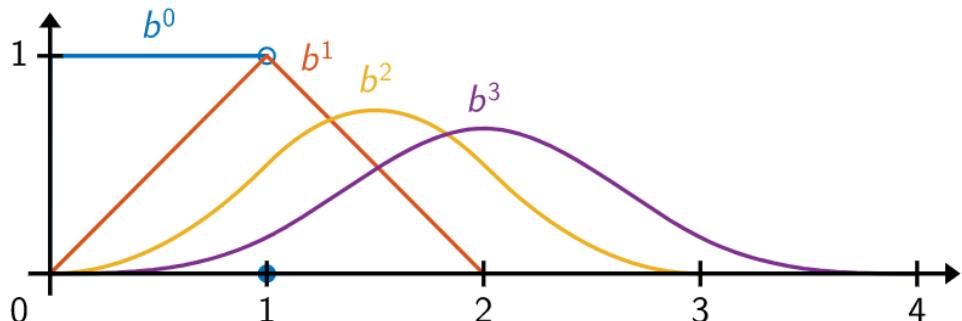
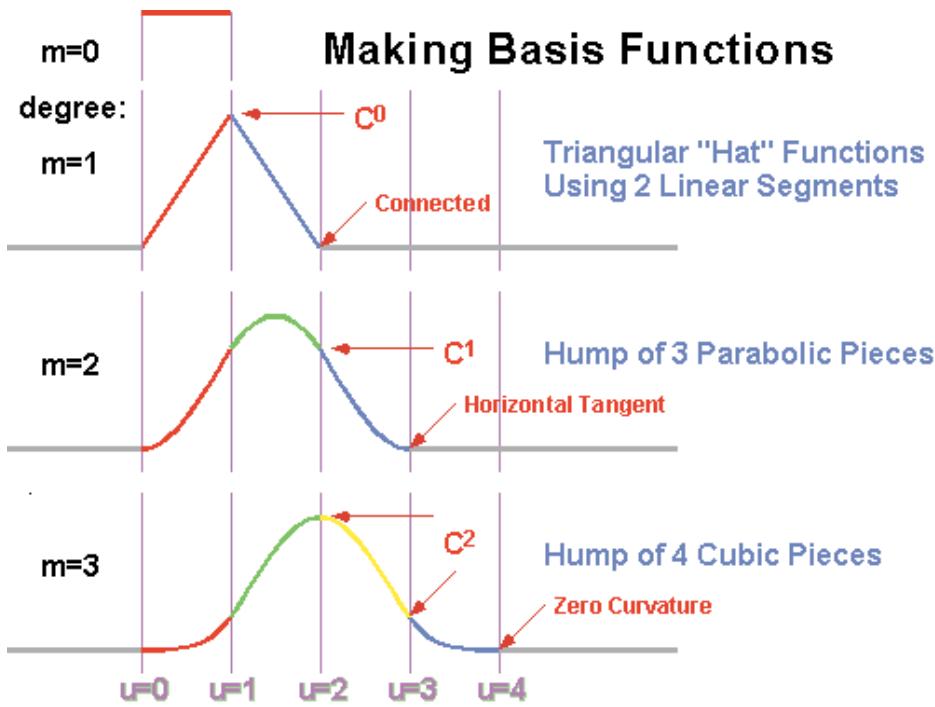


B-spline Basis Functions (1)



$$N_{i,1}(u) = \begin{cases} 1 & \text{if } u_i \leq u < u_{i+1} \\ 0 & \text{else} \end{cases}$$

$$N_{i,2}(u) = \frac{u - u_i}{u_{i+1} - u_i} N_{i,1}(u) + \frac{u_{i+2} - u}{u_{i+2} - u_{i+1}} N_{i+1,1}(u)$$

$$= u N_{i,1}(u) + (2-u) N_{i+1,1}(u) = \begin{cases} u & \text{if } u_i \leq u < u_{i+1} \\ 2-u & \text{if } u_{i+1} \leq u < u_{i+2} \end{cases}$$

$$N_{i,3}(u) = \frac{u - u_i}{u_{i+2} - u_i} N_{i,2}(u) + \frac{u_{i+3} - u}{u_{i+3} - u_{i+1}} N_{i+1,2}(u)$$

$$= \frac{u}{2} [u N_{i,1}(u) + (2-u) N_{i+1,1}(u)] +$$

$$\frac{3-u}{2} [(u-1) N_{i+1,1}(u) + (3-u) N_{i+2,1}(u)]$$

$$= \frac{u^2}{2} N_{i,1}(u) + \frac{-2u^2 + 6u - 3}{2} N_{i+1,1}(u) + \frac{(u-3)^2}{2} N_{i+2,1}(u)$$

$$b^0 : (1 \quad 1)$$

$$b^1 : (0 \quad 1 \quad 0)$$

$$b^2 : \left(0 \quad \frac{1}{2} \quad \left(\frac{3}{4} \right) \quad \frac{1}{2} \quad 0 \right)$$

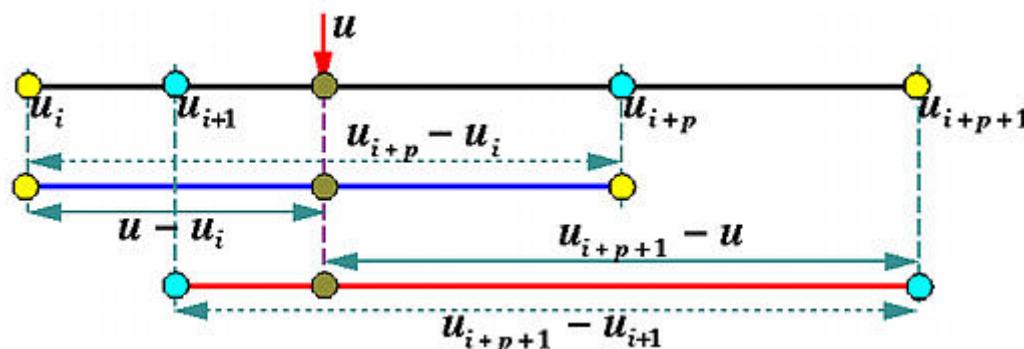
$$b^3 : \left(0 \quad \frac{1}{6} \quad \frac{4}{6} \quad \frac{1}{6} \quad 0 \right)$$

$$\begin{cases}
N_{i,2}(u) = uN_{i,1}(u) + (2-u)N_{i+1,1}(u) \\
N_{i+1,2}(u) = (u-1)N_{i+1,1}(u) + (3-u)N_{i+2,1}(u) \\
N_{i+2,2}(u) = (u-2)N_{i+2,1}(u) + (4-u)N_{i+3,1}(u) \\
\\
N_{i,3}(u) = \frac{u}{2}N_{i,2}(u) + \frac{3-u}{2}N_{i+1,2}(u) \\
= \frac{u}{2}[uN_{i,1}(u) + (2-u)N_{i+1,1}(u)] + \frac{3-u}{2}[(u-1)N_{i+1,1}(u) + (3-u)N_{i+2,1}(u)] \\
= \frac{u^2}{2}N_{i,1}(u) + \frac{-2u^2 + 6u - 3}{2}N_{i+1,1}(u) + \frac{(u-3)^2}{2}N_{i+2,1}(u) \\
\\
N_{i+1,3}(u) = \frac{u-1}{2}N_{i+1,2}(u) + \frac{4-u}{2}N_{i+2,2}(u) \\
= \frac{u-1}{2}[(u-1)N_{i+1,1}(u) + (3-u)N_{i+2,1}(u)] + \frac{4-u}{2}[(u-2)N_{i+2,1}(u) + (4-u)N_{i+3,1}(u)] \\
= \frac{(u-1)^2}{2}N_{i+1,1}(u) + \frac{-2u^2 + 10u - 8}{2}N_{i+2,1}(u) + \frac{(u-4)^2}{2}N_{i+3,1}(u) \\
\\
N_{i,4}(u) = \frac{u}{3}N_{i,3}(u) + \frac{4-u}{3}N_{i+1,3}(u) \\
= \frac{u}{3}\left[\frac{u^2}{2}N_{i,1}(u) + \frac{-2u^2 + 6u - 3}{2}N_{i+1,1}(u) + \frac{(u-3)^2}{2}N_{i+2,1}(u)\right] + \frac{4-u}{3}\left[\frac{(u-1)^2}{2}N_{i+1,1}(u) + \frac{-2u^2 + 10u - 8}{2}N_{i+2,1}(u) + \frac{(u-4)^2}{2}N_{i+3,1}(u)\right] \\
= \frac{u^3}{6}N_{i,1}(u) + \frac{u(-2u^2 + 6u - 3) + (4-u)(u-1)^2}{6}N_{i+1,1}(u) + \frac{u(u-3)^2 + (4-u)(-2u^2 + 10u - 8)}{6}N_{i+2,1}(u) + \frac{-(u-4)^3}{2}N_{i+3,1}(u)
\end{cases}$$

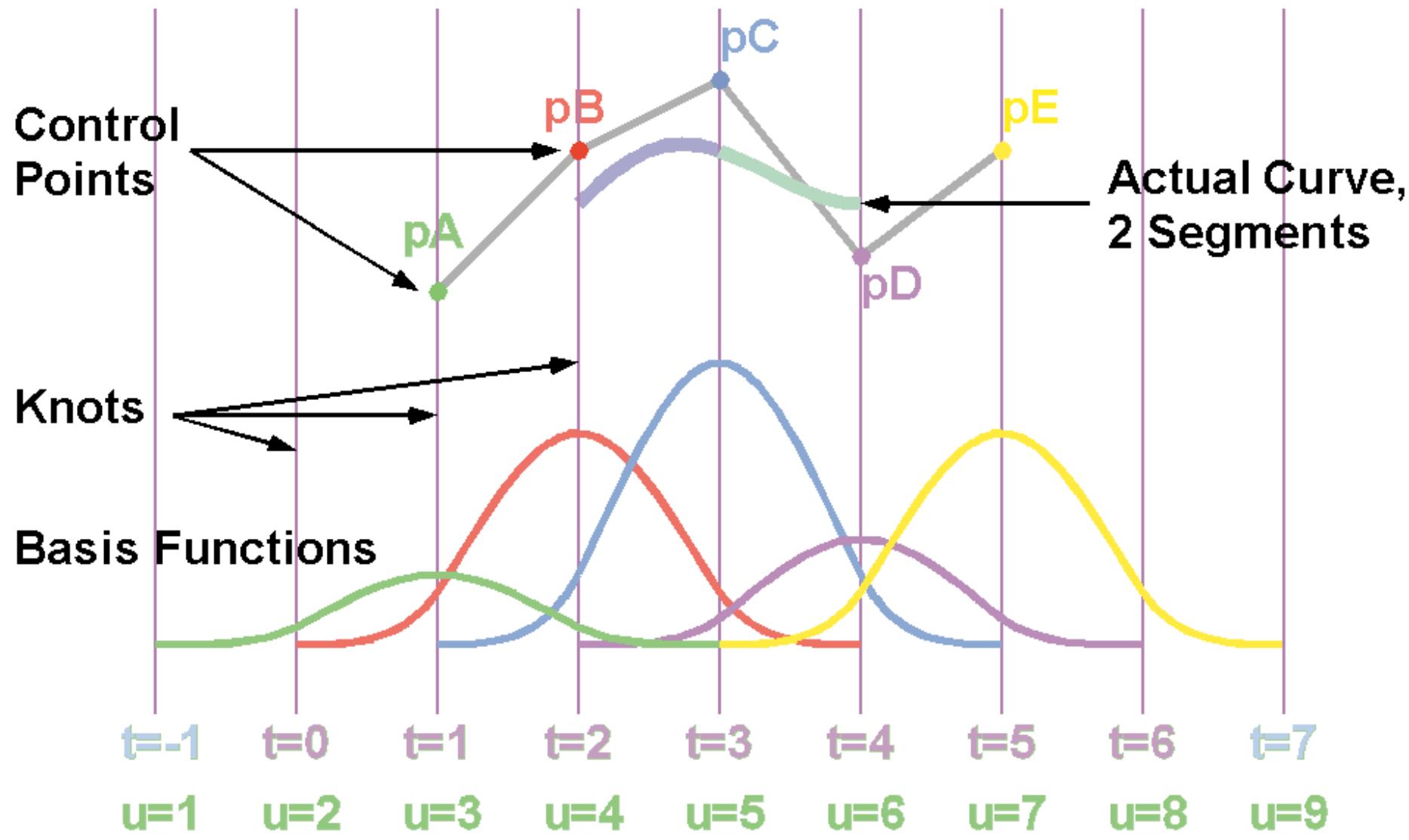
B-spline Basis Functions (2)

- Basis function $N_{i,k}(u)$ is non-zero on
 - $[u_i, u_{i+k+1}]$
 - $(k+1)$ knot span: $[u_i, u_{i+1}), [u_{i+1}, u_{i+2}), \dots, [u_{i+k}, u_{i+k+1})$
- On any knot span $[u_i, u_{i+1})$, at most k basis functions are non-zero
- Meaning of coefficients?

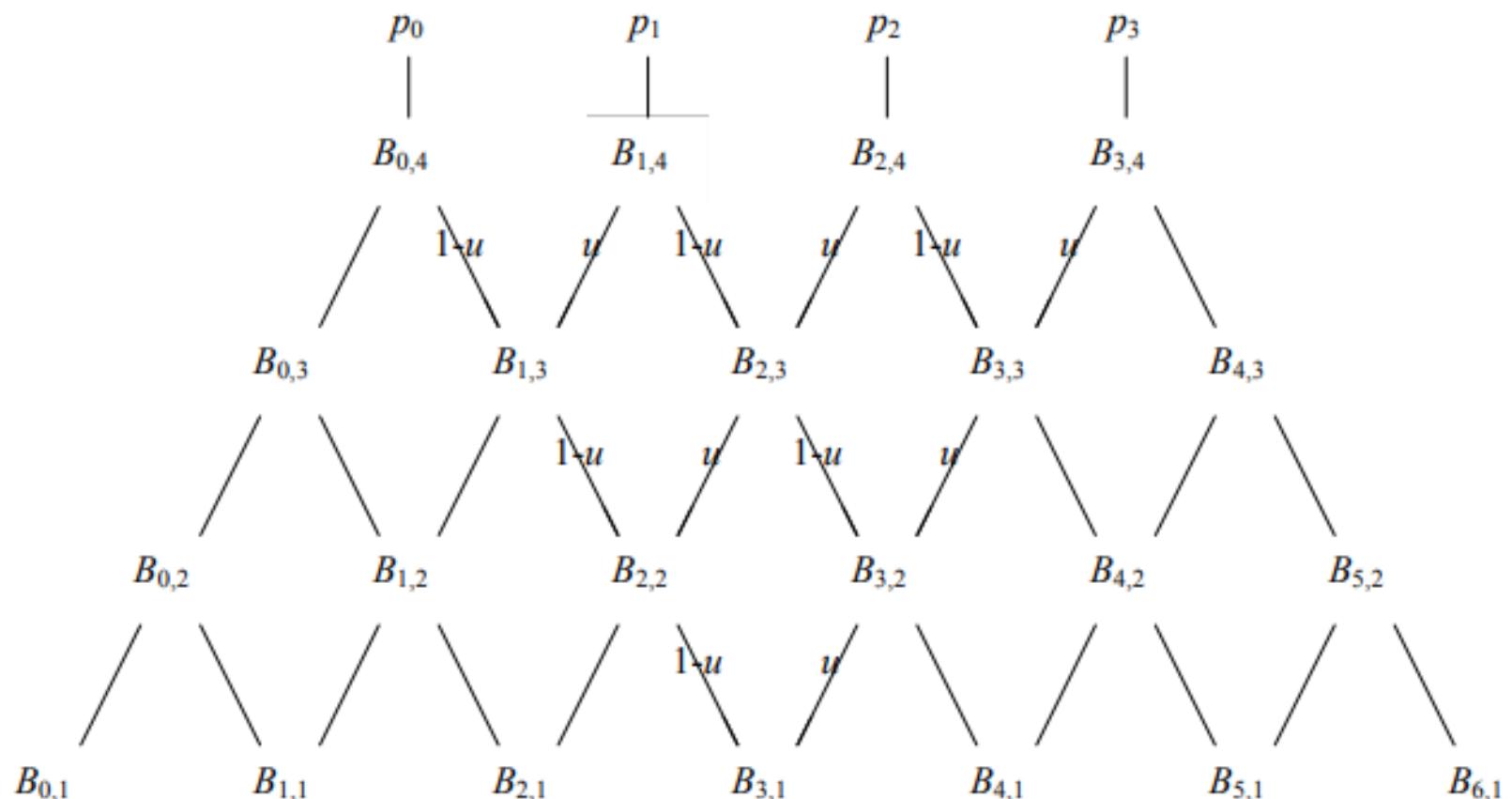
$$\begin{cases} d = 1 \sim: N_{i,d}(u) = \frac{(u - u_i)}{u_{i+d} - u_i} N_{i,d-1}(u) + \frac{(u_{i+d+1} - u)}{u_{i+d+1} - u_{i+1}} N_{i+1,d-1}(u) \\ k = 2 \sim: N_{i,k}(u) = \frac{(u - u_i)}{u_{i+k-1} - u_i} N_{i,k-1}(u) + \frac{(u_{i+k} - u)}{u_{i+k} - u_{i+1}} N_{i+1,k-1}(u) \end{cases}$$



Cubic (4-th Order) B-Spline Basics

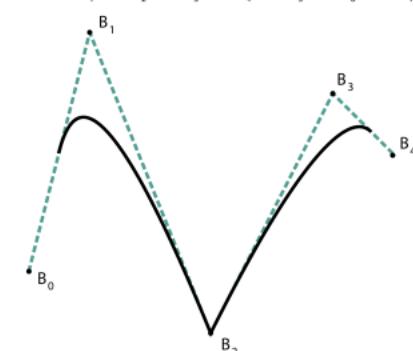
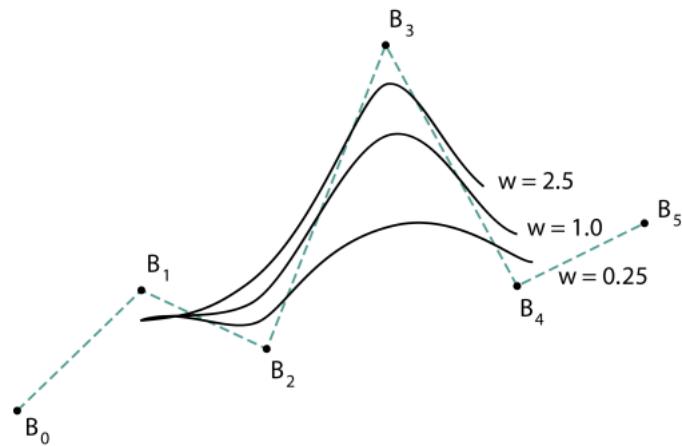
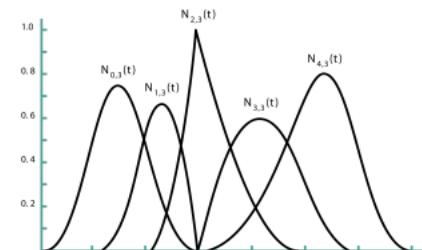
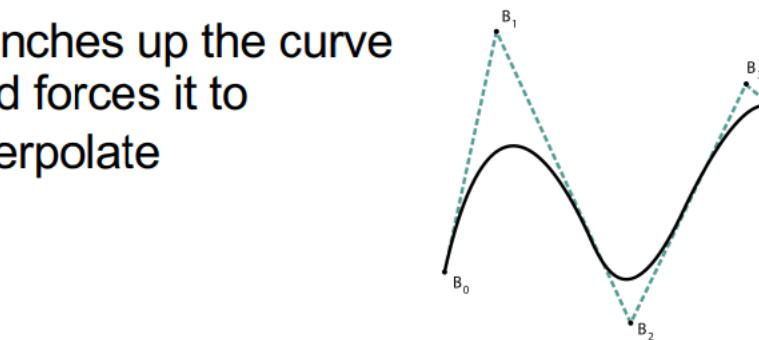
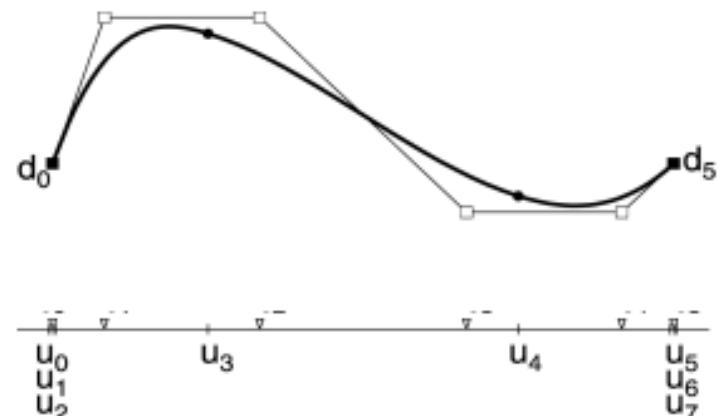
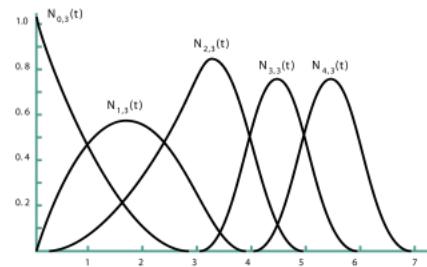


SLIDE: order 4; controlpointlist (pA pB pC pD pE); {uses knots 9}



$$\left. \begin{array}{l} N_{0,4}(u) = (1-u)^3 \\ N_{1,4}(u) = 3u(1-u)^2 \\ N_{2,4}(u) = 3u^2(1-u) \\ N_{3,4}(u) = u^3 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} P(0) = P_0 \\ P(1) = P_1 \\ P'(0) = 3(P_1 - P_0) \\ P'(1) = 3(P_3 - P_2) \end{array} \right.$$

- Knot Vector
 $\{0.0, 0.0, 0.0, 3.0, 4.0, 5.0, 6.0, 7.0\}$
- Several consecutive knots get the same value
- Bunches up the curve and forces it to interpolate



- Knot Vector
 $\{0.0, 1.0, 2.0, \mathbf{3.0}, \mathbf{3.0}, 5.0, 6.0, 7.0\}$
- Several consecutive knots get the same value
- Bunches up the curve and forces it to interpolate
- Can be done midcurve