

Submit the compressed file as (ID)\_(name).zip to [ftp://cdl.hanyang.ac.kr → CAE/Final\_Lab] folder. It should contain the final results of each problem (equations and graphs) using PowerPoint (ID.ppt) and COMSOL files (problem#-#.mph).

1. [PDE solving] Solve the mixed boundary value problem for a Poisson equation as follows. Plot the surface of solution 'u' and compute the solution values at the four points. (15 pts)

**Mixed Boundary Value Problem for a Poisson Equation**

Solve the mixed boundary value problem for the Poisson equation

$$\nabla^2 u = u_{xx} + u_{yy} = f(x, y) = 12xy$$

shown in Fig. 457a.

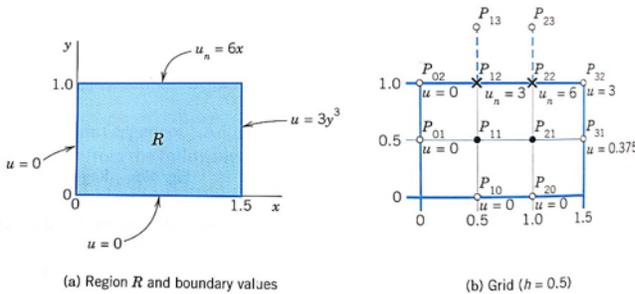


Fig. 457. Mixed boundary value problem in Example 1

$$u_{xx} + u_{yy} = f(x, y) = 12xy$$

Dirichlet B.C

$$u(0, y) = u(y, 0) = 0, u(1.5, y) = 3y^3$$

Neumann B.C

$$u_y(x, 1) = 6x$$

Exact sol.

$$u_{11} = 0.125, u_{12} = 1$$

$$u_{21} = 0.250, u_{22} = 2$$

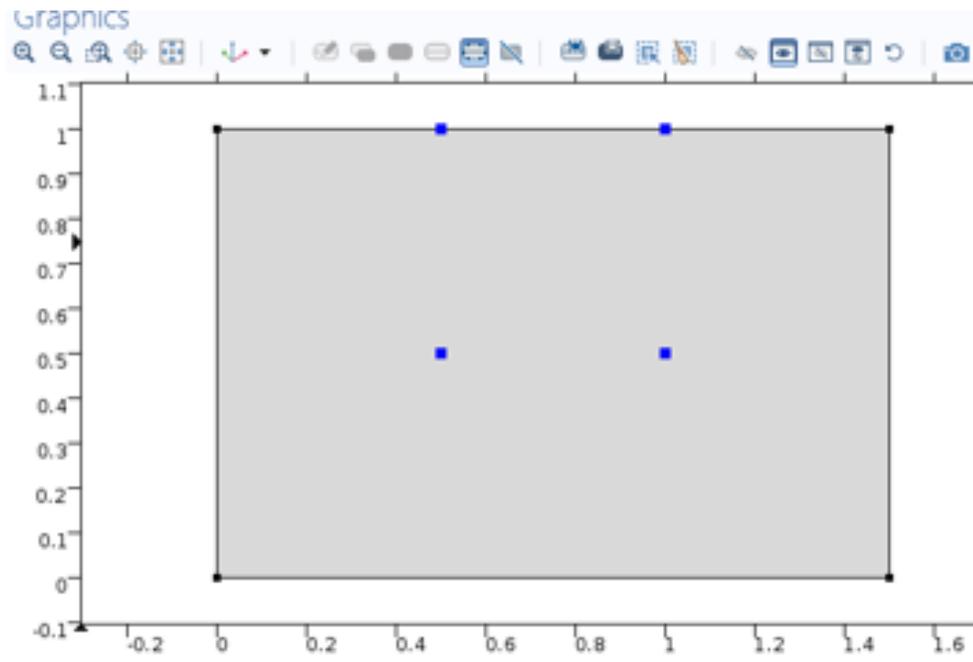
Flux/Source 1

Equation

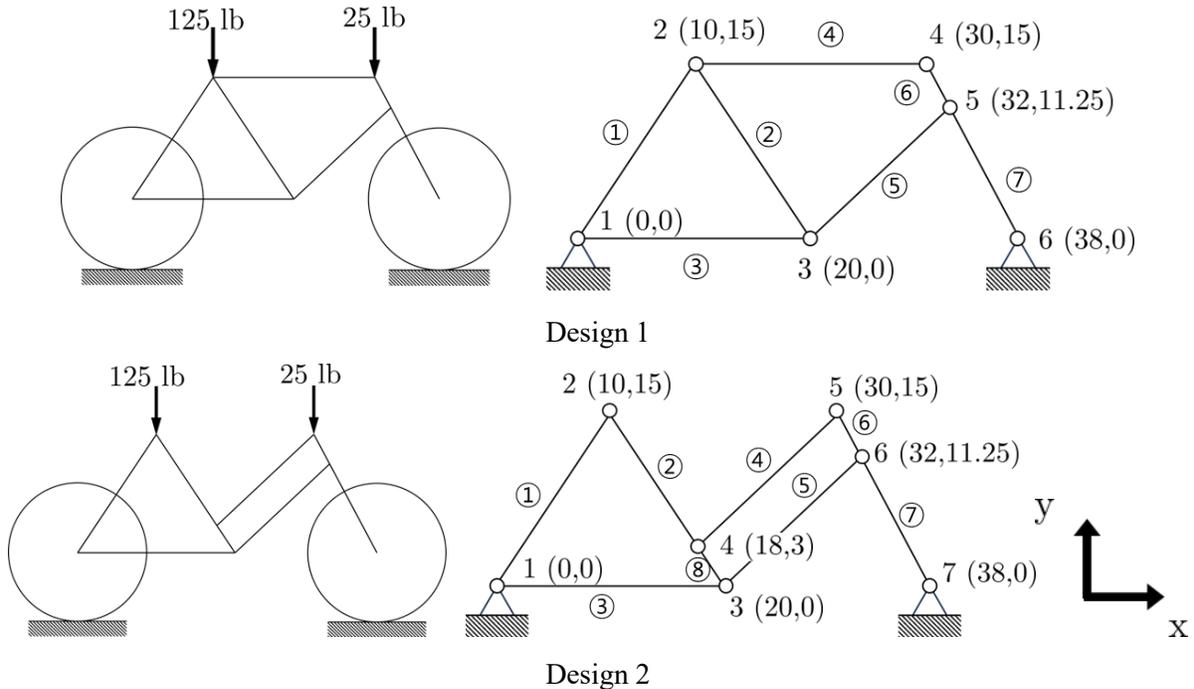
Show equation assuming:

Study 1, Stationary

$$-\mathbf{n} \cdot (-c \nabla u - \alpha u + \gamma) = g - qu$$

$$\nabla = \left[ \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right]$$


2. [2D Beam] Consider the following two bicycle frame models. Answer the following questions based on your analysis of these models:



[Material property]

Design 1

$$E = 30 \times 10^6 \text{ psi}$$

Design 2

$$E = 10 \times 10^6 \text{ psi}$$

[Crosssection data]

$$A_1 = 0.1 \text{ [in}^2\text{]}$$

$$A_2 = A_3 = A_4 = A_5 = 0.15 \text{ [in}^2\text{]}$$

$$A_6 = A_7 = A_8 = 0.3 \text{ [in}^2\text{]}$$

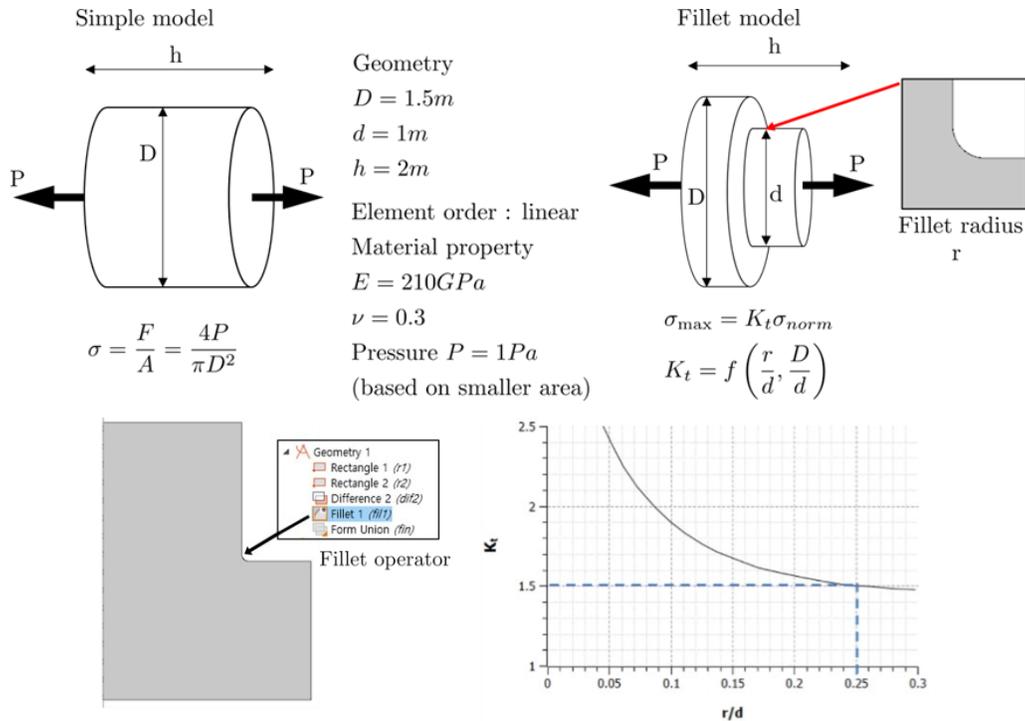
$$I_1 = 0.01 \text{ [in}^4\text{]}$$

$$I_2 = I_3 = I_4 = I_5 = 0.02 \text{ [in}^4\text{]}$$

$$I_6 = I_7 = I_8 = 0.1 \text{ [in}^4\text{]}$$

- 1) Plot the Total displacement (line type) for each model and evaluate the y-direction displacements at the node at 2. (15 pts)
- 2) Plot the von-mises stress (line type) for two bicycle frame models and determine which design is preferable based on structural stiffness, providing an explanation for your choice (10 pts)

3. [Stress concentration] The following cylinder bar with fillet (30 pts total)



(1) Construct the full 3D model and plot the surface of the first principal stress and the Calculate norm stress (10 pts)

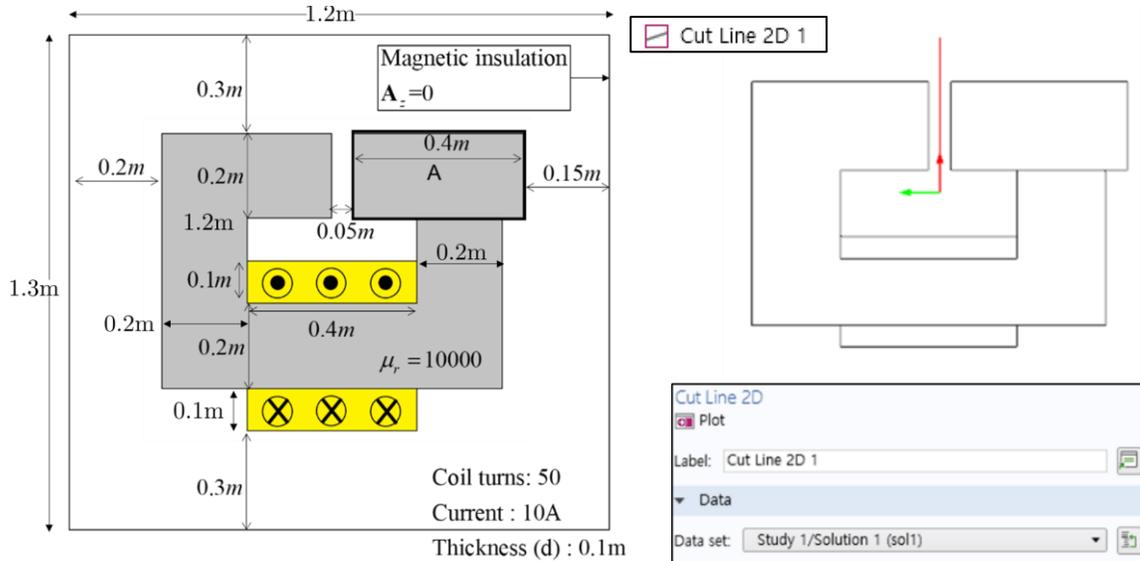
(hint: use Domain Probe 1 (dom1) average value of first principal stress “solid.sp1”)

(2) Construct the 2d axisymmetric fillet ( $r=0.25$ ) model, Compute maximum value of the first principal stress. Check the stress by mesh dependency applying free triangle. Plot the graph as d.o.fs vs stress changing mesh size. (mesh option: extremely coarse ~ extremely fine) (10 pts)

(3) Calculate the value of K when changing the fillet radius (0.05 ~ 0.25 m) as shown in the table. Compare the values of K between the analytic and COMSOL results and fill the table. (mesh option: normal) (10 pts)

4. [Magnetic Actuator] For the actuator and boundary conditions shown, solve the Poisson equation by

**Magnetic Fields (mf)**. (Use **Fine** discretization level.) (30 pts total)



1) Draw a Magnetic flux density norm distribution along the **Cut Line 2D 1** by **1D Plot Group** **Line Graph** (15 pts total)

2) Evaluate the magnetic force using Maxwell stress tensor method by **Boundary Probe 1 (bnd1)** **Boundary Probe 2 (bnd2)** **Global Variable Probe 2 (var2)** (15 pts total) Analytic solution : 1.4289 N

**Hint:**

Magnetic force (Maxwell Stress Tensor Method)

$$f_x = \frac{d}{\mu_0} \left[ \int \frac{B_x B_x}{2} dy + \int B_x B_y dx \right]$$

$d$  : thickness

$\mu_0$  : vacuum permeability

$B$  : flux density

$f$  : force

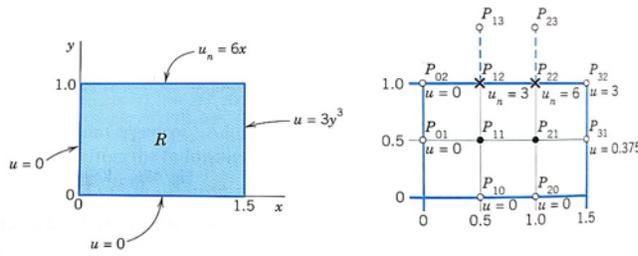
Proble1\_Sol)

**Mixed Boundary Value Problem for a Poisson Equation**

Solve the mixed boundary value problem for the Poisson equation

$$\nabla^2 u = u_{xx} + u_{yy} = f(x, y) = 12xy$$

shown in Fig. 457a.



(a) Region  $R$  and boundary values (b) Grid ( $h=0.5$ )

**Fig. 457. Mixed boundary value problem in Example 1**

$$u_{xx} + u_{yy} = f(x, y) = 12xy$$

Dirichlet B.C

$$u(0, y) = u(y, 0) = 0, u(1.5, y) = 3y^3$$

Neumann B.C

$$u_y(x, 1) = 6x$$

Exact sol.

$$u_{11} = 0.125, u_{12} = 1$$

$$u_{21} = 0.250, u_{22} = 2$$

Flux/Source 1

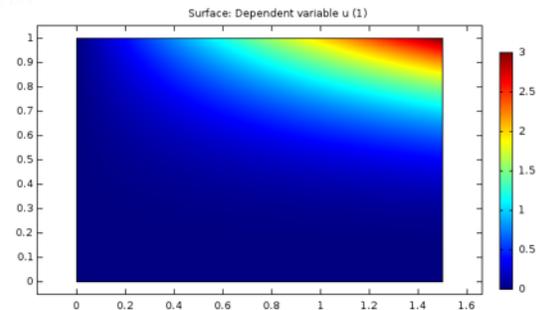
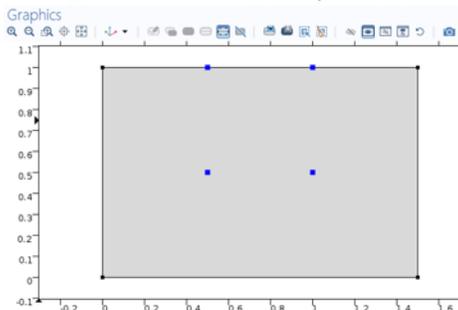
Equation

Show equation assuming:

Study 1, Stationary

$$-\mathbf{n} \cdot (-c \nabla u - \alpha u + \gamma) = g - qu$$

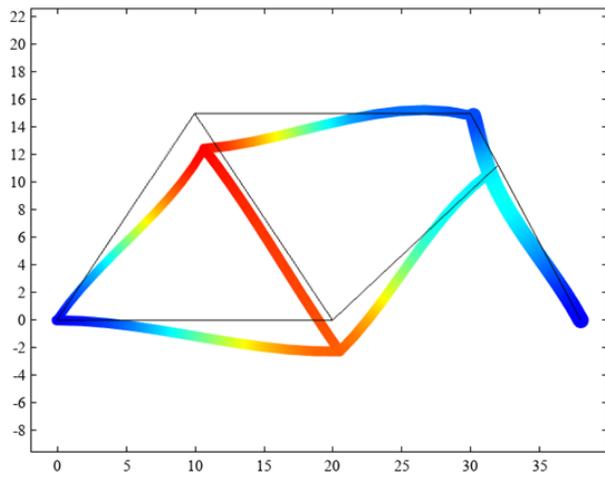
$$\nabla = \left[ \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right]$$



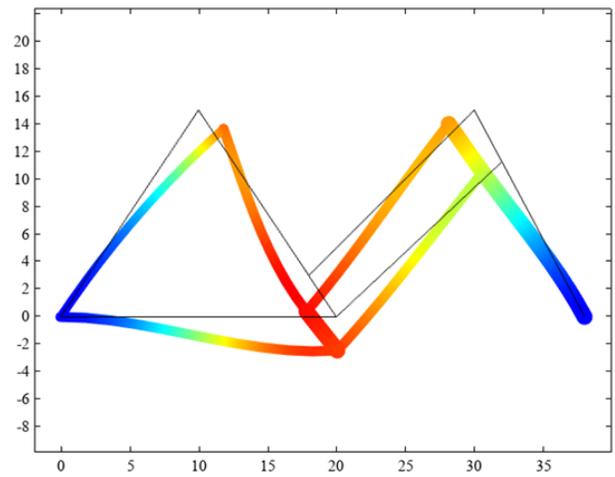
Messages Progress Log Table 1

Dependent variable u (1), Point: 3	Dependent variable u (1), Point: 4	Dependent variable u (1), Point: 5	Dependent variable u (1), Point: 6
0.12500	0.99999	0.25000	2.0000

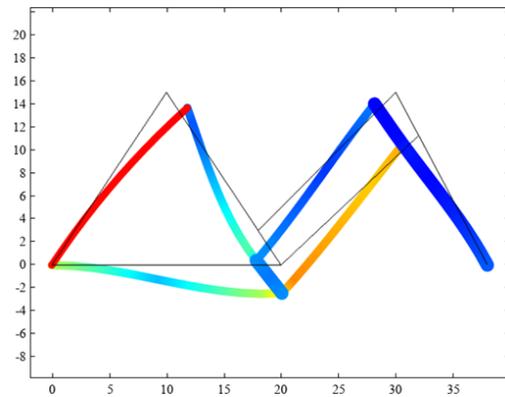
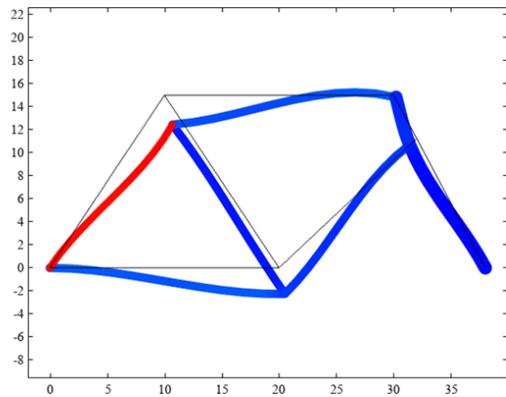
Proble2\_Sol) Fixed B.C case



Displacement field, y component (in), Point: 2  
-0.0010084

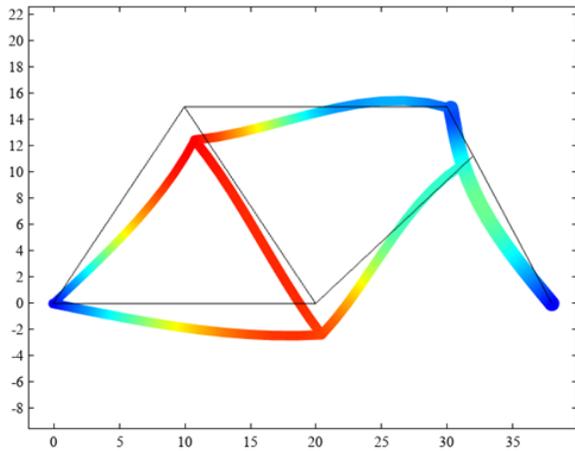


Displacement field, y component (in), Point: 2  
-0.019624

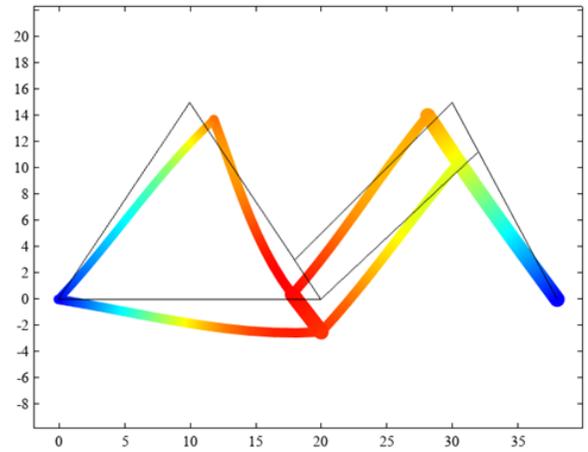


Von-mises 응력 분포를 보면 design 1의 경우 1번 frame에 응력이 집중되어 최대 응력이 1160psi 이고 design2의 경우 938 psi이다. 외력을 frame들에 분배하는 구조임을 알 수 있다. Design 2가 더 나은 구조강성을 갖는다.

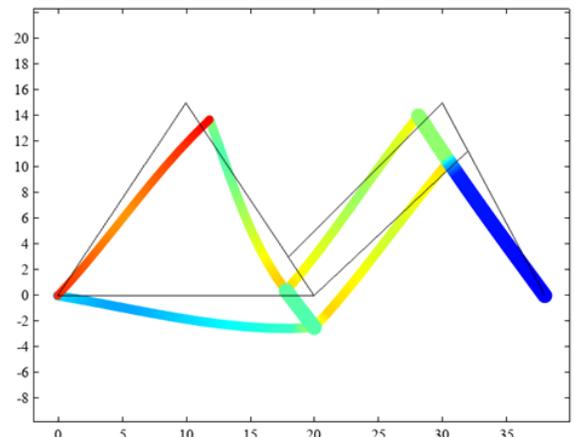
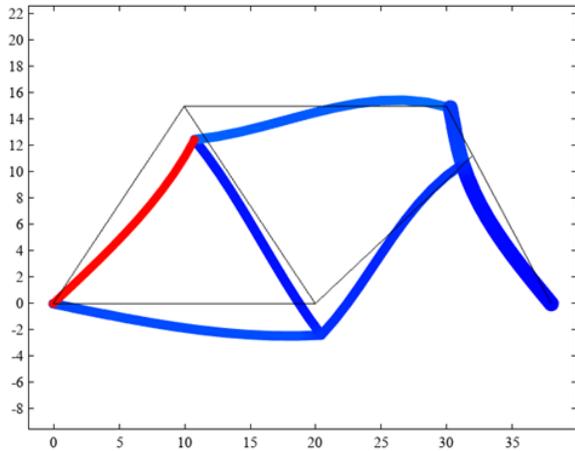
Proble2\_Sol) Pinned B.C case



Displacement field, y component (in), Point: 2  
-0.0010601

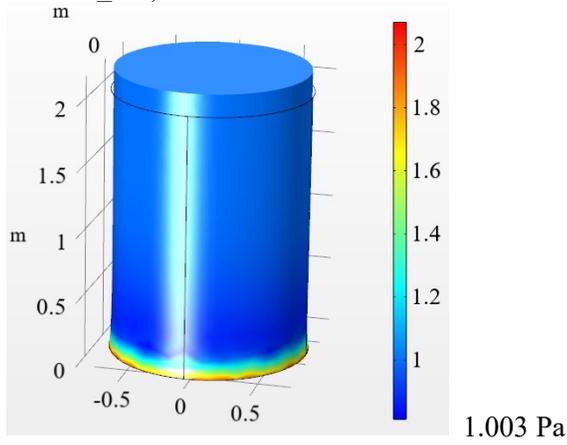


Displacement field, y component (in), Point: 2  
-0.042696

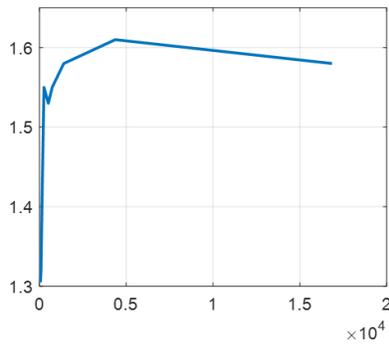


Von-mises 응력 분포를 보면 design 1의 경우 최대 응력이 1180 psi이고 design2의 경우 1300 psi이다.  
최대 응력을 기준으로 Design 1이 더 나은 구조강성을 갖는다.

Proble3\_Sol)

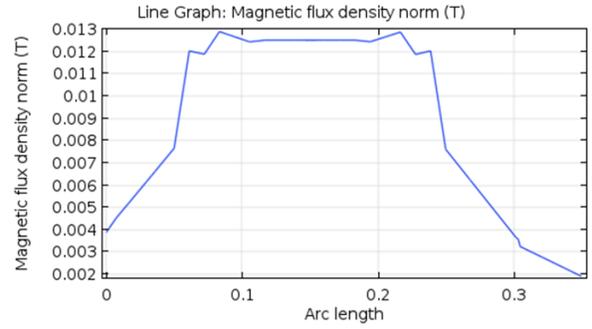
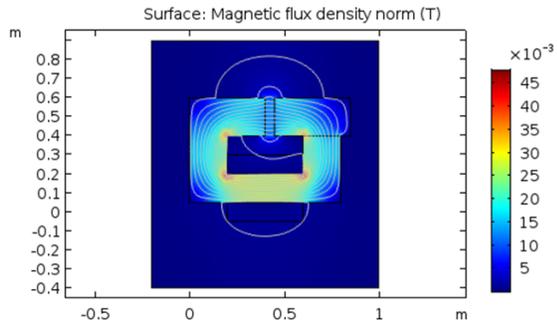


dofs	sigma_max
58	1.305
88	1.32
168	1.43
260	1.55
512	1.53
732	1.55
1398	1.58
4366	1.61
16850	1.58



R(m)	Analytic Kt	FEM Kt
0.05	2.4	2.46
0.1	1.9	1.98
0.15	1.68	1.76
0.2	1.57	1.63
0.25	1.5	1.53

Proble4\_Sol)



4(2) 1.2421 N

# Final Exam Evaluation Criteria

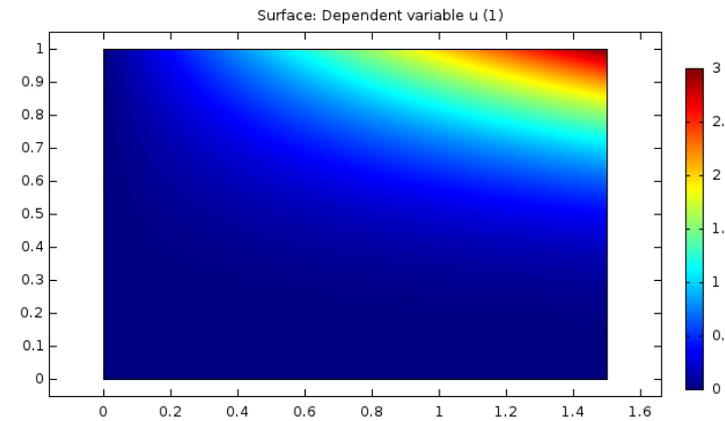
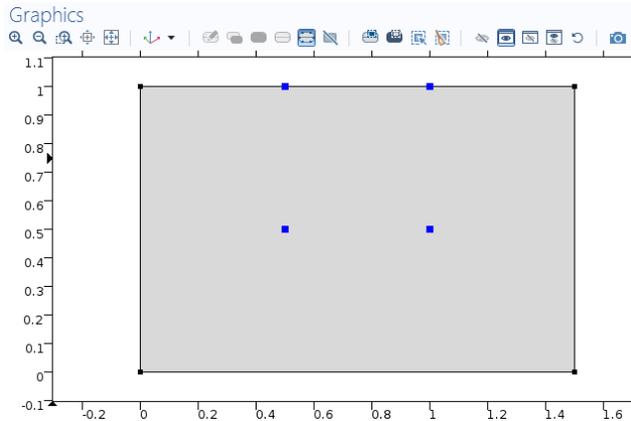
Computational Design Laboratory  
Department of Automotive Engineering  
Hanyang University, Seoul, Korea



한양대학교  
HANYANG UNIVERSITY



# PROBLEM.1 EVALUATION CRITERIA



Messages Progress Log Table 1

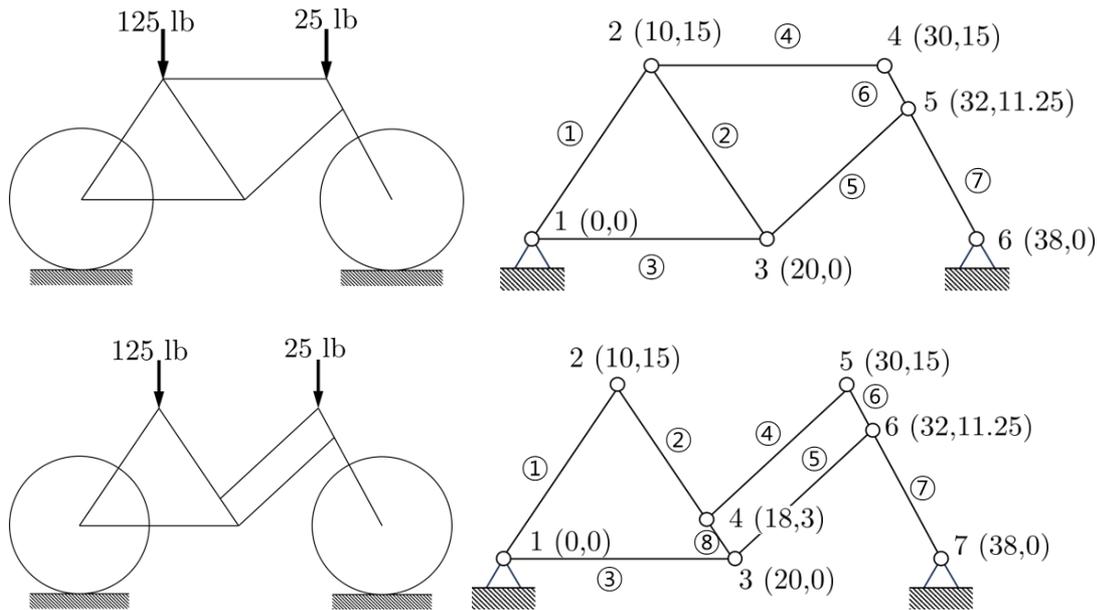
8.85 e-12 AUTO 8.5 e-1 850 e-3 0.85

Dependent variable u (1), Point: 3	Dependent variable u (1), Point: 4	Dependent variable u (1), Point: 5	Dependent variable u (1), Point: 6
0.12500	0.99999	0.25000	2.0000

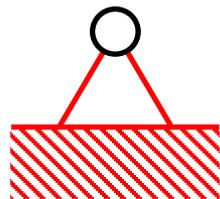
채점 기준

- 1.(1) Surface plot 5점
- (2) Point evaluation 10점

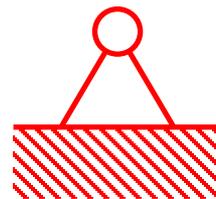
# PROBLEM.2 EVALUATION CRITERIA



(복수 정답 인정) - 문제에서 주어진 경계조건이 보기에 따라 두가지 경계조건으로 해석될 여지가 있어, 두 경우 모두 정답으로 인정합니다. 사전에 검토되지 못한 점 죄송합니다.



Fixed B.C



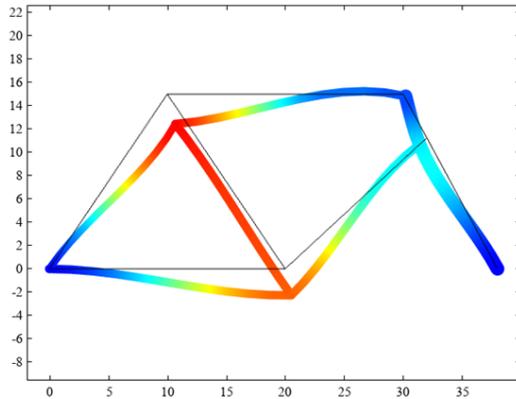
Pinned B.C

2. 2) 구조 강성 측면에서의 설계 평가에 대한 설명도 각각의 해석 결과에 따른 설명이 부합하는 경우, 모두 정답으로 인정

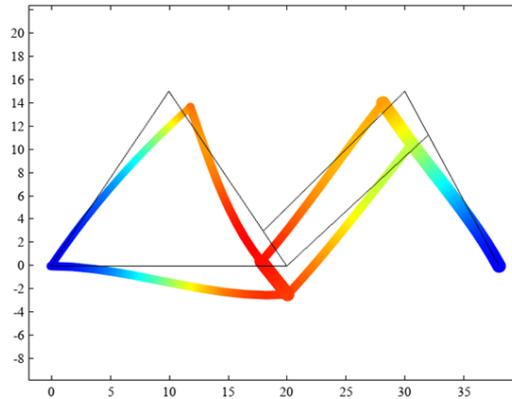
# PROBLEM.2 EVALUATION CRITERIA

Fixed B.C

(1)

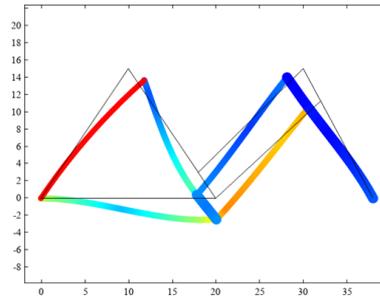
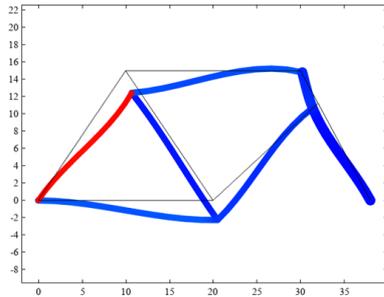


Displacement field, y component (in), Point: 2  
-0.0010084



Displacement field, y component (in), Point: 2  
-0.019624

(2)



Von-mises 응력 분포를 보면 design 1의 경우 최대 응력이 1160 psi이고 design2의 경우 938 psi이다. 최대 응력을 기준으로 Design 1이 더 나은 구조강성을 갖는다.

채점 기준 (단위 설정 오류 시 각 -5점) COMSOL unit system 옵션 결과도 인정  
(보정 없이 사용된 lb·in/s<sup>2</sup> 단위는 감점)

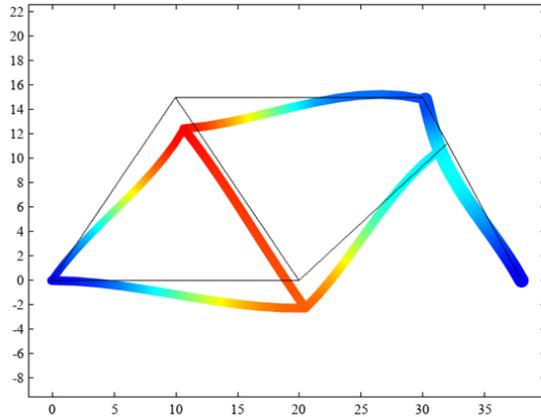
2.(1) 모델링 5점 / Line plot 5점 / displacement 5점)

(2) stress plot 2점 / 설명 8점

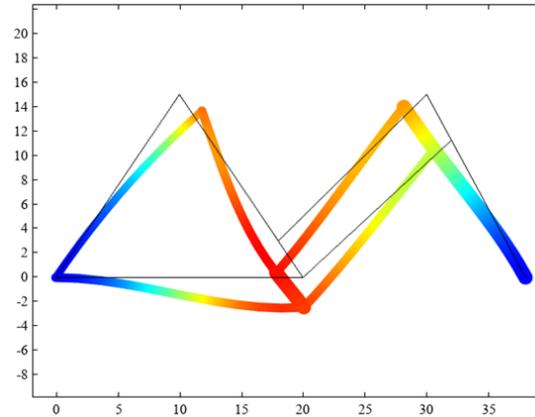
# PROBLEM.2 EVALUATION CRITERIA

## Pinned B.C

(1)

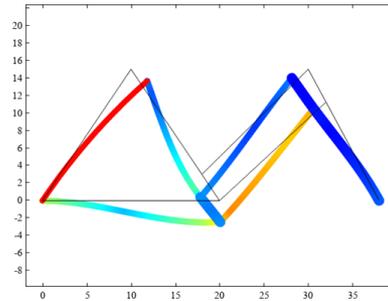
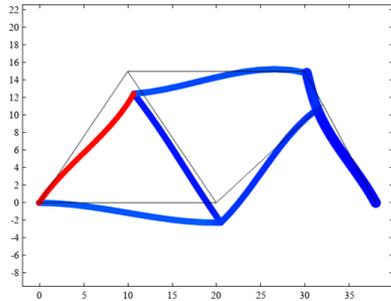


Displacement field, y component (in), Point: 2  
-0.0010084



Displacement field, y component (in), Point: 2  
-0.0065413

(2)



Von-mises 응력 분포를 보면 design 1의 경우 최대 응력이 1180 psi이고 design2의 경우 1300 psi이다. 최대 응력을 기준으로 Design 1이 더 나은 구조강성을 갖는다.

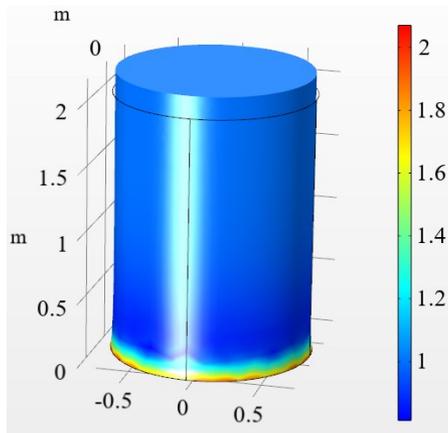
채점 기준 (단위 설정 오류 시 각 -5점) COMSOL unit system 옵션 활용한 결과도 인정

2.(1) 모델링 5점 / Line plot 5점 / displacement 5점)

(2) stress plot 2점 / 설명 8점

# PROBLEM.3 EVALUATION CRITERIA

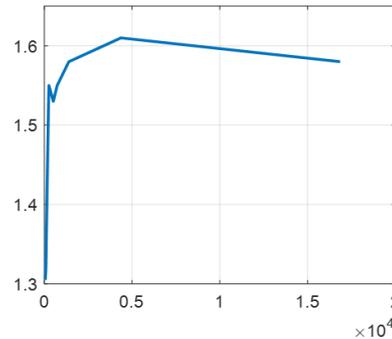
(1)



First principal stress (N/m<sup>2</sup>), Domain Probe 1  
1.0031

(2)

dofs	sigma_max
58	1.305
88	1.32
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512	1.53
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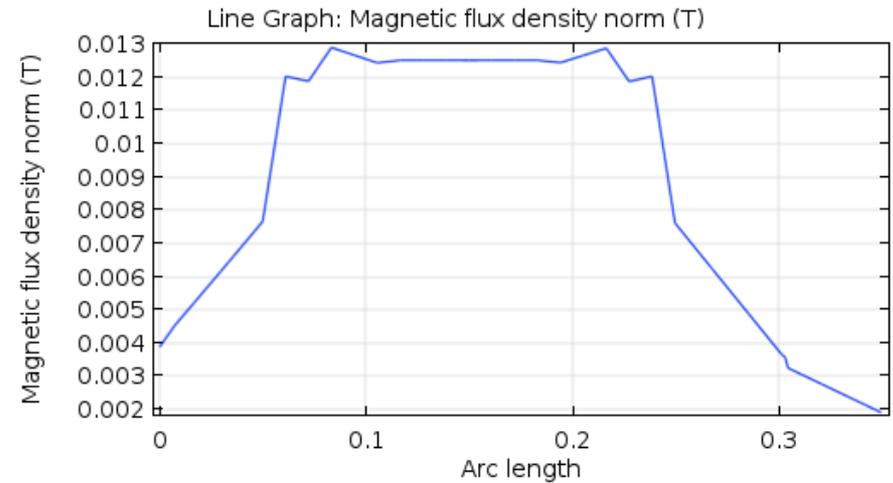
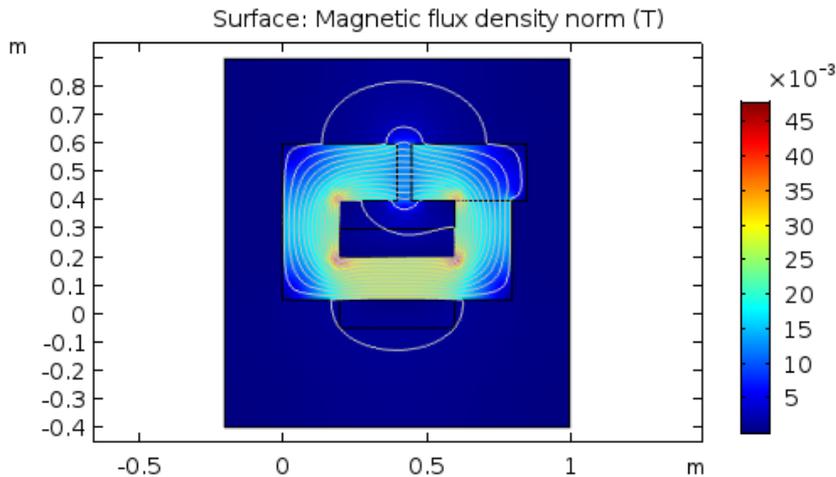
(3)

R(m)	Analytic Kt	FEM Kt
0.05	2.4	2.46
0.1	1.9	1.98
0.15	1.68	1.76
0.2	1.57	1.63
0.25	1.5	1.53

## 채점 기준

- 3.(1) Surface plot 5점 / 응력 계산 5점 (symmetry B.C / Fixed B.C 모두 인정)
- (2) 2D axisymmetric 모델 5점 / Plot graph 5점
- (3) table 10점 (최대 응력만 계산 시 5점)

# PROBLEM.4 EVALUATION CRITERIA



```
0.1*(bnd1+bnd2) (kg^2*m/(s^4*A^2))
1.2481
```

채점 기준

- 4.(1) 올바른 모델링(Geomtry-5/ B.C-3/재료물성-2) 10 / 올바른 플롯5  
 (2) 수식 설정 5/ 적분 영역 설정5 (부분점수 3) / 결과5