the plate: material, curved elements, foam filling



Techniques to Inhibit Buckling (3)

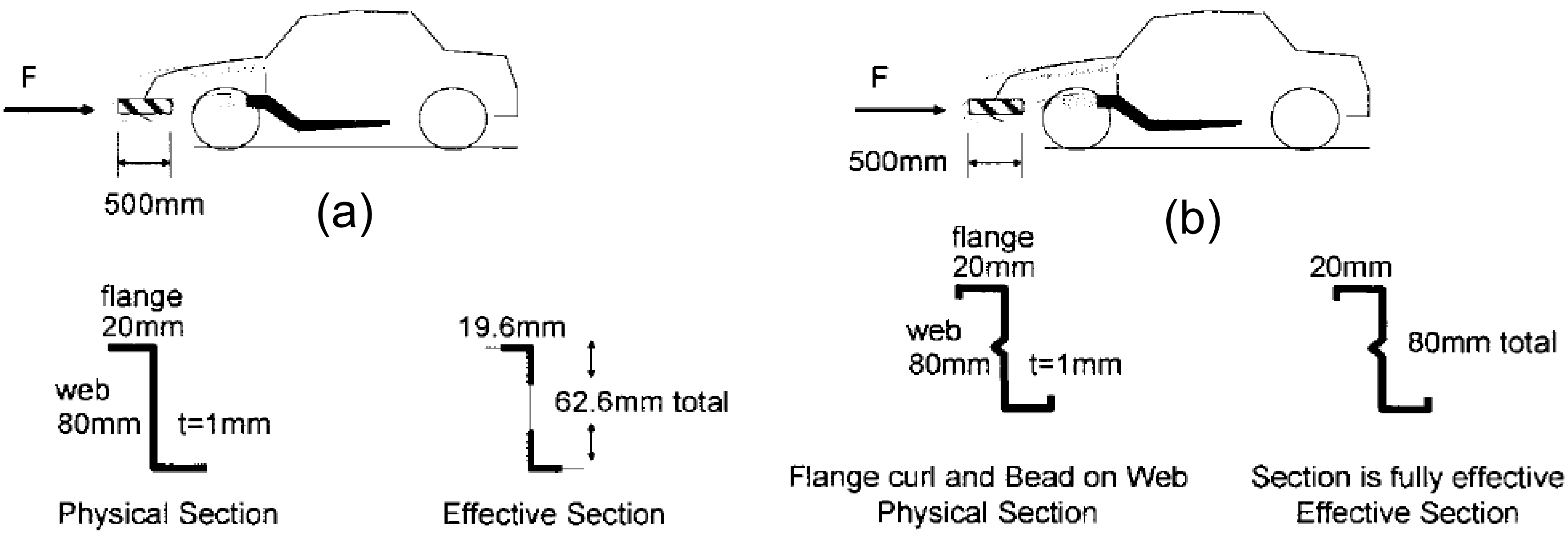
– Width-to-thickness ratio: reducing width with beads and added edges (while maintaining moment of inertia)





Example: Z section

- Part of a bumper reaction structure
- Calculate the ultimate compressive load
  - For the section (a)
  - For the section (b) with two buckling inhibiting techniques: a flange curl and a central bead on the web
  - What if high strength steel (σ_Y = 650 N/mm^2) is replaced?



E	207000	N/mm2			
ν	0.3				
t	1	mm			
sigma_Y	207	N/mm2	--> HHS	650	?

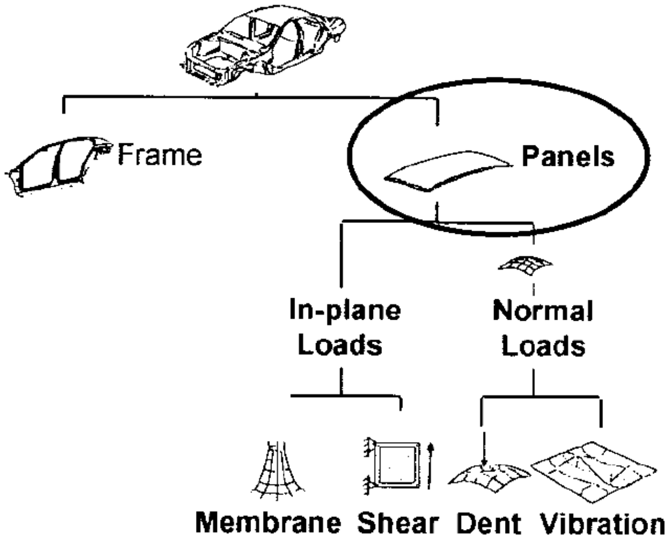
	(a) buckling		(b) no buckling		
	flange	web	flange	web	
k	0.425	4	4	4	
b	20	80	20	40	mm
sigma_CR	199	117	1871	468	N/mm2
w	19.6	62.6			mm
Pye	21073		24840		N

$$\sigma_Y = 270 \rightarrow 650\, N/mm^2 > \sigma_{cr} = 468\, N/mm^2$$

$$\rightarrow \text{effective width (not fully effective)/add buckling inhibitors}$$

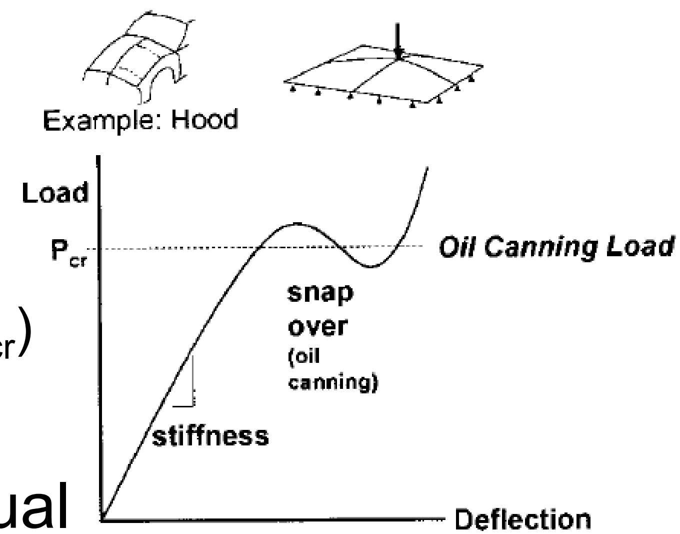
3.6 Automotive Body Panel: Plate/Membrane

- Flat or curved surface with thin thickness
  - Bending stiffness: quite low
  - In-plane stiffness: quite high
  - Highly curved panel: stiffness to out-of-plane loads
- Type of load acting on
  - Normal loading of curved panels
  - In-plane loading of flat or curved panels



Curved Panel with Normal Loading

- Exterior panels
  - Influenced by overall styling
  - Structural performance is not the shape defining function
  - Reaction to normal point loading
    - Stiffness
    - Critical oil-canning load
    - Dent resistance
  - Solidness: pushing with a thumb (K, P_{cr})



- AISI Automotive Steel Design Manual
  - Simply supported boundary conditions
  - Combination of analytical and empirical considerations

Normal Stiffness of Panels

- Theoretical stiffness of a spherical shape under a concentrated load

$$K = \frac{CEt^2}{R\sqrt{1-\nu^2}} \quad \text{where curvature } \frac{1}{R} = \frac{\left(\frac{L_1^{\ 2}}{R_1}\right)+\left(\frac{L_2^{\ 2}}{R_2}\right)}{2L_1L_2}$$

$$\begin{cases} C: \text{ constant} \\ t: \text{ panel thickness} \\ R: \text{ spherical radius} \\ L_1, L_2: \text{ rectangluar panel dimensions} \\ R_1, R_2: \text{ panel radii of curvature in orthogonal directions} \end{cases}$$

- Theoretical shell stiffness

$$K = 1.466\frac{\pi^2 Et^2}{\sqrt{1-\nu^2}}\frac{H_c}{L_1L_2} \quad \text{for } 20 \le \frac{H_c}{t} \le 60 \quad \text{where crown height } H_c = \left(\frac{L_1^{\ 2}}{8R_1}\right)+\left(\frac{L_2^{\ 2}}{8R_2}\right)$$

$$\text{valid over the range } \frac{R_1}{L_1} \text{ and } \frac{R_2}{L_2} > 2, \quad \frac{1}{3} < \frac{L_2}{L_1} < 3, \quad L_1L_2 < 0.774m^2$$

Oil-can Load

- Load where a hard snap over occurs
- Curvature inversion
  - Soft: surface stays in contact with the load applicator
  - Hard: surface snaps over and looses contact with the load applicator

$$P_{cr} = \frac{C R_{cr} \pi^2 E t^4}{L_1 L_2 \left(1-\nu^2\right)} \quad \text{where} \begin{cases} R_{cr} = 45.929 - 34.183\lambda + 6.397\lambda^2 \\ \lambda = 0.5\sqrt{\dfrac{L_1 L_2}{t}\sqrt{\dfrac{12\left(1-\nu^2\right)}{R_1 R_2}}} \\ C = 0.645 - 7.75\times 10^{-7} L_1 L_2 \end{cases}$$

$$\text{valid over the range } \frac{R_1}{L_1} \text{ and } \frac{R_2}{L_2} > 2, \quad \frac{1}{3} < \frac{L_2}{L_1} < 3, \quad L_1 L_2 < 0.774 m^2$$

Dent Resistance

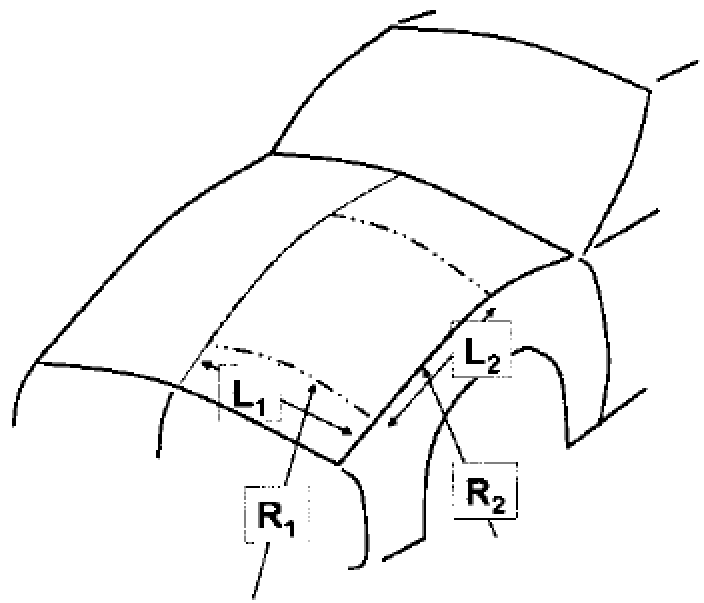
- Kinetic energy of a dart, directed normal to a surface which leaves a permanent dent in the panel
  - W: minimum energy to dent the surface (0.025 mm permanent deformation in the panel)
  - Yield at a dynamic strain rate (10~100/sec)
    - Static tensile test strain rate (0.001/sec)

$$W = 56.8 \frac{\left(\sigma_{yd} t^2\right)^2}{K}$$

$$\begin{cases} K : \text{ panel normal stiffness (theoretical shell stiffness)} \\ \sigma_{yd} : \text{ yield strength at a dynamic strain rate } \left(298 \text{ N/mm}^2\right) \end{cases}$$

Example: Automobile Hood Outer Panel

- Simply supported boundary conditions
- Dynamic yield stress: σ_{yd} = 298 N/mm^2
- Panel stiffness
- Oil-can load
- Denting energy



L_1= 660mm
L_2= 762mm
R_1= 5715mm
R_2= 15750
t= 0.71mm