

# 목 차

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- 구조최적설계
- 위상최적설계
  - 정식화
  - 사례: 자동차 분야

# 구조최적설계: 개념

- 설계변수 (d)
  - 부재크기 (두께, 단면적, 길이), 경계 (절점/조절점 좌표)

- 상태변수 (U)
  - 중량, 응력, 변위, 온도, 고유진동수, 좌굴하중
  - 목적함수, 제약조건

상태방정식  
지배방정식



- 결정사항
  - 해석 종류: 상/편 미분방정식
  - 설계 공간 (설계변수)
  - 목적함수 및 제약함수 (구조 거동, 구조 기하)

# 구조최적설계: 설계변수

## ■ 치수 (Size)

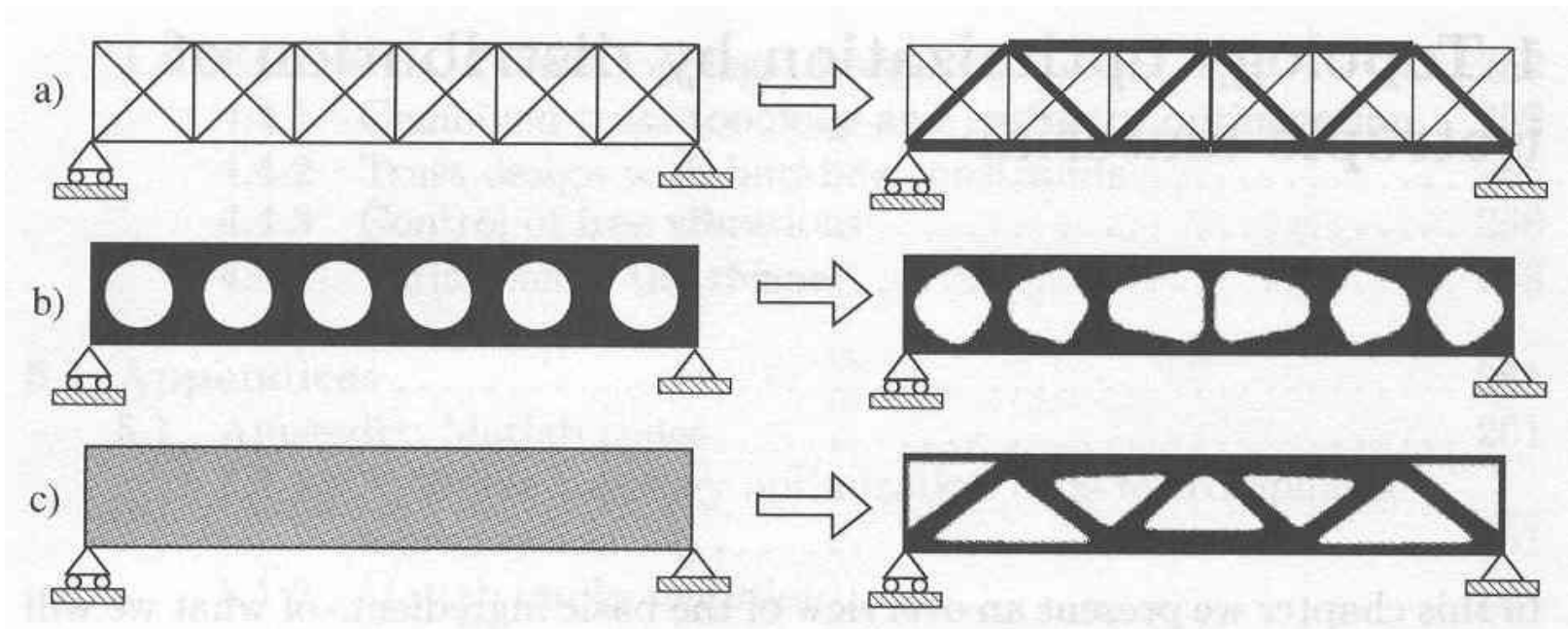
- 부재크기
- 두께, 단면적, 길이

## ■ 형상 (Shape)

- 경계
- 절점/조절점 좌표

## ■ 위상 (Topology)

- 재료 유/무
- 요소밀도



M.P.Bendsøe and O. Sigmund, Topology Optimization: Theory, Methods and Applications, Springer, 2003

# 구조최적설계: 정식화

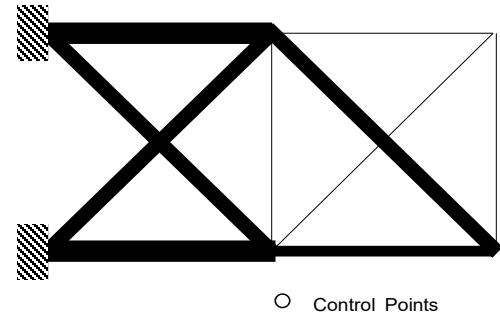
- 설계변수 :
- 목적함수 : 부피(중량) 최소화
- 제약조건 : 상태방정식(해석), 최대응력, 최대변위

$$\min_d \int_{\Omega} \rho d\Omega$$

$$\text{subject to } \begin{cases} a(\mathbf{u}, \mathbf{v}) = f(\mathbf{v}) \forall \mathbf{v} \\ \bar{\sigma} \leq \sigma_{\max} \\ |\mathbf{u}| \leq u_{\max} \end{cases}$$

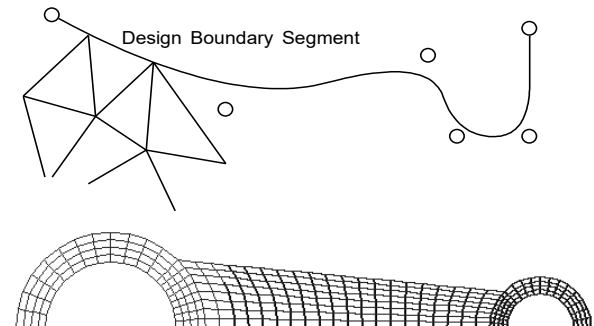
## ■ 치수최적설계

- 항공/토목 구조물: 트러스, 빔, 프레임
- 기계구조물: 형상과 관련



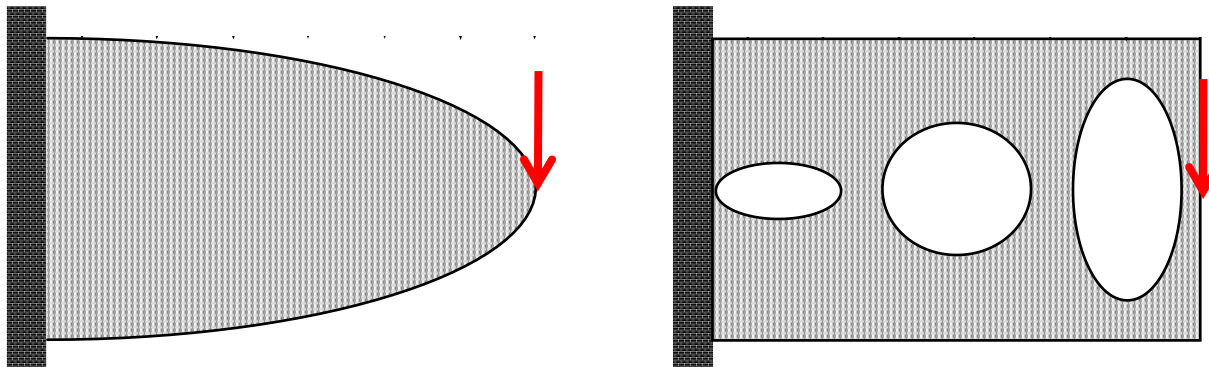
## ■ 형상최적설계

- 파라메트릭 기하 표현: 자동 요소망 생성 필요
- 기저형상 활용



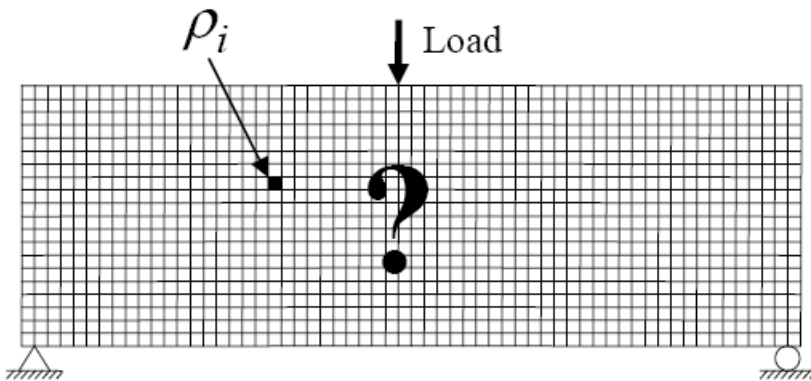
# 구조물 경량화

- 치수(size) 및 형상(shape) 최적설계
  - 초기설계에 크게 의존
    - 최적설계에 의한 경량화 효과 적음 (우수한 초기설계인 경우)
- 효과적인 구조물 경량화: 구조물 내 구멍 생성
  - 구멍의 위치/크기/형상: 패러다임 변화 필요
  - 위상(topology)최적설계 제안



# 새로운 패러다임: 위상최적설계

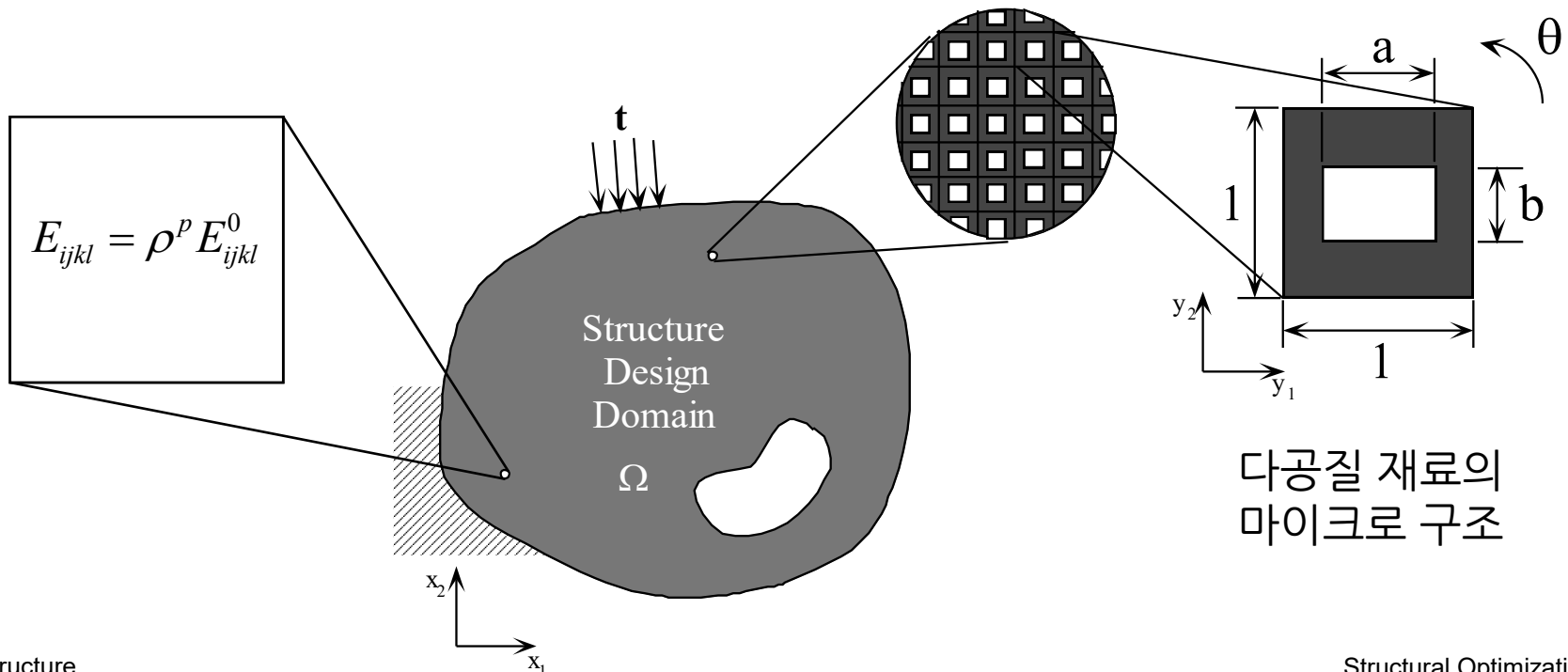
- 형상최적설계의 문제점
  - 형상변화에 따른 해석모델 재생성
  - 설계변화계산의 어려움, 제한적 형상변화
- 아이디어?
  - 해석모델 고정: 구조최적설계방법이 유한요소생성과 별개
  - 형상을 유한요소의 밀도로 표현(pixel, voxel 개념)



# 위상최적설계: 개념

- 설계변수

- 구조물의 재료분포(탄성계수)를 표현하는 값
- 균질화설계법: 다공질구조로 모델링, 복합재역학이론으로 균질화된 물성 계산
- 밀도법: 유한요소의 밀도로 물성계산



# 위상최적설계: 정적 문제

- 설계변수 ( $\rho$ )
  - 이산화한 각 유한요소의 밀도
- 문제 정식화
  - 부피 최소화, 응력/변위 제약조건: 주어진 강도/강성을 만족하는 경량화 설계
  - 응력: 트러스, 빔, 프레임 구조물에서는 유한값, 연속체에서는 국부적 물리량
  - 전역적 물리량 (평균컴플라이언스) 도입: 강성 표현

$$\min_{\rho} \int_{\Omega} \rho d\Omega \rightarrow \sum_{e=1}^N v_e \rho_e$$

$$\text{subject to } \begin{cases} a(\mathbf{u}, \mathbf{v}) = f(\mathbf{v}) \forall \mathbf{v} \rightarrow \mathbf{K}(\rho) \mathbf{U} = \mathbf{F} \\ \bar{\sigma} \leq \sigma_{\max} \\ |\mathbf{u}| \leq u_{\max} \end{cases} \rightarrow \mathbf{U}^T \mathbf{F} \leq l_{\max}$$



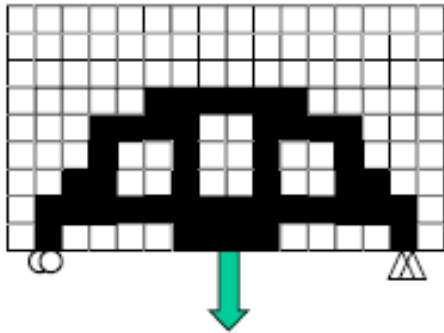
$$\min_{\rho} \mathbf{U}^T \mathbf{F}$$

$$\text{subject to } \begin{cases} \mathbf{K}(\rho) \mathbf{U} = \mathbf{F} \\ \sum_{e=1}^N v_e \rho_e \leq V^* \\ 0 < \rho_{\min} \leq \rho_e \leq 1 \end{cases}$$

# 위상최적설계: 밀도법

## ■ 정수(0/1) 문제

- 모든 조합? Ill-conditioned



$$\min_{\rho} U^T F$$

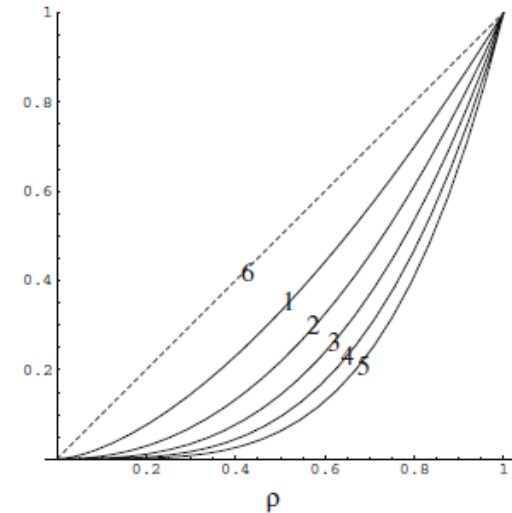
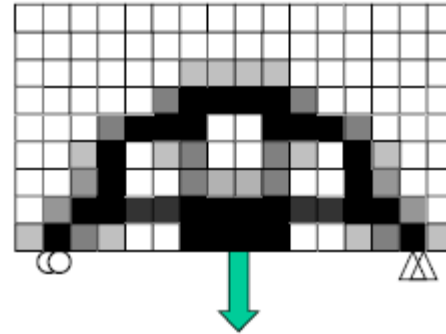
$$\text{subject to } \begin{cases} K(\rho)U = F \\ \sum_{e=1}^N v_e \rho_e \leq V^* \\ 0 < \rho_{\min} \leq \rho_e \leq 1 \end{cases}$$

$$E(\rho_e) = \rho_e^p E_0$$

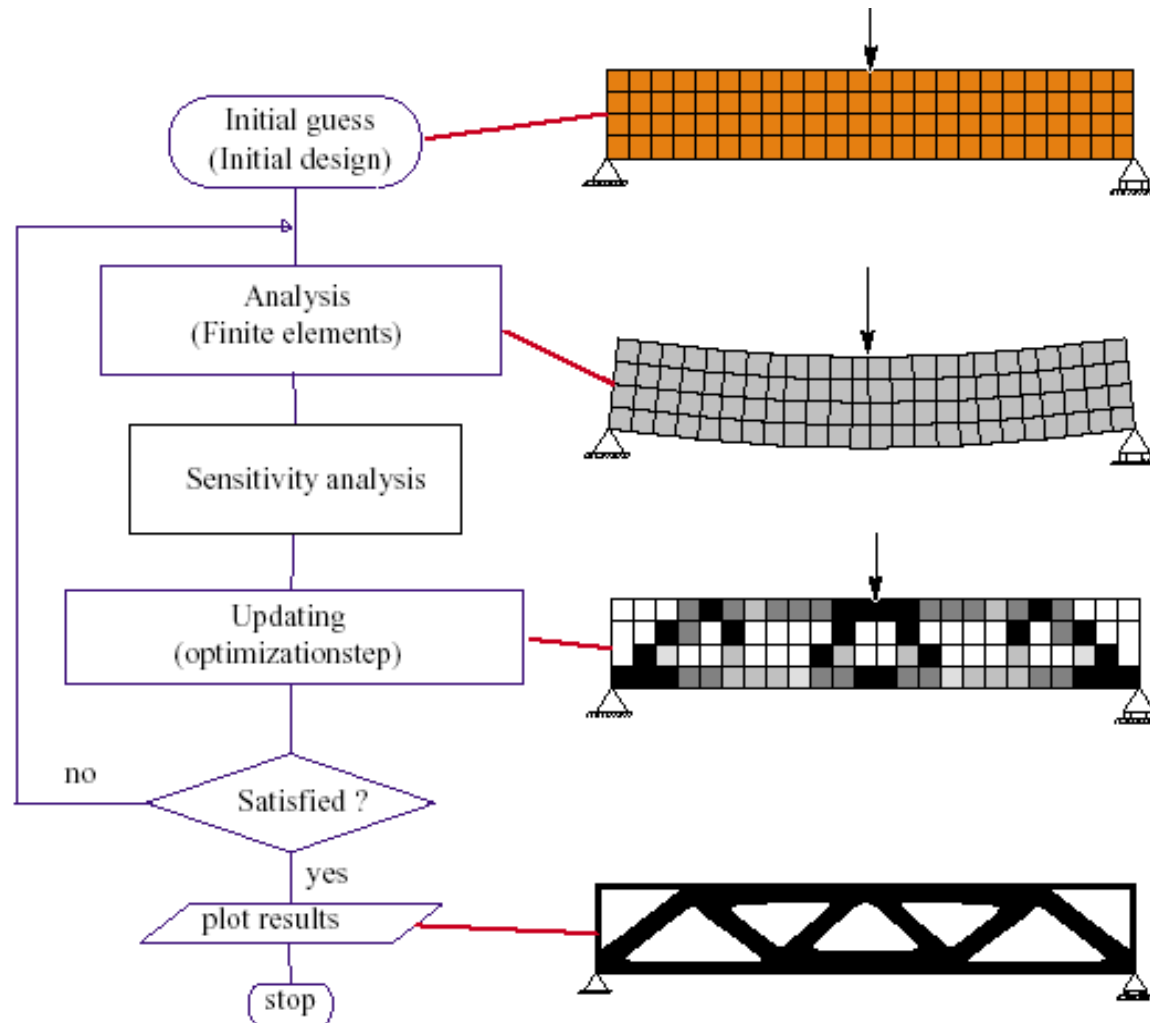
$$p > 1$$

## ■ 실수(0~1) 문제

- 완화, 중간밀도?



# 위상최적설계: 흐름도



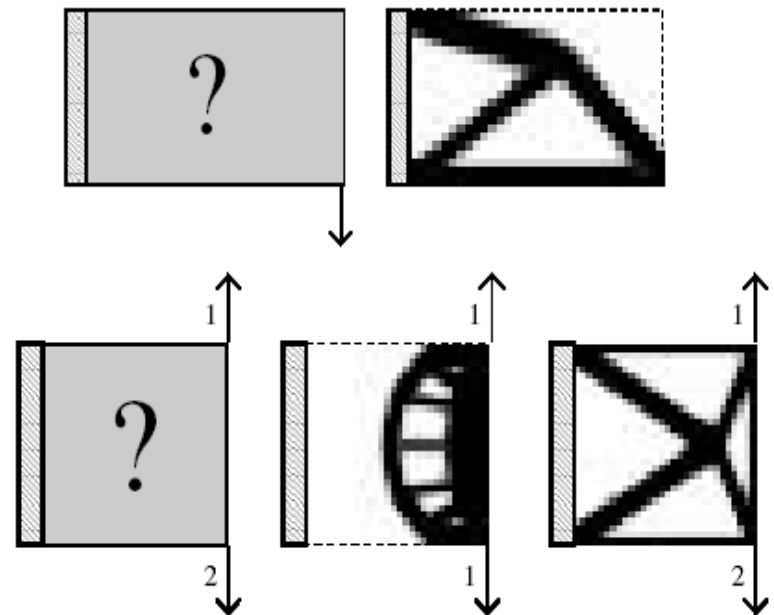
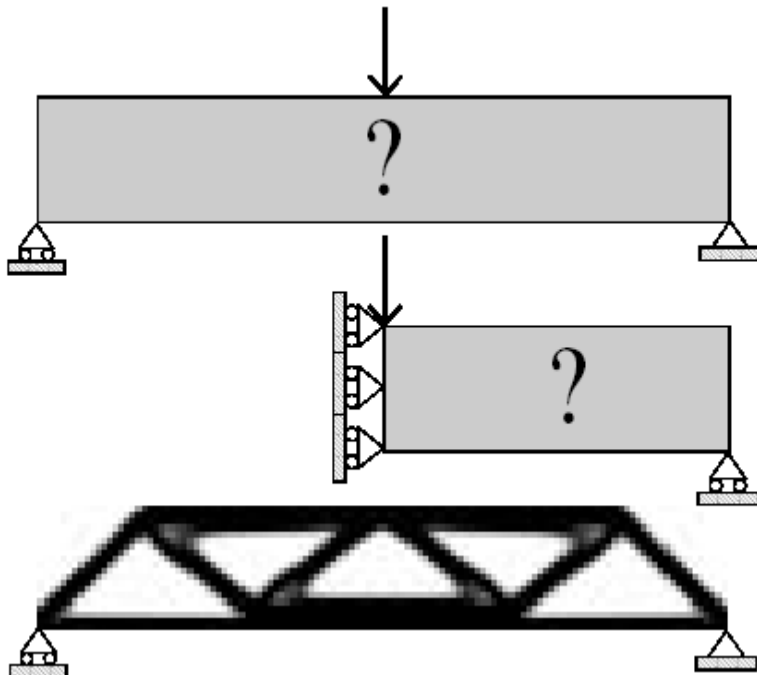
# Topology Optimization: Formulation

$$\begin{aligned}
 & \left. \begin{aligned}
 & \min_{DV} \int_{\Omega} \rho d\Omega \rightarrow \sum_{e=1}^N v_e \rho_e \\
 & \text{subject to } a(\mathbf{u}, \mathbf{v}) = f(\mathbf{v}) \forall \mathbf{v} \rightarrow \mathbf{K}(\rho) \mathbf{U} = \mathbf{F} \\
 & \left. \begin{aligned}
 & \bar{\sigma} \leq \sigma_{\max} \\
 & |\mathbf{u}| \leq u_{\max}
 \end{aligned} \right\} \rightarrow \mathbf{u}^T \mathbf{f} \rightarrow \mathbf{U}^T \mathbf{F}
 \end{aligned} \right\} \xrightarrow[E(\rho_e) = \rho_e^p E_0]{\text{dual problem}} \left\{ \begin{aligned}
 & \min_{\rho} \mathbf{U}^T \mathbf{F} = \phi(\mathbf{U}(\rho)) \\
 & \text{subject to } \left\{ \begin{aligned}
 & \mathbf{K}(\rho) \mathbf{U} = \mathbf{F} \\
 & \sum_{e=1}^N v_e \rho_e \leq V^* \\
 & 0 < \rho_e \leq 1
 \end{aligned} \right.
 \end{aligned} \right. \\
 \\
 & L(\rho, \mathbf{U}, \lambda, \Lambda, \lambda_e^-, \lambda_e^+) = \phi + \lambda^T (\mathbf{K} \mathbf{U} - \mathbf{F}) + \Lambda \left( \sum_{e=1}^N v_e \rho_e - V^* \right) + \sum_{e=1}^N \lambda_e^- (-\rho_e) + \sum_{e=1}^N \lambda_e^+ (\rho_e - 1) \\
 \\
 & \left. \begin{aligned}
 & \frac{\partial L}{\partial \rho} = \underbrace{\frac{d\phi}{d\rho} + \frac{\partial \phi}{\partial \mathbf{U}} \frac{\partial \mathbf{U}}{\partial \rho}}_{\frac{d\phi}{d\rho} + \left( \frac{\partial \phi}{\partial \mathbf{U}} + \lambda^T \mathbf{K} \right) \frac{\partial \mathbf{U}}{\partial \rho} + \lambda^T \frac{\partial \mathbf{K}}{\partial \rho} \mathbf{U} - \frac{\partial \mathbf{F}}{\partial \rho}} + \Lambda v_e - \lambda_e^- + \lambda_e^+ = 0 \\
 & \frac{\partial L}{\partial \mathbf{U}} = \frac{\partial \phi}{\partial \mathbf{U}} + \lambda^T \mathbf{K} = 0 \rightarrow \mathbf{F} + \mathbf{K}^T \lambda = 0 \rightarrow \lambda = -\mathbf{U} \\
 & -\mathbf{U} \frac{\partial \mathbf{K}}{\partial \rho} \mathbf{U} \rightarrow -\mathbf{U}_e^T \frac{\partial \left( \sum_e \mathbf{K}_e \right)}{\partial \rho_e} \mathbf{U}_e = -\mathbf{U}_e^T \left( p \rho_e^{p-1} \mathbf{K}_e^0 \right) \mathbf{U}_e
 \end{aligned} \right\} \rightarrow \left\{ \begin{aligned}
 & -\mathbf{U}_e^T \left( p \rho_e^{p-1} \mathbf{K}_e^0 \right) \mathbf{U}_e + \Lambda v_e = 0 \\
 & 1 = \frac{\mathbf{U}_e^T \left( p \rho_e^{p-1} \mathbf{K}_e^0 \right) \mathbf{U}_e}{\Lambda v_e} = B_e \\
 & \rho_e^{\text{new}} = \rho_e (B_e)^{\eta}
 \end{aligned} \right.
 \end{aligned}$$

# Educational Design Tool (1)

## – TOPOPT ([www.topopt.dtu.dk](http://www.topopt.dtu.dk))

- A 99 line topology optimization code written in Matlab, Struct Multidisc Optim 21, pp.120-127, 2001
- A web-based topology optimization program, Struct Multidisc Optim 22, pp.179-187, 2001
- Interactive topology optimization on hand-held devices, Struct Multidisc Optim 47, pp.1-6, 2013



# Educational Design Tool (2)

The screenshot displays the 'Educational Design Tool' interface within a Netscape browser window. The main window shows a design problem titled 'Try your own design!' with instructions: 'Just construct your own design problem and press 'Submit' button.' The design area features a rectangular plate with a central square void, supported by two pin supports at the bottom corners. Two upward-pointing arrows represent single loading conditions. A toolbar on the left includes various design tools like selection, movement, and deletion. Below the design area are sliders for 'Load case' and 'Volume fraction' (set to 0.25), and 'Reset' and 'Submit' buttons. Three optimization result windows are overlaid:

- Result 9:** Shows a single loading condition. The objective value is 28.21. The status is 'Stopped'.
- Result 10:** Shows a multiple loading condition. The objective value is 39.81. The status is 'Finished'.
- Result 12:** Shows a multiple loading condition with a void area. The objective value is 56.74. The status is 'Finished'.

The optimization results windows display the optimized density distribution (Densities) and the resulting structural layout. The 'Multiple loading with void area' result (Result 12) shows a more complex internal structure with a central void area.

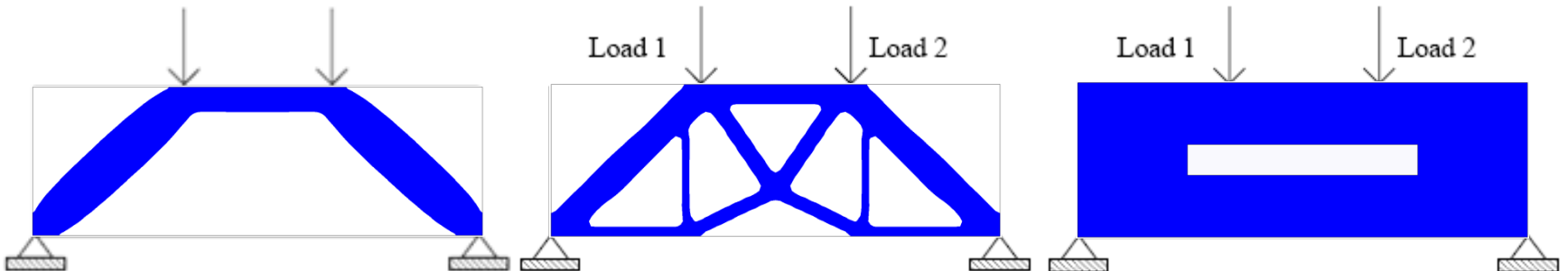
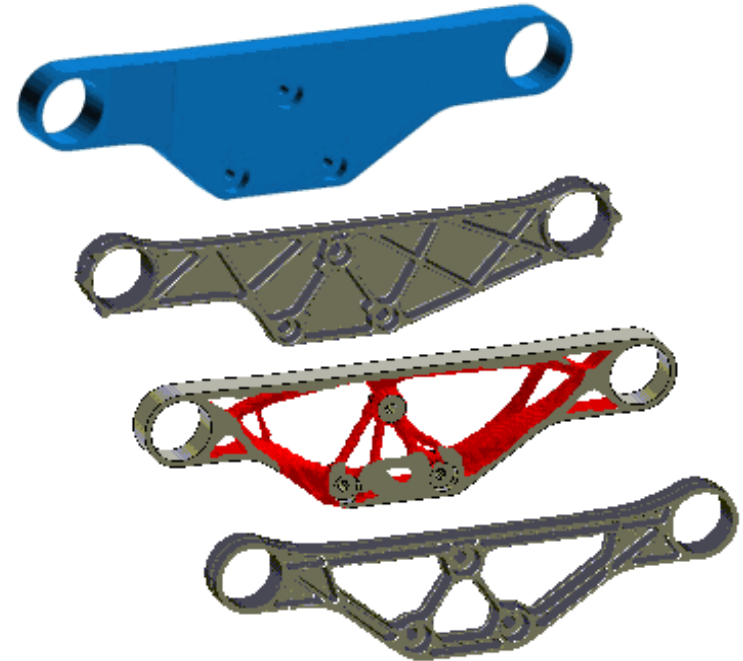
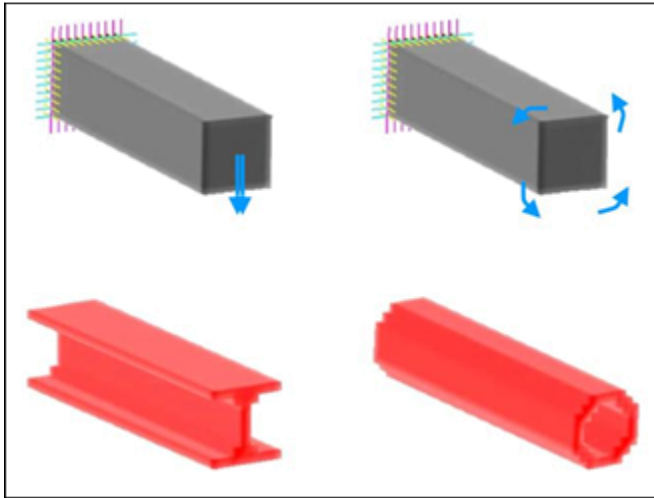
**Single loading**

**Multiple loading**

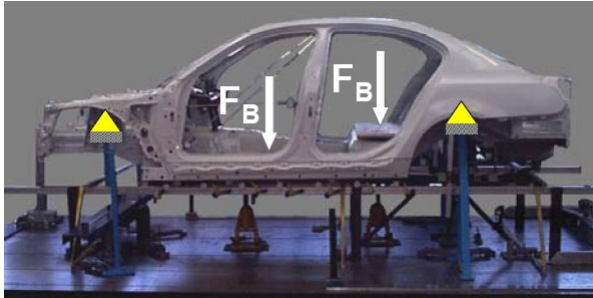
**Multiple loading with void area**

# 정적 문제: 예제

## ■ 고체역학 검증: 하중 경로

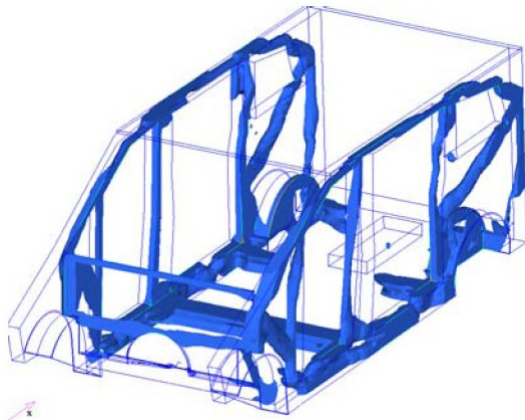
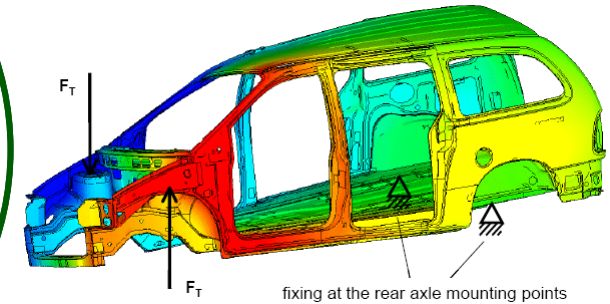
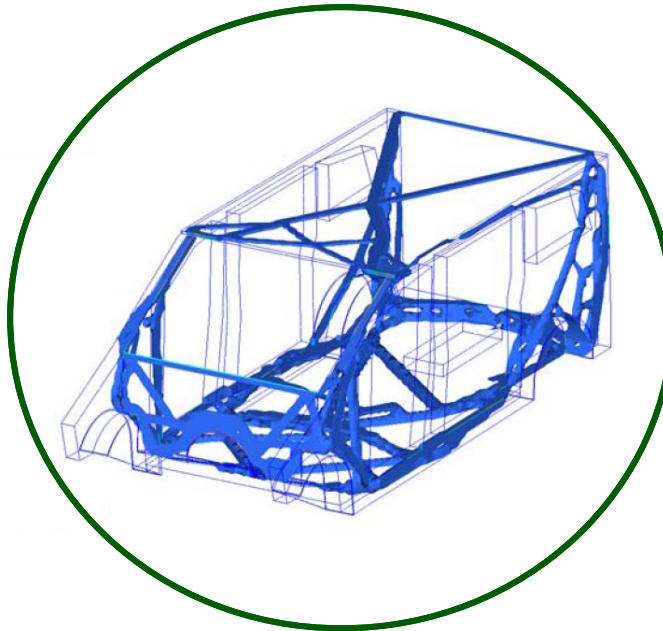
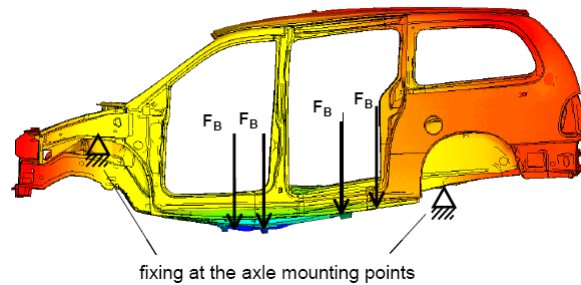


# 위상최적설계: 개념 설계에 활용

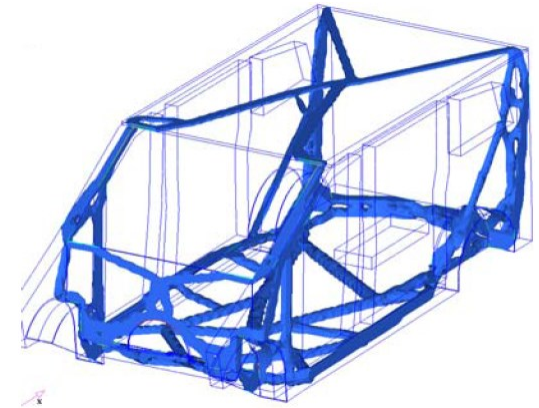


굽힘

비틀림



굽힘 + 비틀림



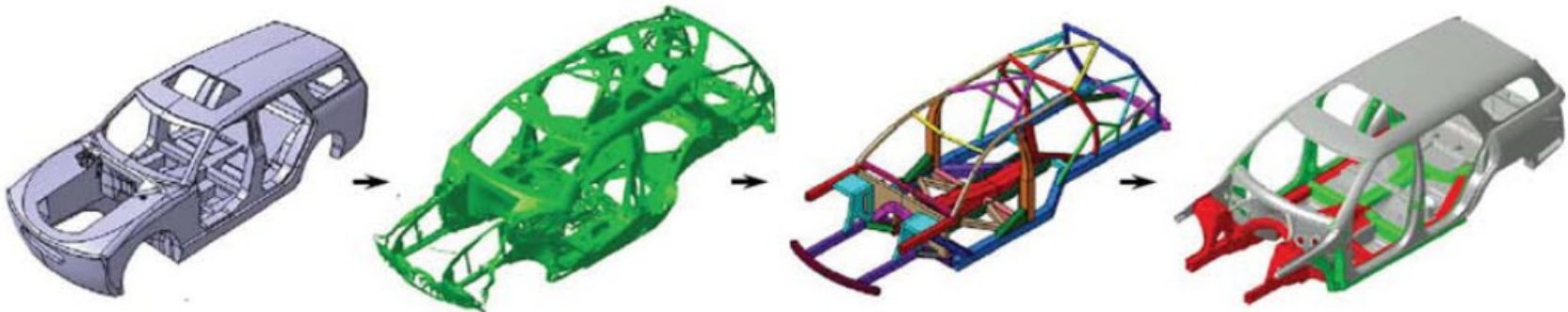
# 위상최적설계: 차체구조설계 사례



## **Light Weight Body Structural Optimization Process**

2008 GDIS

**Topology Optimization used to define Major Load Paths**



### Packaging Volume

Holistic Drivers

- Safety
- NVH Refinement
- Durability

### Topology Results

Spatial load image

### Beam Model

87 paths

### BIW Design

202 components

# Chassis: Ferrari F458 Italia

- 설계 목적

경량화

- 설계 변수 (cascade)

Topology optimization: 밀도

Topometry optimization : 절점 높이

Size optimization : 쉘 두께

- 성능 요구조건

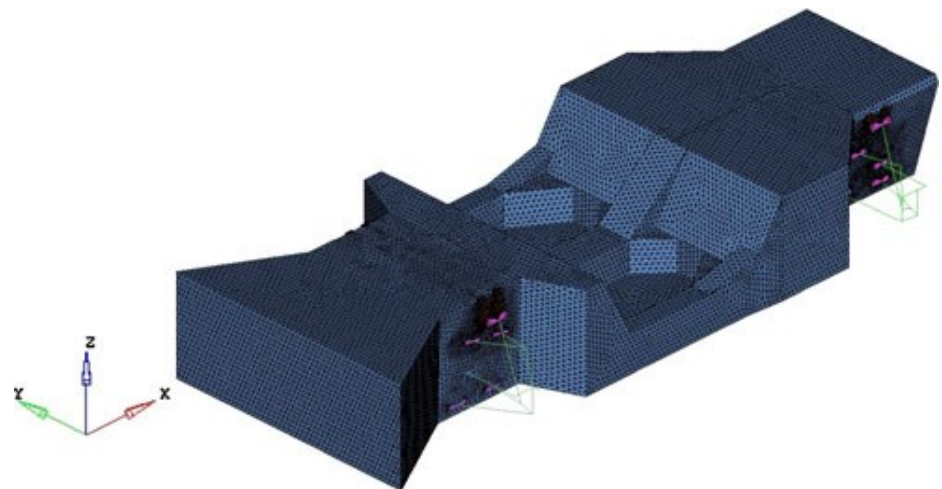
|                                                        |
|--------------------------------------------------------|
| global bending stiffness                               |
| global torsional stiffness                             |
| Local stiffness of the suspensions, engine and gearbox |
| Modal response                                         |
| Crash linearization                                    |

Struct Multidisc Optim (2011) 44:45–56  
DOI 10.1007/s00158-010-0578-7

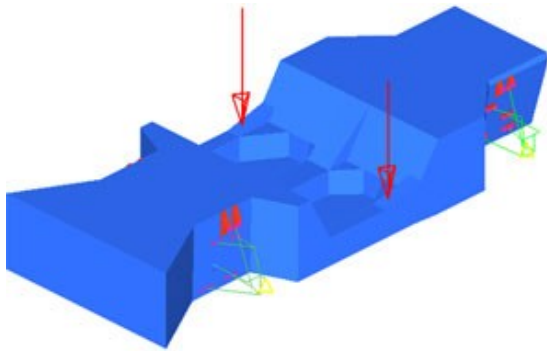
## INDUSTRIAL APPLICATION

### High performance automotive chassis design: a topology optimization based approach

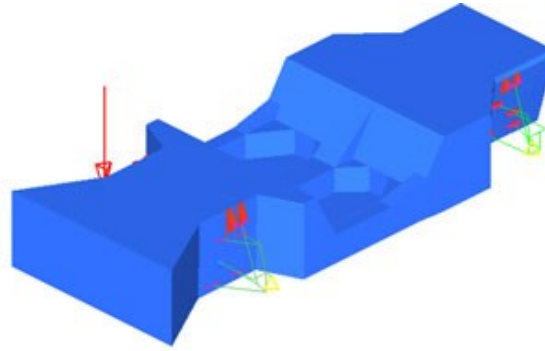
Marco Cavazzuti · Andrea Baldini · Enrico Bertocchi ·  
Dario Costi · Enrico Torricelli · Patrizio Moruzzi



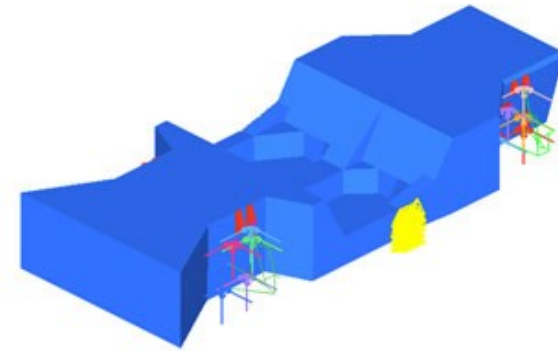
# Chassis: 하중조건



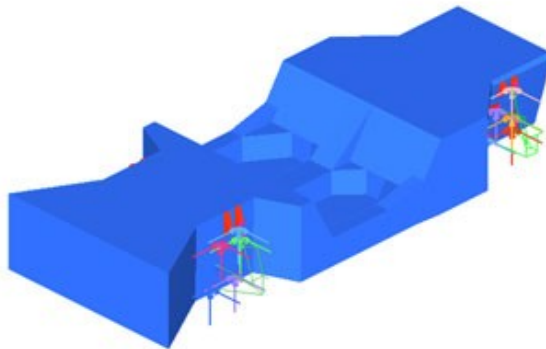
bending stiffness (1 FEA)



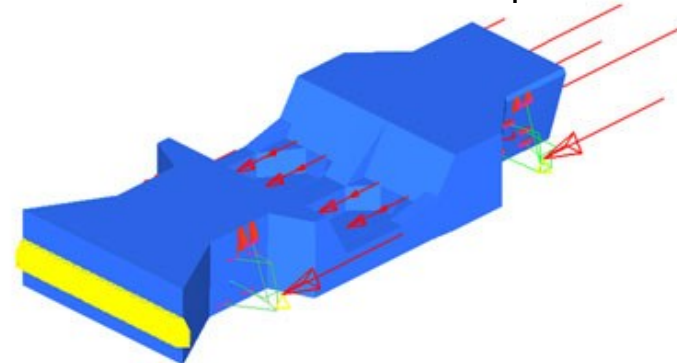
torsional stiffness (1 FEA)



local stiffness with  
clamped sills (32 FEA)

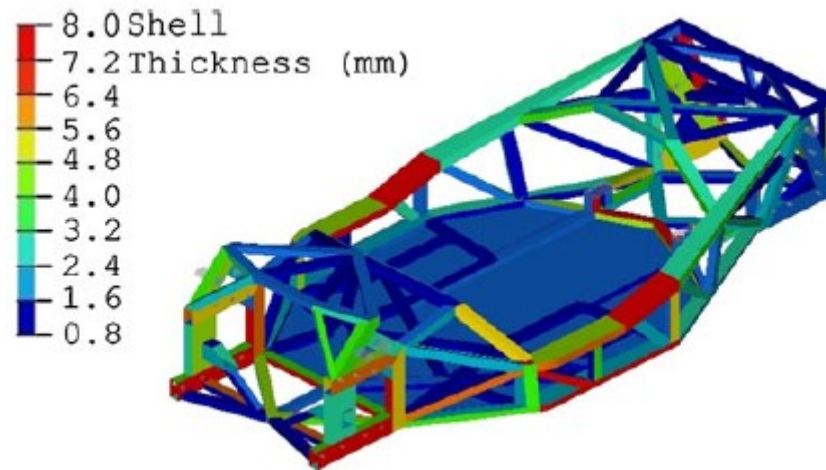
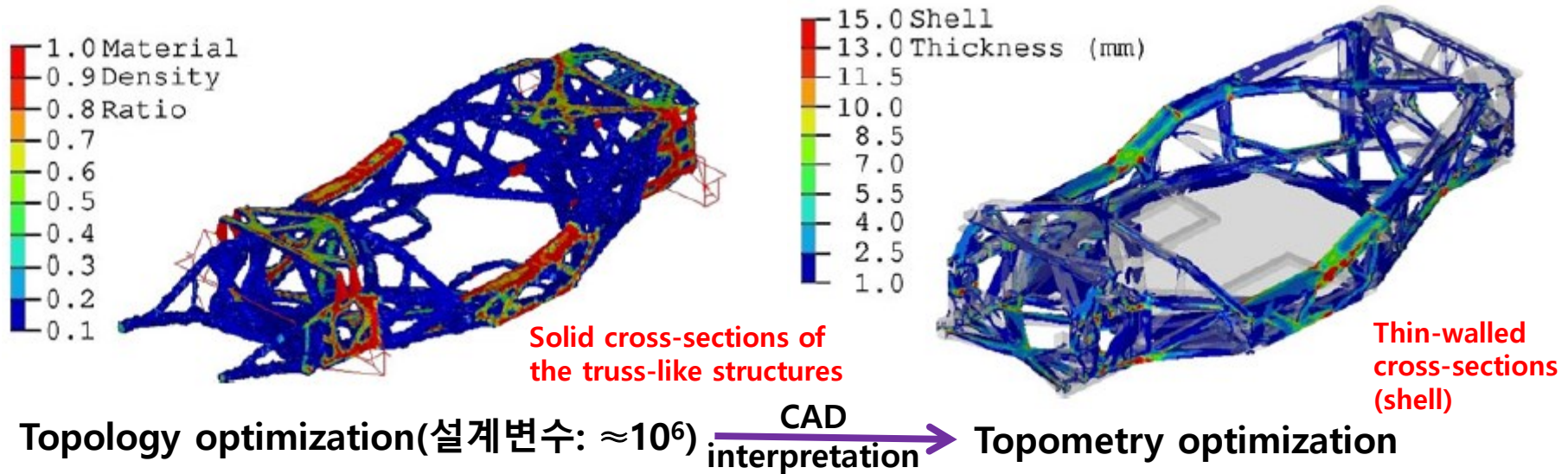


local stiffness with  
inertia relief (32 FEA)



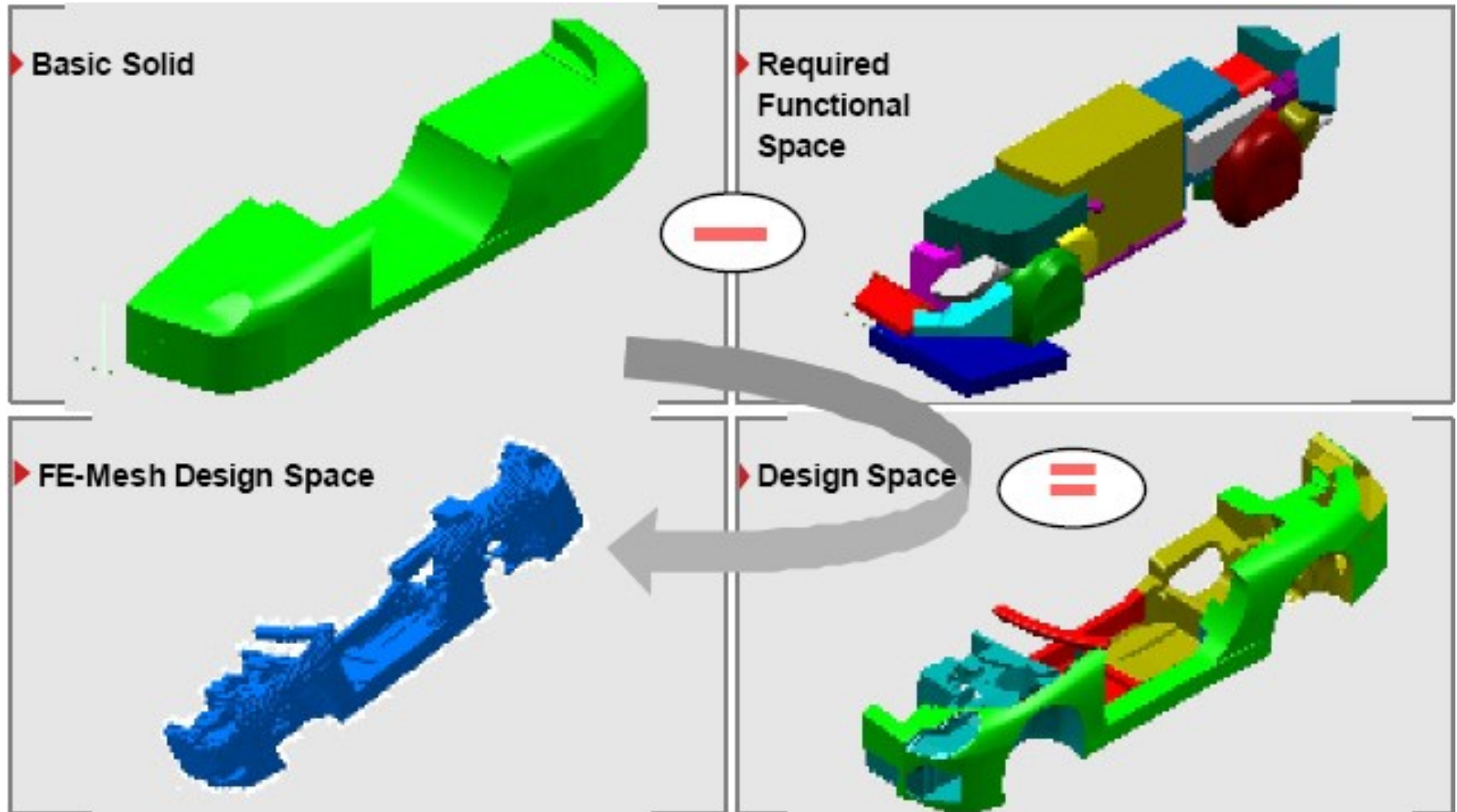
crash linearization (1 FEA)

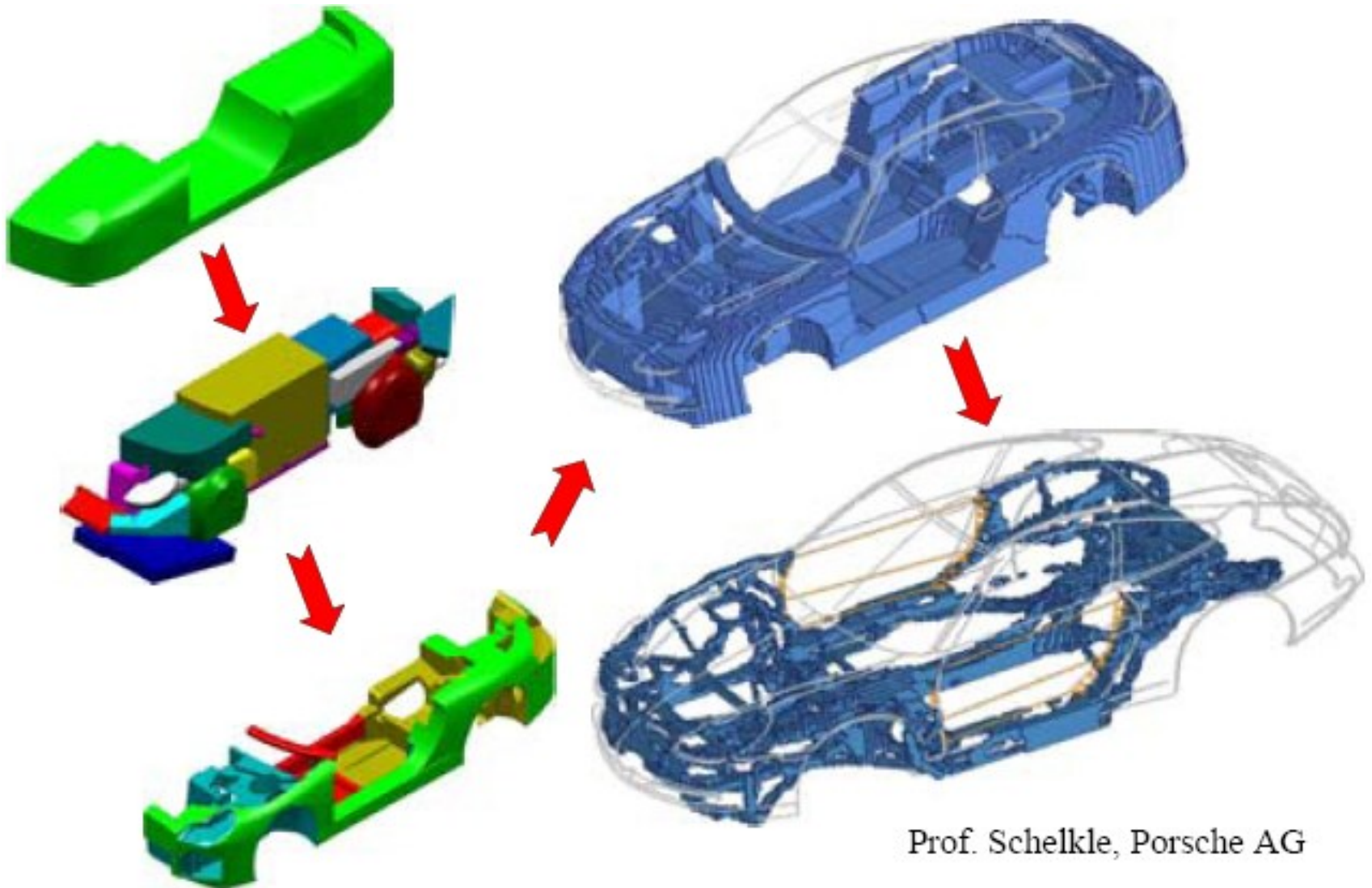
# Chassis: 최적설계



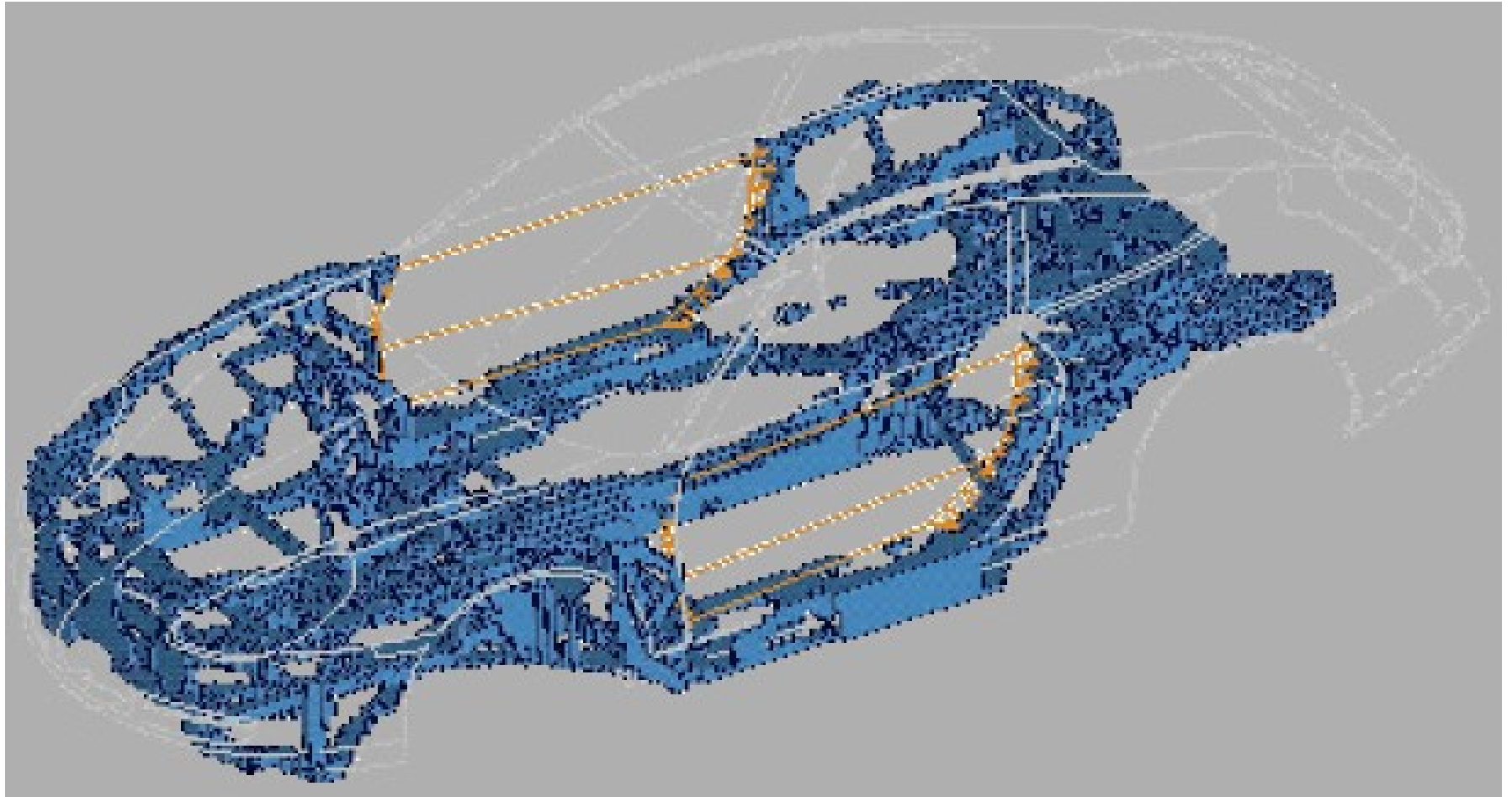
Size optimization(설계변수:  $\approx 330$ )

# Load Path Analysis Sequence

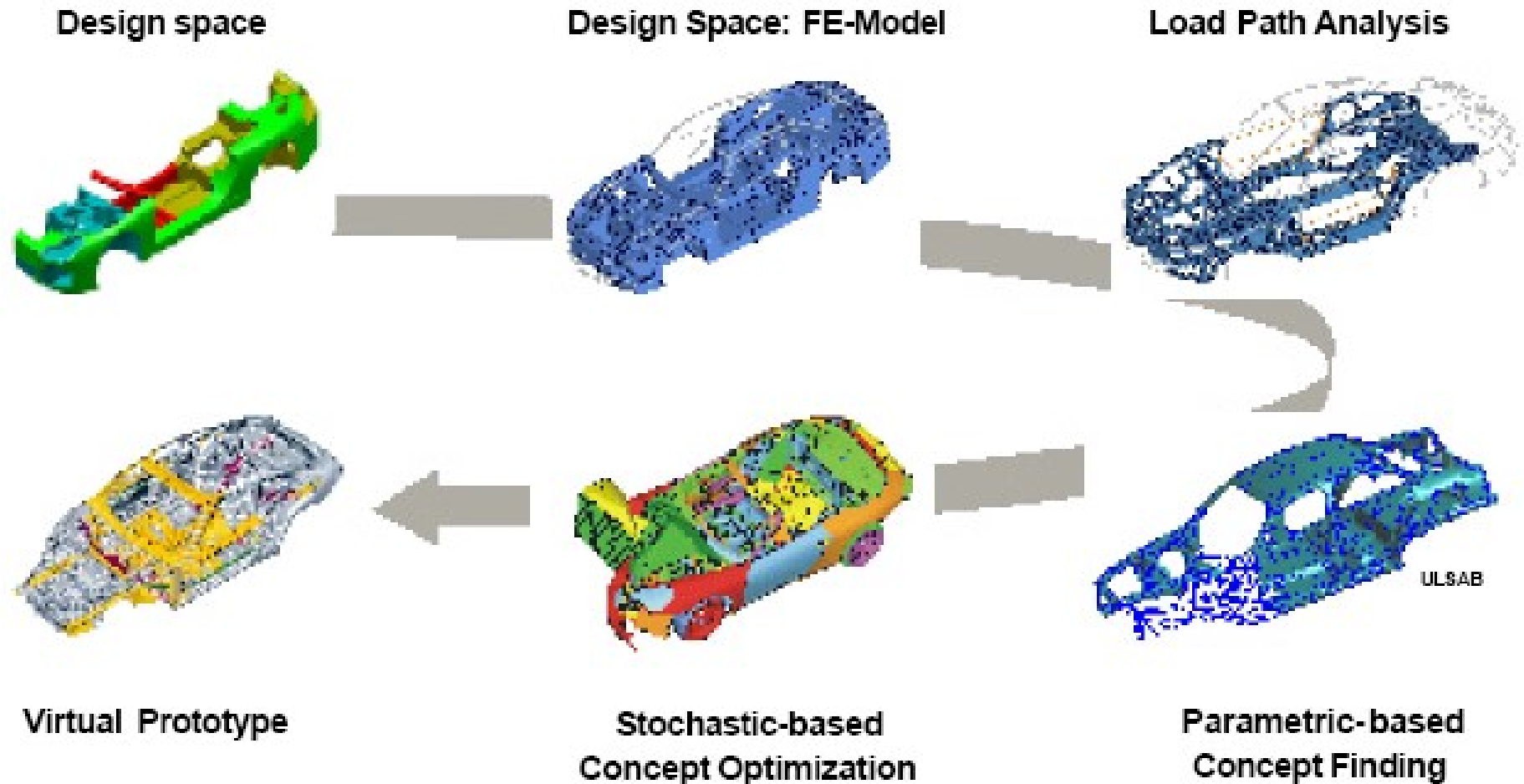




Prof. Schelkle, Porsche AG



# Concept Stage of Vehicle Development



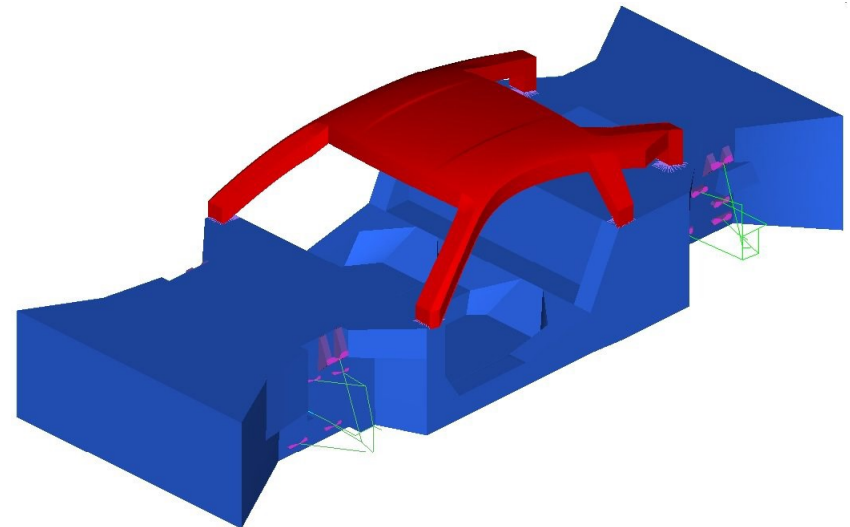
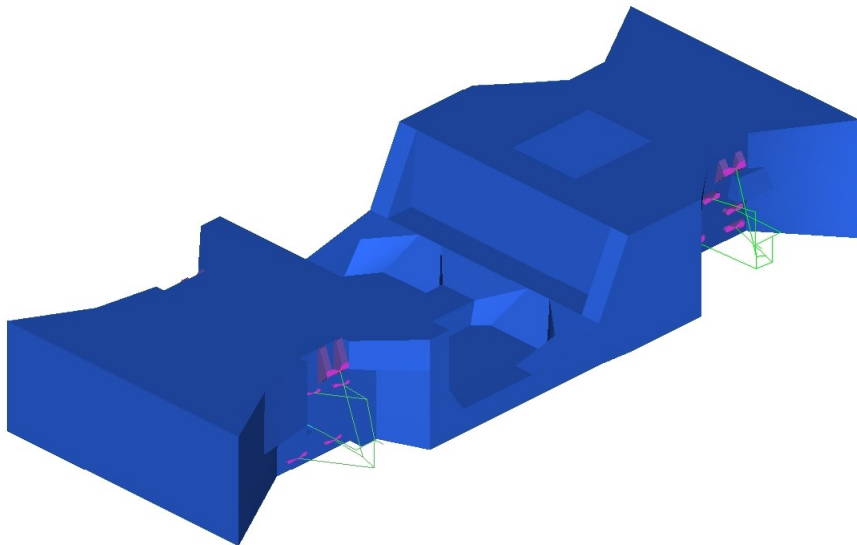
# Automotive Chassis

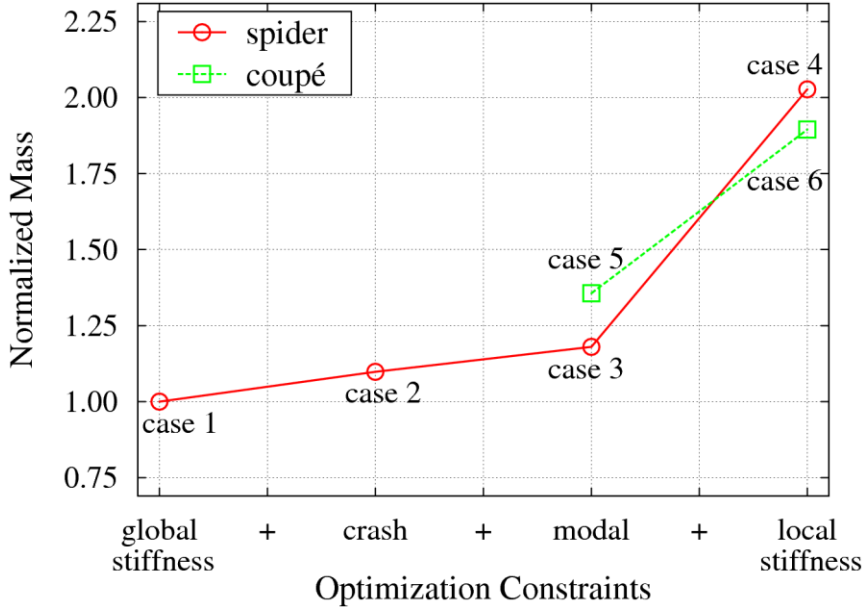
Proceedings of the World Congress on Engineering 2011 Vol III  
WCE 2011, July 6 - 8, 2011, London, U.K.

## Automotive Chassis Topology Optimization: a Comparison Between Spider and Coupé Designs

Marco Cavazzuti, Dario Costi, Andrea Baldini, Patrizio Moruzzi

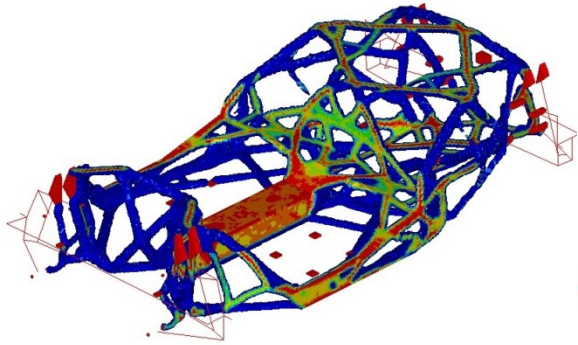
- Comparison Between Spider and Coupe Designs



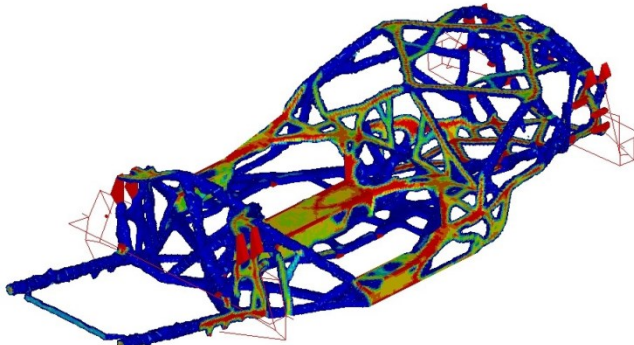


| Active<br>Optimization<br>Constraints   | Cases         |               |                |                 |                |                 |
|-----------------------------------------|---------------|---------------|----------------|-----------------|----------------|-----------------|
|                                         | Spider        |               |                |                 | Coupé          |                 |
|                                         | 1             | 2             | 3              | 4               | 5              | 6               |
| Global bending stiffness                | ✓             | ✗             | ✓              | ✓               | ✓              | ✓               |
| Global torsion stiffness                | ✓             | ✓             | ✓              | ✓               | ✓              | ✗               |
| Crash seat joints displacement          | –             | ✗             | ✗              | ✗               | ✓              | ✗               |
| Crash engine joints displacement        | –             | ✓             | ✓              | ✗               | ✓              | ✓               |
| Crash A-pillar displacement             | –             | ✓             | ✓              | ✓               | ✓              | ✓               |
| Crash pedal displacement                | –             | ✗             | ✓              | ✗               | ✗              | ✗               |
| Crash flame shield displacement         | –             | ✓             | ✓              | ✗               | ✗              | ✗               |
| Crash dashboard joints displacement     | –             | ✗             | ✗              | ✗               | ✗              | ✓               |
| Crash compliance                        | –             | ✓             | ✓              | ✓               | ✓              | ✓               |
| First natural mode                      | –             | –             | ✓              | ✓               | ✓              | ✓               |
| Local front wheel stiffness along $x$   | –             | –             | –              | ✓               | –              | ✓               |
| Local front wheel stiffness along $y$   | –             | –             | –              | ✓               | –              | ✓               |
| Local front wheel stiffness along $z$   | –             | –             | –              | ✓               | –              | ✓               |
| Local rear wheel stiffness along $x$    | –             | –             | –              | ✓               | –              | ✓               |
| Local rear wheel stiffness along $y$    | –             | –             | –              | ✓               | –              | ✓               |
| Local rear wheel stiffness along $z$    | –             | –             | –              | ✗               | –              | ✗               |
| Local engine joint stiffness along $z$  | –             | –             | –              | ✓               | –              | ✓               |
| Local gearbox joint stiffness along $z$ | –             | –             | –              | ✓               | –              | ✗               |
| Total                                   | $\frac{2}{2}$ | $\frac{5}{9}$ | $\frac{8}{10}$ | $\frac{12}{18}$ | $\frac{7}{10}$ | $\frac{12}{18}$ |

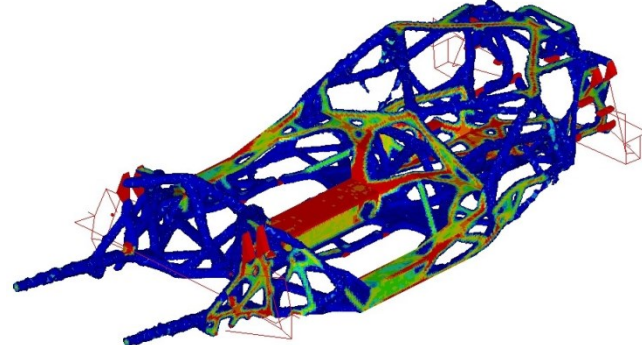
# Results (1)



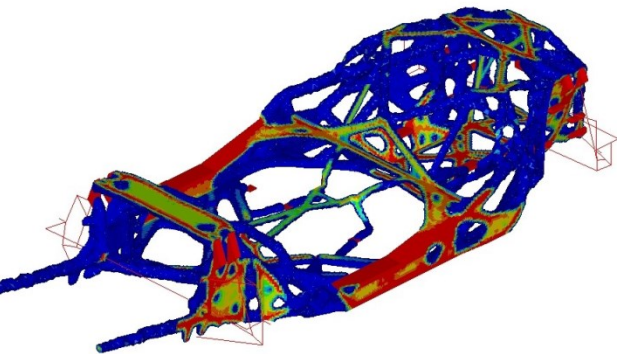
Case 1



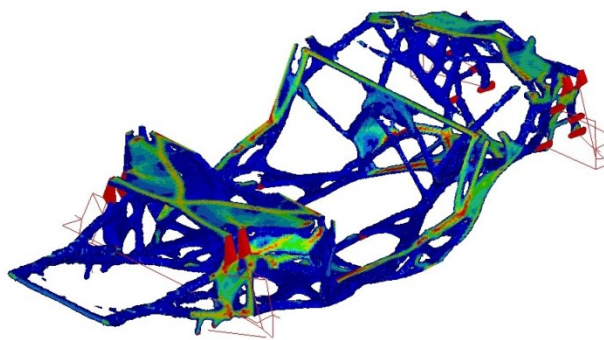
Case 2



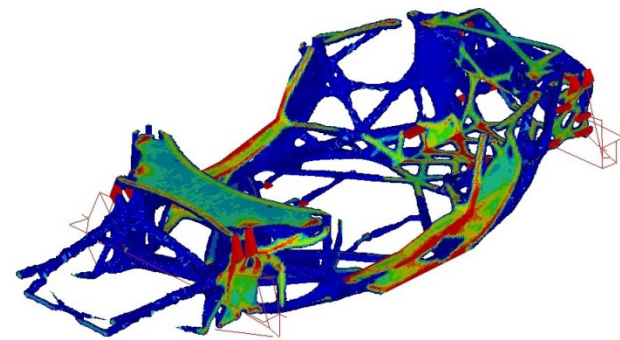
Case 3



Case 4

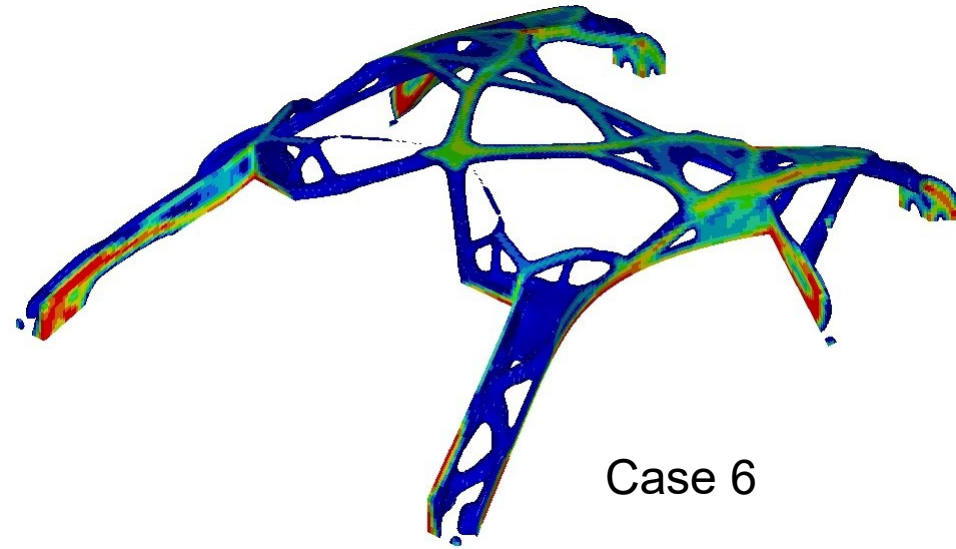
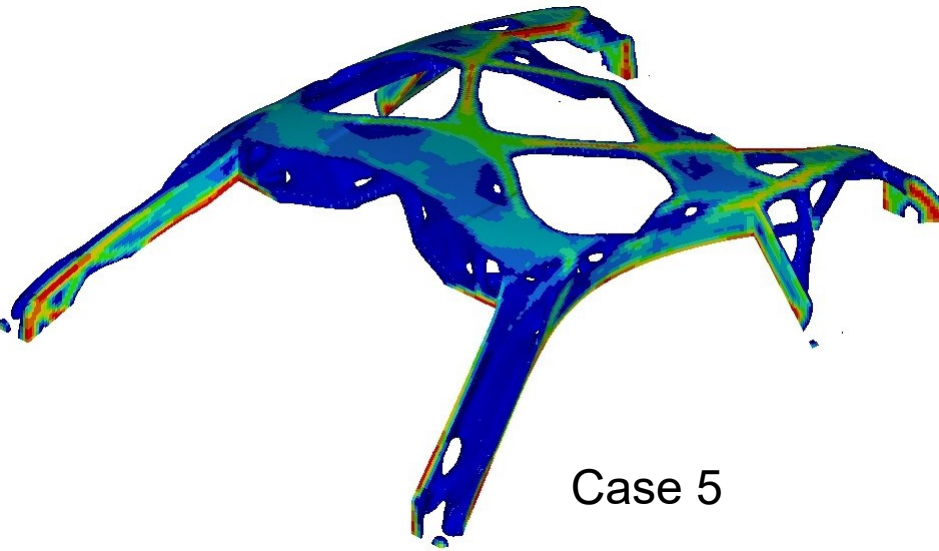


Case 5



Case 6

## Results (2)



## 설계프로세스에 적용

## FutureSteelVehicle



**ULSAB**  
**UltraLight Steel Auto Body**



**ULSAC**  
**UltraLight Steel Auto Closures**



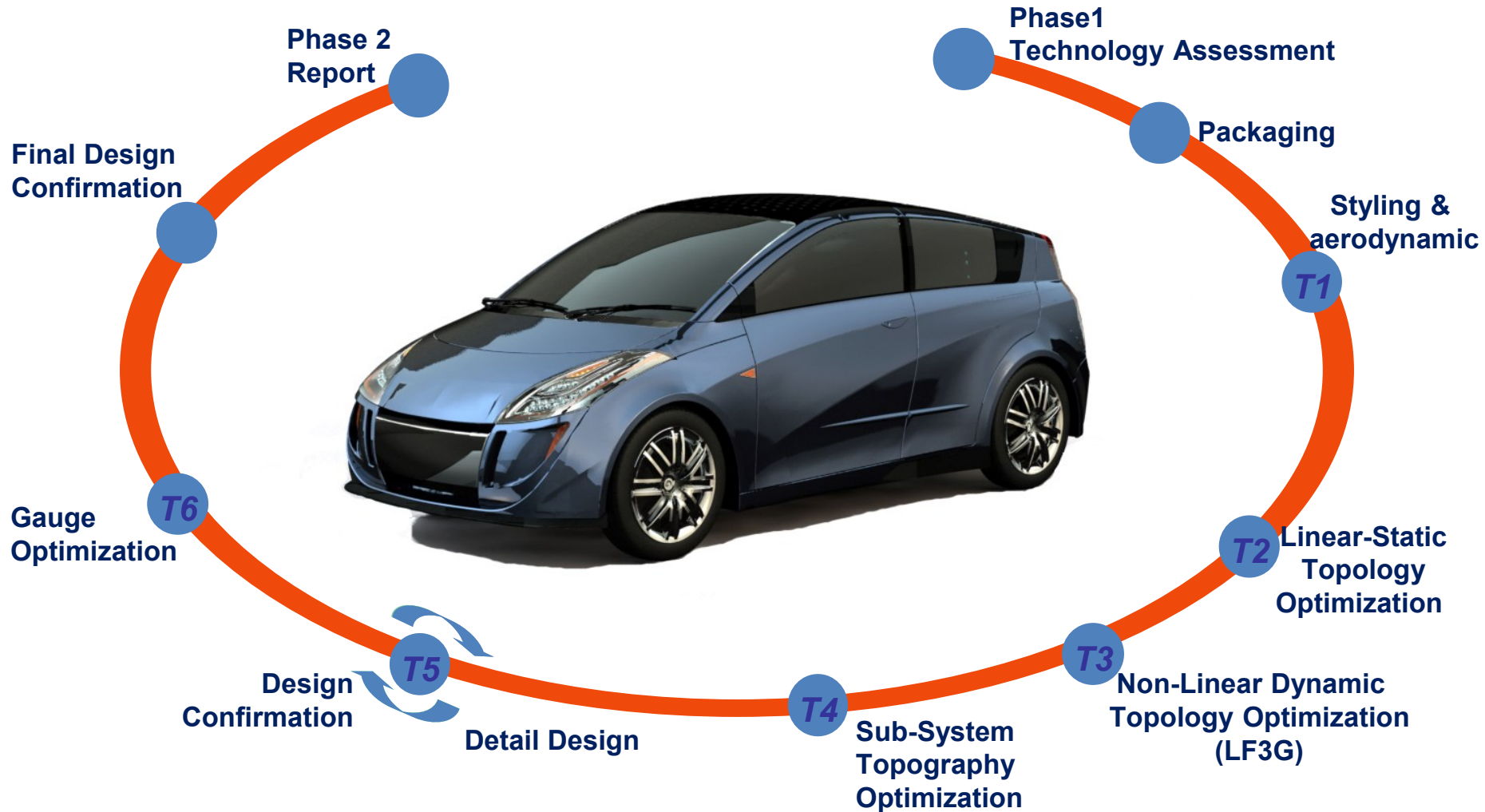
**ULSAS**  
**UltraLight Steel Auto Suspensions**

*Nature's Way to Mobility*

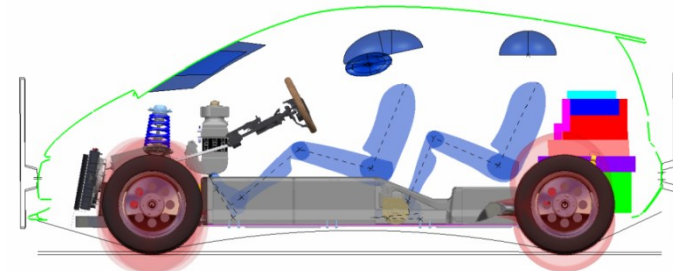
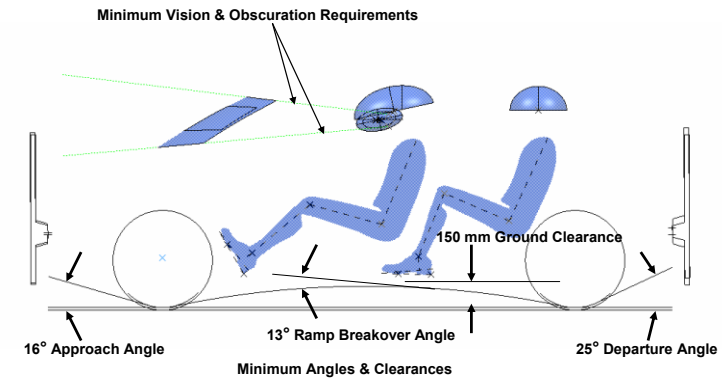
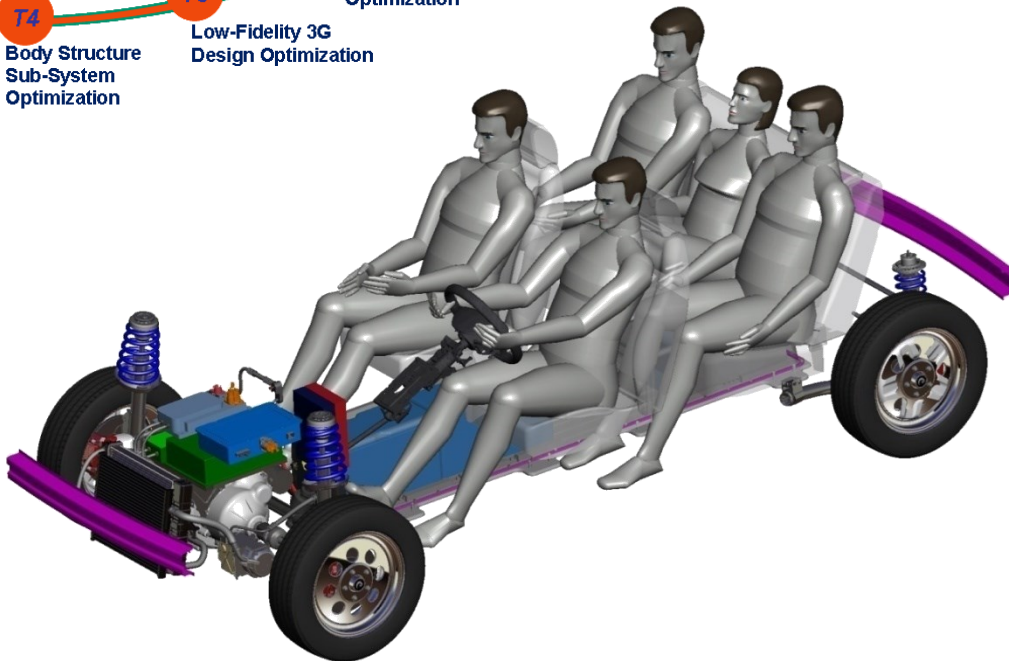
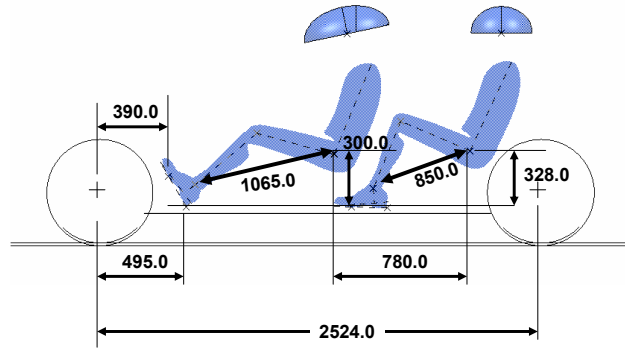
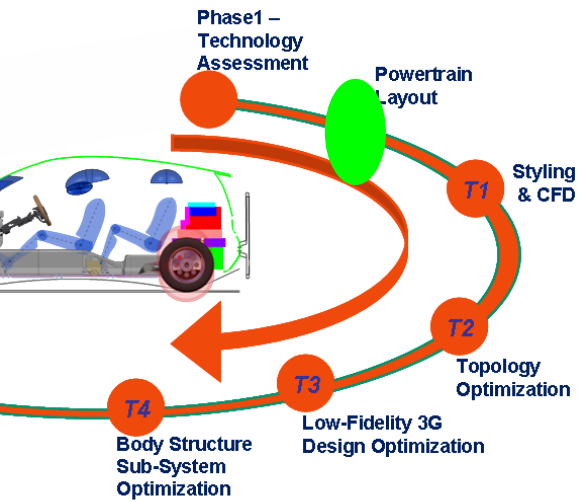
**ULSAB-AVC**  
**Advanced Vehicle Concepts**



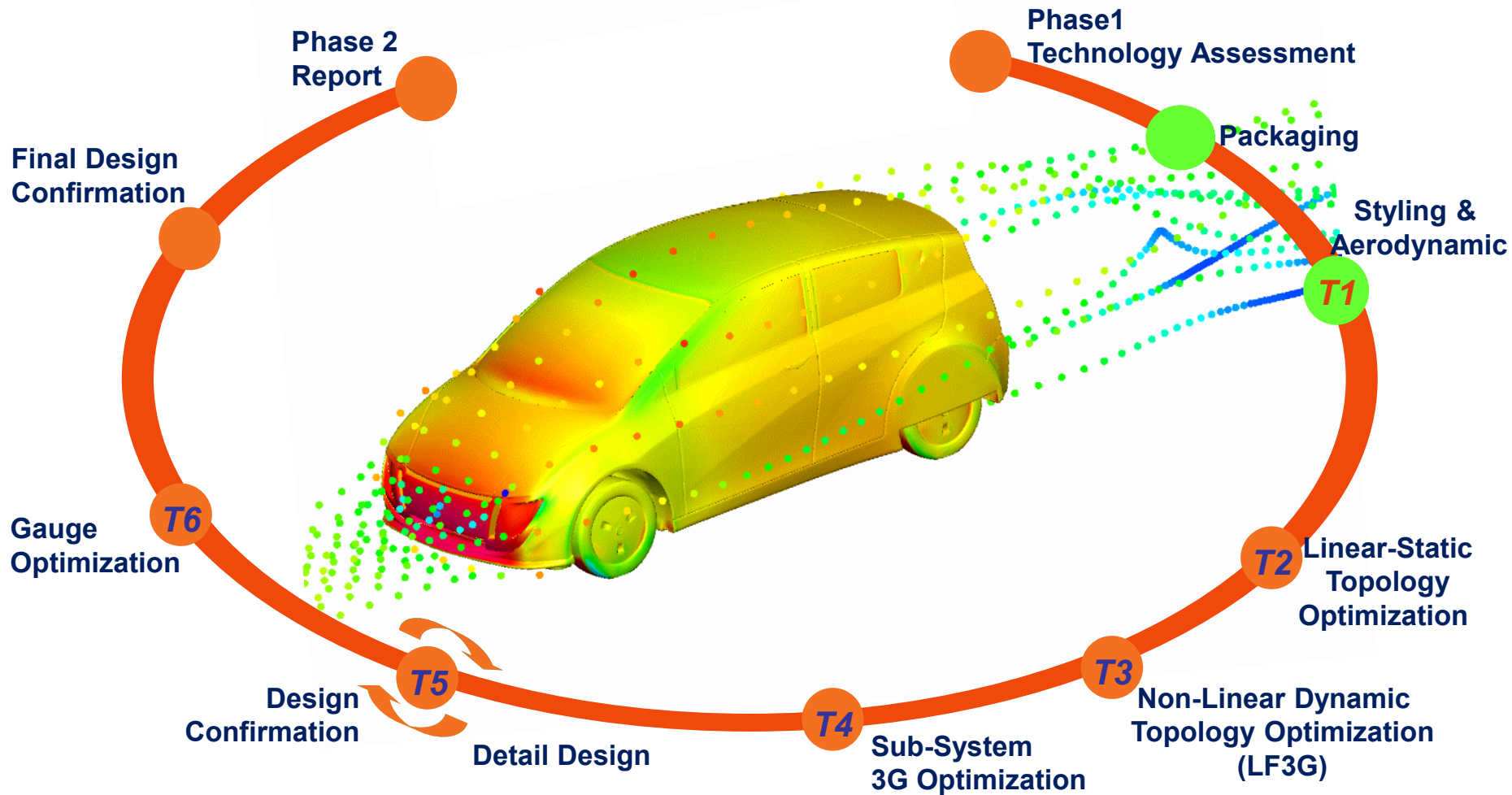
# FSV Design Methodology



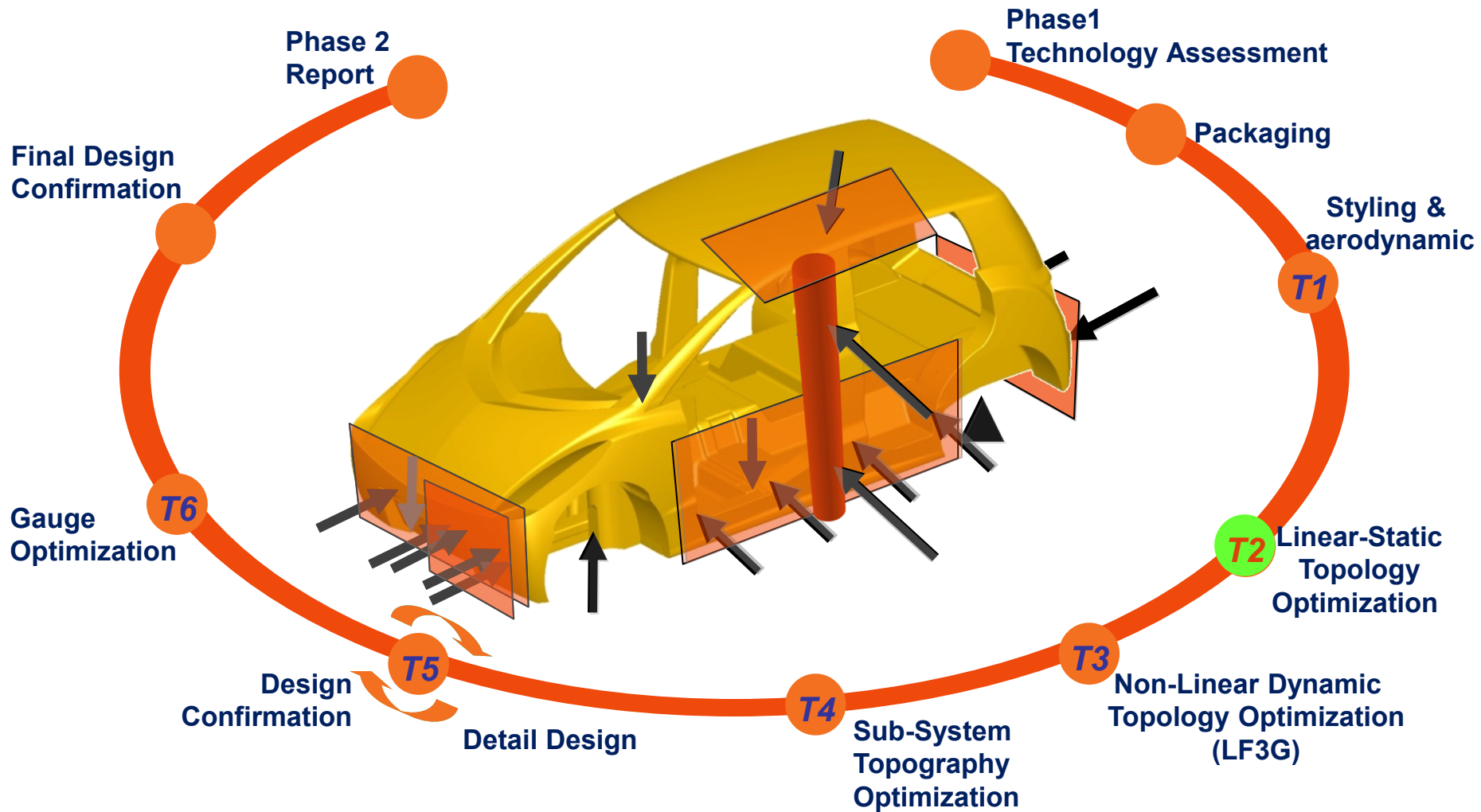
# FSV BEV Packaging



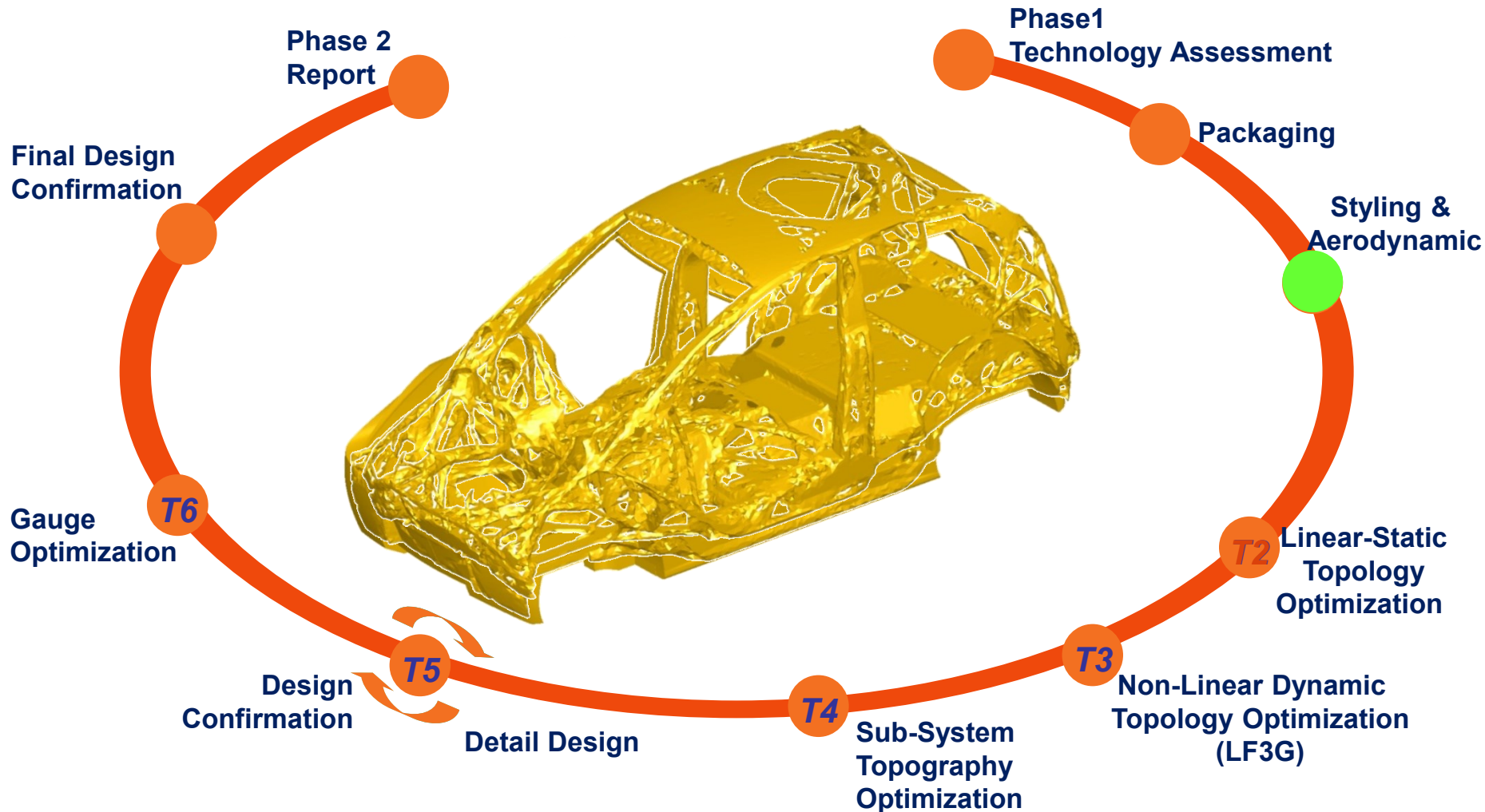
# Aerodynamics & Styling



# Topology Optimization Load Cases

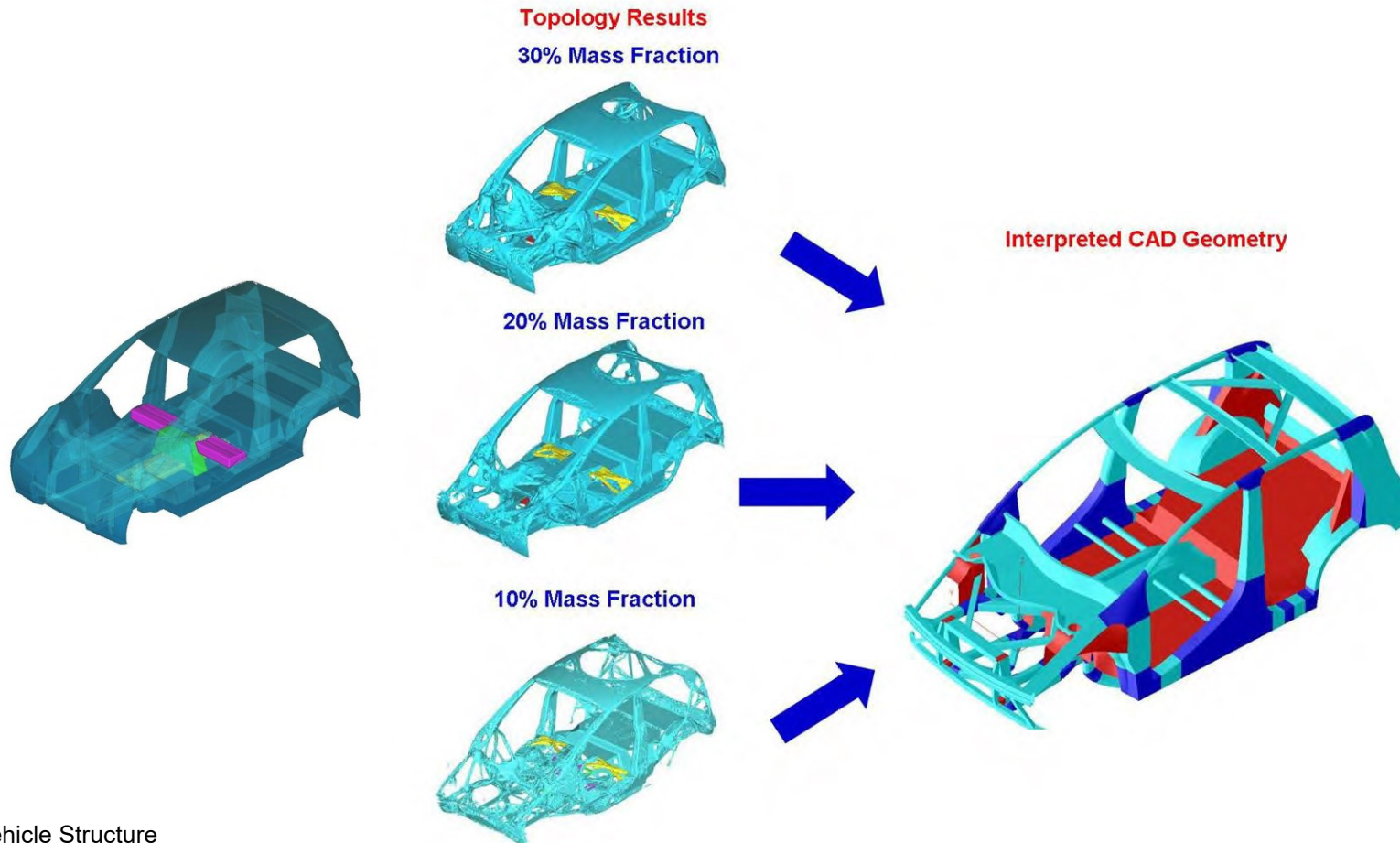


# Linear-Static Topology Optimization

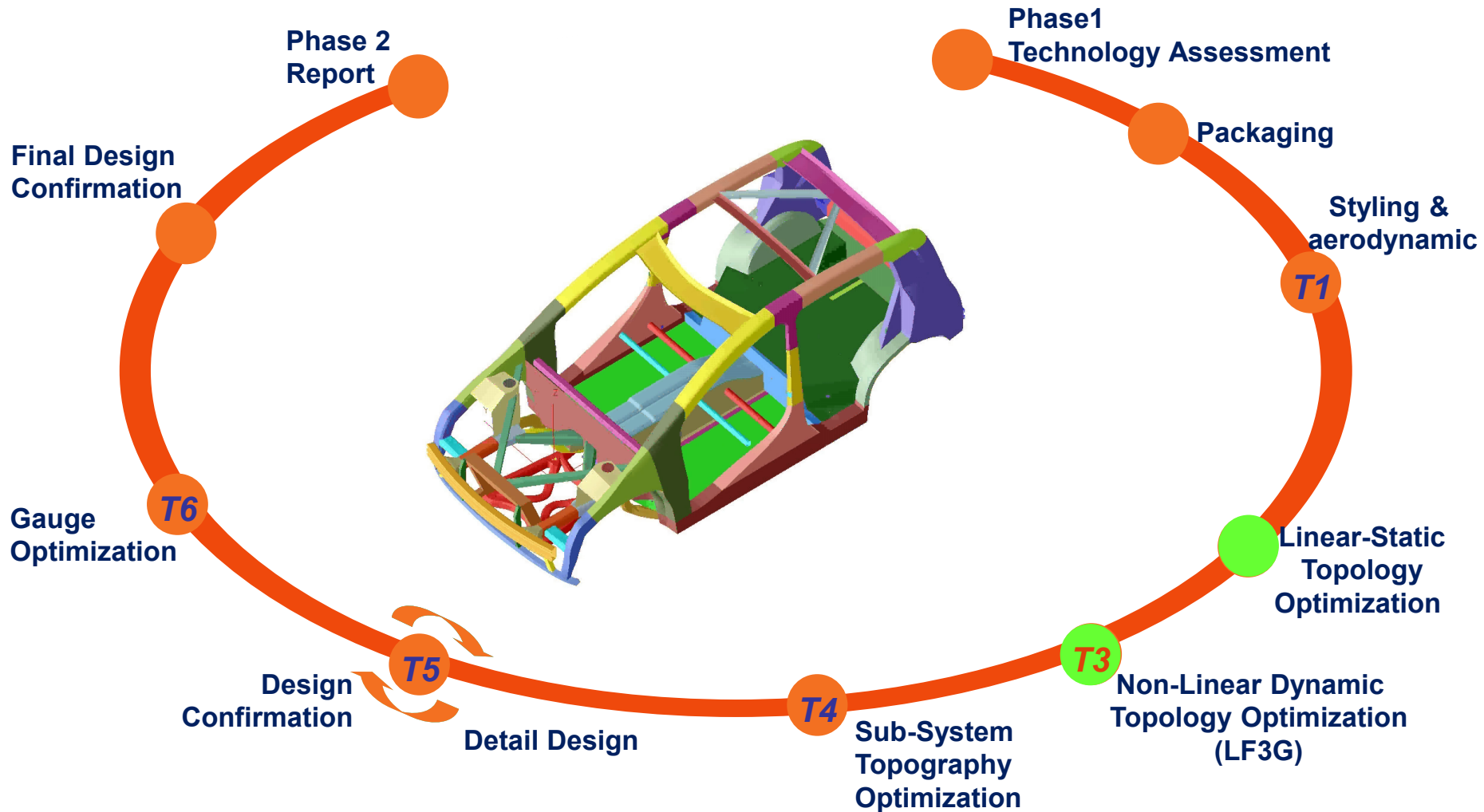


# T2: Topology Optimization

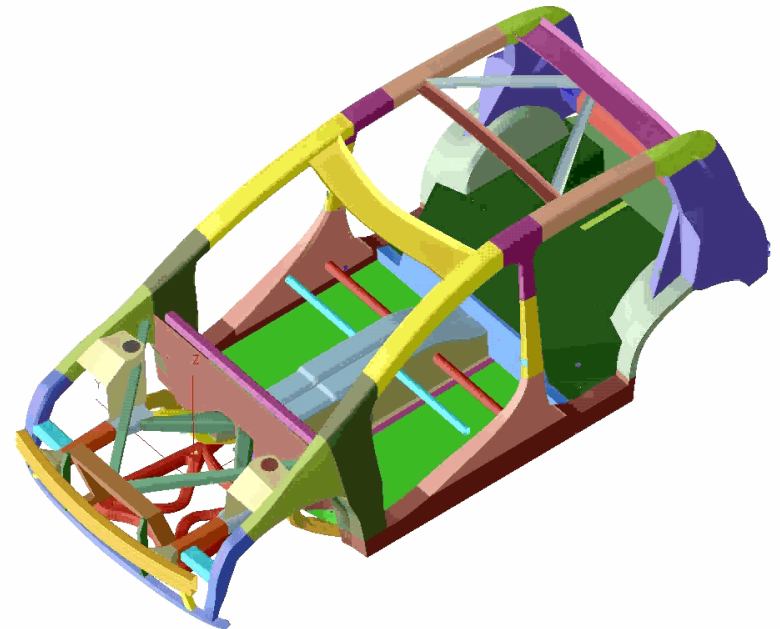
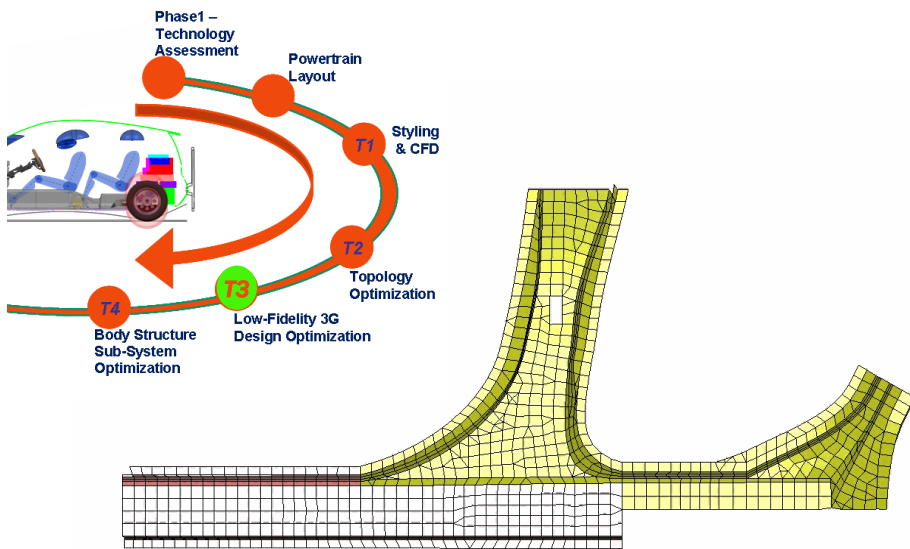
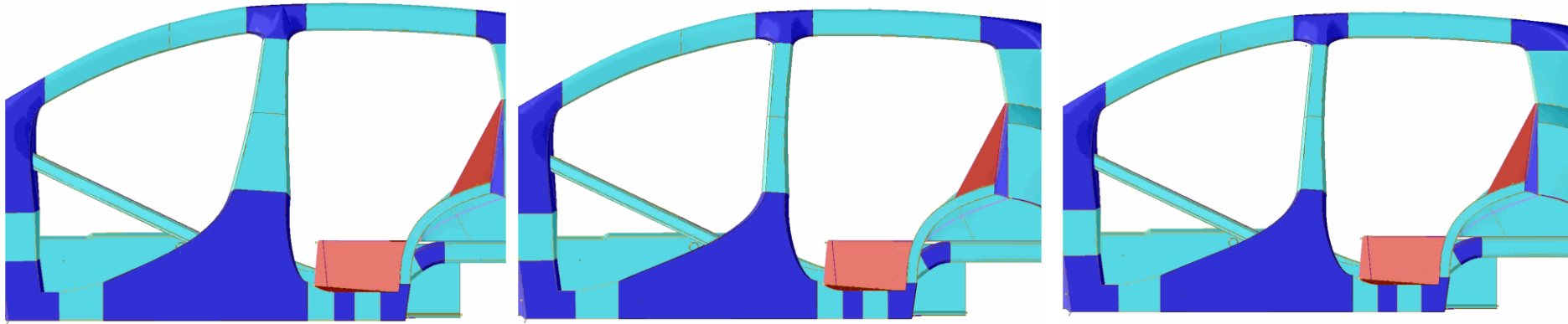
- geometry developed by the topology optimization was manually interpreted into a CAD (Computer Aided Design) model using engineering judgment



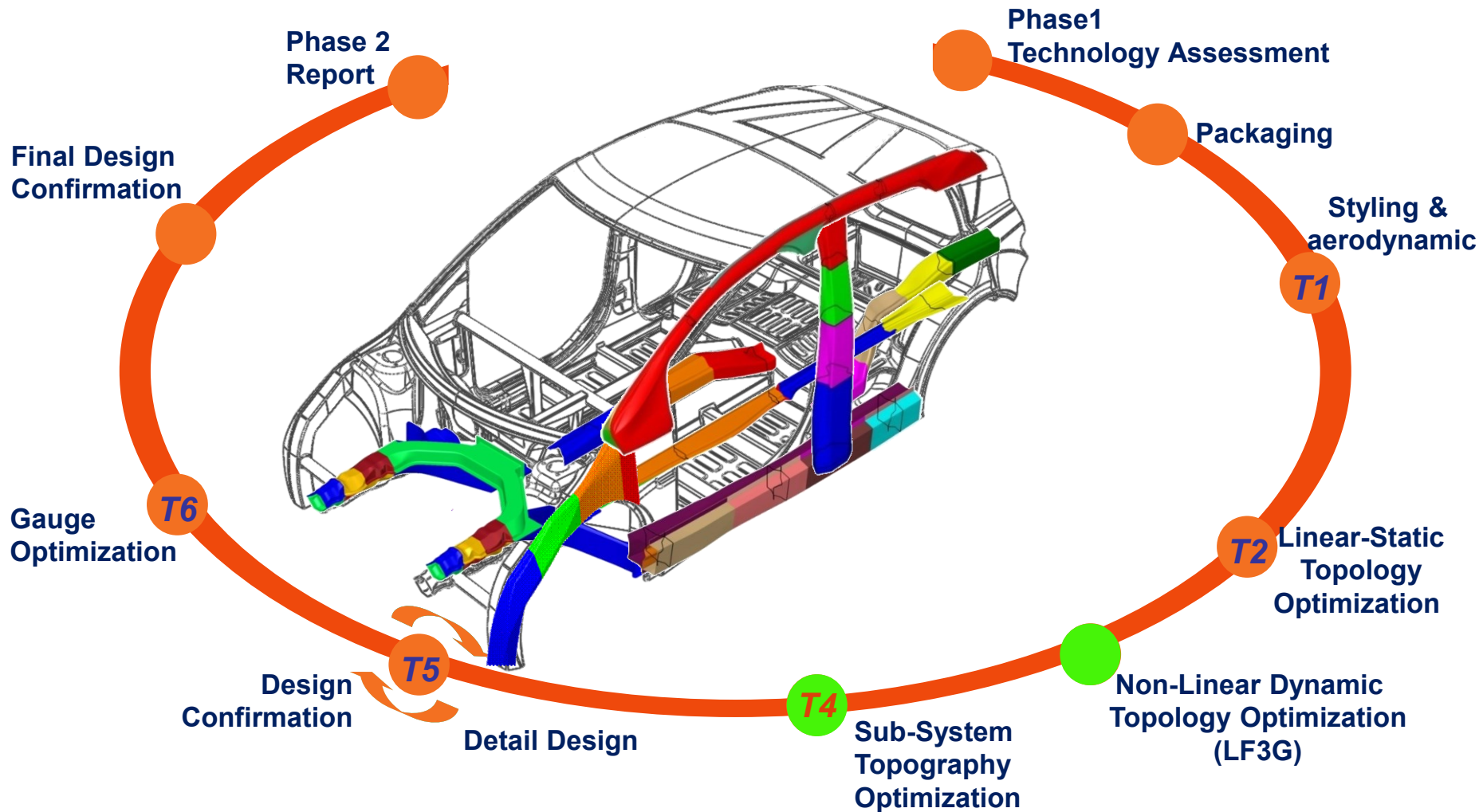
# LF3G Load Path and 3G Optimization



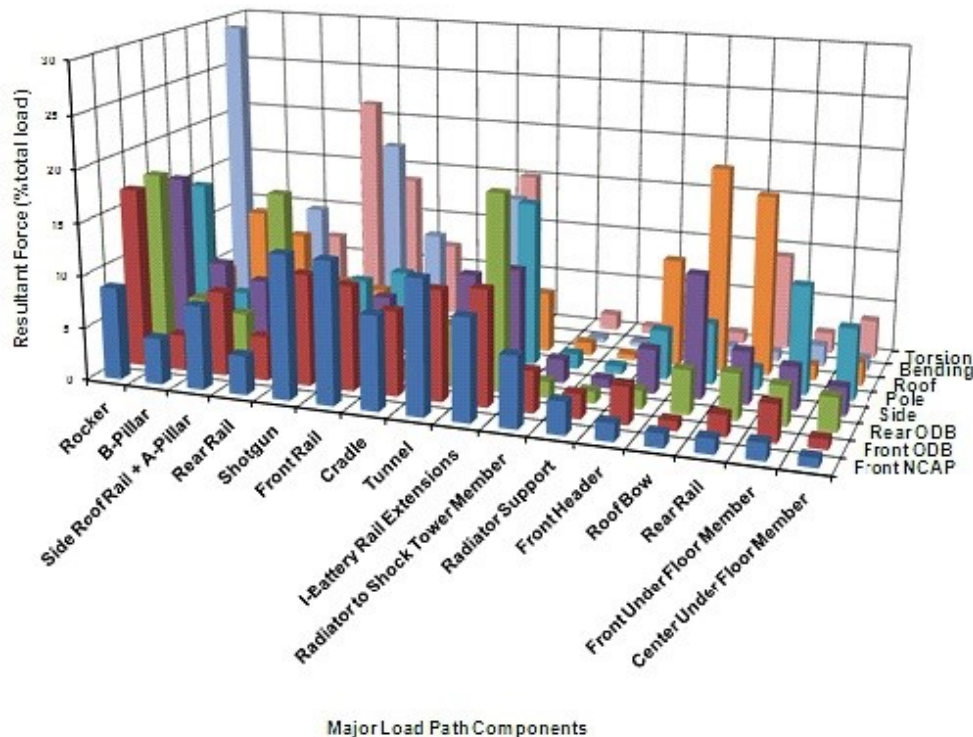
# Low Fidelity 3G (Geometry, Gauge & Grade) Optimization



# Sub-Systems 3G Optimization

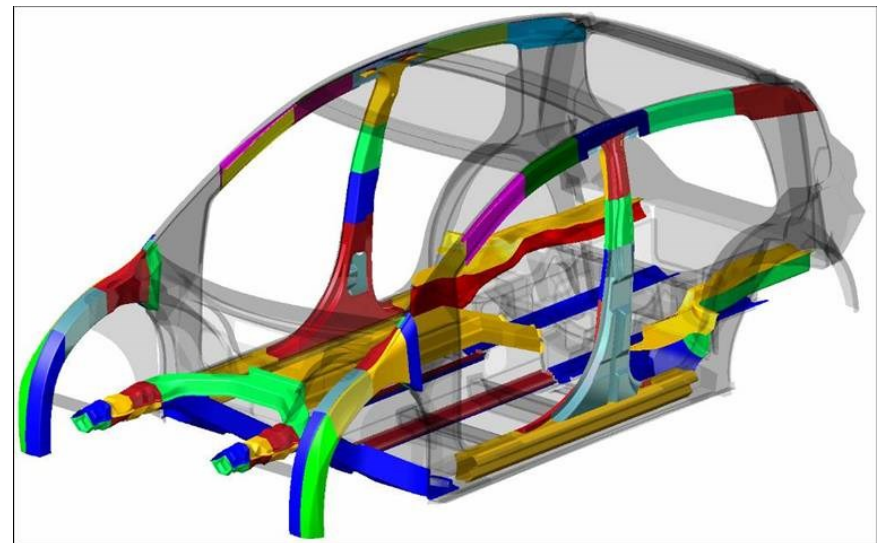


# T4 Load Path Mapping

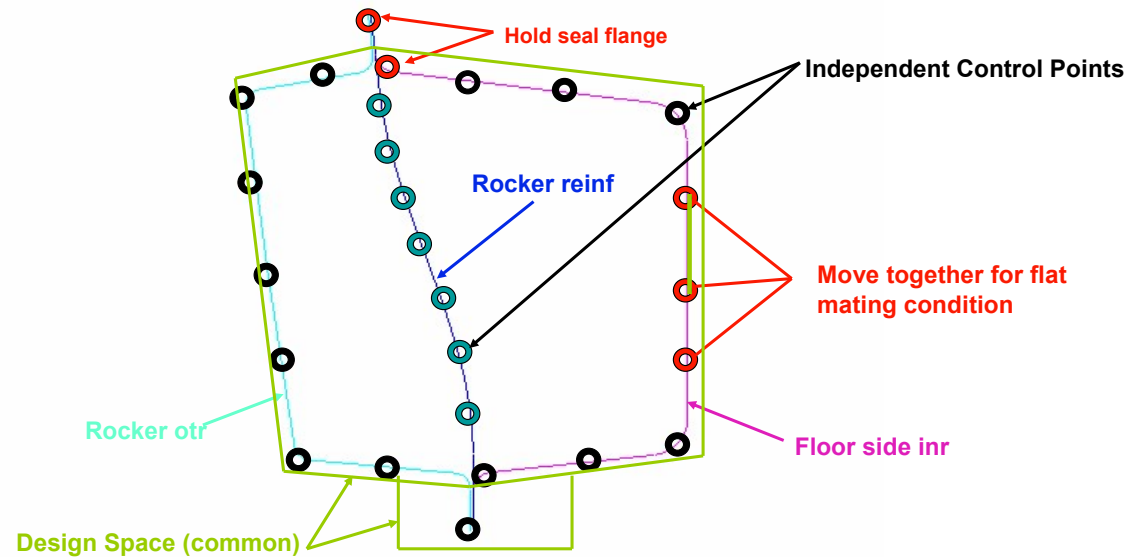
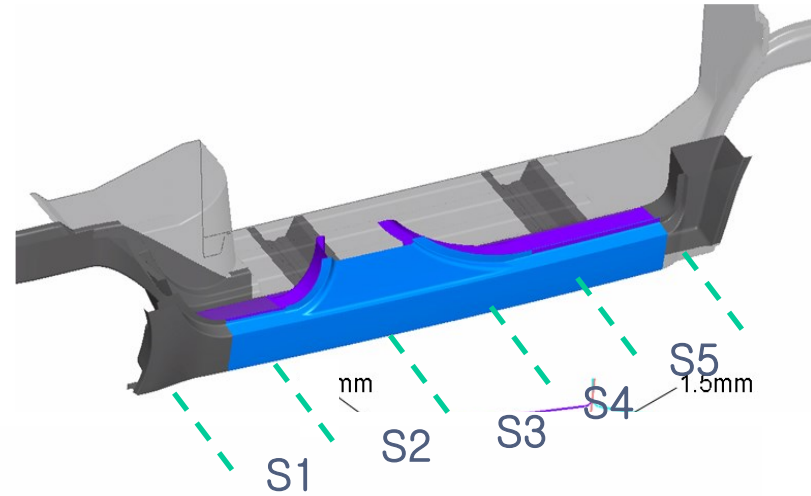
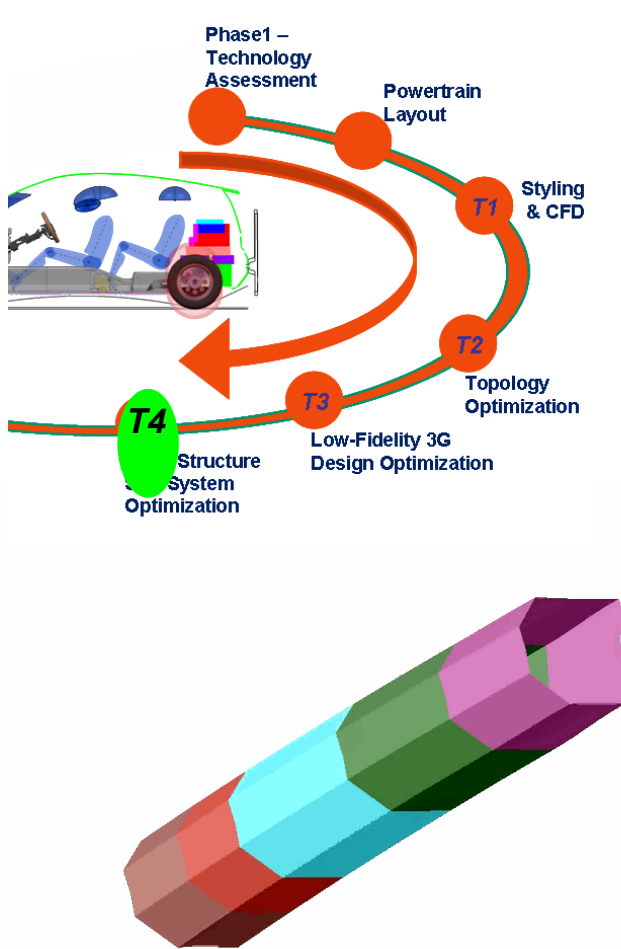


## Selected Sub-Systems

- Front Rail
- Shot Gun
- Rocker
- B-Pillar
- Rear Rail
- Roof Rail
- Tunnel Reinforcement



# Body Structure – Sub-System 3G Optimization



# 정리: 위상최적설계

- 구조물 설계에 있어서 CAE 주도 설계: 위상최적설계가 최적격이자 필수
- 초기설계 없이도 구조물 최적설계 가능
  - 개념설계단계에서 효과적
  - 선행(Up-Front) CAE에 적합
  - 구조물 경량화에 큰 공헌: 혁신설계
- 설계자에게 사용 가능한 도구로 발전하기 위한 주안점

