27.4 The second-order ODE can be expressed as the following pair of first-order ODEs,

$$\frac{dy}{dx} = z$$

$$\frac{dz}{dx} = \frac{2z + y - x}{7}$$

These can be solved for two guesses for the initial condition of z. For our cases we used −1 and −0.5. We solved the ODEs with the Heun method without iteration using a step size of 0.125. The results are

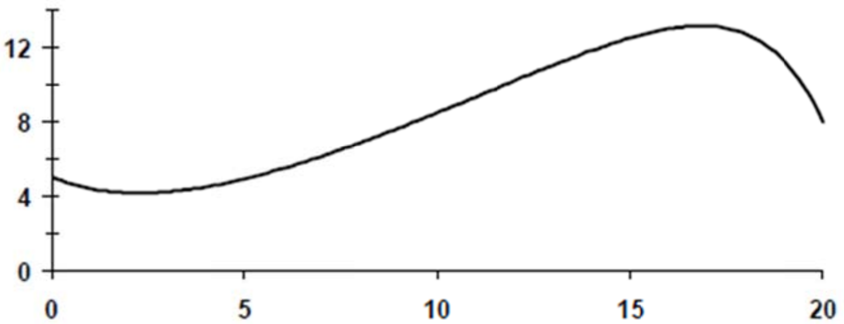
$z(0)$	−1	−0.5
$y(20)$	−11,837.64486	22,712.34615

Clearly, the solution is quite sensitive to the initial conditions. These values can then be used to derive the correct initial condition,

$$z(0) = -1 + \frac{-0.5+1}{22712.34615 - (-11837.64486)}(8 - (-11837.64486)) = -0.82857239$$

The resulting fit is displayed below:

x	y
0	5
2	4.151601
4	4.461229
6	5.456047
8	6.852243
10	8.471474
12	10.17813
14	11.80277
16	12.97942
18	12.69896
20	8



27.5 Centered finite differences can be substituted for the second and first derivatives to give,

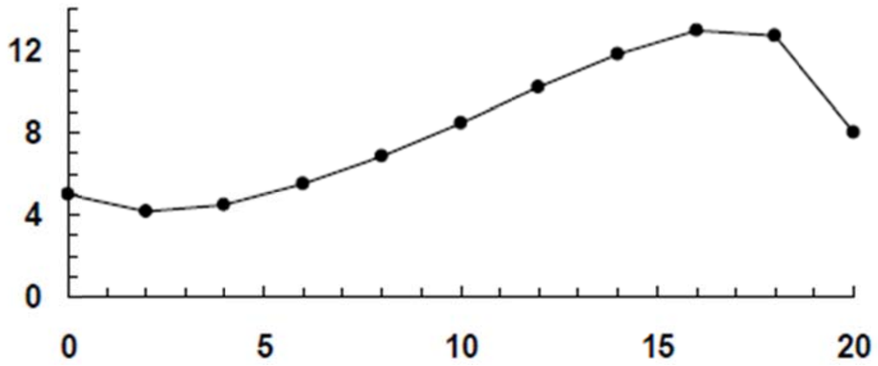
$$7\frac{y_{i+1}-2y_i+y_{i-1}}{\Delta x^2}-2\frac{y_{i+1}-y_{i-1}}{2\Delta x}-y_i+x_i=0$$

or substituting $\Delta x = 2$ and collecting terms yields

$$-2.25y_{i-1}+4.5y_i-1.25y_{i+1}=x_i$$

This equation can be written for each node and solved with methods such as the Tridiagonal solver, the Gauss-Seidel method or LU Decomposition. The following solution was computed using Excel's Minverse and Mmult functions:

x	*y*
0	5
2	4.199592
4	4.518531
6	5.507445
8	6.893447
10	8.503007
12	10.20262
14	11.82402
16	13.00176
18	12.7231
20	8



27.27 (a) The exact solution is

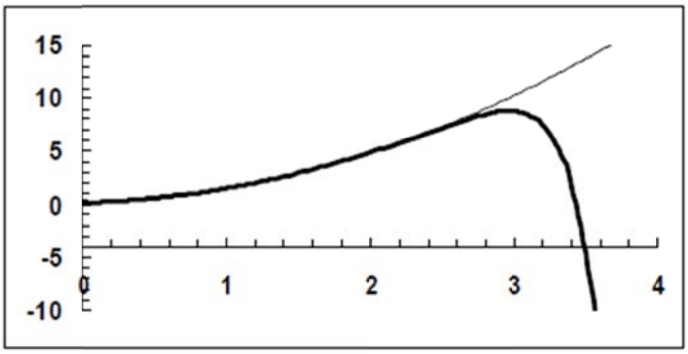
$$y = Ae^{5t} + t^2 + 0.4t + 0.08$$

If the initial condition at $t = 0$ is 0.8, $A = 0$,

$$y = t^2 + 0.4t + 0.08$$

Note that even though the choice of the initial condition removes the positive exponential terms, it still lurks in the background. Very tiny round off errors in the numerical solutions bring it to the fore. Hence all of the following solutions eventually diverge from the analytical solution.

(b) 4th order RK. The plot shows the numerical solution (bold line) along with the exact solution (fine line).



(c)

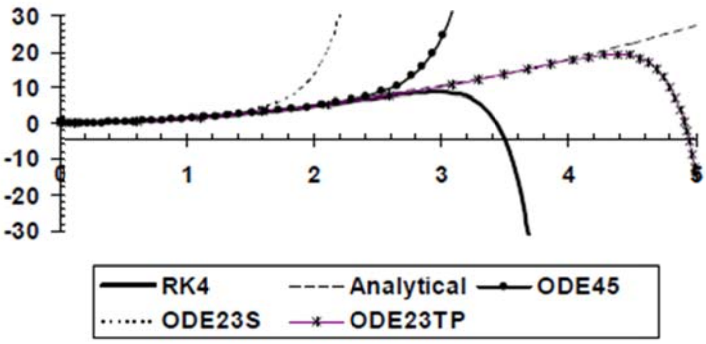
```
function yp=dy(t,y)
yp=5*(y-t^2);
>> tspan=[0,5];
>> y0=0.08;
>> [t,y]=ode45('dy1',tspan,y0);
```

(d)

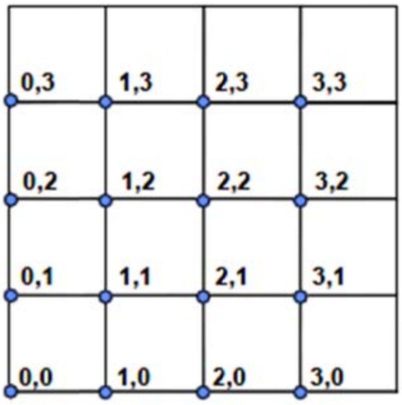
```
>> [t,y]=ode23S('dy1',tspan,y0);
```

(e)

```
>> [t,y]=ode23TB('dy1',tspan,y0);
```



29.8 The nodes to be simulated are



Simple Laplacians are used for all interior nodes. Balances for the edges must take insulation into account. For example, node 1,0 is modeled as

$$4T_{1,0} - T_{0,0} - T_{2,0} - 2T_{1,1} = 0$$

The corner node, 0,0 would be modeled as

$$4T_{0,0} - 2T_{1,0} - 2T_{0,1} = 0$$

The resulting set of equations can be solved for

0	25	50	75	100
23.89706	32.16912	45.58824	60.29412	75
31.25	34.19118	39.88971	45.58824	50
32.72059	33.45588	34.19118	32.16912	25
32.72059	32.72059	31.25	23.89706	0

The fluxes can be computed as

J_x

-1.225	-1.225	-1.225	-1.225	-1.225
-0.40533	-0.53143	-0.68906	-0.72059	-0.72059
-0.14412	-0.21167	-0.27923	-0.2477	-0.21618
-0.03603	-0.03603	0.031526	0.225184	0.351287
0	0.036029	0.216176	0.765625	1.170956

J_y

1.170956	0.351287	-0.21618	-0.72059	-1.225
0.765625	0.225184	-0.2477	-0.72059	-1.225
0.216176	0.031526	-0.27923	-0.68906	-1.225
0.036029	-0.03603	-0.21167	-0.53143	-1.225
0	-0.03603	-0.14412	-0.40533	-1.225

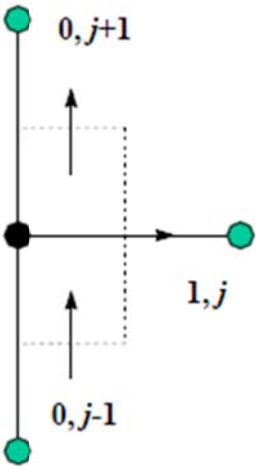
J_n

1.694628	1.274373	1.243928	1.421222	1.732412
0.866299	0.577174	0.732232	1.019066	1.421222
0.259812	0.214008	0.394888	0.732232	1.243928
0.050953	0.050953	0.214008	0.577174	1.274373
0	0.050953	0.259812	0.866299	1.694628

θ (degrees)

136.2922	163.999	-169.992	-149.534	-135
117.8973	157.0362	-160.228	-135	-120.466
123.6901	171.5289	-135	-109.772	-100.008
135	-135	-81.5289	-67.0362	-73.999
0	-45	-33.6901	-27.8973	-46.2922

29.11 The control volume is drawn as in



A flux balance around the node can be written as (note $\Delta x = \Delta y = h$)

$$-kh\Delta z \frac{T_{1,j} - T_{0,j}}{h} + k(h/2)\Delta z \frac{T_{0,j} - T_{0,j-1}}{h} - k(h/2)\Delta z \frac{T_{0,j+1} - T_{0,j}}{h} = 0$$

Collecting and canceling terms gives

$$4T_{0,j} - T_{0,j-1} - T_{0,j+1} - 2T_{1,j} = 0$$