1. (1)

$$\left.\begin{array}{l} \text{design variables:}\begin{cases} b = \text{base of the container} \\ h = \text{height of the container} \end{cases} \\ \text{objective function:}\dfrac{\text{round-trip cost of shipping the container}}{\text{one-way cost of shipping the contents}} = \dfrac{2(18)(80)(2b^2+4bh)}{18(150)(b^2h)} \\ \text{constraints: } 0 \le b \le 10, 0 \le h \le 18 \end{array}\right\}$$

$$\rightarrow \begin{cases} \underset{b,h}{\text{Min}}\ f = \left(\dfrac{32}{15}\right)\left(\dfrac{1}{h}+\dfrac{2}{b}\right) \\ \text{subject to}\begin{cases} g_1 = -b \le 0 \\ g_2 = b-10 \le 0 \\ g_3 = -h \le 0 \\ g_4 = h-18 \le 0 \end{cases} \end{cases} \quad \text{(10 points)}$$

(2)

$$\begin{cases} \underset{x_1,x_2,x_3,x_4}{\text{maximize}} \quad 2x_1 + x_2 + 7x_3 + 4x_4 \ \text{(5 pts)} \\ \text{subject to} \ x_1 + x_2 + x_3 + x_4 = 26 \ \text{(3 pts)} \\ \qquad x_1, x_2, x_3, x_4 \ge 0 \ \text{(2 pts)} \end{cases}$$

(3)

resource 1 per day: $x_A + 0.5x_B$

resource 2 per day: $0.2x_A + 0.5x_B$

total cost of resource 1 and 2 per day

$$C = (x_A + 0.5x_B)[0.375 - 0.00005(x_A + 0.5x_B)] + (0.2x_A + 0.5x_B)[0.75 - 0.0001(0.2x_A + 0.5x_B)]$$

return per day from the sale of products A and B

$$R = x_A(2.00 - 0.0005x_A - 0.00015x_B) + x_B(3.50 - 0.0002x_A - 0.0015x_B)$$

$$\begin{cases} \underset{x_A,x_B}{\text{maximize}} \quad \text{profit } P = R - C \ \text{(7 pts)} \\ \text{subject to} \left.\begin{array}{l} x_A + 0.5x_B \le 1000 \ [\text{requirement of resource 1 per day}] \\ 0.2x_A + 0.5x_B \le 250 \ [\text{requirement of resource 2 per day}] \end{array}\right\} \text{(2 pts)} \\ \qquad x_A, x_B \ge 0 \ \text{(1 pts)} \end{cases}$$

(4)

$$\text{From F.B.D.}\begin{cases} T_3 = T_4 = \dfrac{W_2}{2} \le 100 \\ T_1 + T_2 - W_1 - T_3 = 0 \rightarrow T_1 = \dfrac{1}{2}W_1 + \dfrac{1}{8}W_2 \le 120 \\ 8W_1 + 12T_3 - 16T_2 = 0 \rightarrow T_2 = \dfrac{1}{2}W_1 + \dfrac{3}{8}W_2 \le 160 \end{cases}$$

Minimize $f = W_1 + W_2$ (3 pts)

$$\text{subject to}\quad \left.\begin{matrix} W_2 \le 200 \ \ \text{(Cables 3 and 4)} \\ 4W_1 + 3W_2 \le 1280 \ \ \text{(Cable 2} \\ 4W_1 + W_2 \le 960 \ \ \text{(Cable 1)} \end{matrix}\right\} \ (5 \text{ pts})$$

$W_1 \ge 0,\ W_2 \ge 0$ (2 pts)

(5)

$$x_{ij} = \begin{cases} 1 & \text{if city } j \text{ is visited after city } i \\ 0 & \text{otherwise} \end{cases}$$

Find $\{x_{ij}\}$ to minimize $f = \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij}$ (5 pts)

$$\text{subject to}\quad \left.\begin{matrix} \sum_{i=1}^{n} x_{ij} = 1 \ \ (i = 1,\ldots,n),\ i \ne j \\ \sum_{j=1}^{n} x_{ij} = 1 \ \ (j = 1,\ldots,n),\ j \ne i \end{matrix}\right\} \ (5 \text{ pts})$$

2. 변수 5, 목적함수 5, 최적조건 5, 최적해 5

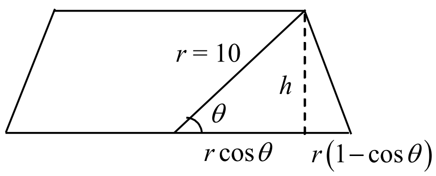
$h = r\sin\theta$

$$f = (2r\cos\theta)h + hr(1-\cos\theta) = hr(1+\cos\theta) = r^2\sin\theta(1+\cos\theta)$$

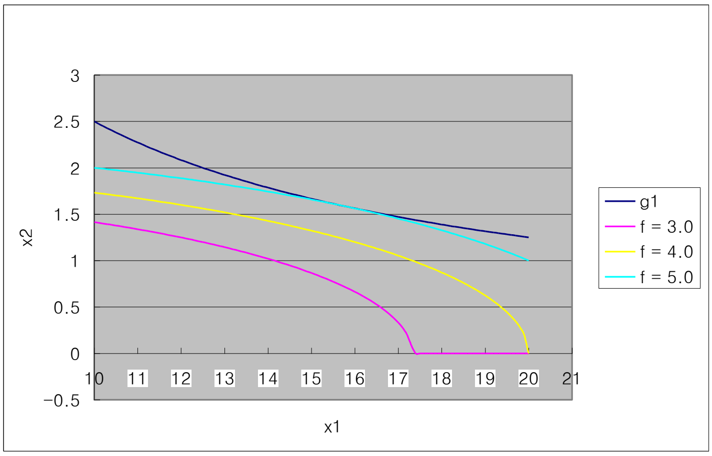
$$\frac{df}{d\theta} = \cos\theta(1+\cos\theta) - \sin^2\theta = 2\cos^2\theta + \cos\theta - 1 = (\cos\theta+1)(2\cos\theta-1) = 0$$

$$\xrightarrow{0^\circ \le \theta \le 90^\circ} 2\cos\theta - 1 = 0 \rightarrow \cos\theta = \frac{1}{2} \rightarrow \theta^* = 60^\circ, h^* = 5\sqrt{3} \approx 8.66, f^* = 75\sqrt{3} \approx 129.9$$

$$\left.\frac{d^2 f}{d\theta^2}\right|_{\theta^*} = -4\cos\theta\sin\theta - \sin\theta - 1 < 0 \rightarrow \text{maximum}$$



3. (1) graphical solution



(2) solutions using KKT conditions

$$L = 0.01x_1^2 + x_2^2 + \mu_1(25 - x_1x_2) + \mu_2(2 - x_1) - \mu_3 x_1 - \mu_4 x_2$$

$$optimality: \begin{cases} \dfrac{\partial L}{\partial x_1} = 0.02x_1 - \mu_1 x_2 - \mu_2 - \mu_3 = 0 \\ \dfrac{\partial L}{\partial x_2} = 2x_2 - \mu_1 x_1 - \mu_4 = 0 \end{cases}$$

$$switching: \mu_1(25 - x_1x_2) = 0, \mu_2(2 - x_1) = 0, \mu_3 x_1 = 0, \mu_4 x_2 = 0$$

$$nonnegativity: \mu_i \ge 0$$

$$feasibility: 25 - x_1x_2 \le 0, 2 - x_1 \le 0, x_1 \ge 0, x_2 \ge 0$$

$\rightarrow 2^4 = 16$ cases, try only feasbile cases, graphical illustration gives insights

$\rightarrow \mu_1 > 0, \mu_2 = \mu_3 = \mu_4 = 0$

$$\left.\begin{array}{l} 0.02x_1 - \mu_1 x_2 = 0 \\ 2x_2 - \mu_1 x_1 = 0 \\ 25 - x_1 x_2 = 0 \end{array}\right\} \rightarrow x_1 = 15.81\left(= 5\sqrt{10}\right), x_2 = \frac{25}{x_1} = 1.58\left(= \frac{\sqrt{10}}{2}\right), \mu_1 = 0.2, f = 5.0$$

(3) sufficient condition

$$\nabla^2 L = \begin{bmatrix} 0.02 & -0.2 \\ -0.2 & 2 \end{bmatrix} \rightarrow \text{positive defintie on the tangent subspace?}$$

$$\nabla g_1^T \mathbf{y} = \mathbf{0} \rightarrow \begin{bmatrix} -1.581 & -15.81 \end{bmatrix} \mathbf{y} = \mathbf{0} \rightarrow \mathbf{y} = c\begin{bmatrix} 10 \\ -1 \end{bmatrix}$$

$$\mathbf{y}^T \nabla^2 L \mathbf{y} = c^2 \begin{bmatrix} 10 & -1 \end{bmatrix} \begin{bmatrix} 0.02 & -0.2 \\ -0.2 & 2 \end{bmatrix} \begin{bmatrix} 10 \\ -1 \end{bmatrix} = 8c^2 > 0 \rightarrow \text{strictly minimum}$$

(4) $26 - x_1x_2 \le 0 \rightarrow 25 - x_1x_2 \le -1 \rightarrow f^{new} \approx f^* - (0.2)(-1) = 5.2$

exact solution: $x_1 = 16.12, x_2 = 1.61, f = 5.19$