Final Exam

12/17/2024

1. Spring system (변위값 틀리고 나머지 맞으면 15 점, 다른 것도 틀리면 10 점)

$$(1)\ k_1 = k_3 = \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix},\ k_2 = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix}$$

$$K = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 100+200 & -200 & 0 \\ 0 & -200 & 200+100 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} = \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 300 & -200 & 0 \\ 0 & -200 & 300 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix}$$

$$KU = F \rightarrow \begin{bmatrix} 100 & -100 & 0 & 0 \\ -100 & 300 & -200 & 0 \\ 0 & -200 & 300 & -100 \\ 0 & 0 & -100 & 100 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ u_3 \\ 0 \end{bmatrix} = \begin{bmatrix} f_1 \\ 0 \\ 500 \\ f_4 \end{bmatrix}$$

$$(2)\ \begin{bmatrix} 300 & -200 \\ -200 & 300 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 500 \end{bmatrix} \xrightarrow{u_2 = \frac{2}{3}u_3} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} mm$$

$$(3)\ \begin{cases} f_1 = -200N \\ f_4 = -300N \end{cases}$$

(4) for spring element 2

$$\begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} f_2 \\ f_3 \end{bmatrix} \rightarrow F = f_3 = -f_2 = 200N$$

2. two bar

$$k_1 = \frac{2EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix},\ k_2 = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \text{(5 pts)}$$

$$\frac{EA}{L} \begin{bmatrix} 2 & -2 & \\ -2 & 3 & -1 \\ & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ u_2 \\ 0 \end{bmatrix} = \begin{bmatrix} f_1 \\ P \\ f_3 \end{bmatrix} \text{(5 pts)} \rightarrow \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{PL}{3EA} \\ 0 \end{bmatrix} \text{(5 pts)}$$

$$\left.\begin{aligned} \sigma_1 &= E\varepsilon_1 = E\mathbf{B}\mathbf{u}_1 = E\begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \frac{P}{3A} \\ \sigma_2 &= E\varepsilon_2 = E\mathbf{B}\mathbf{u}_2 = E\begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = -\frac{P}{3A} \end{aligned}\right\} \text{(5 pts) 부호틀리면 } -2$$

3. BVP

(1) $x^2 \dfrac{d^2 y}{dx^2} + y + 1 = 0$ for $1 < x < 2,\ y(1) = 2,\ y'(2) = 3$

$\int_1^2 \left(x^2 y'' + y + 1\right) v dx = 0$ (5 pts, 이후 계산하면 8 pts, 정답자 없음)

$$\left(x^2 y' v\right)' = 2xy'v + x^2 y'' v + x^2 y' v'$$

$$\left[x^2 y' v\right]_{x=1}^{x=2} - \int_1^2 \left(2xy'v + x^2 y' v'\right) dx + \int_1^2 (y+1) v dx = 0$$

$$\xrightarrow[y'(2)=3]{y(1)=2 \to v(0)=0} \int_1^2 \left(2xy'v + x^2 y' v' - yv\right) dx = \int_1^2 v dx + 12v(2)$$

(2) element 1 $\begin{cases} x : 1 \sim 1.25 \\ \zeta : 0 \sim 1 \end{cases} \to 0.25 : 1 = (x-1) : \zeta \to \zeta = 4(x-1) \to \begin{cases} N_1^1(\zeta) = 1 - \zeta \to N_1^1(x) = 4(1.25 - x) \\ N_2^1(\zeta) = \zeta \to N_2^1(x) = 4(x-1) \end{cases}$

(3) $y(x) \approx Y(x) = \sum_{i=1}^{2} N_i^1(x) Y_i^1,\ v(x) \approx V(x) = \sum_{i=1}^{2} N_i^1(x) V_i^1$

$$k_{ij}^1 = \int_1^{1.25} \left(2x \frac{dN_i^1(x)}{dx} N_i^1(x) + x^2 \frac{dN_i^1(x)}{dx} \frac{dN_j^1(x)}{dx} - N_i^1(x) N_j^1(x) \right) dx$$

4. 수업시간에 풀었던 문제인데 정답자 없음, 형상함수 요건 갖추지 않고 구했으면 7 점

4-node bilinear quadrilateral element

$$\tilde{N}_1^e = \frac{1}{4}(1-\xi)(1-\eta),\ \tilde{N}_2^e = \frac{1}{4}(1+\xi)(1-\eta),\ \tilde{N}_3^e = \frac{1}{4}(1+\xi)(1+\eta),\ \tilde{N}_4^e = \frac{1}{4}(1-\xi)(1+\eta)$$

$$N_5^e = c(1-\xi)(1+\xi)(1-\eta)(1+\eta) \xrightarrow[\eta=0]{\xi=0} 1 = c \to N_5^e = \left(1-\xi^2\right)\left(1-\eta^2\right)$$

$$N_i^e = \tilde{N}_i^e + c_5 N_5^e\ (i = 1, \dots, 4) \to N_1^e = \tilde{N}_1^e + c_5 N_5^e = \frac{1}{4}(1-\xi)(1-\eta) + c_5\left(1-\xi^2\right)\left(1-\eta^2\right)$$

$$\xrightarrow[\eta=0]{\xi=0} 0 = \frac{1}{4} + c_5 \to c_5 = -\frac{1}{4}$$

$$N_i^e = \tilde{N}_i^e - \frac{1}{4} N_5^e\ (i = 1, \dots, 4), N_5^e = \left(1-\xi^2\right)\left(1-\eta^2\right) \to \sum_{i=1}^{5} N_i^e = \underbrace{\sum_{i=1}^{4} \tilde{N}_i^e}_{=1} + 4 \times \left(-\frac{1}{4} N_5^e\right) + N_5^e = 1$$

5.

(1)
$$\left|\begin{array}{l}\begin{cases}x:-3\sim3\\ \xi:-1\sim1\end{cases}\rightarrow\begin{cases}2:6=(\xi+1):(x+3)\\ \xi=\dfrac{1}{3}(x+3)+1\end{cases}\\ \begin{cases}y:-2\sim2\\ \eta:-1\sim1\end{cases}\rightarrow\begin{cases}2:4=(\xi+1):(y+2)\\ \eta=\dfrac{1}{2}(y+2)+1\end{cases}\end{array}\right.\begin{cases}N_1^e=\dfrac{1}{4}(1-\xi)(1-\eta)\rightarrow N_1^e=\dfrac{1}{4}\left(1+\dfrac{x}{3}\right)\left(1+\dfrac{y}{2}\right)\\ N_2^e=\dfrac{1}{4}(1+\xi)(1-\eta)\rightarrow N_2^e=\dfrac{1}{4}\left(1-\dfrac{x}{3}\right)\left(1+\dfrac{y}{2}\right)\\ N_3^e=\dfrac{1}{4}(1+\xi)(1+\eta)\rightarrow N_3^e=\dfrac{1}{4}\left(1-\dfrac{x}{3}\right)\left(1-\dfrac{y}{2}\right)\\ N_4^e=\dfrac{1}{4}(1-\xi)(1+\eta)\rightarrow N_4^e=\dfrac{1}{4}\left(1+\dfrac{x}{3}\right)\left(1-\dfrac{y}{2}\right)\end{cases}$$

$$\mathbf{u}=\begin{bmatrix}u(x,y)\\ v(x,y)\end{bmatrix}=\mathbf{NU}=\begin{bmatrix}N_1&0&N_2&0&N_3&0&N_4&0\\ 0&N_1&0&N_2&0&N_3&0&N_4\end{bmatrix}\begin{bmatrix}u_1\\ v_1\\ u_2\\ v_2\\ u_3\\ v_3\\ u_4\\ v_4\end{bmatrix}(10\text{ pts})$$

(2)
$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}=\begin{bmatrix}\partial u/\partial x\\ \partial v/\partial y\\ \partial u/\partial y+\partial v/\partial x\end{bmatrix}=\begin{bmatrix}\partial/\partial x&0\\ 0&\partial/\partial y\\ \partial/\partial y&\partial/\partial x\end{bmatrix}\mathbf{u}=\begin{bmatrix}\partial/\partial x&0\\ 0&\partial/\partial y\\ \partial/\partial y&\partial/\partial x\end{bmatrix}\mathbf{NU}$$

$$=\begin{bmatrix}\frac{1}{12}\left(1+\frac{y}{2}\right)&0&\frac{1}{12}\left(1+\frac{y}{2}\right)&0&\frac{1}{12}\left(1-\frac{y}{2}\right)&0&\frac{1}{12}\left(1-\frac{y}{2}\right)&0\\ 0&\frac{1}{8}\left(1+\frac{x}{3}\right)&0&\frac{1}{8}\left(1-\frac{x}{3}\right)&0&\frac{1}{8}\left(1-\frac{x}{3}\right)&0&\frac{1}{8}\left(1+\frac{x}{3}\right)\\ \frac{1}{8}\left(1+\frac{x}{3}\right)&\frac{1}{12}\left(1+\frac{y}{2}\right)&\frac{1}{8}\left(1-\frac{x}{3}\right)&\frac{1}{12}\left(1+\frac{y}{2}\right)&\frac{1}{8}\left(1-\frac{x}{3}\right)&\frac{1}{12}\left(1-\frac{y}{2}\right)&\frac{1}{8}\left(1+\frac{x}{3}\right)&\frac{1}{12}\left(1-\frac{y}{2}\right)\end{bmatrix}\mathbf{U}=\mathbf{BU}(10\text{ pts})$$

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\frac{E}{1-\nu^2}\begin{bmatrix}1&\nu&0\\ \nu&1&0\\ 0&0&\frac{1-\nu}{2}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{bmatrix}\rightarrow\boldsymbol{\sigma}=\mathbf{E}\boldsymbol{\varepsilon}$$

$$\mathbf{K}=\int_\Omega\mathbf{B}^T\mathbf{EB}d\Omega=\int_{y=-2}^{y=2}\int_{x=-3}^{x=3}\mathbf{B}^T\mathbf{EB}dxdy(5\text{ pts})$$

$$\mathbf{R}_B=\int_\Omega\mathbf{N}^T\mathbf{f}^Bd\Omega=\int_{y=-2}^{y=2}\int_{x=-3}^{x=3}\mathbf{N}^T\begin{bmatrix}4+x\\ 0\end{bmatrix}dxdy(5\text{ pts})$$