

Size Optimization

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CDL Computational
Design
Lab

OUTLINE

- **Lecture Goals**

- ✓ Bar truss 구조의 부피 최소화 문제 및 위상 최적화와 결합한 설계 문제의 치수 최적화를 수행하고 제약 조건에 따른 설계 결과를 분석한다.

- **Contents**

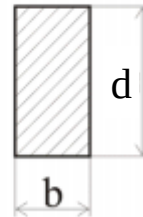
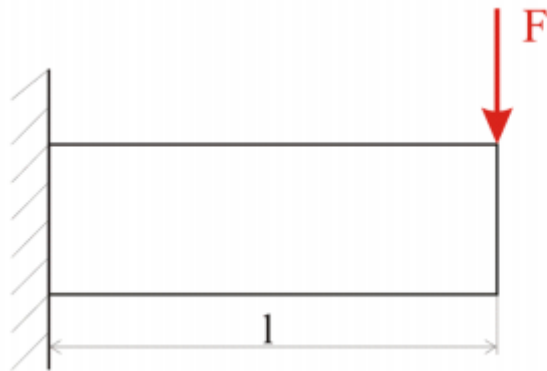
- ✓ Beam design problem using Matlab
- ✓ 5 bar truss 구조 치수 최적화
: volume minimization problem
- ✓ 3 bar truss 구조 치수 최적화
: topology + size optimization

- 해석 프로세스

- Geometry 생성
- Materials and properties
- Mesh 생성
- Load collectors-boundary conditions 설정
- Load collectors-load 설정
- Load steps 정의
- 최적설계 문제 정식화 및 최적설계 실행
- 후처리

BEAM DESIGN PROBLEM USING MATLAB

예제: BEAM DESIGN PROBLEM



$$M = 40 \text{ kN} \cdot \text{mm}$$

$$V = 150 \text{ kN}$$

$$(\sigma_a)_{bending} = 10 \text{ MPa}$$

$$(\tau_a)_{shear} = 2 \text{ MPa}$$

Minimize mass
 b, d

subject to

$$\begin{cases} g_1 = \sigma(b, d) \leq (\sigma_a)_{bending} \\ g_2 = \tau(b, d) \leq (\tau_a)_{shear} \\ g_3 = d \leq 2b \\ g_4 = b \geq 0 \\ g_5 = d \geq 0 \end{cases}$$



Minimize $f = bd$
 b, d

subject to

$$\begin{cases} g_1 = \frac{6M}{bd^2} - (\sigma_a)_{bending} \leq 0 \\ g_2 = \frac{3V}{2bd} - (\tau_a)_{shear} \leq 0 \\ g_3 = d - 2b \leq 0 \\ g_4 = -b \leq 0 \\ g_5 = -d \leq 0 \end{cases}$$

EXCEL을 이용한 최적화



	Initial	Optimum	
b	400	474.3416	설계 변수
d	200	237.1708	
f	80000	112499.9928	
g1	-9.9850	-9.9910	
g2	0.8125	0.0000	
g3	-600	-711.5125	<=0

목적함수 및 구속조건
(설계 변수에 대한 함수 형태)



해 찾기

분석



목적함수

목적 설정: (T)

대상: ☐ 최대값 (M) ☒ 최소 (N) ☐ 지정값 (V) **설계목적**

변수 셀 변경: (B) **설계 변수**

제한 조건에 종속: (U)

구속조건

\$C\$56 <= 0
\$C\$57 <= 0
\$C\$58 <= 0

☒ 제한되지 않는 변수를 음이 아닌 수로 설정 (K)

해법 선택: (E) **최적화방법**

해법
완전한 비선형으로 구성된 해 찾기 문제에 대해서는 GRG Nonlinear 엔진을 선택합니다.
비선형 문제에 대해서는 LP Simplex 엔진을 선택하고 완전하지 않은 비선형으로 구성된 해 찾기 문제에 대해서는 Evolutionary 엔진을 선택합니다.

도움말 (H) **해 찾기 (S)** 닫기 (Q)



변경될 변수의 값 및 목적함수/구속조건 설정

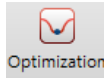


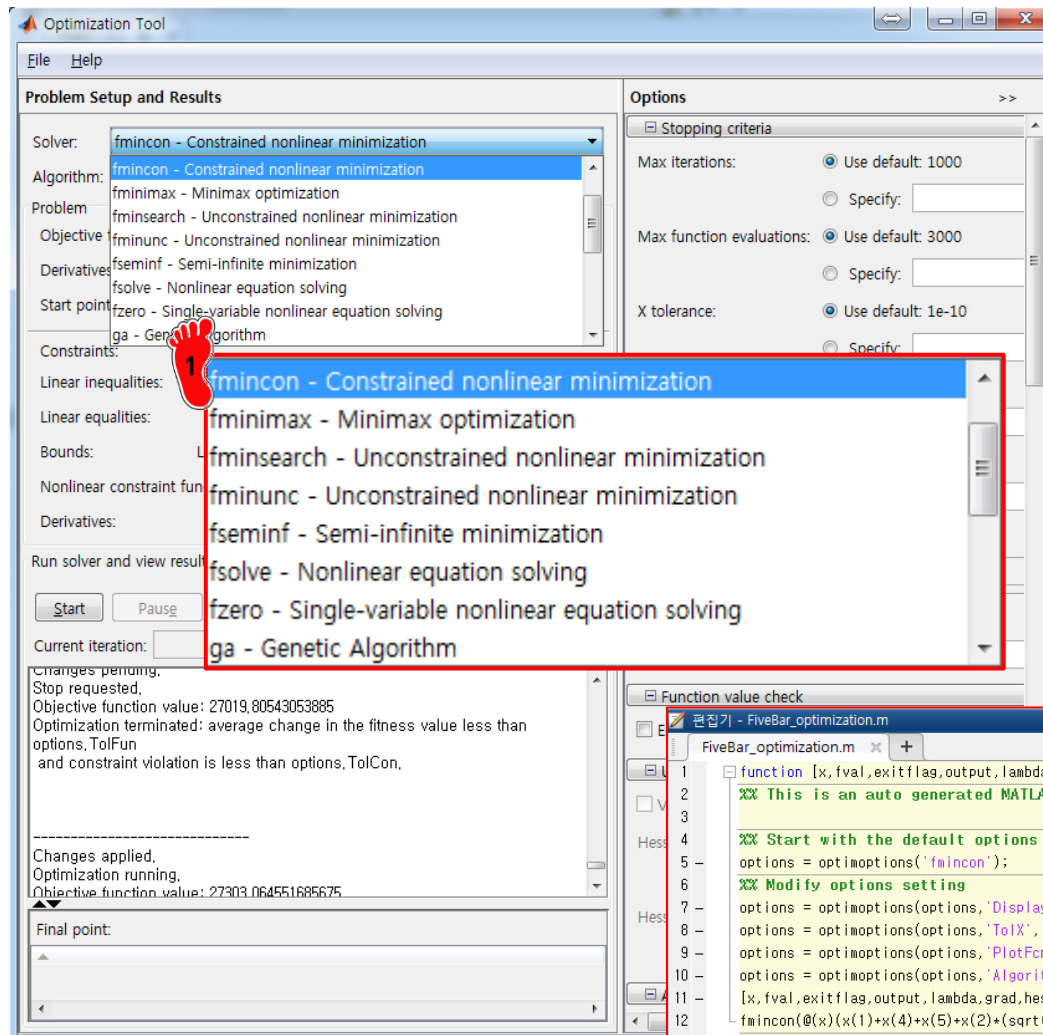
해 찾기 선택
(데이터 → 분석)



확인

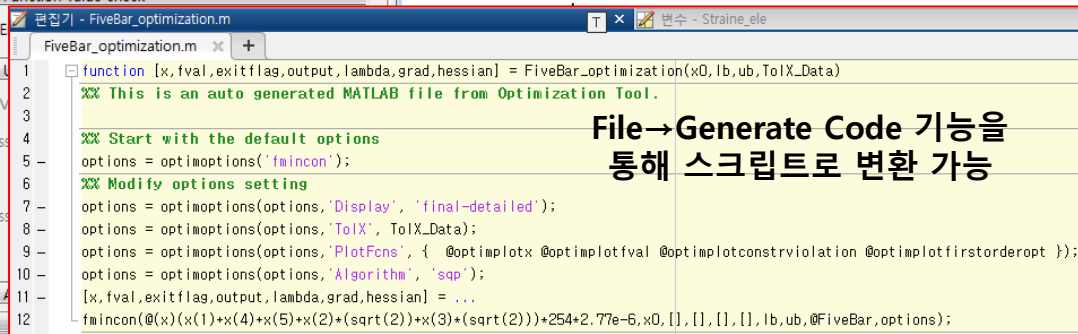
MATLAB을 이용한 최적화

- MATLAB의 Optimization tool 기능을 이용
(명령어 'optimtool' 또는 아이콘 )



MATLAB의 Optimization Tool을 이용하면 다양한 솔버를 이용한 최적화 가능

본 문제와 같이 제약조건이 있는 문제의 경우에는 민감도 기반의 'fmincon'을 많이 사용하며 민감도 계산이 어렵거나 전역 최적해가 필요한 경우 'GA (genetic algorithm)' 를 사용



File→Generate Code 기능을 통해 스크립트로 변환 가능

OPTIMTOOL 이용

최적화 알고리즘

Solver: fmincon - Constrained nonlinear minimization

Algorithm: SQP

목적함수

Objective function: $@(x) x(1)*x(2)$

Derivatives: Approximated by solver

초기 설계값

Start point: [1 1]

선형 구속조건 ($Ax \leq B$)

Linear inequalities: A: [-2 1] b: [0]

Linear equalities: Aeq: beq:

변수의 상/하한값

Bounds: Lower: [0 0] Upper:

Nonlinear constraint function: @beam_cons

Derivatives: Approximated by solver

비선형 구속조건

Run solver and view results

Start Pause Stop

Current iteration: 19 Clear Results

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions, to within the default value of the optimality tolerance, and constraints are satisfied to within the default value of the constraint tolerance.

최적화 결과

Final point:

1	2
335.41	335.41

Quick Reference

fmincon Solver

Find a minimum of a constrained nonlinear multivariable function.

Click to expand the section below corresponding to your task.

Problem Setup and Results

- Solver and Algorithm
- Problem
- Constraints
- Run solver and view results

Options

- Stopping criteria
- Function value check
- User-supplied derivatives
- Approximated derivatives
- Algorithm settings
- Plot functions
- Output function
- Display to command window

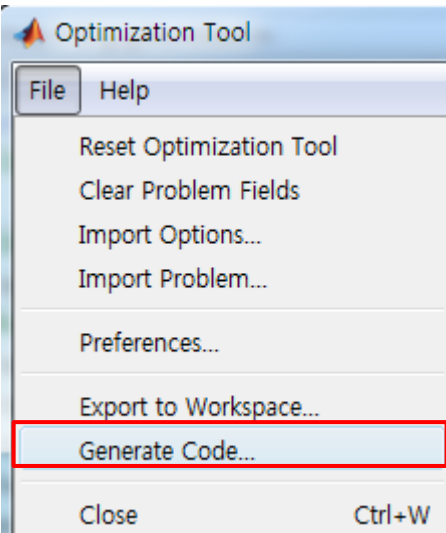
Suggested Next Steps

- Overview of Next Steps
- When the Solver Fails
- When the Solver Might Have Succeeded
- When the Solver Succeeds

More Information

- User Guide
- Function equivalent

GENERATE CODE 이용



```
function [x,fval,exitflag,output,lambda,grad,hessian] = untitled(x0,Aineq,bineq,lb)
%% This is an auto generated MATLAB file from Optimization Tool.

%% Start with the default options
options = optimoptions('fmincon');

%% Modify options setting
options = optimoptions(options,'Display','off');
options = optimoptions(options,'PlotFcn',{ @optimplotx @optimplotfunccount @optimplotfval });
options = optimoptions(options,'Algorithm','sqp');
[x,fval,exitflag,output,lambda,grad,hessian] = ...
fmincon(@(x)x(1)*x(2),x0,Aineq,bineq,[],[],lb,[],@beam_cons,options);
```

→ 일부 상수의 경우 넘어오지 않으므로 수정 필요



```
%% Start with the default options
options = optimoptions('fmincon');

%% Modify options setting
options = optimoptions(options,'Display','off');
options = optimoptions(options,'PlotFcn',{ @optimplotx @optimplotfunccount @optimplotfval });
options = optimoptions(options,'Algorithm','sqp');
[x,fval,exitflag,output] = ...
fmincon(@(x)x(1)*x(2),[1 1],[-2 1],[0],[0 0],[0 0],[0 0],[0 0],@beam_cons,options)
```

MATLAB 최적화 결과값의 의미

```
>> untitled
```

```
x =
```

```
335.4102 335.4102
```

```
fval =
```

```
1.1250e+05
```

```
exitflag =
```

```
1
```

```
output =
```

다음 필드를 포함한 [struct](#):

```
iterations: 19
funcCount: 60
algorithm: 'sqp'
message: 'Local minimum found that satisfies the constraints....'
constrviolation: 2.4825e-13
stepsize: 9.9144e-05
lssteplength: 1
firstorderopt: 3.1236e-06
```

All algorithms:

- 1 First order optimality conditions satisfied.
- 0 Too many function evaluations or iterations.
- 1 Stopped by output/plot function.
- 2 No feasible point found.

Trust-region-reflective, interior-point, and sqp:

- 2 Change in X too small.

Trust-region-reflective:

- 3 Change in objective function too small.

Active-set only:

- 4 Computed search direction too small.
- 5 Predicted change in objective function too small.

Interior-point and sqp:

- 3 Problem seems unbounded.

MATLAB 코드

최적화를 실행하는
메인 스크립트

```
%% Start with the default options
options = optimoptions('fmincon');
%% Modify options setting
options = optimoptions(options,'Display','final-detailed');
options = optimoptions(options,'Algorithm','sqp'); %
x0 = [500 500]; % Start point
[x,fval,exitflag,output] = ...
fmincon(@(x) x(1)*x(2),x0,[-2 1],[0],[],[0 0],[1],@beam_cons,options)
```

```
function [c ceq]=beam_cons(x) [x, fval] = fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlcon,options)
```

```
M = 4e7; % Nmm
V = 15e4; % N
sig_a = 10; %N/mm^2
tau_a = 2; %N/mm^2

b = x(1);
d = x(2);

c(1) = 6*M/b/d^2-sig_a; %g1
c(2) = 3*V/2/b/d-tau_a; %g2
ceq=[];
```

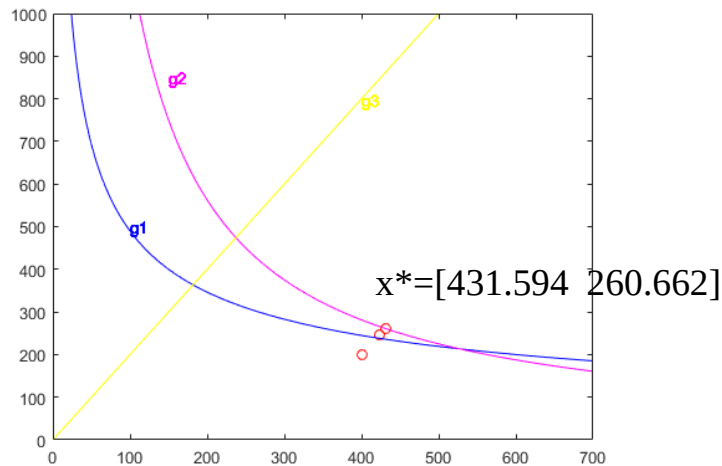
x: design variables
c: inequality constraint
ceq: equality constraint

```
%% Plot
[bb,dd]=meshgrid(0:700,0:1000);
c1 = 6*M./bb./(dd.^2)-sig_a;
c2 = 3*V/2./bb./dd-tau_a;
c3 = dd-2*bb;
contour(bb,dd,c1,[0 0],'b');text(100,500,'g1','color','b');drawnow;
hold on
contour(bb,dd,c2,[0 0],'m');text(150,850,'g2','color','m');drawnow;
hold on
contour(bb,dd,c3,[0 0],'y');text(400,800,'g3','color','y');drawnow;
hold on
plot(x(1),x(2),'o','color','r');drawnow;
```

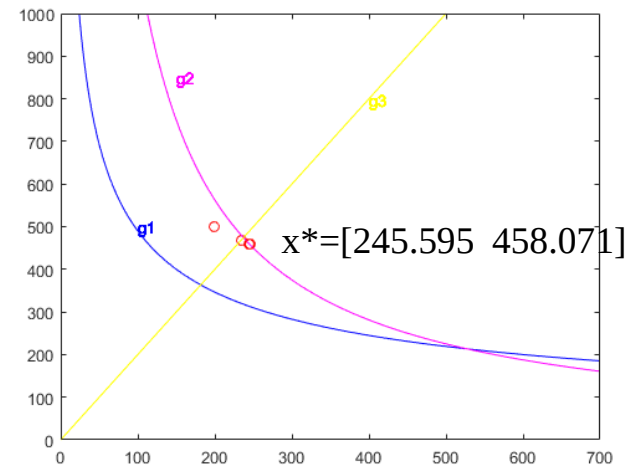
파라미터 및 제한조건의
계산이 포함된
함수 스크립트

최적화 결과

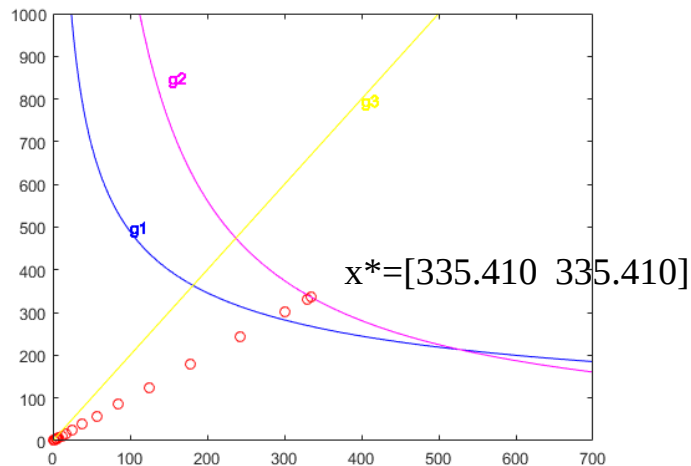
$x_0 = [400 \ 200], f = 112500$



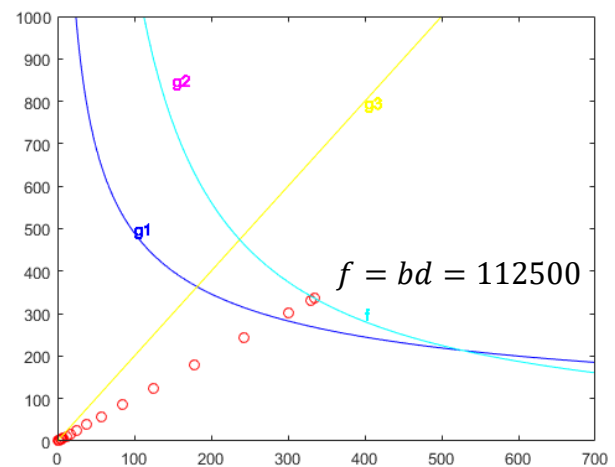
$x_0 = [200 \ 500], f = 112500$



$x_0 = [1 \ 1], f = 112500$



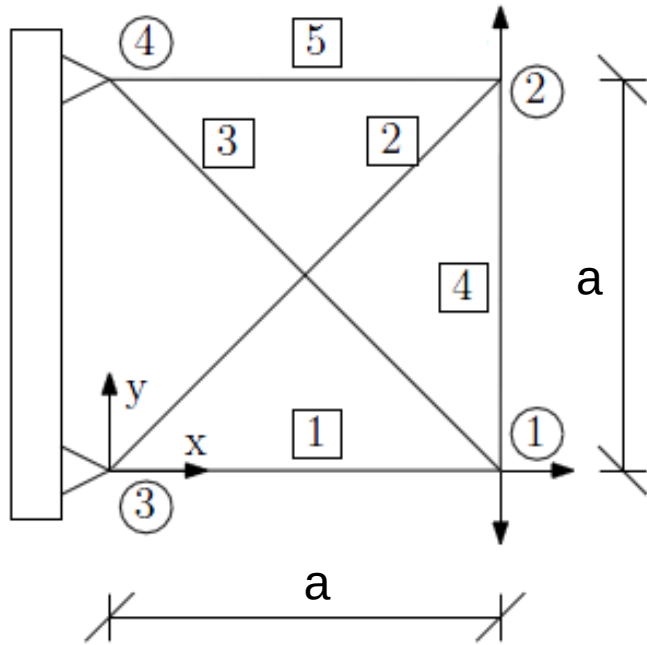
최종 수렴 위치에 대한 목적함수 그래프



목적함수 그래프가 g_2 함수와 같아짐
→ 최적값을 갖는 결과가 다수 존재함

5 BAR TRUSS STRUCTURE (VOLUME MINIMIZATION PROBLEM) 로드 요소

예제: 5 BAR STRUCTURE



Material : aluminum

탄성계수 : 6.895E4 N/mm²

프와송비 : 0.3

질량밀도 : $2.77\text{E-}6 \text{ kg/mm}^3$

$$a = 254 \text{ mm}$$
$$F_{1x} = 22.241 \text{ kN} , F_{1y} = -13.344 \text{ kN}$$
$$F_{2y} = 4.448 \text{ kN}$$

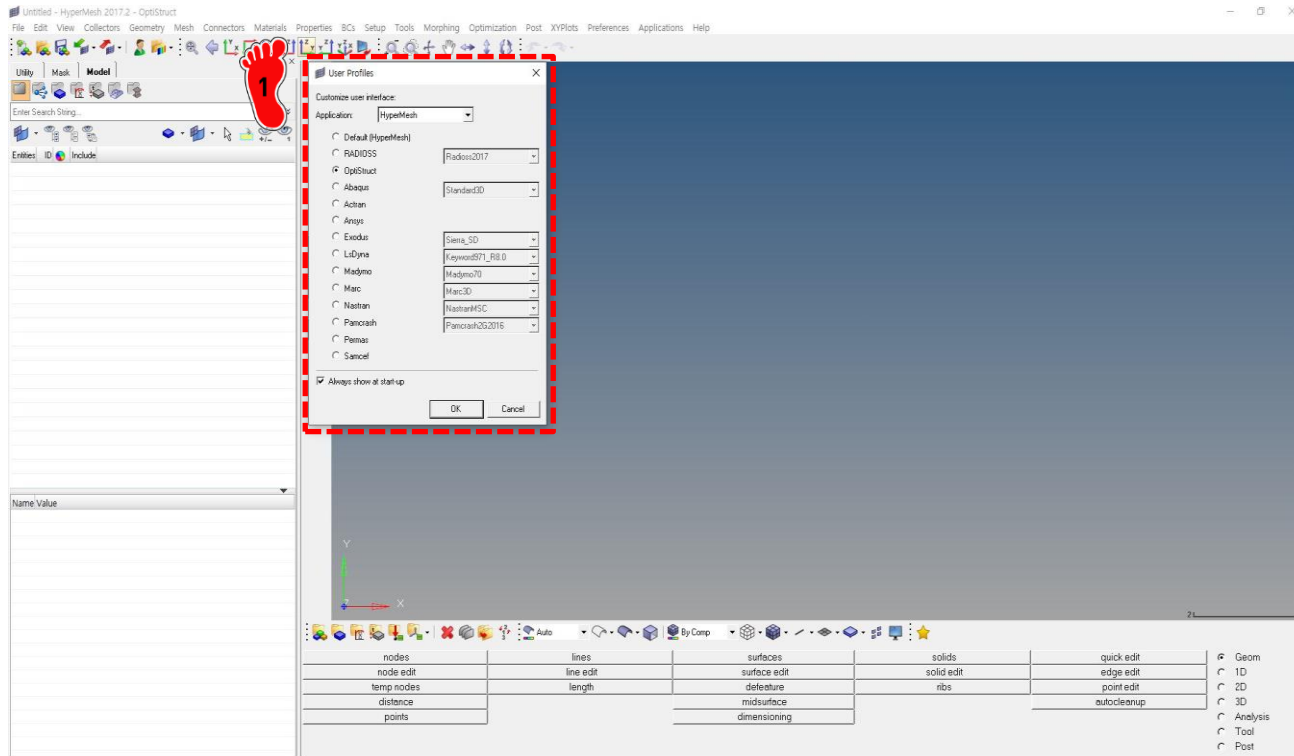
Referenced optimum values*

부재 번호	1	2	3	4	5
단면적 [mm ²]	32.26	6.45	30.39	10.77	6.45

* A. Pospisilova and M. Leps, *ACTA PLYTECHNICA*, 2012

	설계 예제 (1)	설계 예제 (2)	설계 예제 (3)
설계변수	각 부재의 단면적		
설계목적	부피 최소화		
제약조건: 부재 최대응력	413.684 MPa 이하	–	413.684 MPa 이하
제약조건: 절점 최대변위	–	1.524 mm 이하(x,y dir.)	1.524 mm 이하(x,y dir.)
제약조건: 부재 단면적	$6.4516 \text{ mm}^2 \leq \text{부재 단면적} \leq 64.516 \text{ mm}^2$		

기하형상 생성 (1)



1 OptiStruct 선택, OK

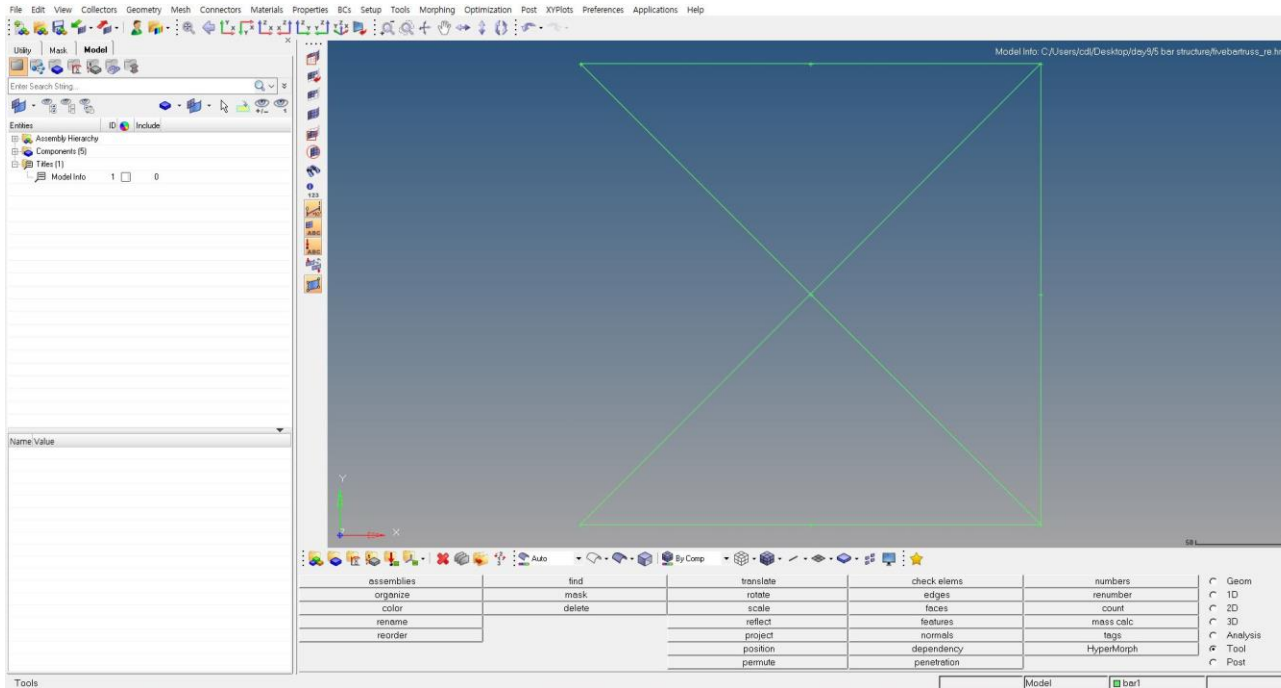
기하형상 생성 (2)



문제에 주어진 5-bar truss
생성

좌표(x,y,z)

(0,0,0)
(254,0,0)
(254,254,0)
(0,254,0)



재료 물성 및 특성 입력

Name	Value
Solver Keyword	MAT1
Name	material1
ID	1
Color	
Include	[Master Model]
Defined	☒
Card Image	MAT1
User Comments	Hide In Menu/Export
E	68950.0
G	
NU	0.3
RHO	2.77e-006

밀도 : [ton/mm³]

Properties (5)			
property1	1		0
property2	2		0
property3	3		0
property4	4		0
property5	5		0

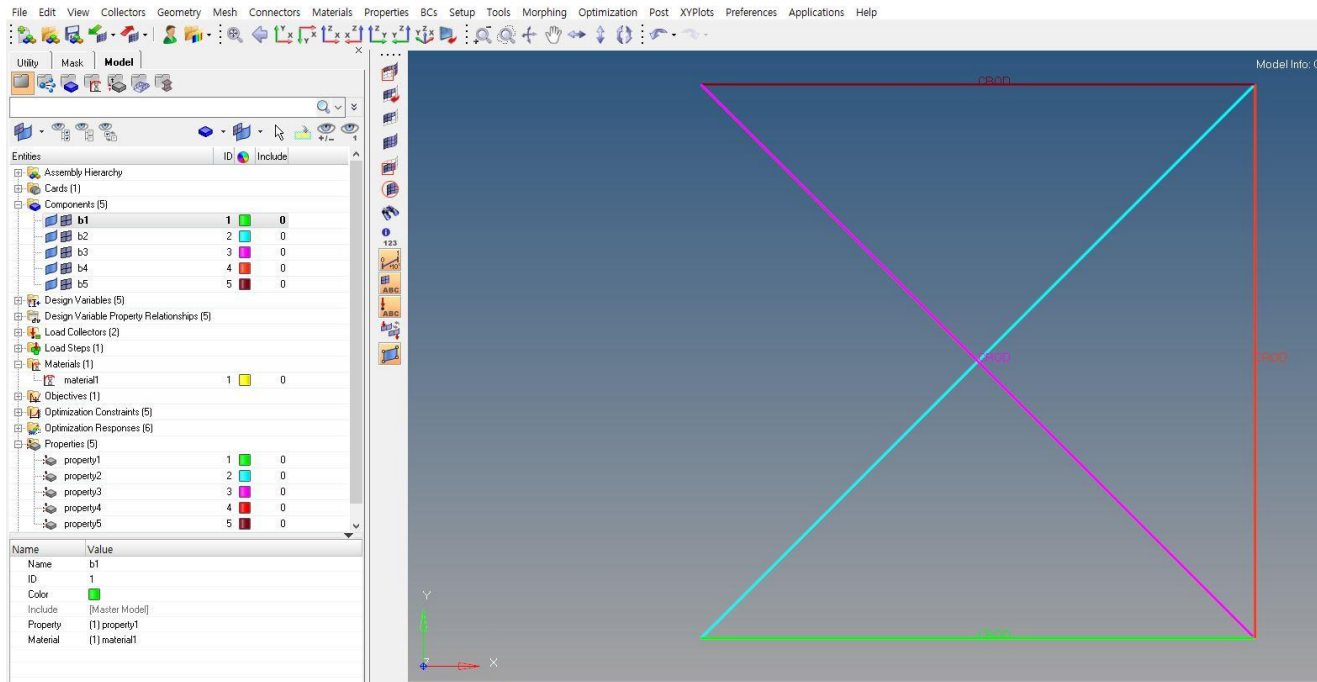
Name	Value
Solver Keyword	PROD
Name	property1
ID	1
Color	
Include	[Master Model]
Defined	☒
Card Image	PROD
Material	(1) material1
User Comments	Hide In Menu/Export
beamsec	<Unspecified>
A	20.0
J	
C	
NSM	



문제에 주어진 Aluminum
물성치 입력
(탄성계수 : 6.895E4 N/mm²
프와송비 : 0.3
밀도 : 2.77E-6 kg/mm³)

5개의 PROD 특성 생성
(초기 단면적 : 20 mm²)

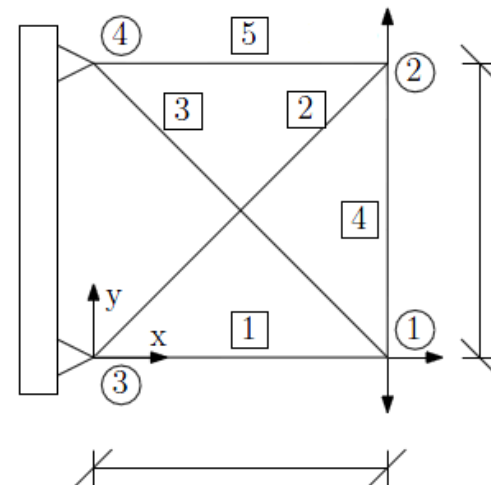
요소망 생성



분할수 1인 1D CROD 요소
5개 생성

1D> Elem types> rod>
CROD 선택

Node 중복을 방지하기 위
해 하나의 component에서
요소를 만든 후
'Tool' -> 'organize'를 이용
하여 새로운 component로
move



설계변수 설정

1

☒ desvar
☐ generic relationship
☐ function relationship

desvar = x 1

initial value =	2	0	0	0	0
lower bound =	6	4	5	2	
upper bound =	6	4	5	1	6

☐ move limit default
☐ no ddval

2

☒ desvar
☐ generic relationship
☐ function relationship

name = x 1

C0 = 0 0 0 0

☒ property1
☐ property2
☐ property3
☐ property4
☐ property5

☒ x1
☐ x2
☐ x3
☐ x4
☐ x5

prop
 Area A
 designvars

	1	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0
	0	0	0	0

1

'Analysis' -> 'optimization'
-> size(parameter) 클릭하
여 변수 생성

설계변수 이름 및 범위 설정

설계변수는 단면적이므로
최소값 : (6.4516 mm²)
최대값 : (64.516 mm²)

5개 부재에 해당하는 5개의
설계변수 생성

2

생성한 변수에 물리적 의미
를 부여하는 과정

각각의 변수를 대응하는 부
재의 특성과 연결

5개의 연결관계 생성

응답 설정

1

response = T 1 x

response type
static displacement

nodes

☐ numbers

dof1
dof2
dof3
total disp

1

'Analysis' -> 'optimization'
-> 'response' 클릭

조인트 1, 2에 대한 절점 변위 응답 설정 (Tx, Ty): 4개

2

전체 요소에 대한 응력 응답 설정

3

전체 부피 응답 설정

2

response = s t r e s s _

response type
static stress

property1
property2
property3
property4
property5

props

excluding:
elems

3

response = v o l _

response type
volume

total

치수 최적화 설정: (1) 응력 제약조건

The screenshot shows the OptiStruct software interface with the following settings and steps indicated:

- Step 1 (Red foot icon):** In the 'response =' field, 'stress' is selected from the list of responses.
- Step 2 (Red foot icon):** In the 'loadsteps' section, 'loadstep1' is checked.
- Step 3 (Yellow foot icon):** The objective is set to 'min'.
- Step 4 (Green foot icon):** In the 'control cards' list, 'SCREEN' is selected.

Additional interface details include:

- Constraint Definition:** 'constraint =' is set to 'c s'. The lower bound is $-1.0000e+20$ and the upper bound is 4.13680 .
- Run Options:** The 'run options' dropdown is set to 'optimization'.
- Control Cards:** A list of control cards (RESTARTW, RESULTS, SCREEN, SENSITIVITY, SENSOUT, SHAPE, SHRES) is shown, with 'SCREEN' highlighted.
- Buttons:** 'delete', 'disable', 'enable', 'next', 'prev', and 'return' buttons are visible.
- Input File:** The input file path is `< > day 9 / 5 bar structure / five bar truss . fem`.
- Export Options:** The 'export options' dropdown is set to 'all'.
- Memory Options:** The 'memory options' dropdown is set to 'memory default'.
- Include Connectors:** A checkbox labeled 'include connectors' is present.

1 'Analysis' -> 'optimization'
-> 'dconstraints' 클릭

'response'에서 응력 응답
선택

'loadsteps' 설정

2 최대응력 제한조건 설정

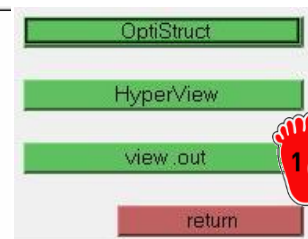
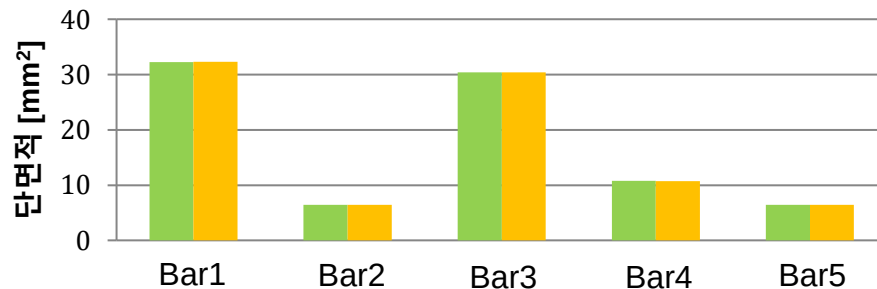
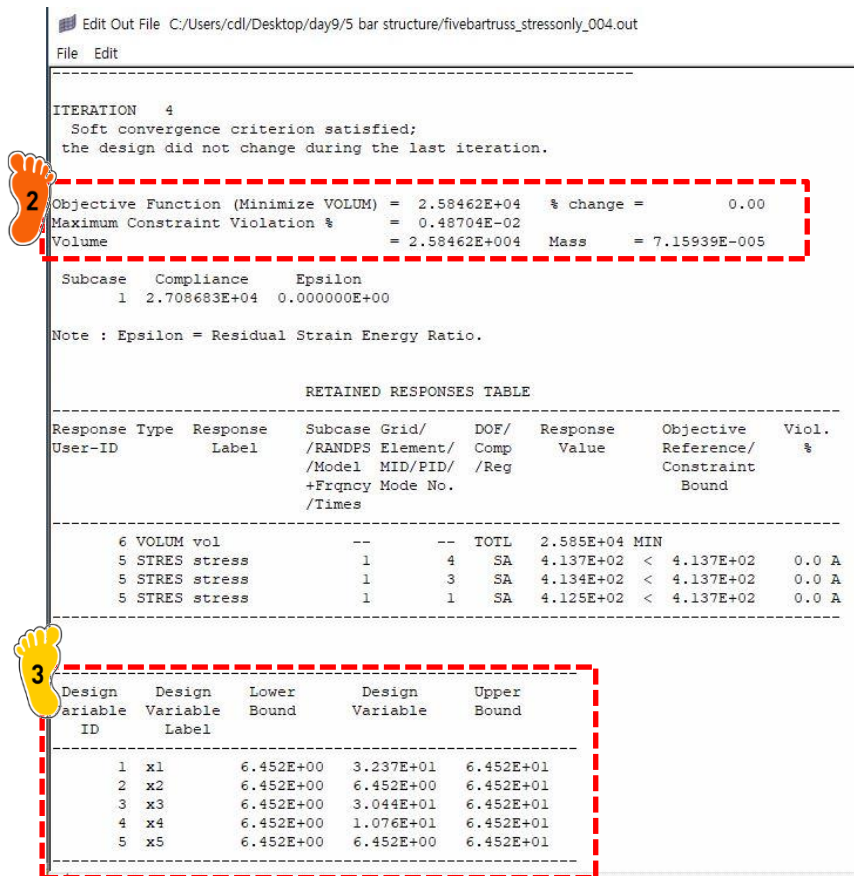
3 'Analysis' -> 'optimization'
-> 'objective' 클릭

부피 최소화 설정

4 'control cards' -> 'screen'
활성화

최적화 수행

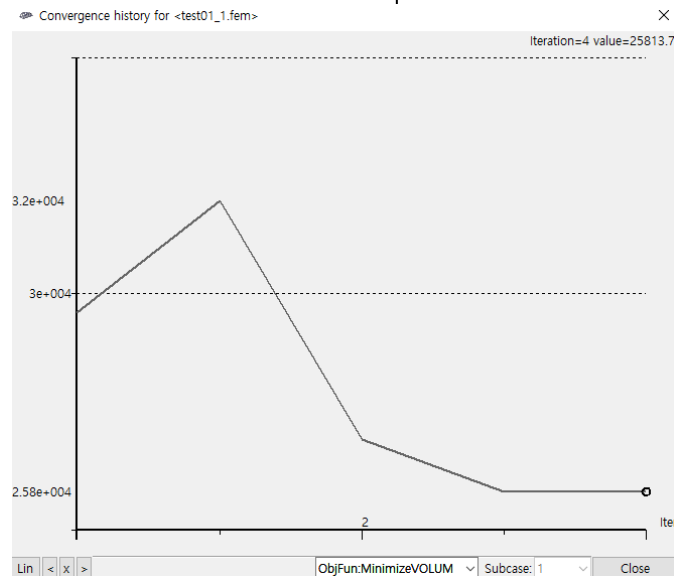
최적화 결과 확인: (1) 응력 제약조건



1 'view.out' 클릭하여 해석결과 확인

2 최적설계 결과 목적함수 값 확인

3 최적설계 결과 설계변수 값 확인

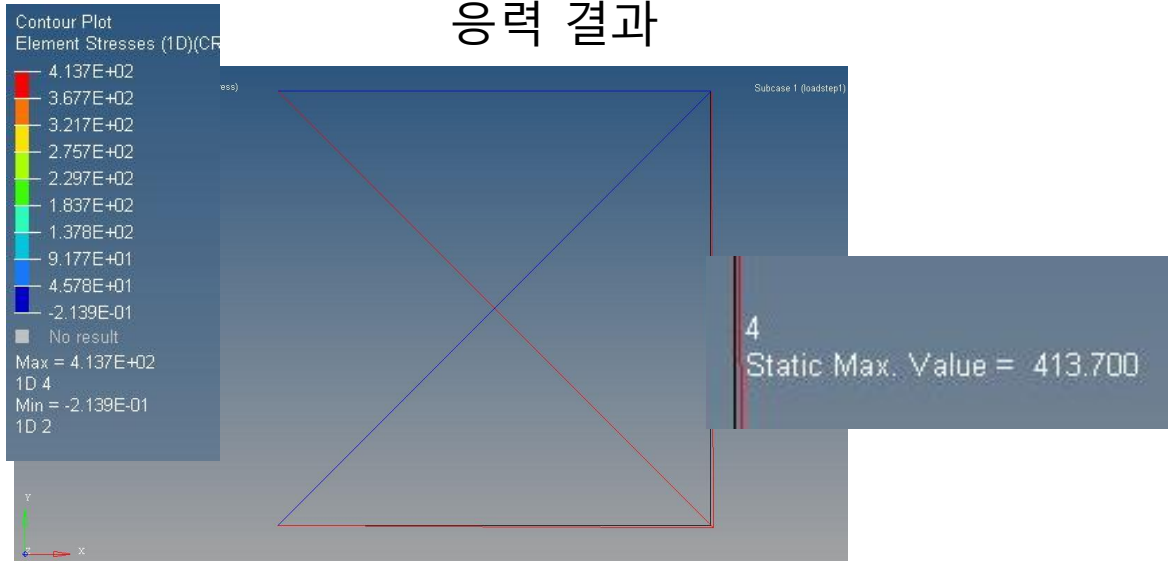


■ : Reference (vol = 2.6e4 mm³)

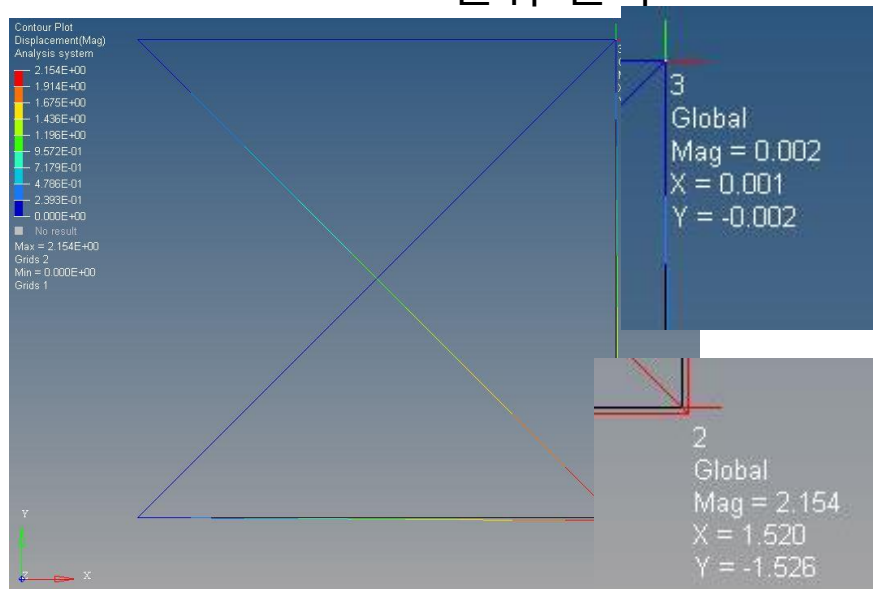
■ : HM (vol = 2.6e4 mm³)

후처리: (1) 응력 제약조건

응력 결과



변위 결과



변위 제약조건을 부여하지
않음.
절점 2의 y축 방향 변위가
위배됨 ($|Ty| > 1.524\text{mm}$)

*  Measures → Add
(Measure Group 3) →
Relative Displacement 선택
→ X, Y 체크 → Nodes 클릭

치수 최적화 설정: (2) 변위 제약조건

The screenshot displays the OptiStruct software interface for setting up displacement constraints. Key elements include:

- Constraint Definition:** A table with columns for constraint name, lower bound, and upper bound. The first row is labeled 'c d 1'.
- Response Selection:** A list of responses to be optimized, including T1x, T1y, T2x, T2y, stress, and vol.
- Loadstep Configuration:** A section for defining loadsteps, with 'loadstep1' selected.
- Optimization Options:** A section for setting optimization parameters, including 'run options' and 'memory options'.

Red dashed boxes and foot icons (1, 2, 3, 4) highlight the following steps:

- Step 1:** Selecting the response to be optimized (e.g., T1x, T1y, T2x, T2y, stress, vol).
- Step 2:** Setting the maximum displacement constraint (e.g., 'loadstep1').
- Step 3:** Setting the optimization options (e.g., 'run options' and 'memory options').
- Step 4:** Selecting the 'screen' control card for optimization.

1 'Analysis' -> 'optimization'
-> 'dconstraints' 클릭

'response'에서 변위 응답
선택

'loadsteps' 설정

2 최대변위 제한조건 설정

총 4개의 변위 제약조건

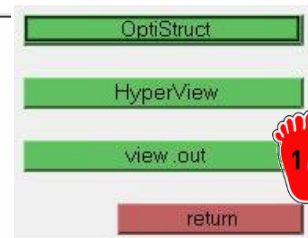
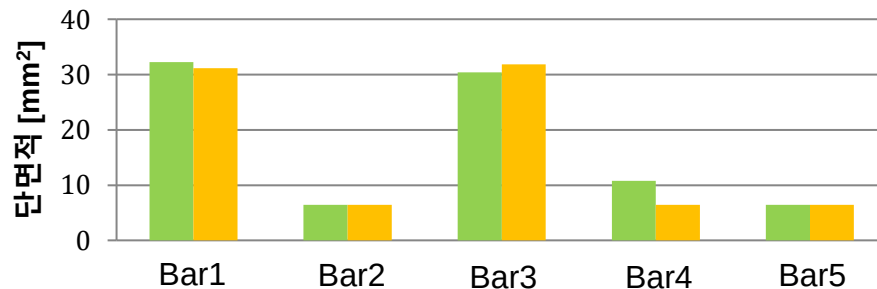
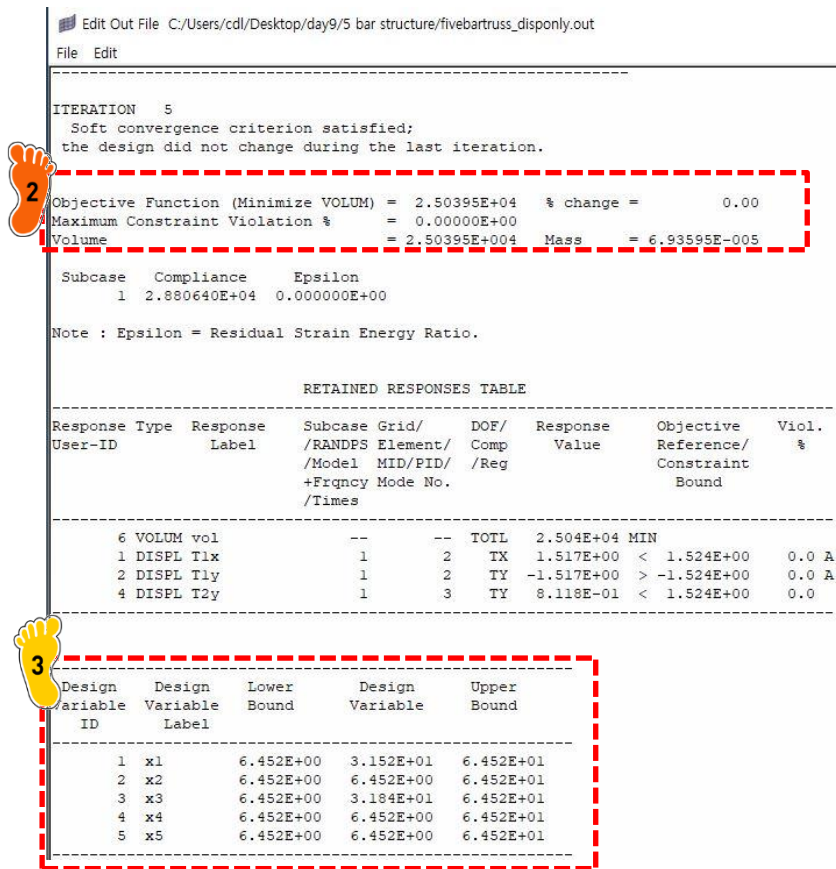
3 'Analysis' -> 'optimization'
-> 'objective' 클릭

부피 최소화 설정

4 'control cards' -> 'screen'
활성화

최적화 수행

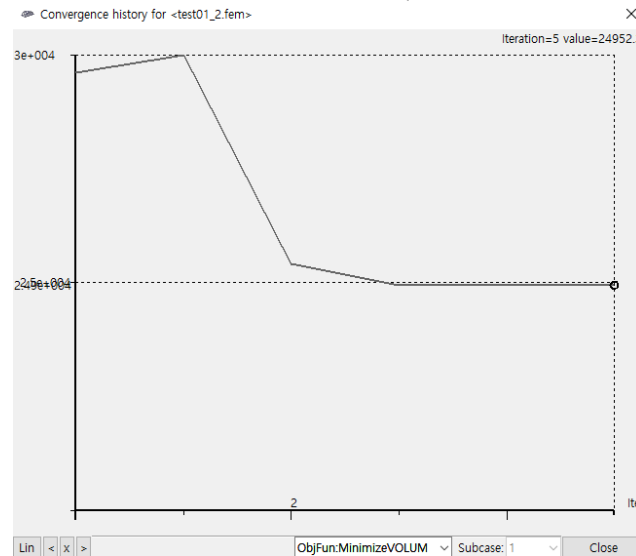
최적화 결과 확인: (2) 변위 제약조건



1 'view.out' 클릭하여 해석결과 확인

2 최적설계 결과 목적함수 값 확인

3 최적설계 결과 설계변수 값 확인

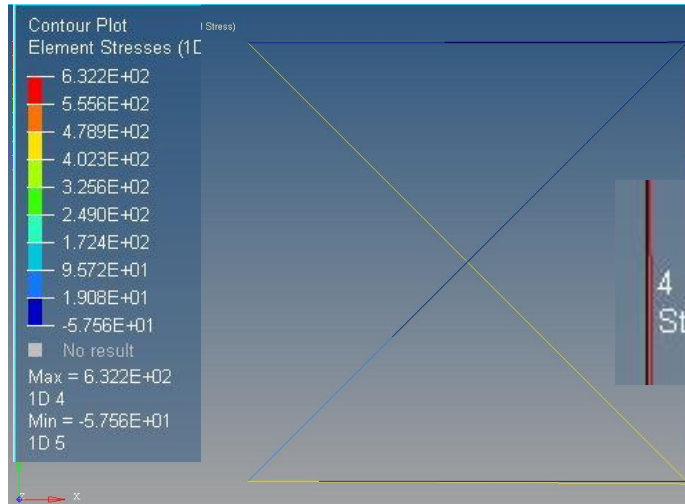


■ : Reference (vol = 2.6e4 mm³)

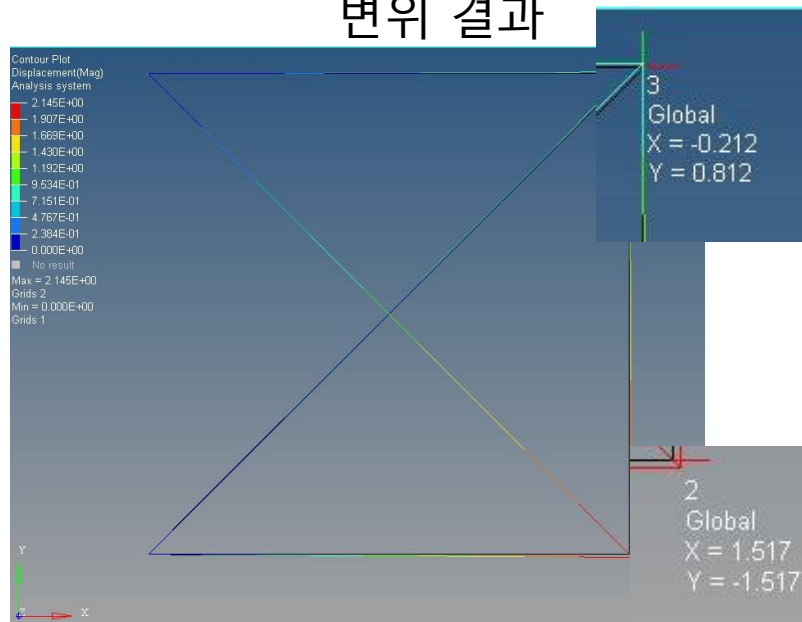
■ : HM (vol = 2.5e4 mm³)

후처리: (2) 변위 제약조건

응력 결과



변위 결과



변위조건은 모두 만족하나
응력조건을 위반한 결과가
나타남 (>413.684 MPa)

*  Measures → Add
(Measure Group 3) →
Relative Displacement 선택
→ X, Y 체크 → Nodes 클릭

치수 최적화 설정: (3) 응력 & 변위

The screenshot displays the OptiStruct software interface with several key areas highlighted by numbered foot icons:

- Foot 1 (Red):** Points to the 'response =' field and the list of response types (T1x, T1y, T2x, T2y, stress, vol).
- Foot 2 (Orange):** Points to the 'constraint =' field and the lower/upper bound settings.
- Foot 3 (Yellow):** Points to the 'loadsteps' field and the 'loadstep1' checkbox.
- Foot 4 (Green):** Points to the 'control cards' menu, specifically the 'screen' option.

At the bottom, the 'run options' are set to 'optimization', and the 'memory options' are set to 'memory default'.

1 'Analysis' -> 'optimization'
-> 'dconstraints' 클릭

'response'에서 응력 및 변위 응답 선택

'loadsteps' 설정

2 최대변위 제한조건 설정

총 5개의 변위 제약조건

3 'Analysis' -> 'optimization'
-> 'objective' 클릭

부피 최소화 설정

4 'control cards' -> 'screen' 활성화

최적화 수행

최적화 결과 확인: (3) 응력 & 변위

Edit Out File C:/Users/cdl/Desktop/day9/5 bar structure/fivebartruss_003.out

File Edit

ITERATION 4
Soft convergence criterion satisfied:
the design did not change during the last iteration.

Objective Function (Minimize VOLUM) = 2.58437E+04 % change = 0.00
Maximum Constraint Violation % = 0.00000E+00
Volume = 2.58437E+004 Mass = 7.15870E-005

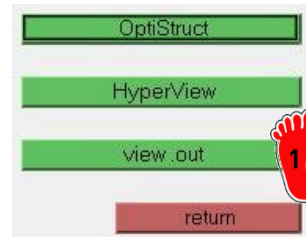
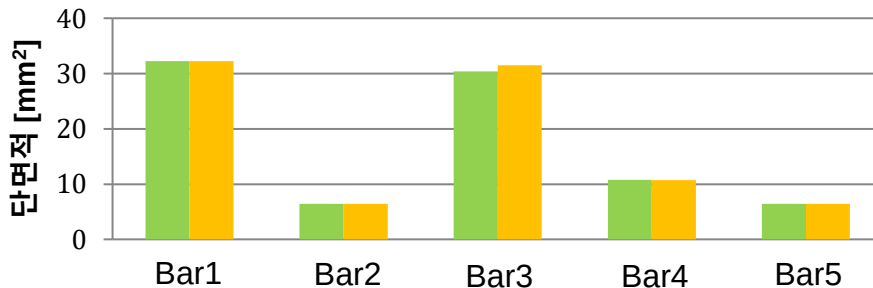
Subcase Compliance Epsilon
1 2.708991E+04 0.000000E+00

Note : Epsilon = Residual Strain Energy Ratio.

RETAINED RESPONSES TABLE

Response User-ID	Type	Response Label	Subcase /RANDPS /Model MID/PID/ +Frgncy Mode No. /Times	Grid/ Element/ /Reg	DOF/ Comp /Reg	Response Value	Objective Reference/ Constraint Bound	Viol. %
6	VOLUM	vol	--	--	TOTL	2.584E+04	MIN	
1	DISPL	Tlx	1	2	TX	1.523E+00	< 1.524E+00	0.0 A
2	DISPL	Tly	1	2	TY	-1.519E+00	> -1.524E+00	0.0 A
5	STRES	stress	1	4	SA	4.137E+02	< 4.137E+02	0.0 A
5	STRES	stress	1	3	SA	4.128E+02	< 4.137E+02	0.0 A
5	STRES	stress	1	1	SA	4.134E+02	< 4.137E+02	0.0 A

Design Variable ID	Design Variable Label	Lower Bound	Design Variable	Upper Bound
1	x1	6.452E+00	3.229E+01	6.452E+01
2	x2	6.452E+00	6.452E+00	6.452E+01
3	x3	6.452E+00	3.050E+01	6.452E+01
4	x4	6.452E+00	1.075E+01	6.452E+01
5	x5	6.452E+00	6.452E+00	6.452E+01

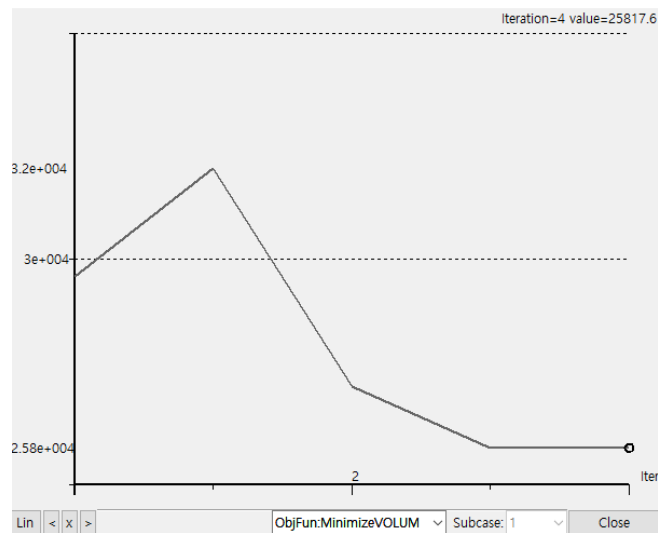


1 'view.out' 클릭하여 해석결과 확인

2 최적설계 결과 목적함수 값 확인

3 최적설계 결과 설계변수 값 확인

Convergence history for <test01_3.fem>



■ : Reference (vol = 2.6e4 mm³)

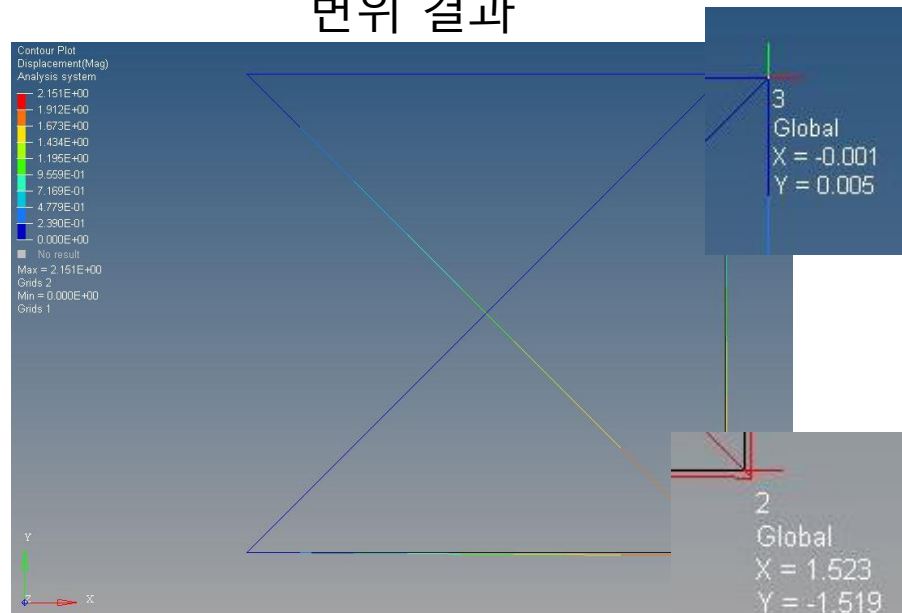
■ : HM (vol = 2.6e4 mm³)

후처리: (3) 응력 & 변위

응력 결과



변위 결과



응력 및 범위조건 모두 만족
함

*  Measures → Add
(Measure Group 3) →
Relative Displacement 선택
→ X, Y 체크 → Nodes 클릭

치수 최적화 결과

HyperMesh를 이용한 치수 최적화의 경우 참고 문헌의 결과와 유사한 결과가 도출된 것을 알 수 있음

		최적화 문제			
		참고 문헌	(1) 응력 제약	(2) 변위 제약	(3) 응력 & 변위
부피[mm ³]		2.6e4	2.6e4	2.5e4	2.6e4
최대 응력[MPa]		414.09	413.70	632.19	413.67
변위 [mm]	절점 1,x	1.52	1.52	1.52	1.52
	절점 1,y	-1.52	-1.53	-1.52	-1.52
	절점 2,x	0.001	0.001	-0.21	-0.001
	절점 2,y	-0.004	-0.002	0.81	0.005
변수 [mm ²]	A1	32.26	32.37	31.52	32.29
	A2	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>
	A3	30.39	30.44	31.84	30.50
	A4	10.77	10.76	<u>6.45</u>	10.75
	A5	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>

MATLAB을 이용한 최적화: fmincon

• 최적화 세팅

최적화 알고리즘

Solver: fmincon - Constrained nonlinear minimization

Algorithm: SQP

Objective function: $@(x) (x(1)+x(4)+x(5)+x(2)*(\sqrt{2})+x(3)*(\sqrt{2}))*254*2.77e-6$

Derivatives: Approximated by solver

Start point: $\text{rand}(5,1)*64.516*0.9+6.4516$

Constraints:

Linear inequalities: A: [] b: []

Linear equalities: Aeq: [] beq: []

Bounds: Lower: $6.4516*\text{ones}(5,1)$ Upper: $6.4516*\text{ones}(5,1)$

Nonlinear constraint function: @FiveBar

Derivatives: Approximated by solver

Run solver and view results

Start Pause Stop

Current iteration: 32 Clear Results

options.MaxFunEvals = 500 (the default value).

Optimization running.
Objective function value: 0.0715409861874495
Converged to an infeasible point.

fmincon stopped because the size of the current step is less than the default value of the step size tolerance but constraints are not satisfied to within the default value of the constraint tolerance.

최적화 결과

1	2	3	4	5
32.251	6.452	30.484	10.745	6.452

목적함수

초기 설계값

변수의 경계값

구속조건 계산을 위한 함수 (구조해석포함)

fmincon - Constrained nonlinear minimization

Trust region reflective

Interior point

SQP

Active set

Trust region reflective

Max function evaluations: Use default: 100*numberOfVariables

Specify: 5000

Use default: 1e-6

Specify: 1e-15

Function tolerance: Use default: 1e-6

Specify: 1e-10

Constraint tolerance: Use default: 1e-6

Specify: 0.01

Unboundedness threshold: Use default: -1e20

Specify:

Function value check

Error if user-supplied function returns Inf, NaN or complex

User-supplied derivatives

Validate user-supplied derivatives

Hessian sparsity pattern: Use default: sparse(ones(numberOfVariables))

Specify:

Hessian multiply function: Use default: No multiply function

Specify:

Approximated derivatives

fmincon Solver

Find a minimum of a cons multivariable function.

Click to expand the section corresponding to your task

Problem Setup and Results

Solver and Algorithm

Problem

Constraints

Run solver and view results

Options

Stopping criteria

Function value check

User-supplied derivative

Approximated derivative

Algorithm settings

Plot functions

Output function

Display to command window

Suggested Next Step

Overview of Next Steps

When the Solver Fails

When the Solver Might Fail

When the Solver Succeeds

More Information

User Guide

Function equivalent

MATLAB을 이용한 최적화: fmincon

		최적화 문제			
		참고 문헌	(1) 응력 제약	(2) 변위 제약	(3) 응력 & 변위
부피[mm ³]		2.6e4	2.6e4	2.5e4	2.6e4
최대 응력[MPa]		414.09	413.68	632.33	414.70
변위 [mm]	절점 1,x	1.52	1.52	1.52	1.52
	절점 1,y	-1.52	-1.52	-1.52	-1.52
	절점 2,x	0.001	3.92e-7	-0.21	-8.30e-6
	절점 2,y	-0.004	-1.50e-6	0.81	3.19e-5
변수 [mm ²]	A1	32.26	32.26	31.37	32.26
	A2	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>
	A3	30.39	30.41	31.67	30.41
	A4	10.77	10.75	<u>6.45</u>	10.75
	A5	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>

MATLAB의 fmincon 솔버의 경우 민감도를 이용한 알고리즘을 사용하며 크게 4가지 방법을 사용

본 문제에 적용 가능한 방법은 'Interior point', 'SQP', 'Active set' 3가지이며, 본 문제의 경우 결과에 큰 차이가 없으므로 기본으로 되어 있는 'Interior point'를 이용한 결과 이용함

참고 문헌의 결과와 유사한 결과를 나타냄

MATLAB을 이용한 최적화: GA

• 최적화 세팅

Optimization Tool

File Help

최적화 알고리즘

Problem Setup and Results

Solver: **ga - Genetic Algorithm**

Problem

Fitness function: **@(x) (x(1)+x(4)+x(5)+x(2)*(sqrt(2))+x(3)*(sqrt(2)))*254** **목적함수**

Number of variables: **5** **설계 변수 개수**

Constraints:

Linear inequalities: A: [] b: []

Linear equalities: Aeq: [] beq: []

Bounds: Lower: **6.4516*ones(5,1)** Upper: **64.516*ones(5,1)** **변수의 경계값**

Nonlinear constraint function: **@FiveBar** **구속조건 계산**

Integer variable indices: []

Run solver and view results

☐ Use random states from previous run

Start Pause Stop

Current iteration: 4 Clear Results

and constraint violation is less than options.TolCon.

Changes applied.
Optimization running.
Objective function value: 27303.064551685675
Optimization terminated: average change in the fitness value less than options.TolFun
and constraint violation is less than options.TolCon.

최적화 결과

Final point:

1	2	3	4	5
35.354	6.455	31.588	11.884	6.453

Options

Specify:

Use default: -Inf

Specify:

Stall generations:

Use default: 50

Specify:

Stall time limit:

Use default: Inf

Specify:

Stall test: **average change**

Function tolerance:

Use default: 1e-6

Specify:

Constraint tolerance:

Use default: 1e-3

Specify:

Plot functions

Plot interval:

☒ Best fitness ☐ Best individual ☐ Distance

☐ Expectation ☐ Genealogy ☐ Range

☐ Score diversity ☒ Scores ☐ Selection

☐ Stopping ☒ Max constraint

Custom function:

Output function

Custom function:

Display to command window

Level of display: **off**

User function evaluation

Evaluate fitness and constraint functions: **in serial**

Quick Reference

Genetic Algorithm

This tool corresponds to the

Click to expand the section corresponding to your task

Problem Setup and Results

Problem

Constraints

Run solver and view results

Options

Specify options for the Genetic Algorithm

Population

Fitness scaling

Selection

Reproduction

Mutation

Crossover

Migration

Constraint parameters

Hybrid function

Stopping criteria

Plot Functions

Output function

Display to command window

User function evaluation

More Information

User Guide

Function equivalent

MATLAB을 이용한 최적화: GA

		최적화 문제			
		참고 문헌	(1) 응력 제약	(2) 변위 제약	(3) 응력 & 변위
부피[mm ³]		2.6e4	2.6e4	2.7e4	2.6e4
최대 응력[MPa]		414.09	414.02	593.93	414.10
변위 [mm]	절점 1,x	1.52	1.52	1.39	1.52
	절점 1,y	-1.52	-1.52	-1.39	-1.52
	절점 2,x	0.001	9.0e-7	-0.21	1.71e-5
	절점 2,y	-0.004	-3.4e-6	0.8	-6.53e-5
변수 [mm ²]	A1	32.26	32.23	34.45	32.23
	A2	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>
	A3	30.39	30.39	34.78	30.38
	A4	10.77	10.74	6.87	10.74
	A5	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>	<u>6.45</u>

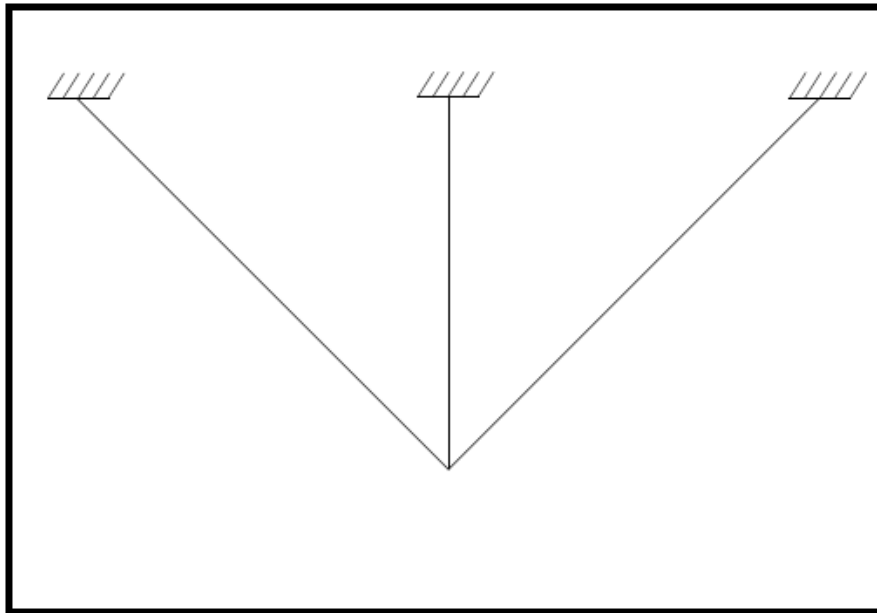
GA의 경우 최적화 계산과정에서 랜덤 요인이 포함되기 때문에 100번을 수행하여 제일 좋은 결과를 도출함

참고 문헌의 결과와 유사한 결과를 나타냄

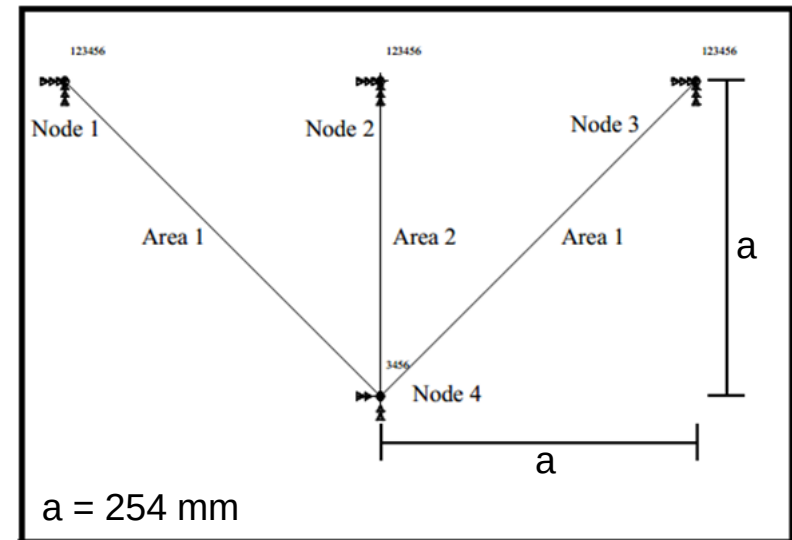
응력 제약 문제의 경우 변위 제약조건을 고려하지 않음에도 변위 제약조건을 만족하는 것을 알 수 있음

숙제: EIGENVALUE 최적화

- Weight (volume) minimization of a three bar truss



Modeling



Objectives :

Minimize the weight of the truss

1st eigen frequency must be between

1,500 Hz – 1,550 Hz

(모드해석 케이스 필요)

(센서→시스템→모드)

Material : aluminum

Initial value of each area

Area 1 : 645.16 mm²

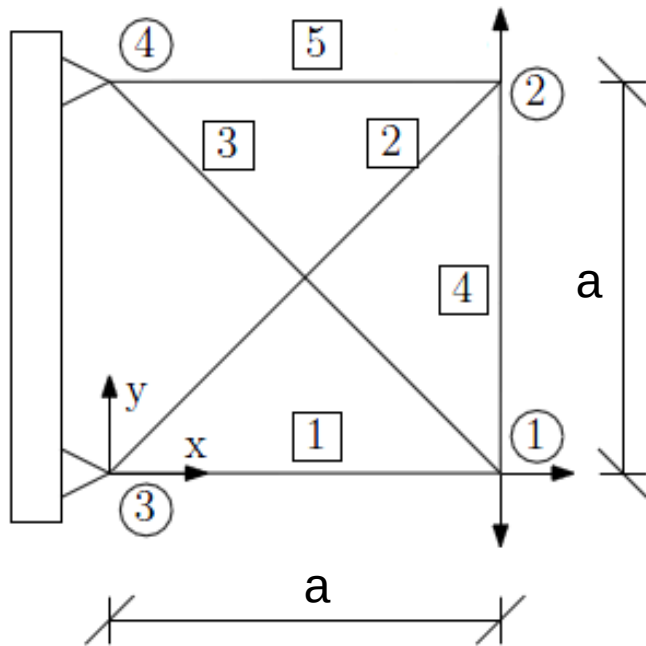
Area 2 : 1290.32 mm²

Upper and lower bound of section area

$$64.516 \leq \text{Area 1 or 2} \leq 6451.6$$

3 BAR TRUSS STRUCTURE (TOPOLOGY + SIZE OPTIMIZATION)

예제: 5 BAR STRUCTURE



Material : aluminum

탄성계수 : $6.895 \times 10^4 \text{ N/mm}^2$

프와송비 : 0.3

질량밀도 : $2.77 \times 10^{-6} \text{ kg/mm}^3$

$a = 254 \text{ mm}$

$F_{1x} = 22.241 \text{ kN}$, $F_{1y} = -13.344 \text{ kN}$

$F_{2y} = 4.448 \text{ kN}$

문제정식화

설계변수 : 각 부재의 단면적

설계목적 : 부피 최소화

설계제약조건

부재 최대응력 413.684 MPa 이하

절점 최대변위(x,y방향) 1.524 mm 이하

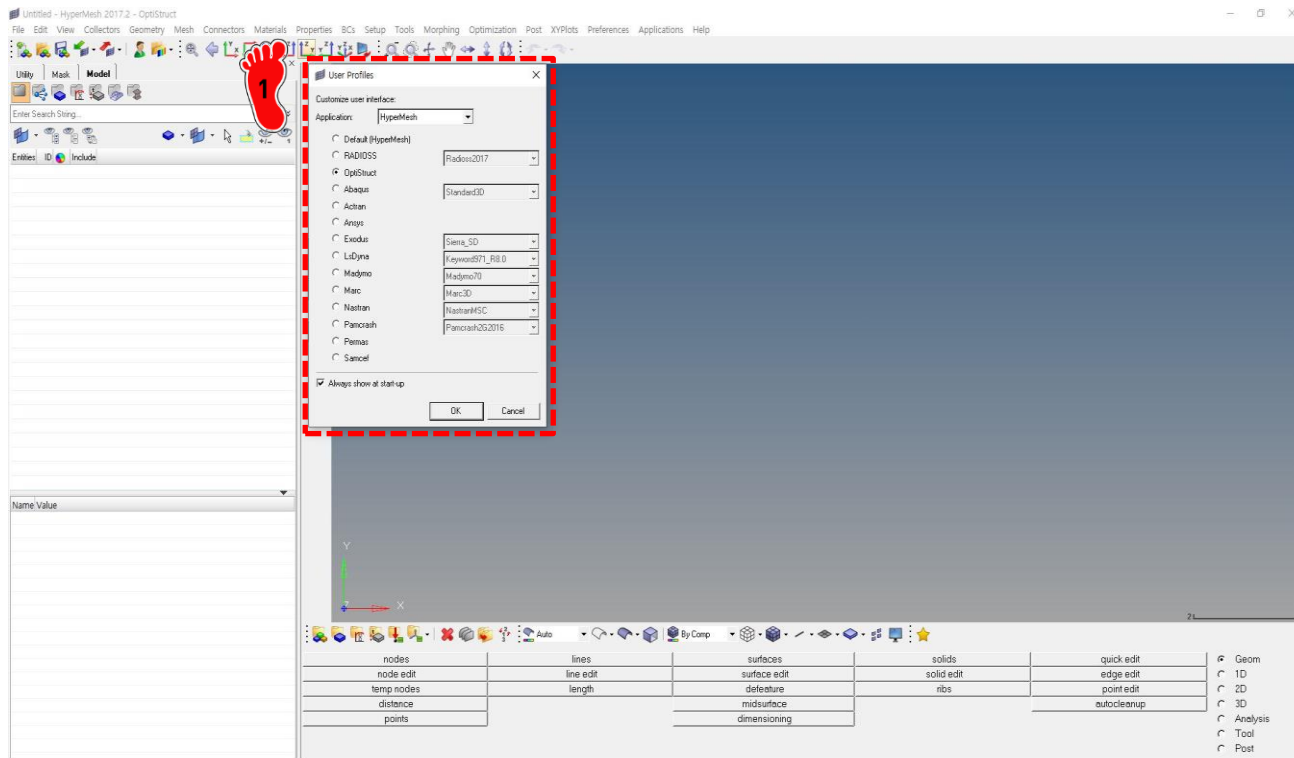
$6.4516 \text{ mm}^2 \leq \text{부재 단면적} \leq 64.516 \text{ mm}^2$

Referenced optimum values*

부재 번호	1	2	3	4	5
단면적 [mm ²]	32.26	6.45	30.39	10.77	6.45

* A. Pospisilova and M. Leps, ACTA PLYTECHNICA, 2012

기하형상 생성 (1)

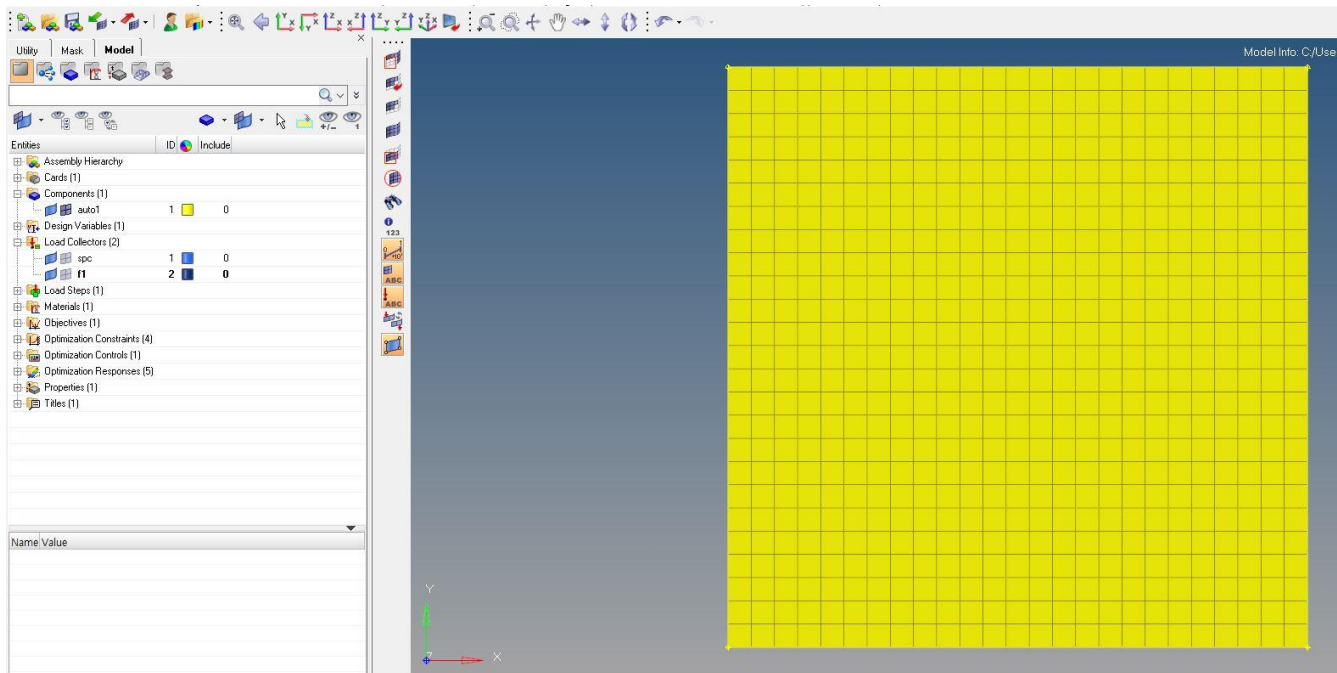


1 OptiStruct 선택, OK

기하형상 생성 (2)



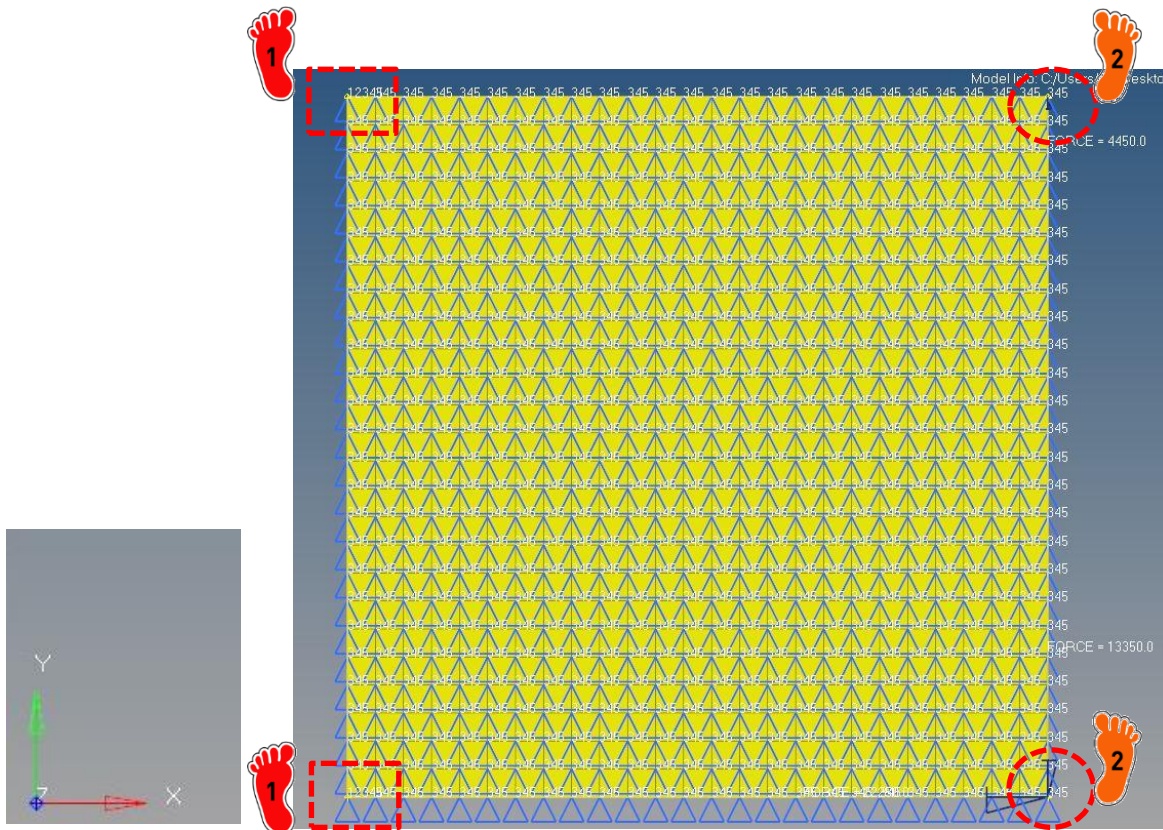
254*254 평면 생성
(설계영역)



재료 aluminum
(탄성계수 : $6.895 \times 10^4 \text{ N/mm}^2$
프와송비 : 0.3
밀도 : $2.77 \times 10^{-6} \text{ kg/mm}^3$)

PSHELL → 두께 5.5 mm,
요소 크기 10 mm인 판 요소 생성

구속조건 및 하중조건 설정



1 고정 구속 및 핀구속

2 문제에 주어진 하중 입력

$$\begin{aligned} F_{1x} &= 22.241 \text{ kN}, \\ F_{1y} &= -13.344 \text{ kN} \\ F_{2y} &= 4.448 \text{ kN} \end{aligned}$$

Load step 생성

설계변수 및 응답설정

1



1

'Analysis' -> 'optimization'
-> 'topology' 클릭하여 변
수 생성

설계변수 이름 및 범위 설정

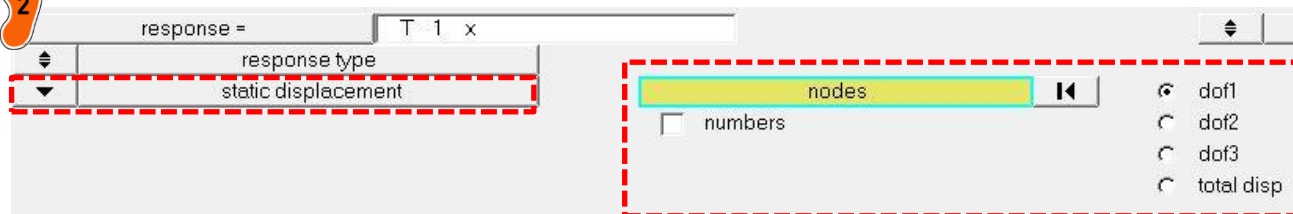
설계변수는 판의 두께

2

'Analysis' -> 'optimization'
-> 'response' 클릭

조인트 1, 2에 대한 절점 변
위 응답 설정 (Tx, Ty): 4개

2

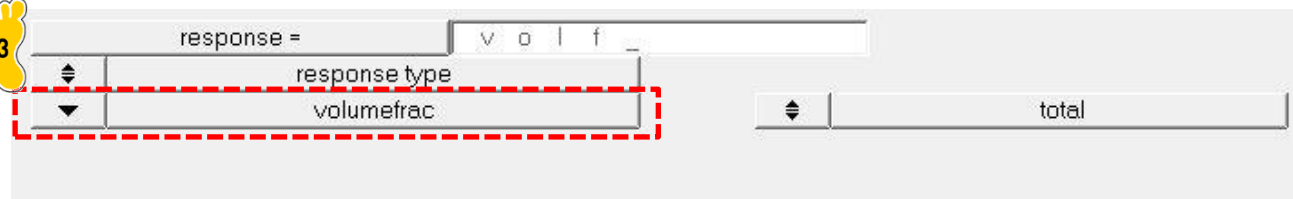


3

Volume fraction 설정

전체 부피 응답 설정

3



치수 최적화 설정: 변위 제약조건

The screenshot shows the OptiStruct software interface with the following elements highlighted:

- Red Foot 1:** Points to the 'response =' field and the 'loadsteps' section.
- Red Foot 2:** Points to the 'constraint =' field and the 'lower bound =' and 'upper bound =' fields.
- Yellow Foot 3:** Points to the 'min' dropdown and the 'response =' field.
- Green Foot 4:** Points to the 'opti control' and 'constr screen' fields.

1 'Analysis' -> 'optimization'
-> 'dconstraints' 클릭

'response'에서 변위 응답
선택

'loadsteps' 설정

2 최대변위 제한조건 설정

총 4개의 변위 제약조건

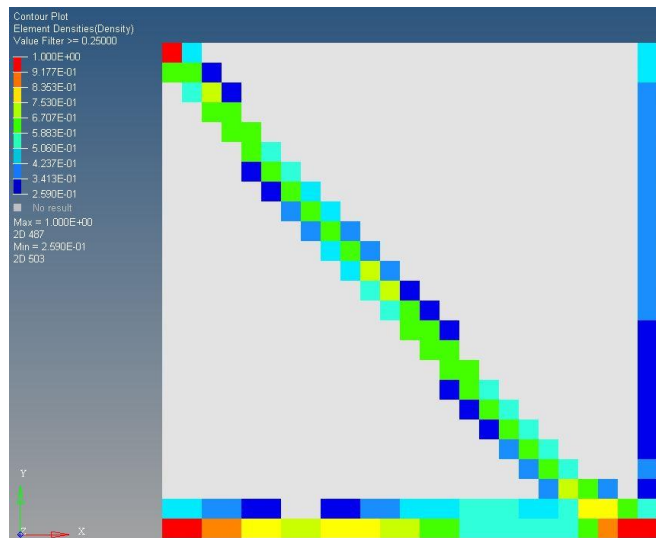
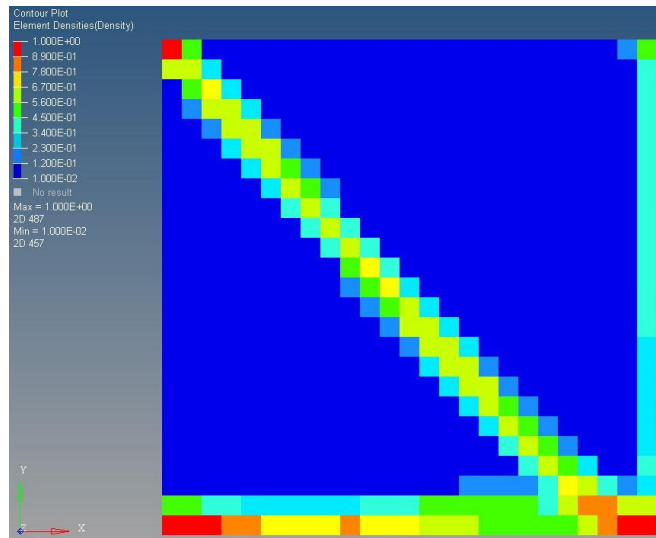
3 'Analysis' -> 'optimization'
-> 'objective' 클릭

부피 최소화 설정

4 'Analysis' -> 'optimization'
-> 'opti control' 기본값 설정

최적화 수행

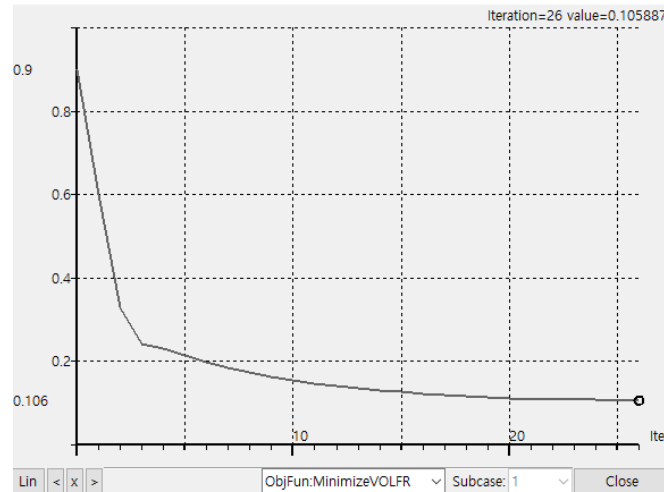
위상 최적화 결과 확인



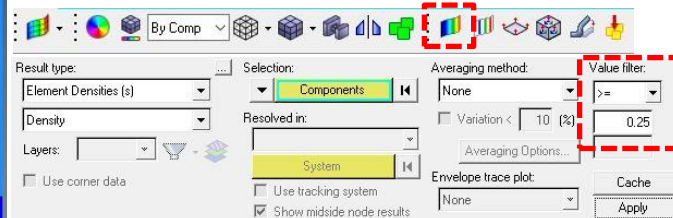
Optimization summary:

Convergence history for <test02_1.fem>

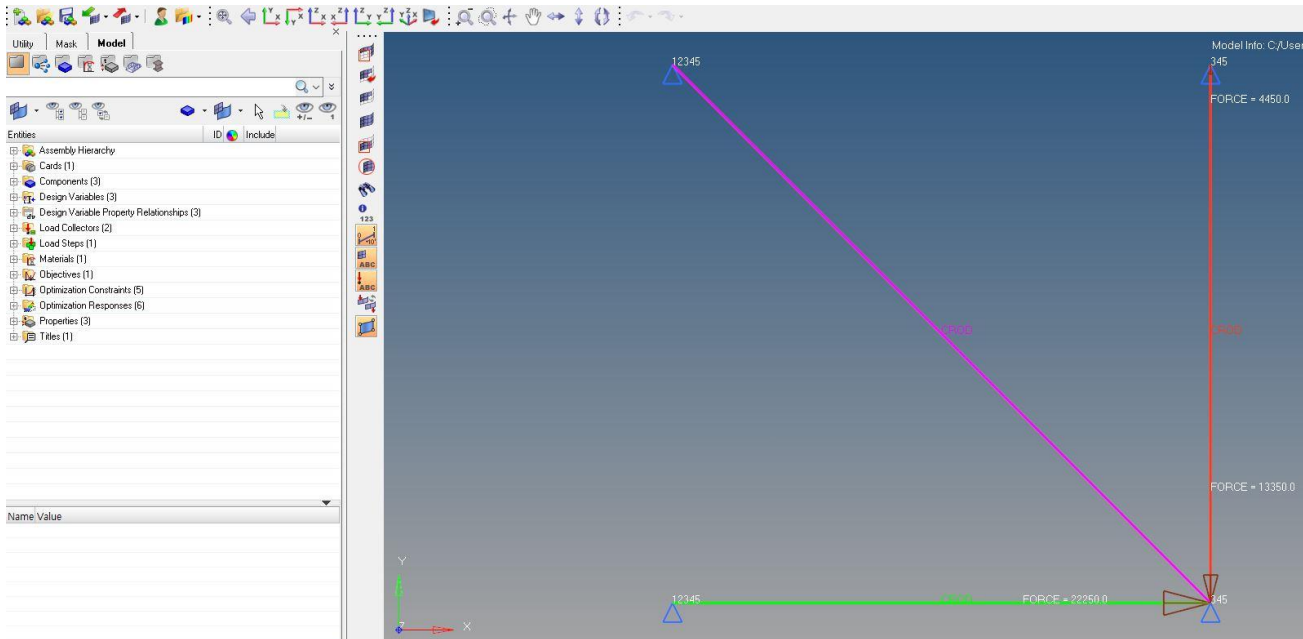
Graph



1 재료밀도 0.25 이상인 결과 확인 시
1, 3, 4번 부재만 표현되는 것을 확인



치수 최적화 설정



1 위상최적화에서 도출된 1, 3, 4번 부재만을 모델링하여 치수최적설계 수행

재료 물성치, 단면적 모델링, 경계조건 설정 등은 앞선 실습 예제와 동일하게 설정

치수최적설계 모델링 후, 최적설계 수행

치수 최적화 결과 확인



치수최적화 결과 확인

Objective Function (Minimize VOLUM) = 2.18683E+04 % change = 0.00
 Maximum Constraint Violation % = 0.12040E+03
 Volume = 2.18683E+004 Mass = 6.05753E-005

Subcase Compliance Epsilon
 1 2.711363E+04 2.683505E-16

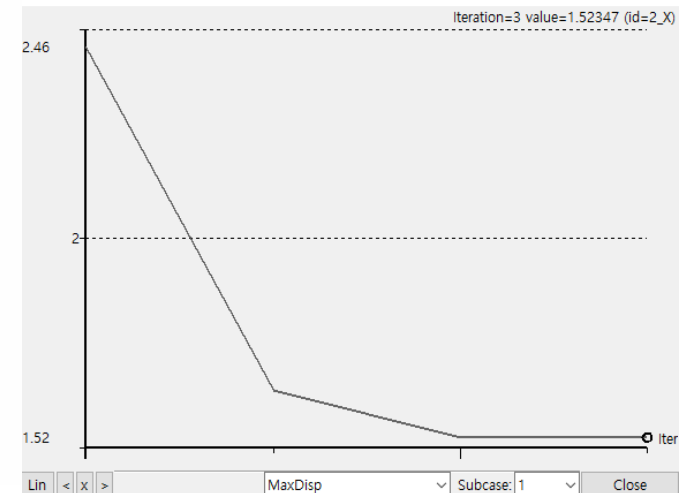
Note : Epsilon = Residual Strain Energy Ratio.

RETAINED RESPONSES TABLE

Response Type	Response	Subcase	Grid/	DOF/	Response	Objective	Viol.
User-ID	Label	/RANDPS	Element/	Comp	Value	Reference/	%
		/Model	MID/PID/	/Reg		Constraint	
		+Frqncy	Mode No.			Bound	
		/Times					
6	VOLUM vol	--	--	TOTL	2.187E+04	MIN	
1	DISPL Tlx	1	2	TX	1.523E+00	< 1.524E+00	0.0 A
2	DISPL Tly	1	2	TY	-1.522E+00	> -1.524E+00	0.0 A
5	STRES stress	1	4	SA	4.137E+02	< 4.137E+02	0.0 A
5	STRES stress	1	3	SA	4.134E+02	< 4.137E+02	0.0 A
5	STRES stress	1	1	SA	4.136E+02	< 4.137E+02	0.0 A

Design Variable ID	Design Variable Label	Lower Bound	Design Variable	Upper Bound
1	x1	6.452E+00	3.228E+01	6.452E+01
2	x2	6.452E+00	3.045E+01	6.452E+01
3	x3	6.452E+00	1.076E+01	6.452E+01

Convergence history for <test02_3.fem>



치수 최적화 결과

		최적화 문제			
		참고 문헌	(1) 응력 제약	(2) 변위 제약	(3) 응력 & 변위
부피[mm ³]		2.6e4	2.2e4	2.1e4	2.2e4
최대 응력[MPa]		414.09	413.68	689.75	413.68
변위 [mm]	절점 1,x	1.52	1.52	1.52	1.52
	절점 1,y	-1.52	-1.52	-1.52	-1.52
	절점 2,x	0.001	0	0	0
	절점 2,y	-0.004	-0.001	1.02	0.002
변수 [mm ²]	A1	32.26	32.30	32.30	32.28
	A2	6.45	-	-	-
	A3	30.39	30.43	30.47	30.45
	A4	10.77	10.76	<u>6.45</u>	10.76
	A5	6.45	-	-	-

변위 제약조건 문제의 경우
변위는 만족하나 응력은 만
족하지 못함

응력, 변위를 모두 고려한
경우 제약조건을 모두 만족
하며, 앞서 5-bar 문제에 비
해 부피도 감소한 것을 알
수 있음 (2.6e4→2.2e4)