1.

$$\frac{dy_1}{dx} = -0.5y_1, \frac{dy_2}{dx} = 4 - 0.3y_2 - 0.1y_1$$

$$y_1(0) = 4,\ y_2(0) = 6,\ \Delta x = 0.5$$

(1) Euler's explicit method (5 pts)

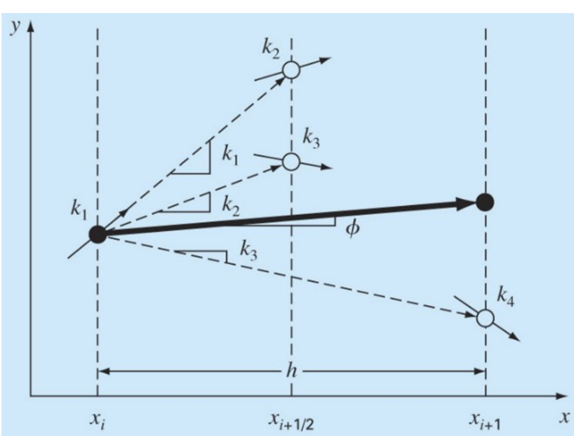
$$\begin{cases} y_1(0.5) = 4 + [-0.5(4)](0.5) = 3 \\ y_2(0.5) = 6 + [4 - 0.3(6) - 0.1(4)](0.5) = 6.9 \end{cases}$$

(2) 4-th order Runge-Kutta ($k_{i,j}$ 4개, y_j 각 3pts)

$$\begin{cases} k_{1,1} = f_1(0,4,6) = -0.5(4) = -2 \rightarrow y_1 + k_{1,1}\frac{h}{2} = 4 + (-2)\frac{0.5}{2} = 3.5 \\ k_{1,2} = f_2(0,4,6) = 4 - 0.3(6) - 0.1(4) = 1.8 \rightarrow y_2 + k_{1,2}\frac{h}{2} = 6 + (1.8)\frac{0.5}{2} = 6.45 \\ k_{2,1} = f_1(0.25, 3.5, 6.45) = -1.75 \rightarrow y_1 + k_{2,1}\frac{h}{2} = 4 + (-1.75)\frac{0.5}{2} = 3.5625 \\ k_{2,2} = f_2(0.25, 3.5, 6.45) = 1.715 \rightarrow y_2 + k_{2,2}\frac{h}{2} = 6 + (1.715)\frac{0.5}{2} = 6.42875 \\ k_{3,1} = f_1(0.25, 3.5625, 6.42875) = -1.78125 \rightarrow y_1 + k_{3,1}h = 4 + (-1.78125)(0.5) = 3.109375 \\ k_{3,2} = f_2(0.25, 3.5625, 6.42875) = 1.715125 \rightarrow y_2 + k_{3,2}h = 6 + (1.715125)(0.5) = 6.857563 \\ k_{4,1} = f_1(0.5, 3.109375, 6.857563) = -1.554688 \\ k_{4,2} = f_2(0.5, 3.109375, 6.857563) = 1.631794 \end{cases}$$

$$y_1 = y_1(0) + \frac{1}{6}(k_{1,1} + 2k_{2,1} + 2k_{3,1} + k_{4,1})\Delta x = 4 + \frac{1}{6}[-2 + 2(-1.75 - 1.78125) - 1.554688](0.5) = 3.115234$$

$$y_2 = y_2(0) + \frac{1}{6}(k_{1,1} + 2k_{2,1} + 2k_{3,1} + k_{4,1})\Delta x = 6 + \frac{1}{6}[1.8 + 2(1.715 + 1.715125) + 1.631794](0.5) = 6.857670$$



2.

(1) $\dfrac{d^2u}{dx^2} = 1 \rightarrow u = \dfrac{1}{2}x^2 + c_1 x + c_2 \xrightarrow[u(1)=0 \rightarrow c_1 = -1/2]{u(0)=0 \rightarrow c_2 = 0} = \dfrac{1}{2}x^2 - \dfrac{1}{2}x$ (3 pts)

(2)

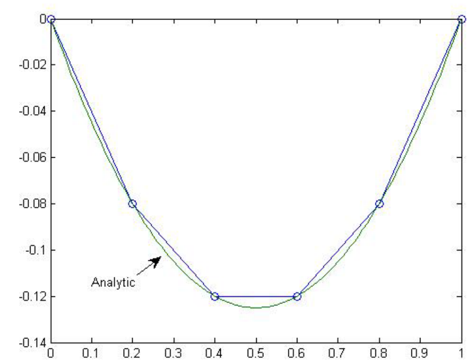
$$\frac{d^2u}{dx^2} \approx \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta x)^2} \xrightarrow[u_0=0,u_5=0]{\Delta x=0.2} \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta x)^2} = 1 \text{ for } i = 1,2,3,4$$

$$\left.\begin{array}{l} i=1:\ u_0 - 2u_1 + u_2 = (\Delta x)^2 \\ i=2:\ u_1 - 2u_2 + u_3 = (\Delta x)^2 \\ i=3:\ u_2 - 2u_3 + u_4 = (\Delta x)^2 \\ i=4:\ u_3 - 2u_4 + u_5 = (\Delta x)^2 \end{array}\right\} \rightarrow \begin{bmatrix} -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0.04 \\ 0.04 \\ 0.04 \\ 0.04 \end{Bmatrix} \quad \text{(5 pts)}$$

(3) 5 pts + 2 pts (graph)

$$\left.\begin{array}{l} -2u_1 + u_2 = 0.04 \rightarrow u_2 = 0.04 + 2u_1 \\ u_1 - 2u_2 + u_3 = 0.04 \rightarrow u_3 = 0.04 + 2u_2 - u_1 = 3(0.04) + 3u_1 \\ u_2 - 2u_3 + u_4 = 0.04 \rightarrow u_4 = 0.04 + 2u_3 - u_2 = 6(0.04) + 4u_1 \\ u_3 - 2u_4 = 0.04 \rightarrow 5u_1 = -10(0.04) \end{array}\right\}$$

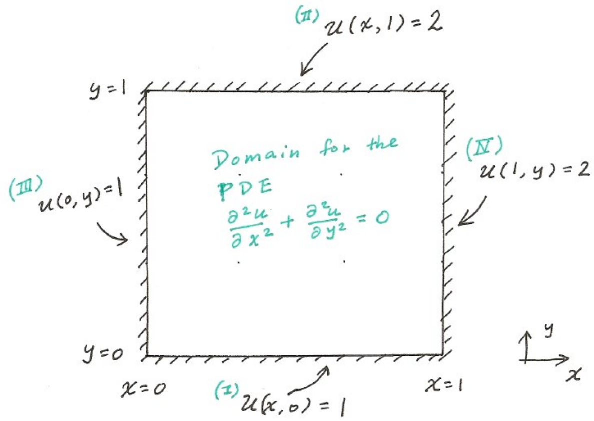
$$\rightarrow \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} -2 \\ -3 \\ -3 \\ -2 \end{Bmatrix} (0.04)$$

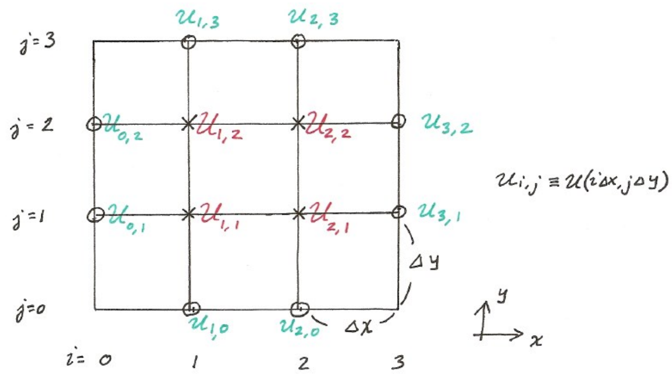


(4) $\dfrac{du}{dx} \approx \dfrac{u_{i+1} - u_i}{\Delta x} \rightarrow \dfrac{u_1 - u_0}{\Delta x} = 0.5,\ u_5 = 2$

$$\left.\begin{array}{l} i=1:\ u_1 - 0.5(\Delta x) - 2u_1 + u_2 = (\Delta x)^2 \\ i=2:\ u_1 - 2u_2 + u_3 = (\Delta x)^2 \\ i=3:\ u_2 - 2u_3 + u_4 = (\Delta x)^2 \\ i=4:\ u_3 - 2u_4 + u_5 = (\Delta x)^2 \end{array}\right\} \rightarrow \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0.14 \\ 0.04 \\ 0.04 \\ -1.96 \end{Bmatrix} \quad \text{(5 pts)}$$

3.





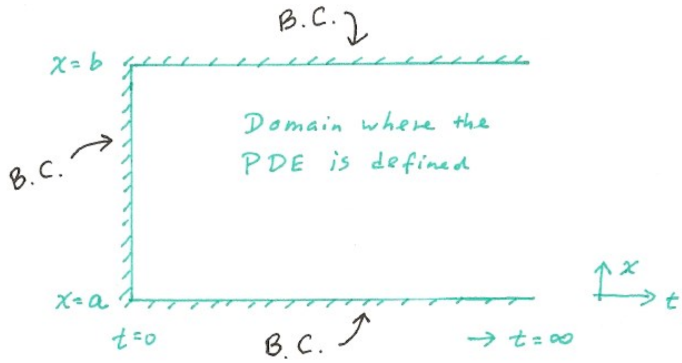
$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} \approx \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta x)^2}, \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} \approx \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\Delta y)^2} \quad \text{(5 pts)}$$

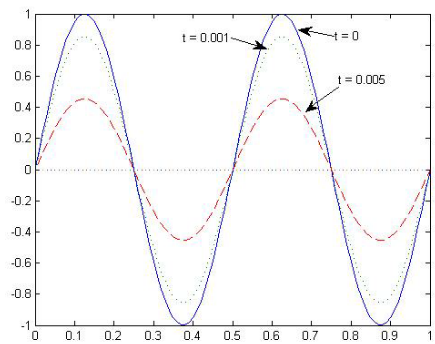
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \rightarrow \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\Delta x)^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\Delta y)^2} = 0$$

$$\Delta x = \Delta y \rightarrow -4u_{i,j} + u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} = 0 \quad \text{(5 pts)}$$

$$\underbrace{\left.\begin{aligned} i=1, j=1 &: -4u_{1,1} + u_{2,1} + u_{1,2} + u_{0,1} + u_{1,0} = 0 \\ i=1, j=2 &: -4u_{1,2} + u_{1,1} + u_{2,2} + u_{0,2} + u_{1,3} = 0 \\ i=2, j=1 &: -4u_{2,1} + u_{2,2} + u_{1,1} + u_{3,1} + u_{2,0} = 0 \\ i=2, j=2 &: -4u_{2,2} + u_{1,2} + u_{2,1} + u_{3,2} + u_{2,3} = 0 \end{aligned}\right\}}_{\text{(5 pts)}} \xrightarrow[\substack{u_{0,1}=u_{0,2}=1 \\ u_{3,1}=u_{3,2}=2}]{\substack{u_{1,0}=u_{2,0}=1 \\ u_{1,3}=u_{2,3}=2}} \underbrace{\begin{bmatrix} -4 & 1 & 1 & 0 \\ 1 & -4 & 0 & 1 \\ 1 & 0 & -4 & 1 \\ 0 & 1 & 1 & -4 \end{bmatrix} \begin{Bmatrix} u_{1,1} \\ u_{1,2} \\ u_{2,1} \\ u_{2,2} \end{Bmatrix} = \begin{Bmatrix} -2 \\ -3 \\ -3 \\ -4 \end{Bmatrix}}_{\text{(5 pts)}}$$

4. 중간과정은 틀렸지만 끝까지 전개하면 15점





$$u(x,t)=G(x)H(t)\rightarrow\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}\rightarrow G\frac{dH}{dt}=H\frac{d^2G}{dx^2}\rightarrow\frac{1}{G}\frac{d^2G}{dx^2}=\frac{1}{H}\frac{dH}{dt}=(\text{constant})=-k^2\quad(5\text{ pts})$$

$$u(0,t)=G(0)H(t)=0\rightarrow G(0)=0$$

$$u(1,t)=G(1)H(t)0=0\rightarrow G(1)=0$$

$$\begin{cases}\frac{1}{G}\frac{d^2G}{dx^2}=-k^2\rightarrow G(x)=A\cos(kx)+B\sin(kx)\xrightarrow[G(1)=0]{G(0)=0}\left\{\begin{matrix}A=0\\ \sin k=0\rightarrow k=n\pi\end{matrix}\right\}\rightarrow G_n(x)=\sin(n\pi x)\quad(5\text{ pts})\\ \frac{1}{H}\frac{dH}{dt}=-k^2\rightarrow H(t)=\exp(-k^2t)\quad(5\text{ pts})\end{cases}$$

$$u_n(x,t)=G_n(x)H_n(t)\rightarrow u_n(x,t)=\sin(n\pi x)\exp(-n^2\pi^2t)\rightarrow u(x,t)=\sum_{n=1}^{\infty}a_nu_n(x,t)$$

$$u(x,0)=\sum_{n=1}^{\infty}a_nu_n(x,0)=\sum_{n=1}^{\infty}a_n\sin(n\pi x)=\sin(4\pi x)\rightarrow a_4=1,\ a_n=0\text{ for all }n\neq 4$$

$$u(x,t)=\sin(4\pi x)\exp(-16\pi^2t)\quad(5\text{ pts})$$

5.

(1) (5 pts each)

Round-ff error: ①Common arithmetic operations (+,-,*,/), ②Large computations, ③Adding a large and a small number, ④Subtractive cancellation, ⑤Smearing, ⑥Inner products

Truncation error: Taylor series approximation

(2) (5 pts each) a 로 설명해야 함

$$\frac{dy}{dt}=-ay\rightarrow\begin{cases}\text{explicit (forward): }\frac{y_{i+1}-y_i}{\Delta t}=-ay_i\rightarrow y_{i+1}=(1-a\Delta t)y_i\rightarrow\text{stability: }|1-a\Delta t|\leq 1\\ \text{implicit (backward): }\frac{y_{i+1}-y_i}{\Delta t}=-ay_{i+1}\rightarrow y_{i+1}=\frac{1}{1+a\Delta t}y_i\rightarrow\text{unconditionally stable}\end{cases}$$

6.

(1) $\omega_S + \frac{Z_R}{Z_S}\omega_R - \frac{Z_R + Z_S}{Z_S}\omega_C = 0 \rightarrow GR_{TM} = \frac{\omega_S}{\omega_C} = \frac{Z_R + Z_S}{Z_S} = \frac{80+40}{40} = 3$ (2 pts)

$J_{vehicle} = m_{body} R_{tire}^2 = 1500 \times 0.3^2 = 135 \text{ kg}\bullet\text{m}^2 s$ (1 pt)

$J_{eq} = \left(J_{eng} \times GR_{TM}^2 + J_{motor}\right) \times GR_F^2 + J_{vehicle} = \left(0.2 \times 3^2 + 0.1\right) \times 4^2 + 135 = 165.4\,\text{kg}\bullet\text{m}^2$ (2 pts)

(2) $T_{whl} = (T_{eng} \times GR_{TM} + T_{mot}) \times GR_F = \left(50 \times 3 + 80\right) \times 4 = 920\,\text{Nm}$ (2 pts)

$v = 36 \text{ km/h} = \frac{36}{3.6} = 10\,\text{m/s}$, (1 pt) $F_{res} = \frac{1}{2} C_d A_{front} \rho_{air} v^2 + \mu_{roll} m_{body} g = 183 \text{ N}$ (2 pts)

$\alpha_{whl} = \frac{T_{whl} - F_{res} R_{tire}}{J_{eq}} = \frac{920 - 183 \times 0.3}{165.4} = 5.23 \text{ rad/s}^2$, $\therefore\ a_{veh} = \alpha_{whl} R_{itre} = 1.57\,\text{m/s}^2$ (3 pts)

(3) $v = 36 \text{ km/h} = 10\,\text{m/s}$, $\omega_{mot} = \frac{v}{R_{tire}} GR = \frac{10}{0.3} \times 4 = 133.33 \text{ rad/s}$ (1 pt)

$T_{mot}\omega_{mot} = 80 \times 133.33 = 10,667\text{ W}$ (1 pt)

$\eta_{motor} V_{bat} I_{bat} = T_{mot}\omega_{mot} \leftarrow \left(V_{bat} = V_{OCV} - R_{in} I_{bat} = 350 - 0.1 I_{bat}\right)$

$\rightarrow\ 0.09 I_{bat}^2 - 315 I_{bat} + 10667 = 0 \rightarrow\ I_{bat} = 34.2 \text{ A}$ (3 pts)

$\therefore\ SOC = SOC_{ini} - \frac{100}{C_{nom}} \int_0^{5\,\text{min}} I_{bat}\, dt = 50 - \frac{100}{50,000} \times 34.2 \times 300 = 29.5\,\%$ (2 pts)

(4) $\omega_{eng} = \frac{v}{R_{tire}} \times GR_{TM} \times GR_F = \frac{10}{0.3} \times 3 \times 4 = 400 \text{ rad/s}$ (1 pts)

At $T_{eng} = 50 \text{ Nm},\ \omega_{eng} = 400\frac{30}{\pi} = 3820\,\text{RPM} \rightarrow\ \text{BSFC} = 320 \text{ g/kWh}$ (2 pts)

Fuel rate: $\frac{320\,\text{g}}{\text{kWh}} \times \frac{50 \times 400}{1000} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1\,\text{L}}{860\,\text{g}} = 0.00207\,\text{L/s}$ (2 pts)

Fuel consumption: $\int_0^{5\,\text{min}} 0.00207\,\text{L/s}\ dt = 0.62\,\text{L}$ (1 pts)

$\therefore\ \text{FE} = \frac{10\,\text{m/s} \times 300\,\text{s}}{0.62\,\text{L}} \times \frac{1\,\text{m}}{1000\,\text{km}} = 4.84\,\text{km/L}$ (2 pts)

(5) $1\,\text{km/L} = \frac{\text{km}}{1.609}\frac{3.785}{\text{L}} = 2.352\,\text{MPG}$ $\quad\therefore\ \text{FE} = 4.84\,\text{km/L} \times 2.35\frac{\text{MPG}}{\text{km/L}} = 11.4\,\text{MPG}$ (2 pts)