목 차

- 구조최적설계
- 위상최적설계
  - 정식화
  - 사례: 자동차 분야

구조최적설계: 개념

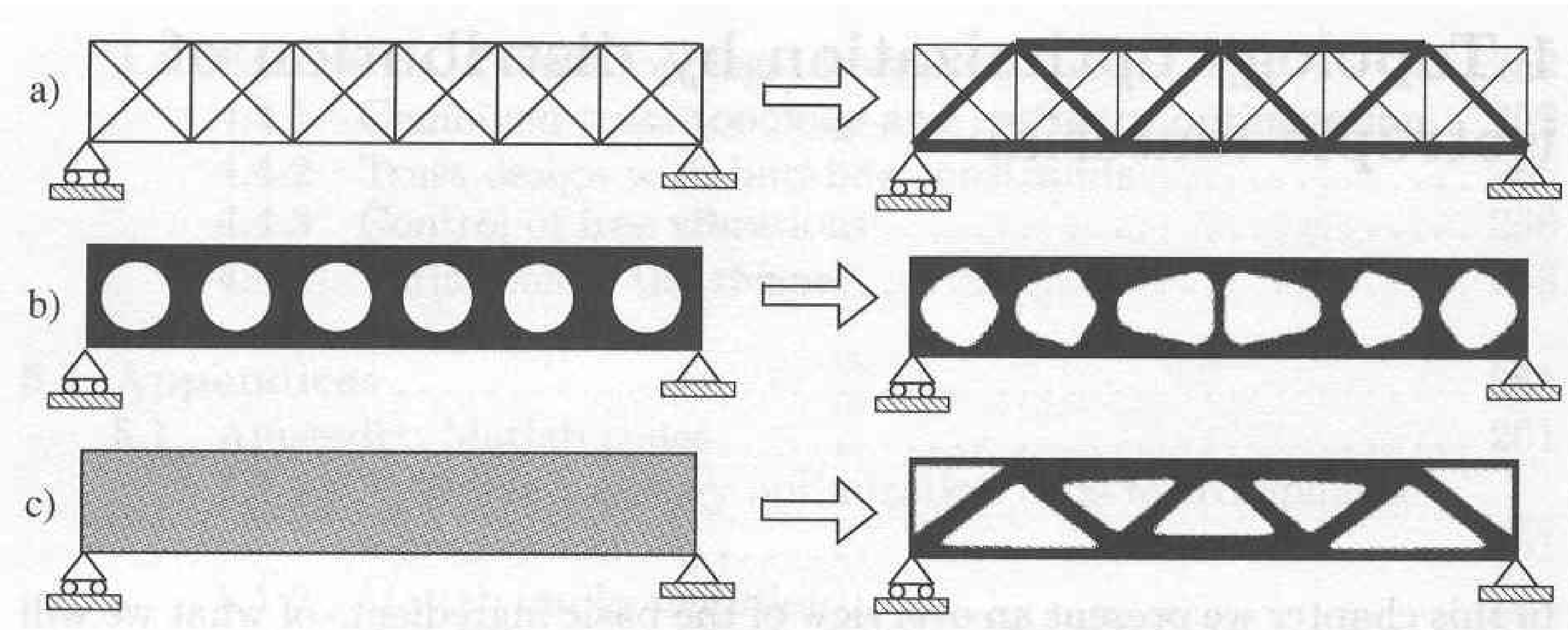
- 설계변수 (d)
  - 부재크기 (두께, 단면적, 길이), 경계 (절점/조절점 좌표)
- 상태변수 (U)
  - 중량, 응력, 변위, 온도, 고유진동수, 좌굴하중
  - 목적함수, 제약조건

상태방정식
지배방정식

- 결정사항
  - 해석 종류: 상/편 미분방정식
  - 설계 공간 (설계변수)
  - 목적함수 및 제약함수 (구조 거동, 구조 기하)

구조최적설계: 설계변수

- **치수 (Size)**
  - 부재크기
  - 두께, 단면적, 길이
- **형상 (Shape)**
  - 경계
  - 절점/조절점 좌표
- **위상 (Topology)**
  - 재료 유/무
  - 요소밀도



M.P.Bendsøe and O. Sigmund, Topology Optimization: Theory, Methods and Applications, Springer, 2003

구조최적설계: 정식화

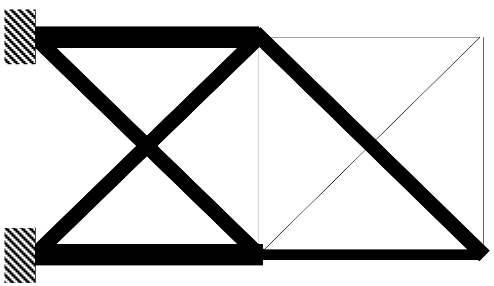
- 설계변수 :
- 목적함수 : 부피(중량) 최소화
- 제약조건 : 상태방정식(해석), 최대응력, 최대변위

$$\min_{d} \int_{\Omega} \rho \, d\Omega$$

$$\text{subject to} \begin{cases} a(\boldsymbol{u},\boldsymbol{v}) = f(\boldsymbol{v}) \, \forall \boldsymbol{v} \\ \bar{\sigma} \le \sigma_{\max} \\ |\boldsymbol{u}| \le u_{\max} \end{cases}$$

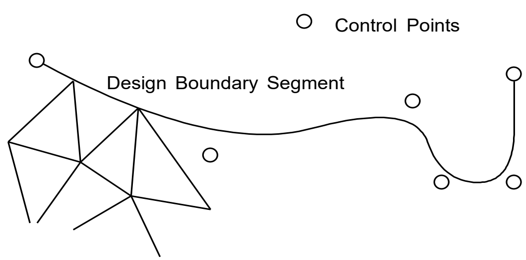
치수최적설계

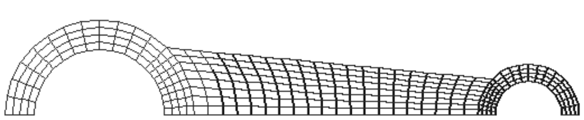
- 항공/토목 구조물: 트러스, 빔, 프레임
- 기계구조물: 형상과 관련



형상최적설계

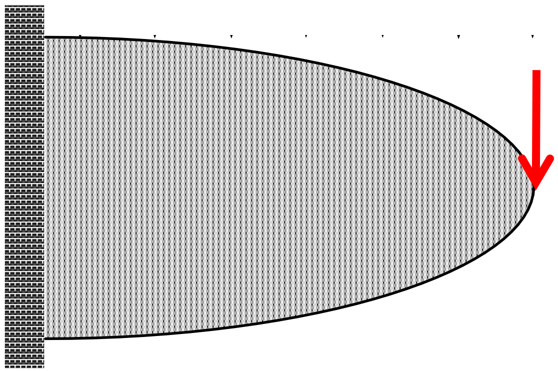
- 파라메트릭 기하 표현: 자동 요소망 생성 필요
- 기저형상 활용

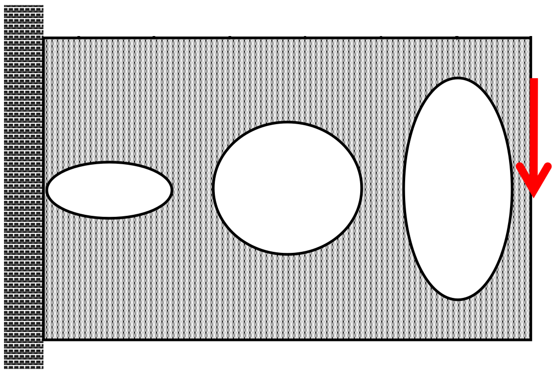




구조물 경량화

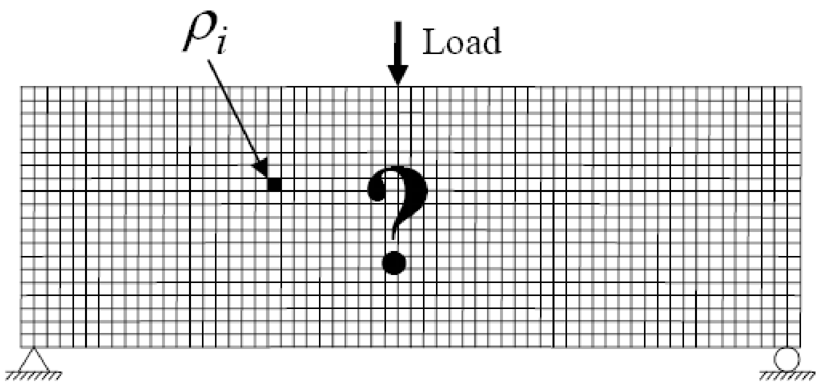
- 치수(size) 및 형상(shape) 최적설계
  - 초기설계에 크게 의존
  - 최적설계에 의한 경량화 효과 적음 (우수한 초기설계인 경우)
- 효과적인 구조물 경량화: 구조물 내 구멍 생성
  - 구멍의 위치/크기/형상: 패러다임 변화 필요
  - 위상(topology)최적설계 제안

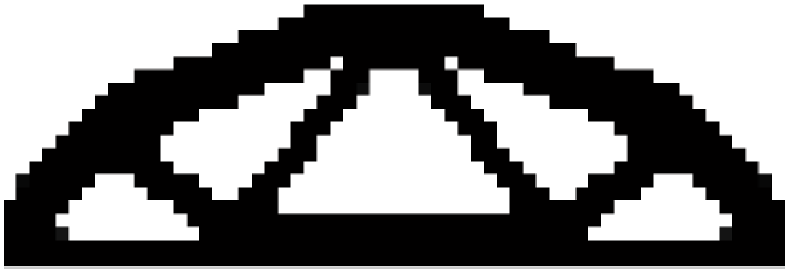




새로운 패러다임: 위상최적설계

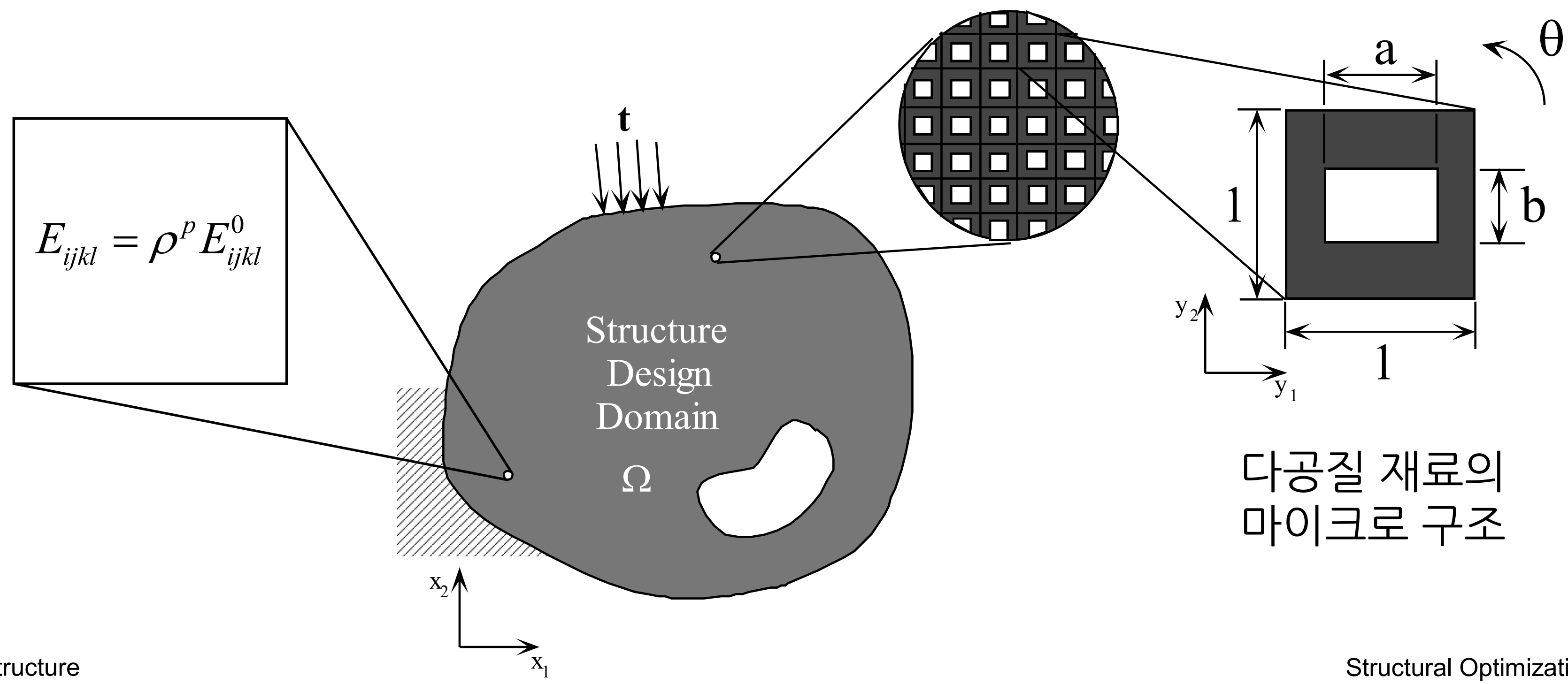
- 형상최적설계의 문제점
  - 형상변화에 따른 해석모델 재생성
  - 설계변화계산의 어려움, 제한적 형상변화
- 아이디어?
  - 해석모델 고정: 구조최적설계방법이 유한요소생성과 별개
  - 형상을 유한요소의 밀도로 표현(pixel, voxel 개념)





위상최적설계: 개념

- 설계변수
  - 구조물의 재료분포(탄성계수)를 표현하는 값
  - 균질화설계법: 다공질구조로 모델링, 복합재역학이론으로 균질화된 물성 계산
  - 밀도법: 유한요소의 밀도로 물성계산



다공질 재료의
마이크로 구조

위상최적설계: 정적 문제

- 설계변수 (ρ)
  - 이산화한 각 유한요소의 밀도
- 문제 정식화
  - 부피 최소화, 응력/변위 제약조건: 주어진 강도/강성을 만족하는 경량화 설계
  - 응력: 트러스, 빔, 프레임 구조물에서는 유한값, 연속체에서는 국부적 물리량
  - 전역적 물리량 (평균컴플라이언스) 도입: 강성 표현

$$\min_{\rho} \quad \int_{\Omega} \rho \, d\Omega \rightarrow \sum_{e=1}^{N} v_e \rho_e$$

$$\text{subject to} \begin{cases} a(\boldsymbol{u},\boldsymbol{v}) = f(\boldsymbol{v}) \, \forall \boldsymbol{v} \rightarrow \boldsymbol{K}(\rho)\boldsymbol{U} = \boldsymbol{F} \\ \left.\begin{array}{l} \bar{\sigma} \le \sigma_{\max} \\ |\boldsymbol{u}| \le u_{\max} \end{array}\right\} \rightarrow \boldsymbol{U}^T \boldsymbol{F} \le l_{\max} \end{cases}$$

$\Rightarrow$

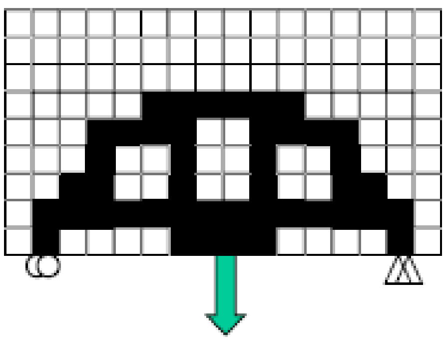
$$\min_{\rho} \quad \boldsymbol{U}^T \boldsymbol{F}$$

$$\text{subject to} \begin{cases} \boldsymbol{K}(\rho)\boldsymbol{U} = \boldsymbol{F} \\ \sum_{e=1}^{N} v_e \rho_e \le V^* \\ 0 < \rho_{\min} \le \rho_e \le 1 \end{cases}$$

위상최적설계: 밀도법

정수(0/1) 문제

- 모든 조합? Ill-conditioned

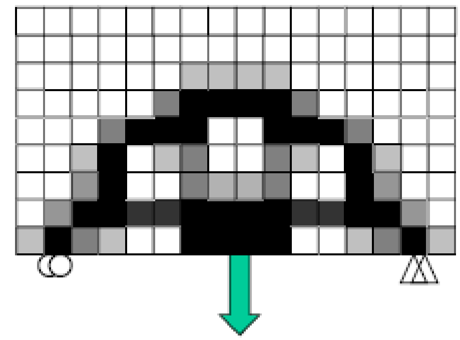


$$\min_{\rho} \quad \boldsymbol{U}^T \boldsymbol{F}$$

$$\text{subject to} \quad \begin{cases} \boldsymbol{K}(\rho)\boldsymbol{U} = \boldsymbol{F} \\ \sum_{e=1}^{N} v_e \rho_e \le V^* \\ 0 < \rho_{\min} \le \rho_e \le 1 \end{cases}$$

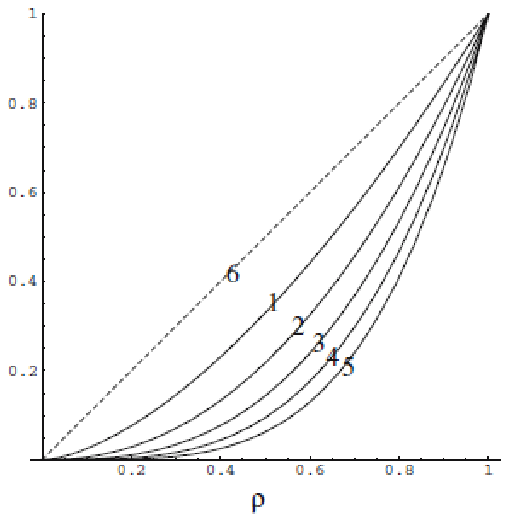
실수(0~1) 문제

- 완화, 중간밀도?

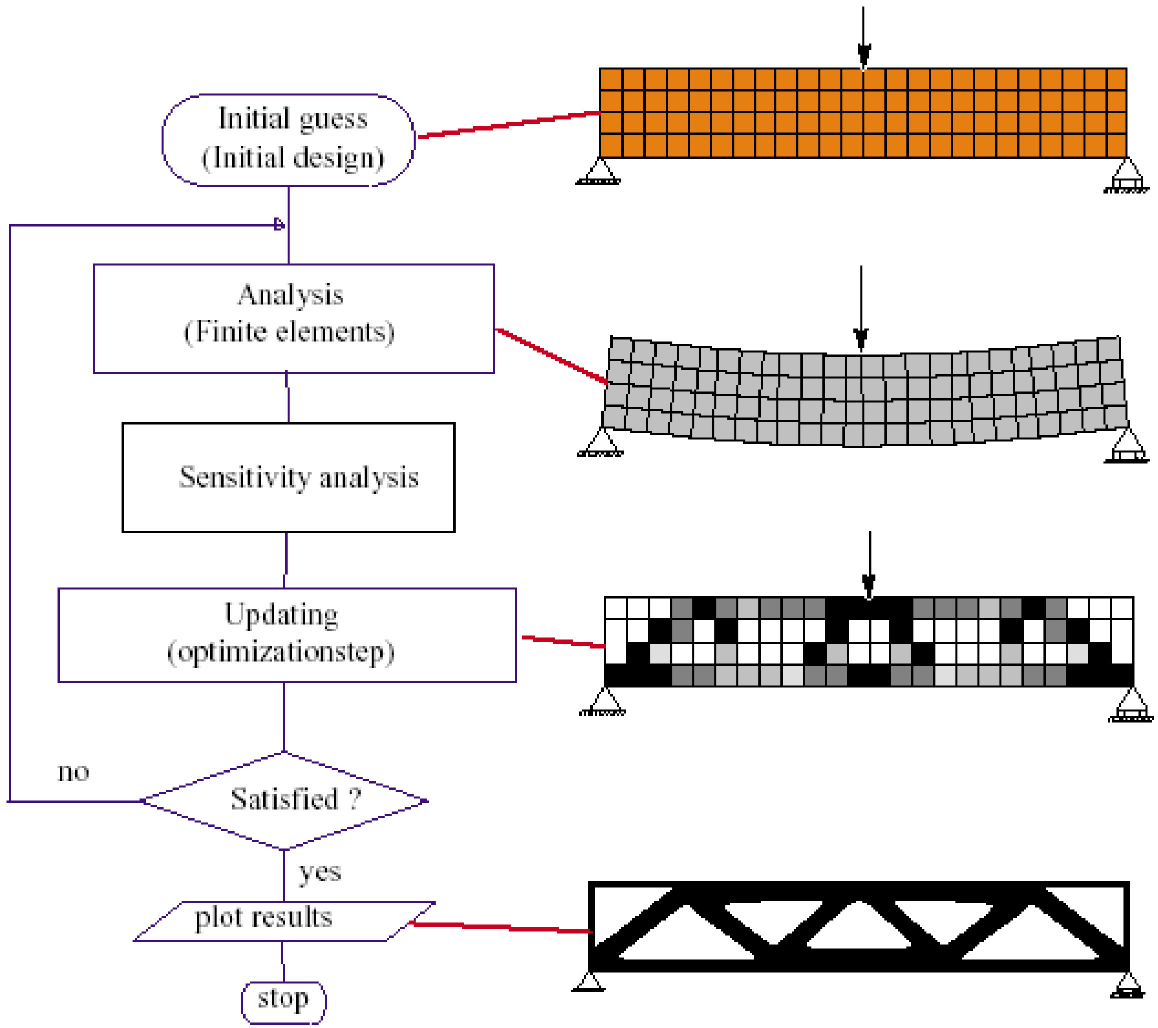


$$E(\rho_e) = \rho_e^{\,p} E_0$$

$$p > 1$$



위상최적설계: 흐름도



M.P.Bendsøe and O. Sigmund, Topology Optimization: Theory, Methods and Applications, Springer, 2003

Topology Optimization: Formulation

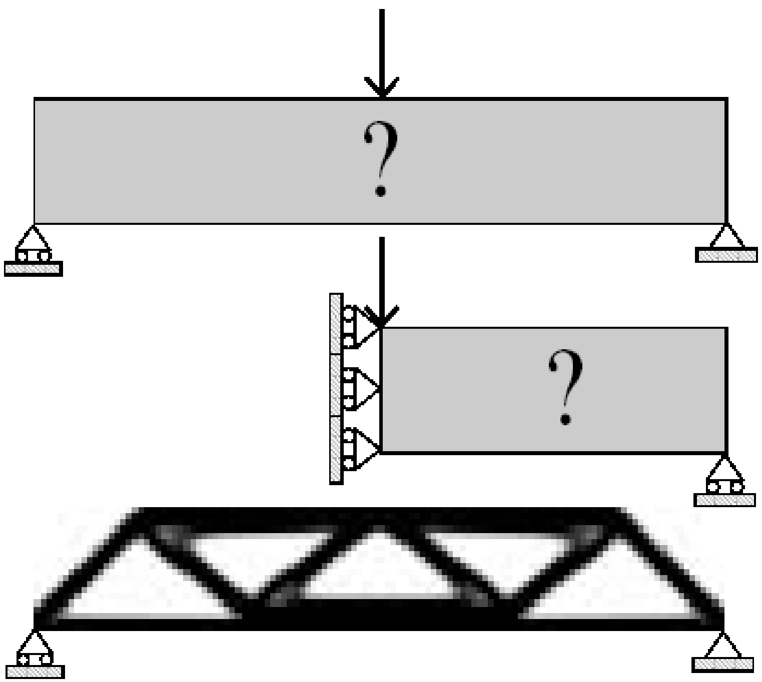
$$\left.\begin{aligned}
&\min_{DV} \int_\Omega \rho \, d\Omega \rightarrow \sum_{e=1}^{N} v_e \rho_e \\
&\text{subject to } a(\mathbf{u},\mathbf{v}) = f(\mathbf{v})\,\forall \mathbf{v} \rightarrow \mathbf{K}(\rho)\mathbf{U} = \mathbf{F} \\
&\qquad \left.\begin{aligned} &\bar{\sigma} \le \sigma_{\max} \\ &|\mathbf{u}| \le u_{\max} \end{aligned}\right\} \rightarrow \mathbf{u}^T\mathbf{f} \rightarrow \mathbf{U}^T\mathbf{F}
\end{aligned}\right\} \xrightarrow[E(\rho_e)=\rho_e{}^p E_0]{\text{dual problem}} \left\{\begin{aligned}
&\min_{\rho} \mathbf{U}^T\mathbf{F} = \phi\big(\mathbf{U}(\rho)\big) \\
&\text{subject to } \left\{\begin{aligned} &\mathbf{K}(\rho)\mathbf{U} = \mathbf{F} \\ &\sum_{e=1}^{N} v_e \rho_e \le V^* \\ &0 < \rho_e \le 1 \end{aligned}\right.
\end{aligned}\right.$$

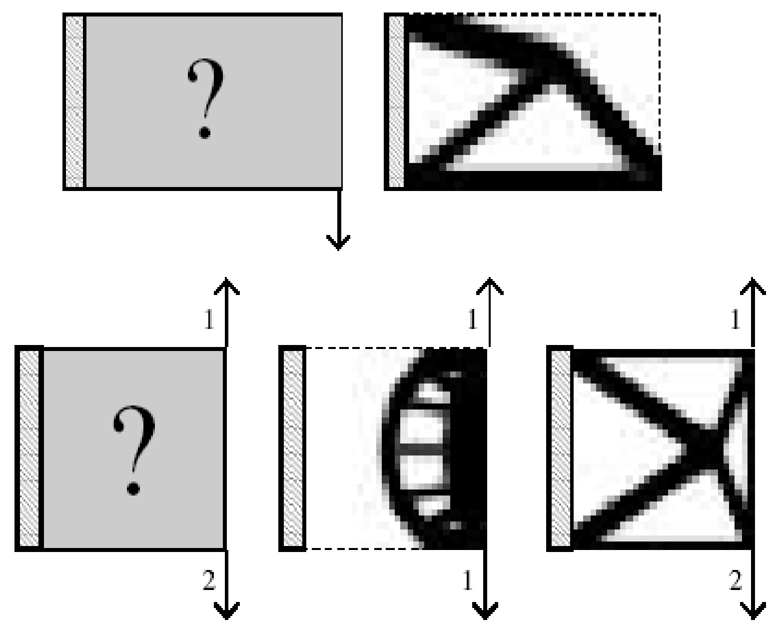
$$L\left(\rho,\mathbf{U},\lambda,\Lambda,\lambda_e^-,\lambda_e^+\right) = \phi + \lambda^T(\mathbf{K}\mathbf{U}-\mathbf{F}) + \Lambda\left(\sum_{e=1}^{N} v_e\rho_e - V^*\right) + \sum_{e=1}^{N}\lambda_e^-(-\rho_e) + \sum_{e=1}^{N}\lambda_e^+(\rho_e - 1)$$

$$\left.\begin{aligned}
&\frac{\partial L}{\partial \rho} = \underbrace{\frac{d\phi}{d\rho} + \frac{\partial \phi}{\partial \mathbf{U}}\frac{\partial \mathbf{U}}{\partial \rho} + \lambda^T\left(\frac{\partial \mathbf{K}}{\partial \rho}\mathbf{U} + \mathbf{K}\frac{\partial \mathbf{U}}{\partial \rho} - \frac{\partial \mathbf{F}}{\partial \rho}\right)}_{\frac{d\phi}{d\rho} + \left(\frac{\partial \phi}{\partial \mathbf{U}} + \lambda^T\mathbf{K}\right)\frac{\partial \mathbf{U}}{\partial \rho} + \lambda^T\frac{\partial \mathbf{K}}{\partial \rho}\mathbf{U} - \frac{\partial \mathbf{F}}{\partial \rho}} + \Lambda v_e - \lambda_e^- + \lambda_e^+ = 0 \\
&\frac{\partial L}{\partial \mathbf{U}} = \frac{\partial \phi}{\partial \mathbf{U}} + \lambda^T\mathbf{K} = 0 \rightarrow \mathbf{F} + \mathbf{K}^T\lambda = 0 \rightarrow \lambda = -\mathbf{U} \\
&-\mathbf{U}\frac{\partial \mathbf{K}}{\partial \rho}\mathbf{U} \rightarrow -\mathbf{U}_e^T \frac{\partial\left(\sum_e \mathbf{K}_e\right)}{\partial \rho_e}\mathbf{U}_e = -\mathbf{U}_e^T\left(p\rho_e^{p-1}\mathbf{K}_e^0\right)\mathbf{U}_e
\end{aligned}\right\} \rightarrow \left\{\begin{aligned}
&-\mathbf{U}_e^T\left(p\rho^{p-1}\mathbf{K}_e^0\right)\mathbf{U}_e + \Lambda v_e = 0 \\
&1 = \frac{\mathbf{U}_e^T\left(p\rho_e^{p-1}\mathbf{K}_e^0\right)\mathbf{U}_e}{\Lambda v_e} = B_e \\
&\rho_e^{new} = \rho_e\left(B_e\right)^\eta
\end{aligned}\right.$$

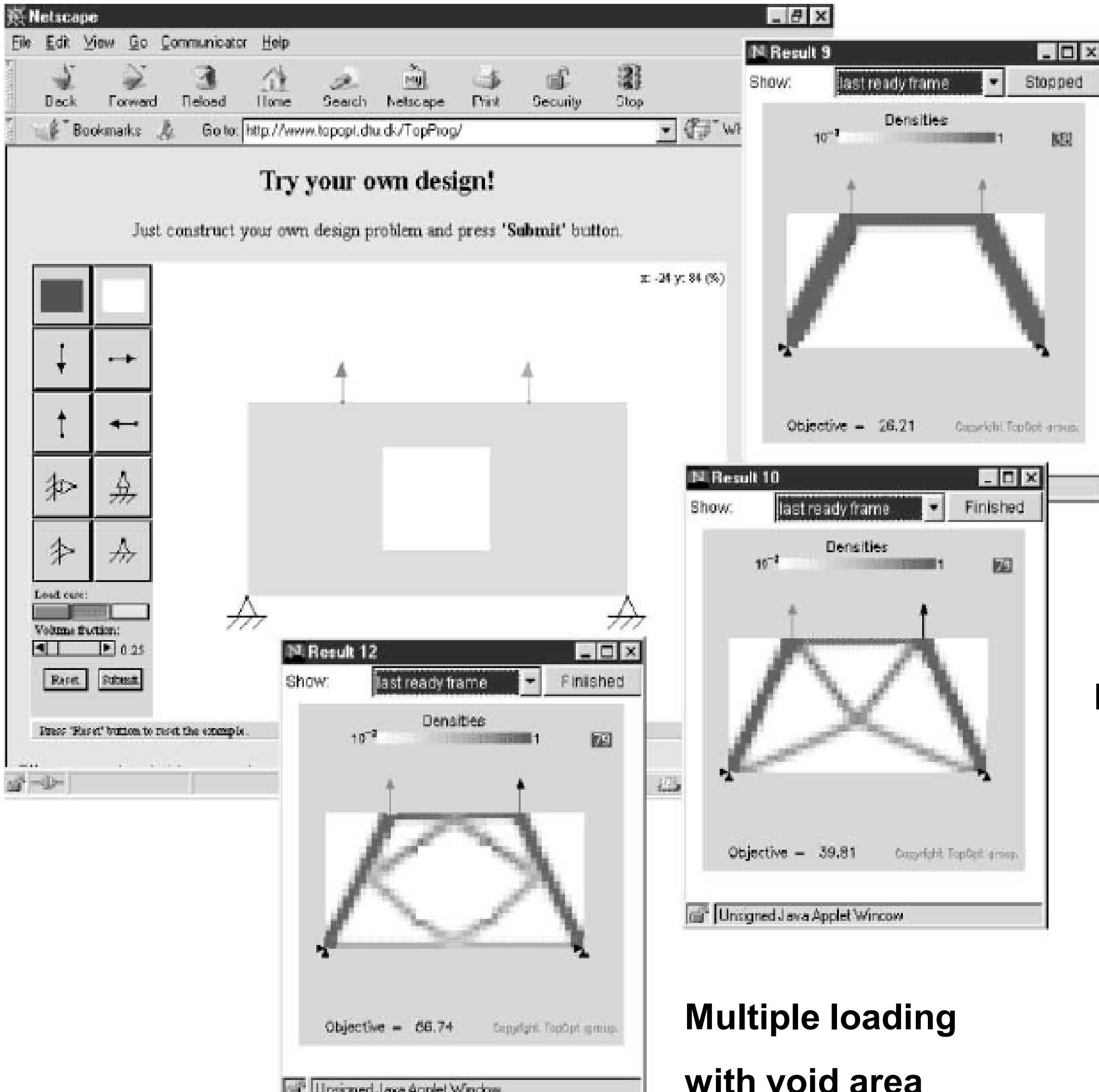
Educational Design Tool (1)

- TOPOPT (www.topopt.dtu.dk)
  - A 99 line topology optimization code written in Matlab, Struct Multidisc Optim 21, pp.120-127, 2001
  - A web-based topology optimization program, Struct Multidisc Optim 22, pp.179-187, 2001
  - Interactive topology optimization on hand-held devices, Struct Multidisc Optim 47, pp.1-6, 2013





Educational Design Tool (2)



Single loading

Multiple loading

Multiple loading with void area

99 Lines of Matlab Code (2001)

Educational article

Struct Multidisc Optim 21, 120–127

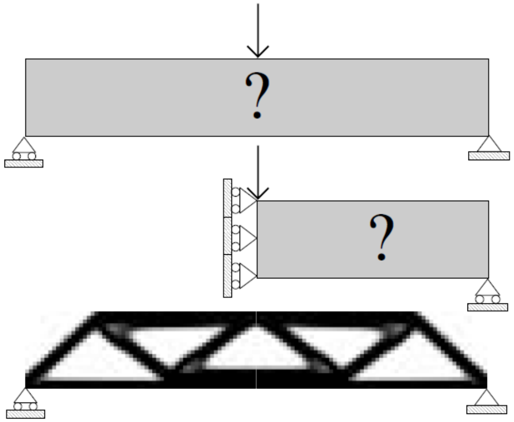
A 99 line topology optimization code written in Matlab

O. Sigmund

```
top(nelx,nely,volfrac,penal,rmin)
```

– nelx and nely: number of elements in the horizontal and vertical directions
– volfrac: volume fraction
– penal: penalization power (=3)
– rmin: filter size(divided by element size)

\>>top(60,20,0.5,3.0,1.5)



Alternative '88 line Matlab code' (2011)
New 99 line Matlab code (2020)

Struct Multidisc Optim (2011) 43:1–16
DOI 10.1007/s00158-010-0594-7

EDUCATIONAL ARTICLE

Efficient topology optimization in MATLAB using 88 lines of code

Erik Andreassen · Anders Clausen · Mattias Schevenels · Boyan S. Lazarov · Ole Sigmund

```
top88(nelx,nely,volfrac,penal,rmin,ft)
```

Structural and Multidisciplinary Optimization
https://doi.org/10.1007/s00158-020-02629-w

EDUCATIONAL PAPER

A new generation 99 line Matlab code for compliance topology optimization and its extension to 3D

Federico Ferrari[1] · Ole Sigmund[1]

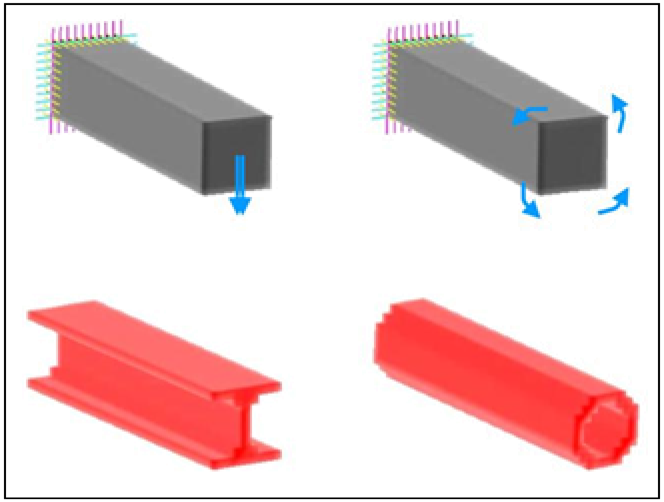
Received: 18 February 2020 / Revised: 2 May 2020 / Accepted: 10 May 2020

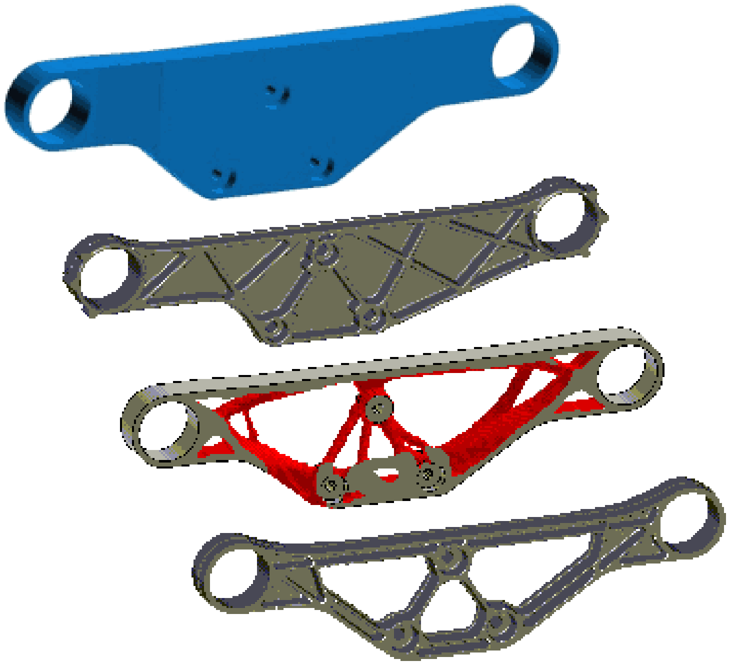

```
top99neo(nelx,nely,volfrac,penal,rmin,ft,ftBC,eta,beta,move,maxit)
```

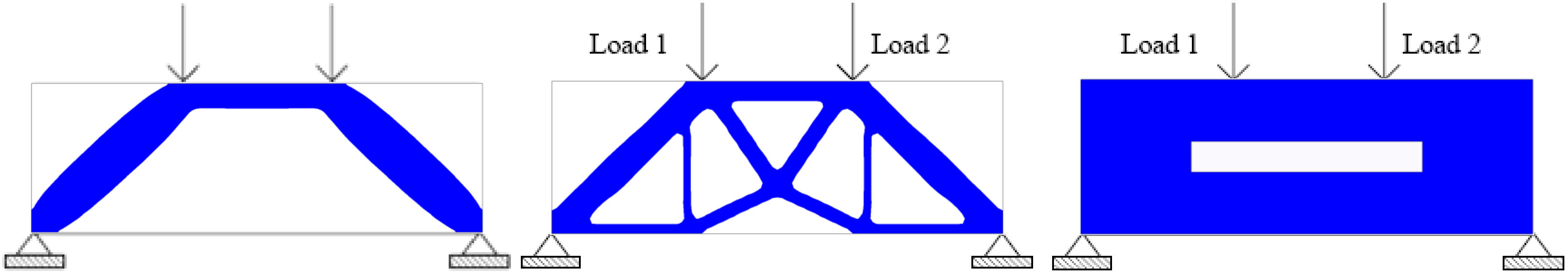
```
top3D125(nelx,nely,nelz,volfrac,penal,rmin,ft,ftBC,eta,beta,move,maxit)
```

정적 문제: 예제

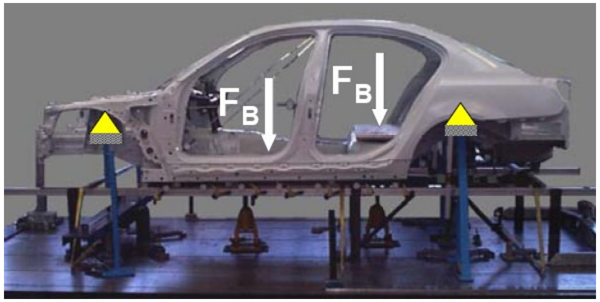
고체역학 검증: 하중 경로





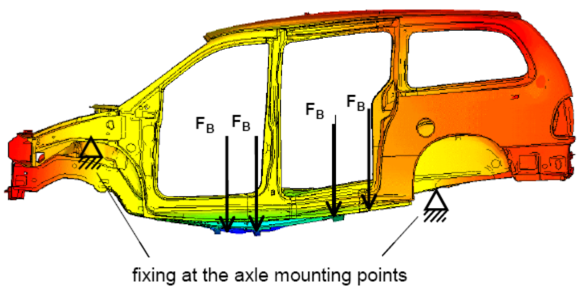


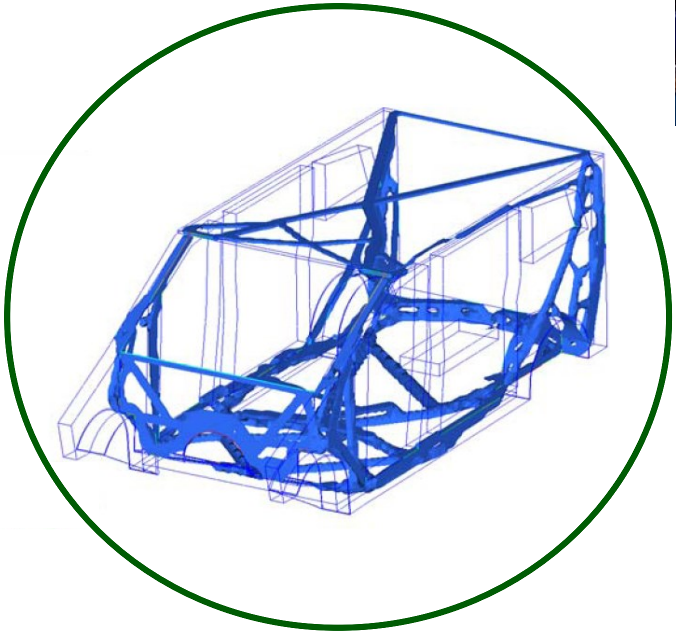
위상최적설계: 개념 설계에 활용

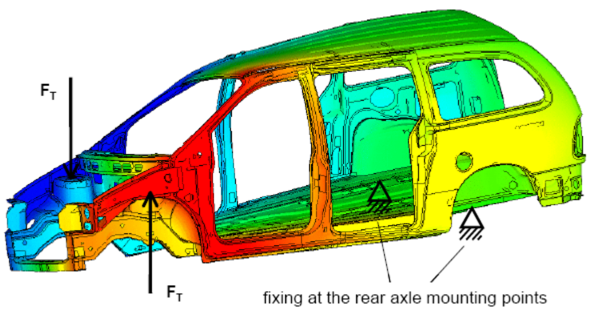


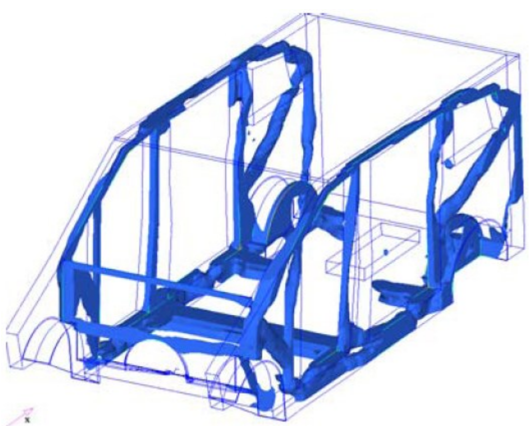
굽힘

비틀림

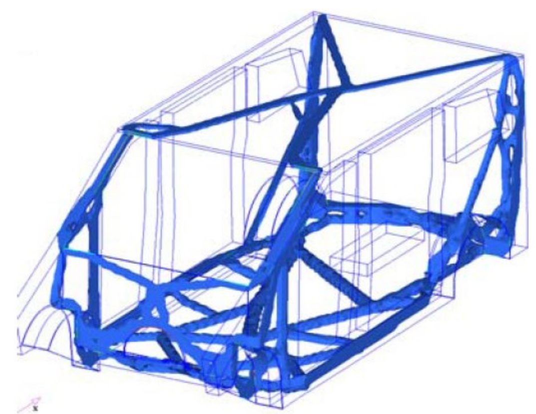








굽힘 + 비틀림



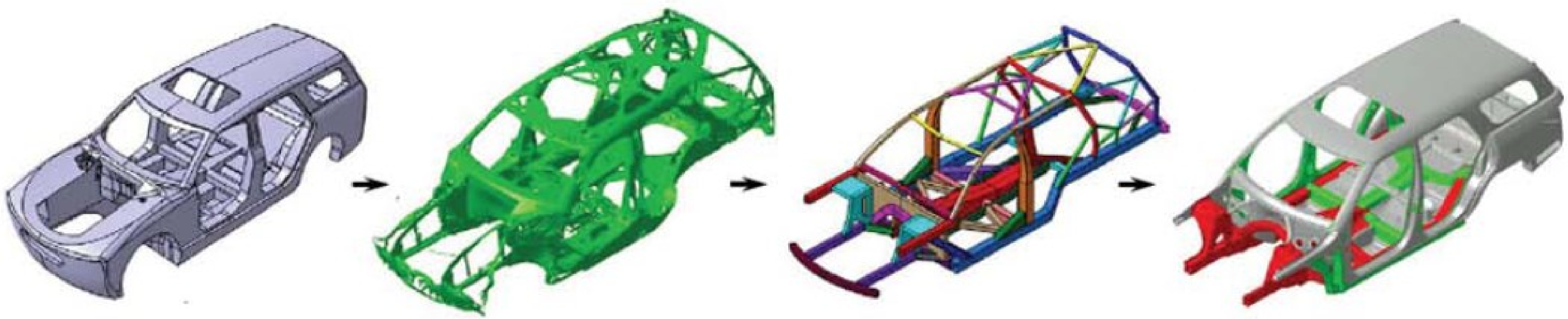
위상최적설계: 차체구조설계 사례

CHRYSLER

Light Weight Body
Structural Optimization Process

2008 GDIS

Topology Optimization used to define Major Load Paths



Packaging Volume
Holistic Drivers
- Safety
- NVH Refinement
- Durability

Topology Results
Spatial load image

Beam Model
87 paths

BIW Design
202 components

Chassis: Ferrari F458 Italia

- 설계 목적

  경량화

- 설계 변수 (cascade)

  Topology optimization: 밀도

  Topometry optimization : 절점 높이

  Size optimization : 쉘 두께

- 성능 요구조건

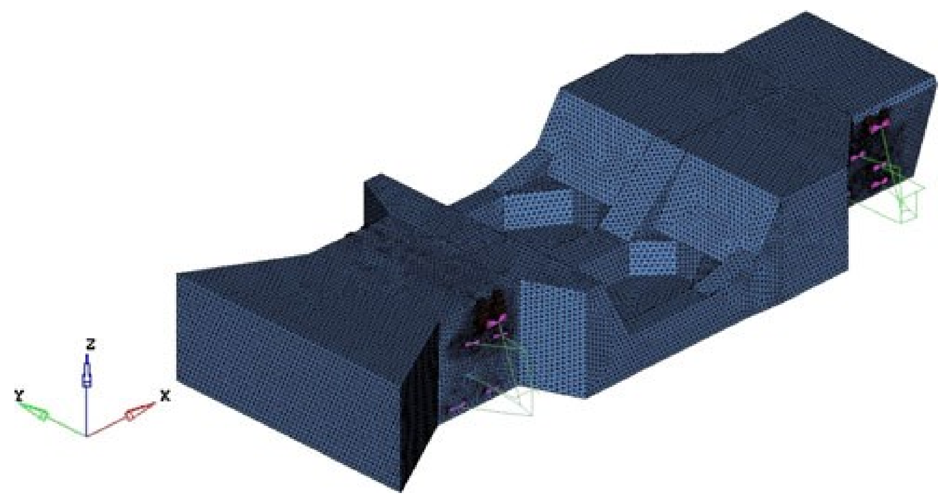
global bending stiffness
global torsional stiffness
Local stiffness of the suspensions, engine and gearbox
Modal response
Crash linearization

Struct Multidisc Optim (2011) 44:45–56
DOI 10.1007/s00158-010-0578-7

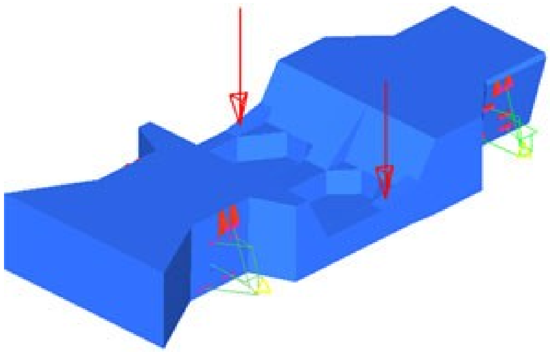
INDUSTRIAL APPLICATION

High performance automotive chassis design: a topology optimization based approach

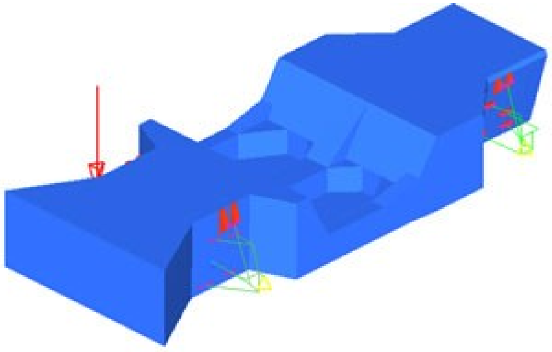
Marco Cavazzuti · Andrea Baldini · Enrico Bertocchi · Dario Costi · Enrico Torricelli · Patrizio Moruzzi



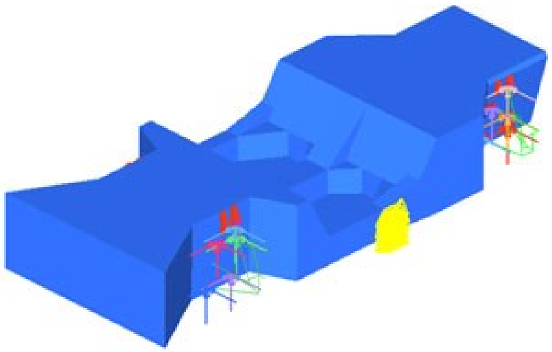
Chassis: 하중조건



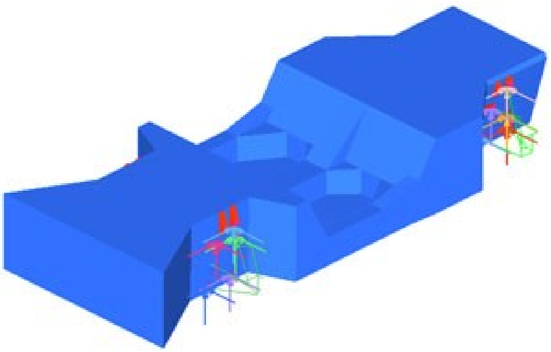
bending stiffness (1 FEA)



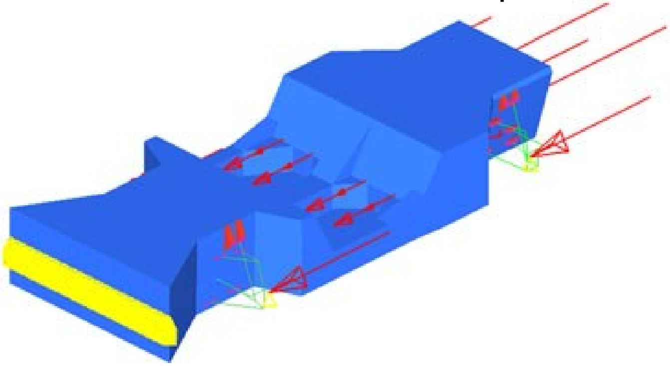
torsional stiffness (1 FEA)



local stiffness with
clamped sills (32 FEA)

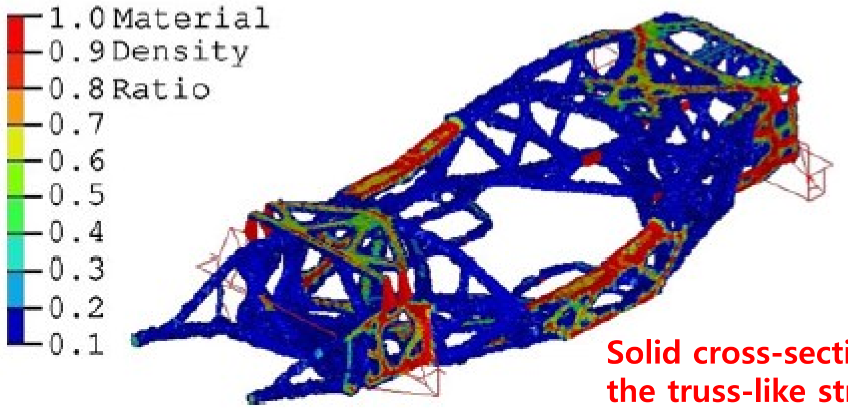


local stiffness with
inertia relief (32 FEA)

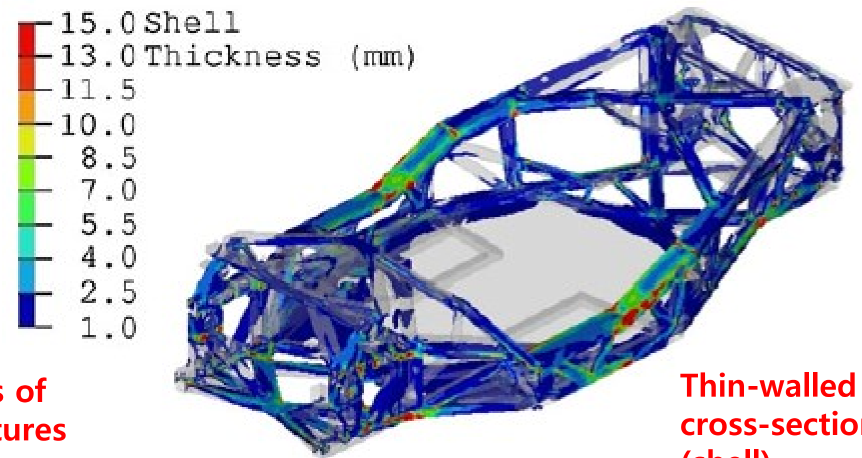


crash linearization (1 FEA)

Chassis: 최적설계

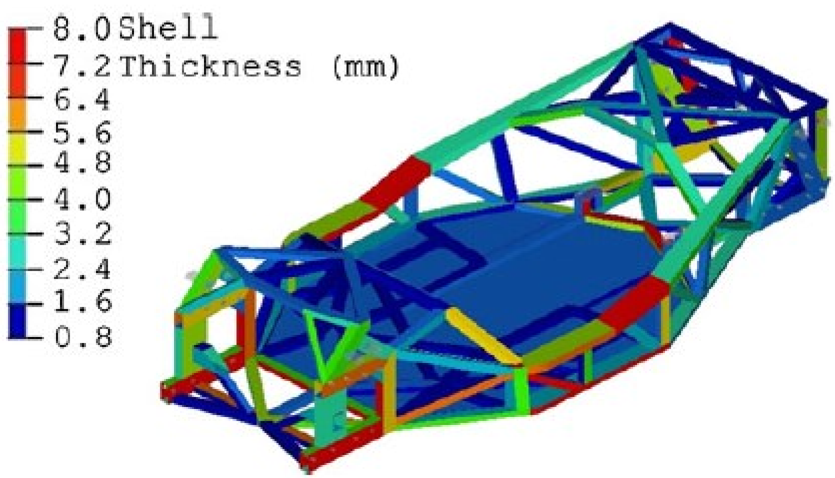


Solid cross-sections of the truss-like structures



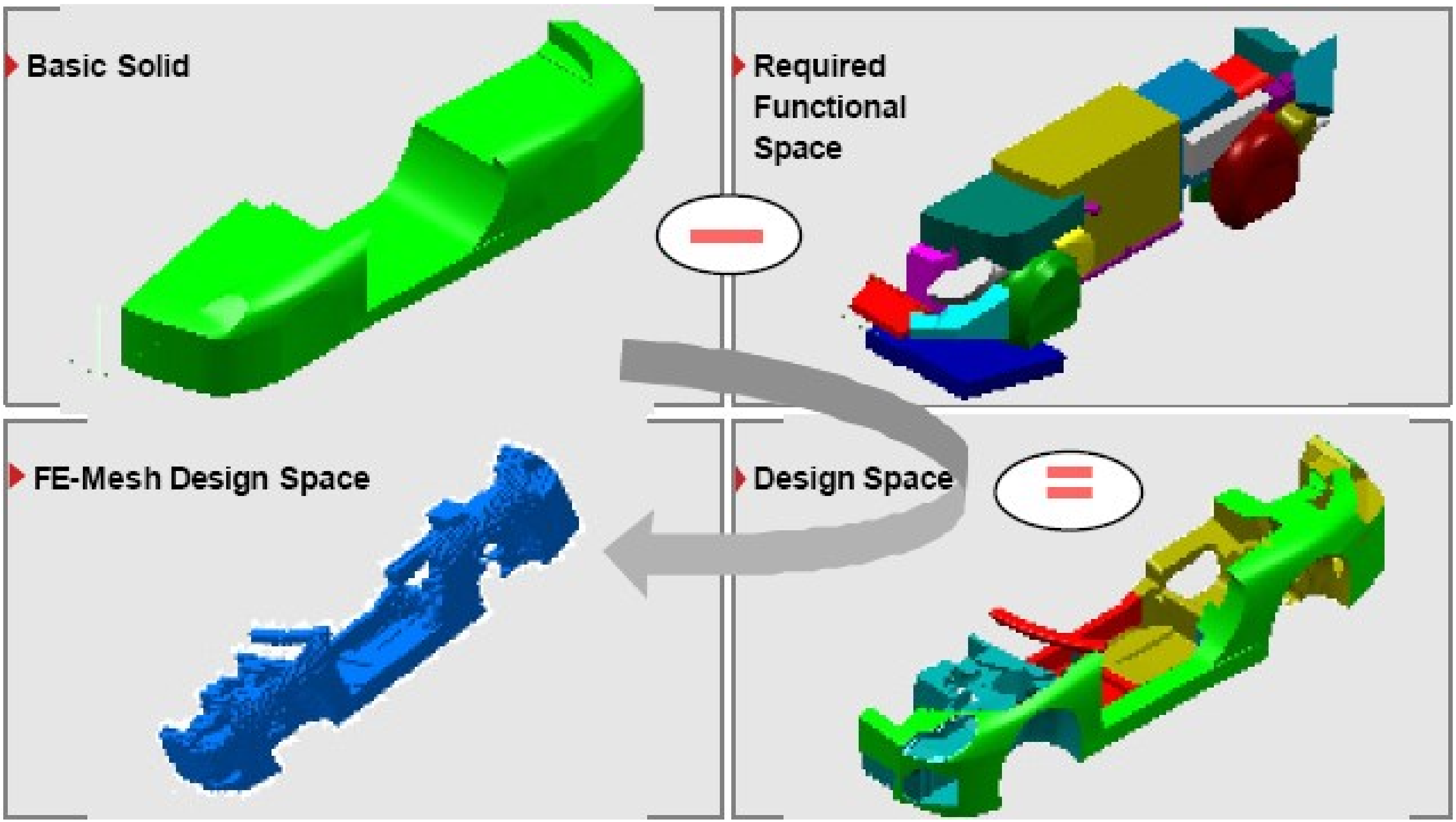
Thin-walled cross-sections (shell)

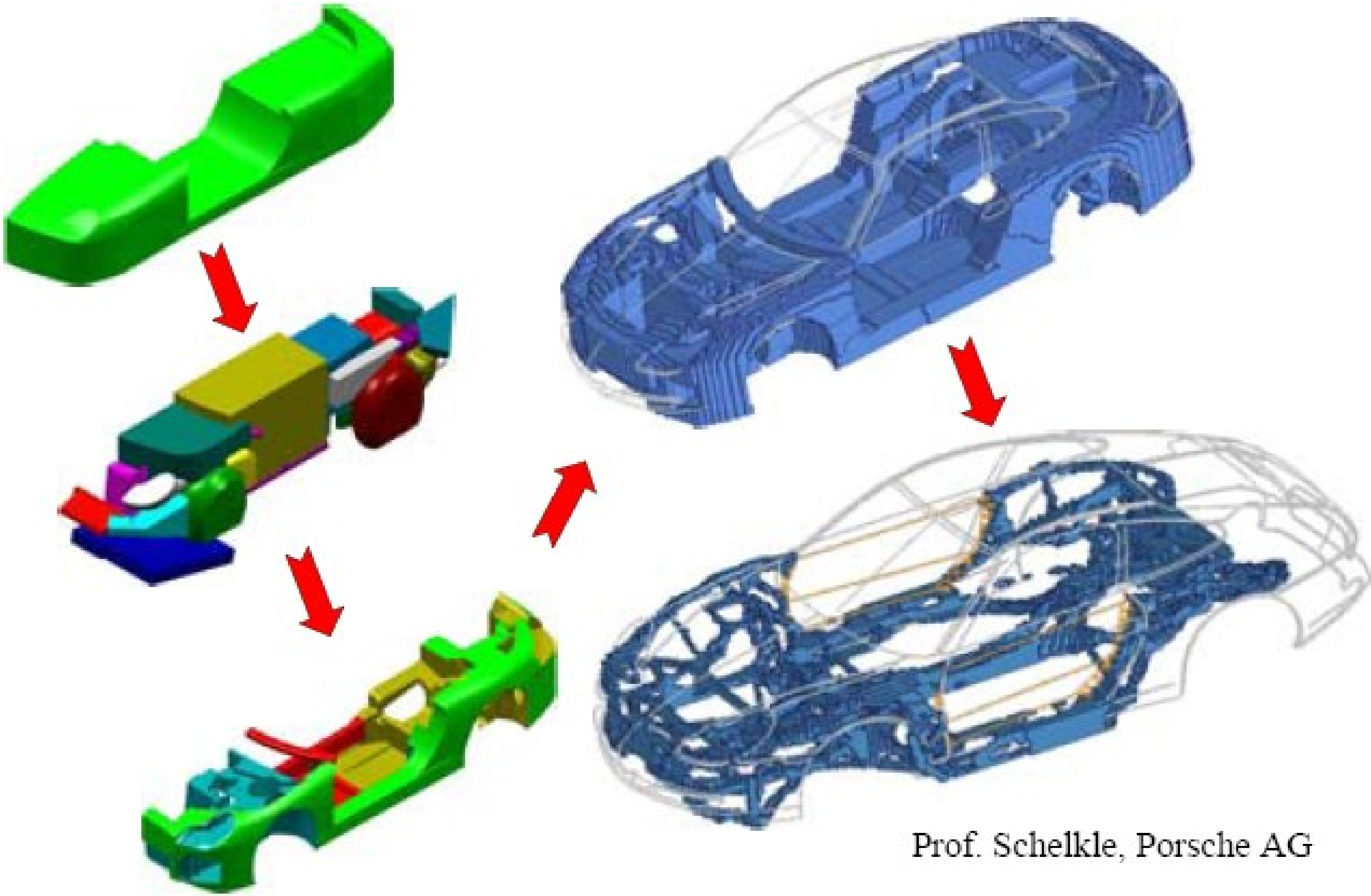
Topology optimization(설계변수: ≈10^6) → CAD interpretation → **Topometry optimization**

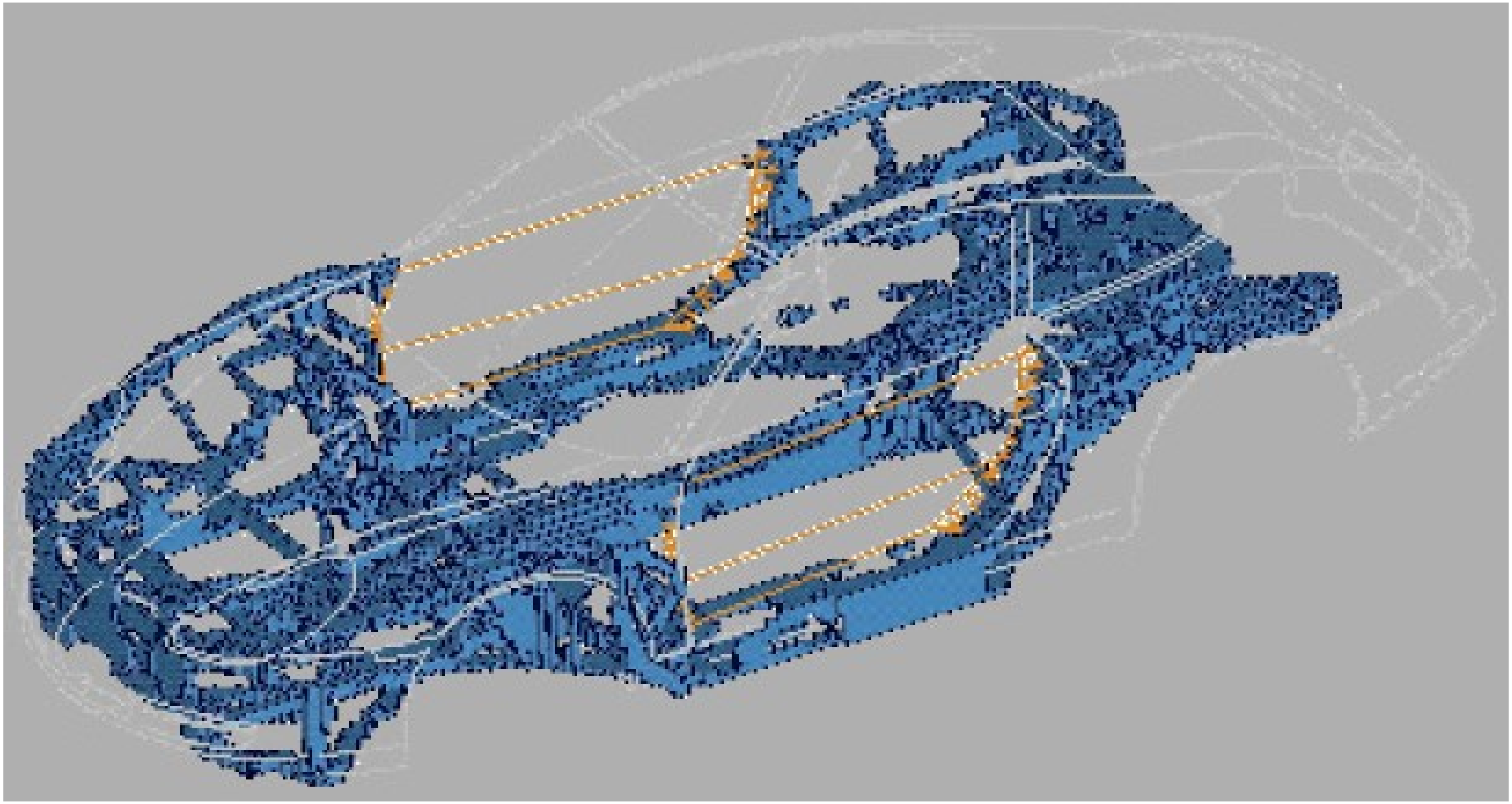


Size optimization(설계변수: ≈330)

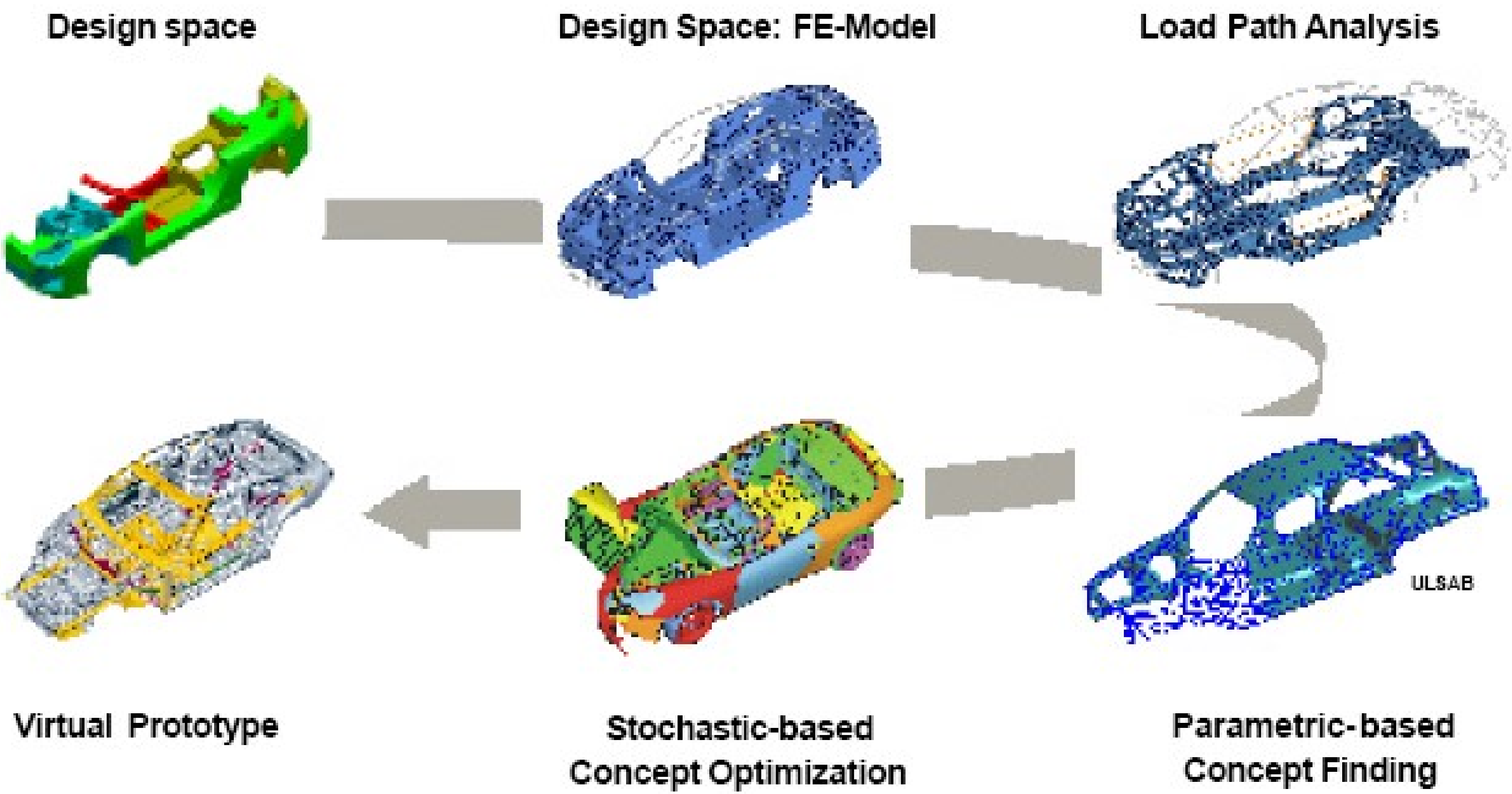
Load Path Analysis Sequence




Prof. Schelkle, Porsche AG



Concept Stage of Vehicle Development



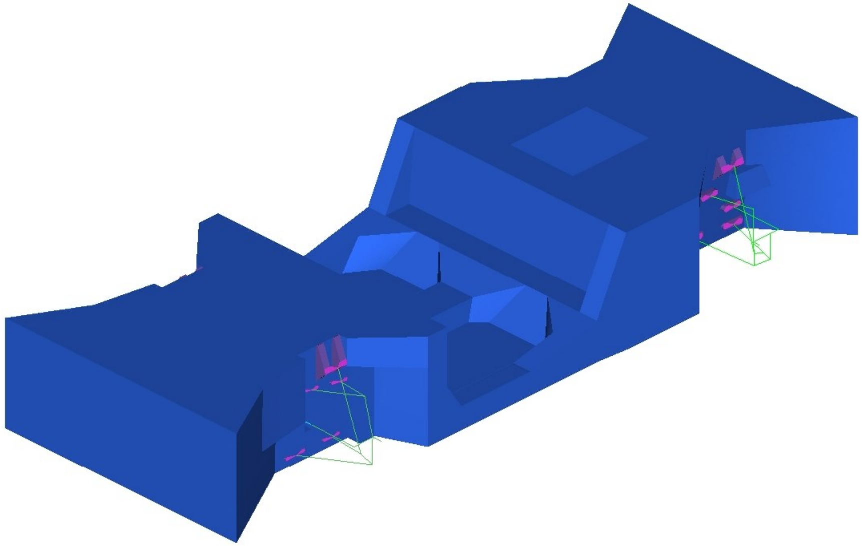
Automotive Chassis

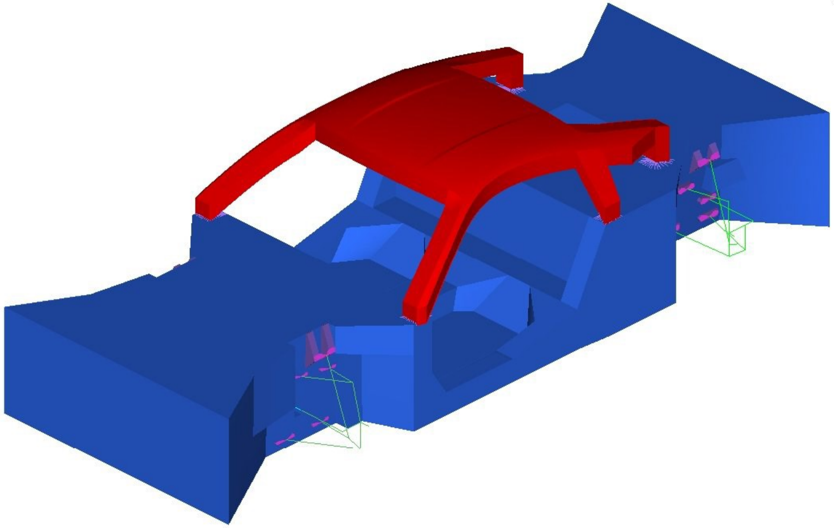
Proceedings of the World Congress on Engineering 2011 Vol III
WCE 2011, July 6 - 8, 2011, London, U.K.

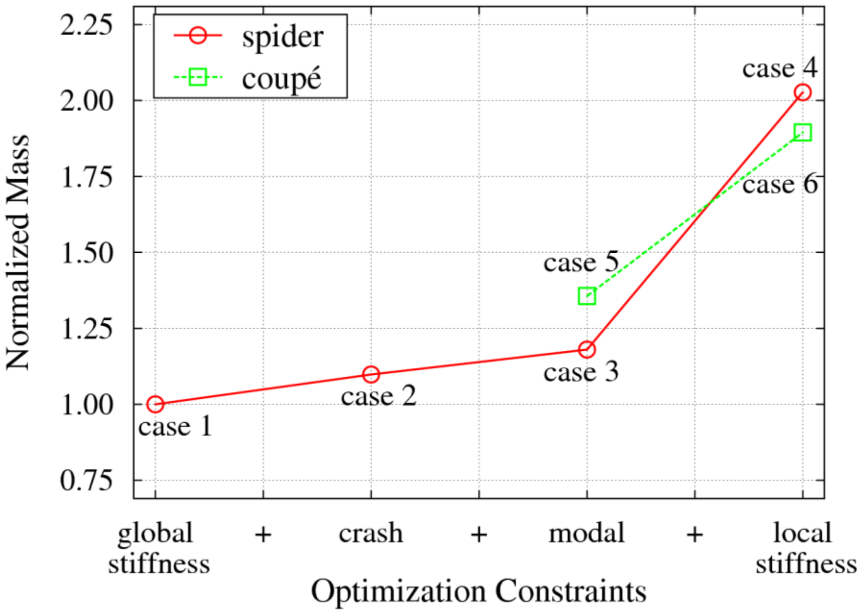
Automotive Chassis Topology Optimization: a Comparison Between Spider and Coupé Designs

Marco Cavazzuti, Dario Costi, Andrea Baldini, Patrizio Moruzzi

- Comparison Between Spider and Coupe Designs

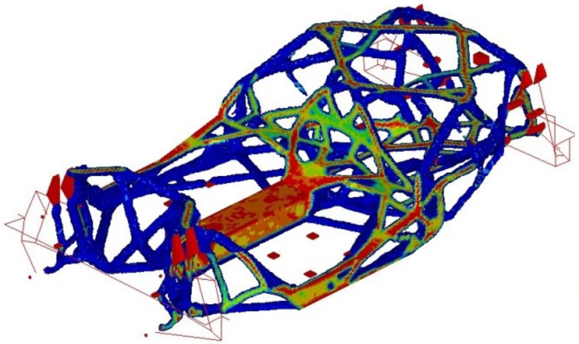




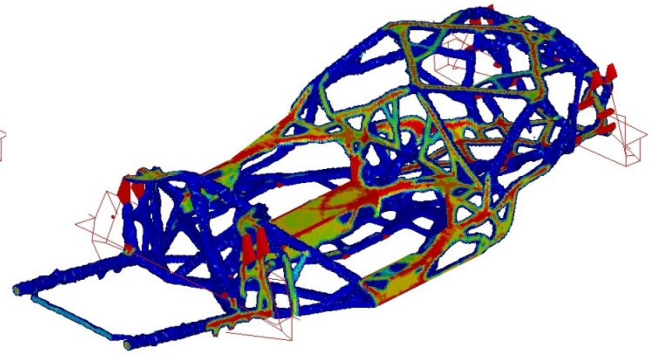


Active Optimization Constraints	Cases					
	Spider				Coupé	
	1	2	3	4	5	6
Global bending stiffness	✓	✗	✓	✓	✓	✓
Global torsion stiffness	✓	✓	✓	✓	✓	✗
Crash seat joints displacement	–	✗	✗	✗	✓	✗
Crash engine joints displacement	–	✓	✓	✗	✓	✓
Crash A-pillar displacement	–	✓	✓	✓	✓	✓
Crash pedal displacement	–	✗	✓	✗	✗	✗
Crash flame shield displacement	–	✓	✓	✗	✗	✗
Crash dashboard joints displacement	–	✗	✗	✗	✗	✓
Crash compliance	–	✓	✓	✓	✓	✓
First natural mode	–	–	✓	✓	✓	✓
Local front wheel stiffness along x	–	–	–	✓	–	✓
Local front wheel stiffness along y	–	–	–	✓	–	✓
Local front wheel stiffness along z	–	–	–	✓	–	✓
Local rear wheel stiffness along x	–	–	–	✓	–	✓
Local rear wheel stiffness along y	–	–	–	✓	–	✓
Local rear wheel stiffness along z	–	–	–	✗	–	✗
Local engine joint stiffness along z	–	–	–	✓	–	✓
Local gearbox joint stiffness along z	–	–	–	✓	–	✗
Total	$\frac{2}{2}$	$\frac{5}{9}$	$\frac{8}{10}$	$\frac{12}{18}$	$\frac{7}{10}$	$\frac{12}{18}$

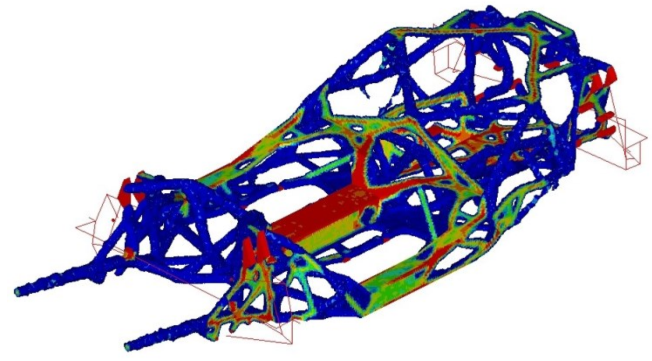
Results (1)



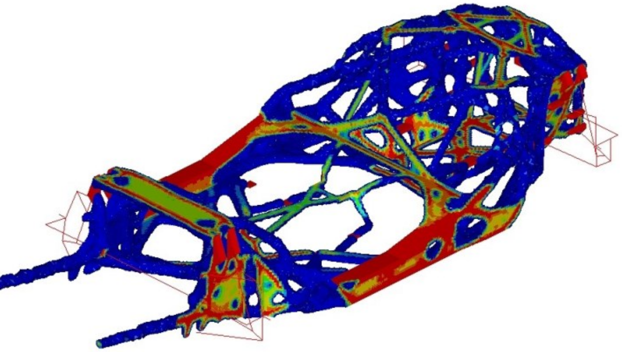
Case 1



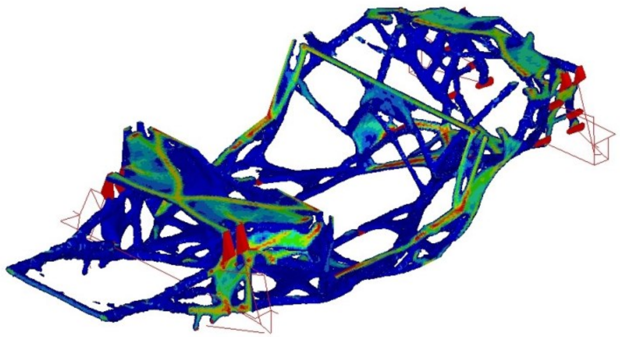
Case 2



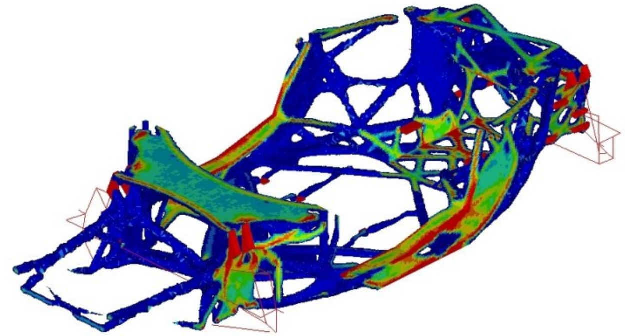
Case 3



Case 4

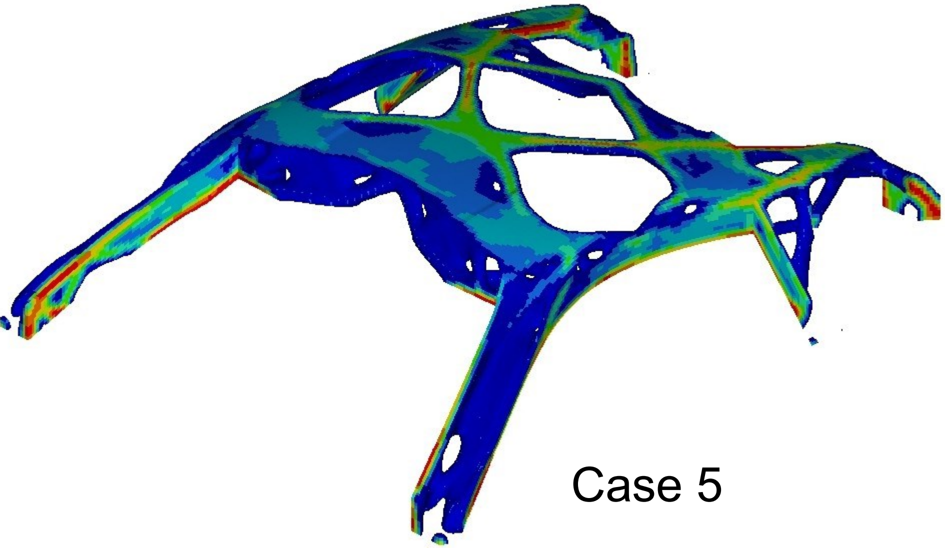


Case 5

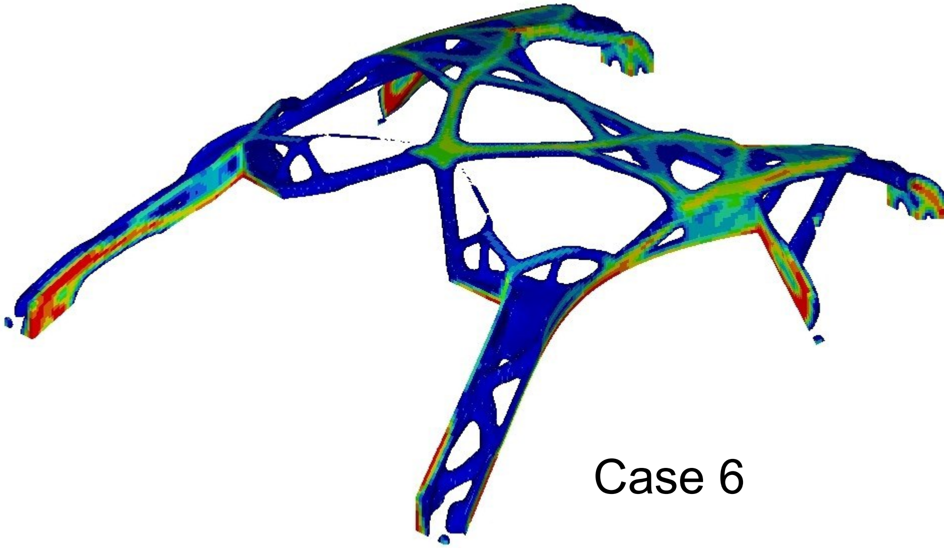


Case 6

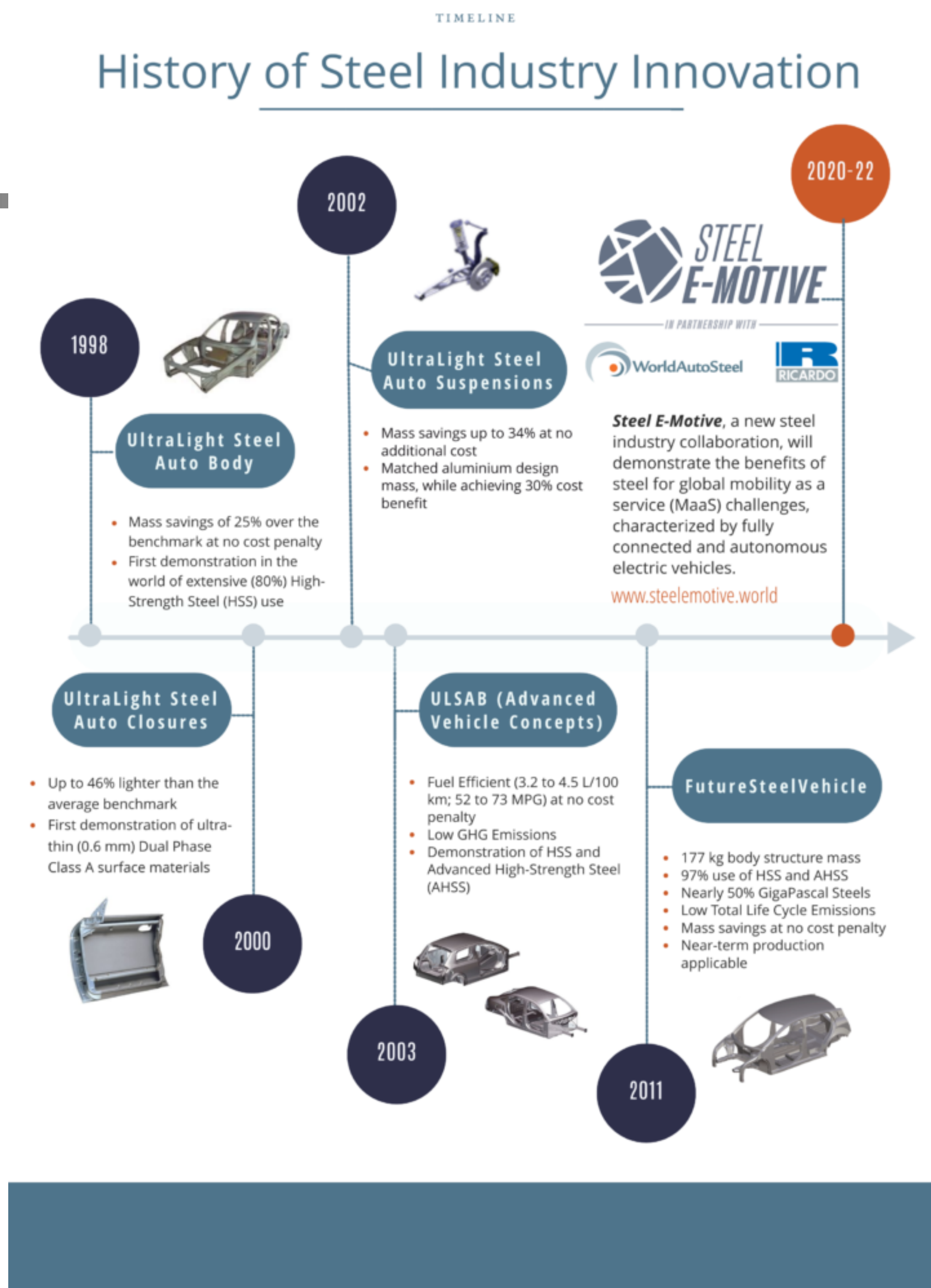
Results (2)



Case 5



Case 6


TIMELINE
History of Steel Industry Innovation
1998
UltraLight Steel Auto Body
Mass savings of 25% over the benchmark at no cost penalty
First demonstration in the world of extensive (80%) High-Strength Steel (HSS) use
2000
UltraLight Steel Auto Closures
Up to 46% lighter than the average benchmark
First demonstration of ultra-thin (0.6 mm) Dual Phase Class A surface materials
2002
UltraLight Steel Auto Suspensions
Mass savings up to 34% at no additional cost
Matched aluminium design mass, while achieving 30% cost benefit
2003
ULSAB (Advanced Vehicle Concepts)
Fuel Efficient (3.2 to 4.5 L/100 km; 52 to 73 MPG) at no cost penalty
Low GHG Emissions
Demonstration of HSS and Advanced High-Strength Steel (AHSS)
2011
FutureSteelVehicle
177 kg body structure mass
97% use of HSS and AHSS
Nearly 50% GigaPascal Steels
Low Total Life Cycle Emissions
Mass savings at no cost penalty
Near-term production applicable
2020-22
STEEL E-MOTIVE
IN PARTNERSHIP WITH
WorldAutoSteel
RICARDO
Steel E-Motive, a new steel industry collaboration, will demonstrate the benefits of steel for global mobility as a service (MaaS) challenges, characterized by fully connected and autonomous electric vehicles.
www.steelemotive.world

YouTube

설계프로세스에 적용

ULSAB
UltraLight Steel Auto Body

ULSAC
UltraLight Steel Auto Closures

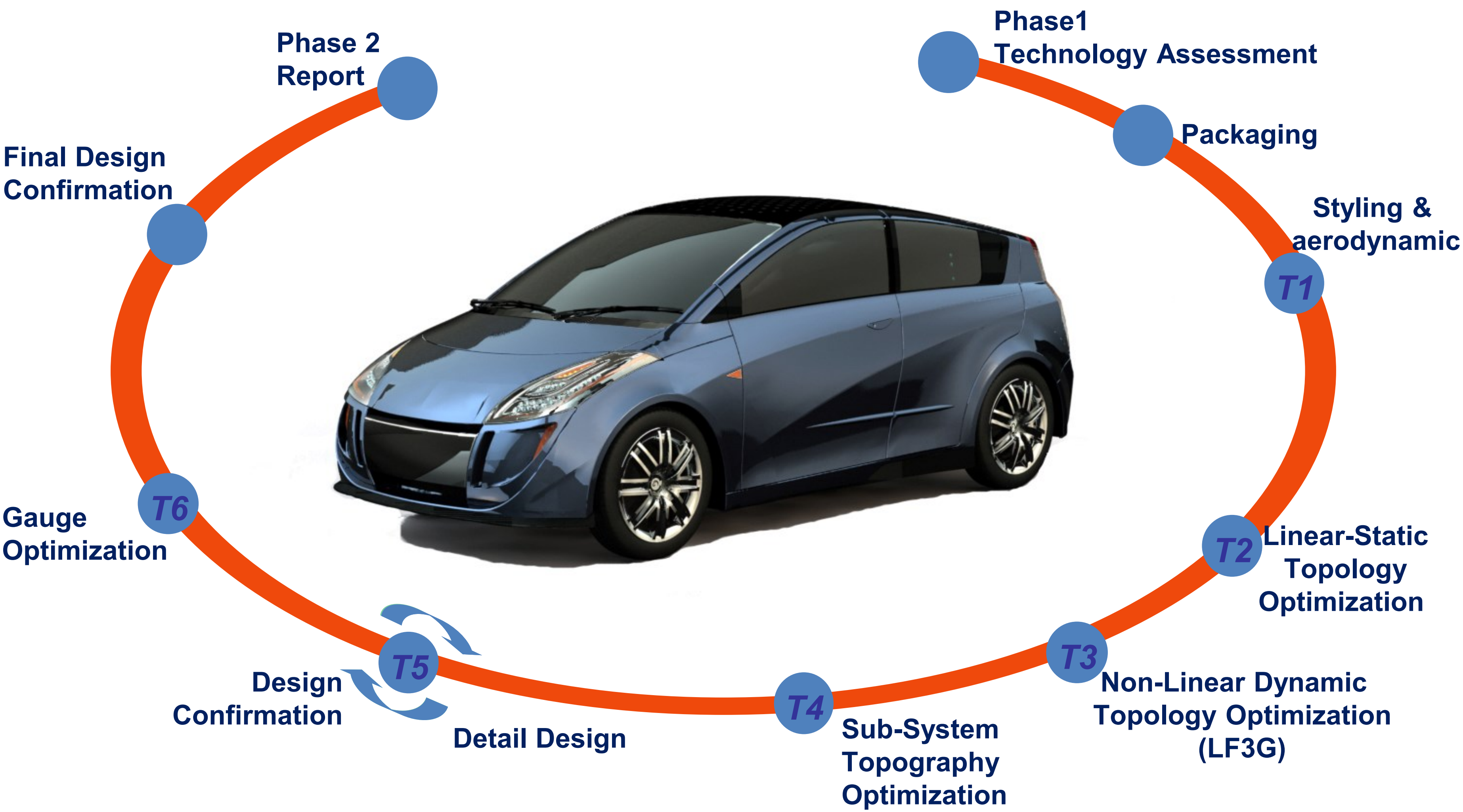
FutureSteelVehicle

Nature's Way to Mobility

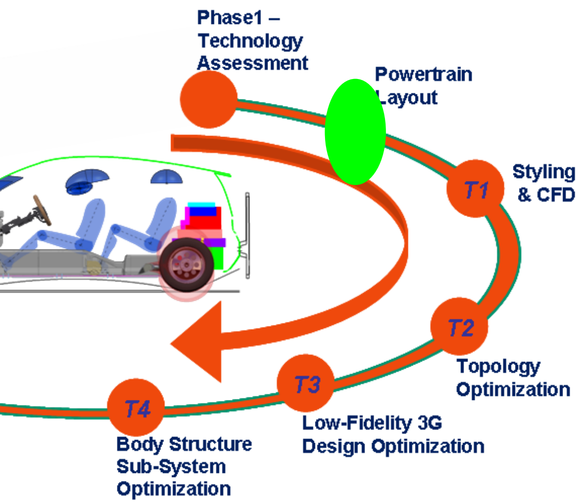
ULSAS
UltraLight Steel Auto Suspensions

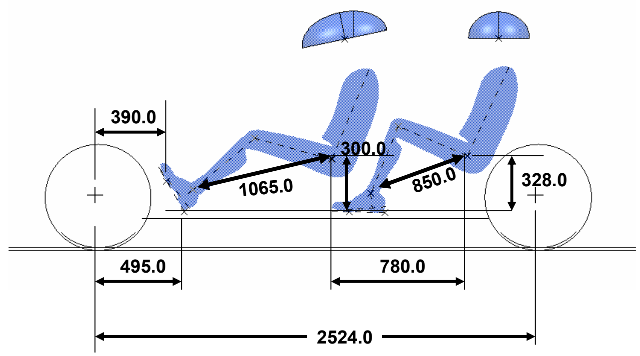
ULSAB-AVC
Advanced Vehicle Concepts

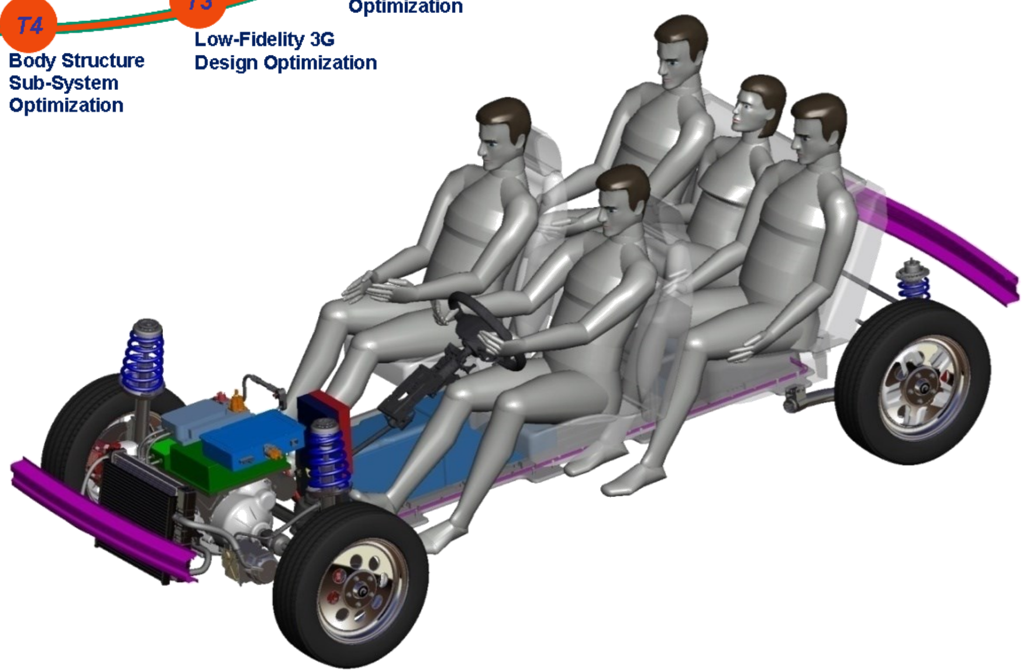
FSV Design Methodology

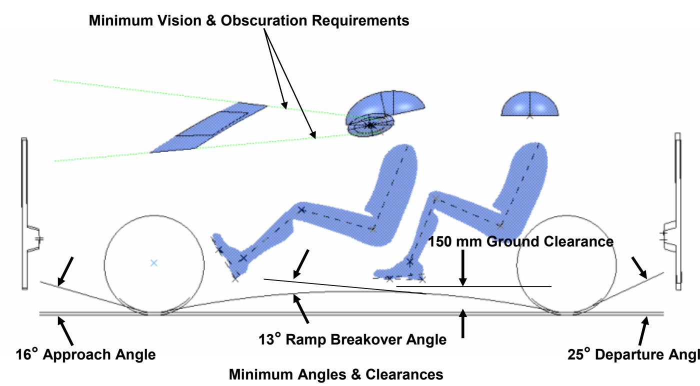


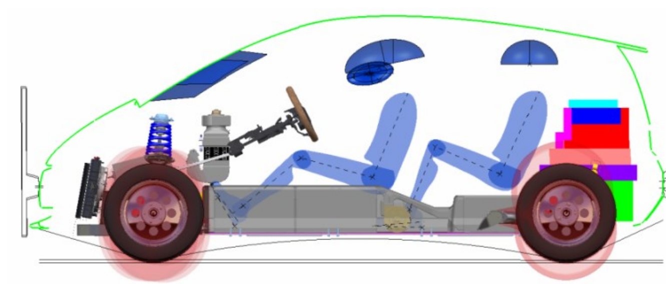
FSV BEV Packaging



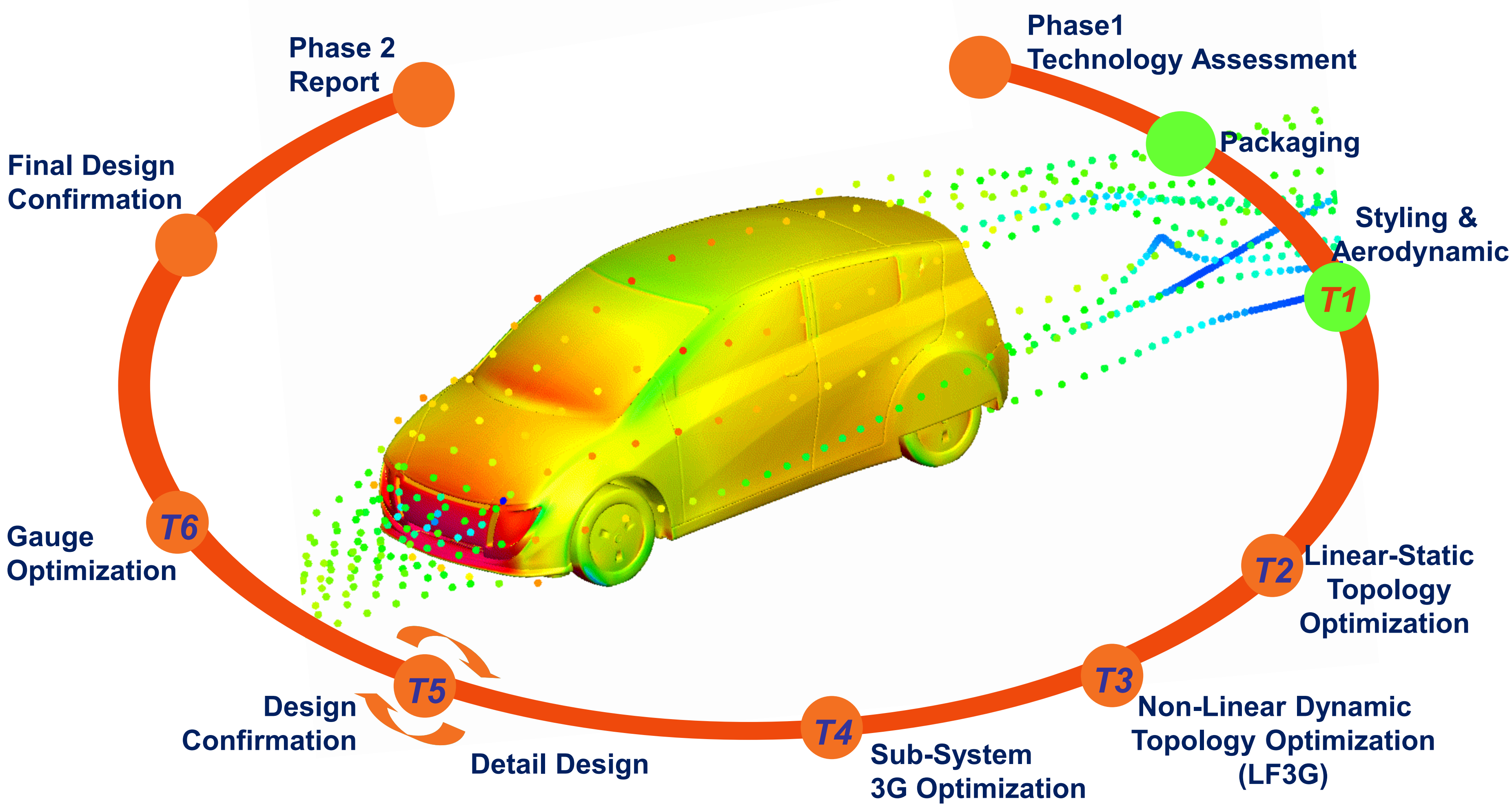




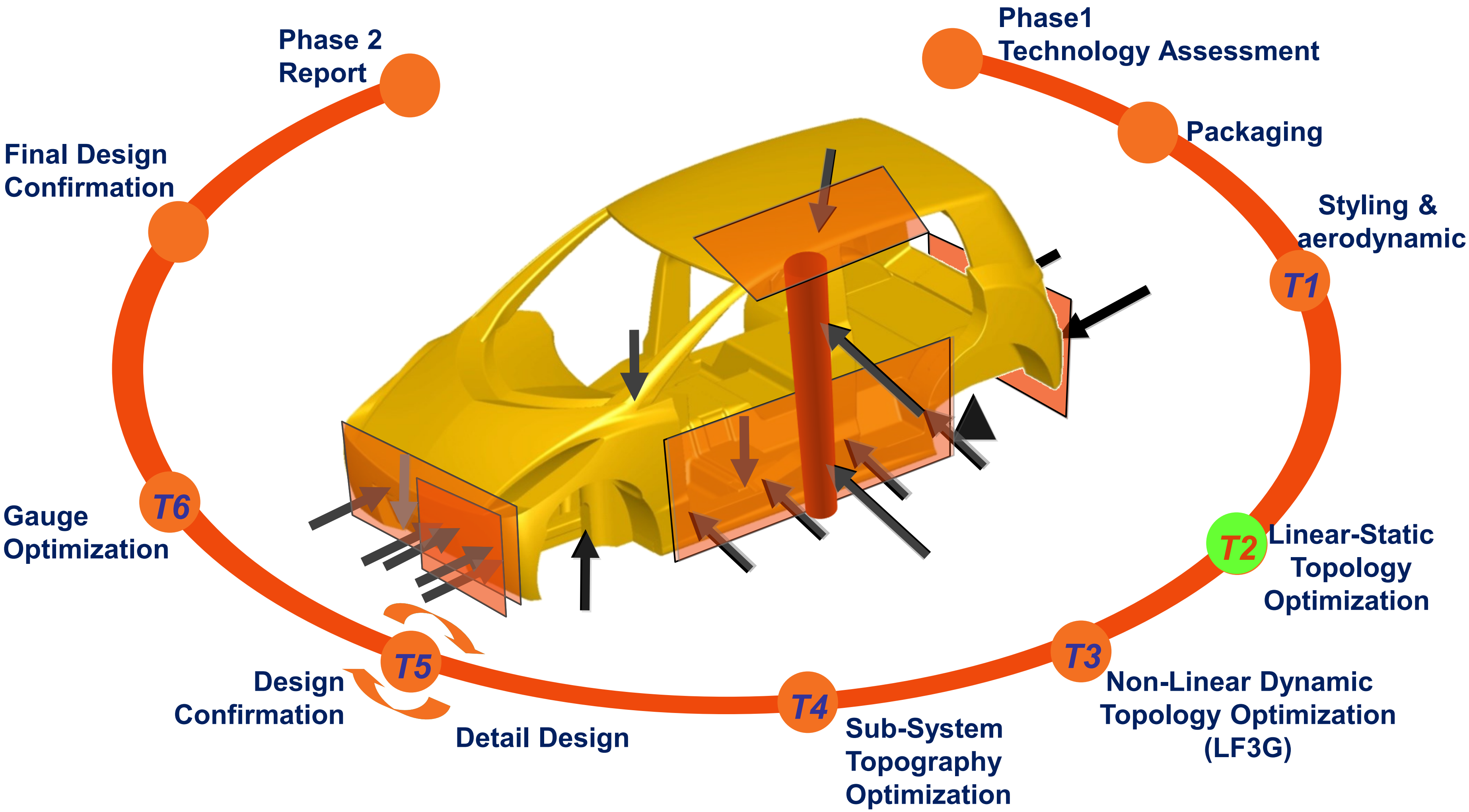




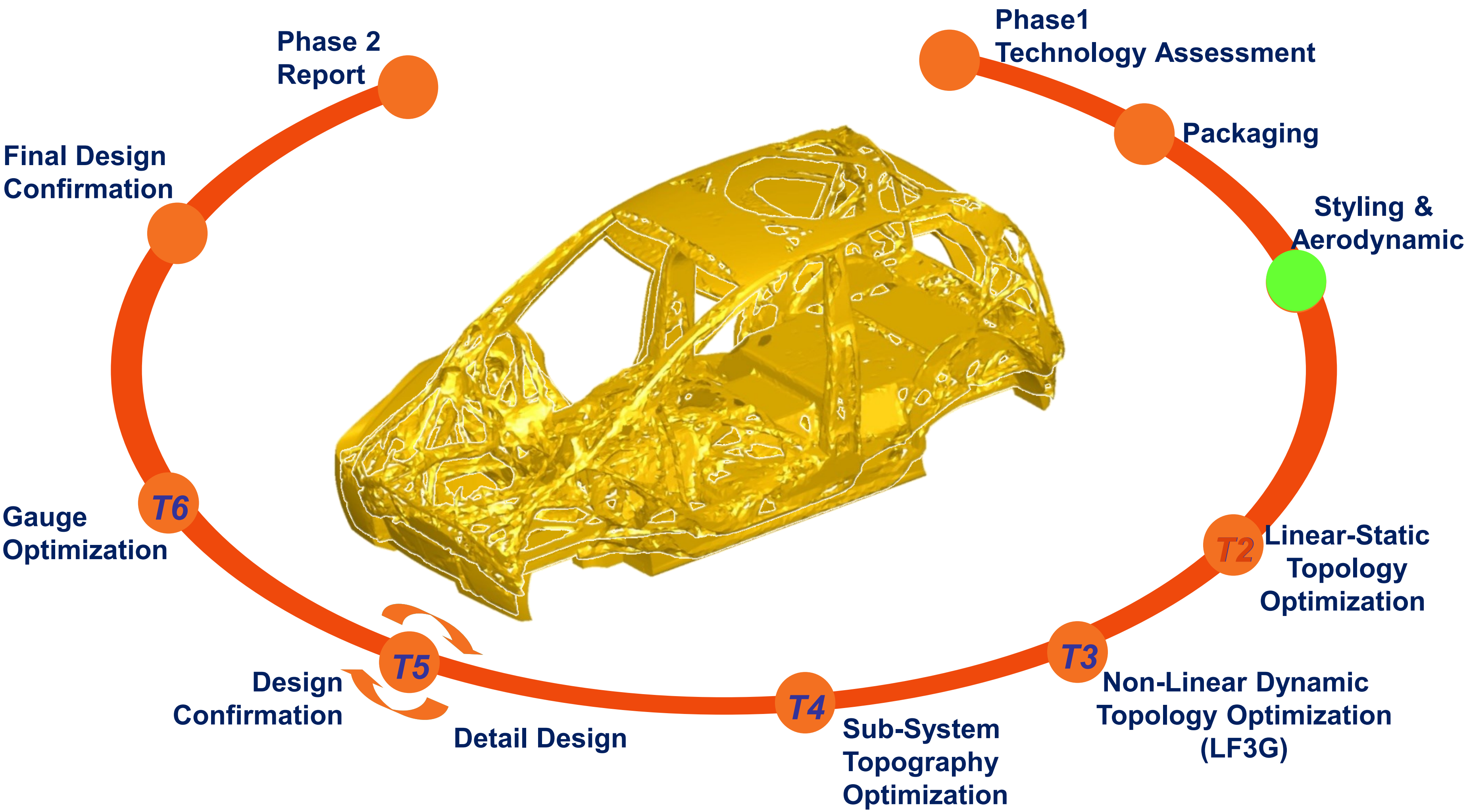
Aerodynamics & Styling



Topology Optimization Load Cases

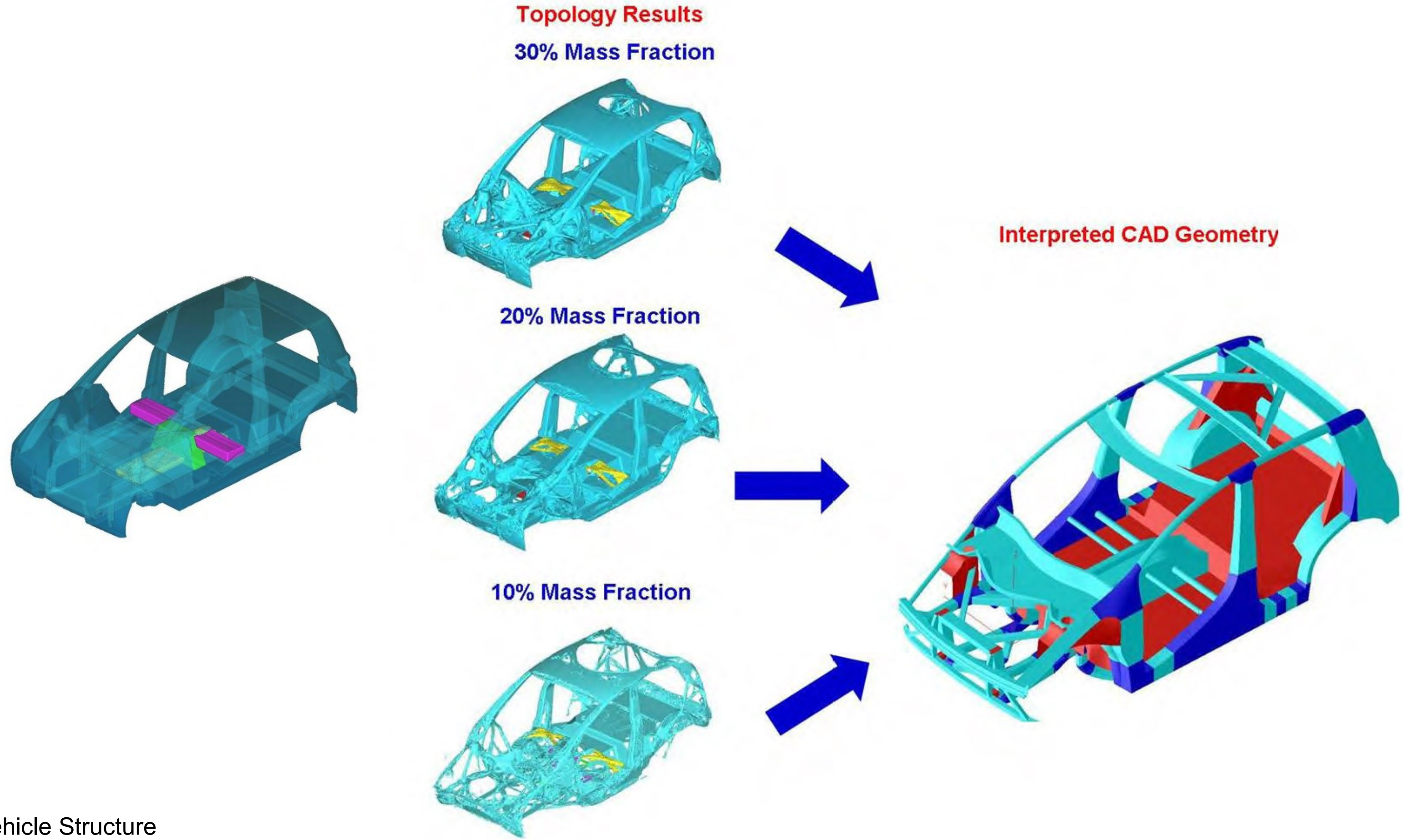


Linear-Static Topology Optimization

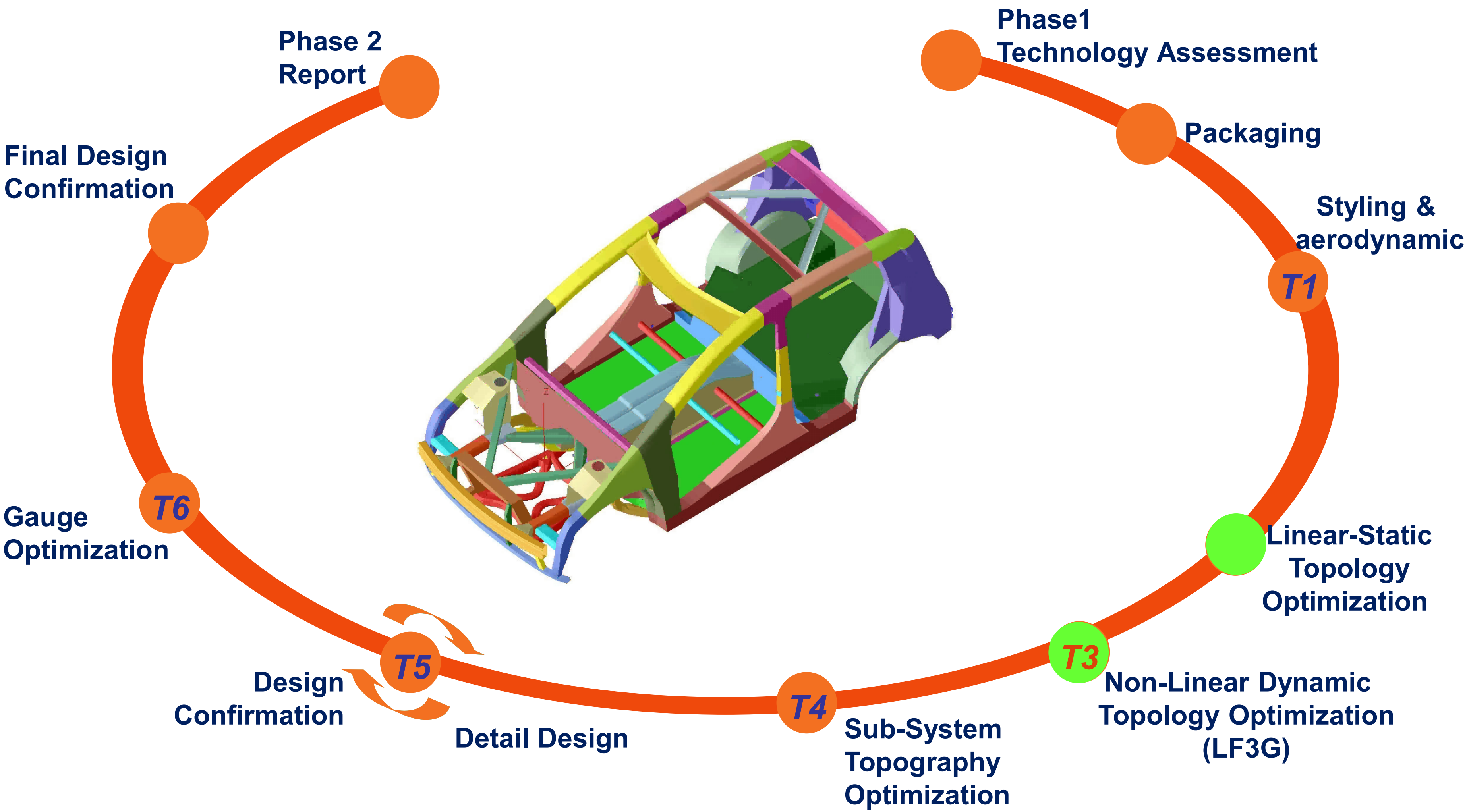


T2: Topology Optimization

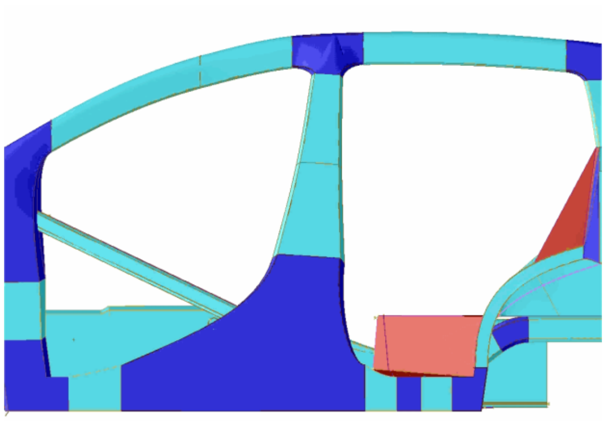
– geometry developed by the topology optimization was manually interpreted into a CAD (Computer Aided Design) model using engineering judgment

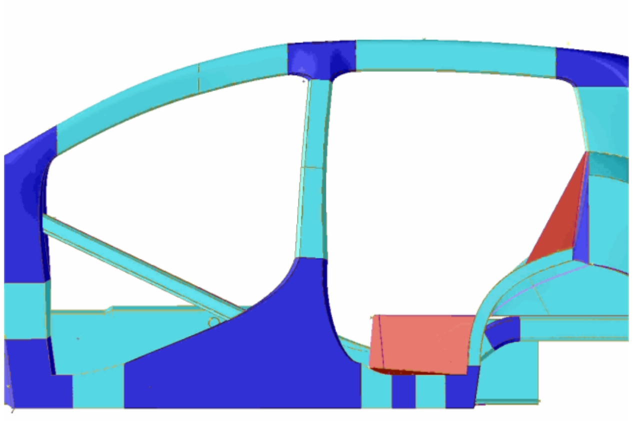


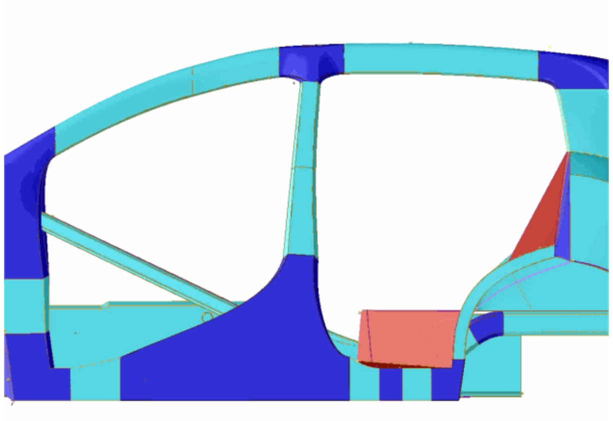
LF3G Load Path and 3G Optimization

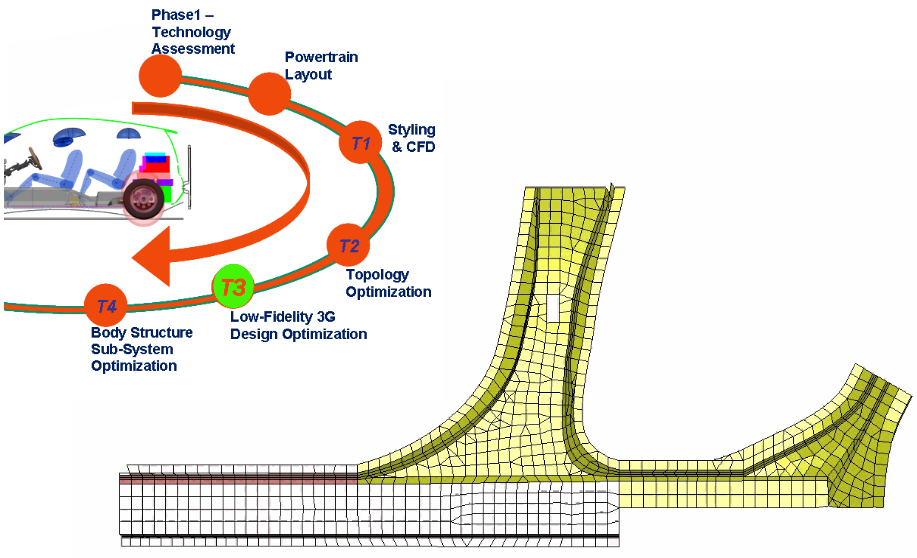


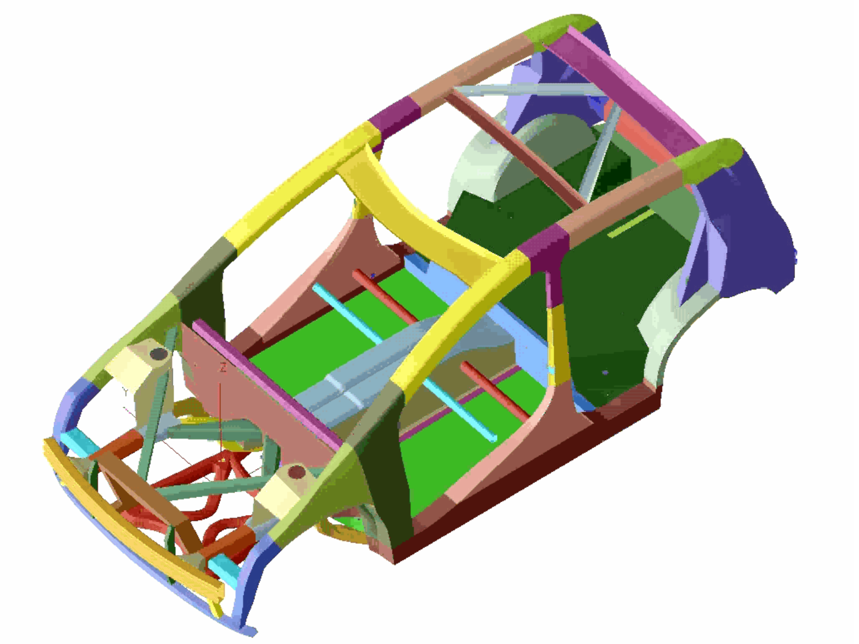
Low Fidelity 3G (Geometry, Gauge & Grade) Optimization



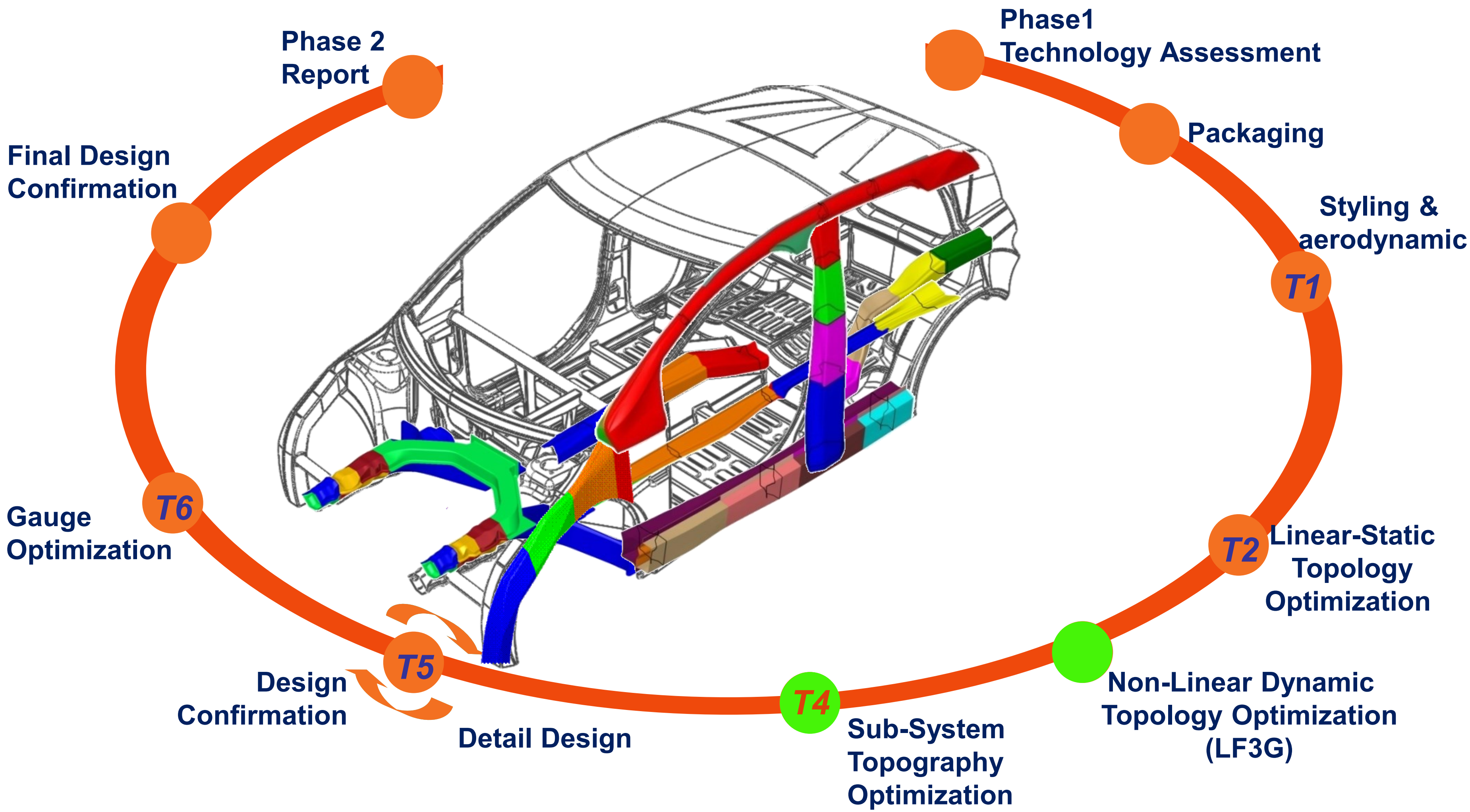




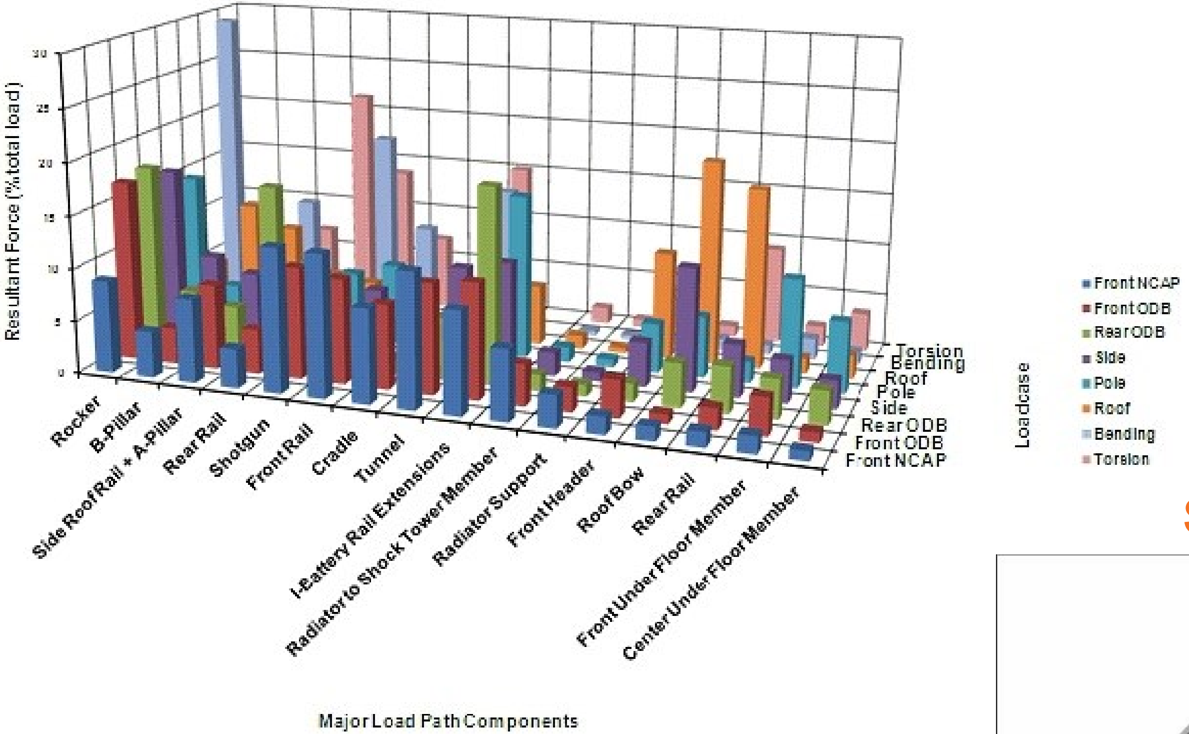




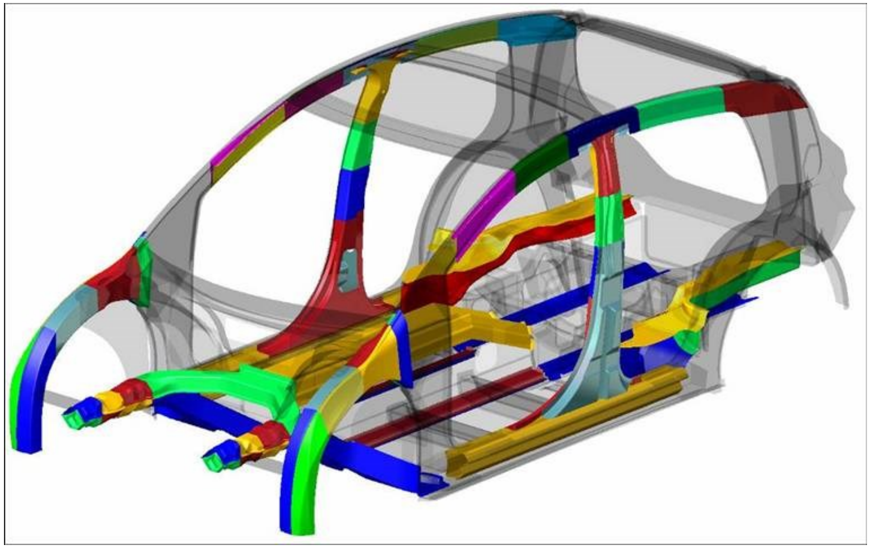
Sub-Systems 3G Optimization



T4 Load Path Mapping

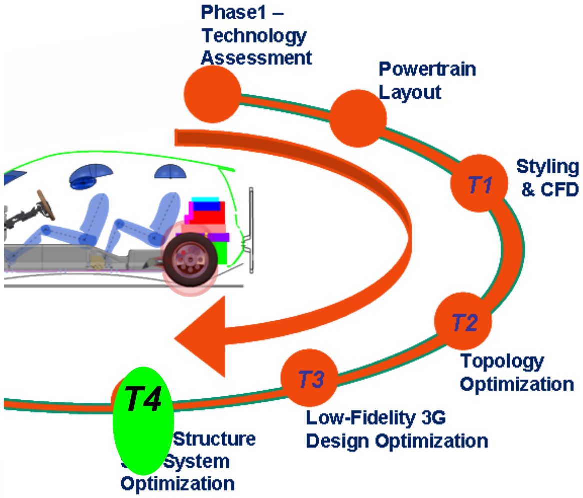


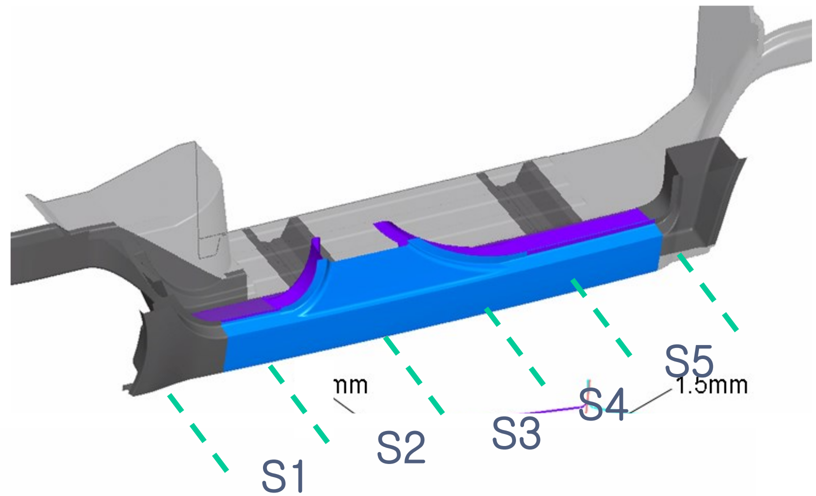
Selected Sub-Systems

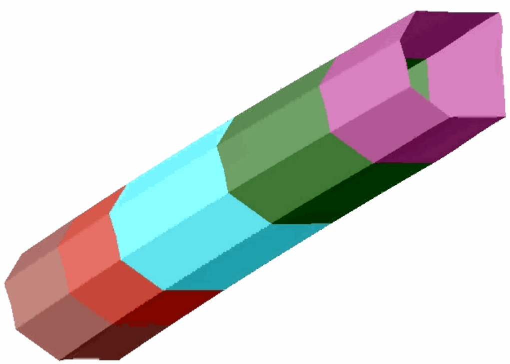


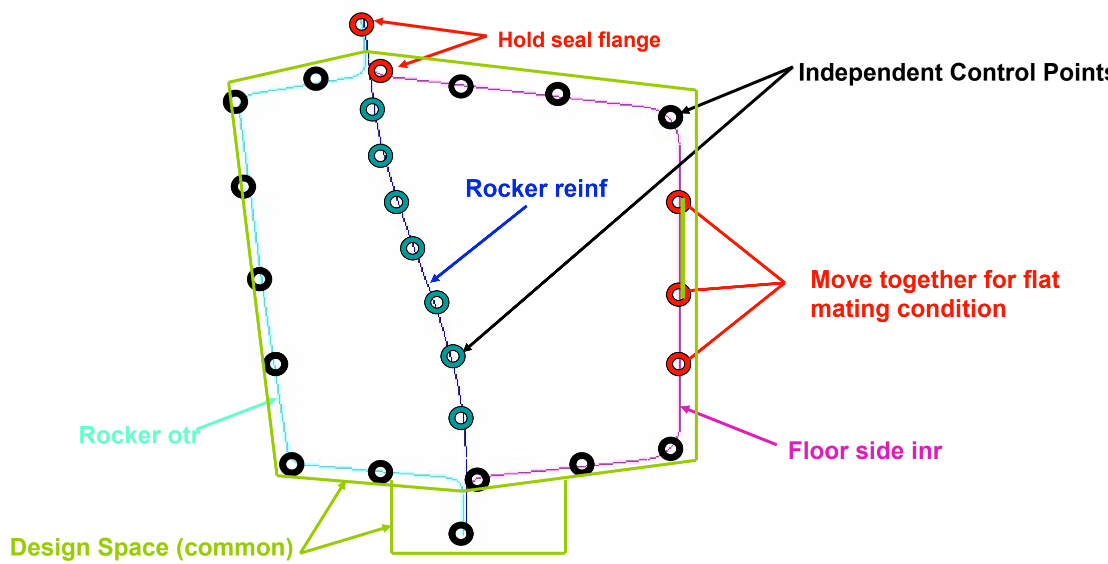
- **Front Rail**
- **Shot Gun**
- **Rocker**
- **B-Pillar**
- **Rear Rail**
- **Roof Rail**
- **Tunnel Reinforcement**

Body Structure – Sub-System 3G Optimization









정리: 위상최적설계

- 구조물 설계에 있어서 CAE 주도 설계: 위상최적설계 가 최적격이자 필수
- 초기설계 없이도 구조물 최적설계 가능
  - 개념설계단계에서 효과적
  - 선행(Up-Front) CAE에 적합
  - 구조물 경량화에 큰 공헌: 혁신설계
- 설계자에게 사용 가능한 도구로 발전하기 위한 주안점

1. 위상최적설계 결과 (역학적으로 최적인 구조)
2. 설계자가 사용할 수 있는 CAD 모델로 쉽게 변환
3. 설계자가 수정한 CAD 모델로 자동 재해석: 검증 완료