30.2 Because we now have derivative boundary conditions, the boundary nodes must be simulated. For node 0,

$$T_0^{l+1} = T_0^l + \lambda(T_1^l - 2T_0^l + T_{-1}^l) \qquad (i)$$

This introduces an exterior node into the solution at $i = -1$. The derivative boundary condition can be used to eliminate this node,

$$\left.\frac{dT}{dx}\right|_0 = \frac{T_1 - T_{-1}}{2\Delta x}$$

which can be solved for

$$T_{-1} = T_1 - 2\Delta x \frac{dT_0}{dx}$$

which can be substituted into Eq. (*i*) to give

$$T_0^{l+1} = T_0^l + \lambda\left(2T_1^l - 2T_0^l - 2\Delta x \frac{dT_0^l}{dx}\right)$$

For our case, $dT_0/dx = 1$ and $\Delta x = 2$, and therefore $T_{-1} = T_1 - 4$. This can be substituted into Eq. (*i*) to give,

$$T_0^{l+1} = T_0^l + \lambda(2T_1^l - 2T_0^l - 4)$$

A similar analysis can be used to embed the zero derivative in the equation for the n^{th} node,

$$T_n^{l+1} = T_n^l + \lambda(T_{n+1}^l - 2T_n^l + T_{n-1}^l) \qquad (ii)$$

This introduces an exterior node into the solution at $n + 1$. The derivative boundary condition can be used to eliminate this node,

$$\left.\frac{dT}{dx}\right|_n = \frac{T_{n+1} - T_{n-1}}{2\Delta x}$$

which can be solved for

$$T_{n+1} = T_{n-1} + 2\Delta x \frac{dT_n}{dx}$$

which can be substituted into Eq. (*ii*) to give

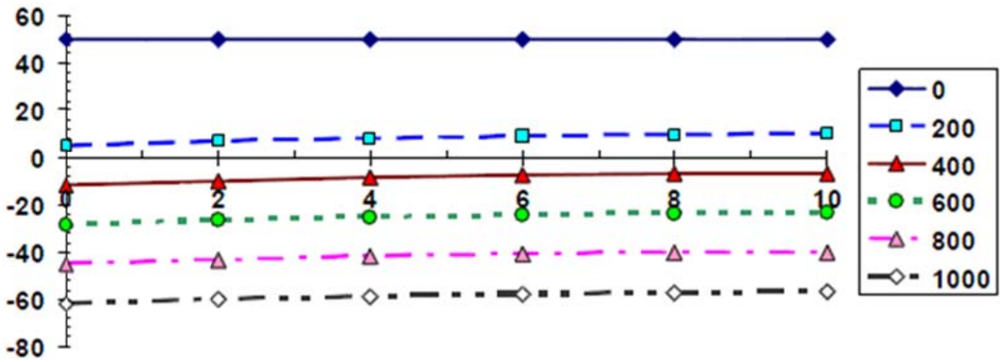
$$T_n^{l+1} = T_n^l + \lambda\left(2T_{n-1}^l - 2T_n^l + 2\Delta x \frac{dT_n}{dx}\right)$$

For our case, $n = 5$ and $dT_n/dx = 0$, and therefore

$$T_5^{l+1} = T_5^l + \lambda(2T_4^l - 2T_5^l)$$

Together with the equations for the interior nodes, the entire system can be solved with a step of 0.1 s. The results for some of the early steps along with some later selected values are tabulated below. In addition, a plot of the later results is also shown

t	*x* = 0	*x* = 2	*x* = 4	*x* = 6	*x* = 8	*x* = 10
0	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000
0.1	49.9165	50.0000	50.0000	50.0000	50.0000	50.0000
0.2	49.8365	49.9983	50.0000	50.0000	50.0000	50.0000
0.3	49.7597	49.9949	50.0000	50.0000	50.0000	50.0000
0.4	49.6861	49.9901	49.9999	50.0000	50.0000	50.0000
0.5	49.6153	49.9840	49.9997	50.0000	50.0000	50.0000
•						
•						
•						
200	5.000081	6.800074	8.200059	9.200048	9.800042	10.00004
400	-11.6988	-9.89883	-8.49883	-7.49882	-6.89881	-6.69881
600	-28.4008	-26.6008	-25.2008	-24.2008	-23.6007	-23.4007
800	-45.1056	-43.3056	-41.9056	-40.9056	-40.3056	-40.1056
1000	-61.8104	-60.0104	-58.6104	-57.6104	-57.0104	-56.8104



Notice what's happening. The rod never reaches a steady state, because of the heat loss at the left end (unit gradient) and the insulated condition (zero gradient) at the right.

30.16 We will solve this problem with the simple explicit method. Therefore, the interior nodes are handled in a standard fashion as

$$T_i^{l+1} = T_i^l + \lambda(T_{i+1}^l - 2T_i^l + T_{i-1}^l)$$

For the *n*th-node, the insulated condition can be developed by writing the balance as

$$T_n^{l+1} = T_n^l + \lambda(2T_{n-1}^l - 2T_n^l)$$

Finally, the convective boundary condition at the first node ($i = 0$) can be represented by first writing the general balance as

$$T_0^{l+1} = T_0^l + \lambda(T_1^l - 2T_0^l + T_{-1}^l) \qquad (i)$$

This introduces an exterior node into the solution at $i = -1$. The boundary condition can be used to eliminate this node. To do this, a finite difference representation of the condition can be written as

$$-k'\frac{T_1 - T_{-1}}{2\Delta x} = h\left(T_a - T_0\right)$$

which can be solved for

$$T_{-1} = T_1 + \frac{2\Delta x h}{k'}\left(T_a - T_0\right)$$

which can be substituted into Eq. (i) to give

$$T_0^{l+1} = T_0^l + \lambda\left(2T_1^l - 2T_0^l + \frac{2h\Delta x}{k'}(T_a - T_0^l)\right)$$

The entire system can be solved with a step of 1 s. A plot of the results is shown below. After sufficient time, the rod will approach a uniform temperature of 50°C.