Design for Vibration

– To create an acceptable vibration environment for the automobile passengers

YouTube SDOF resonance vibration test
MDOF system forced vibration

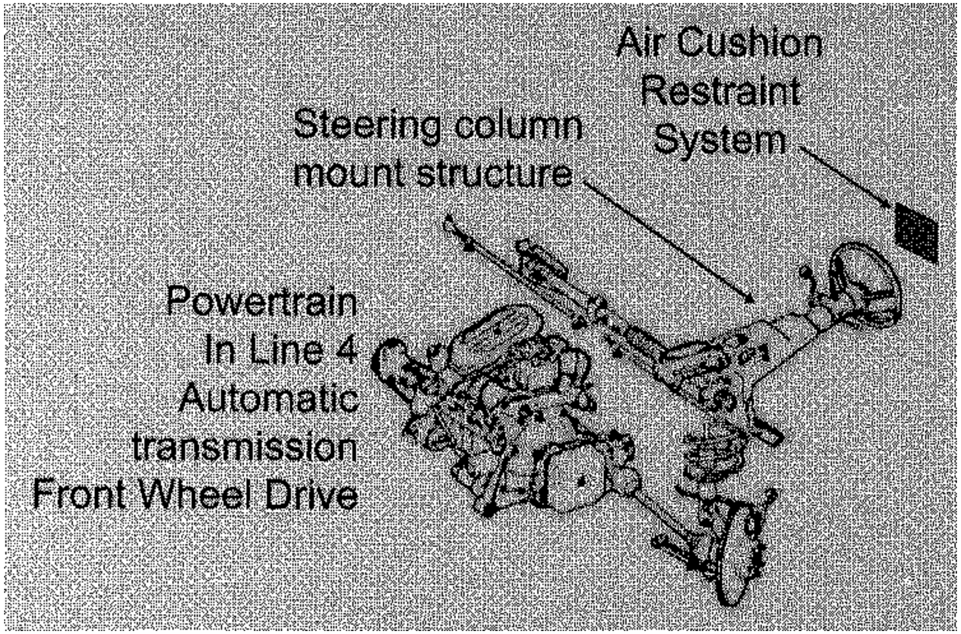
- First-order vibration modeling
- Source-path-receiver model of vibrating systems
- Frequency response of Single-Degree-of-Freedom System
- SDOF models of vehicle vibration systems
- Strategies for design for vibration
- Body structure vibration testing
- Modeling body structure resonant behavior

7.1 First-Order Vibration Modeling

- Body structure: resonant system w/ infinite number of natural frequencies
- Avoid resonance at the wrong frequency
- Identify desirable vibration behavior
- Assumption
  - Amplitude will not be large to the receiver of the vibration
  - Uncoupled vibration: frequency of the vibration source ≠ resonance in the vibration path
  - Well-designed vehicle: set of independent single-degree-of-freedom oscillators

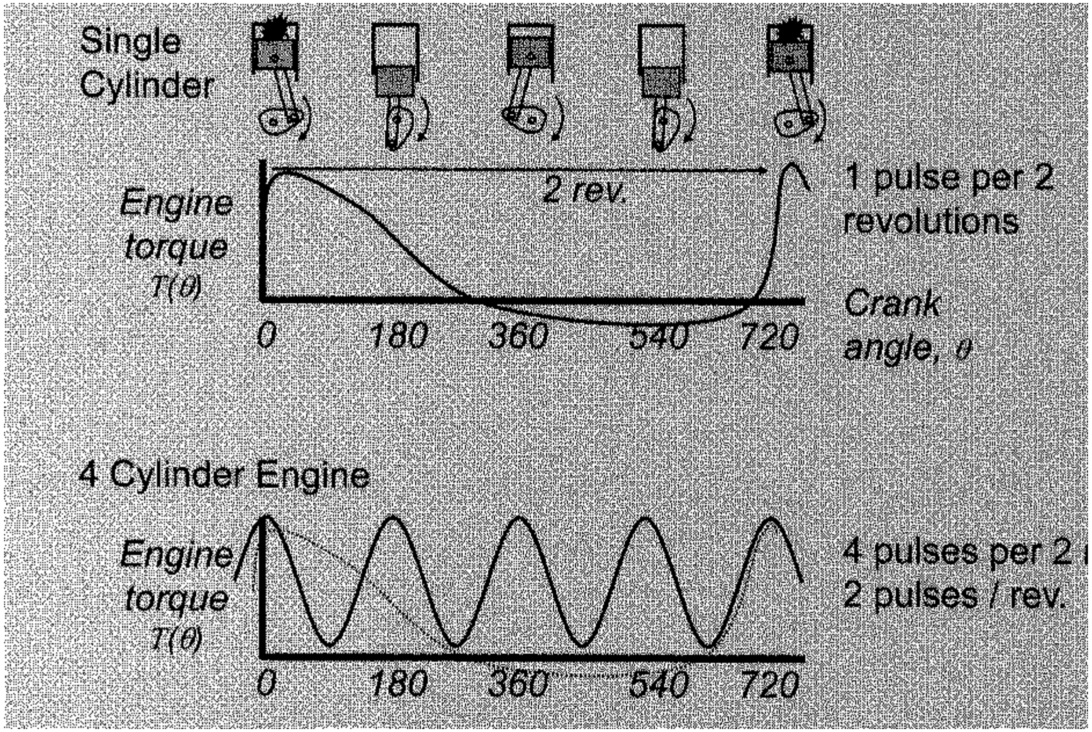
Example: Vibration System

- Powertrain + Steering Column + driver ACRS(Air Cushion Restraint System)
  - Vibration source: Powertrain
  - Vibration model 1: Steering Column Mount
  - Vibration model 2: Steering Column + ACRS(Air Cushion Restraint System)



Example: Vibration Source

- Four-cylinder engine in a transverse front-wheel-drive configuration with an automatic transmission
- Engine torque pulse



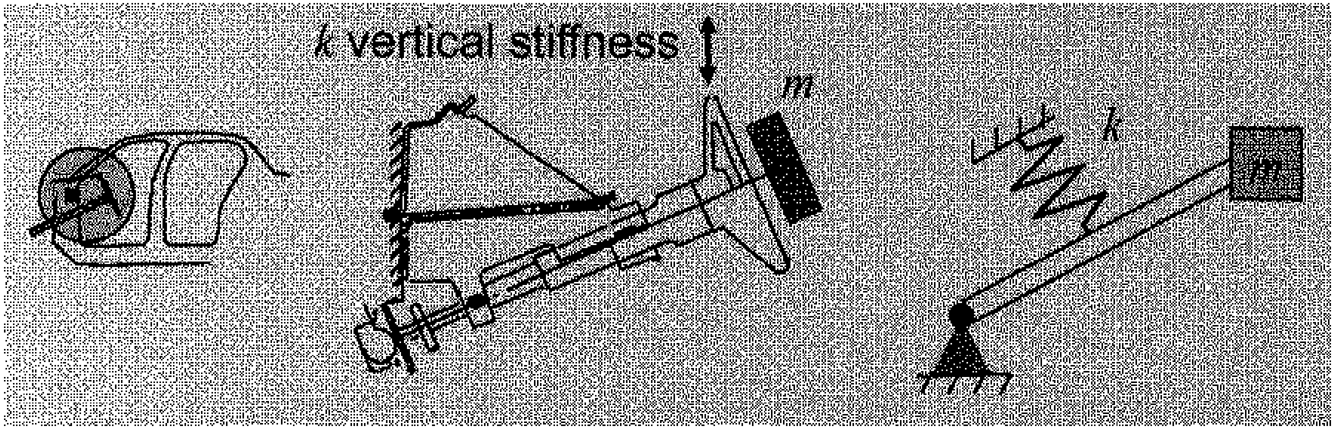
$$\Omega = \left(\frac{1 \text{ pulse}}{2 \text{ rev}}\right)\left(\frac{N \text{ rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)(4 \text{ cylinders})$$

$$= \frac{N}{30} \text{Hz}$$

$$\text{@idle: } N = 700\text{rpm} \rightarrow \Omega = 23.3\text{Hz}$$

Example: Vibration Model 1

- Steering Column Mount

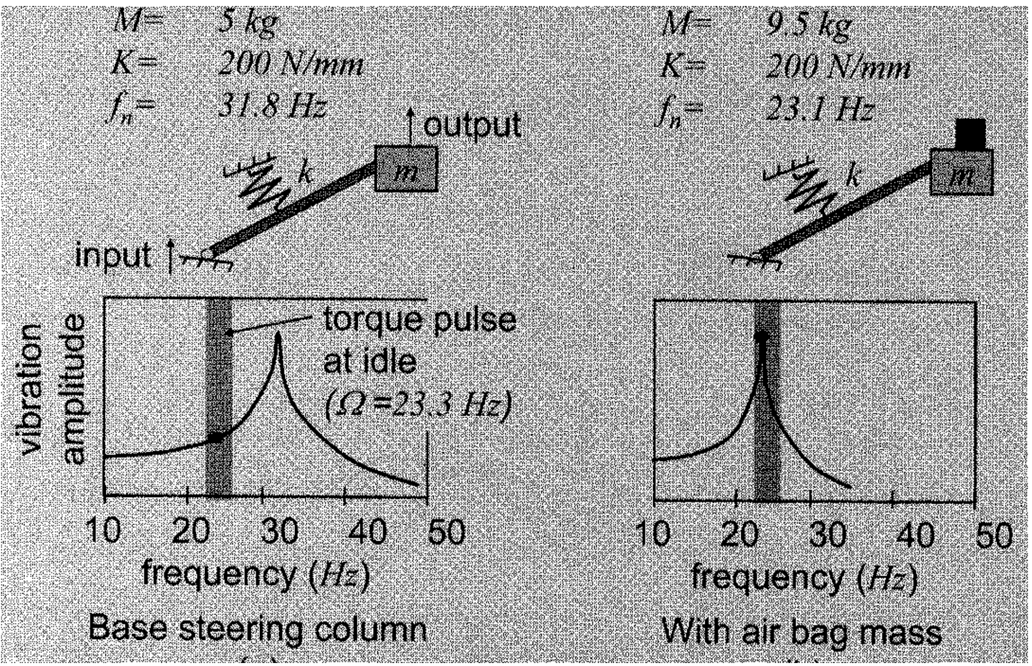


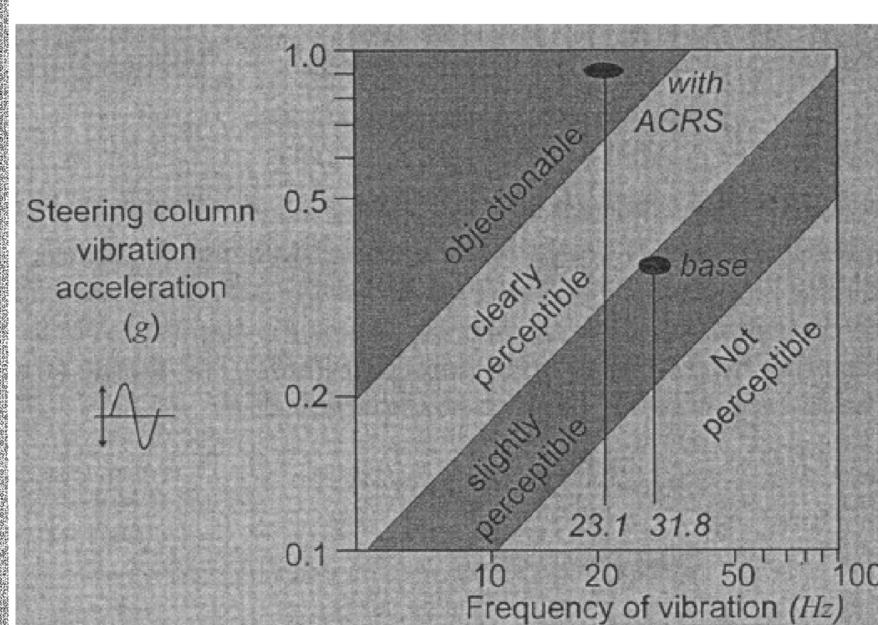
$$k = 200\frac{N}{mm},\quad m = 5\text{kg} \;\Big\}\rightarrow \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\dfrac{\text{kg}\cdot m/\sec^2}{mm}\dfrac{1000mm}{m}}{5\text{kg}}} = 200\frac{rad}{\sec} = 2\pi f$$

$$f = \omega_n \frac{rad}{\sec}\left(\frac{1\ \text{cycle}}{2\pi\ rad}\right) = \frac{\omega_n}{2\pi}\left(\frac{\text{cycle}}{\sec}\right) = \frac{\omega_n}{2\pi}\text{Hz} = 31.8\text{Hz}$$

Example: Vibration Model 2

- Coupled resonance: Steering column + ACRS (Air Cushion Restraint System)

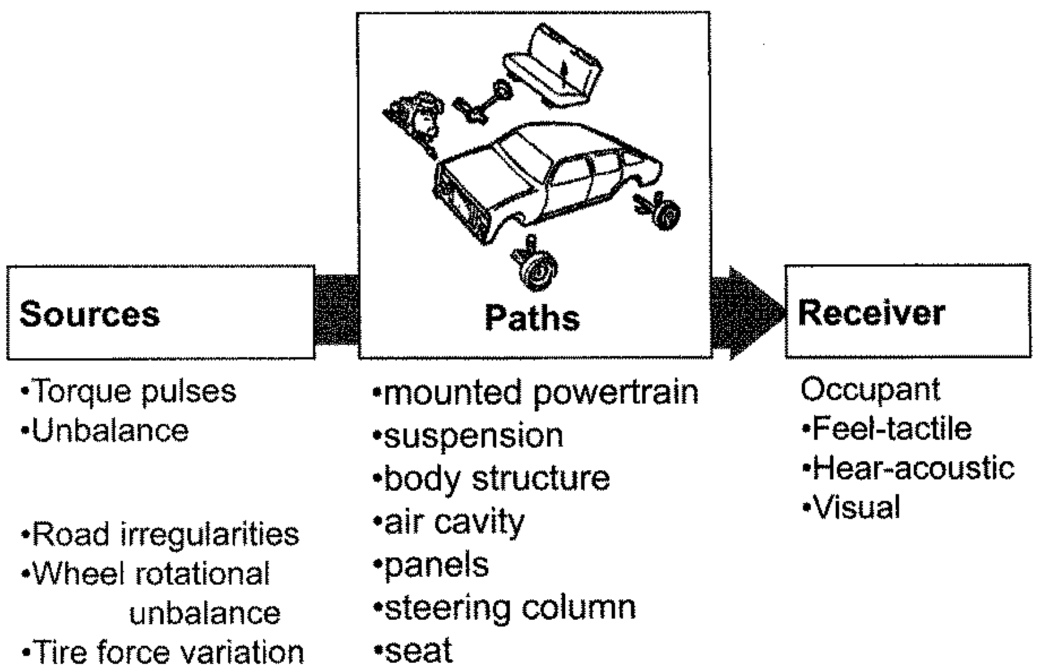




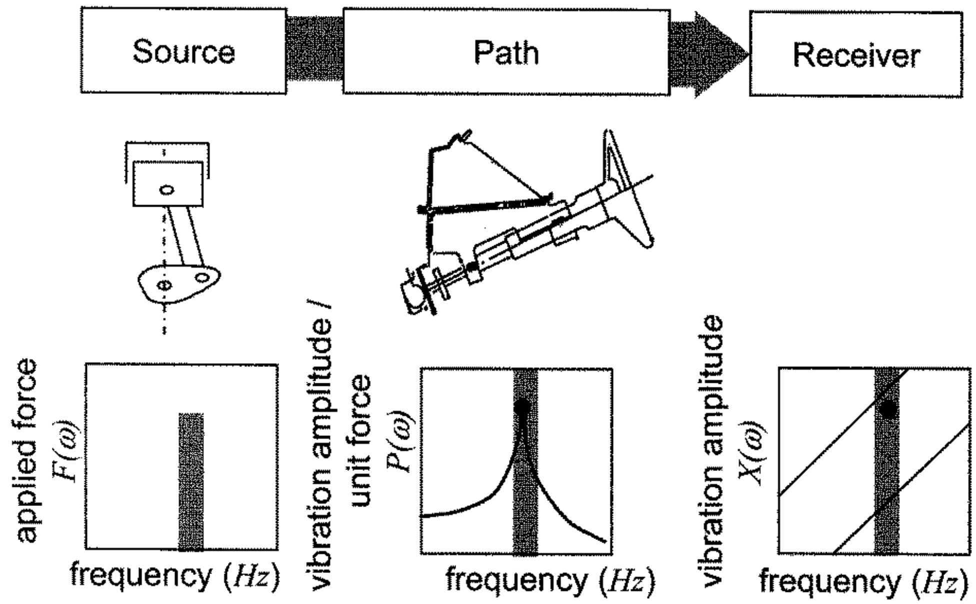
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\,\dfrac{\mathrm{kg}\cdot m/\sec^2}{mm}\,\dfrac{1000mm}{m}}{(5+4.5)\,\mathrm{kg}}} = 145\,\frac{rad}{\sec} \rightarrow f = 23.1\,\mathrm{Hz}$$

7.2 Source-Path-Receiver Model

- Source of vibration energy (engine torque pulses)
- Path for the vibration: series of subsystems (steering column with ACRS)
- Receiver which determines the acceptability of the vibration level (driver's hands)



Vibration Characteristics



$$F(\omega)\left[\frac{X(\omega)}{F(\omega)}\right]=X(\omega)\rightarrow \underset{\text{source}}{F(\omega)}\left[\underset{\text{path transfer function}}{P(\omega)}\right]=\underset{\text{response /receiver}}{X(\omega)}$$

$$F(\omega)\left[\left(\frac{F_T(\omega)}{F(\omega)}\right)\left(\frac{X(\omega)}{F_T(\omega)}\right)\right]=X(\omega)\rightarrow \underset{\text{source}}{F(\omega)}\left[\underbrace{T(\omega)P(\omega)}_{\text{path}}\right]=\underset{\text{receiver}}{X(\omega)}$$

$F_T(\omega)$: force transmitted through a subsystem of the path

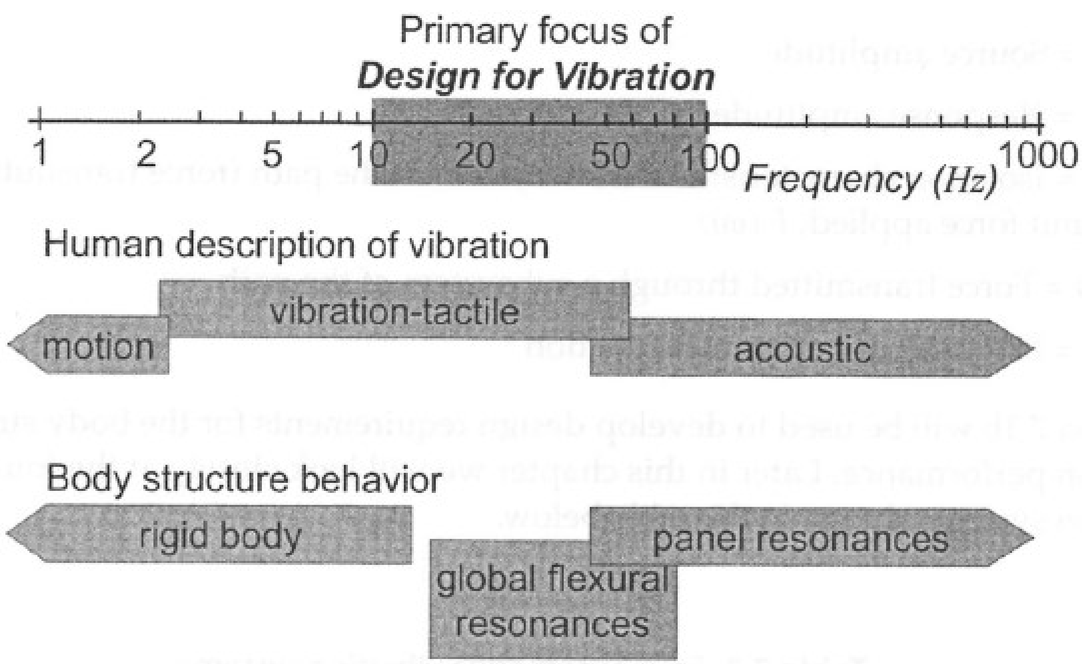
$T(\omega)$: isolated characteristic of a subsystem in the path

Automobile Vibration Systems

	Source	Isolator	Force into body	Body transfer function	Body deflection
	$F(\omega)$	$T(\omega)$	$F_T(\omega)$	$P(\omega)$	$X(\omega)$
1 (7.4.1)	Powertrain unbalance force	Mounted powertrain	Force through engine mounts	Body structure	Deflection at seat, steering column
2 (7.4.2)	Force at suspension spindle	suspension	Force through shock absorber and ride spring	Body structure	Deflection at seat, steering column
3 (7.4.3)	Road deflection at tire patch	suspension	Force through shock absorber and ride spring	Body structure	Deflection at seat, steering column
4 (7.8.3)	High frequency chassis deflections	Chassis links with end bushings	Body panel vibrations	Passenger compartment acoustic resonances	Interior sound pressure

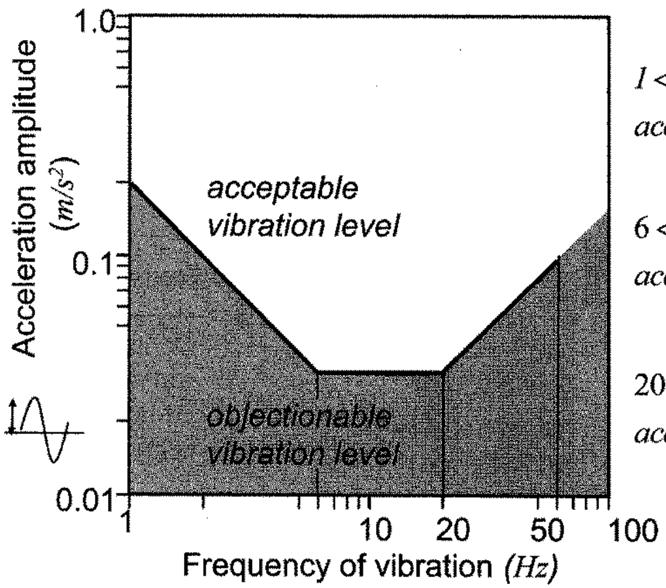
Automobile Vibration Spectrum

- Body structure behavior
  - ~ 10 Hz: rigid body
  - 10 ~ 100Hz: primary bending and torsion resonances (overall body architecture, hard to change in the later stages)
  - 100 Hz ~: localized and influenced by structural details



Human Response to Vibration

- Subjective test → U-shape iso-comfort curve
  - Imperceptible / just perceptible/ annoying
  - 6~20 Hz: least tolerated area



$1 < f < 6Hz$

$accel(g) \le 0.204 / f$

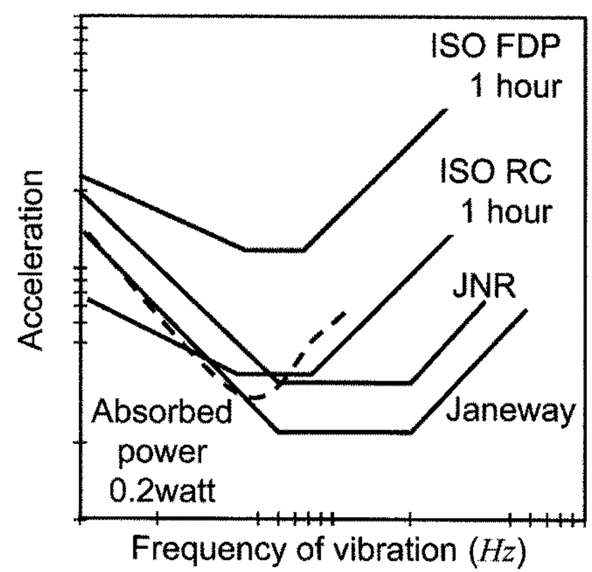
$6 < f < 20Hz$

$accel(g) \le .0341$

$20 < f < 60Hz$

$accel(g) \le 0.0017 f$

Janeway vertical seat vibration criteria



ISO: International Standards Organization

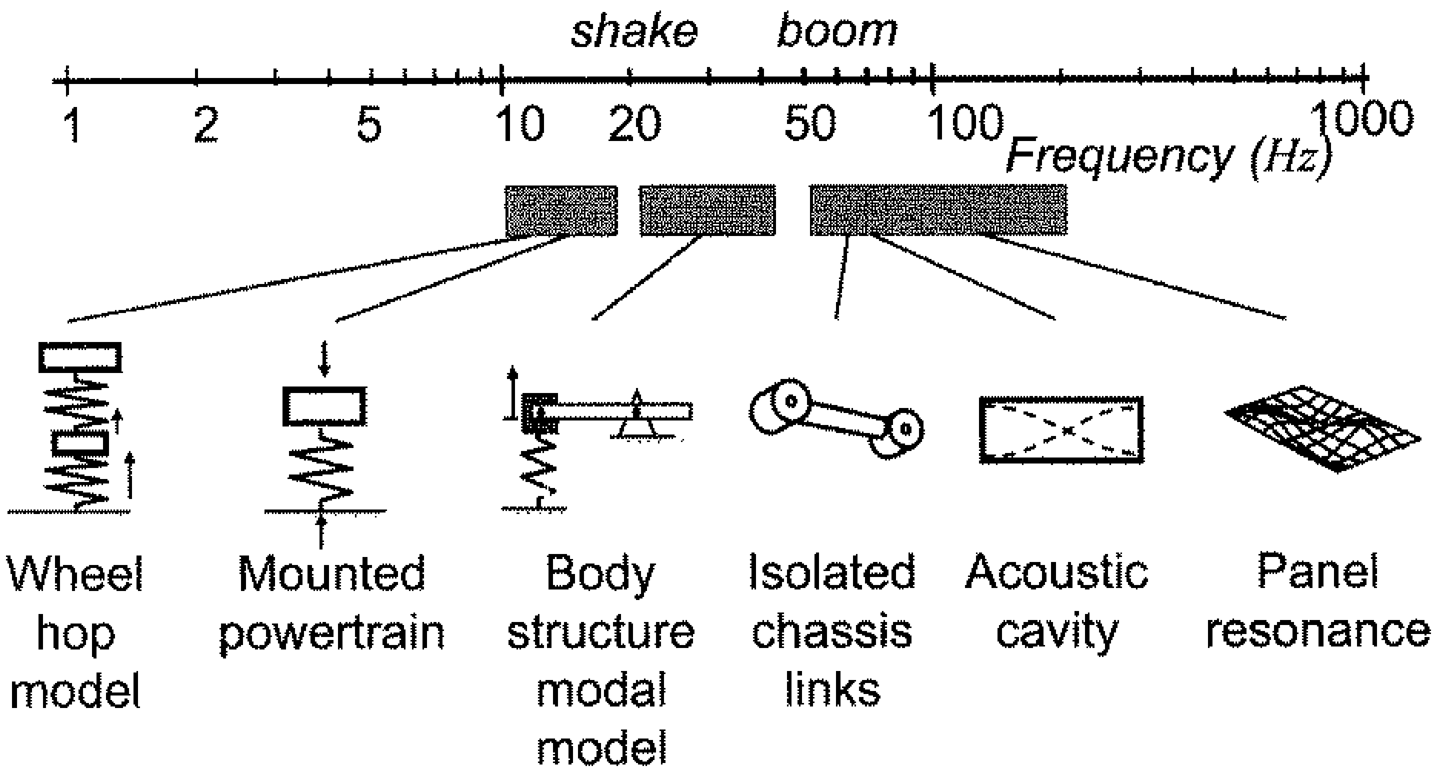
FDP: Fatigue Decreased Proficiency

RC: Reduced Comfort

JNR: Japan National Railway

Comparison of vibration limits

Major Vibratory Systems



7.3 Frequency Response of SDOF System

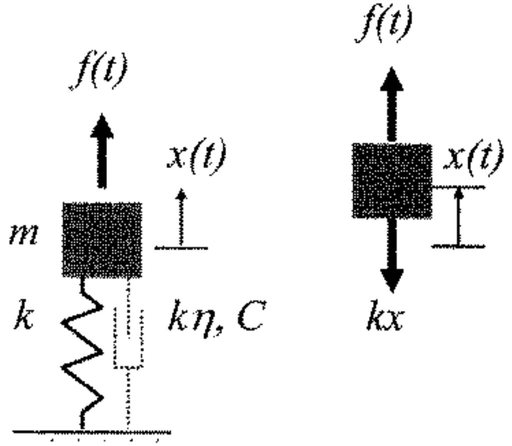
- Equations of motion

$$f(t) = F\sin(\omega t) \rightarrow x(t) = X\sin(\omega t)$$

$$f(t) - kx(t) = m\frac{d^2x}{dt^2} \rightarrow F\sin(\omega t) = kX\sin(\omega t) - mX\omega^2\sin(\omega t)$$

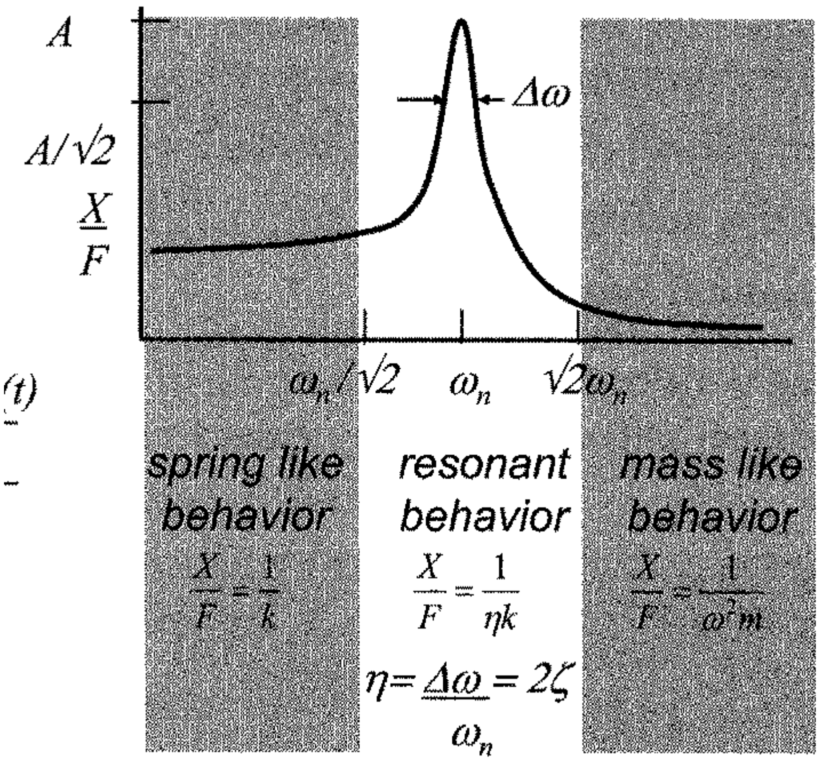
$$F = kX - m\omega^2 X \rightarrow \frac{X}{F} = \frac{1}{k - m\omega^2} = \frac{1/k}{1-\left(\frac{m}{k}\right)\omega^2} = \frac{1/k}{1-\left(\frac{\omega}{\omega_n}\right)^2} = P(\omega)$$

- Relation of vibration amplitudes
  - Displacement amplitude: X
  - Velocity amplitude: $X\omega$
  - Acceleration amplitude: $X\omega^2$



Regions of Vibration Behavior

$$\begin{cases} \omega \ll \omega_n : \text{ spring-like behavior } \left(F = kX \rightarrow \left| \dfrac{X}{F} \right| = \dfrac{1}{k} \right) \\ \omega = \omega_n : \text{ vibration amplitude grows very large} \\ \omega \gg \omega_n : \text{ mass-like behavior } \left(F = m\left(-\omega^2 X\right) \rightarrow \left| \dfrac{X}{F} \right| = \dfrac{1}{m\omega^2} \right) \end{cases}$$



Amplitude at Resonance

- Viscous damping ($\propto$ velocity)

$$F_D = C(velocity),\ \ \zeta = \frac{C}{2\sqrt{km}} \quad \begin{cases} C: \text{ viscous damping coefficient (for shock absorber, } C = 2) \\ \zeta: \text{ viscous damping factor} \end{cases}$$

$$|F_D| = C(\omega X) = 2\zeta\sqrt{km}(\omega X) = 2\zeta k\sqrt{\frac{m}{k}}(\omega X)$$

$$\left|\frac{X}{F_D}\right| = \frac{1}{2\zeta k(\omega/\omega_n)} \xrightarrow{\omega=\omega_n} \left|\frac{X}{F_D}\right| = \frac{1}{2\zeta k}$$

- Structural damping ($\propto$ deflection)

$$|F_D| = \eta(kX) \xrightarrow{\omega=\omega_n} \left|\frac{X}{F_D}\right| = \frac{1}{\eta k},\quad \eta: \text{ damping factor}$$

base metal: $0.00001 < \eta < 0.001$

spot-welded automobile body: $0.03 < \eta < 0.1$

- Relation of viscous and structural damping

$$\eta = 2\zeta \rightarrow \eta = \frac{\Delta\omega}{\omega_n}$$

$\Delta\omega$: bandwidth measured at $\underbrace{\text{the half-power amplitude}}_{(\text{amplitude at resonance})/\sqrt{2}}$

Example: Deflection Amplitude

$$m = 100\text{kg} \ \left(\text{4-cylinder automatic transmission powertrain}\right)$$

$$k = 600\,\text{N}/\text{mm} \ \left(\text{combined engine mount vertical stiffness}\right)$$

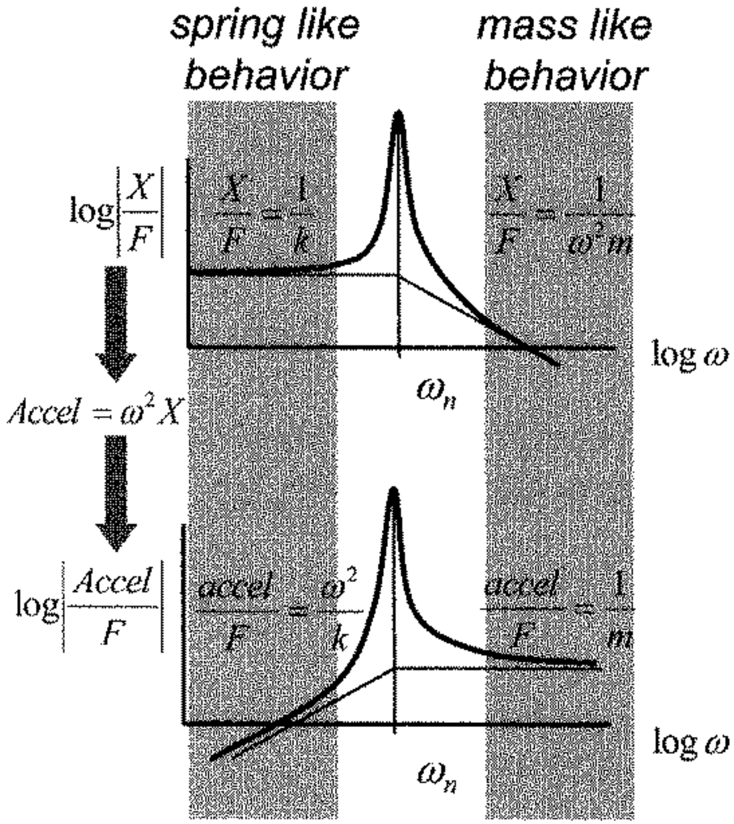
$$F = 500\text{N} \ \left(\text{amplitude of vertical sinusoidal force}\right)$$

- (1) mounted powertrain vertical bounce
  - Operating frequency: 15 Hz
- (2) amplitude recorded by accelerometer (10g)
  - Operating frequency: 40 Hz
- (3) at resonance
  - Damping ratio: 0.1

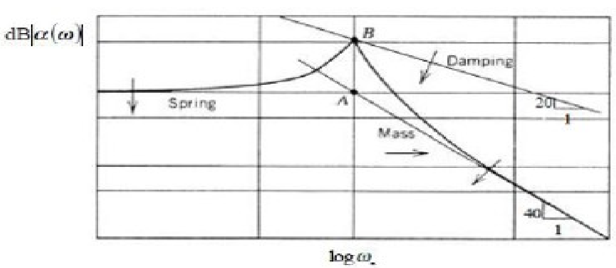
Transfer Function

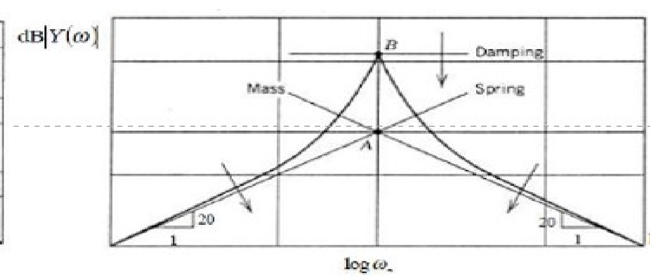
– Log (displacement output: X/F) vs. Log (frequency)
– Log (velocity output: V/F) vs. Log (frequency)
– Log (acceleration output: A/F) vs. Log (frequency)

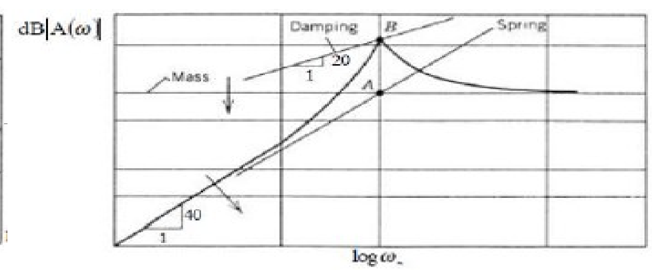
$$
\begin{cases}
\omega \ll \omega_n : \ F = kX \rightarrow \left|\dfrac{\omega^2 X}{F}\right| = \dfrac{\omega^2}{k} \\
\omega = \omega_n : \ \text{vibration amplitude grows very large} \\
\omega \gg \omega_n : \ F = m\left(-\omega^2 X\right) \rightarrow \left|\dfrac{\omega^2 X}{F}\right| = \dfrac{1}{m}
\end{cases}
$$



FRF		receptance	mobility	inertance
		$\frac{X}{F}=\frac{1}{k-\omega^2 m+i\omega c}$	$\frac{V}{F}=\frac{i\omega}{k-\omega^2 m+i\omega c}$	$\frac{A}{F}=\frac{-\omega^2}{k-\omega^2 m+i\omega c}$
Linear Scale	Stiffness	$\frac{1}{k}$	$\frac{i\omega}{k}$	$-\frac{\omega^2}{k}$
	Mass	$-\frac{1}{\omega^2 m}$	$-\frac{i}{\omega m}$	$\frac{1}{m}$
Logarithmic Scale	Stiffness	$-\log k$	$\log\omega-\log k$	$2\log\omega-\log k$
	Mass	$-2\log\omega-\log m$	$-\log\omega-\log m$	$-\log m$

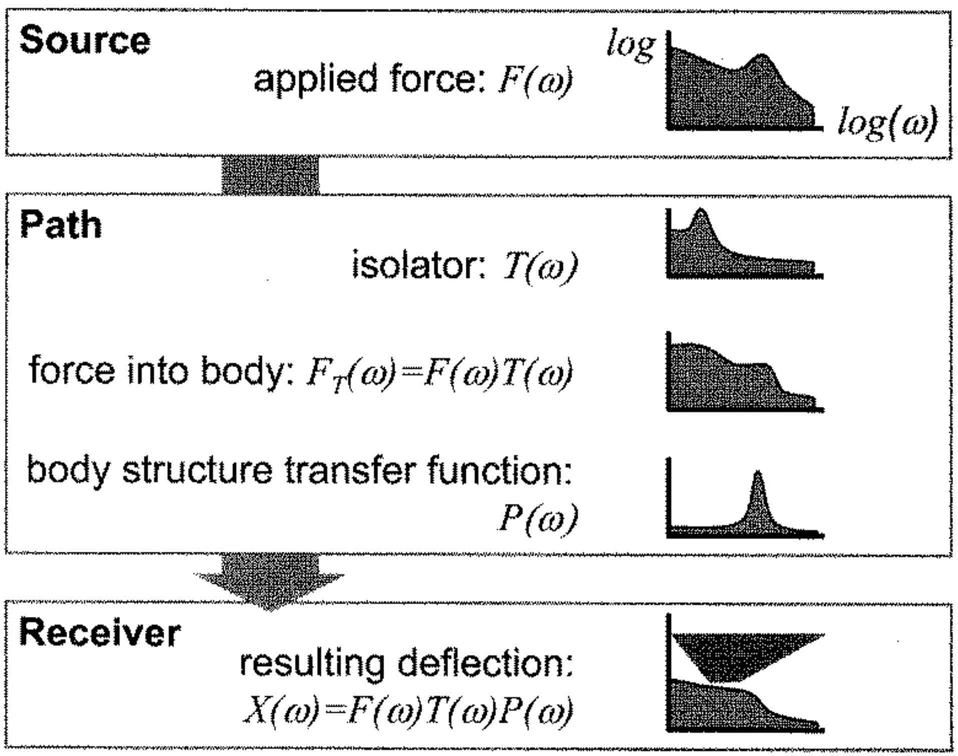






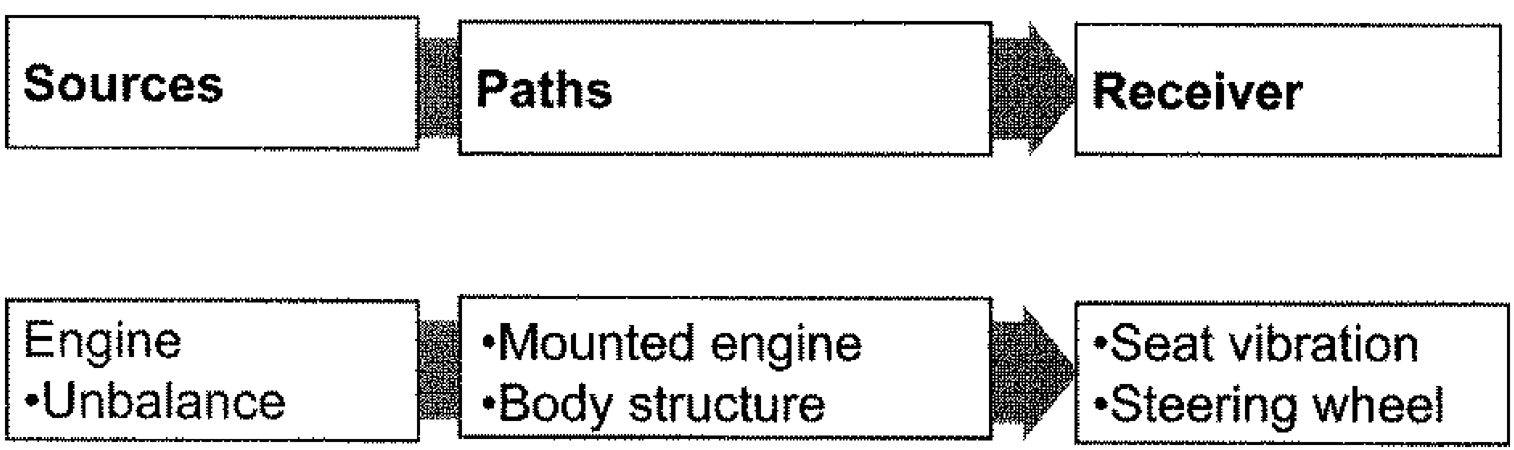
7.4 SDOF Models of Vehicle Vibration Systems

- Powertrain path: reciprocating unbalance
- Suspension path: load at spindle
- Suspension path: deflection at tire patch



Powertrain Path Vibration System

source	isolator	Force into body	Body transfer function	Body deflection
F(ω)	T(ω)	F_T(ω)	P(ω)	X(ω)
Powertrain unbalance force	Mounted powertrain	Force through engine mounts	Body structure	Deflection at seat, steering column

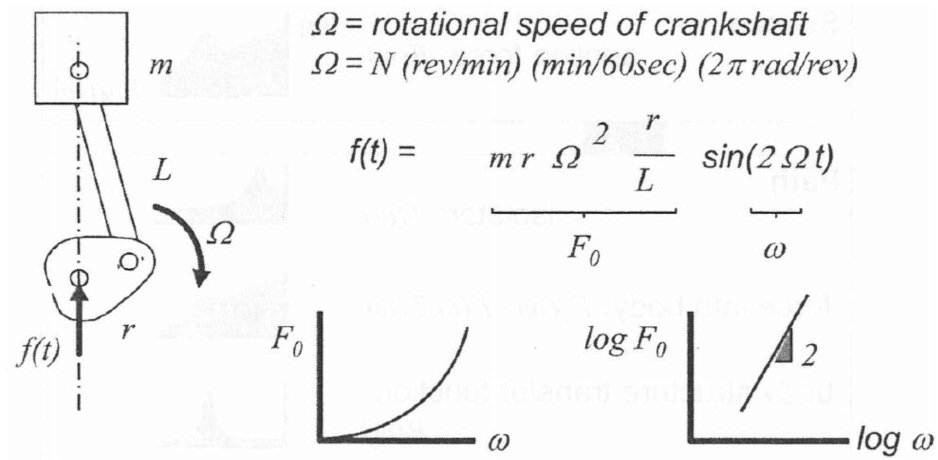


Powertrain Path: Reciprocating Unbalance

- Source

$$f(t) = mr\Omega^2 \frac{r}{L}\sin(2\Omega t) = F_0 \sin(\omega t)$$

$$\Omega = N\left(\frac{rev}{\min}\right)\left(\frac{2\pi \text{ rad}}{rev}\right)\left(\frac{1\min}{60\sec}\right) = \frac{2\pi}{60}N\left(rad/s\right)$$



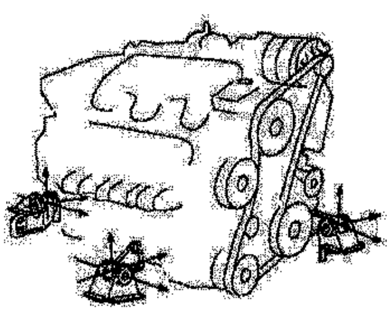
- Path

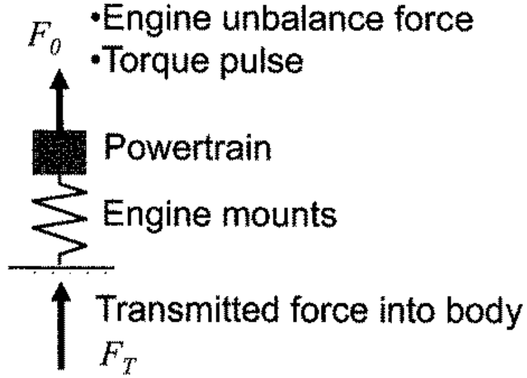
$$F_T = kX$$

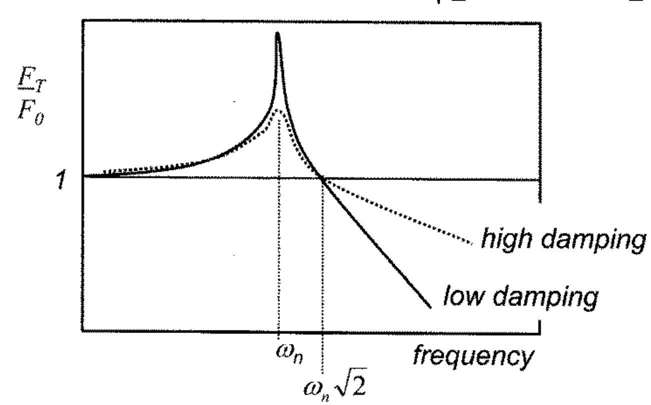
$$\left|\frac{F_T}{F}\right| = \left|T(\omega)\right| = \left|\frac{1}{1-(\omega/\omega_n)^2}\right| < 1 \rightarrow \omega > \omega_n\sqrt{2}$$

$$k^* = k + i\eta k \quad (\eta: \text{ loss factor})$$

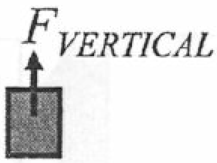
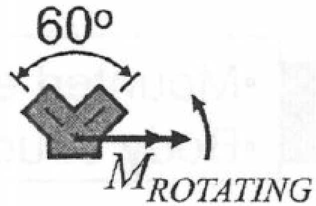
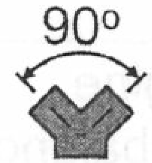
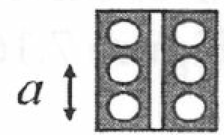
$$\left|\frac{F_T}{F}\right| = \left|T(\omega)\right| = \frac{\sqrt{1+\eta^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\eta^2}}$$







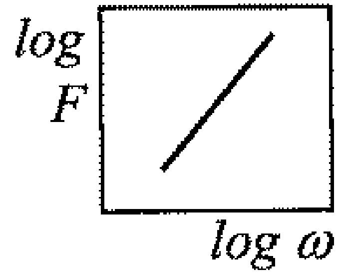
Unbalance Forcing Function

	In line 4	V6	V8
	 planar crankshaft	 even 120⁰ crankshaft	 even 90⁰ crankshaft
Excitation amplitude (2 x engine speed)	$F_{VERTICAL} = 4mr\frac{r}{l}\Omega^2$	$M_{ROT} = \frac{3}{2}mr\frac{r}{l}\Omega^2 a$ cylinder spacing 	*None*
Balance strategy	$F_{VERTICAL}$ may be eliminated with dual counter rotating balance shafts at 2 x engine speed	crankshaft counter weights balance the primary rotating couple leaving the above moment	crankshaft counter weights balance the primary rotating couple

Transfer Function Model of Powertrain

Source

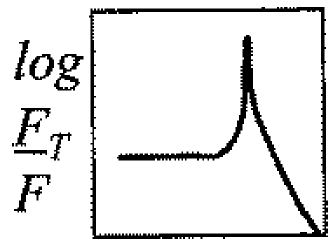
Unbalance force $F(\omega)$

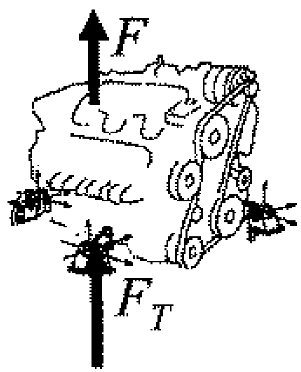


X

Path

Force transmitted per force applied $T(\omega)$





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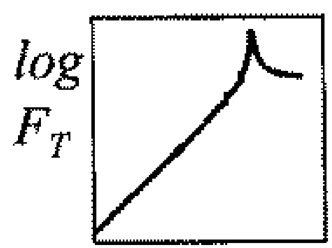
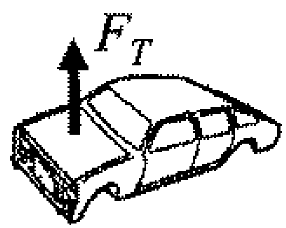
Force into body





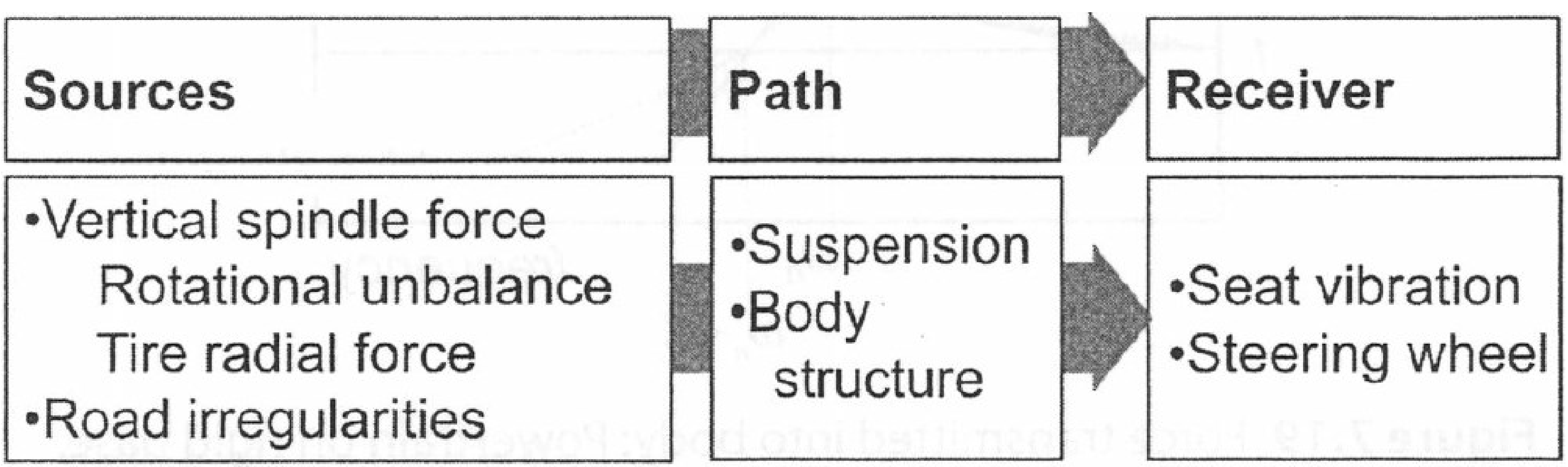
$F_T=[F(\omega)][T(\omega)]$

Example

- A four-cylinder, automatic-transmission powertrain has a mass of 100kg. The engine mount system constrains motion to the vertical. The combined engine mount vertical stiffness is 600N/mm. For an evaluation, the mounted powertrain is placed on a bed plate (ground) and a sinusoidal vertical force is applied to the center of mass.
  - Determine the bounce natural frequency.
  - At what frequency does isolation of unbalance forces begin?
  - What is the engine speed at which isolation begins?

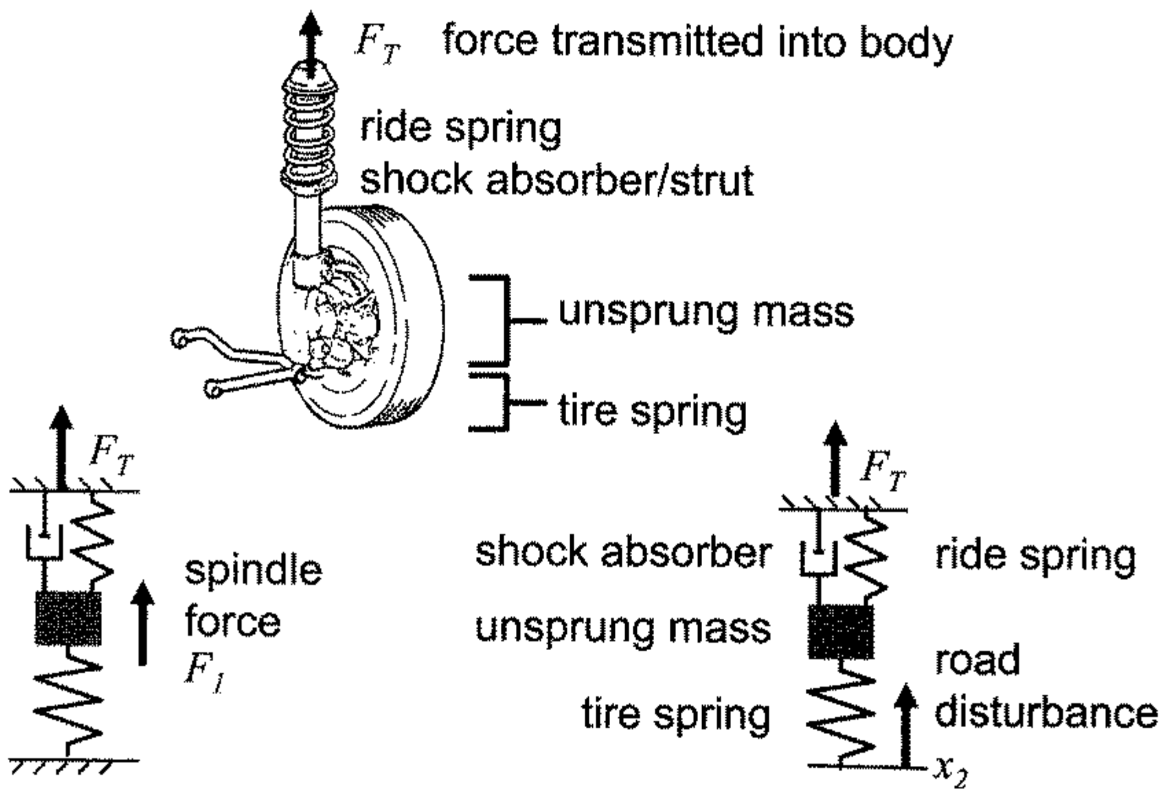
Suspension Path Vibration System

source	isolator	Force into body	Body transfer function	Body deflection
Force at suspension spindle	suspension	Force through shock absorber and ride spring	Body structure	Deflection at seat, steering column
Road deflection at tire patch	suspension	Force through shock absorber and ride spring	Body structure	Deflection at seat, steering column



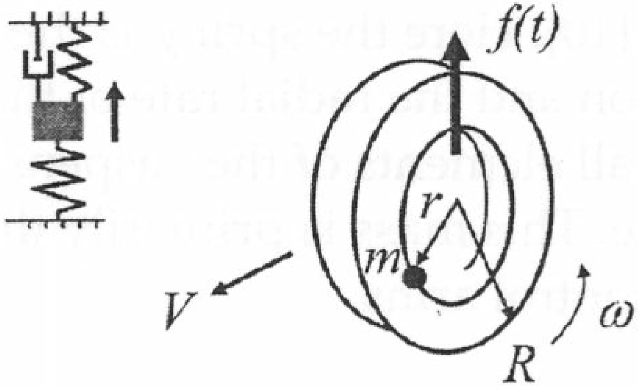
Suspension Model

- Mass: wheel, knuckle, brakes, control arms
- Spring: parallel combination of ride spring of suspension and radial rate of tire



Spindle force input (wheel unbalance) Road input

Suspension Vibration Sources: Load at Spindle

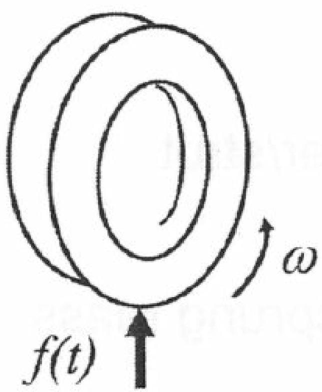


(a) Tire unbalance force

$f(t)=mr\ \omega^2 sin(\omega\ t)$

$\omega = \dfrac{V}{R}$

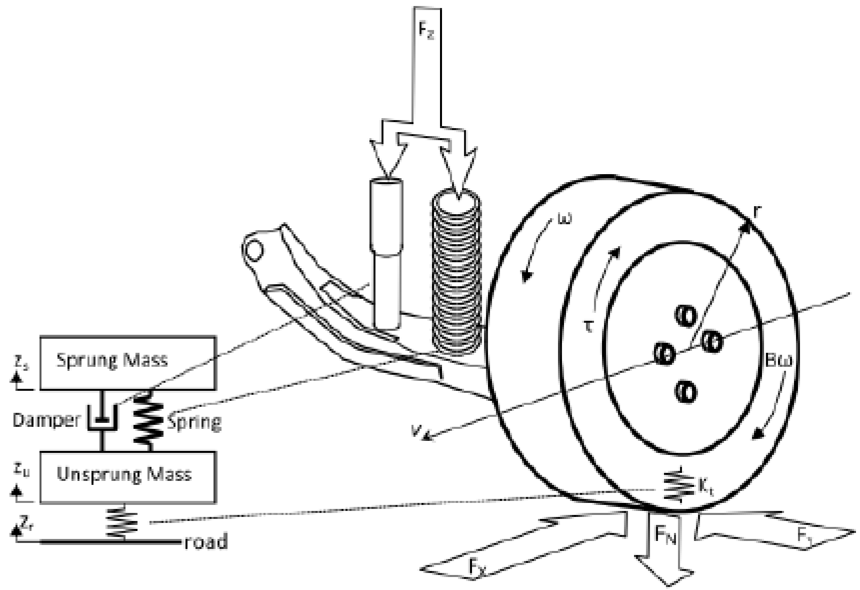
where m unbalance mass
r radial position of mass
V Vehicle velocity
R Tire rolling radius

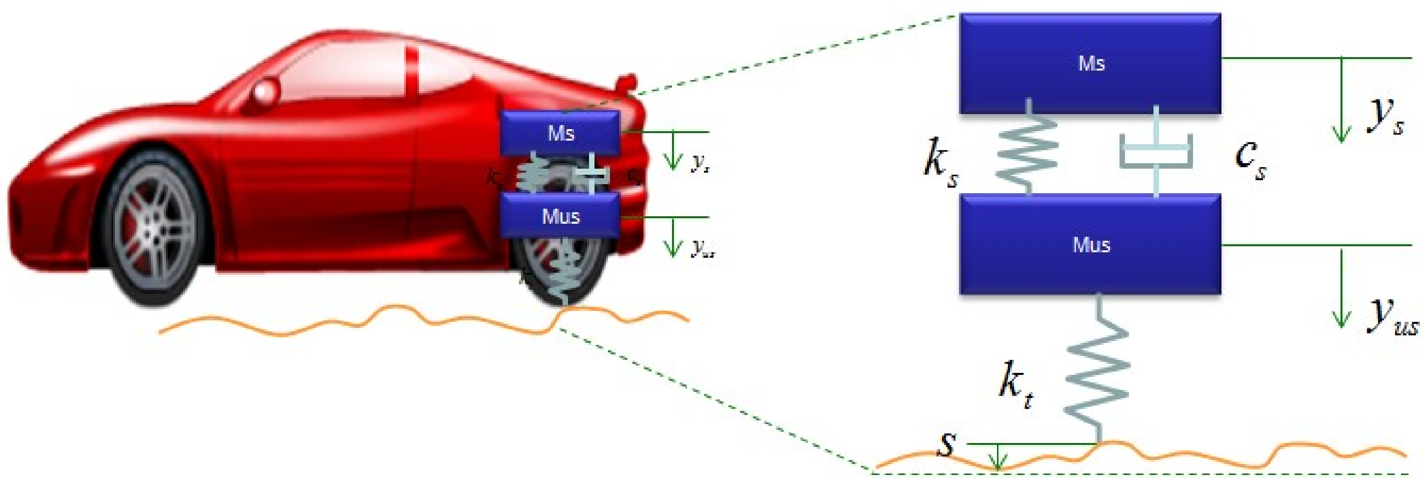


(b) Tire radial force variation

$f(t)=F_n sin(n\omega\square t)$,
n order of variation
$n=1, 2, 3, \ldots$

Quarter Car Model





Suspension Analysis: Load at Spindle

$$\underbrace{-k_1 X_1 - k_2 X_1 - iC\omega X_1 + F}_{\text{forces on unsprung mass}} = m\left(-\omega^2 X_1\right)$$

C: shock absorber viscous damping factor

$(1000 \sim 2000\, Ns/m)$

$$\frac{X_1}{F} = \frac{1}{k_1 + k_2 - m\omega^2 + iC\omega}$$

$$\frac{X_1}{F} = \frac{\dfrac{1}{k_1 + k_2}}{\left[1 - \left(\dfrac{\omega}{\omega_n}\right)^2\right] + i\left(\dfrac{C\omega}{k_1 + k_2}\right)}$$

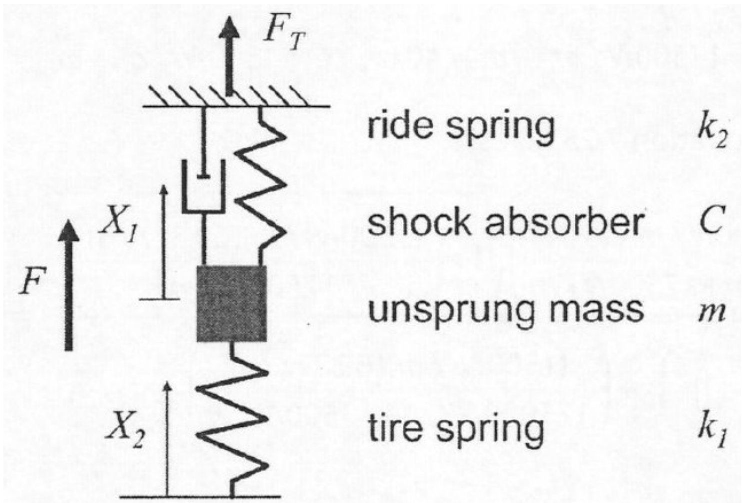
where $\omega_n{}^2 = \dfrac{k_1 + k_2}{m}$ $\left(\rightarrow f_n : \text{wheel hop frequency}\right)$

$$\left|\frac{X_1}{F}\right| = \frac{\dfrac{1}{k_1 + k_2}}{\sqrt{\left[1 - \left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left(\dfrac{C\omega}{k_1 + k_2}\right)^2}}$$

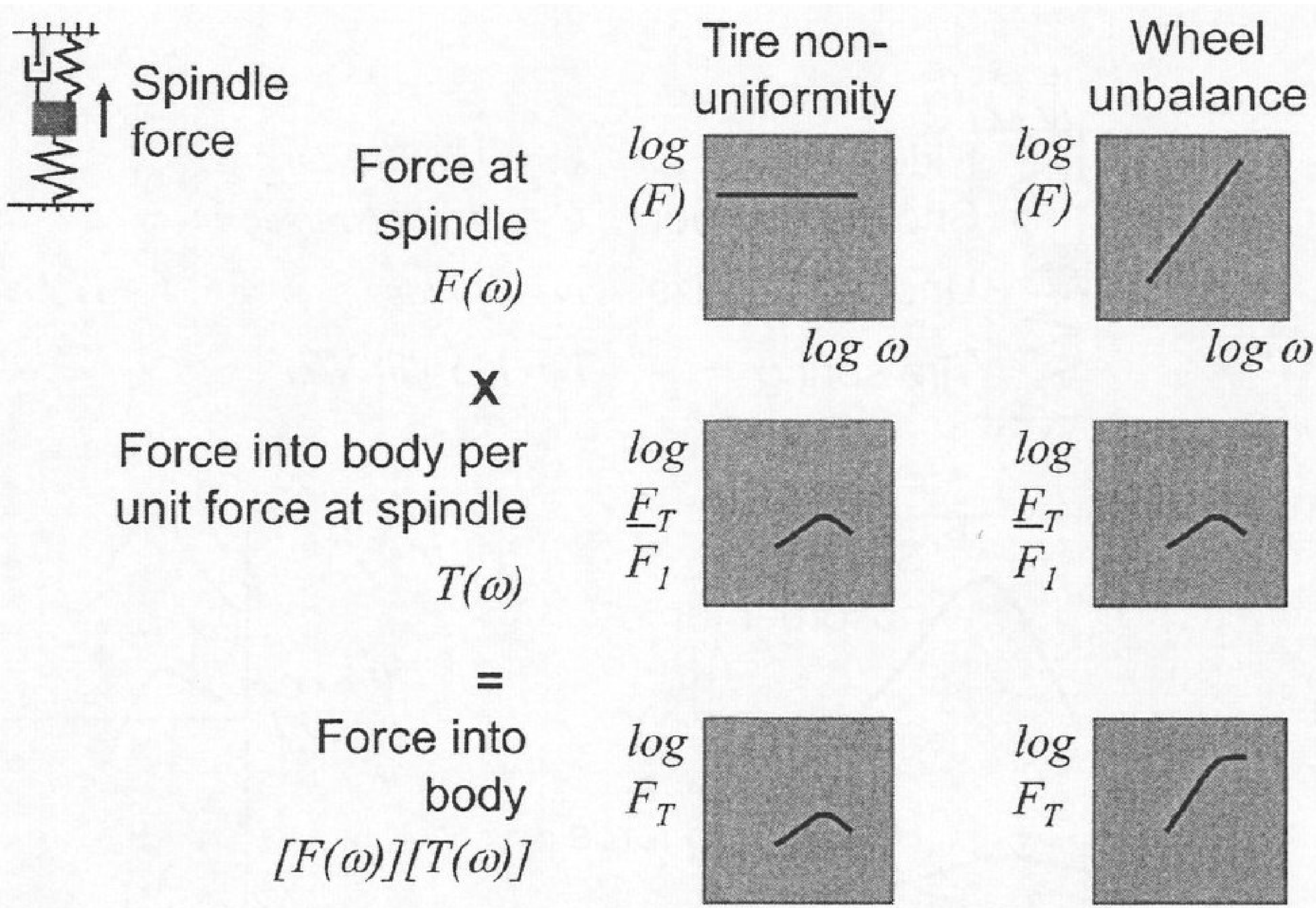
$$F_T = X_1\left(k_2 + iC\omega\right) \rightarrow \left|\frac{F_T}{X_1}\right| = \sqrt{k_2{}^2 + \left(C\omega\right)^2}$$

force transmitted to body through shock absorber and ride spring

$$\left|\frac{F_T}{F}\right| = \frac{\left(\dfrac{k_2}{k_1 + k_2}\right)\sqrt{1 + \left(\dfrac{C\omega}{k_2}\right)^2}}{\sqrt{\left[1 - \left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left(\dfrac{C\omega}{k_1 + k_2}\right)^2}} = \left|T\left(\omega\right)\right|$$

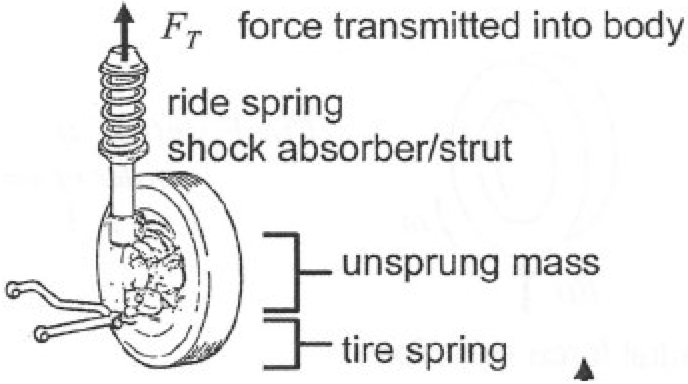


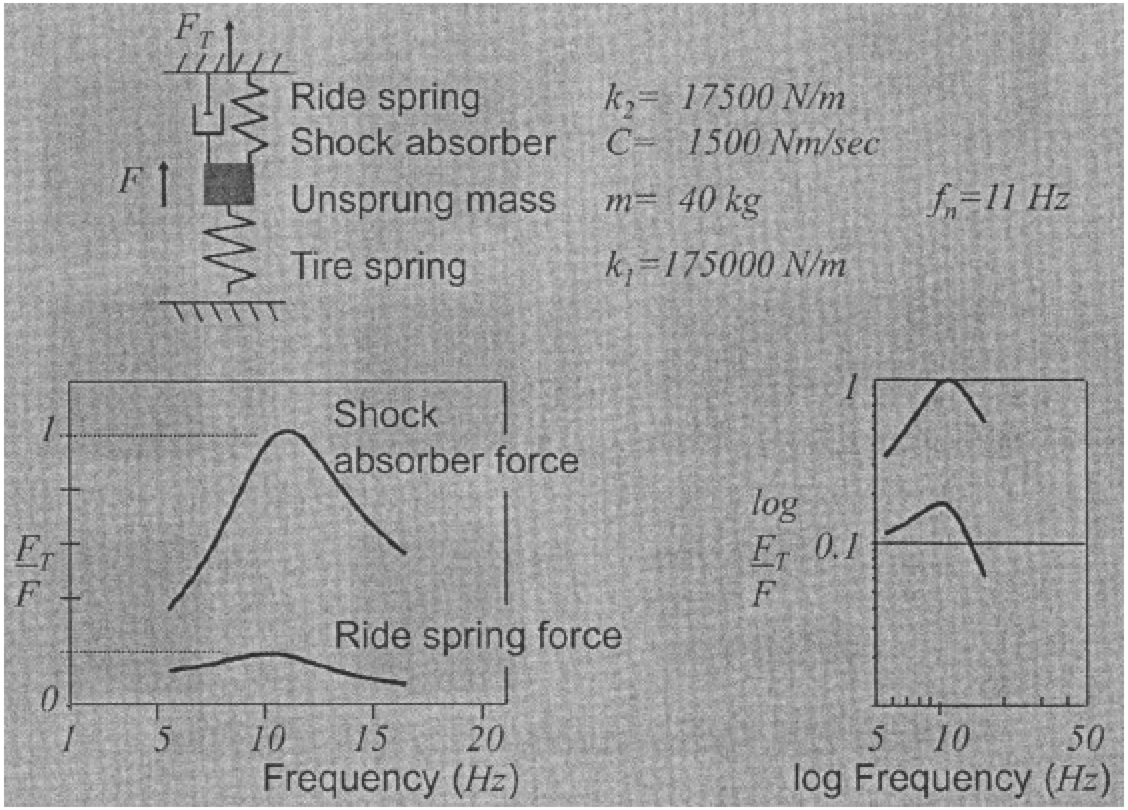
Force into Body due to Force at Spindle



Example

– A suspension with rolling radius R=300mm has an unbalance of 1oz(28.35g) at the tire rim of radius r=170mm. The vehicle is travelling at 70mph.
  - What is the unbalance force frequency and magnitude?
– For McPherson strut front suspension, typical tire radial rate k_1=175N/mm, ride rate k_2=17.5N/mm, unsprung mass m_1=40kg, C=1500Ns/m
  - Wheel hop frequency (moving at the vehicle speed)
  - Vehicle speed at resonance
  - Force transmitted to the body per unit force at the spindle at the vehicle speed corresponding to wheel hop





Force applied to the spindle will be passed un attenuated into the body at the wheel hop frequency. Above this frequency, the suspension will attenuate spindle forces

Suspension Path: Deflection at Tire Patch

- Dynamic characteristics for typical roads: Power Spectral Density (PSD) of the displacement
  - Means to characterize a random signal
  - Visualize as mean-square value of the signal as filtered through a 1Hz bandwidth filter at a center frequency f

$$G = G_0 \frac{\left[1+\left(\frac{\upsilon_0}{\upsilon}\right)\right]}{\left(2\pi\upsilon\right)^2}$$

$$G:\ \text{Power Spectral Density}\ \left[m^2/\left(cycle/m\right)\right],\ G_0 = \begin{cases} 1.35\times10^{-4}\ (\text{rough roads}) \\ 1.35\times10^{-5}\ (\text{smooth roads}) \end{cases}$$

$$\upsilon:\ \text{Wave number}\ \left(cycle/m\right),\ \upsilon_0 = \begin{cases} 0.015(\text{bituminous roads}) \\ 0.0061(\text{concrete roads}) \end{cases}$$

$$f = \upsilon V$$

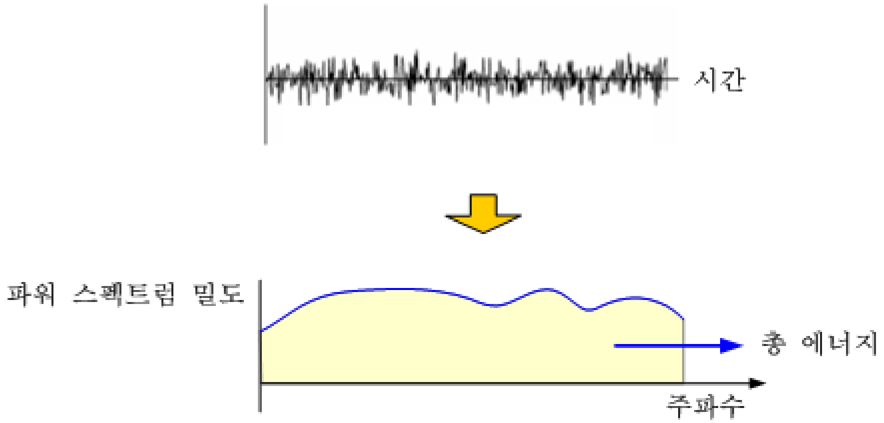
$$f:\ \text{temporal frequency}\ \left(Hz\right)$$

$$\upsilon:\ \text{spatial frequency}\ \left(cycle/m\right)$$

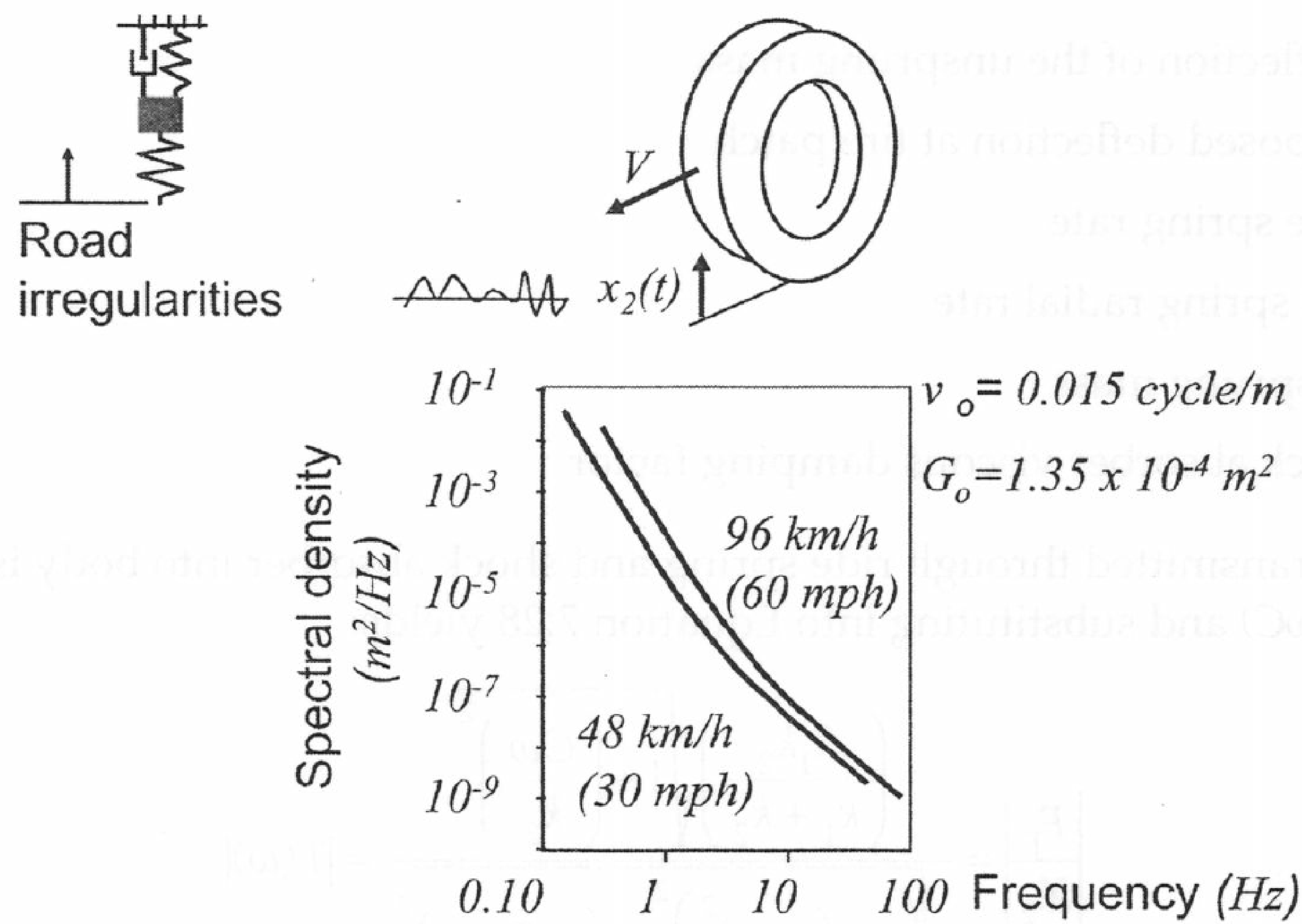
$$V:\ \text{vehicle speed}\ \left(m/\text{sec}\right)$$

Power Spectral Density (PSD)

- 에너지 스펙트럼 밀도
  - 시간 함수로 표현되는 에너지를 푸리에 변환(Fourier transform)을 통해 주파수 함수로 변환하였을 경우, 각 주파수 별 에너지의 크기
  - 각 주파수 별 에너지 값을 모든 주파수에 대하여 합산을 하게 되면 외란을 통해 전달되는 총 에너지를 구할 수 있다



Suspension Vibration Sources: Road Deflections



Suspension Analysis: Road Deflections

$$\underbrace{-k_1\left(X_1 - X_2\right) - k_2 X_1 - iC\omega X_1}_{\text{forces on unsprung mass}} = m\left(-\omega^2 X_1\right)$$

C : shock absorber viscous damping factor $\left(1000 \sim 2000\, Ns/m\right)$

$$k_1 X_2 = \left(k_1 + k_2 - m\omega^2 + iC\omega\right) X_1$$

$$\frac{X_1}{X_2} = \frac{k_1}{k_1 + k_2 - m\omega^2 + iC\omega}$$

$$\frac{X_1}{X_2} = \frac{\dfrac{k_1}{k_1 + k_2}}{\left[1 - \left(\dfrac{\omega}{\omega_n}\right)^2\right] + i\left(\dfrac{C\omega}{k_1 + k_2}\right)}$$

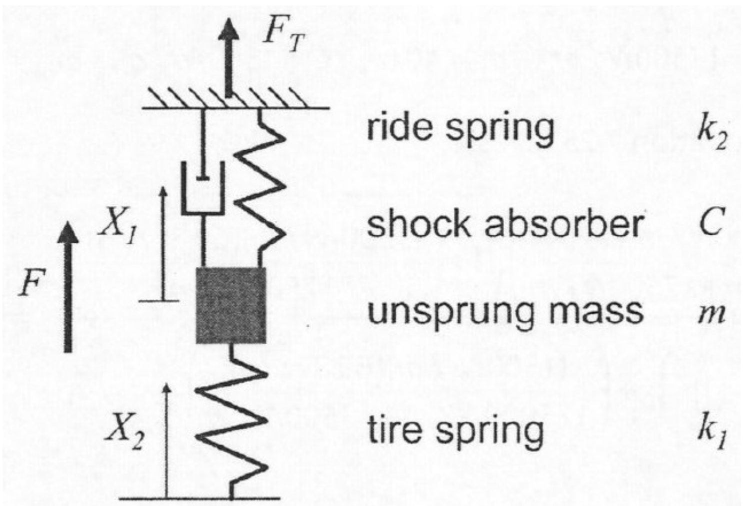
where $\omega_n{}^2 = \dfrac{k_1 + k_2}{m}$

$$\left|\frac{X_1}{X_2}\right| = \frac{\dfrac{k_1}{k_1 + k_2}}{\sqrt{\left[1 - \left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left(\dfrac{C\omega}{k_1 + k_2}\right)^2}}$$

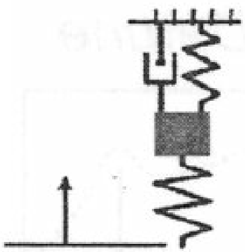
$$F_T = X_1\left(k_2 + iC\omega\right) \rightarrow \left|\frac{F_T}{X_1}\right| = \sqrt{k_2{}^2 + \left(C\omega\right)^2}$$

force transmitted to body through shock absorber and ride spring

$$\left|\frac{F_T}{X_2}\right| = \frac{\left(\dfrac{k_1 k_2}{k_1 + k_2}\right)\sqrt{1 + \left(\dfrac{C\omega}{k_2}\right)^2}}{\sqrt{\left[1 - \left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left(\dfrac{C\omega}{k_1 + k_2}\right)^2}} = \left|T\left(\omega\right)\right|$$



Force into Body due to Road Disturbance

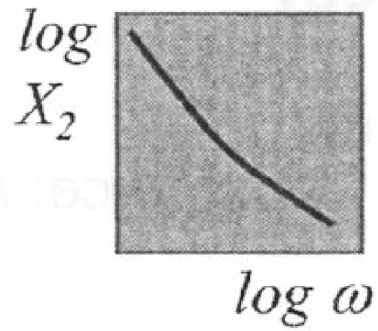


Road irregularities

Source

Typical road deflection

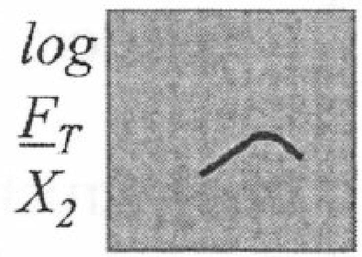
$F(\omega)$



X

Force into body per unit displacement at tire patch

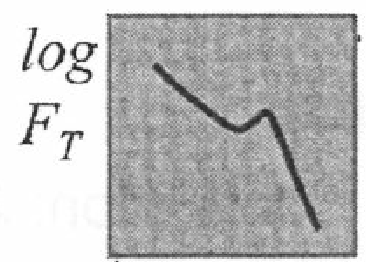
$T(\omega)$



=

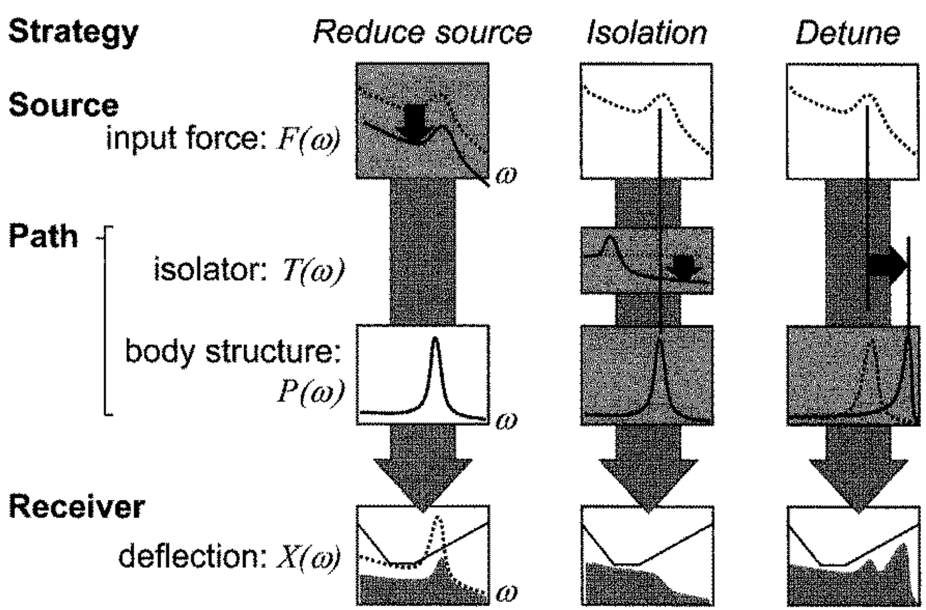
Force into body per typical road spectrum

$[F(\omega)][T(\omega)]$



7.5 Strategies for Design

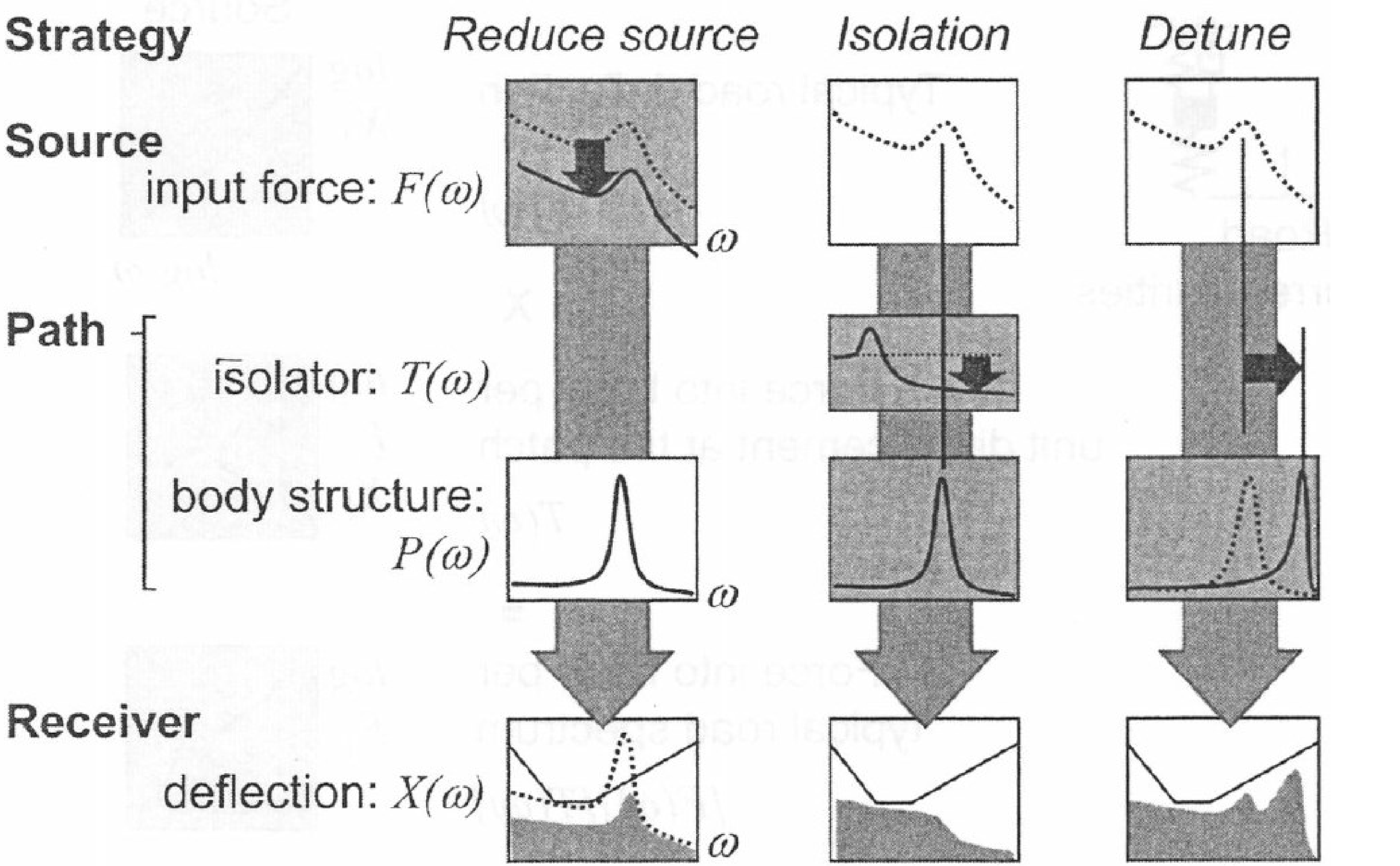
- Objective: minimize the source vibration energy flowing to the receiver with undesirable results
- Three of most important strategies
  - Reduce amplitude of the source
  - Block the flow of energy using isolators in the path
  - Detune resonances in the system



Design for Vibration Strategies

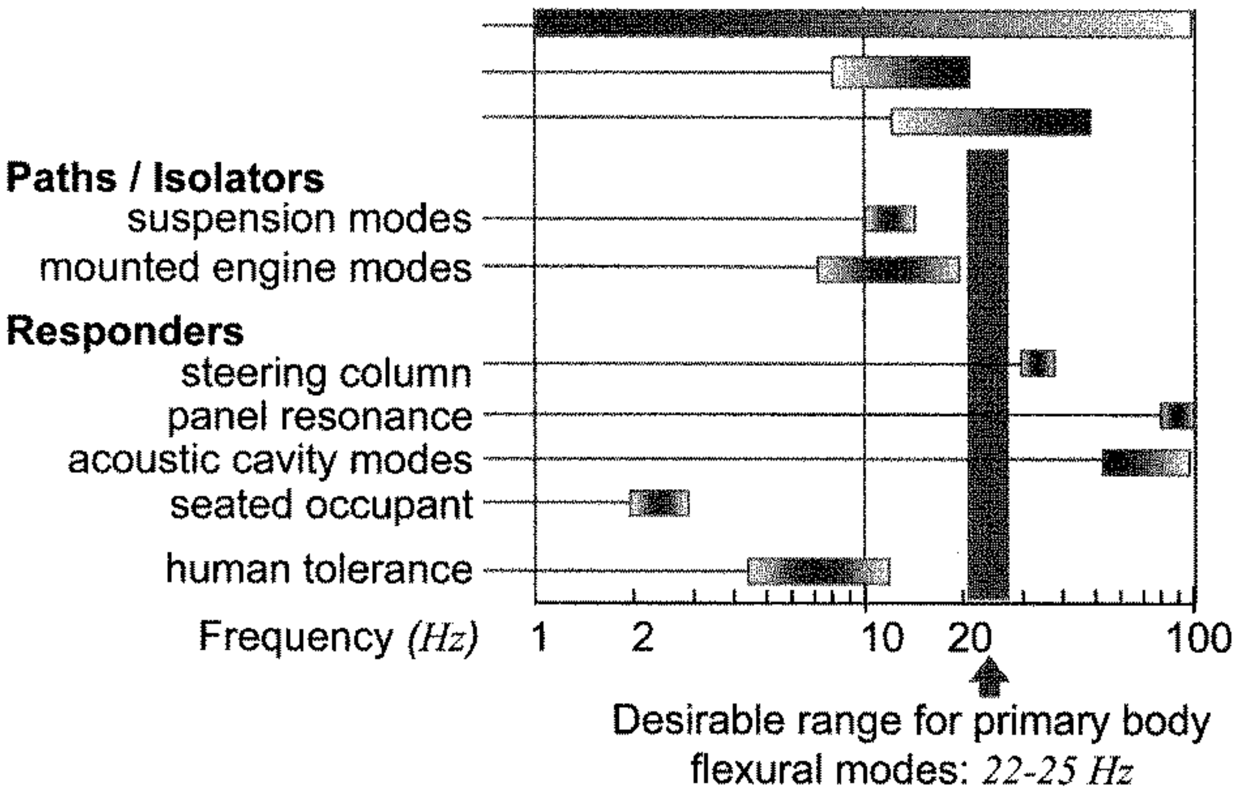
- Reduce amplitude of the source
  - Powertrain
    - Minimize reciprocating mass in engine
    - Add balance shafts to in-line 4 cylinder engine
  - Suspension
    - Balance tires
    - High quality tires with low radial force vibration
    - Minimize shock absorber forces using a linkage ratio ~ 1
- Block the flow of energy using isolators in the path
  - Mounted powertrain at isolator
  - Suspension as isolator
  - Rubber bushings in chassis links at acoustic frequencies
- Detune resonances in the system
  - Position body primary bending and torsion resonances

Vibration Control Strategies



Noise and Vibration Mode Map

- Detune resonances of the body from sources and responders
- Desirable structural resonance band: 22~25 Hz

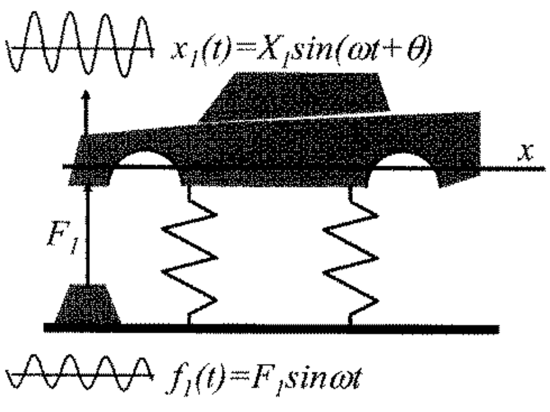


7.6 Body Structure Vibration Testing (1)

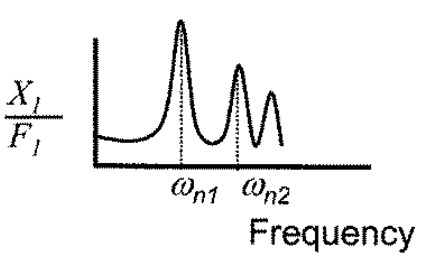
- Result of a vibration test
  - Transfer function: $P(\omega)$
  - Deflected shape (mode shape) for each resonance
- Typical test set-up
  - Support soft springs: inflated inner tubes or elastic cords
    - Rigid body modes at low frequencies (< 3Hz)
  - Electromagnetic or hydraulic shaker: (forcing location) front bumper attachment
    - Excite major modes of vibration (not near a nodal point)
    - Locally stiff (not to locally flexing the structure)
  - Accelerometer: body at the shaker attachment
    - Measure the driving point frequency response
  - input(randomly varying force)→[Fourier Transform]→(out signals)

Body Structure Vibration Testing (2)

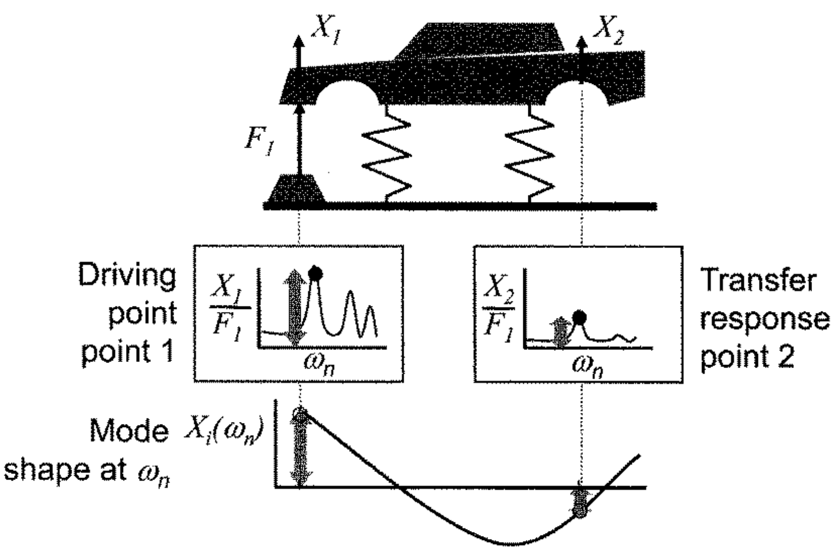
- Driving point response
  - Force amplitude fixed → frequency incremented
- Mode shape
  - Forcing frequency fixed (resonance) → amplitude measured
  - Node(no deflection) / Anti-node(greatest deflection)
  - Lightly damped structure: In-phase / 180° out-of-phase



Vibration test set up

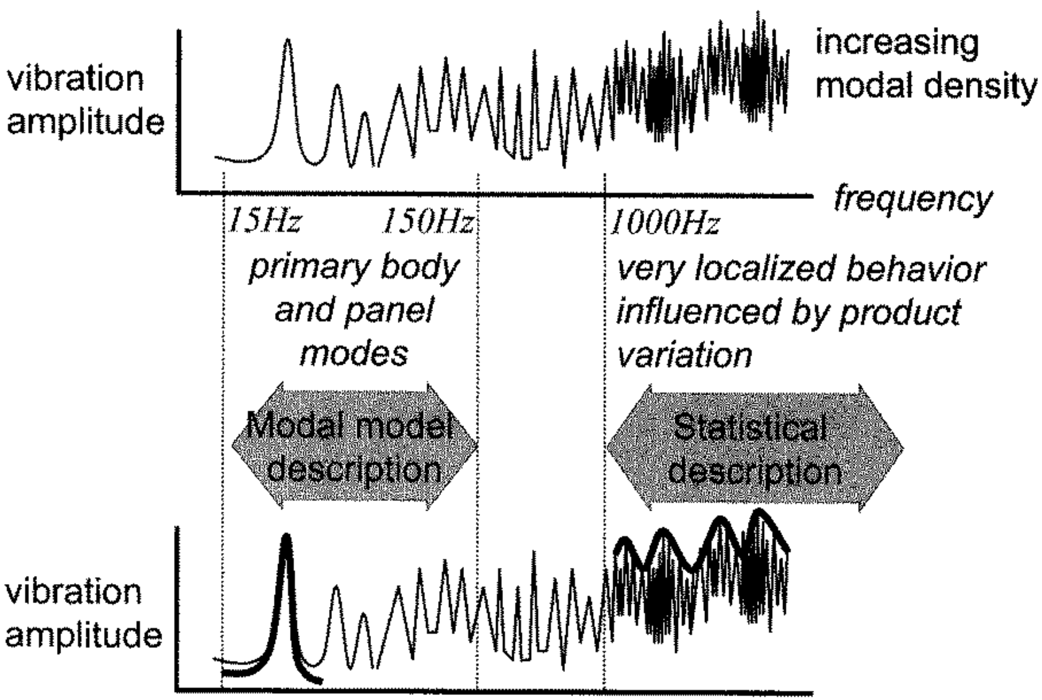


Driving point response



7.7 Modeling Resonant Behavior

- Structure's modal density
  - Number of modes occurring in a fixed bandwidth
  - Increase with increasing frequency
- Lower frequency
  - 10~150Hz
  - individual modes
  - modal model
- High frequency
  - 1000Hz~
  - high modal density
  - statistical approach



Modal Model (1)

- Primary modes of vibration

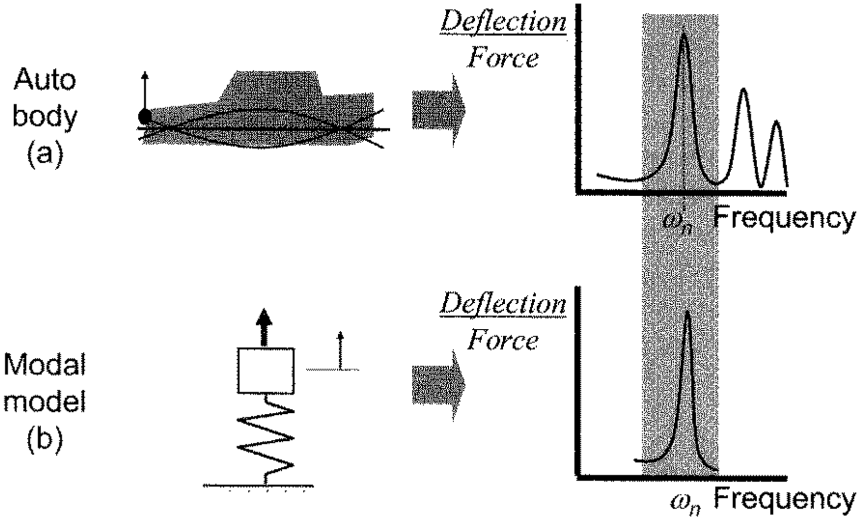
$$F_{\text{physical}} \rightarrow F_{\text{modal}} \rightarrow X_{\text{modal}} \rightarrow X_{\text{physical}}$$

$$F_{\text{modal}} = F_{\text{physical}} \phi_{\text{input}}$$

$$\begin{cases} F_{\text{physical}}: \text{force applied to the physical body} \\ \qquad\qquad \text{structure at the input location} \\ F_{\text{modal}}: \text{force applied to the modal model} \\ \phi_{\text{input}}: \text{influence coefficient at the input} \\ \qquad \text{(determined from mode shape at resonance)} \end{cases}$$

$$X_{\text{physical}} = X_{\text{modal}} \phi_{\text{output}}$$

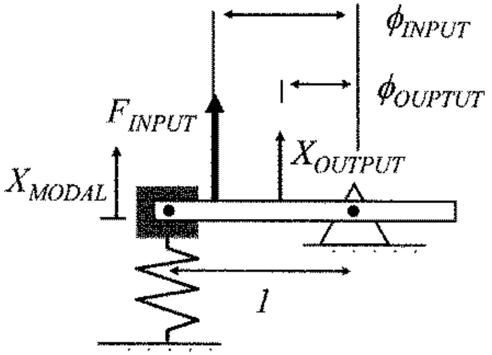
$$\begin{cases} X_{\text{physical}}: \text{deflection of the physical body} \\ \qquad\qquad \text{structure at the output location} \\ X_{\text{modal}}: \text{deflection of the modal model} \\ \phi_{\text{output}}: \text{influence coefficient at the output} \\ \qquad \text{(determined from mode shape at resonance)} \end{cases}$$

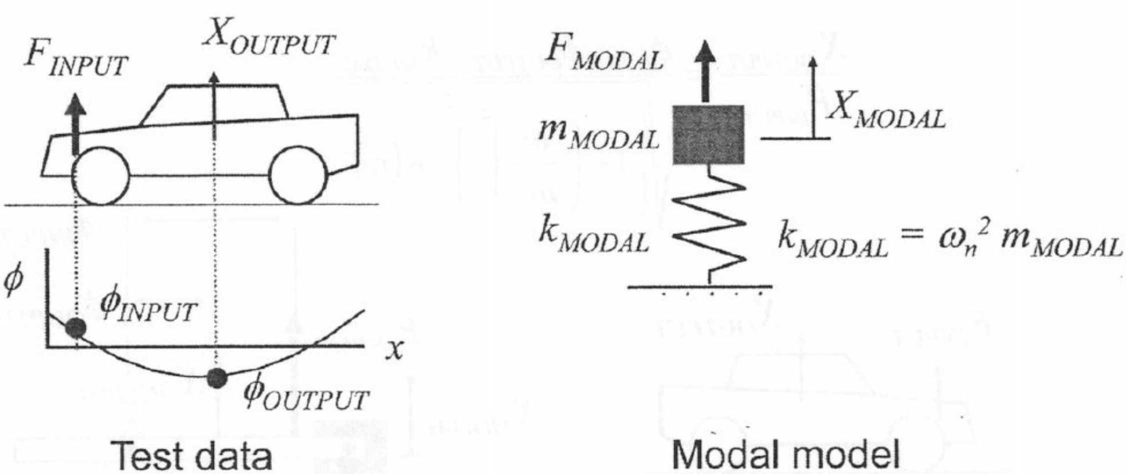


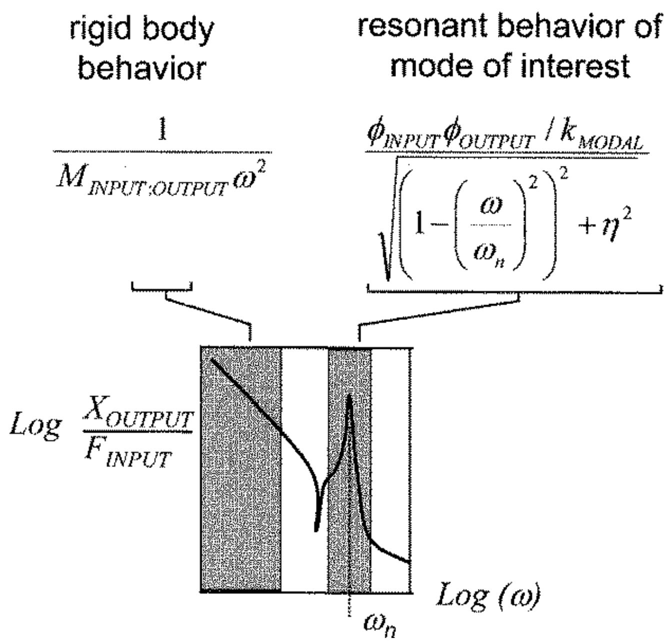
Modal Model (2)

$$F = kX - m\omega^2 X \rightarrow \frac{X}{F} = \frac{1}{k - m\omega^2} = \frac{1/k}{1-\left(\frac{m}{k}\right)\omega^2} = \frac{1/k}{1-\left(\frac{\omega}{\omega_n}\right)^2} = P(\omega)$$

$$\frac{X_{\text{physical}}}{F_{\text{physical}}} = \frac{X_{\text{modal}}\phi_{\text{output}}}{F_{\text{modal}}/\phi_{\text{input}}} = \phi_{\text{input}}\phi_{\text{output}}\frac{X_{\text{modal}}}{F_{\text{modal}}} = \frac{\phi_{\text{input}}\phi_{\text{output}}/k_{\text{modal}}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\eta^2}}$$







Example: Effect of Mass Placement

- Primary body resonance: 22~25 Hz
- Increase the resonant frequency
  - Increased body stiffness
  - Careful placement of subsystem masses
- Selection of battery location: front corner, dash, trunk

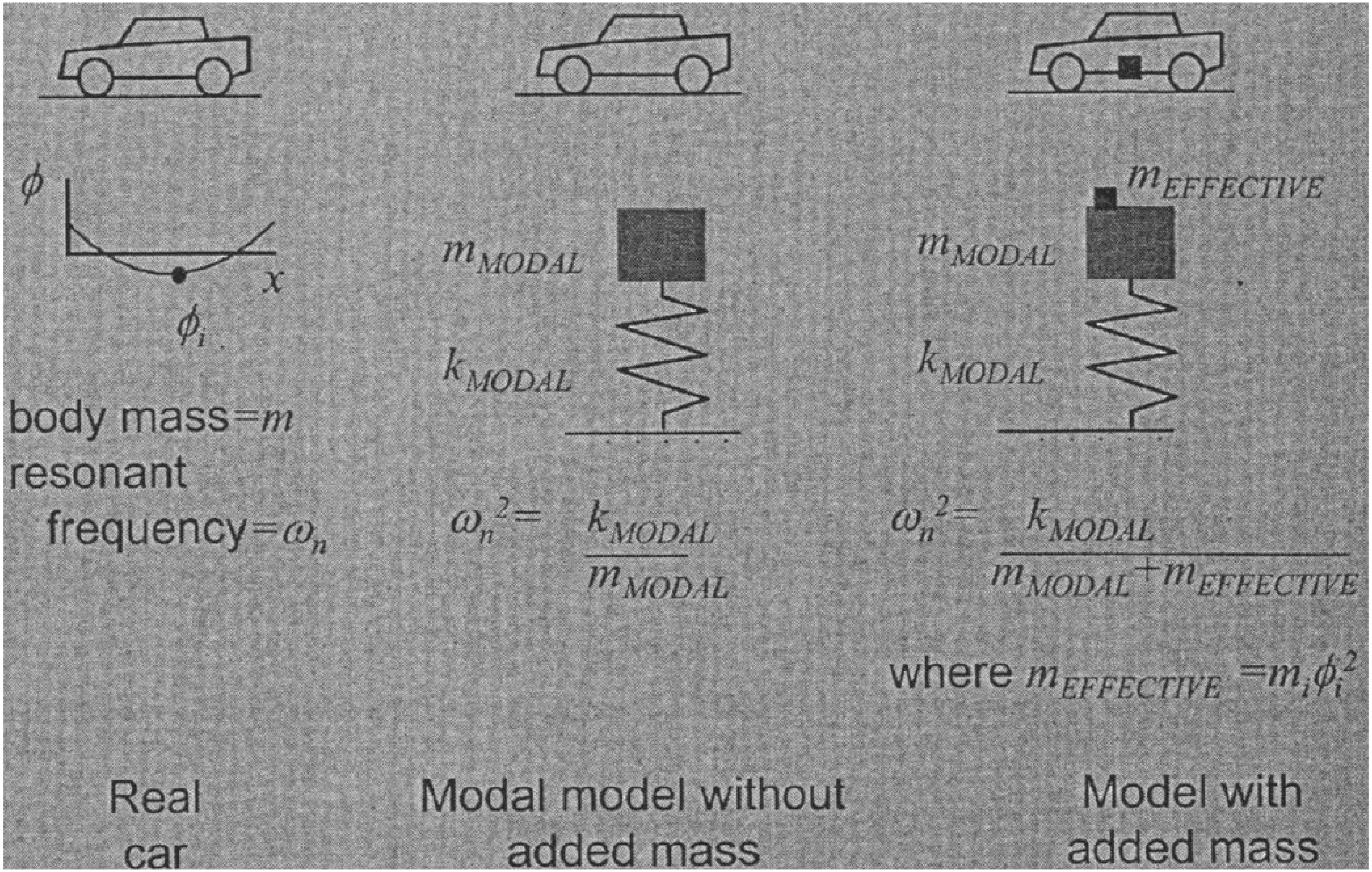
$$\text{primary bending resonance: } f_n = 47\text{Hz}\left(\omega_n = 295.3\,\text{rad/sec}\right) \xrightarrow{\text{influence coefficients}} \phi_i = \begin{cases} 0.9 \text{ @front corner} \\ -0.2 \text{ @dash} \\ 0.15 \text{ @trunk} \end{cases}$$

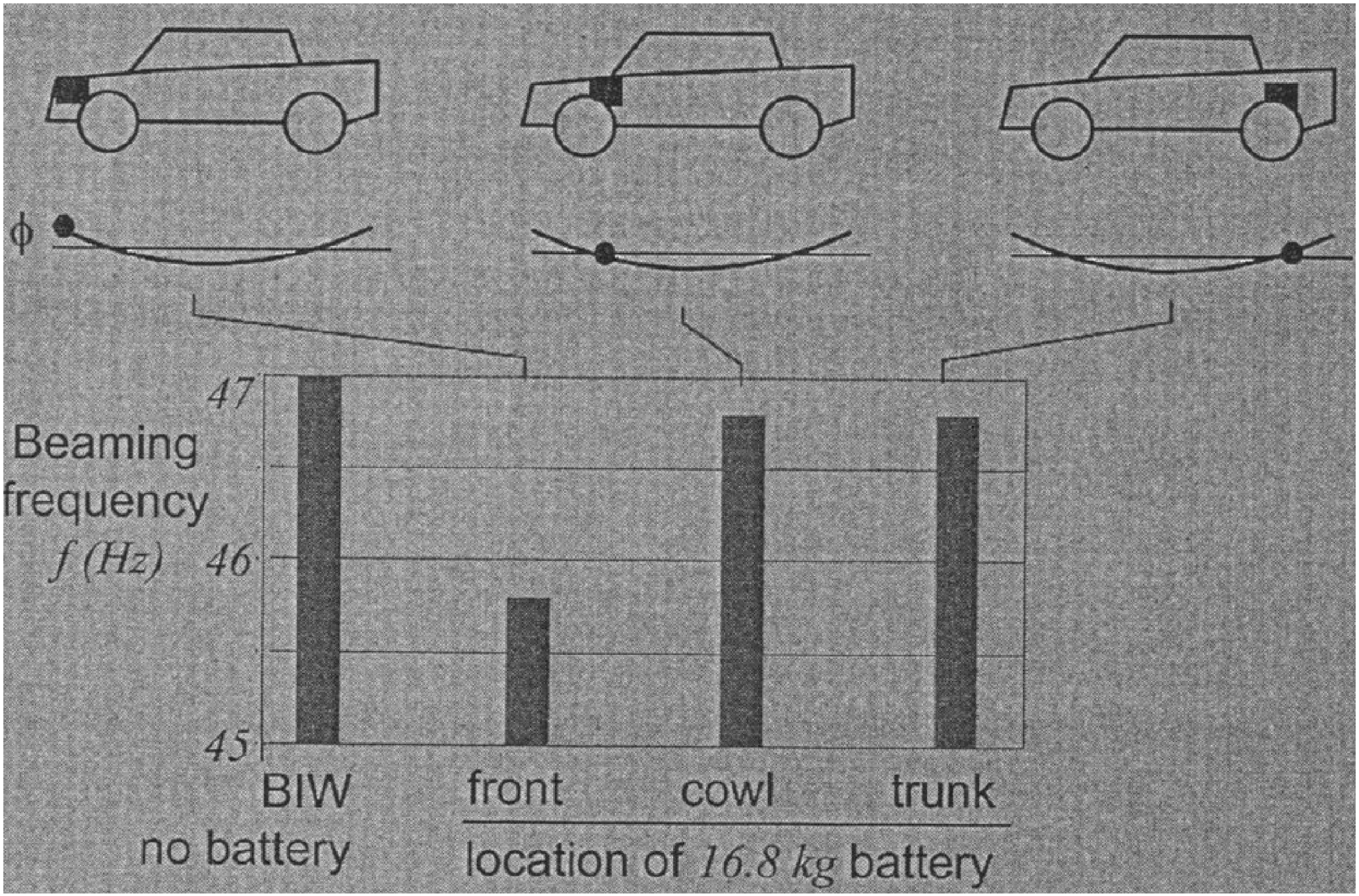
body shell mass: $M = 250\text{kg}\left(= M_{\text{modal}}\right) \rightarrow$ modal stiffness?

battery mass: $m = 16.8\text{kg} \rightarrow$ effective mass of the battery?

primary bending frequency for the battery at each location?

Effect on Resonant Frequency of an Added Mass



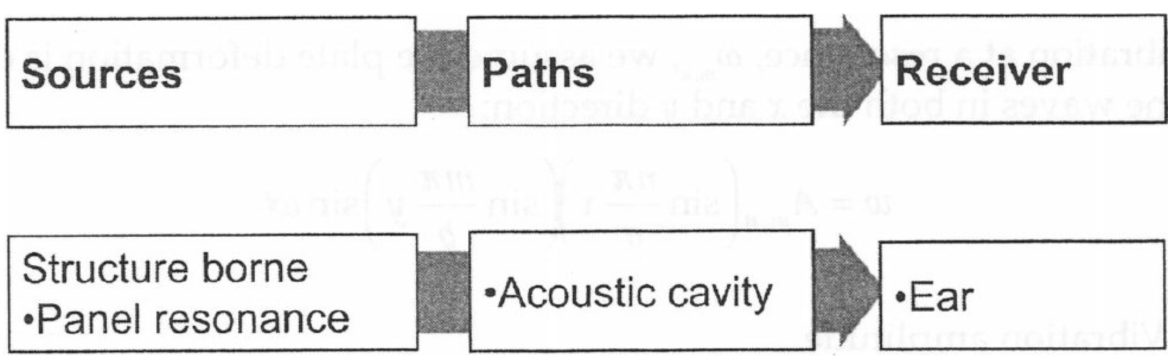


φ
47
46
45
Beaming
frequency
f (Hz)
BIW
no battery
front
cowl
trunk
location of 16.8 kg battery

7.8 Vibration at High Frequency

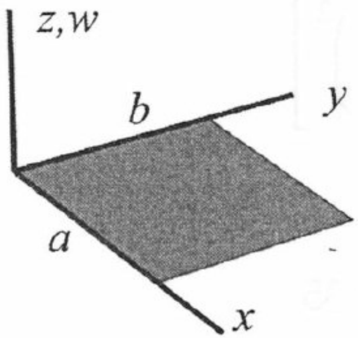
- Primary body structure resonance: 18~50Hz
  - Vibration at the receiver: tactile
- Higher frequencies: 50~400Hz
  - More localized response of body structure, acoustic
- Structure-borne panel vibration system

Source $F(\omega)$	Body structure transfer function $P(\omega)$	Acoustic deflection $X(\omega)$
Body panel vibrations	Passenger compartment acoustic resonances	Interior sound pressure



Body Panel Vibration (1)

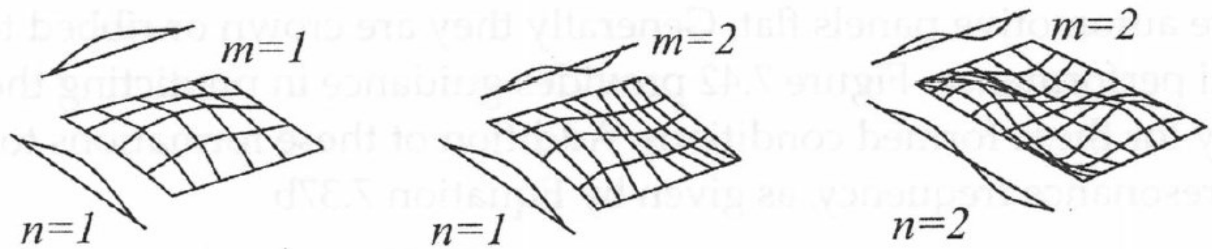
$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{q}{D} = 0 \quad \begin{cases} w(x,y): \text{ normal deflection of the plate} \\ q(x,y): \text{ normal load per unit area} \\ D = \dfrac{Et^3}{12(1-\nu^2)}: \text{ plate bending stiffness} \end{cases}$$



Let deflected shape be:

$w = A_o sin(n\pi x/a) sin(m\pi y/b)\ sin\ \omega_n t$

(note similarity to plate buckling shapes)



$$\omega_n = \sqrt{\frac{Et^3}{12(1-\mu^2)m''}}\left(\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2\right)$$

where
m''=mass per unit area

Body Panel Vibration (2)

$$\left.\begin{aligned} w_{m,n} &= A_{m,n}\left(\sin\frac{n\pi}{a}x\right)\left(\sin\frac{m\pi}{b}x\right)\sin\omega t \\ q &= m''\frac{\partial^2 w}{\partial t^2} \end{aligned}\right\} \rightarrow \frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} + \frac{q}{D} = 0$$

$$\left[\left(\frac{n\pi}{a}\right)^4 + 2\left(\frac{n\pi}{a}\right)^2\left(\frac{m\pi}{b}\right)^2 + \left(\frac{m\pi}{b}\right)^4 - \frac{\omega^2 m''}{D}\right]\left[A_{m,n}\left(\sin\frac{n\pi}{a}x\right)\left(\sin\frac{m\pi}{b}x\right)\sin\omega t\right] = 0$$

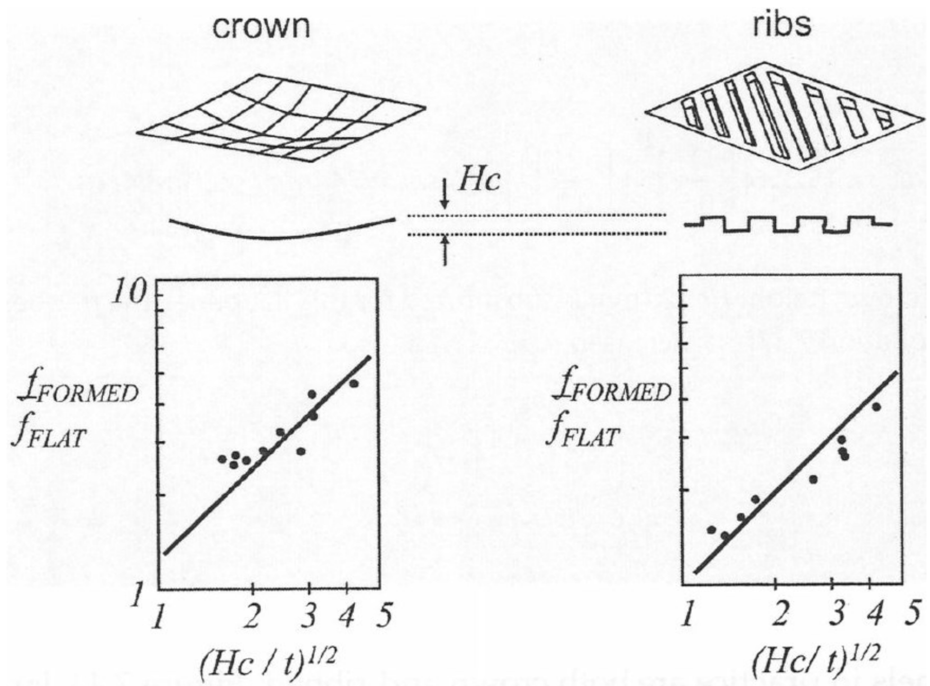
$$\rightarrow \omega_{m,n} = \pi^2\sqrt{\frac{D}{m''}}\left[\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2\right]$$

$$\frac{f_{\text{FORMED}}}{f_{\text{FLAT}}} = C\sqrt{\frac{H_C}{t}}$$

H_C : crown height

t : panel thickness

$$C = \begin{cases} 1.25 \ (\text{crown}) \\ 1 \ (\text{ribbed}) \end{cases}$$

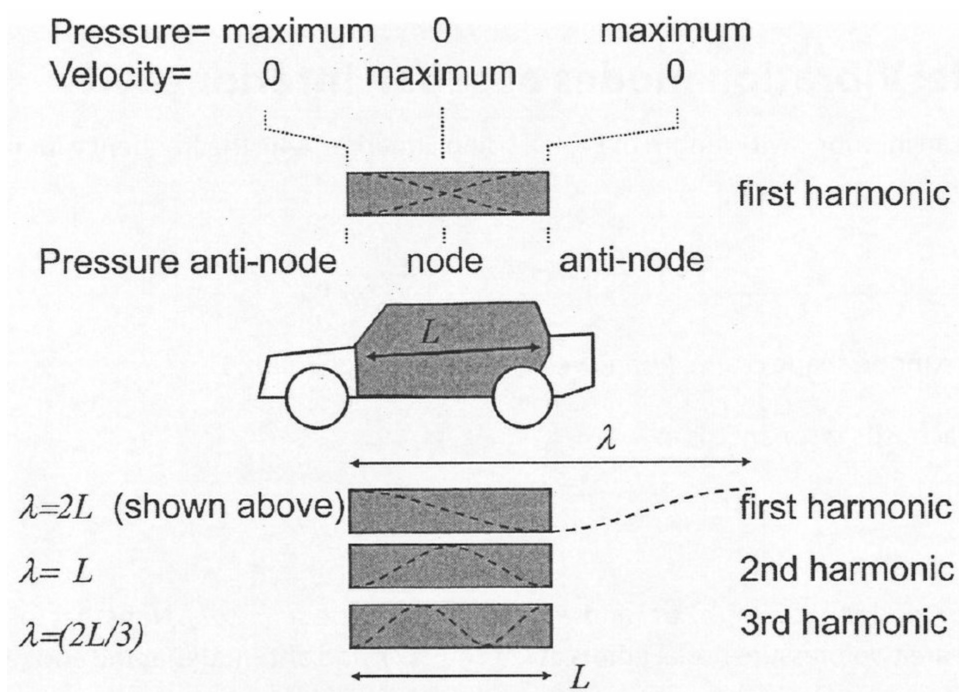


Acoustic Cavity Resonance

- Closed air cavity of passenger compartment
  - Resonate with a standing acoustic wave
  - Closed boundary conditions at either end

$$\left.\begin{aligned} f_n \lambda &= c \\ \lambda &= \frac{2L}{n} \end{aligned}\right\} \rightarrow f_n = c\left(\frac{n}{2L}\right)$$

$$\begin{cases} f_n: \text{ resonant frequency}(Hz) \\ \lambda: \text{ wavelength} \\ c: \text{ speed of sound in air}(330\,m/\text{sec}) \\ L: \text{ cabin length} \\ n: \text{ number of half cosine waves along cabin length} \end{cases}$$



pressure mode shape @each resonance: $\cos\left(\frac{n\pi x}{L}\right)$

notion of sound level

air velocity mode shape @each resonance: $\sin\left(\frac{n\pi x}{L}\right)$

indication of sensitivity of cavity mode to excitation by a panel

Example

- Vibration frequencies for floor pan
- Vibration modes of sedan interior cavity

(1) The floor pan at the rear foot $\rightarrow$ flat panel

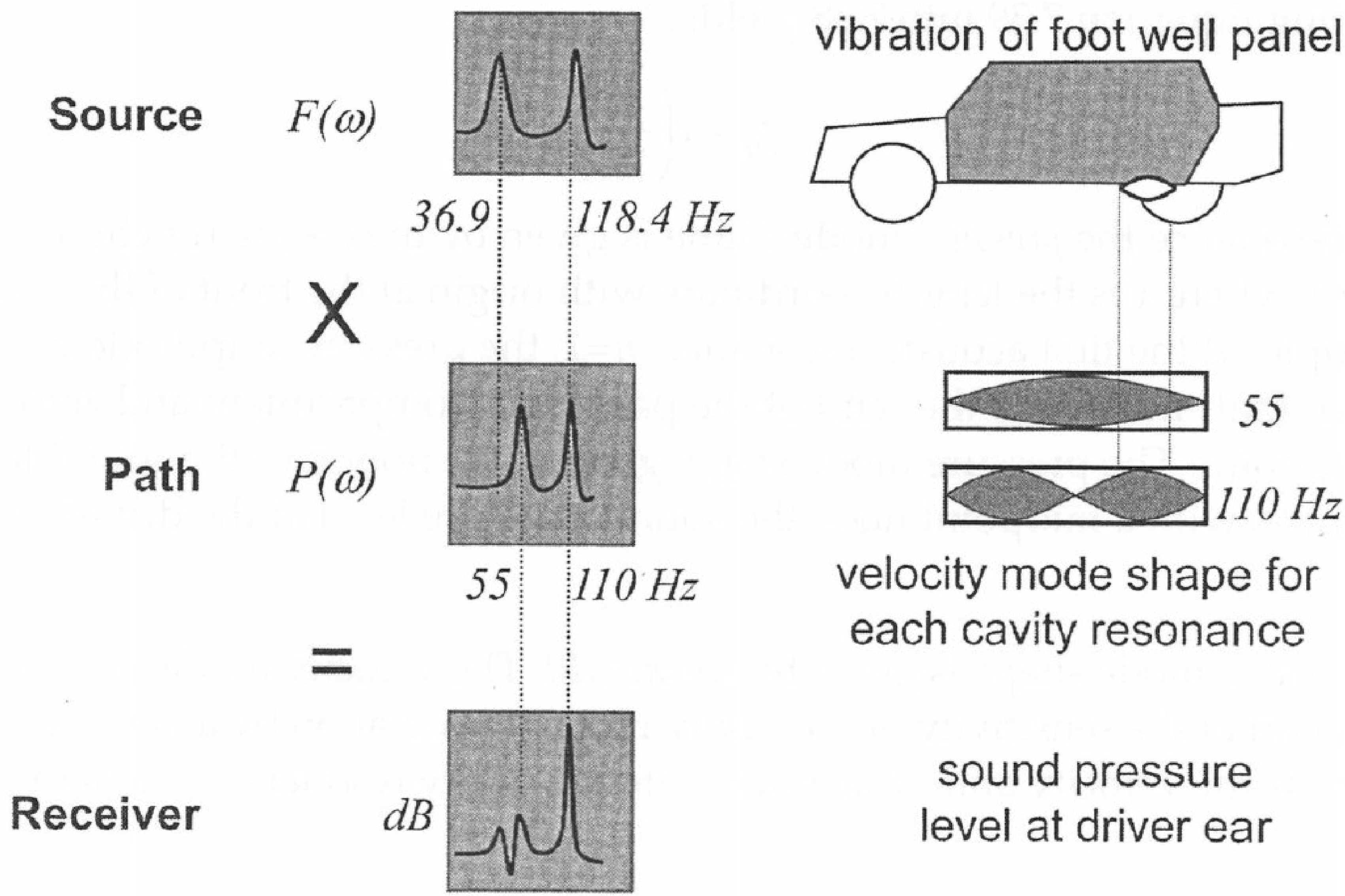
$$\left.\begin{array}{l} a = 500mm \\ b = 300mm \\ t = 1mm \\ \rho = 7.83 \times 10^{-6}\, kg/mm^2 \end{array}\right\} \rightarrow \omega ? \begin{cases} \omega_{1,1} \\ \omega_{1,2} \end{cases}$$

$$H_C = 20mm \rightarrow f_{\mathrm{FORMED}}\ ?$$

(2) A sedan has an interior cavity length of $L = 3m$.

frequency of first and second acoustic resonance?

Panel and Acoustic Cavity



Vibration Isolation through Elastomeric Elements

- Suspension elements due to road impacts → high frequency deflections
- Isolation of higher frequency vibration
  - Elastomeric bushings at the body connections

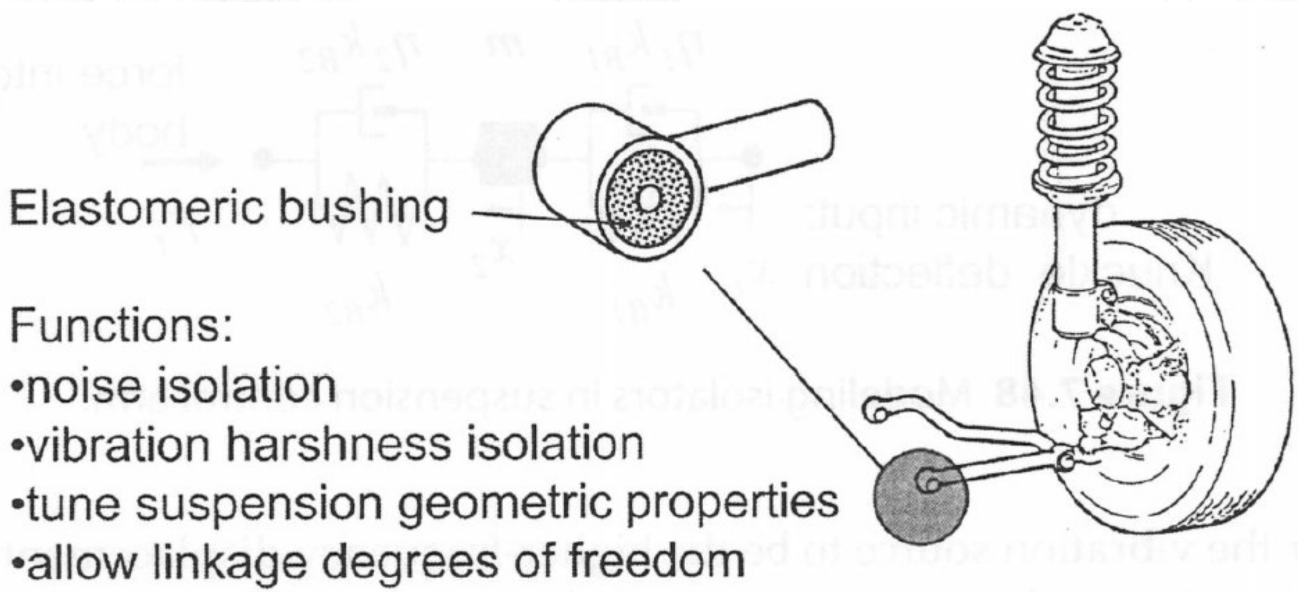
Source	**Isolator**	**Force into body**	**Body transfer function**	**Body deflection**
$F(\omega)$	$T(\omega)$	$F_T(\omega)$	$P(\omega)$	$X(\omega)$
High frequency chassis deflections	Chassis links with end bushings	Body panel vibrations	Passenger compartment acoustic resonances	Interior sound pressure

Suspension Lower Control Arm

High frequency displacement → Isolator → Panel (structure borne noise)

- Suspension noise and harshness
- Chassis noise and harshness

- Suspension links with rubber bushings
- Body mounts



Functions:

•noise isolation
•vibration harshness isolation
•tune suspension geometric properties
•allow linkage degrees of freedom

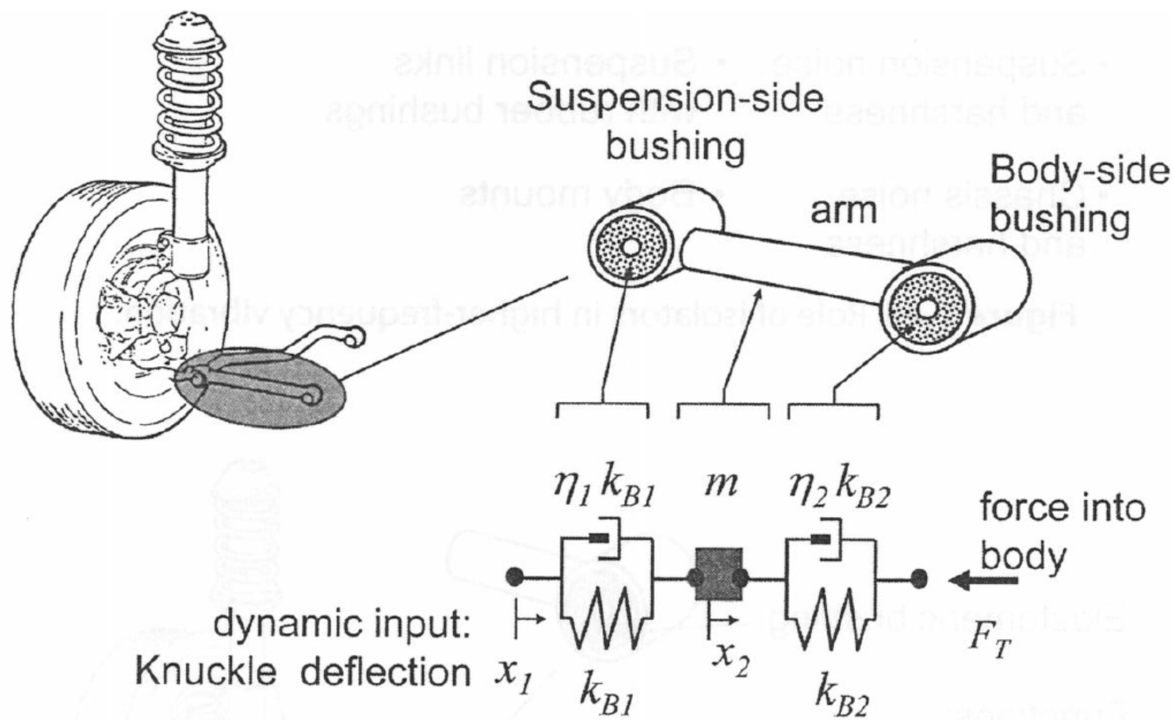
Modeling Isolators

$$F = kX + i\eta kX = k^* X \rightarrow \frac{F}{X} = \underbrace{k}_{\text{stiffness}} + i \underbrace{\eta k}_{\text{damping}} = k^*$$

F : force through the bushing

X : deflection across the bushing

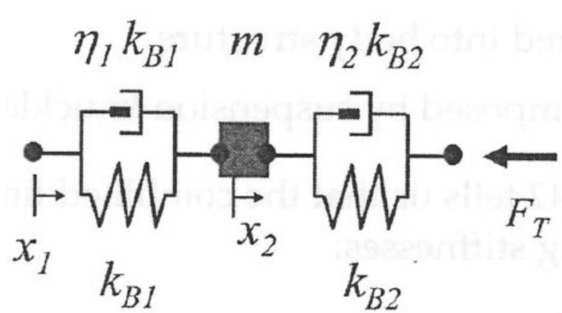
η : loss factor for the elastomeric material

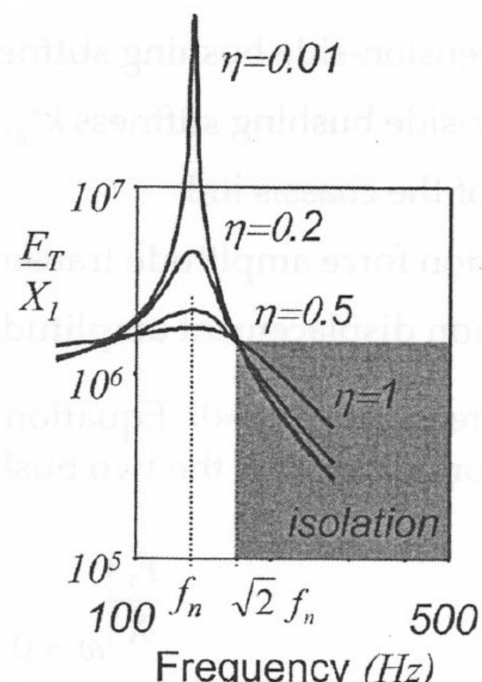


Response of Isolators

$$\frac{F_T}{X_1}=\frac{\dfrac{k_{B1}^*k_{B2}^*}{k_{B1}^*+k_{B2}^*}}{1-\omega^2\dfrac{m}{k_{B1}^*+k_{B2}^*}}\rightarrow\begin{cases}\omega\approx 0\left(\text{static stiffness}\right):\left|\dfrac{F_T}{X_1}\right|=\dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}}\\ \omega_n=\sqrt{\dfrac{k_{B1}+k_{B2}}{m}}:\left|\dfrac{F_T}{X_1}\right|=\left(\dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}}\right)\dfrac{\sqrt{1+\eta^4}}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2+\eta^2}}=\left|T\left(\omega\right)\right|\end{cases}$$

$$\begin{cases}k_{B1}^*:\ \text{suspension-side bushing stiffness, } k_{B1}^*=k_{B1}+i\eta_1k_{B1}\\ k_{B2}^*:\ \text{body-side bushing stiffness, } k_{B2}^*=k_{B2}+i\eta_2k_{B2}\\ m:\ \text{mass of the chassis link}\\ F_T:\ \text{vibration force amplitude transmitted into body structure}\\ X_1:\ \text{vibration displacement amplitude imposed by suspension knuckle}\end{cases}$$





Example: suspension lower control arm

- Source of high frequency vibration
  - gear meshing in the transmission → front wheel drive shaft → suspension knuckle → suspension control arm → body structure

$$\text{mesh frequency: } f = 400Hz$$

$$k_{B1} = k_{B2} = 175000\, N/m$$

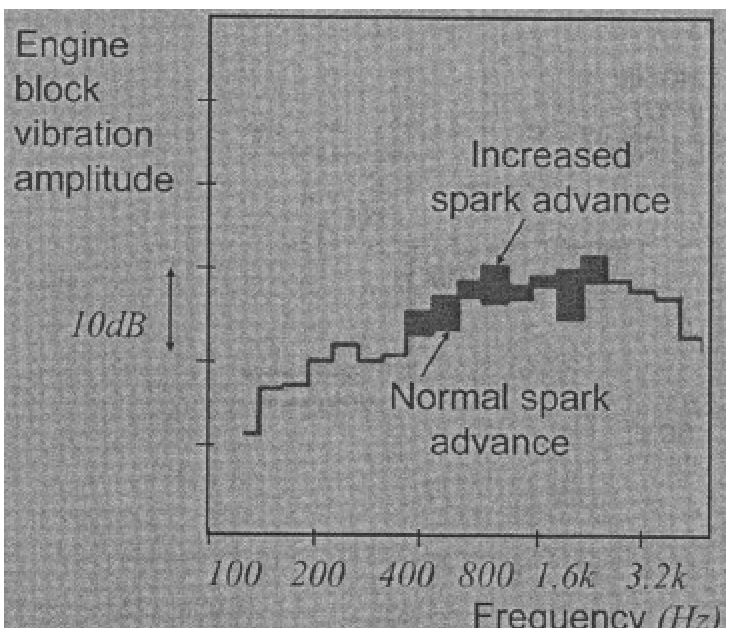
$$\eta = 0.2$$

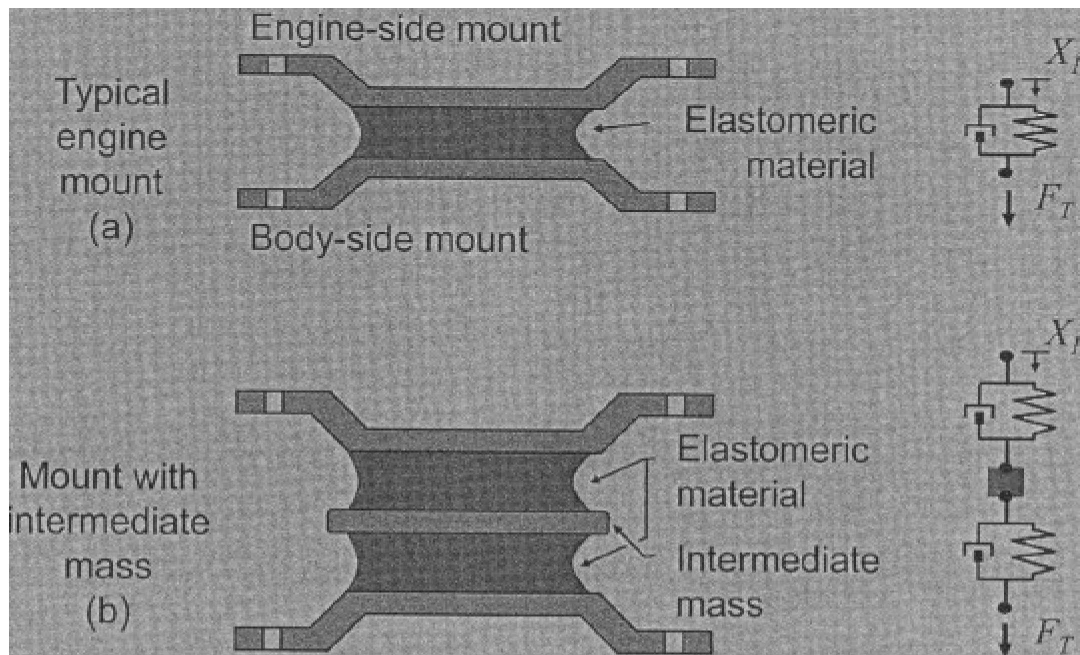
$$m = 5kg$$

$$\begin{cases} \omega \approx 0: \left|\dfrac{F_T}{X_1}\right| = \dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}} = 875000\dfrac{N}{m} \\ \omega_n = \sqrt{\dfrac{k_{B1}+k_{B2}}{m}} = 836.7\dfrac{rad}{s}(133Hz): \\ \left|\dfrac{F_T}{X_1}\right| = \left(\dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}}\right)\dfrac{\sqrt{1+\eta^4}}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2+\eta^2}} = \left(\dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}}\right)\dfrac{\sqrt{1+0.2^4}}{\sqrt{\left[1-\left(\dfrac{400}{133}\right)^2\right]^2+0.2^2}} = \underbrace{0.125\left(\dfrac{k_{B1}k_{B2}}{k_{B1}+k_{B2}}\right)}_{\text{dynamic force into body}} \end{cases}$$

Example: High-Frequency Powertrain Vibration through Engine Mount (1)

- Powertrain → engine mount → body structure: direct mount
- High frequency vibration of engine block: structure-borne noise
- Increase engine spark timing → improve fuel economy
  - Increase dynamic block deflections in 400~2000Hz range
  - To isolate acoustic vibrations, engine mount with free mass





Example: High-Frequency Powertrain Vibration through Engine Mount (2)

- Target static stiffness: 200 N/mm
- Isolation begins at 270 Hz
- Needed intermediate mass?

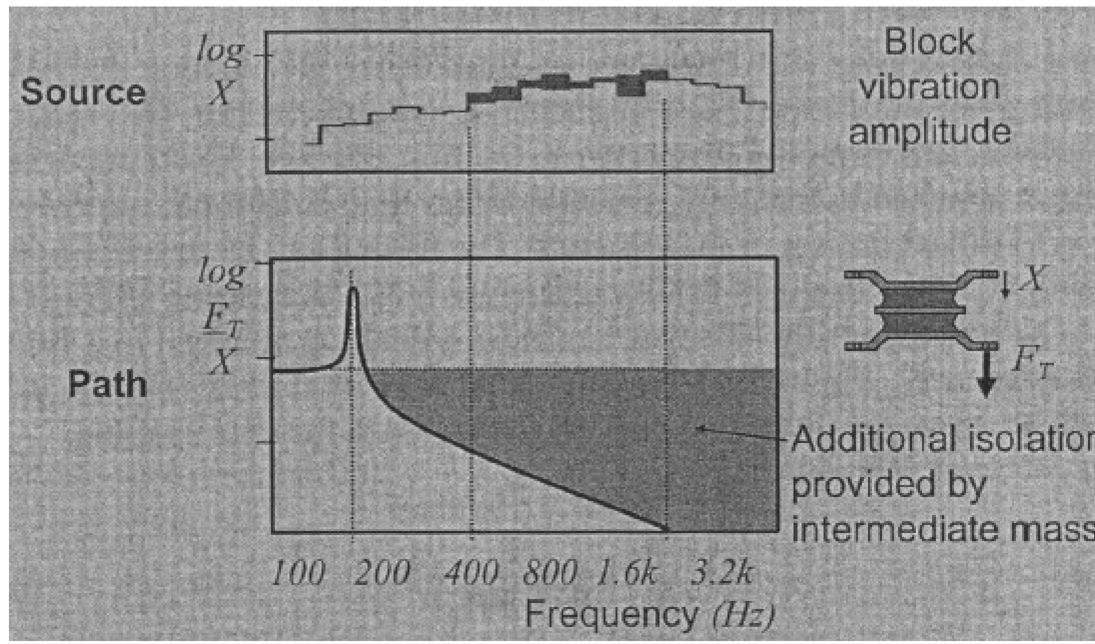
$$\omega \approx 0 : \left| \frac{F_T}{X_1} \right| = \frac{k_{B1} k_{B2}}{k_{B1} + k_{B2}} = 200 \frac{N}{mm}$$

$$\rightarrow k_{B1} = k_{B2} = 400 \frac{N}{mm}$$

$$f_n \sqrt{2} = 270Hz \rightarrow f_n = 190Hz$$

$$\omega_n = \sqrt{\frac{k_{B1} + k_{B2}}{m}}$$

$$\rightarrow m = \frac{k_{B1} + k_{B2}}{\omega_n^{\ 2}} = \frac{2(400)\frac{N}{10^{-3} m}}{\left[2\pi(190)\right]^2} = 0.56kg$$

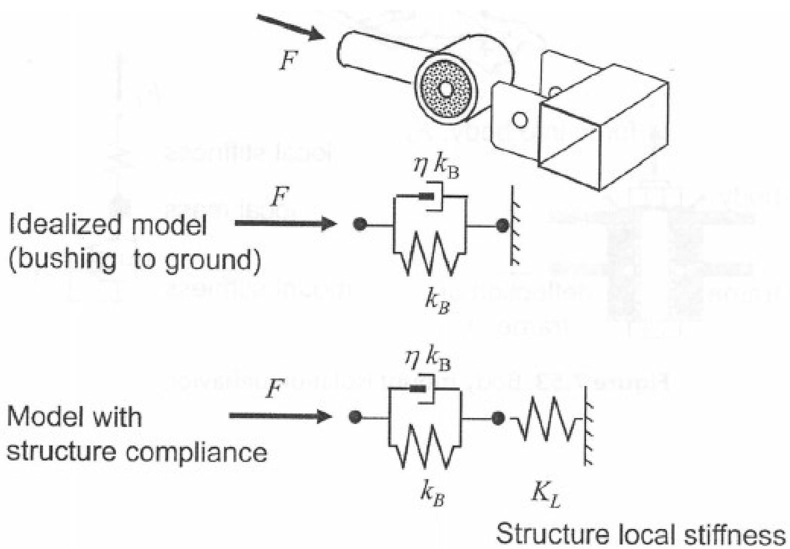


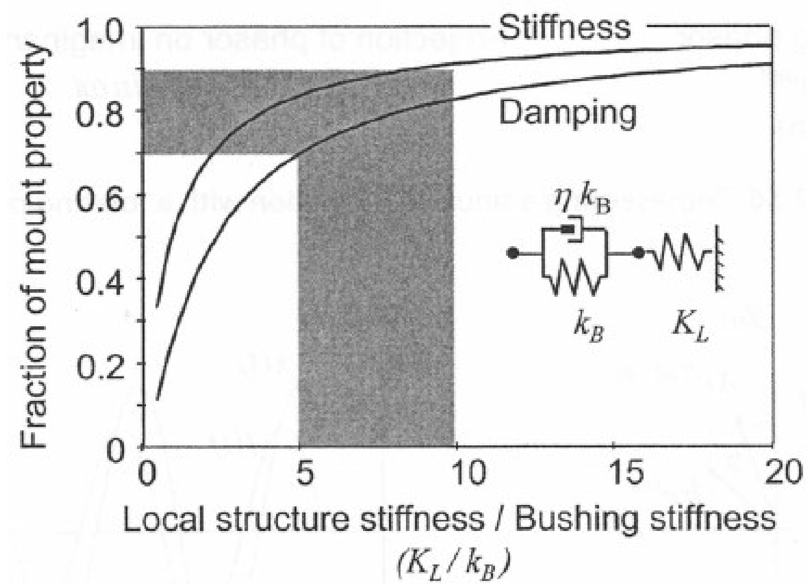
Local Stiffness Effect on Vibration Isolators

- Desired high-frequency-isolation: bush material?
- Localized flexing of structure: local stiffness (K_L)
  - five times the bushing stiffness to maintain 70% of damping

$$X = X_{local} + X_{bushing} = \frac{F}{K_L} + \frac{F}{k_B + i(\eta k_B)} \rightarrow \frac{F}{X} = \frac{k_B K_L + i(K_L \eta k_B)}{K_L + k_B + i(\eta k_B)}$$

$$\frac{F}{X} = \frac{k_B\left[(k_B/K_L)+1+\eta^2(k_B/K_L)\right]+i(\eta k_B)}{\left[(k_B/K_L)+1\right]^2+\left[\eta(k_B/K_L)\right]^2} \xrightarrow{\eta^2\sim 0} \frac{F}{X} = \frac{k_B}{\left[(k_B/K_L)+1\right]} + i\frac{\eta k_B}{\left[(k_B/K_L)+1\right]^2} \Leftrightarrow \frac{F}{X} = k_B + i\eta k_B$$





HYUNDAI MOTOR GROUP
HYUNDAI NGV
KSAE 한국자동차공학회 The Korean Society of Automotive Engineers
2017년
한국자동차공학회
춘계 학술대회 연계
B 공진에 기인한 Buzz
S 마찰에 기인한 Squeak
R 충격으로 발생하는 Rattle
현대자동차그룹 BSR 개선 연구 공모전
주 제 차량 BSR 기술 개발을 위한 관련 학술 연구 및 산업계 적용 연구 공모 (공학 전분야)
BSR 개요 • BSR은 차량에서 발생할 수 있는 이음의 대표적인 유형으로 공진에서 발생한 B (Buzz), 마찰에서 기인한 S (Squeak), 충격으로 발생하는 R (Rattle)을 의미
• 차량 및 부품 단위 등 전반적인 자동차 BSR 기술 개발 대상으로 연구 개발 전 후 개선 사례 또는 학술 연구 논문
참여 대상 국내외 대학(원) 재학생 / 산업계 종사자
접수 일정 및 방법
한국자동차공학회 접수 | 현대엔지비 접수
2월 10일(금) ~ 3월 10일(금) | 2월 10일(금) ~ 4월 7일(금)
한국자동차공학회 홈페이지 접수 (http://www.ksae.org) | 오아시스 홈페이지 접수 (http://oasis.hyundai-ngv.com)
학회 접수 시 한국자동차공학회 춘계학술대회 논문 제출로 인정 | –
※ 택1 하여 접수하면 되며, 학회 접수기간 마감 시 현대엔지비 접수 가능
일 정
접수기간 | 1차 심사결과 | 논문 발표 | 시상식
2월 10일(금) ~ 4월 7일(금) | 4월14일(금) 이메일 통보 | 한국자동차공학회 춘계학술대회 | 현대자동차그룹 BSR 컨퍼런스
5월 18일(목) | 9월 개최 예정
제주도해비치호텔앤리조트 | 롤링힐스호텔
※ 현대자동차그룹 BSR 컨퍼런스 일정 및 장소는 행사 일정에 따라 변동 될 수 있음.
시상 내역
최우수상 1편 (200만원)
우수상 2편 (각 100만원)
장려상 3편 (각 50만원)
문 의 현대엔지비 기술협력팀
buzzs2@hyundai-ngv.com / 02-870-8461
주최 및 주관 현대자동차 / 현대엔지비 / 한국자동차공학회

국가소음정보시스템 (환경부)

- **120dB** · 전투기의 이착륙소음
- **110dB** · 자동차의 경적소음
- **100dB** · 열차 통과시 철도변 소음
- **90dB** · 소음이 심한 공장 안 · 큰소리의 독창
- **80dB** · 지하철의 차내소음
- **70dB** · 전화벨(0.5m) · 시끄러운 사무실
- **60dB** · 조용한 승용차 · 보통회화
- **50dB** · 조용한사무실
- **40dB** · 도서관 · 주간의 조용한 주택
- **30dB** · 심야의 교외 · 속삭이는 소리
- **20dB** · 시계 초침 · 나뭇잎 부딪치는 소리

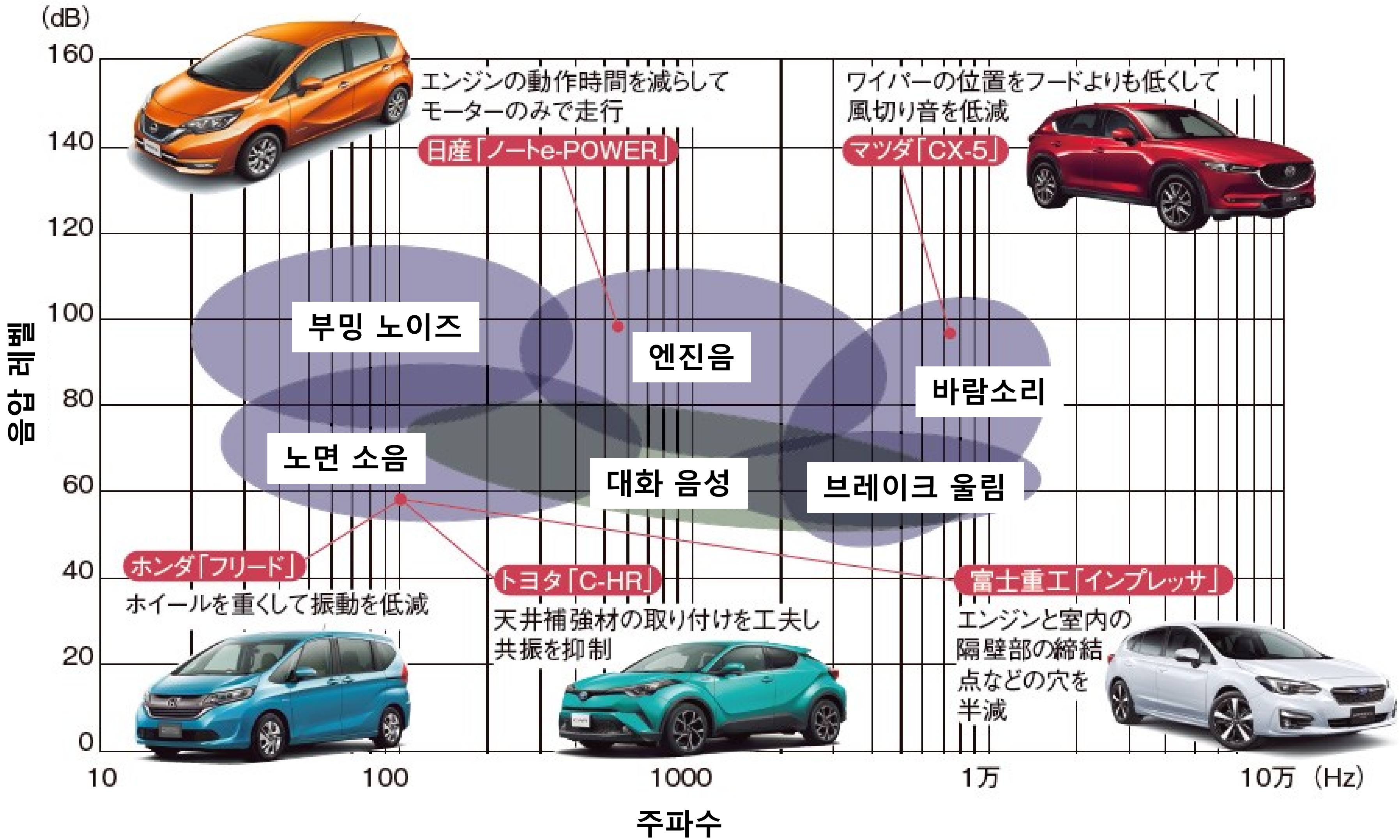
소음도의 인체 영향

소음크기	음원의 예	소음의 영향	비교
20	나뭇잎 부딪히는 소리	쾌적	
30	조용한 농촌, 심야의 교회	수면에 거의 영향없음	
35	조용한 공원	수면에 거의 영향없음	WHO 침실기준
40	조용한 주택의 거실	수면깊이 낮아짐	
50	조용한 사무실	호흡, 맥박수 증가, 계산력 저하	환경기준설정선(주간)
60	보통의 대화소리, 백화점내 소음	수면장애 시작	
70	전화벨소리, 거리	TV·라디오 청취방해	공사장규제기준
80	철로변 및 지하철 소음	청역장애 시작	
90	소음이 심한 공장안	난청증상 시작, 소변량 증가	
100	착암기, 경적소리	작업량저하, 단시간노출시 일시적 난청	

$$L_B = 10\log_{10}\frac{B}{A}\,[\mathrm{dB(decibel)}]$$

$$L_{B_{SPL}} = 20\log_{10}\frac{P}{20\mu\mathrm{Pa}}\,[\mathrm{dB_{SPL}}] \rightarrow 40\mathrm{dB_{SPL}} : \text{최소음압의 } 100\text{배}$$

대화를 방해하는 다양한 소음



기본 대책

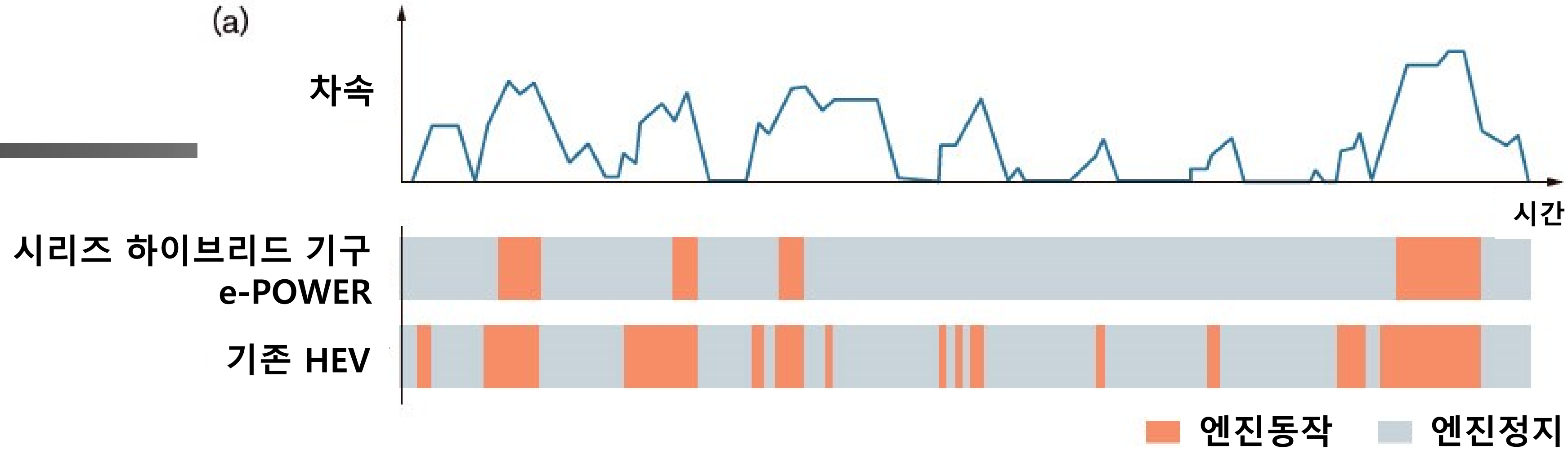
- 좋은 자동차의 정의?
  - 기본 운동성 요소(가속, 회전, 정지)이외에 정숙성
- 정숙성 향상의 필요성
  - 본격적인 전동화 시대의 대비, 중요한 경쟁력
- 소음 발생 원인: 엔진음, 로드노이즈
  - 사람 목소리와 주파수대역이 겹침
  - 노면소음: 타이어로부터의 진동이 휠에 전달되어 소리 발생

- (1) 소리의 원인 자체를 억제
- (2) 소리가 차내로 들어오는 것을 차단
- (3) 차내에 들어온 소리를 정확하게 흡수

Nikkei Automotive 2017년 3월호

Mazda CX-5

(1) 전 모델대비 Cd값을 6% 저감
(2) 타이어로부터의 소음 저감을 위해 화물칸을 중심으로
(3) 천장에 배치한 흡음재

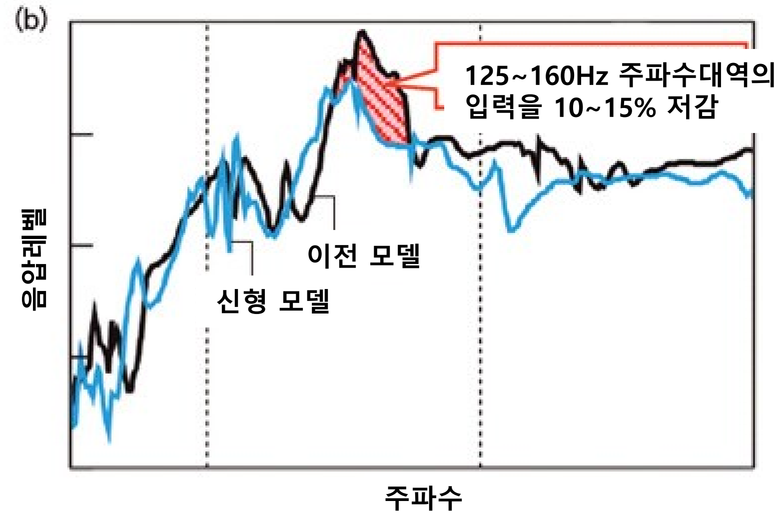


Nissan 신형 Note

(1) 모터를 최대한 활용하여 불쾌한 엔진 소리를 억제 (엔진을 발전기 회전에만 사용) 발전효율이 높은 2,400rpm 전후로 적은 변동 → 소음크기도 적음

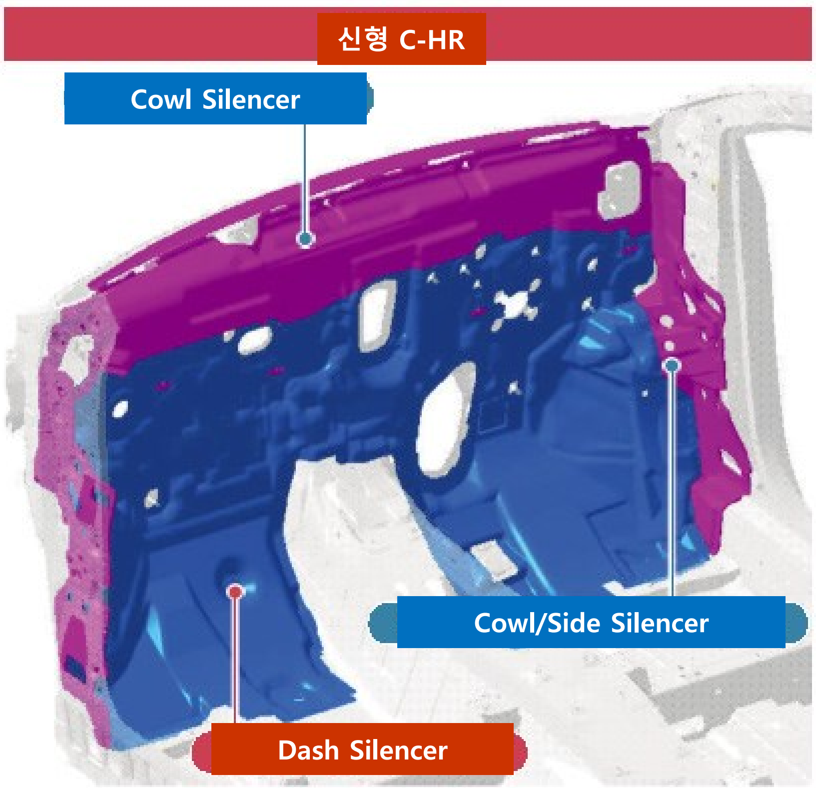
Honda FREED

(1) 휠 1개당 약1kg 무겁게 하여 9kg 전후
휠의 rim외경 두께를 10% 두껍게 → 회전 시 관성력 증가 → 진동 억제
(이전 휠) roll방향으로 진동하여 로드노이즈에 악영향
스프링 하부를 무겁게 하면 조정안정성이 악화됨 → 휠 측면 디스크부 두께를 증가시켜 강성 증가 → 조정안정성 확보



Toyota C-HR (Prius보다 조용)

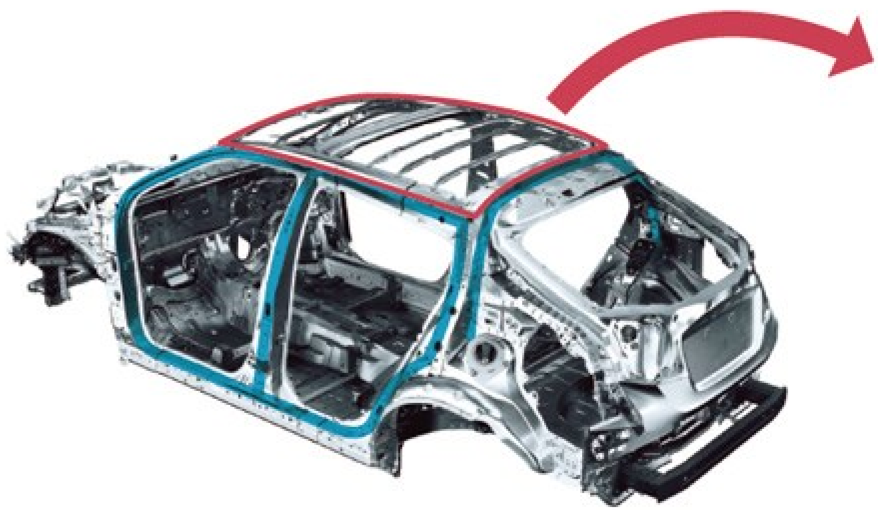
(2) 엔진룸과 차량을 구분하는 dash silencer (dash panel에 장착하여 방음, 흡음, 진동억제 등 담당) → 형상 공통화 (Prius, CHR)
토요타 TNGA, 스바루 SGP (플랫폼 쇄신) → PHEV 대응
기존 모델 대비 구멍 면적은 줄이고(20%) 흡/차음재 두께는 증가(5→20mm)

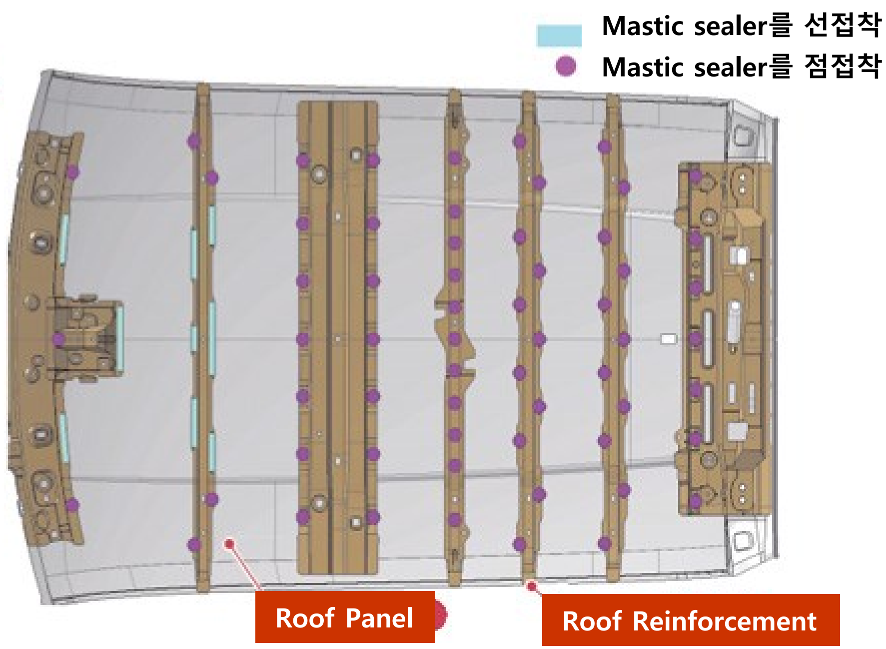


C/D segment의 FF, MC플랫폼

Toyota C-HR (유럽시장)

(2) 루프패널: 노면에 의한 소리(로드 노이즈) 차이를 줄일 대책
유럽노면(포장상태가 제각각): 거친 노면(저주파수"고")과 깨끗한 노면(고주파수"샤")의 반복 변환
루프패널과 루프보강재(진동억제용)를 탄성접착제(mastic sealer)로 고정: 설치길이를 부분적으로 구분, 바디 내에 일부러 진동 발생부위 설계하여 주파수 상쇄





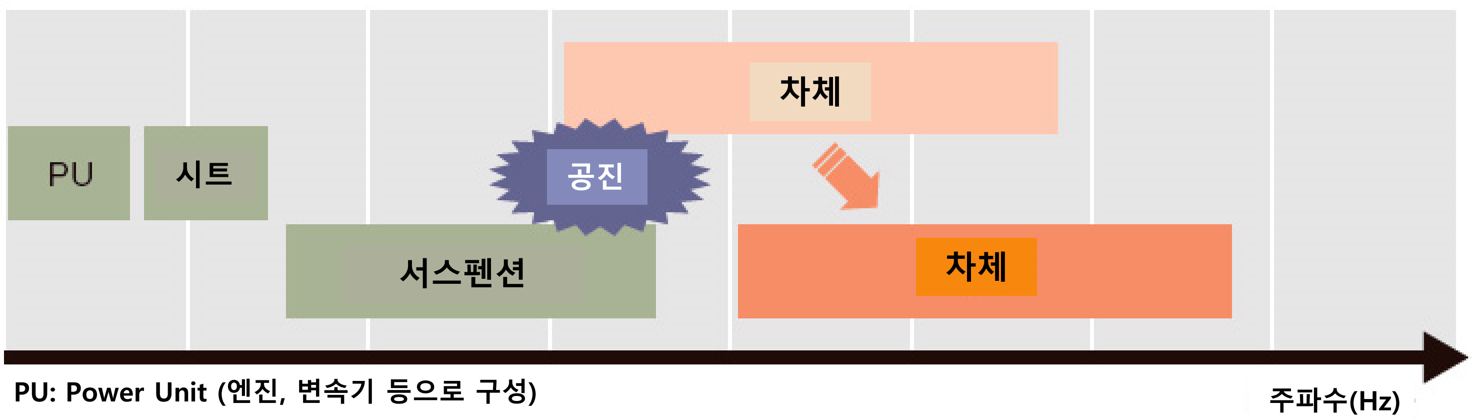
Subaru Impreza: 해석기술로 수치화

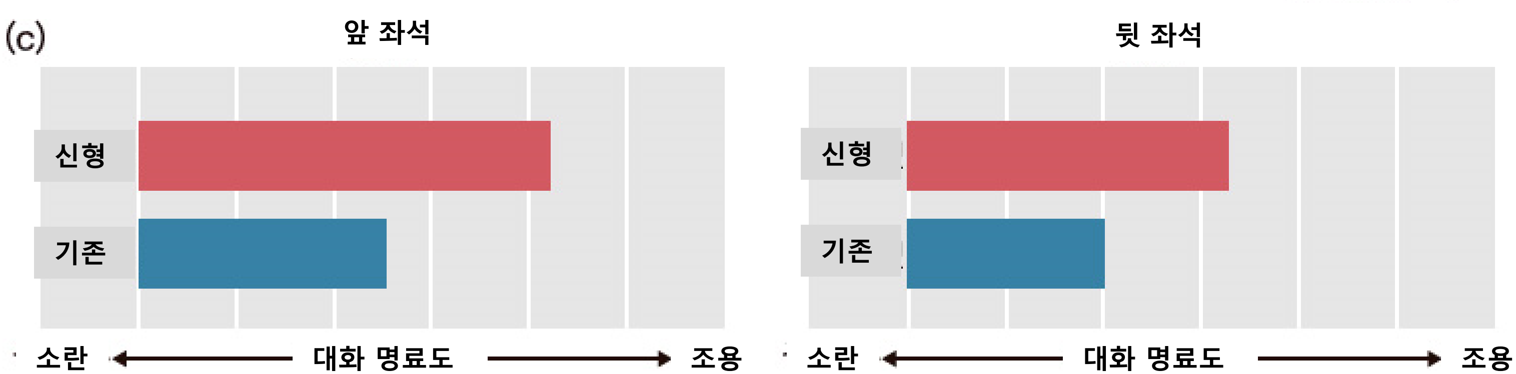
(2) Dash Silencer의 구멍을 줄여 소리의 침입경로를 배제 (41→20)
- Insulator와 주변부품간의 틈을 축소
- 차 실내를 통하는 연료배관을 차 실외로
- Fender Harness를 폐지/통합
- 엔진ECU를 엔진룸 내로 이동
- 바디통합유닛을 instrument panel(IP)로 결합하여 체결점을 폐지
- HVAC(공조)을 IP의 모듈부품으로 하여 체결점을 폐지
- 보안유닛을 HVAC에 붙여서 체결점을 폐지
- Idling stop기구의 부착점을 줄임

(1) 각 부재의 강성을 높여서 진동을 줄임 (이전 모델 대비)
- 차체 전면부의 횡굽힘강성을 90% 향상
- 차체의 비틀림강성을 70% 향상
- Front Suspension의 강성을 70% 향상
- Rear Suspension의 강성을 100% 향상

Subaru Impreza: 해석기술로 수치화

(1) 차체와 서스펜션 일부에서 진동주파수가 일치
→ 공진으로 로드 노이즈 증가





HMC: RANC(Road-noise Active Noise Control)

- 주행 중 소음 원인 (차내 소음 주파수: 20~10kHz)
  - Engine noise
  - Wind noise (풍절음): 500~10kHz
  - Road noise: 가장 큼, 20~500Hz
- 소음제어기술
  - 수동적 방법(소음전달억제): 차체구조 강화, 이중창 채용, 흡음재나 차음재 추가
    - 차음재 대량 사용 → 연비악화, 저주파 노이즈 절감은 차음재만으로는 어렵고 비용 증대
  - 능동적 기술(ANC): 차내 음을 S/W로 분석, 저주파수 (65~125Hz)의 노이즈에 대한 역위상 음파를 발생시켜 소거
    - 노이즈 레벨이 일정하게 예상되는 경우만 가능: 엔진 노이즈

HMC: RANC(Road-noise Active Noise Control)

- 2019.11.11 (개발기간 6년, 차내 소음 3dB↓, 미사용대비 50%↓)
- 가속도센서, 앰프, 마이크, DSP로 구성: 차재 오디오시스템도 가능
  - 가속도센서: 도로→차내 진동 계측, 제어컴퓨터로 분석, 위치중요
  - 음발생→탑승자: 9ms, DSP[광통신(신호전송시간단축)+알고리즘최적화]로 역위상 음파 발생: 2ms
  - 마이크: 항시 로드노이즈 감시, DSP로 정보송신
- 험로에서 발생하는 저주파 노이즈를 대폭 감소
- 기존 물리적 차음 부품 사용 감소 가능성