27.4 The second-order ODE can be expressed as the following pair of first-order ODEs,

$$\frac{dy}{dx} = z$$
$$\frac{dz}{dx} = \frac{2z + y - x}{7}$$

These can be solved for two guesses for the initial condition of z. For our cases we used −1 and −0.5. We solved the ODEs with the Heun method without iteration using a step size of 0.125. The results are

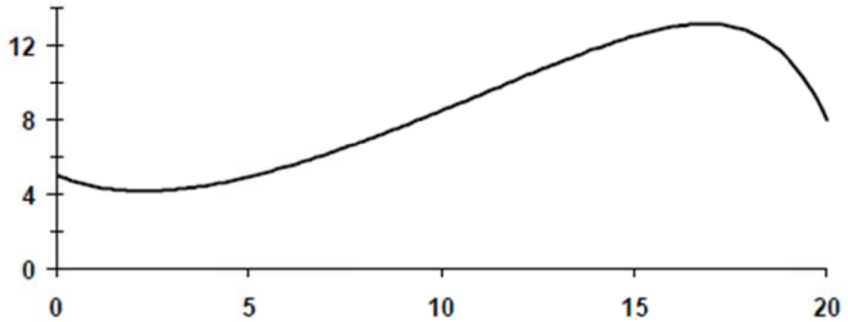
$z(0)$	−1	−0.5
$y(20)$	−11,837.64486	22,712.34615

Clearly, the solution is quite sensitive to the initial conditions. These values can then be used to derive the correct initial condition,

$$z(0) = -1 + \frac{-0.5 + 1}{22712.34615 - (-11837.64486)}(8 - (-11837.64486)) = -0.82857239$$

The resulting fit is displayed below:

x	*y*
0	5
2	4.151601
4	4.461229
6	5.456047
8	6.852243
10	8.471474
12	10.17813
14	11.80277
16	12.97942
18	12.69896
20	8



27.5 Centered finite differences can be substituted for the second and first derivatives to give,

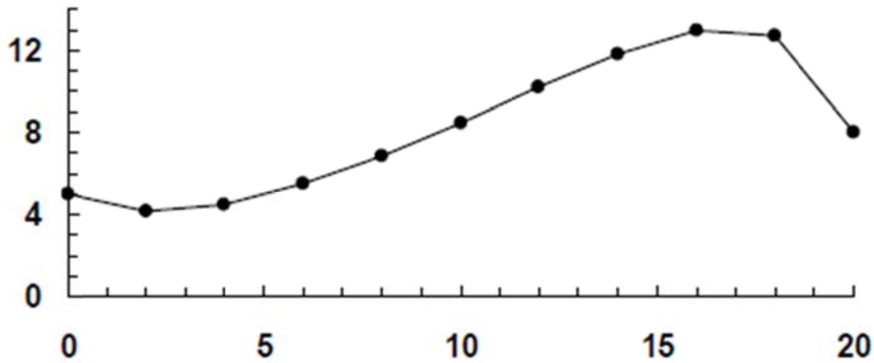
$$7\frac{y_{i+1}-2y_i+y_{i-1}}{\Delta x^2}-2\frac{y_{i+1}-y_{i-1}}{2\Delta x}-y_i+x_i=0$$

or substituting $\Delta x = 2$ and collecting terms yields

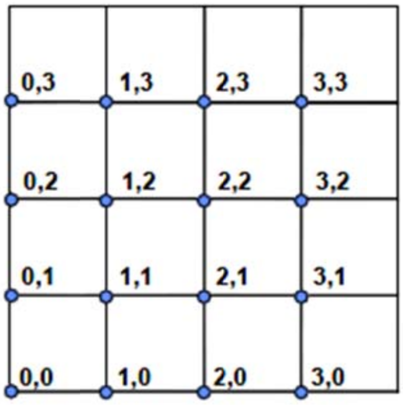
$$-2.25y_{i-1}+4.5y_i-1.25y_{i+1}=x_i$$

This equation can be written for each node and solved with methods such as the Tridiagonal solver, the Gauss-Seidel method or LU Decomposition. The following solution was computed using Excel's Minverse and Mmult functions:

x	*y*
0	5
2	4.199592
4	4.518531
6	5.507445
8	6.893447
10	8.503007
12	10.20262
14	11.82402
16	13.00176
18	12.7231
20	8



29.8 The nodes to be simulated are



Simple Laplacians are used for all interior nodes. Balances for the edges must take insulation into account. For example, node 1,0 is modeled as

$$4T_{1,0} - T_{0,0} - T_{2,0} - 2T_{1,1} = 0$$

The corner node, 0,0 would be modeled as

$$4T_{0,0} - 2T_{1,0} - 2T_{0,1} = 0$$

The resulting set of equations can be solved for

0	25	50	75	100
23.89706	32.16912	45.58824	60.29412	75
31.25	34.19118	39.88971	45.58824	50
32.72059	33.45588	34.19118	32.16912	25
32.72059	32.72059	31.25	23.89706	0

The fluxes can be computed as

J_x

-1.225	-1.225	-1.225	-1.225	-1.225
-0.40533	-0.53143	-0.68906	-0.72059	-0.72059
-0.14412	-0.21167	-0.27923	-0.2477	-0.21618
-0.03603	-0.03603	0.031526	0.225184	0.351287
0	0.036029	0.216176	0.765625	1.170956

J_y

1.170956	0.351287	-0.21618	-0.72059	-1.225
0.765625	0.225184	-0.2477	-0.72059	-1.225
0.216176	0.031526	-0.27923	-0.68906	-1.225
0.036029	-0.03603	-0.21167	-0.53143	-1.225
0	-0.03603	-0.14412	-0.40533	-1.225

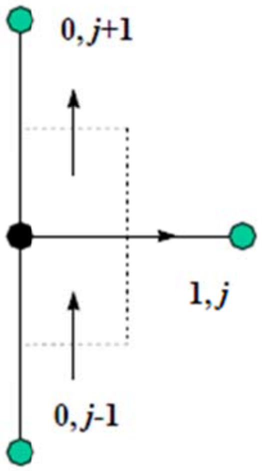
J_n

1.694628	1.274373	1.243928	1.421222	1.732412
0.866299	0.577174	0.732232	1.019066	1.421222
0.259812	0.214008	0.394888	0.732232	1.243928
0.050953	0.050953	0.214008	0.577174	1.274373
0	0.050953	0.259812	0.866299	1.694628

θ (degrees)

136.2922	163.999	-169.992	-149.534	-135
117.8973	157.0362	-160.228	-135	-120.466
123.6901	171.5289	-135	-109.772	-100.008
135	-135	-81.5289	-67.0362	-73.999
0	-45	-33.6901	-27.8973	-46.2922

29.11 The control volume is drawn as in



A flux balance around the node can be written as (note $\Delta x = \Delta y = h$)

$$-kh\Delta z\frac{T_{1,j}-T_{0,j}}{h}+k(h/2)\Delta z\frac{T_{0,j}-T_{0,j-1}}{h}-k(h/2)\Delta z\frac{T_{0,j+1}-T_{0,j}}{h}=0$$

Collecting and canceling terms gives

$$4T_{0,j}-T_{0,j-1}-T_{0,j+1}-2T_{1,j}=0$$

30.2 Because we now have derivative boundary conditions, the boundary nodes must be simulated. For node 0,

$$T_0^{l+1} = T_0^l + \lambda(T_1^l - 2T_0^l + T_{-1}^l) \qquad (i)$$

This introduces an exterior node into the solution at $i = -1$. The derivative boundary condition can be used to eliminate this node,

$$\left.\frac{dT}{dx}\right|_0 = \frac{T_1 - T_{-1}}{2\Delta x}$$

which can be solved for

$$T_{-1} = T_1 - 2\Delta x \frac{dT_0}{dx}$$

which can be substituted into Eq. (*i*) to give

$$T_0^{l+1} = T_0^l + \lambda\left(2T_1^l - 2T_0^l - 2\Delta x \frac{dT_0^l}{dx}\right)$$

For our case, $dT_0/dx = 1$ and $\Delta x = 2$, and therefore $T_{-1} = T_1 - 4$. This can be substituted into Eq. (*i*) to give,

$$T_0^{l+1} = T_0^l + \lambda(2T_1^l - 2T_0^l - 4)$$

A similar analysis can be used to embed the zero derivative in the equation for the n^{th} node,

$$T_n^{l+1} = T_n^l + \lambda(T_{n+1}^l - 2T_n^l + T_{n-1}^l) \qquad (ii)$$

This introduces an exterior node into the solution at $n + 1$. The derivative boundary condition can be used to eliminate this node,

$$\left.\frac{dT}{dx}\right|_n = \frac{T_{n+1} - T_{n-1}}{2\Delta x}$$

which can be solved for

$$T_{n+1} = T_{n-1} + 2\Delta x \frac{dT_n}{dx}$$

which can be substituted into Eq. (*ii*) to give

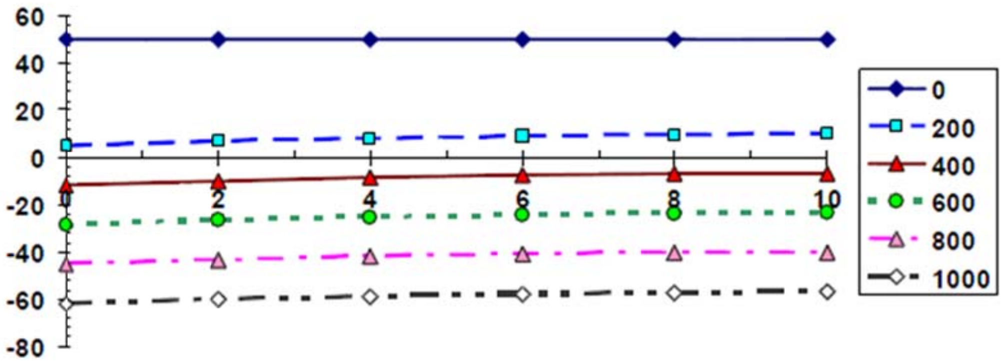
$$T_n^{l+1} = T_n^l + \lambda\left(2T_{n-1}^l - 2T_n^l + 2\Delta x \frac{dT_n}{dx}\right)$$

For our case, $n = 5$ and $dT_n/dx = 0$, and therefore

$$T_5^{l+1} = T_5^l + \lambda(2T_4^l - 2T_5^l)$$

Together with the equations for the interior nodes, the entire system can be solved with a step of 0.1 s. The results for some of the early steps along with some later selected values are tabulated below. In addition, a plot of the later results is also shown

t	*x* = 0	*x* = 2	*x* = 4	*x* = 6	*x* = 8	*x* = 10
0	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
0.1	49.9165	50.0000	50.0000	50.0000	50.0000	50.0000
0.2	49.8365	49.9983	50.0000	50.0000	50.0000	50.0000
0.3	49.7597	49.9949	50.0000	50.0000	50.0000	50.0000
0.4	49.6861	49.9901	49.9999	50.0000	50.0000	50.0000
0.5	49.6153	49.9840	49.9997	50.0000	50.0000	50.0000
•						
•						
•						
200	5.000081	6.800074	8.200059	9.200048	9.800042	10.00004
400	-11.6988	-9.89883	-8.49883	-7.49882	-6.89881	-6.69881
600	-28.4008	-26.6008	-25.2008	-24.2008	-23.6007	-23.4007
800	-45.1056	-43.3056	-41.9056	-40.9056	-40.3056	-40.1056
1000	-61.8104	-60.0104	-58.6104	-57.6104	-57.0104	-56.8104



Notice what's happening. The rod never reaches a steady state, because of the heat loss at the left end (unit gradient) and the insulated condition (zero gradient) at the right.

30.16 We will solve this problem with the simple explicit method. Therefore, the interior nodes are handled in a standard fashion as

$$T_i^{l+1} = T_i^l + \lambda(T_{i+1}^l - 2T_i^l + T_{i-1}^l)$$

For the nth-node, the insulated condition can be developed by writing the balance as

$$T_n^{l+1} = T_n^l + \lambda(2T_{n-1}^l - 2T_n^l)$$

Finally, the convective boundary condition at the first node ($i = 0$) can be represented by first writing the general balance as

$$T_0^{l+1} = T_0^l + \lambda(T_1^l - 2T_0^l + T_{-1}^l) \qquad (i)$$

This introduces an exterior node into the solution at $i = -1$. The boundary condition can be used to eliminate this node. To do this, a finite difference representation of the condition can be written as

$$-k'\frac{T_1 - T_{-1}}{2\Delta x} = h\left(T_a - T_0\right)$$

which can be solved for

$$T_{-1} = T_1 + \frac{2\Delta x h}{k'}\left(T_a - T_0\right)$$

which can be substituted into Eq. (i) to give

$$T_0^{l+1} = T_0^l + \lambda\left(2T_1^l - 2T_0^l + \frac{2h\Delta x}{k'}(T_a - T_0^l)\right)$$

The entire system can be solved with a step of 1 s. A plot of the results is shown below. After sufficient time, the rod will approach a uniform temperature of 50°C.