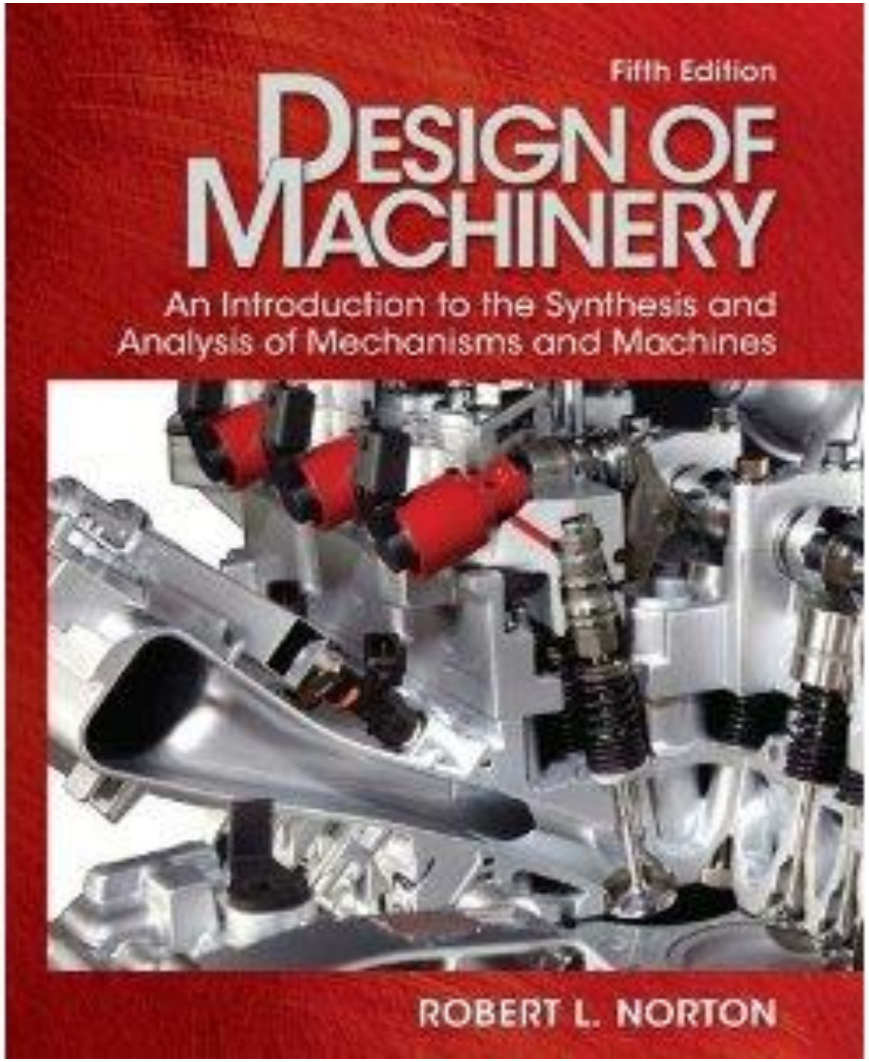

Fifth Edition
DESIGN OF MACHINERY
An Introduction to the Synthesis and Analysis of Mechanisms and Machines
ROBERT L. NORTON

Kinematics and Kinetics

- Kinematics
  - The study of motion without concern of what forces cause the motion, first step in the analysis or design of a mechanism
- Kinetics
  - The study of forces on systems in motion
- Design requires the knowledge both
- Dynamics
  - Combination of kinematics and kinetics

- However, first must know the acceleration before applying Newton’s 2nd Law (F=ma)

- Designs require some finite service life, with this in mind the following must be taken into consideration:
  - The stresses on the system must be within some allowable range
  - The maximum forces on the system must be within some allowable range
  - The acceleration must be known
- How do we accomplish this? Kinematics

Mechanism and Machine

- Mechanism
  - A device that transforms motion into some desirable pattern and typically develops very low forces and transmits very little power
  - Combination of rigid or resilient bodies joined together to provide a specific absolute motion
- Machine
  - Contains mechanisms which are designed to provide significant forces and transmit significant power
  - Mechanism capable of performing useful work or capable of transmitting significant forces

Degrees of Freedom (DOF)

- Number of independent parameters (coordinates) which are needed to uniquely define position in space at any instant in time
  - In a 3-D frame: x, y, z, θ, ϕ, ρ
- Rigid body: essentially nondeformable
  - Assume: rigid & massless
- Types of motion
  - Rotation + Translation = Complex Motion

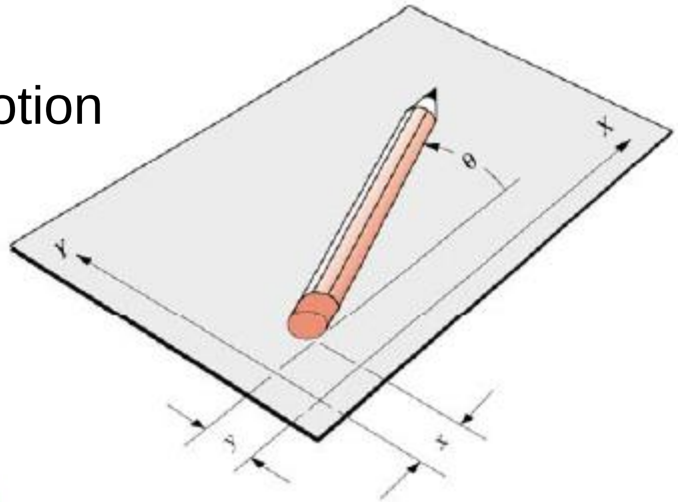
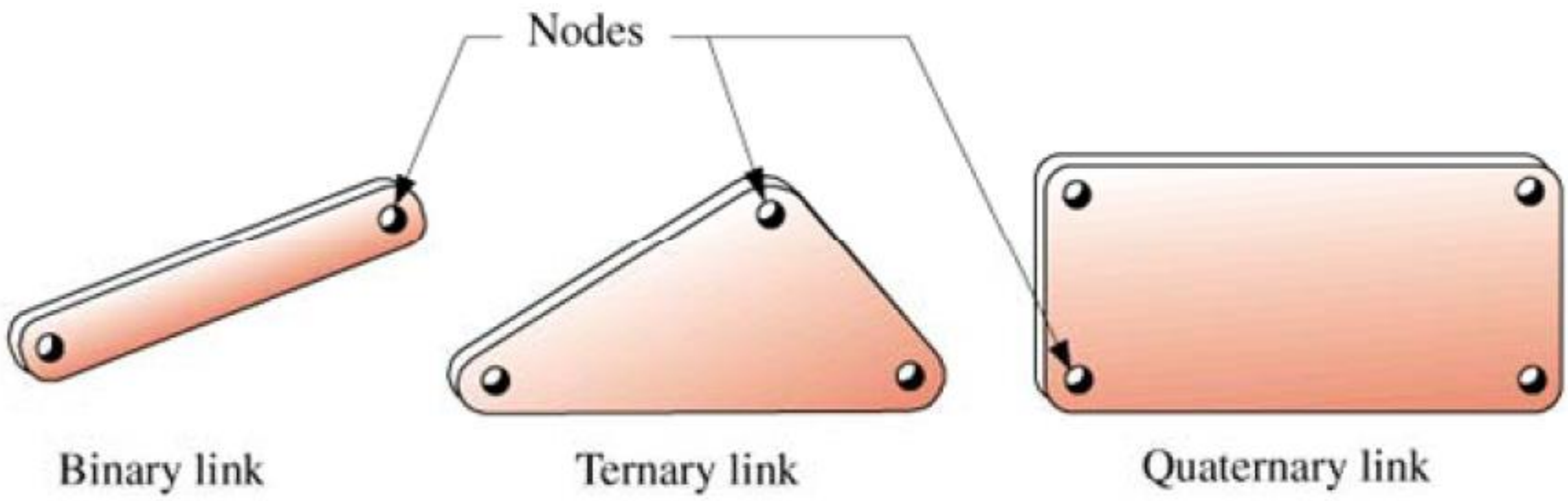


FIGURE 2-1

A rigid body in a plane has three *DOF*

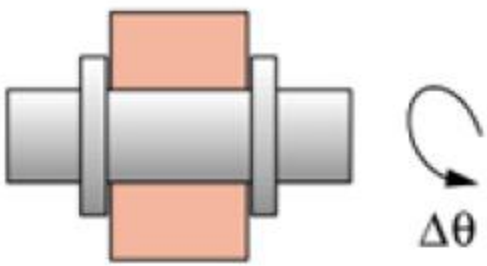
Links, Joints and Kinematic Chains

- Link: rigid or flexible members have at least two nodes (points of attachment)
- Linkage: basic building block of all mechanisms
- Links of different order

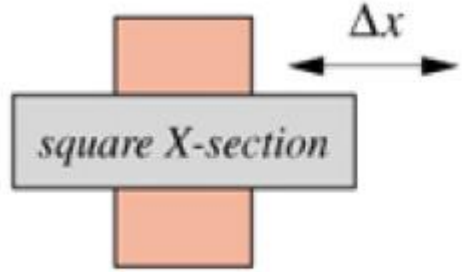


Six Lower Pairs

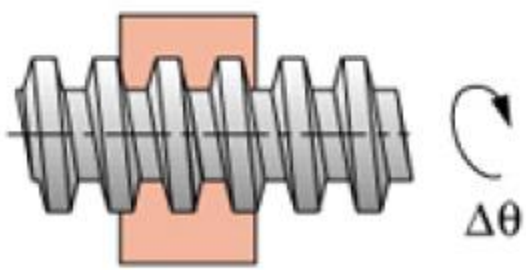
- Joint: connection between two or more links (at their nodes), which allows some motion between the links



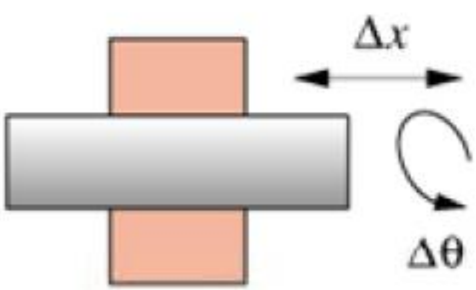
Revolute (R) joint—1 *DOF*



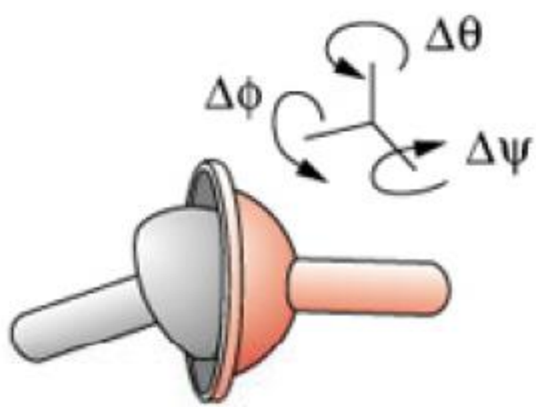
Prismatic (P) joint—1 *DOF*



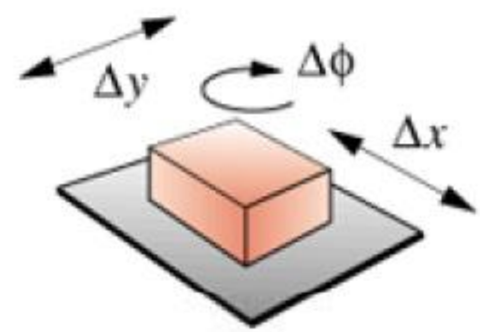
Helical (H) joint—1 *DOF*



Cylindric (C) joint—2 *DOF*



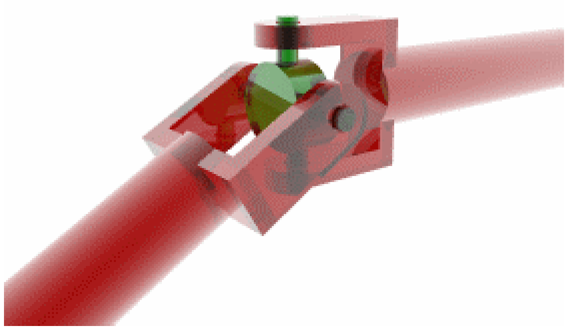
Spherical (S) joint—3 *DOF*



Planar (F) joint—3 *DOF*

Table 1. Freedom of general kinematic pair.

Translation/Rotation	0	1	2	3
0	Fixed	Revolute	Universal	Spherical
1	Translational	Cylindrical		
2		Planar		



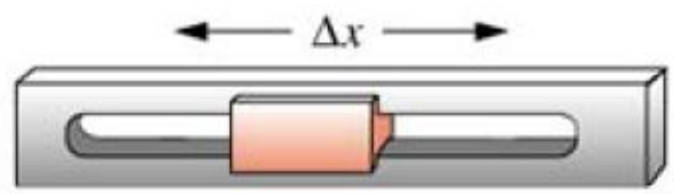
Joint (1)

- Joints are classified into several ways:
  - Number of DOF allowed at a joint
  - By type of contact
  - By type of physical closure (Force or Form closed)
  - Number of links (order of joint)

- Full joints: 1 DOF

Rotating full pin (R) joint (form closed)

pin (or revolute) joint

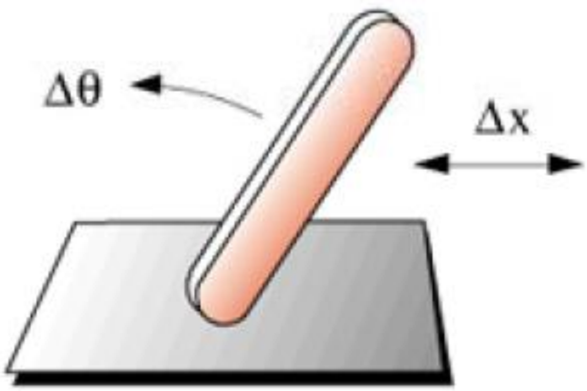


Translating full slider (P) joint (form closed)

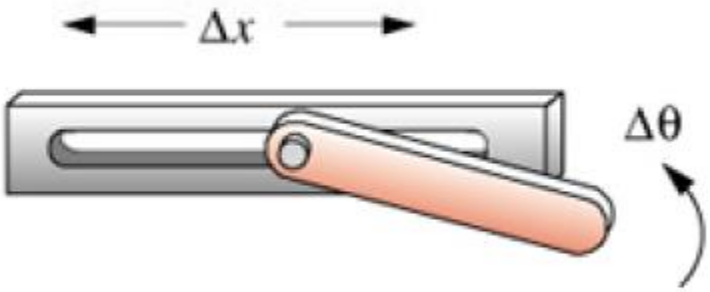
sliding (or prismatic) joint

Joint (2)

- Half (or RP, Roll-Slide) joints: 2 DOF



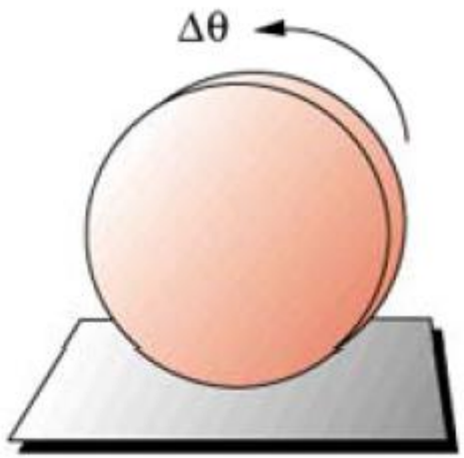
Link against plane (force closed)



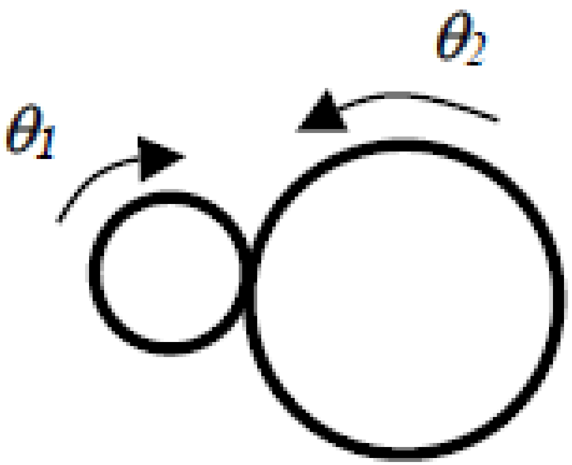
Pin in slot (form closed)

Joint (3)

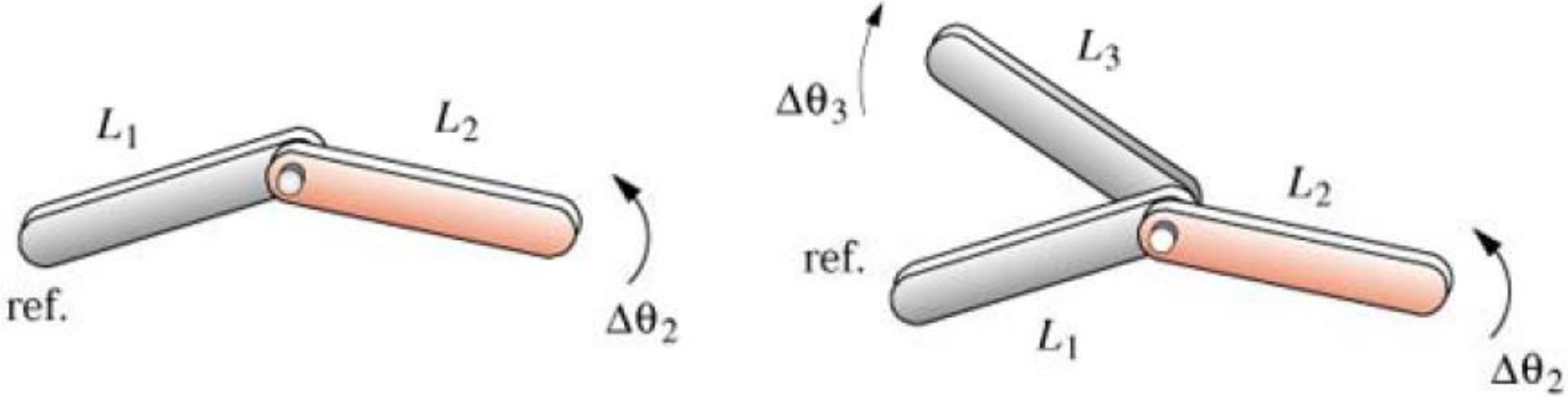
- Rolling joint: 1 or 2 DOF (may roll, slide, or roll-slide depending on friction)
  - Pure-roll (R), pure-slide (P), roll-slide (RP)



May roll, slide, or roll-slide, depending on friction



- Order = (number of links joined) – 1



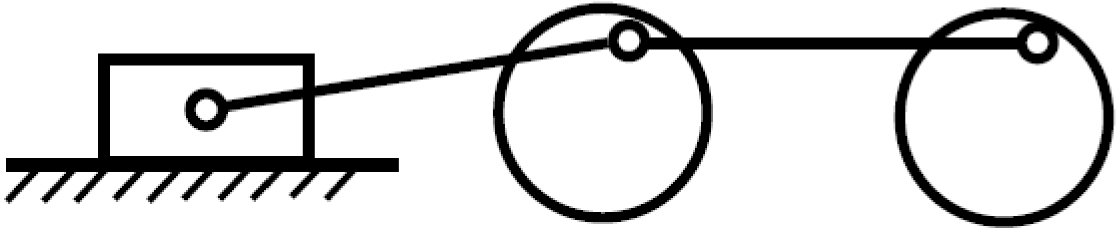
First order pin joint - one *DOF*
(two links joined)

Second order pin joint - two *DOF*
(three links joined)

- Links are combined using joints to form kinematic chain or just linkage
- Linkage with at least one link fixed → mechanism

Planar Motion

- Rectilinear translation
  - Points in the body move in parallel straight lines (piston)
- Curvilinear translation
  - Points in the body move along identical curves
  - Link does not rotate w.r.t. the ground (link connecting two disks)
- Rotation
  - Points in the body rotate about a single point, which is usually fixed to the ground (disks)
- General planar motion
  - General combination of rotation and translation (connecting rod joining piston and disk)

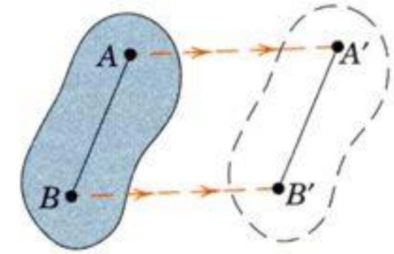
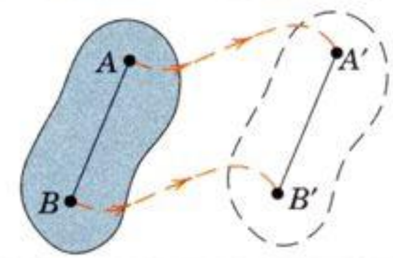
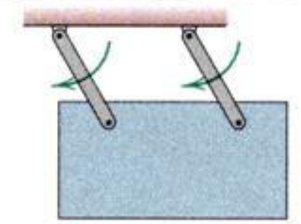
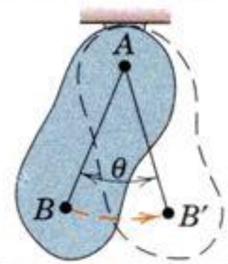
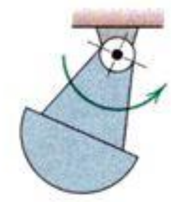
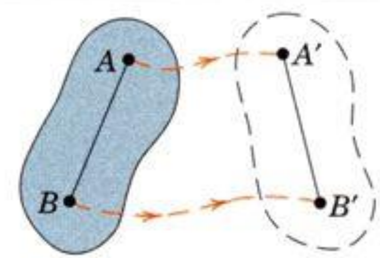
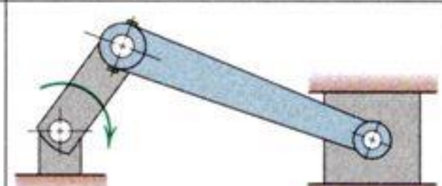


Rigid-Body Plane Motion

I (Pure) Translation

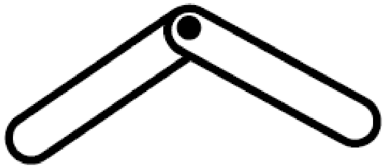
II (Pure) Rotation

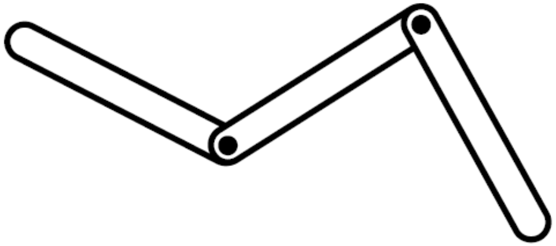
III General Motion

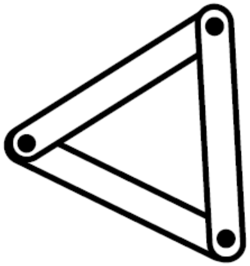
	Type of Rigid-Body Plane Motion	Example
(a) Rectilinear translation		 Rocket test sled
(b) Curvilinear translation		 Parallel-link swinging plate
(c) Fixed-axis rotation		 Compound pendulum
(d) General plane motion		 Connecting rod in a reciprocating engine 4

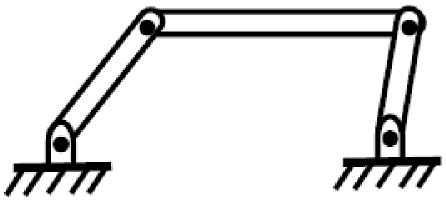
Linkage Classification by Number of Links

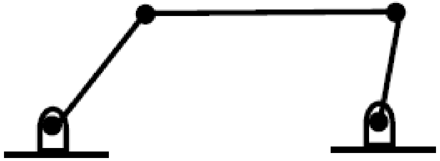
- Two links with one joint (dyad)
- Three links (triad)
- Four links (four-bar mechanism)
  - Joined by 4 pin-joints
  - Slider joint replaces one of the pin joints

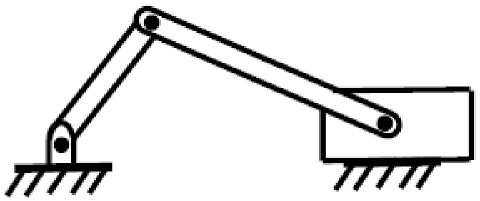


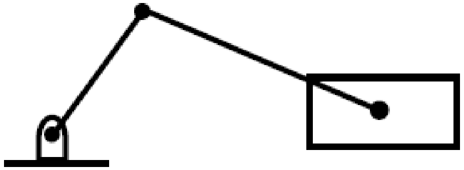




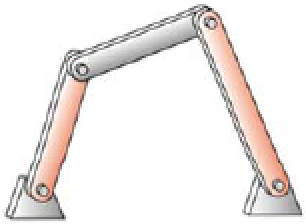






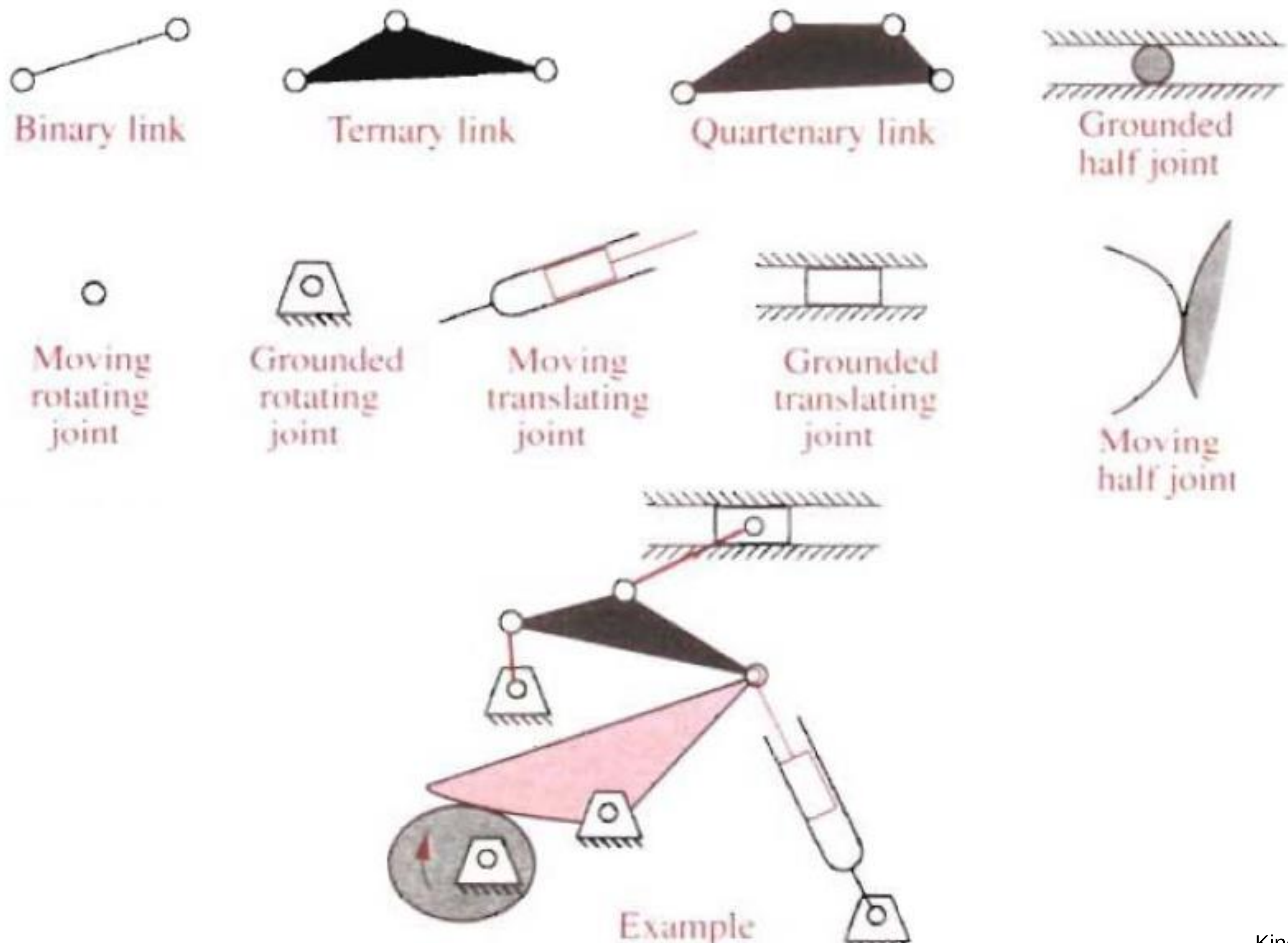


- Kinematic chain
  - An assemblage of links and joints to provide a controlled output motion in response to a supplied input motion
- Crank
  - A link which makes a complete revolution and is pivoted to ground
- Rocker
  - A link which has an oscillatory rotation and is pivoted to ground
- Coupler (aka Connecting Rod)
  - Has a complex motion and is not pivoted to ground
- Ground
  - Any link that is fixed w.r.t. the reference frame
  - Note: reference frame may be in motion



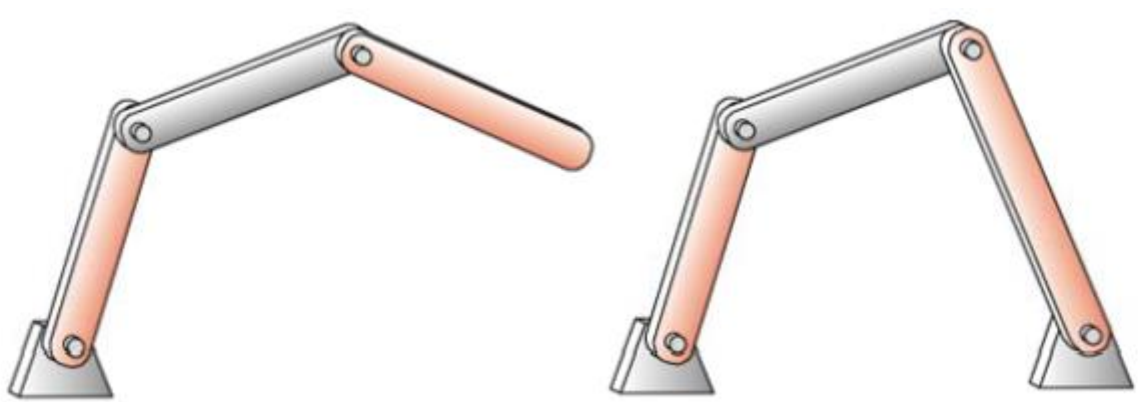
[Crank-Rocker Four-Bar Linkage - YouTube]

Drawing Kinematic Diagrams



Determining DOF (1)

- Kinematic chains may be either OPEN or CLOSED



(*a*) Open mechanism chain (*b*) Closed mechanism chain

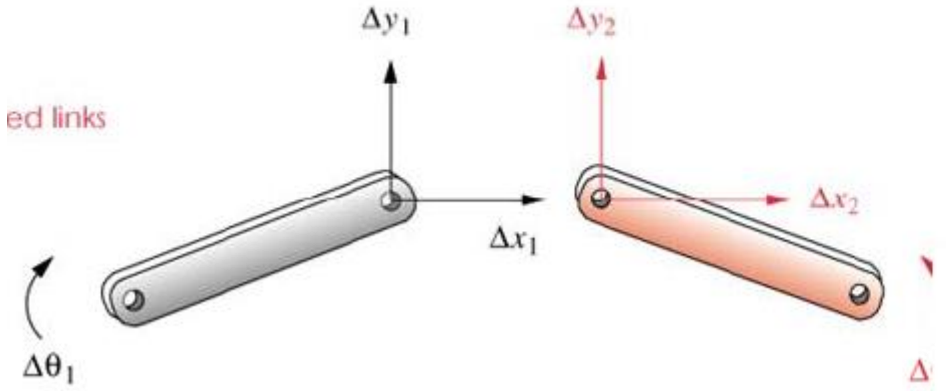
- Dyad
  - An open kinematic chain of two binary links and one joint

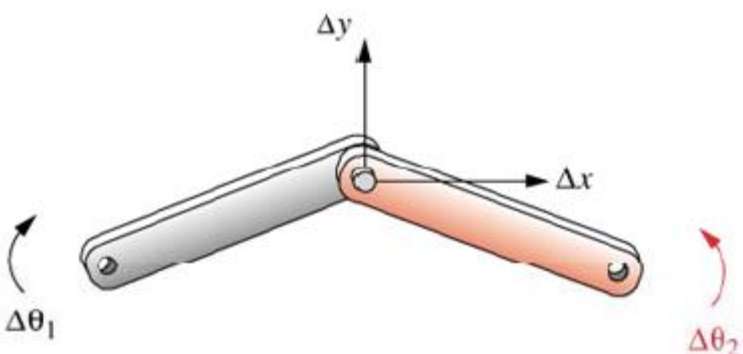
Determining DOF (2)

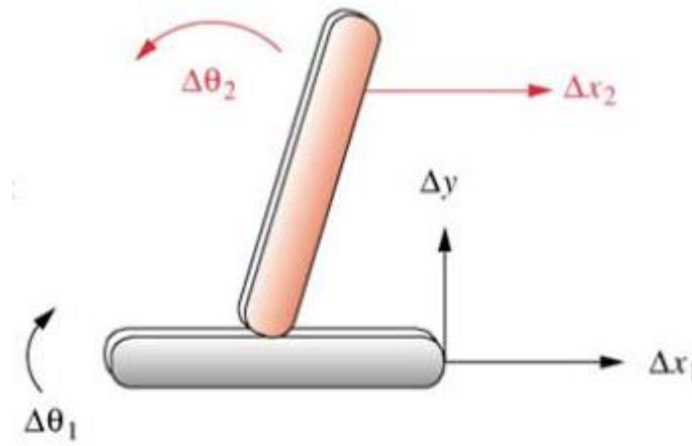
- To determine the overall DOF
  - Account for number of links and joints and their interactions
- Any free link on a plane has 3 DOF: Δx, Δy, $\Delta \theta$
- Gruebler's Equation
  - DOF = 3L - 2J - 3G
  - L: number of links
  - J: number of joints
  - G: number of ground
  - As there is always only one ground plane, G=1 always
  - DOF = 3(L – 1) - 2J
  - Note: J=1/2 for half joints because a half joint only removes one DOF

Determining DOF (3)

- Kutzbach's modification
  - $DOF = 3(L - 1) - 2J_1 - J_2$
  - L: number of links
  - J_1: number of full joints
  - J_2 : number of half joints

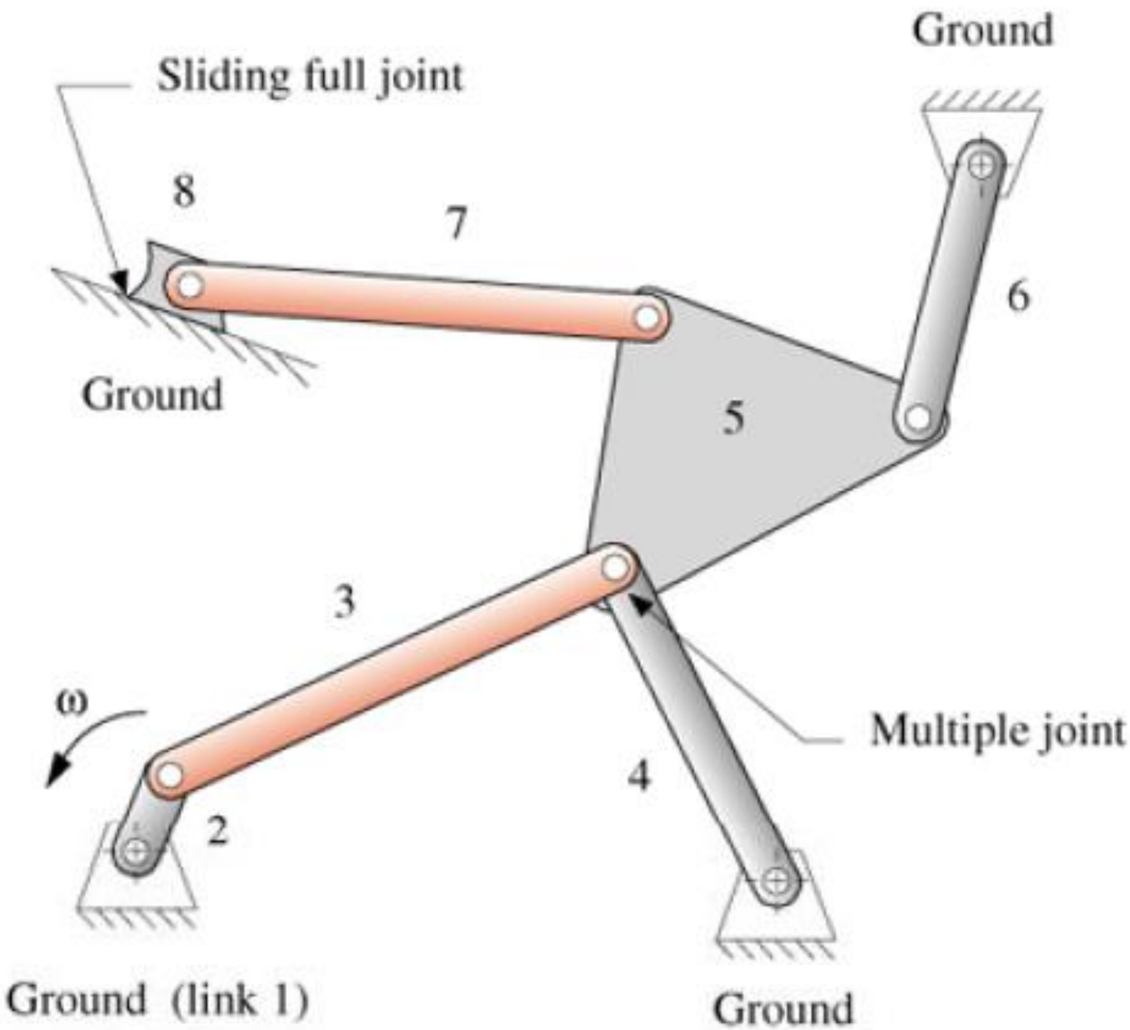




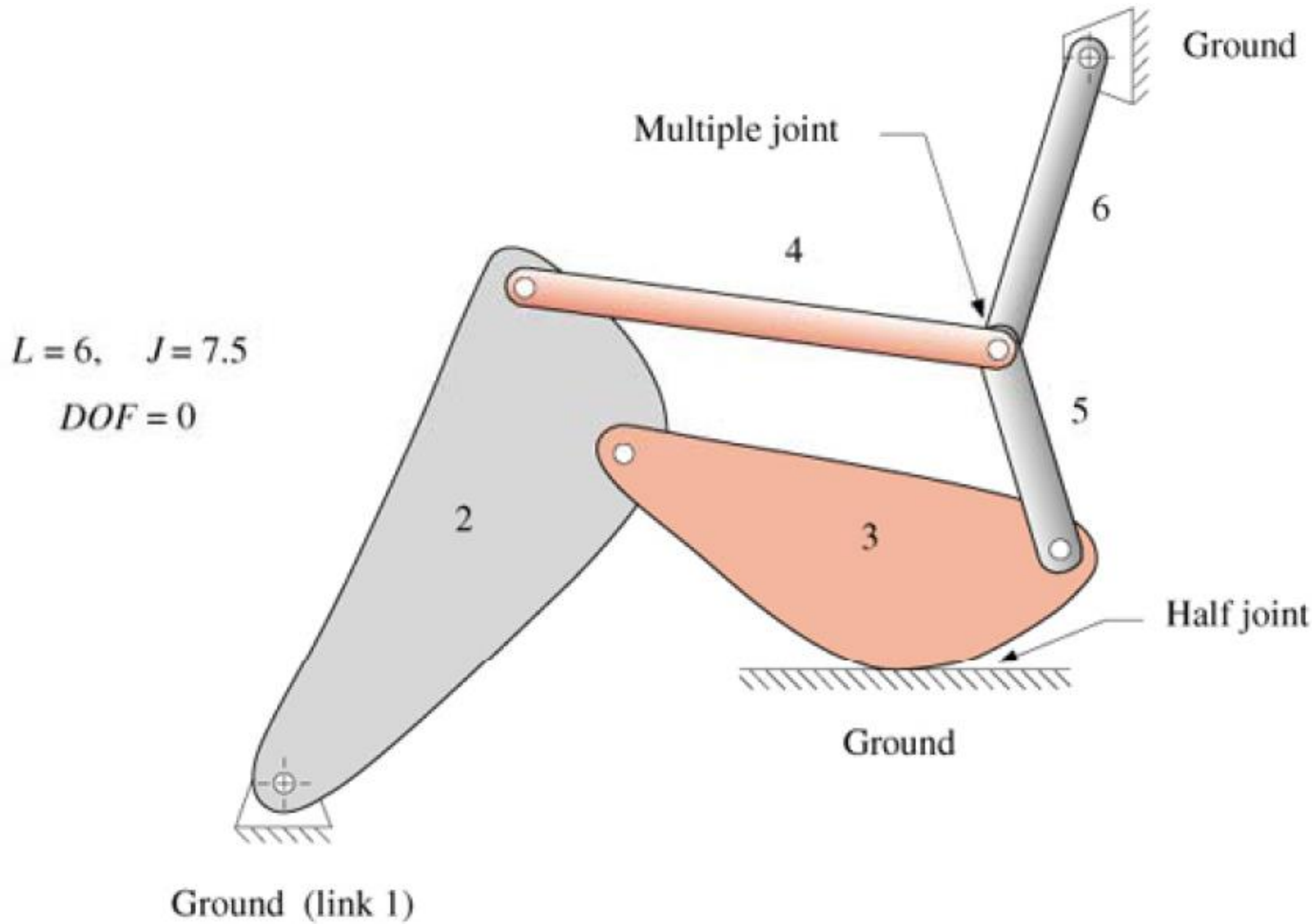


Determining DOF (4)

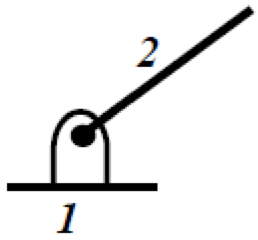
Name (Symbol)	*DOF*	Cont-ains
Revolute (R)	1	R
Prismatic (P)	1	P
Helical (H)	1	RP
Cylindric (C)	2	RP
Spherical (S)	3	RRR
Planar (F)	3	RPP

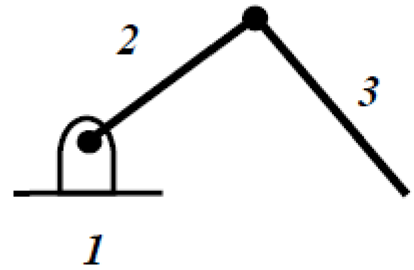


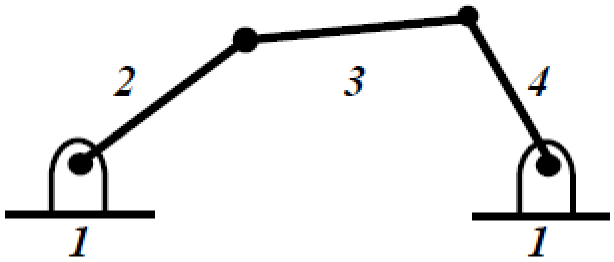
Determining DOF (5)

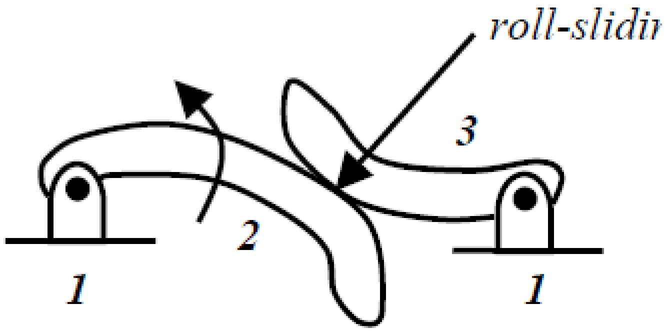


Calculation of Mobility

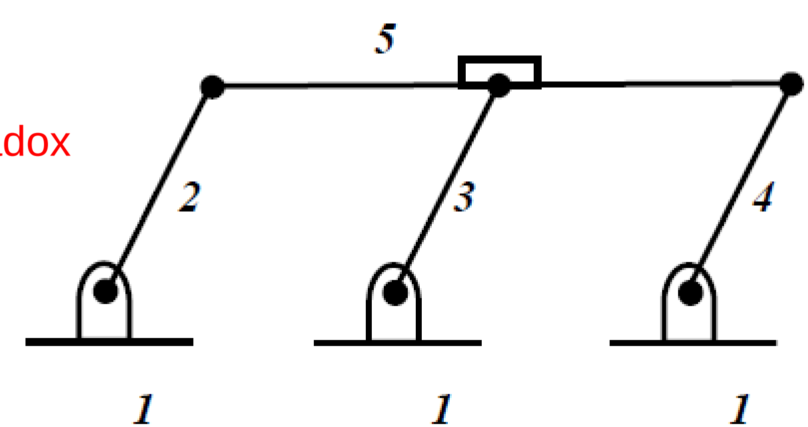


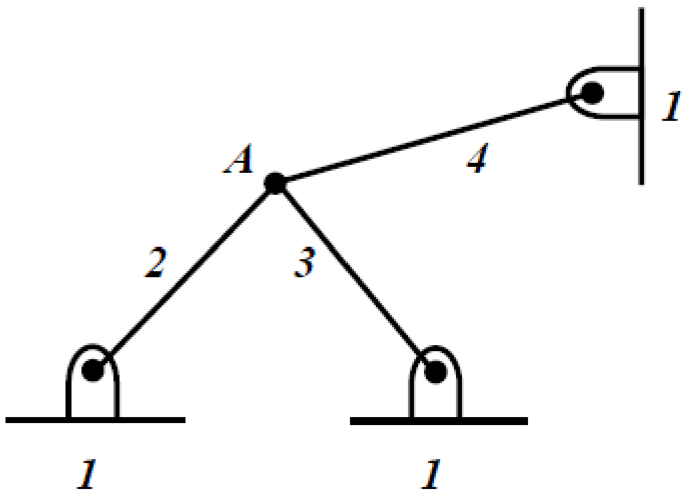






paradox

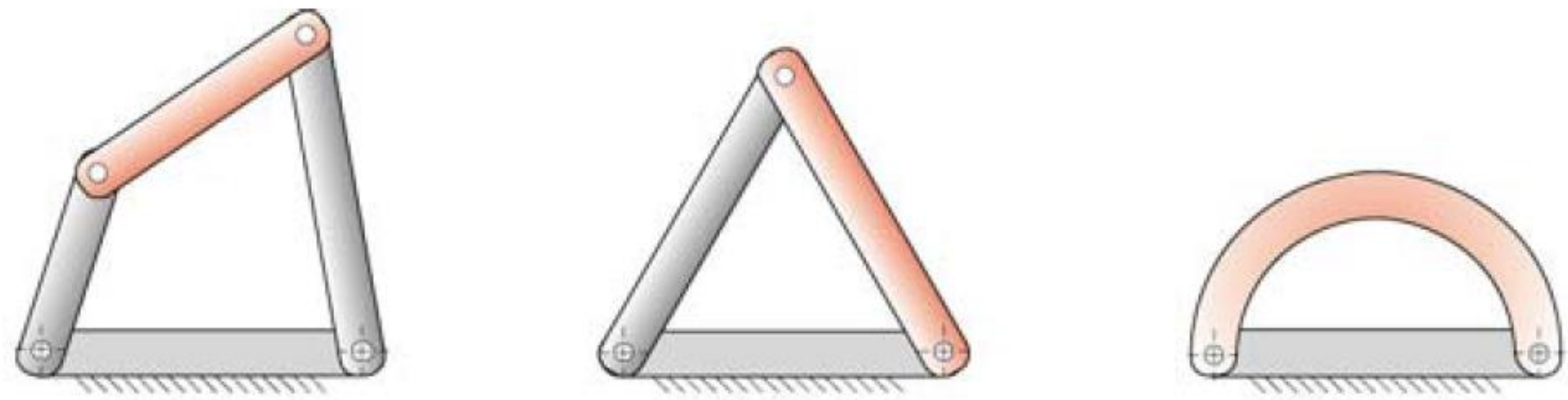




Statically indeterminate structure

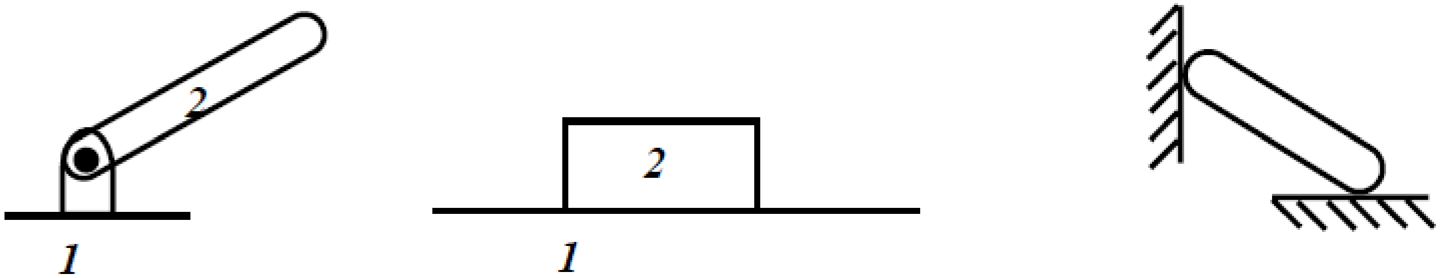
Mechanisms and Structures

- Mechanism: DOF > 0 → possible motion
- Structure: DOF = 0 → no motion possible
- Preloaded structure: DOF < 0 → no motion and possible stresses

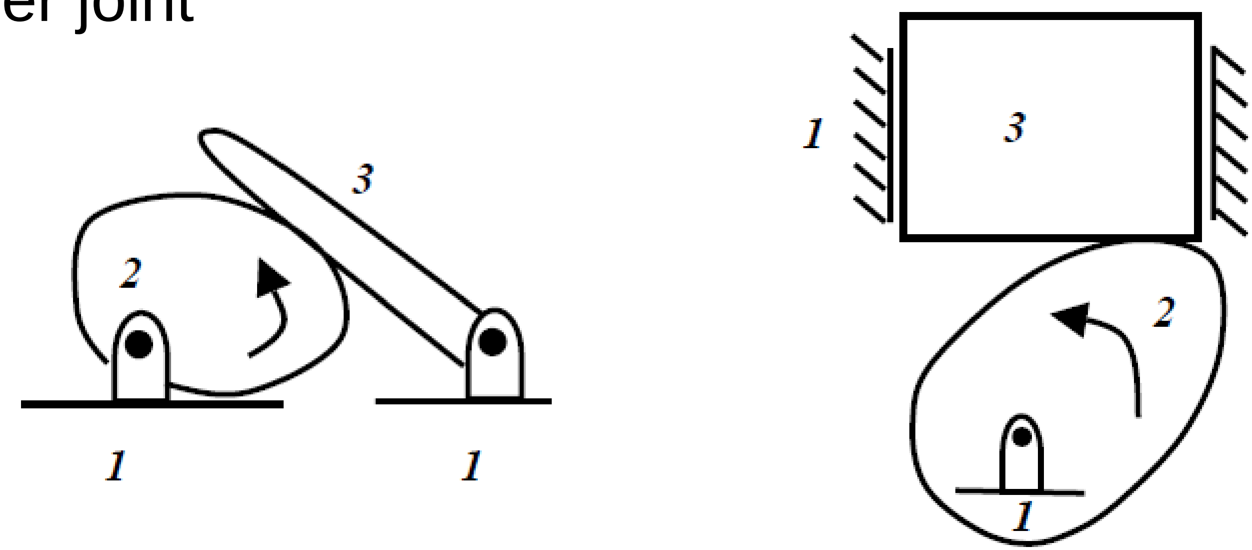


Simple Mechanisms (1)

- 2 links: [3*(2-1)-2*1=1] by adding a single pin or slider joint

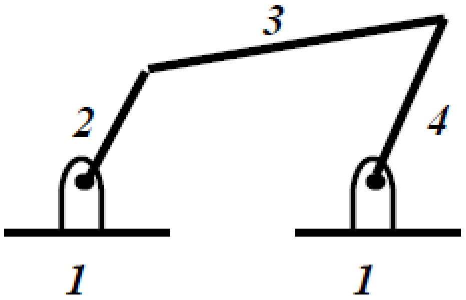


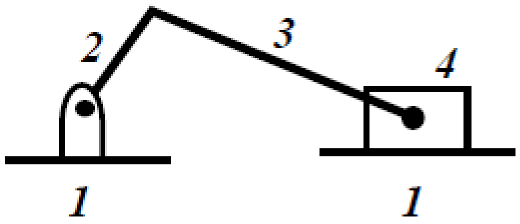
- 3 links: [3*(3-1)-2*2-1=1] by adding two pins and a roll-slider joint



Simple Mechanisms (2)

- 4 bars + 4 pins: 4-bar mechanism [3*(4-1)-2*4=1]

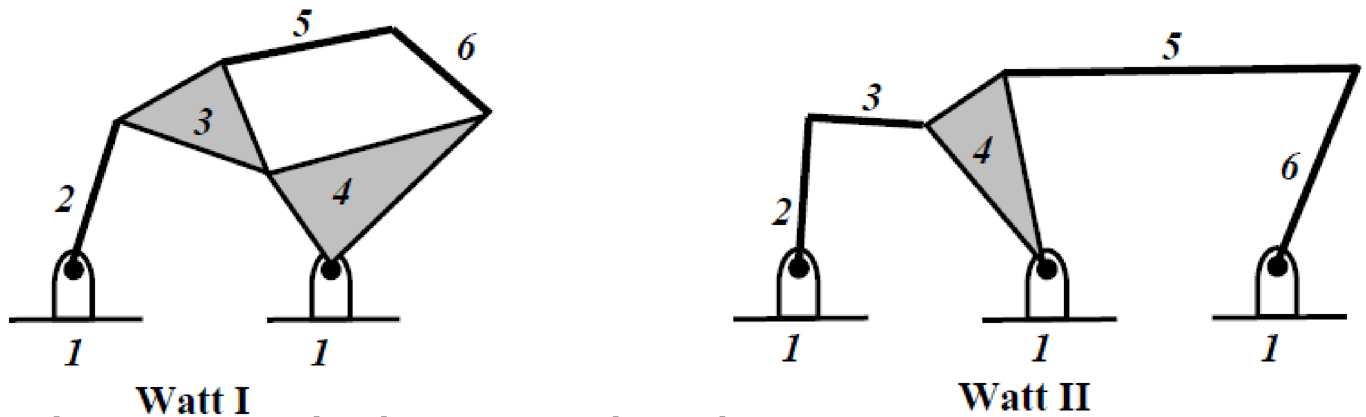




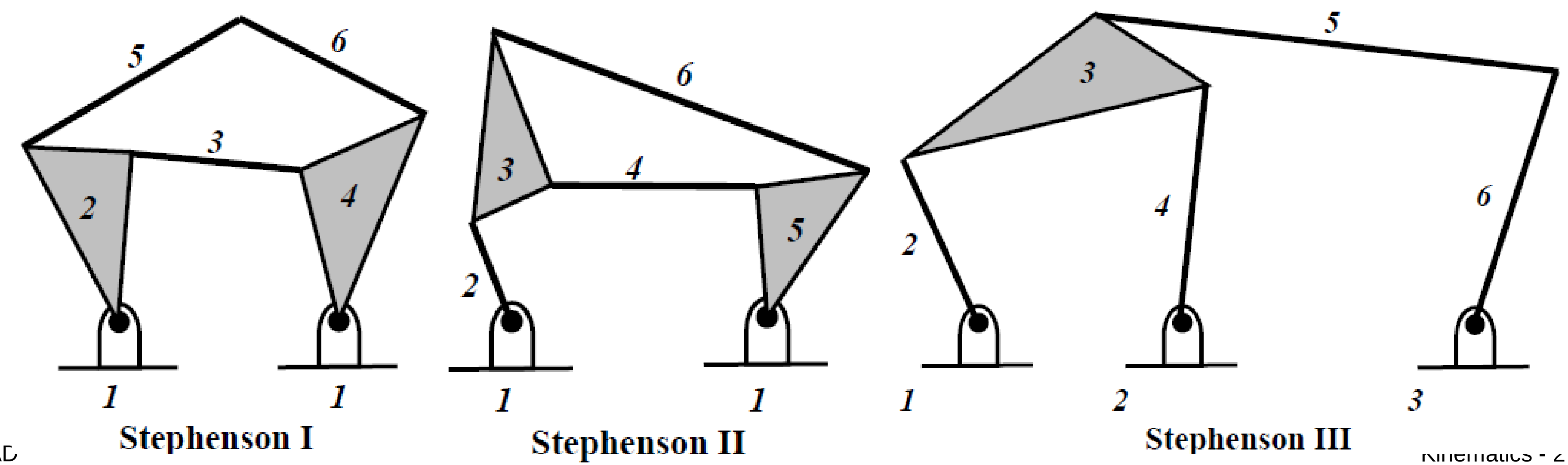
  - https://www.youtube.com/watch?v=KBFFwgCCP0U
- 5 links + pin or slider joints: not possible
- 6-bar linkage: [3*(6-1)-2*7=1]
  - 6 binary links? 6 pin joints (x)
  - 2 ternary (adjacent) + 4 binary links : Watt linkage
  - 2 ternary (separate) + 4 binary links : Stephenson linkage

Simple Mechanisms (3)

- Watt six-bar mechanism



- Stephenson six-bar mechanism



Number Synthesis

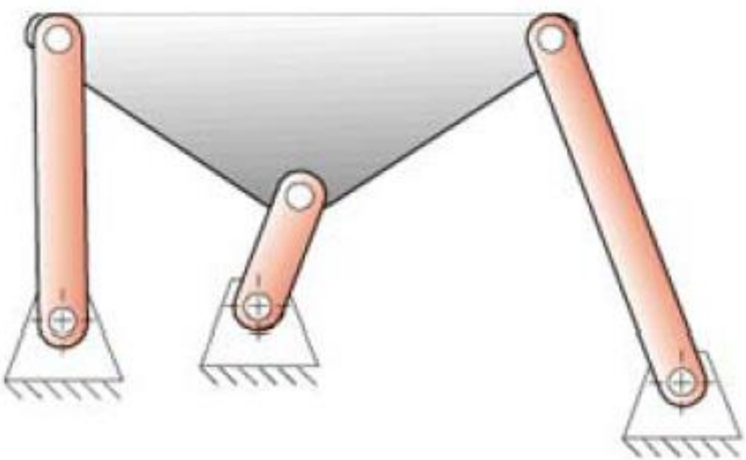
- The determination of the number and order of links and joints necessary to produce motion of a particular DOF
  - Gives all possible combinations of linkage combinations from which to choose
- Order: number of nodes per link
  - Binary, ternary, quaternary
- Example: drive all possible link combinations for one DOF for sets ≤ 8 links and link orders ≤ 6
  - Assume only full rotating joints

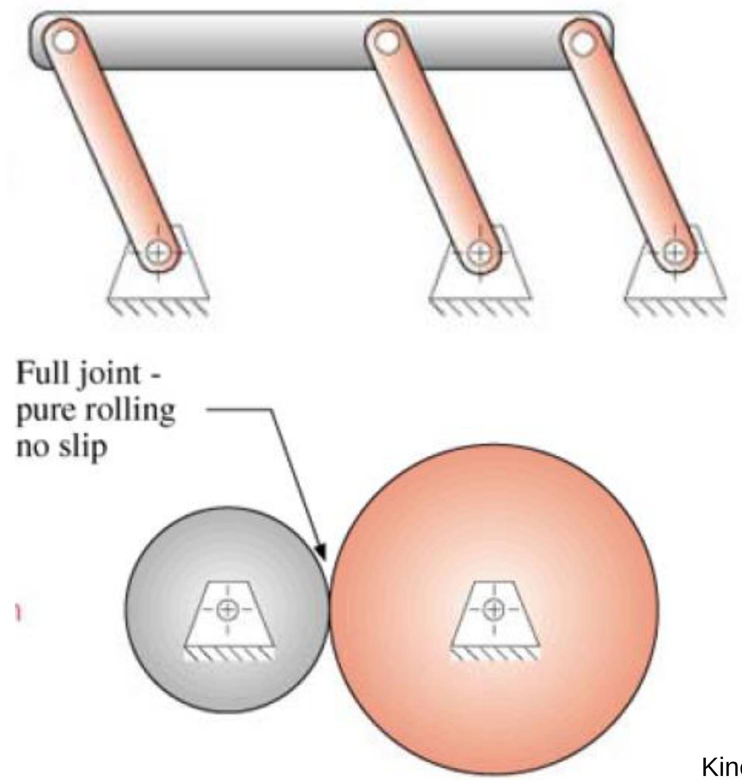
TABLE 2-2 1-*DOF* Planar Mechanisms with Revolute Joints and Up to 8 Links

Total Links	Link Sets				
	Binary	Ternary	Quaternary	Pentagonal	Hexagonal
4	4	0	0	0	0
6	4	2	0	0	0
6	5	0	1	0	0
8	7	0	0	0	1
8	4	4	0	0	0
8	5	2	1	0	0
8	6	0	2	0	0
8	6	1	0	1	0

Paradoxes

- Gruebler Equation ignores link sizes and shapes
  - It can give misleading conclusions for unique geometric configurations
  - Example: E-quintet

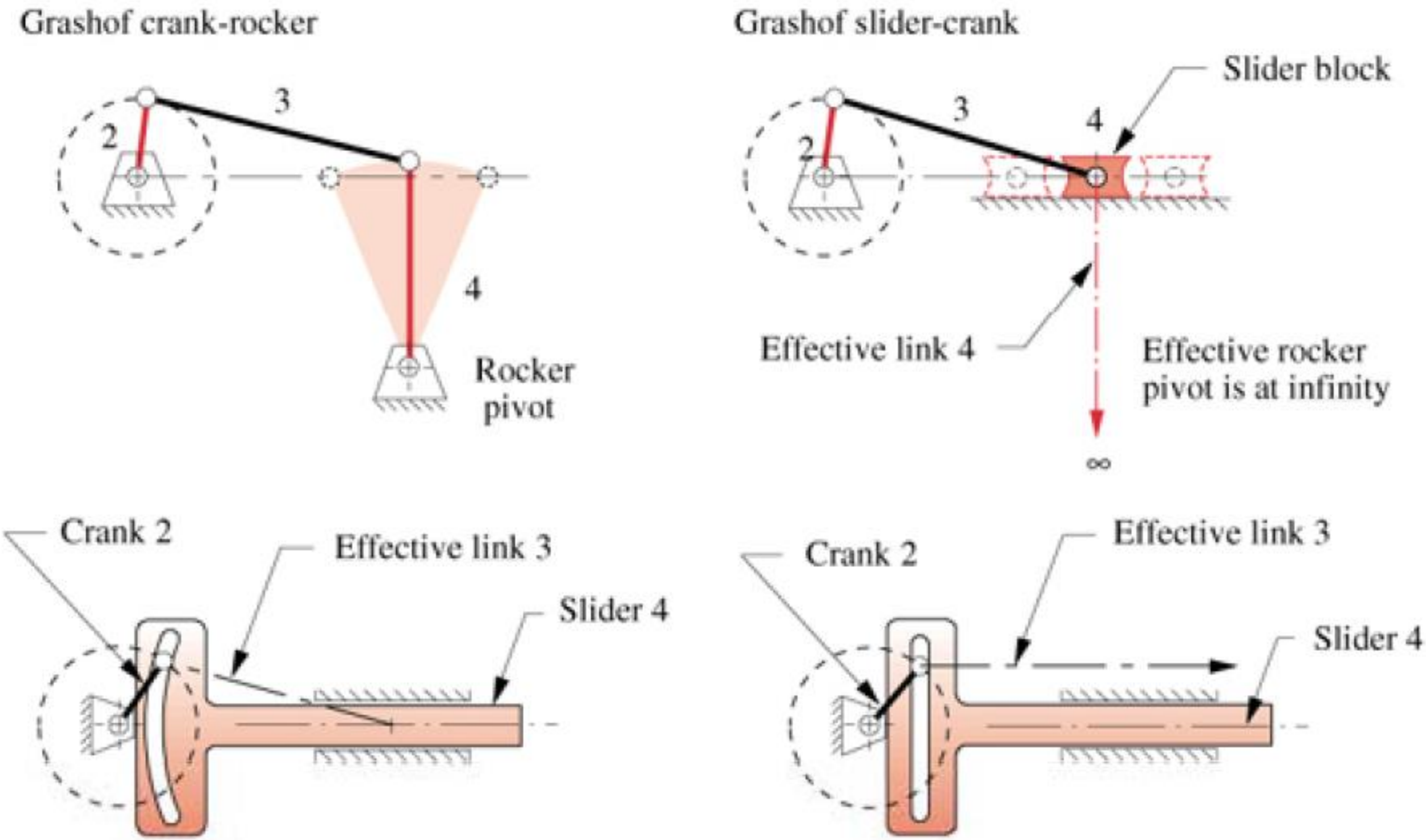




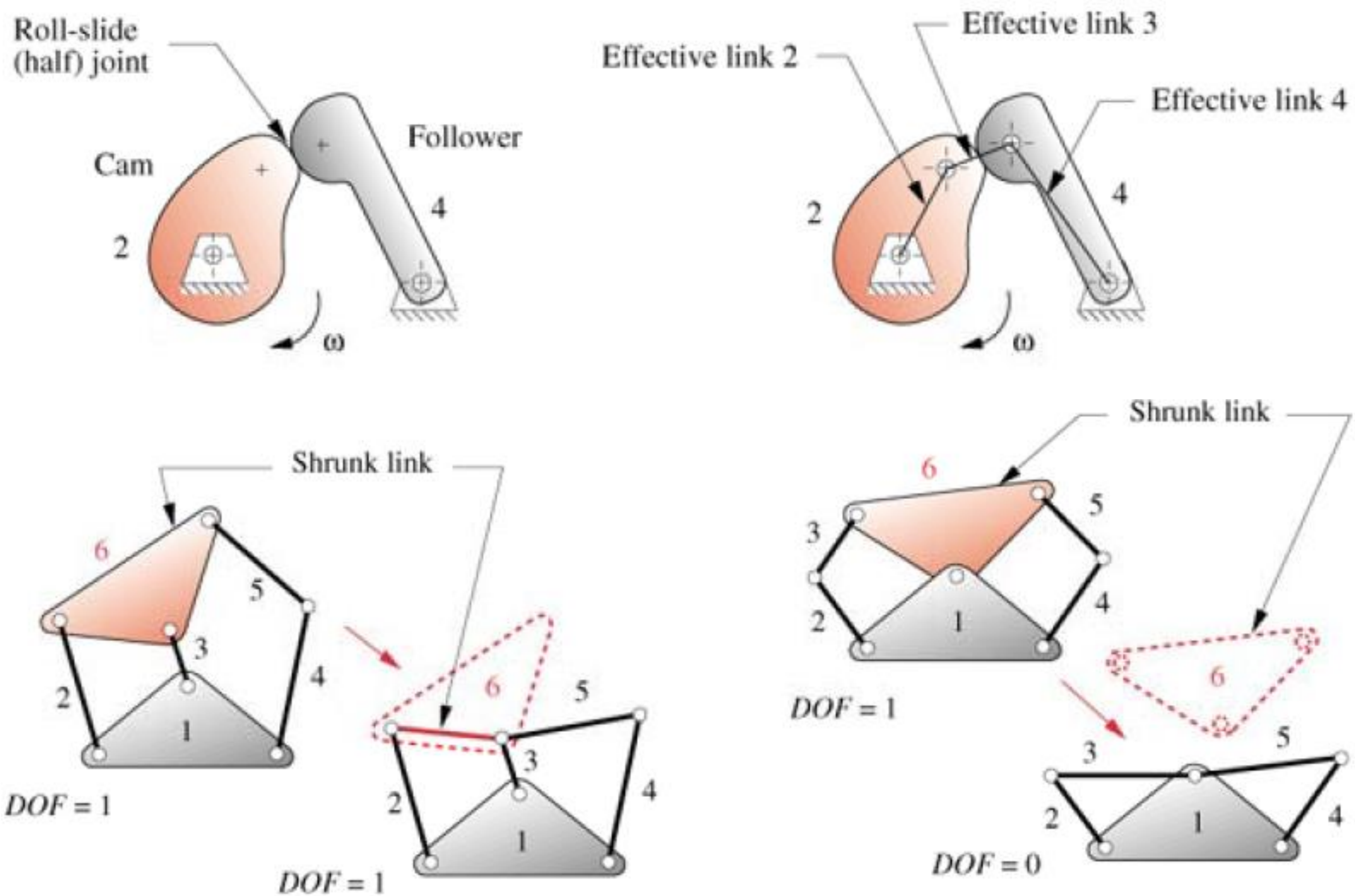
Linkage Transformation (1)

- [1] Any full rotating joint can be replaced by a sliding full joint with no change in DOF of the mechanism
- [2] Any full joint can be replaced by a half joint, but this will increase DOF by 1
- [3] Removal of a link will reduce DOF by 1
- [4] The combination of [2] and [3] will keep the original DOF unchanged
- [5] Any ternary or higher order link can be partially ‘shrunk’ to a lower order link by coalescing nodes
  - Creation of a multiple joint, but will not change DOF
- [6] Complete shrinkage of a link is equivalent to its removal
  - Creation of a multiple joint and DOF will be reduced

Linkage Transformation (2)

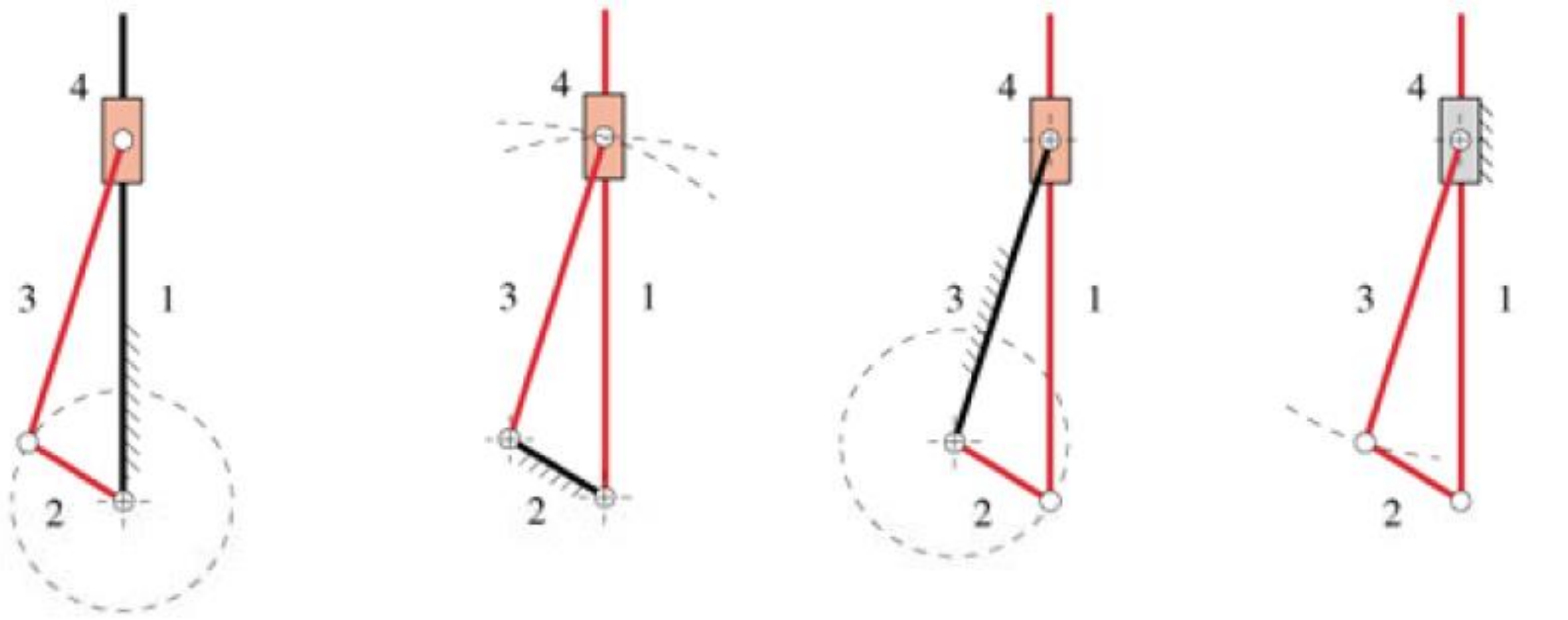


Linkage Transformation (3)

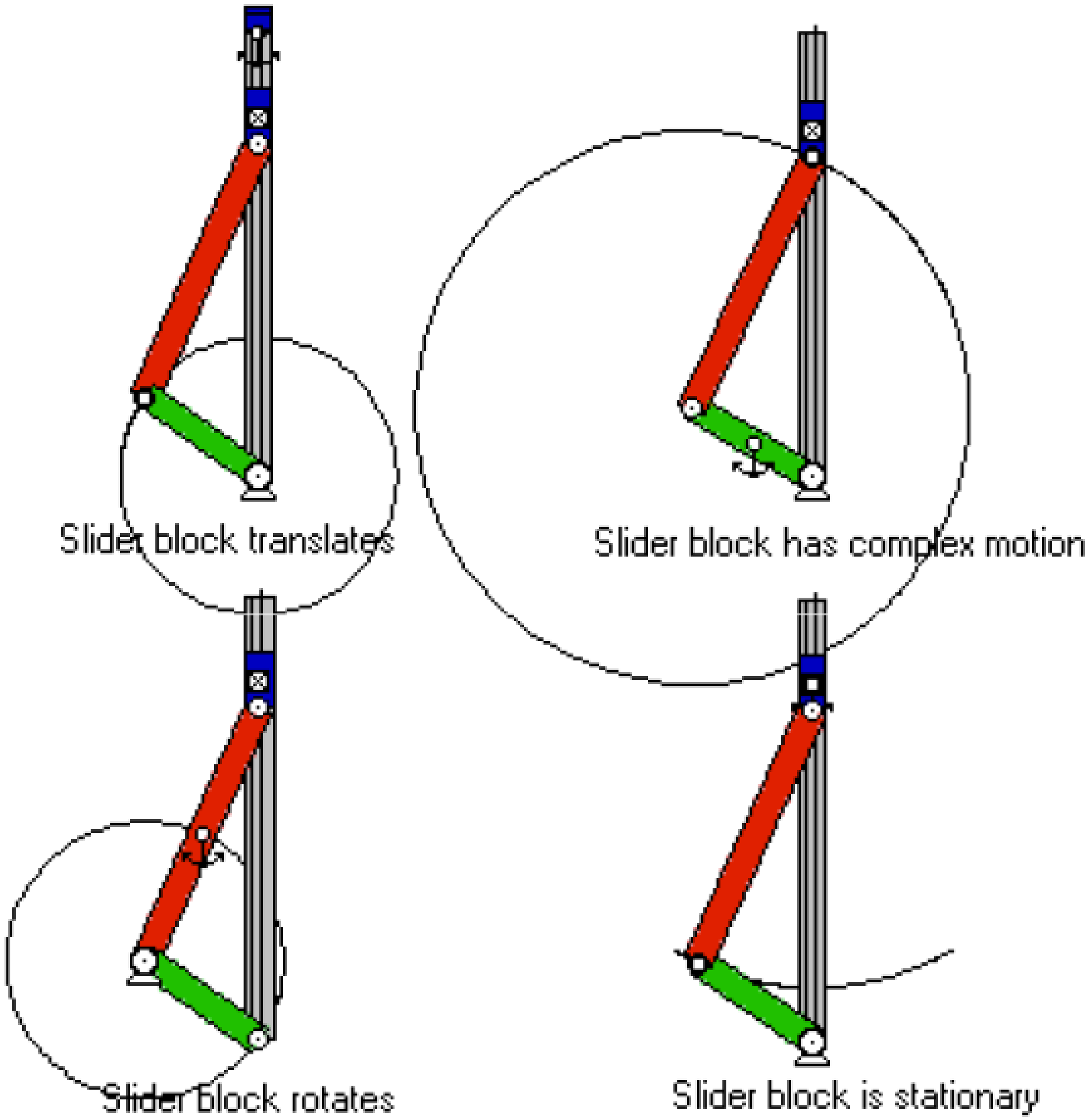


Inversion (1)

- An inversion is created by grounding a different link in the kinematic chain → there are as many inversions of a given linkage as there are links



Inversion (2)



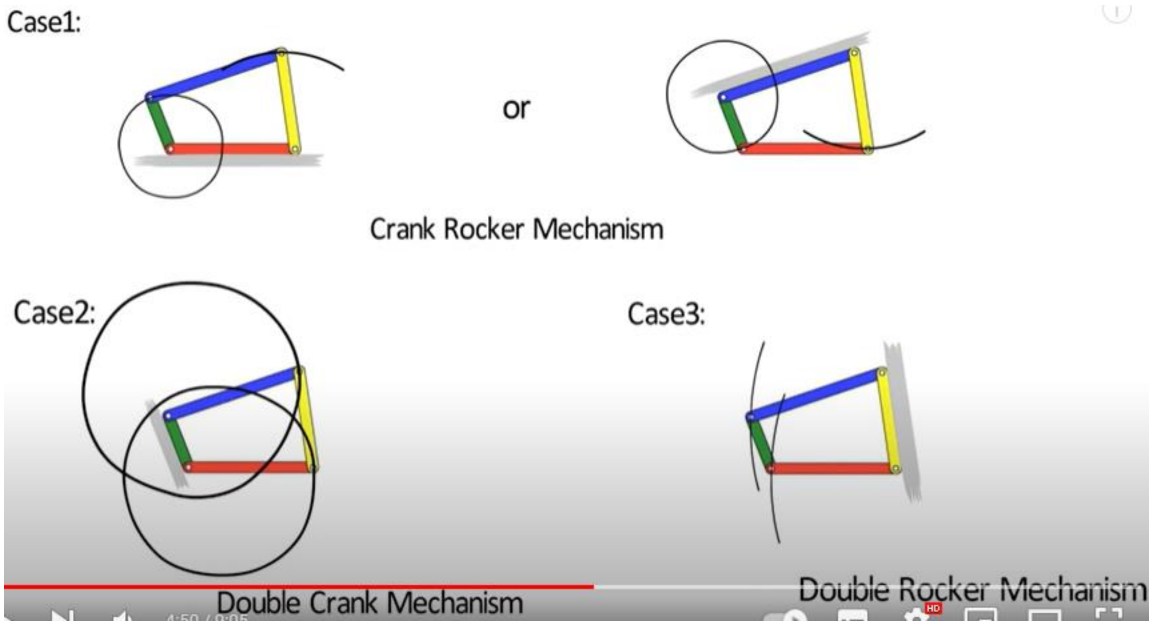
Slider block translates
Slider block has complex motion
Slider block rotates
Slider block is stationary

The Grashof Condition

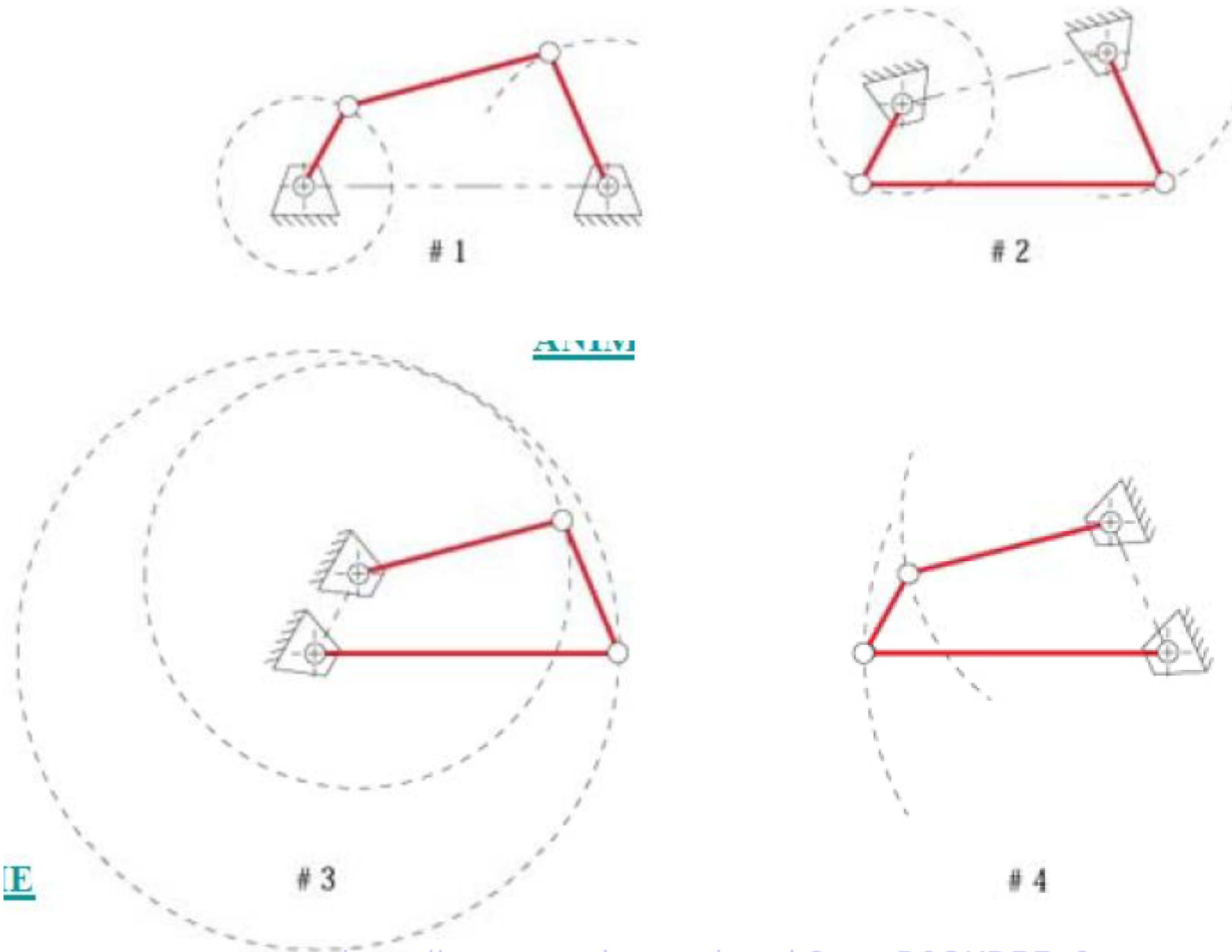
- A Fourbar linkage is the simplest possible pin jointed mechanism for a 1 DOF controlled motion
- Grashof Condition is a simple relation which predicts the behavior of fourbar linkage inversions
  - S : length of shortest link
  - L : length of longest link
  - P : length of one remaining link
  - Q : length of other remaining link
  - If $S + L \leq P + Q$, then the linkage is Grashof → At least one link will be capable of making full revolution w.r.t. the ground
  - If $S + L > P + Q$, then no link will be able to make a complete revolution → triple rocker mechanism

S + L < P + Q

- Case 1: shortest link adjacent to fixed link
  - Crank rocker mechanism
- Case 2: shortest link is the fixed link
  - Double crank mechanism
- Case 3: shortest link is opposite to the fixed link
  - Double rocker mechanism



All Inversions of the Grashof Fourbar Linkage

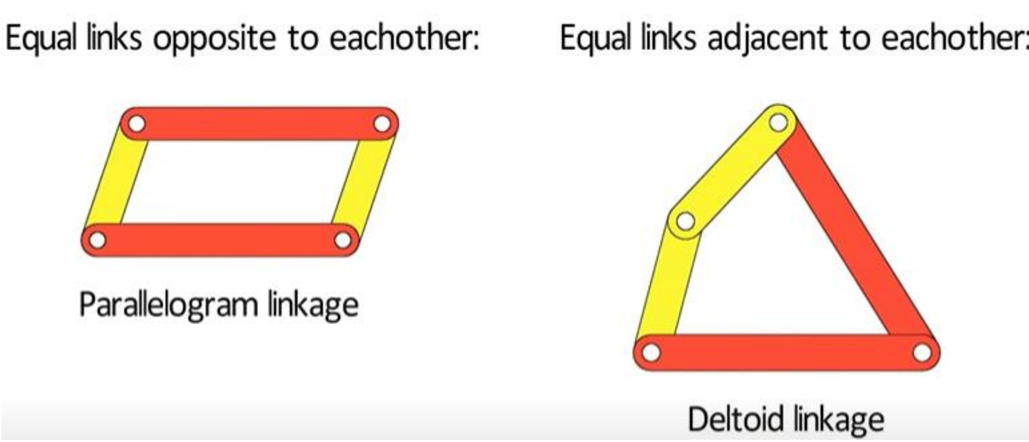


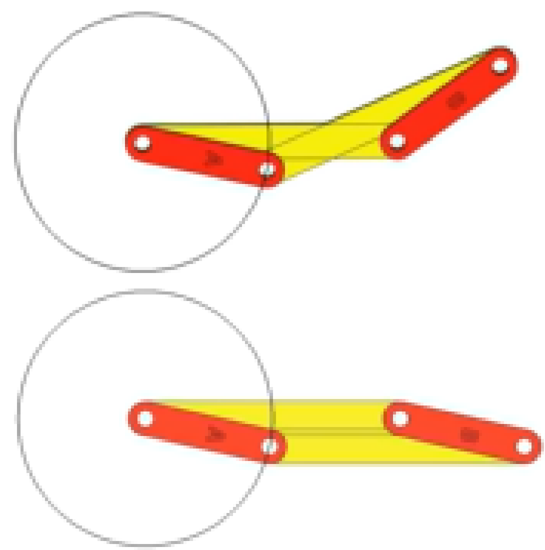
https://www.youtube.com/watch?v=qvD9QKDPRv8

$S + L = P + Q$

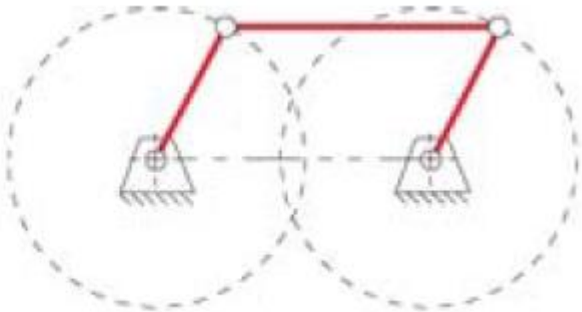
- The links become colinear at least once per revolution of input crank

- Case 1: length of the links are distinct
  - All the inversions obtained are same as in the case $S + L < P + Q$
- Case 2: length of 2 links is same, $P=S \rightarrow Q=L$
  - All the inversions are either crank rocker or double crank
  - When the links become colinear, both the linkages suffer from “ change point condition” i.e. the output behavior becomes indeterminate

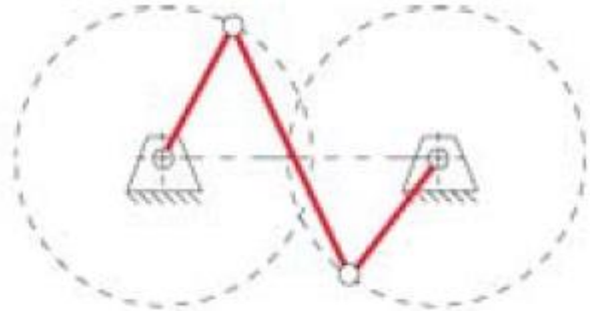




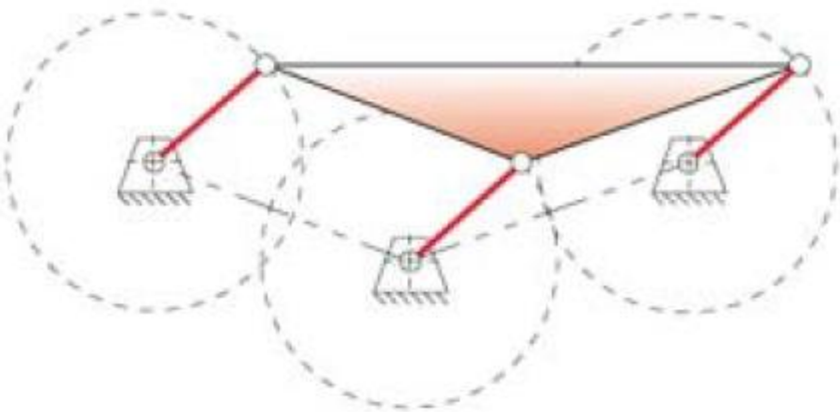
Special-case of Grashof Linkage (S+L=P+Q)



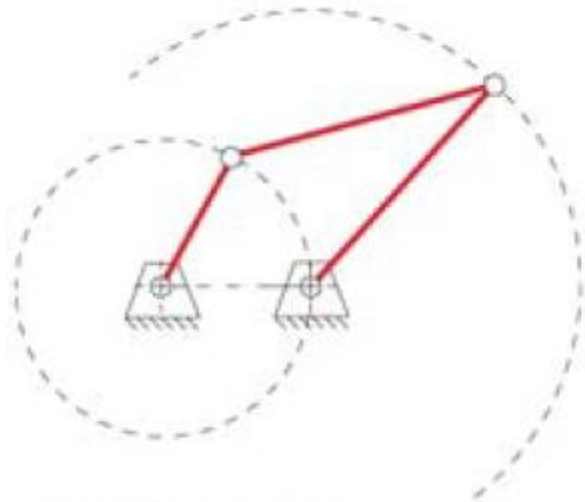
(*a*) Parallelogram form



(*b*) Antiparallelogram form



(*c*) Double-parallelogram linkage gives parallel motion (pure curvilinear translation) to coupler and also carries through the change points

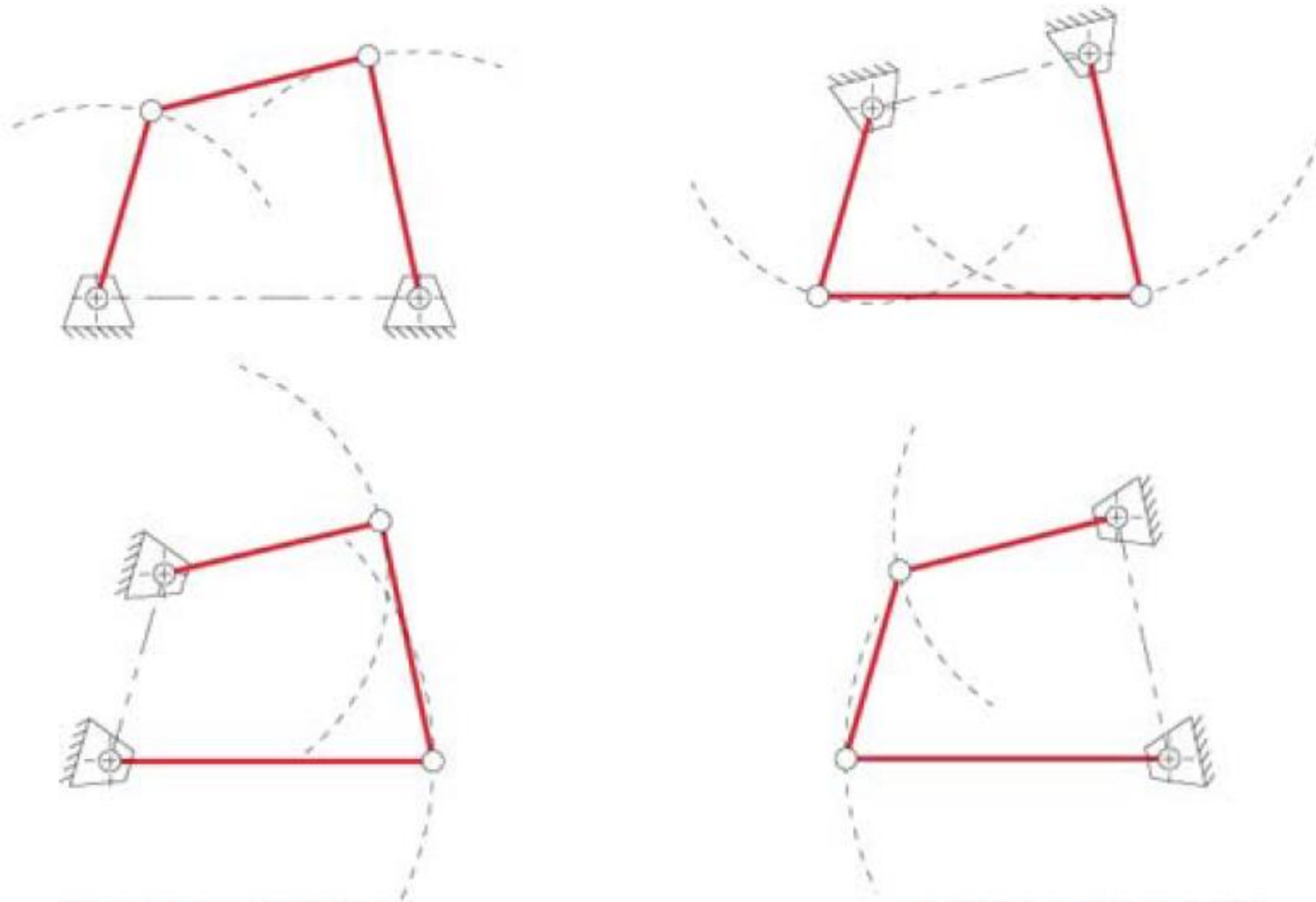


(*d*) Deltoid or kite form

Grashof

- Crank Rocker
  - https://www.youtube.com/watch?v=q0dd4SuUjKk
- Double Rocker
  - https://www.youtube.com/watch?v=mFnwPuUVB0k
- Rotating Coupler
  - https://www.youtube.com/watch?v=1_Yg_KAR7LI
- Special Case 1: Delta/Kite
  - https://www.youtube.com/watch?v=7nXKmvL3mOo
- Special Case 2: Parallel/Antiparallel
  - https://www.youtube.com/watch?v=gY0s0oiEnZI

All Inversions of the non-Grashof Fourbar Linkage



Triple-rocker #3 (RRR3)

(d) Triple-rocker #4 (RRR4)

Non-Grashof

- Double Rocker: Outward/Outward
  - https://www.youtube.com/watch?v=mK51eFIUMAg
- Double Rocker: Inward/Inward
  - https://www.youtube.com/watch?v=AbtCFGzZzHM
- Double Rocker: Outward/Inward
  - https://www.youtube.com/watch?v=mL2HMsEjq_U