

Submit the compressed file as `(ID)_(name).zip` to [<ftp://cdl.hanyang.ac.kr> → Undergraduate\_CAE → lab → midterm] folder. It should contain the final results (graphs) of each problem using PowerPoint (`ID.ppt`), MATLAB file (`problem#_#.m`), Simulink file (`problem#_#.slx`)

1.[MATLAB] The Volume  $V$  of liquid in a hollow horizontal cylinder of radius  $r$  and length  $L$  is related to the depth of the liquid  $h$  by

$$V = \left[ r^2 \cos^{-1} \left( \frac{r-h}{r} \right) - (r-h)\sqrt{2rh - h^2} \right] L$$

Develop an M-file to create a plot of volume versus depth. (20 pts)

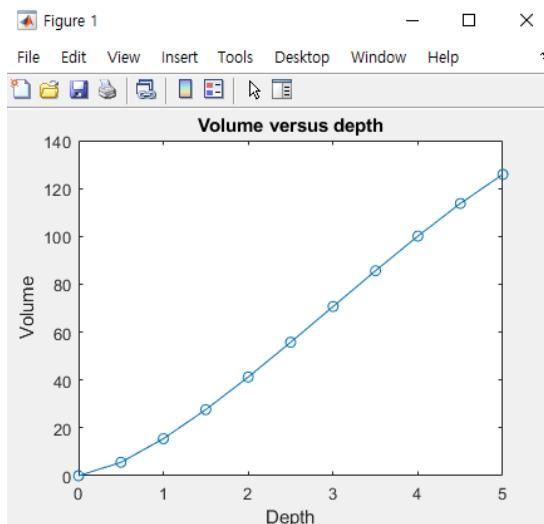
(Satisfy all the post-processing styles of the plot provided below.)

Here are the first few lines:

```
function problem1(r,L,plot_title)
% volume of horizontal cylinder
% inputs :
% r = radius
% L= length
% plot_title=string holding plot title
```

Test your program with

```
>>problem1(3,5,'Volume versus depth')
```



Sol)

```
function problem1(r,L,plot_title)

% volume of horizontal cylinder
% inputs :
% r = radius
% L= length
% plot_title=string holding plot title
H=0:0.5:L;
v= @(h) (r^2*acos((r-h)./r)-(r-h).*sqrt(2*r*h-h.^2))*L;
V=v(H);
plot(H,V,'o-')
title(plot_title)
xlabel('Depth')
ylabel('Volume')
end
```

2.[MATLAB] Solve the Van der pol equation with initial value.

$$\frac{d^2y}{dt^2} - \mu(1-y^2)\frac{dy}{dt} + y = 0$$

initial condition

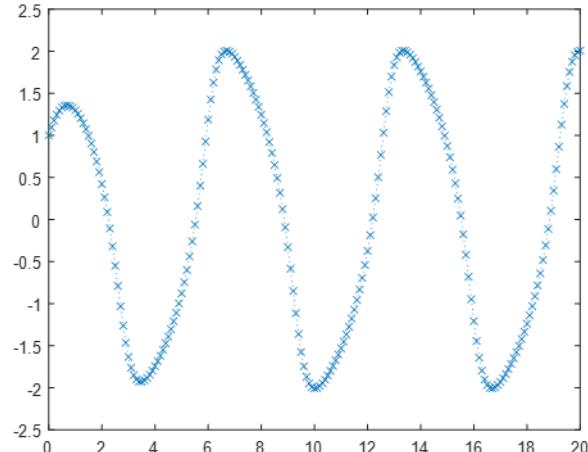
$$\text{at } t=0, y=1 \quad \frac{dy}{dt} = 1$$

- (1) Given the following two scripts (main, vander), Develop a single function script (RK4.m) that covers the fourth-order RK method for second-order ODE and obtain the solution (20 pts)  
 $\mu = 1$ ,  $tspan=[0 20]$ , step size  $h=0.1$  plot style “x:” ex:  $\text{plot}(x,y,\text{'x:')}$ )

[Fourth-order RK]

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$\begin{cases} k_1 = f(x_i, y_i) \\ k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h) \\ k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h) \\ k_4 = f(x_i + h, y_i + k_3 h) \end{cases}$$



[main.m]

```
clc; clear all; close all;

h=0.01;
tf=20;
[t,y]= RK4 (@vander, [0 tf], [1,1],h);

figure(1)
plot(t,y(:,1), 'x:')
```

[vander.m]

```
function yp = vander(t,y)
...
...
```

- (2) Modify main.m and vander.m to obtain MATLAB built in function ODE45 solution with default setting, compare the results, and explain the difference between the two ODE solvers. (5 pts)

[main.m]

```
clc; clear all; close all;
```

```
h=0.1;
tf=20;
[t,y]= RK4(@vander,[0 tf],[1,1],h);
[t,y]= ode45(@vander,[0 tf],[1,1]);
```

```
figure(1)
subplot(2,1,1)
plot(t,y(:,1),'x:')
subplot(2,1,2)
plot(t,y(:,1),'o-')
```

- (3) When solving the equation with parameter setting in  $\mu = 1000$ ,  $tspan=[0 1000]$ , step size  $h=0.1$ , ode45 takes an excessive amount of computation. Explain this phenomenon and Apply MATLAB built-in function solution for it. Compare with the step size with ode45 results and the built-in function. (5 pts)

Sol) 2.

(1)

```
clc; clear all; close all;
h=0.1;
tf=1000;
[t,y]= RK4(@vander,[0 tf],[1,1],h);
[t2,y2]= ode45(@vander,[0 tf],[1,1]);
[t3,y3]= ode23s(@vander,[0 tf],[1,1]);
```

```
figure(1)
subplot(3,1,1)
```

```

plot(t,y(:,1),'x:')

subplot(3,1,2)
plot(t2,y2(:,1),'o-')
subplot(3,1,3)
plot(t3,y3(:,1),'*--')

function [t, y] = RK4(dydt, tspan, y0, h)

t0 = tspan(1); tn = tspan(2);
t = (t0:h:tn)'; n = length(t);
m=length(y0);
y = y0 .* ones(n,m);
for i = 1:n-1

    k1 = dydt(t(i), y(i,:));
    k2 = dydt(t(i) + h/2, y(i,:) + k1'*h/2);
    k3 = dydt(t(i) + h/2, y(i,:) + k2'*h/2);
    k4 = dydt(t(i) + h, y(i,:) + k3'*h);
    slope = (k1 + 2*k2 + 2*k3 + k4)/6;
    y(i+1,:) = y(i,:) + slope'*h;
end
end

function yp = vander(t,y)
mu=100;
yp=[y(2); mu*(1-y(1).^2)*y(2)-y(1)];
end

```

(2) RK45 함수의 경우, 4차, 5차 룽게쿠타법을 이용해 정확도에 따라 적응적으로 step size를 조절하여 높은 계산효율성을 갖는다.

(3) 반데르풀 방정식은 parameter에 따라 system이 stiff해져 수치해석적 근사해 계산에서 과도한 계산량을 갖게된다. MATLAB built-in function인 ode23s, ode23t 등을 활용하여 stiff system에 대한 솔루션을 효율적인 계산으로 얻을 수 있다.

3.[Simulink] Solve the following RLC second order system using signal-based solution (10 pts)

Show the transient current and electrical charge on capacitor using scope block.

Equation of the transient RLC circuit

$$L \frac{di}{dt} + Ri + \frac{q}{C} - E(t) = 0 \text{ and } i = \frac{dq}{dt}$$

$L$  : inductor

$R$  : resistance

$C$  : capacitance

$E$  : voltage source

$q$  : electrical charge

$$E = E_0 \sin(\omega t)$$

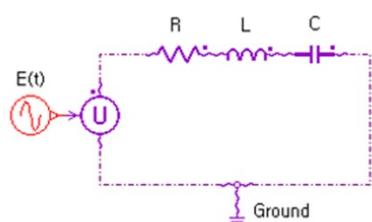
$$L = 2 \text{ H}$$

$$E_0 = 5 \text{ V}$$

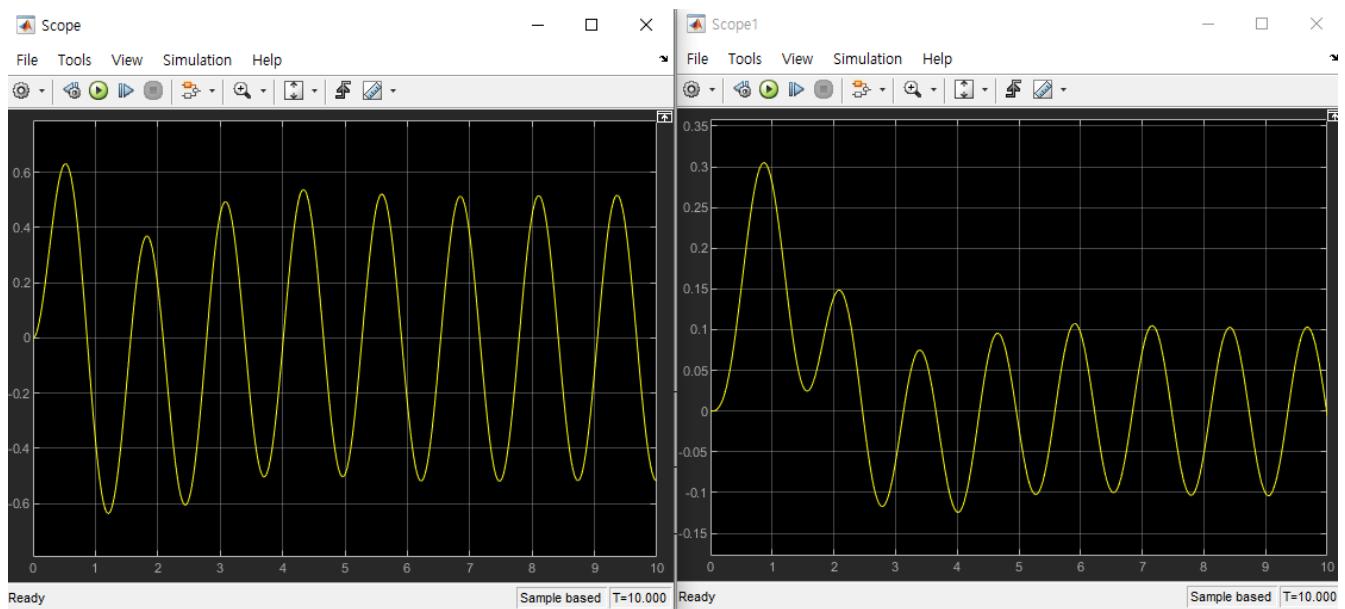
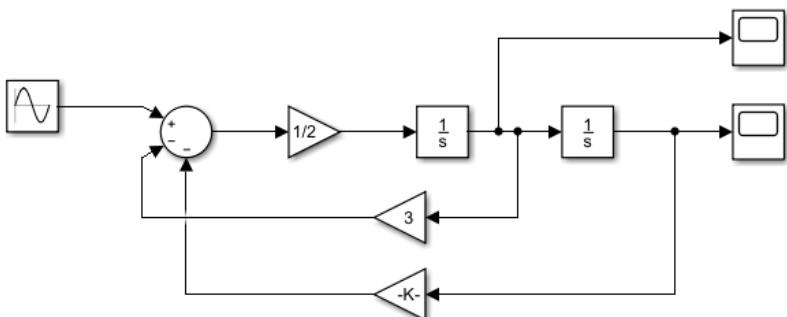
$$C = 0.25 \text{ F}$$

$$\omega = 5 \text{ rad/s}$$

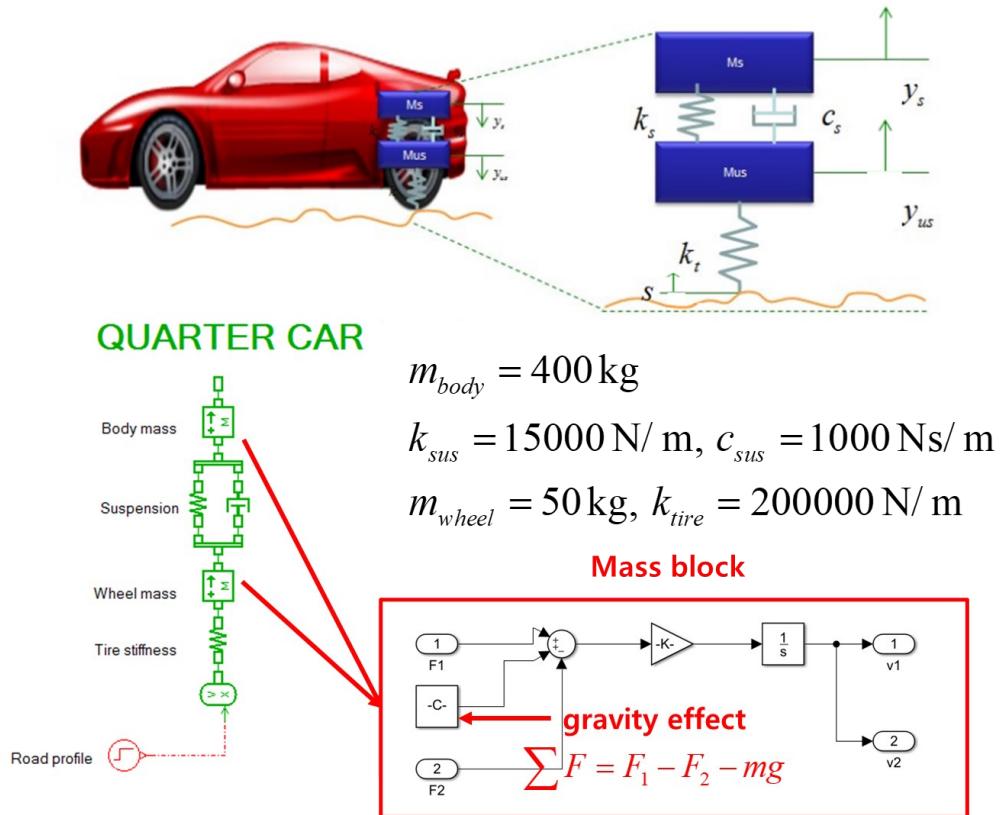
$$R = 3 \Omega$$



Sol)



4. [Simulink] Consider quarter car model by Simulink. The parameter values in the model are as follows.

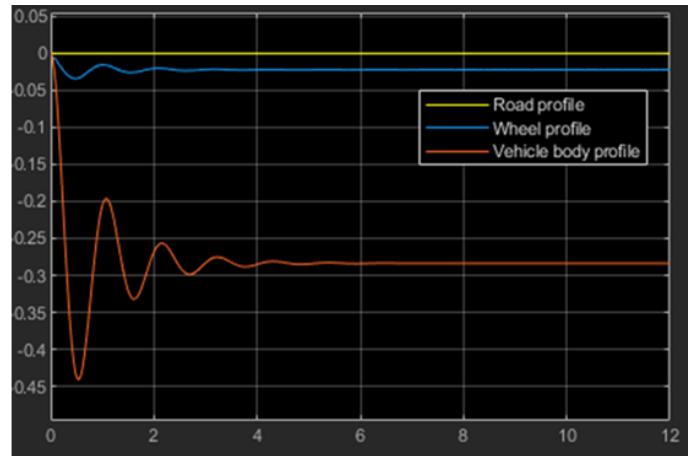
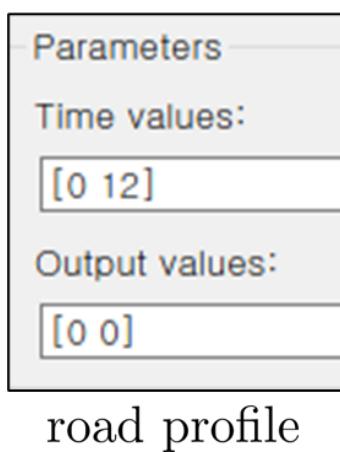


For this problem, you are required to construct a Simulink model of a quarter car to evaluate its suspension behavior under two specific scenarios: (Simulation time: 12 s)

(1) Freefall due to Weight of Car (10 pts)

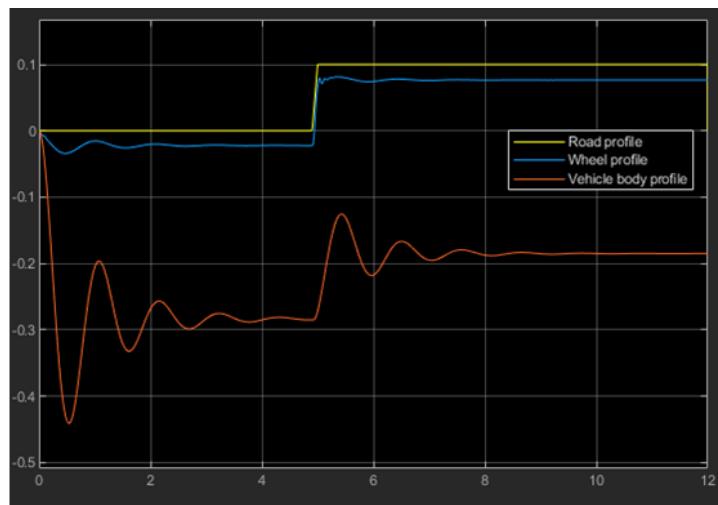
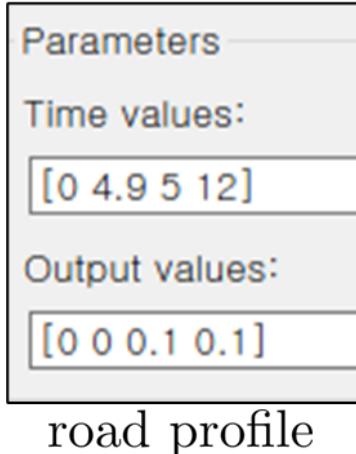
-Assume that the car undergoes a freefall due to its own weight.

-Build a Simulink model to simulate freefall road profile and obtain following scope plot.

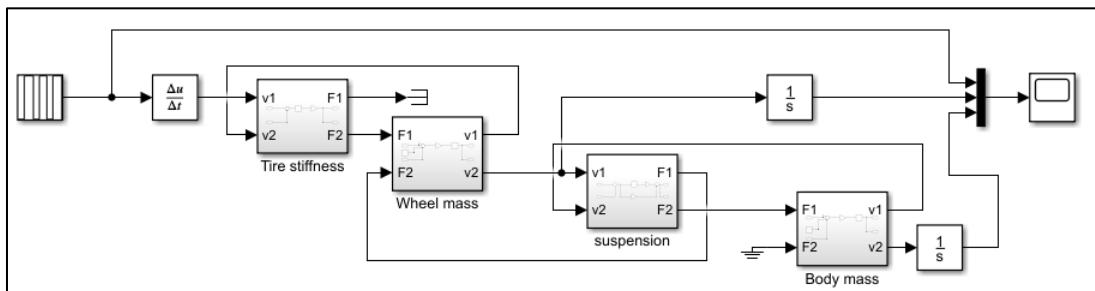


## (2) Road Bump Interaction (10 pts)

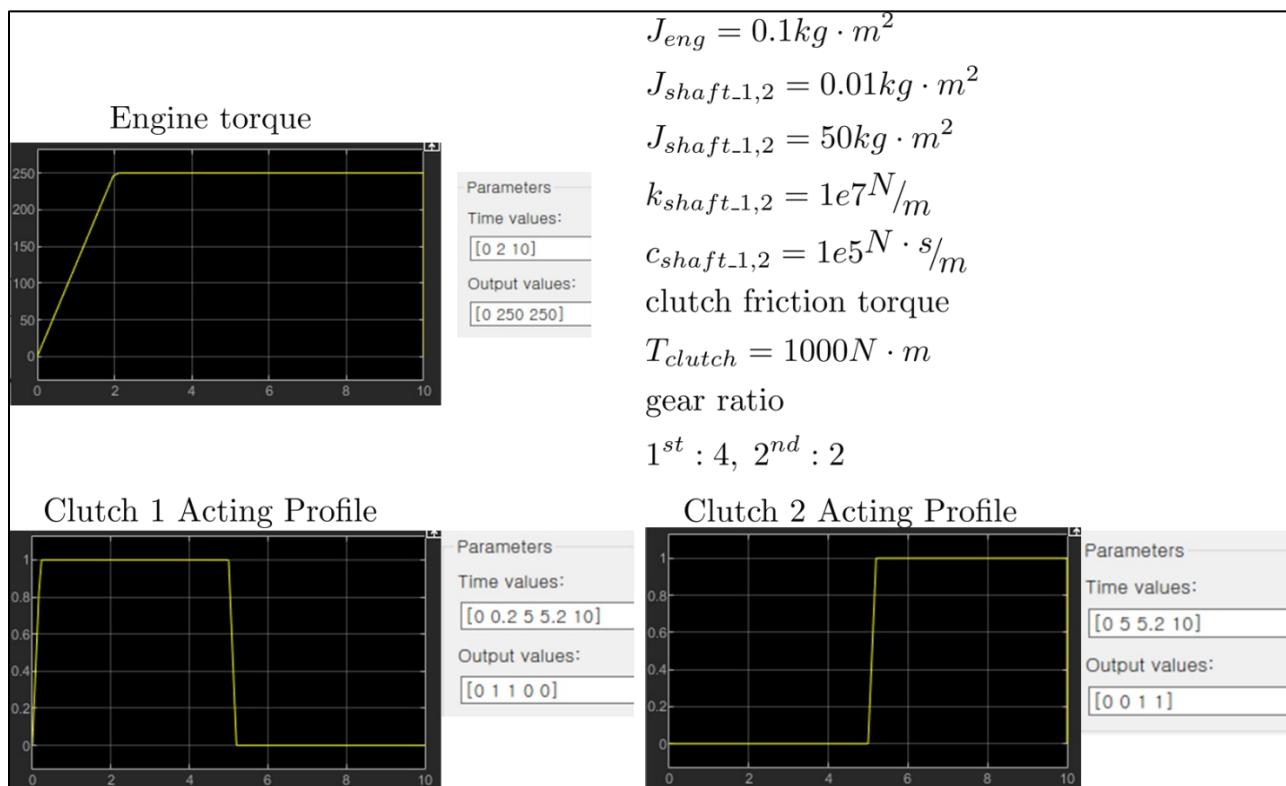
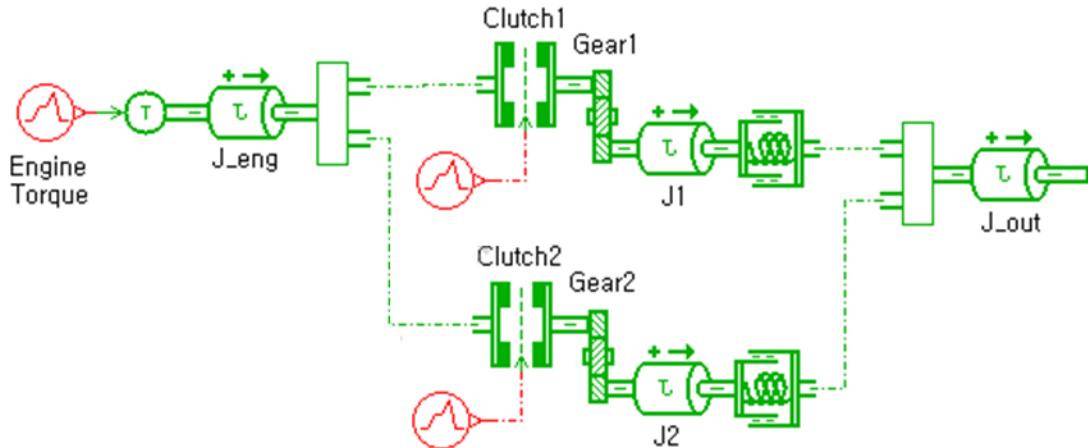
Starting from the car's stable position after the freefall (from Scenario 1), simulate its response when it encounters a 0.1m road bump at 5 s.



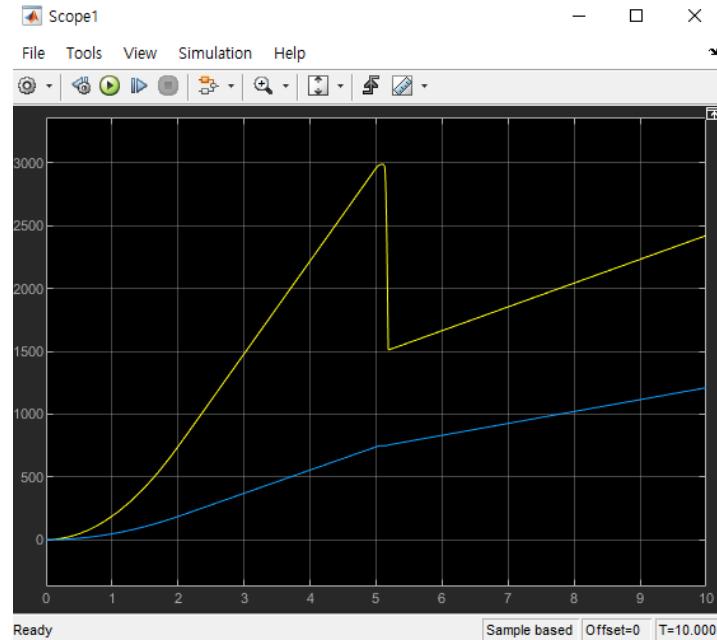
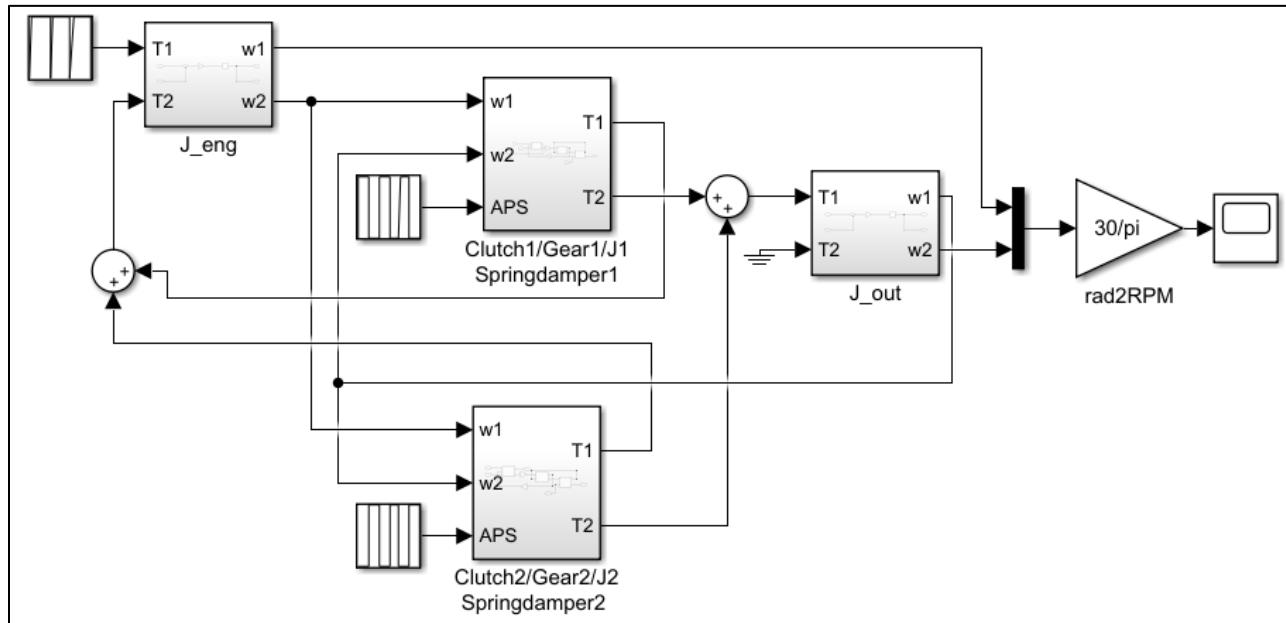
Sol)



5. [Simulink - Powertrain] the Following 2-speed transmission. The parameter values in the model are as follows. Construct a Simulink model for this system. Show the engine( $J_{eng}$ ) and vehicle side( $J_{out}$ ) side RPM. (20 pts)  
 (Simulation time: 10 sec)



Sol)



# Midterm Exam Evaluation Criteria

Computational Design Laboratory  
Department of Automotive Engineering  
Hanyang University, Seoul, Korea

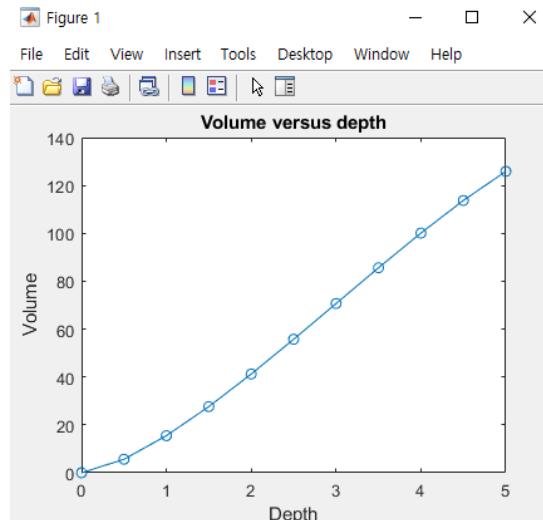


# PROBLEM.1 EVALUATION CRITERIA

```

function problem1(r,L,plot_title)
% volume of horizontal cylinder
% inputs :
% r = radius
% L= length
% plot_title=string holding plot title
H=0:0.5:L;
v= @(h) (r^2*acos((r-h)./r)-(r-h).*sqrt(2*r*h-h.^2))*L;
V=v(H);
plot(H,V,'o-')
title(plot_title)
xlabel('Depth')
ylabel('Volume')
end

```

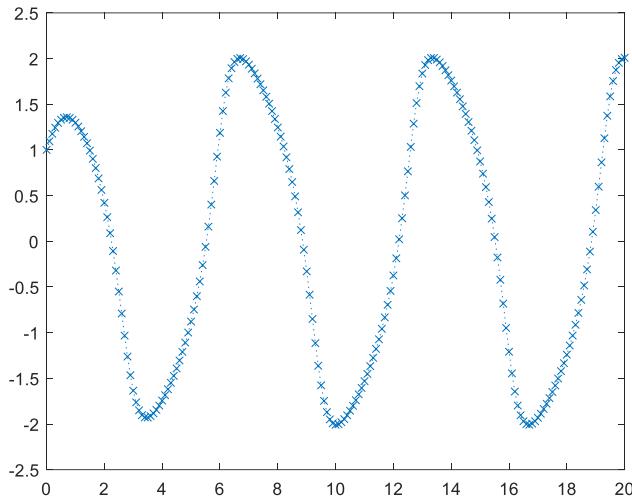


## 채점 기준

1.

- 코드(x,y plot 완성)작성 완성 – 10 점
- x,y label – 각 2점
- title – 2점
- line style – 4점

# PROBLEM.2 EVALUATION CRITERIA

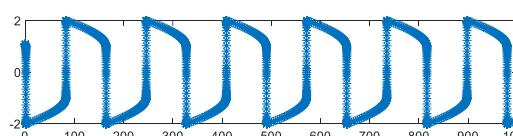
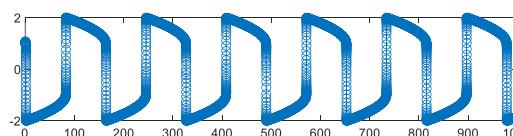
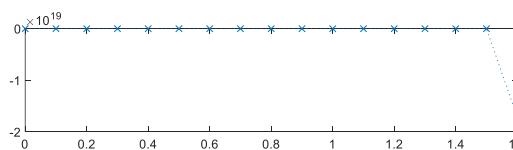
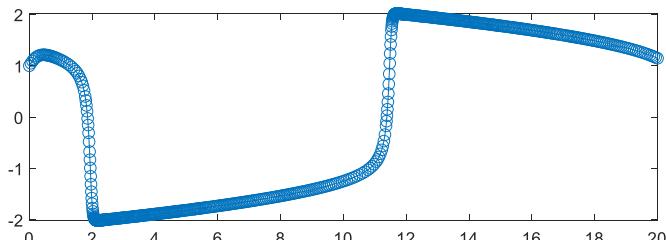
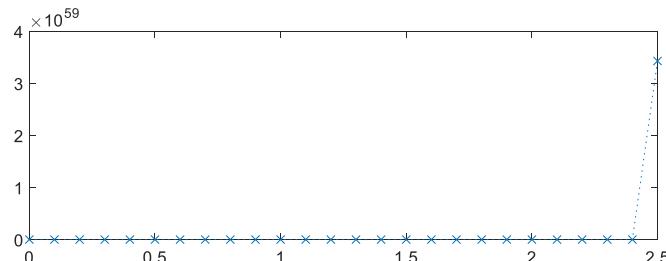


채점 기준

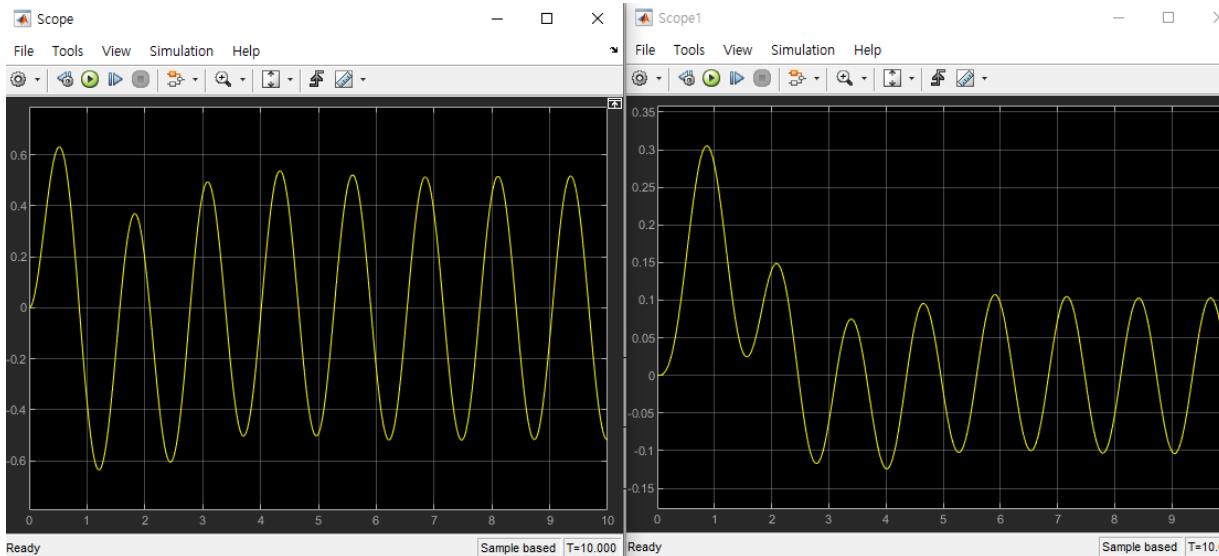
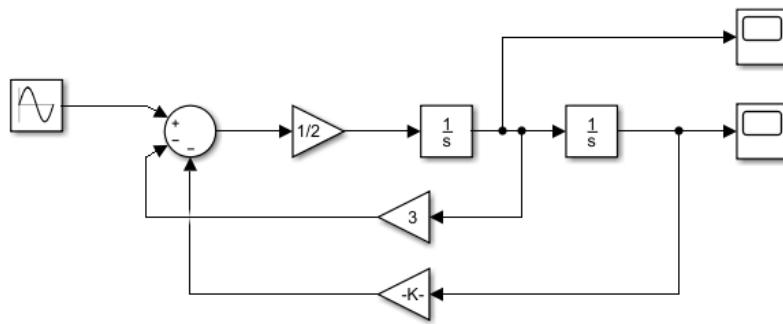
(1) 코드작성 완성 – 20점  
RK function 코드 완성 5점

(2) 코드작성 완성 – 3점  
두 solver 차이점 설명 2점  
RK4는 고정 step size  
Ode45 변동 step size

(3) Stiff 문제 설명 3점  
Stiff 문제를 위한 solver 활용-2점



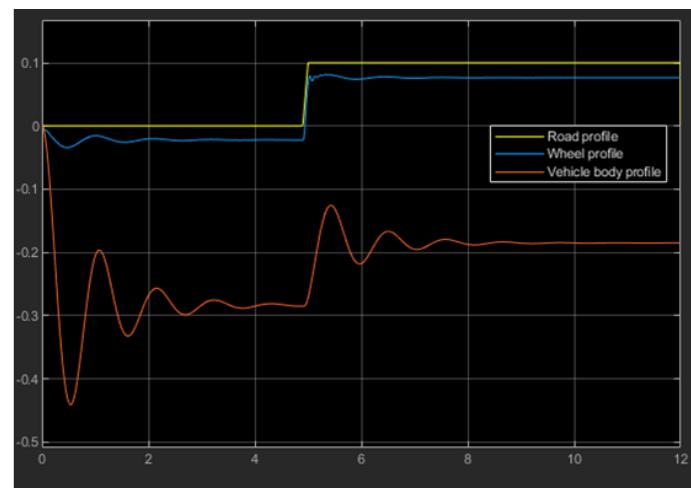
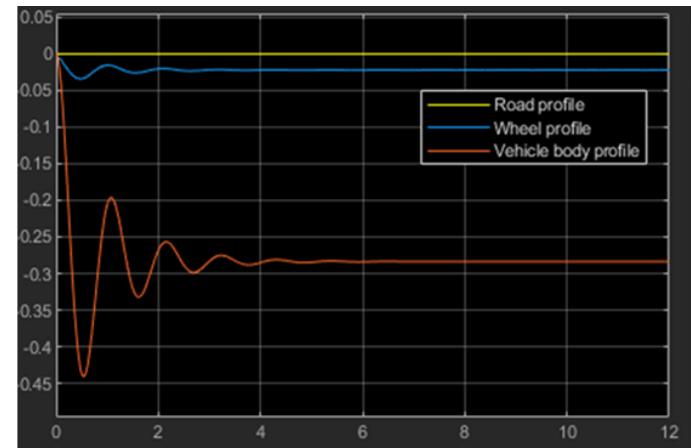
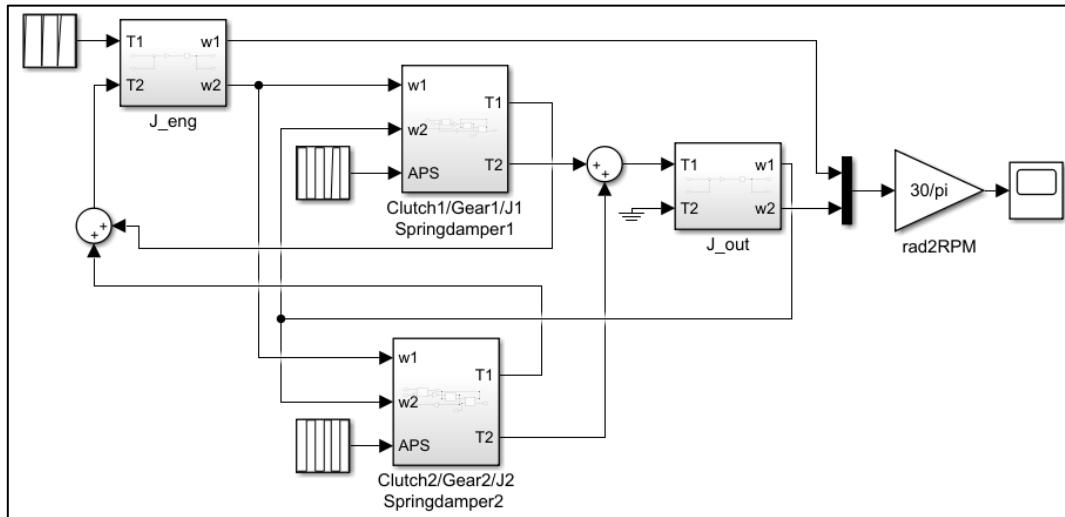
# PROBLEM.3 EVALUATION CRITERIA



채점 기준

3. 블록다이어그램 완성 5점 / 시뮬레이션 결과 5점

# PROBLEM.4 EVALUATION CRITERIA

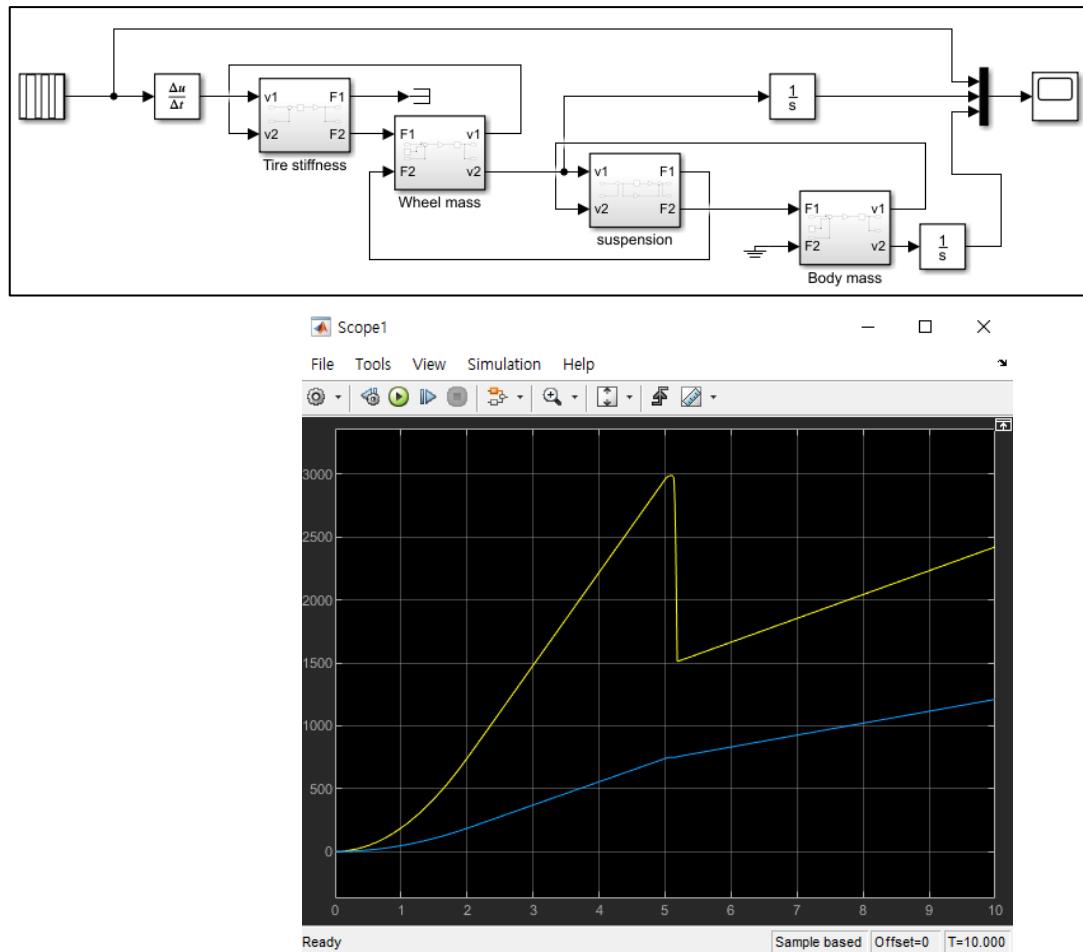


채점 기준

(1) 블록다이어그램 5점  
올바른 해석 결과 5점

(2) 블록다이어그램 5점  
올바른 해석 결과 5점

# PROBLEM.5 EVALUATION CRITERIA



채점 기준

블록다이어그램 10점  
올바른 해석 결과 10점