Profit Maximization Problem (1)

Step 1: Project/problem description. A company manufactures two machines, A and B. Using available resources, either 28 A or 14 B can be manufactured daily. The sales department can sell up to 14 A machines or 24 B machines. The shipping facility can handle no more than 16 machines per day. The company makes a profit of $400 on each A machine and $600 on each B machine. How many A and B machines should the company manufacture every day to maximize its profit?

Step 2: Data and information collection. Data and information are defined in the project statement. No additional information is needed.

Step 3: Definition of design variables. The following two design variables are identified in the problem statement:

x_1 = number of A machines manufactured each day
x_2 = number of B machines manufactured each day

Step 4: Optimization criterion. The objective is to maximize daily profit, which can be expressed in terms of design variables using the data given in step 1 as

$$P = 400x_1 + 600x_2, \$ \tag{a}$$

Step 5: Formulation of constraints. Design constraints are placed on manufacturing capacity, on sales personnel, and on the shipping and handling facility. The constraint on the shipping and handling facility is quite straightforward:

$$x_1 + x_2 \le 16 \text{ (shipping and handling constraint)} \tag{b}$$

Profit Maximization Problem (2)

Constraints on manufacturing and sales facilities are a bit tricky because they are either "this" or "that" type of requirements. First, consider the manufacturing limitation. It is assumed that if the company is manufacturing x_1 A machines per day, then the remaining resources and equipment can be proportionately used to manufacture x_2 B machines, and vice versa. Therefore, noting that $x_1/28$ is the fraction of resources used to produce A and $x_2/14$ is the fraction used to produce B, the constraint is expressed as

$$\frac{x_1}{28} + \frac{x_2}{14} \le 1 \text{ (manufacturing constraint)} \qquad \text{(c)}$$

Similarly, the constraint on sales department resources is given as

$$\frac{x_1}{14} + \frac{x_2}{24} \le 1 \text{ (limitation on sale department)} \qquad \text{(d)}$$

Finally, the design variables must be nonnegative as

$$x_1,\ x_2 \ge 0 \qquad \text{(e)}$$

Graphical Solutions (1)

Profit Maximization Problem

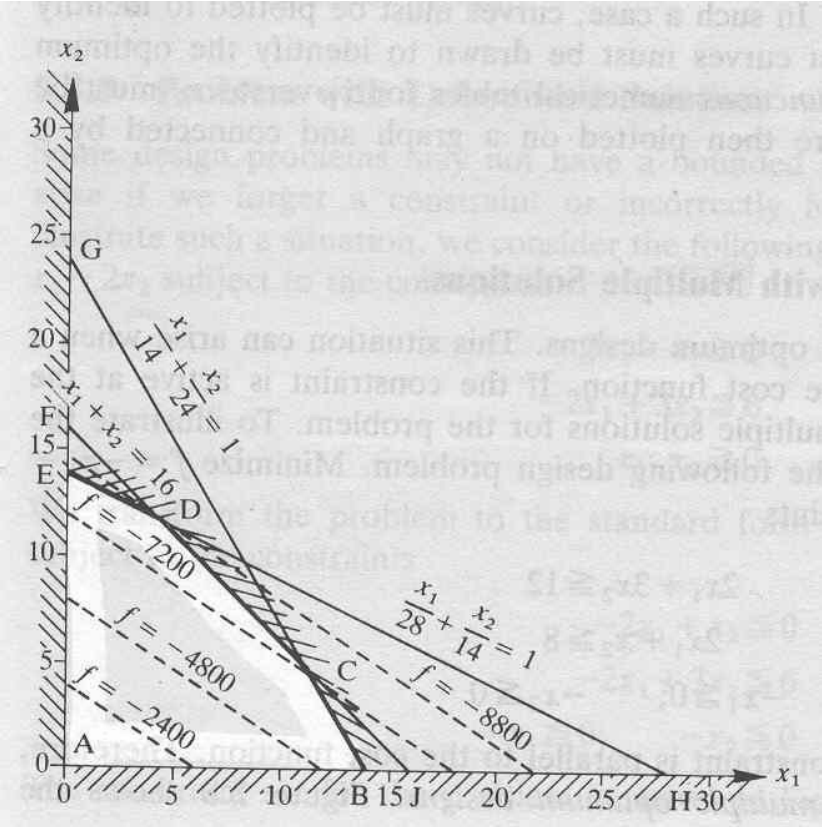
$$\underset{x_1, x_2}{\text{Maximize}} \quad f = 400x_1 + 600x_2$$

subject to

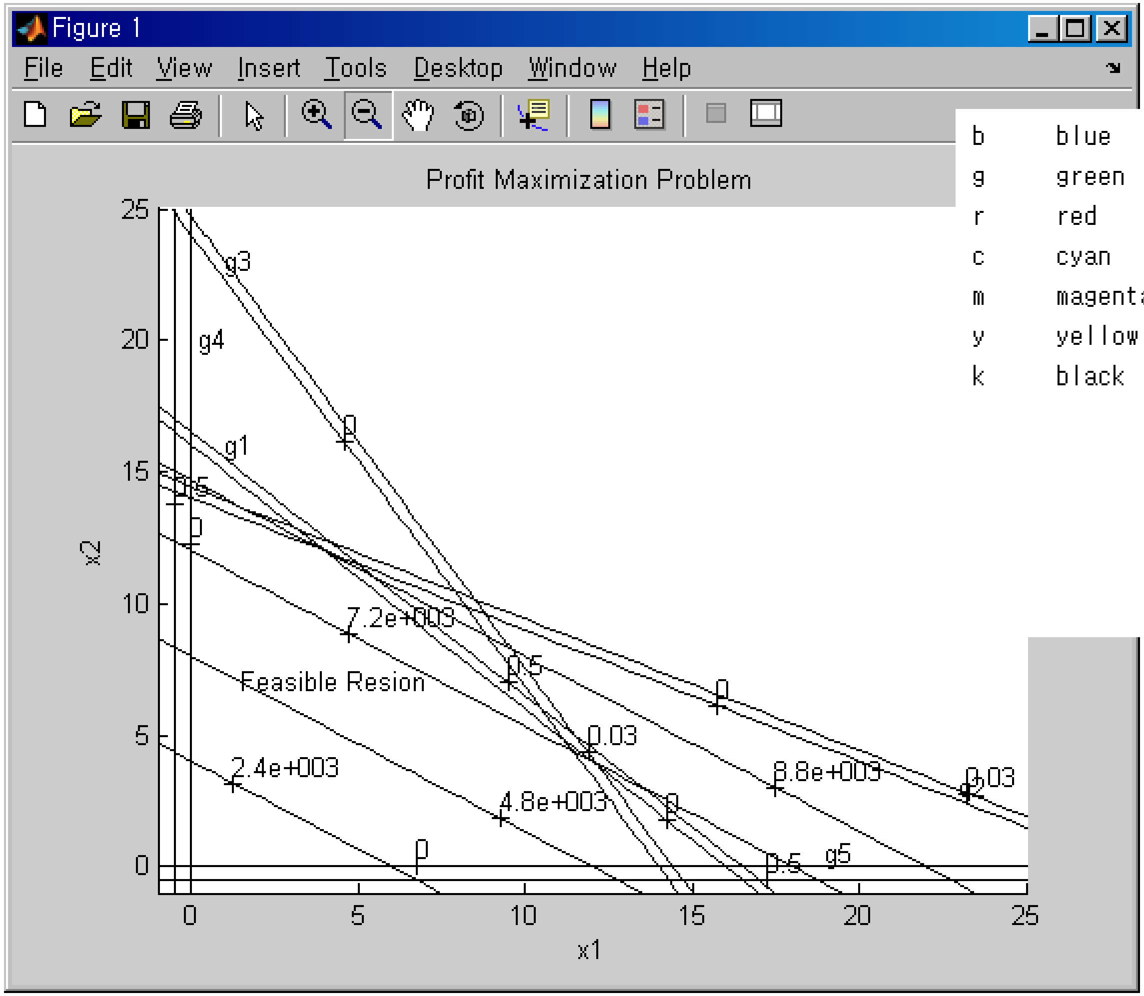
$$\begin{cases} x_1 + x_2 \le 16 & \text{(shipping and handling)} \\ \dfrac{x_1}{28} + \dfrac{x_2}{14} \le 1 & \text{(manufacturing)} \\ \dfrac{x_1}{14} + \dfrac{x_2}{24} \le 1 & \text{(limitations on sales dept.)} \\ x_1, x_2 \ge 0 \end{cases}$$

x_1 = # of A machines manufactured each day

x_2 = # of B machines manufactured each day



Graphical Solutions (2)



b	blue	.	point	-	solid
g	green	o	circle	:	dotted
r	red	x	x-mark	-.	dashdot
c	cyan	+	plus	--	dashed
m	magenta	*	star	(none)	no line
y	yellow	s	square		
k	black	d	diamond		
		v	triangle (down)		
		^	triangle (up)		
		<	triangle (left)		
		>	triangle (right)		
		p	pentagram		
		h	hexagram		

Minimum Weight Tubular Column Design

$$\underset{R,t}{\text{Minimize}}\ f = 2\rho l \pi R t = \left(2.4608\times10^5\right)Rt$$

subject to

$$\begin{cases} \sigma = \dfrac{P}{A} = \dfrac{P}{2\pi Rt} \le \sigma_a \rightarrow g_1 = \dfrac{10\times10^6}{2\pi Rt} - 248\times10^6 \le 0 \\ P \le \dfrac{\pi^2 EI}{4l^2} = \dfrac{\pi^3 ER^3 t}{4l^2} \rightarrow g_2 = 10\times10^6 - \dfrac{\pi^3\left(207\times10^9\right)R^3 t}{4(5)^2} \le 0 \\ R,t \ge 0 \rightarrow \begin{cases} g_3 = -R \le 0 \\ g_4 = -t \le 0 \end{cases} \end{cases}$$

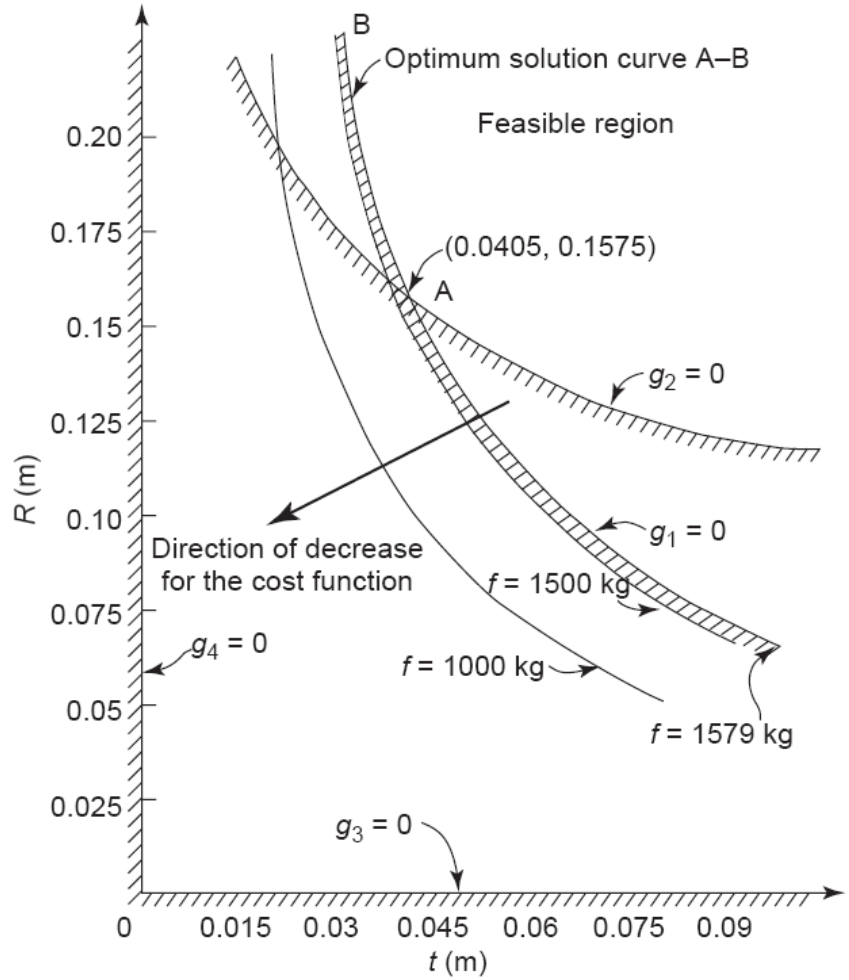
$P = 10MN$

$E = 207GPa$

$\rho = 7833\,kg/m^3$

$l = 5.0m$

$\sigma_a = 248MPa$



* cost function contours run parallel to the stress constraint g1

Beam Design Problem (1)

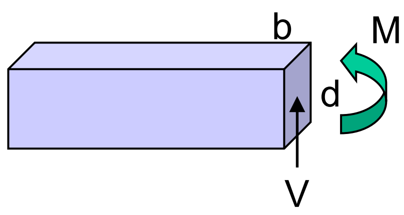
Step 1: Project/problem description. A beam of rectangular cross-section is subjected to a bending moment M (N·m) and a maximum shear force V (N). The bending stress in the beam is calculated as $\sigma = 6M/bd^2$ (Pa), and average shear stress is calculated as $\tau = 3V/2bd$ (Pa), where b is the width and d is the depth of the beam. The allowable stresses in bending and shear are 10 and 2 MPa, respectively. It is also desirable that the depth of the beam does not exceed twice its width and that the cross-sectional area of the beam is minimized. In this section, we formulate and solve the problem using the graphical method.

Step 2: Data and information collection. Let bending moment M = 40 kN·m and the shear force V = 150 kN. All other data and necessary equations are given in the project statement. We shall formulate the problem using a consistent set of units, N and mm.

Step 3: Definition of design variables. The two design variables are

d = depth of beam, mm
b = width of beam, mm

Beam Design Problem (2)



$$\underset{b,d}{\text{Minimize}} \; f = bd$$

subject to

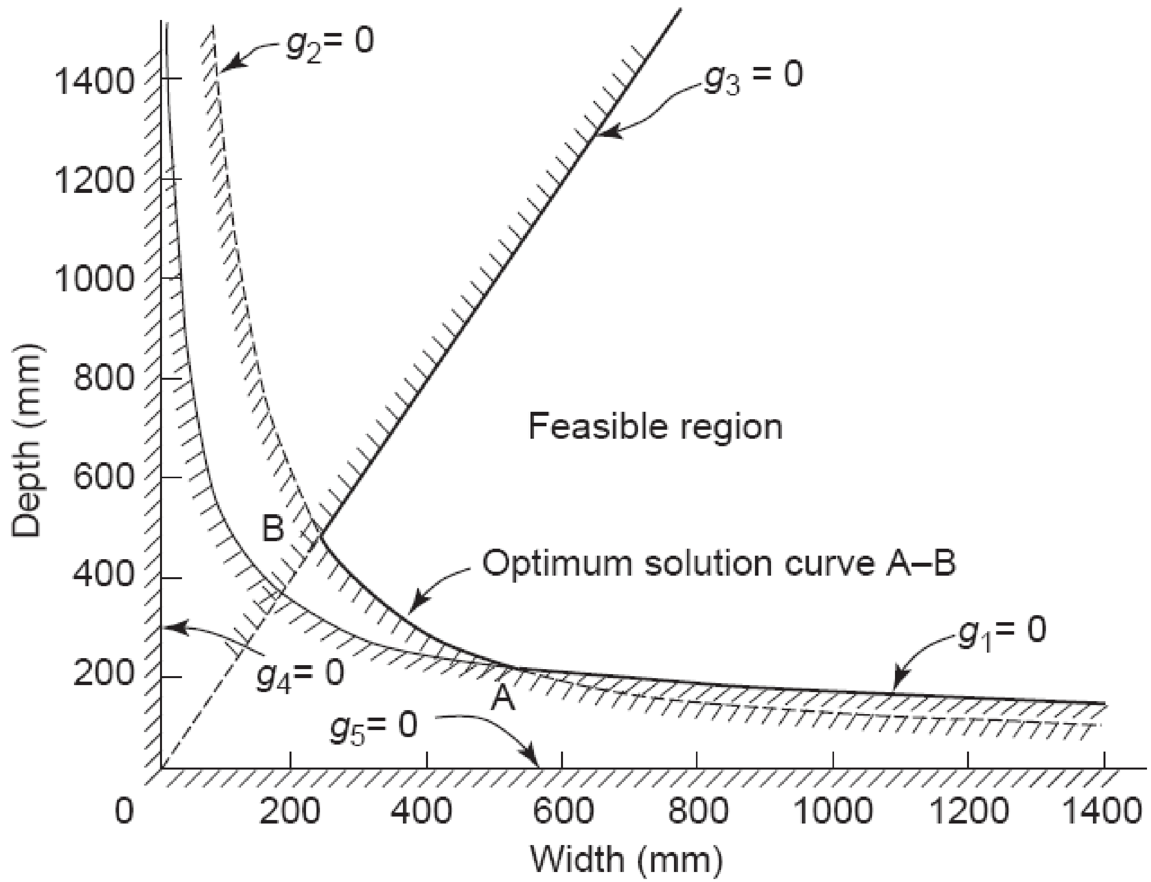
$$\begin{cases} \sigma = \dfrac{6M}{bd^2} \le (\sigma_a)_{bending} \\ \tau = \dfrac{3V}{2bd} \le (\tau_a)_{shear} \\ d \le 2b \\ b, d \ge 0 \end{cases}$$

$$M = 40kN \cdot m$$

$$V = 150kN$$

$$(\sigma_a)_{bending} = 10MPa$$

$$(\tau_a)_{shear} = 2MPa$$

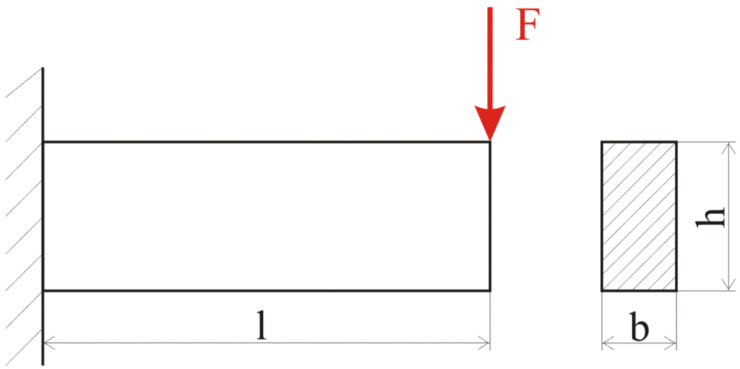


* cost function is parallel to the stress constraint g2

$$\left.\begin{aligned} b^* &= 237mm,\; d^* = 474mm \text{ @ point B} \\ b^* &= 527.3mm,\; d^* = 213.3mm \text{ @ point A} \end{aligned}\right\} \rightarrow f^* = 115{,}000mm^2$$

Beam Design Problem (3)

- Cantilever beam loaded with force F=2400 N. Minimize weight such that stresses do not exceed yield. Further the height h should not be larger than twice the width b.



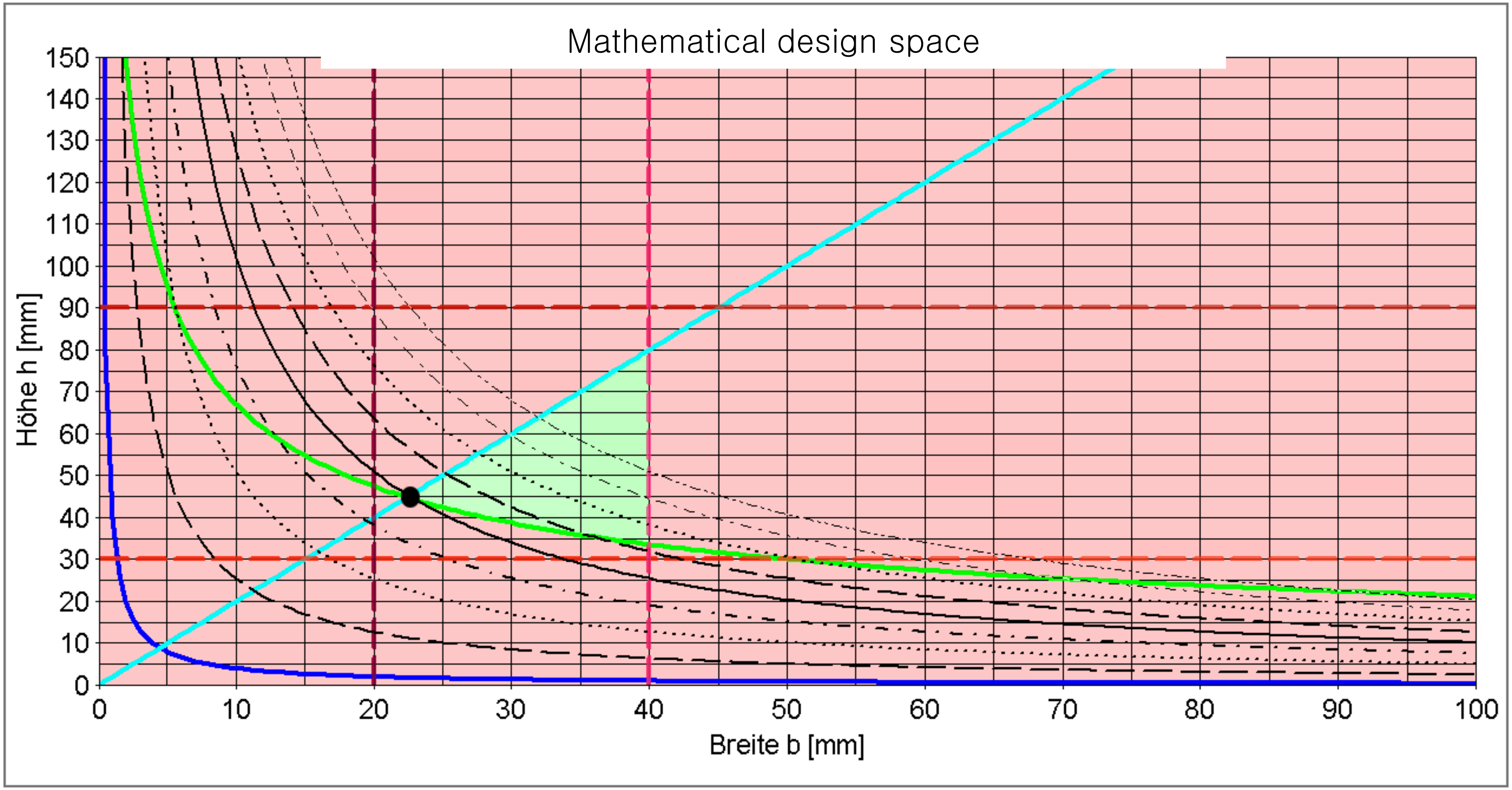
- Objective
  - Weight: Min $m(b,h)$
- Design variables
  - Width: $b^L \leq b \leq b^U$, $20 \leq b \leq 40$
  - Height: $h^L \leq h \leq h^U$, $30 \leq h \leq 90$
- Design constraints:

$$\sigma(b,h) \leq \sigma_{max}, \text{ with } \sigma_{max} = 160 \text{ MPa}$$

$$\tau(b,h) \leq \tau_{max}, \text{ with } \tau_{max} = 60 \text{ MPa}$$

$$h \leq 2 * b$$

Graphical Solution

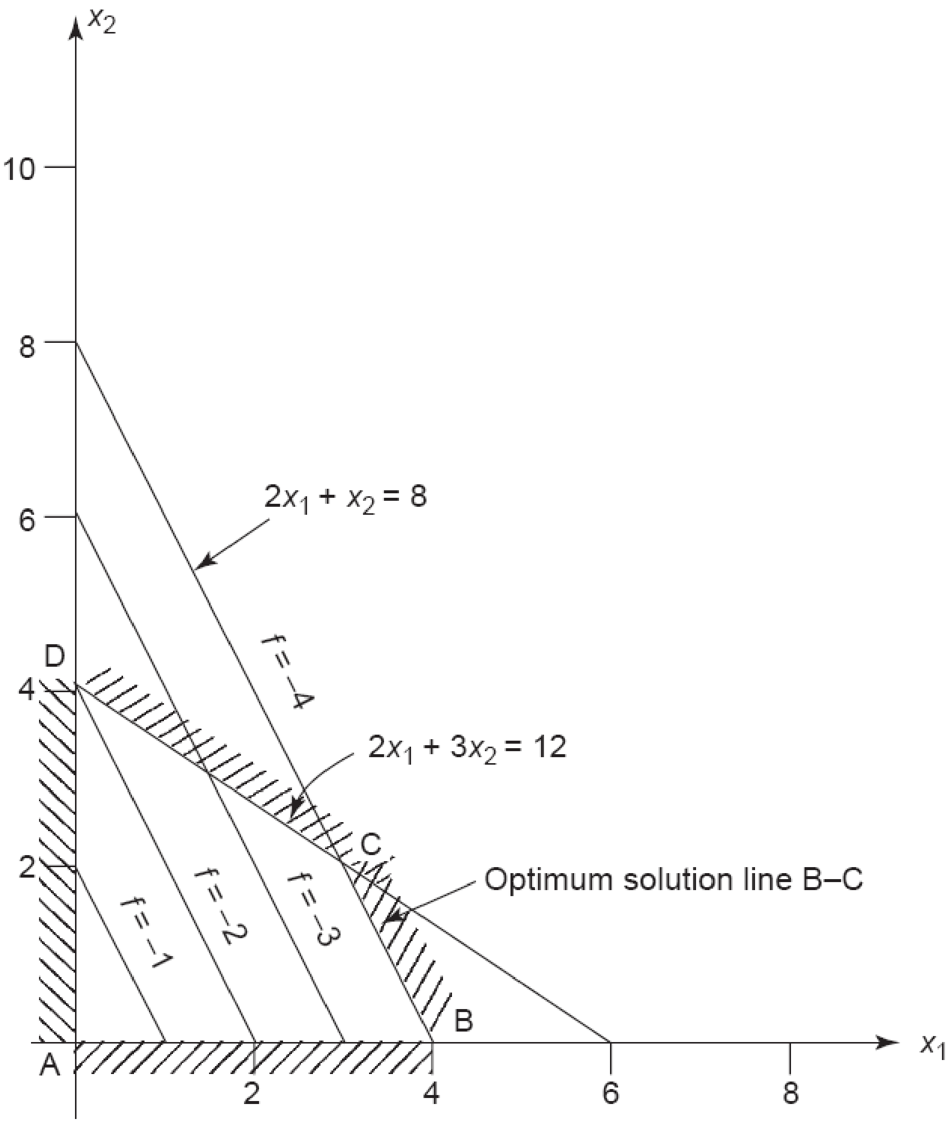


Multiple Solutions

Minimize $f(\mathbf{x}) = -x_1 - 0.5x_2$

subject to

$$\begin{cases} 2x_1 + 3x_2 \leq 12 \\ 2x_1 + x_2 \leq 8 \\ -x_1 \leq 0 \\ -x_2 \leq 0 \end{cases}$$



Unbounded Solutions

Maximize $f(\mathbf{x}) = x_1 - 2x_2$

subject to

$$\begin{cases} 2x_1 - x_2 \geq 0 \\ -2x_1 + 3x_2 \leq 6 \\ x_1, x_2 \geq 0 \end{cases}$$

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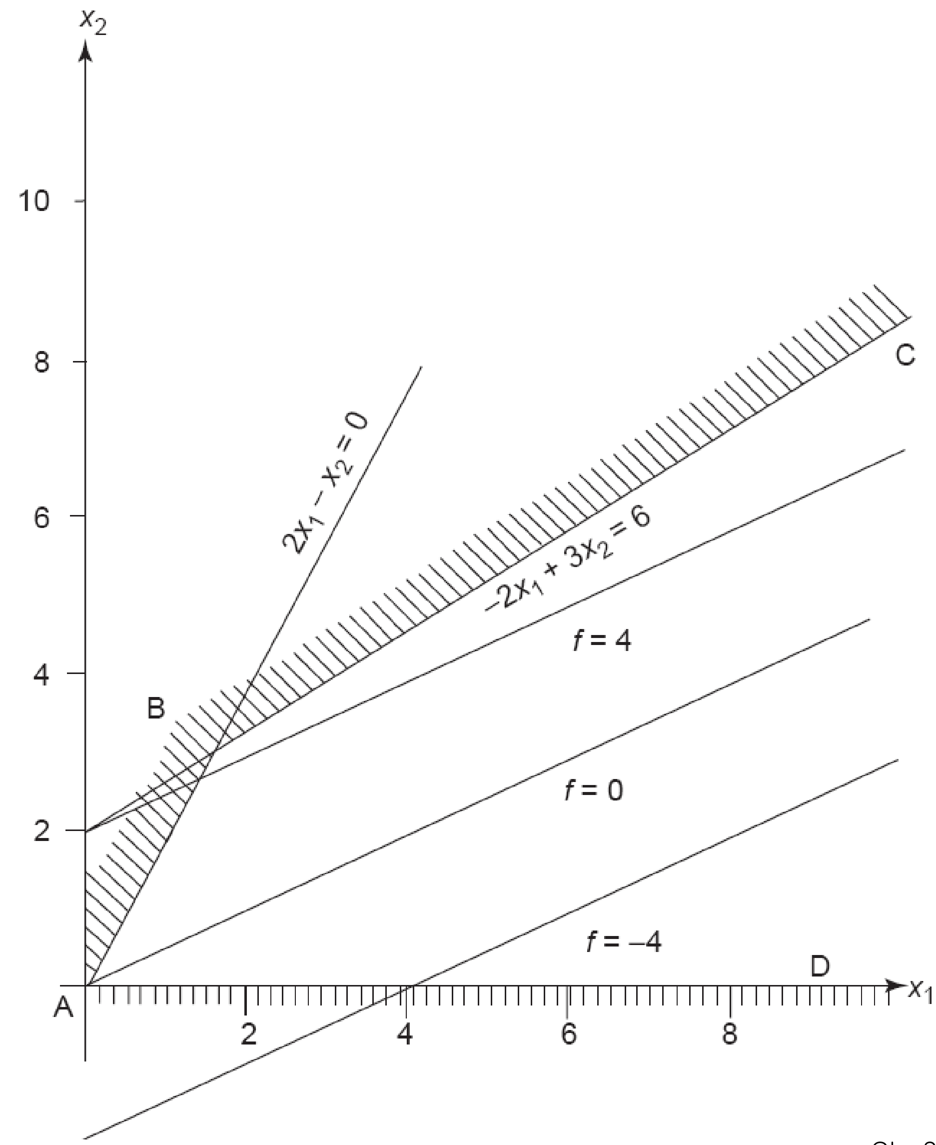
Minimize $f(\mathbf{x}) = -x_1 + 2x_2$

subject to

$$\begin{cases} -2x_1 + x_2 \leq 0 \\ -2x_1 + 3x_2 \leq 6 \\ -x_1 \leq 0 \\ -x_2 \leq 0 \end{cases}$$



Infeasible Problem

- Too many constraints

$$\text{Minimize } f(\mathbf{x}) = x_1 + 2x_2$$

subject to

$$\begin{cases} 3x_1 + 2x_2 \le 6 \\ 2x_1 + 3x_2 \ge 12 \\ x_1 \le 5 \\ x_2 \le 5 \\ x_1, x_2 \ge 0 \end{cases}$$