

Problem Formulation Process (1)

- Step 1: Project/Problem Statement
 - Is the project goal clear?
 - descriptive statement for the project/ problem
 - overall *objectives* of the project and the *requirements* to be met
- Step 2: Data and Information Collection
 - Is all the information available to solve the problem?
 - Performance requirements, resource limits, cost of raw materials
 - Identification of analysis procedures and tools
 - project statement is vague, and assumptions about modeling of the problem need to be made in order to formulate and solve it

Problem Formulation Process (2)

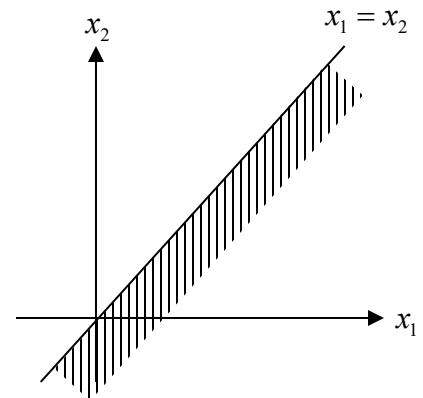
- Step 3: Identification/Definition of Design Variables
 - What are these variables? How do I identify them?
 - identify a set of variables that describe the system, called the *design variables*
 - should be independent of each other, minimum number
 - As many independent parameters as possible should be designated as design variables at the problem formulation phase
- Step 4: Optimization Criterion
 - How do I know that my design is the best?
 - must be a scalar function whose numerical value can be obtained once a design is specified (*function of the design variable vector*)
 - *maximized* or *minimized* depending on problem requirements
 - criterion that is to be minimized is usually called a *cost function* in engineering literature

Problem Formulation Process (3)

- Step 5: Formulation of Constraints
 - What restrictions do I have on my design?
 - All restrictions placed on the design
 - identify all constraints and develop expressions for them
 - must be designed and fabricated with the given *resources* and must meet *performance requirements*

Problem Formulation Steps

- Identification of *design variables*
 - Parameters chosen to describe the design
 - Independent of each other, minimum number
- Identification of an *objective (cost) functions*
 - Criterion to compare various designs
 - As a function of the design variables
 - Single/Multi-objective
- Identification of all *design constraints*
 - All restrictions placed on a design
 - Feasible/Infeasible
 - Explicit/Implicit, Linear/Nonlinear, Equality/Inequality



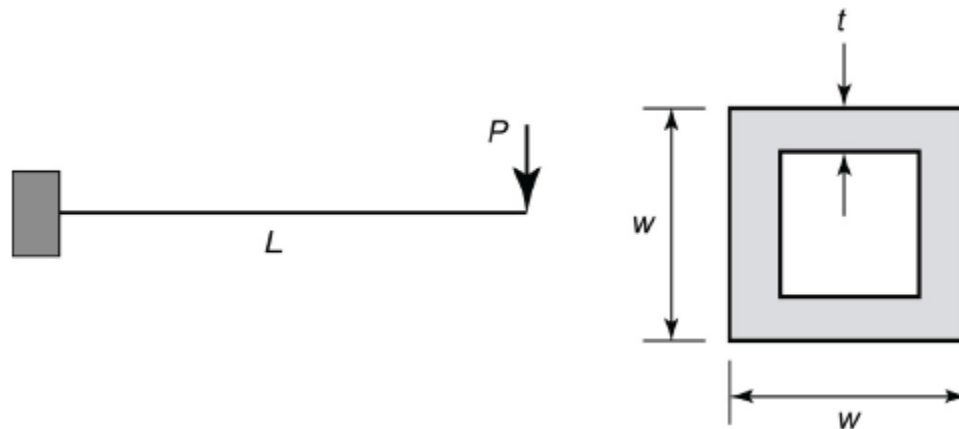
Design of a Cantilever Beam (1)

- Step 1: Problem Statement

Cantilever beams are used in many practical applications in civil, mechanical, and aerospace engineering. To illustrate the step of problem description, we consider the design of a hollow square-cross-section *cantilever beam* to support a load of 20 kN at its end. The beam, made of steel, is 2 m long, as shown in Fig. 2.1. The failure conditions for the beam are as follows: (1) the material should not fail under the action of the load, and (2) the deflection of the free end should be no more than 1 cm. The width-to-thickness ratio for the beam should be no more than 8 to avoid local buckling of the walls. A *minimum-mass* beam is desired. The width and thickness of the beam must be within the following limits:

$$60 \leq \text{width} \leq 300 \text{ mm} \quad (\text{a})$$

$$3 \leq \text{thickness} \leq 15 \text{ mm} \quad (\text{b})$$



Design of a Cantilever Beam (2)

- Step 2: Data and Information Collection

Notation	Data
A	Cross-sectional area, mm^2
E	Modulus of elasticity of steel, $21 \times 10^4 \text{ N/mm}^2$
G	Shear modulus of steel, $8 \times 10^4 \text{ N/mm}^2$
I	Moment of inertia of the cross-section, mm^4
L	Length of the member, 2000 mm
M	Bending moment, N/mm
P	Load at the free end, 20,000 N
Q	Moment about the neutral axis of the area above the neutral axis, mm^3
q	Vertical deflection of the free end, mm
q_a	Allowable vertical deflection of the free end, 10 mm
V	Shear force, N
w	Width (depth) of the section, mm
t	Wall thickness, mm
σ	Bending stress, N/mm^2
σ_a	Allowable bending stress, 165 N/mm^2
τ	Shear stress, N/mm^2
τ_a	Allowable shear stress, 90 N/mm^2

$$A = w^2 - (w - 2t)^2 = 4t(w - t), \text{mm}^2$$

$$I = \frac{1}{12}w \times w^3 - \frac{1}{12}(w - 2t) \times (w - 2t)^3 = \frac{1}{12}w^4 - \frac{1}{12}(w - 2t)^4, \text{mm}^4$$

$$Q = \frac{1}{2}w^2 \times \frac{w}{4} - \frac{1}{2}(w - 2t)^2 \times \frac{(w - 2t)}{4} = \frac{1}{8}w^3 - \frac{1}{8}(w - 2t)^3, \text{mm}^3$$

$$M = PL, \text{N/mm}$$

$$V = P, \text{N}$$

$$\sigma = \frac{Mw}{2I}, \text{N/mm}^2$$

$$\tau = \frac{VQ}{2It}, \text{N/mm}^2$$

$$q = \frac{PL^3}{3EI}, \text{mm}$$

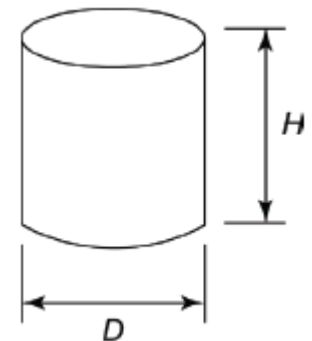
$$Q = \int_A y dA: \text{ first moment of area } \rightarrow \tau = \frac{V(x)Q(y)}{Ib(y)}$$

Design of a Cantilever Beam (3)

- Step 3: Definition of Design Variables
 - w = outside width (depth) of the section, mm
 - t = wall thickness, mm
- Step 4: Optimization Criterion
 - Design a minimum-mass cantilever beam
 - cross-sectional area of the beam:
- Step 5: Formulation of Constraints
 - Bending stress constraint
 - Shear stress constraint
 - Deflection constraint
 - Width-thickness restriction
 - Dimension restrictions

Design of a Can (1)

- Step 1: Problem Statement
 - Design a can to hold at least 400ml of liquid
 - Production in billions → Minimize the manufacturing cost
 - Cost directly related to the surface area of the sheet metal
 - Minimize the sheet metal required to fabricate the can
 - Diameter of the can should be no more than 8 cm. Also, it should not be less than 3.5 cm.
 - Height of the can should be no more than 18 cm and no less than 8 cm.



- Step 2: Data and Information Collection

Design of a Can (2)

- Step 3: Design variables
 - Diameter of the can (cm) / Height of the can (cm)
- Step 4: Cost function

- Total surface area of the sheet metal

$$f(D, H) = \pi DH + 2 \left(\frac{\pi D^2}{4} \right)$$

- Step 5: Constraints

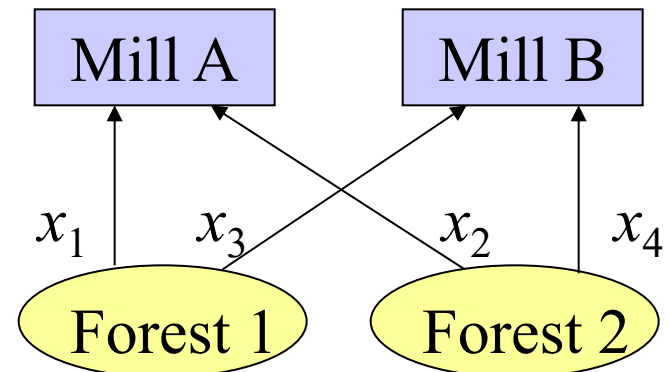
- Volume: $\left(\frac{\pi D^2}{4} \right) H \geq 400$
- Size of the can: side/technological/sizing constraints, simple bounds, upper and lower limits

$$3.5 \leq D \leq 8; \quad 8 \leq H \leq 18$$

Saw Mill Operation (1)

- Step 1: Problem Statement
 - Each forest can yield up to 200 logs/day
 - Cost to transport the logs is estimated at 15 cents/km/log
 - At least 300 logs are needed each day
 - Minimize the cost of transportation of logs each day
- Step 2: Data and Information Collection

Mill	Distance (km)		Capacity /day
	Forest 1	Forest 2	
A	24.0	20.5	240 logs
B	17.2	18.0	300 logs



Saw Mill Operation (2)

- Step 3: Design variables: x_1, x_2, x_3, x_4
- Step 4: Cost function
 - Cost of transportation of logs each day
- Step 5: Constraints
 - Mill capacities:
 - Yield of forests:
 - Daily needs:

Linear Programming problem
→ Integer Programming problem

find x_1, x_2, x_3, x_4 to minimize
 $f = 24x_1 + 20.5x_2 + 17.2x_3 + 18x_4$ subject to
 $x_1 + x_2 \leq 240, x_1 + x_3 \leq 300, x_1 + x_3 \leq 200,$
 $x_2 + x_4 \leq 200, x_1 + x_2 + x_3 + x_4 \geq 300$

Pro Search

- ☒ Formulate the linear programming model with the objective function and constraints
- ☒ Use an optimization library to solve the linear programming problem

Perplexity

Based on the optimization results provided, I can interpret the solution for you:

The optimal solution to minimize the objective function $f = 24x_1 + 20.5x_2 + 17.2x_3 + 18x_4$ subject to the given constraints is:

$x_1 = 0$
 $x_2 = 0$
 $x_3 = 300$
 $x_4 = 0$

This solution results in a minimum value of $f = 5160.0$.

Let's verify that this solution satisfies all the constraints:

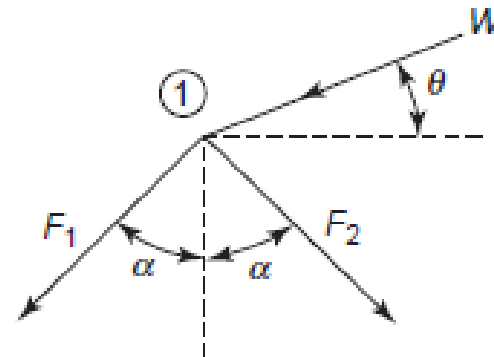
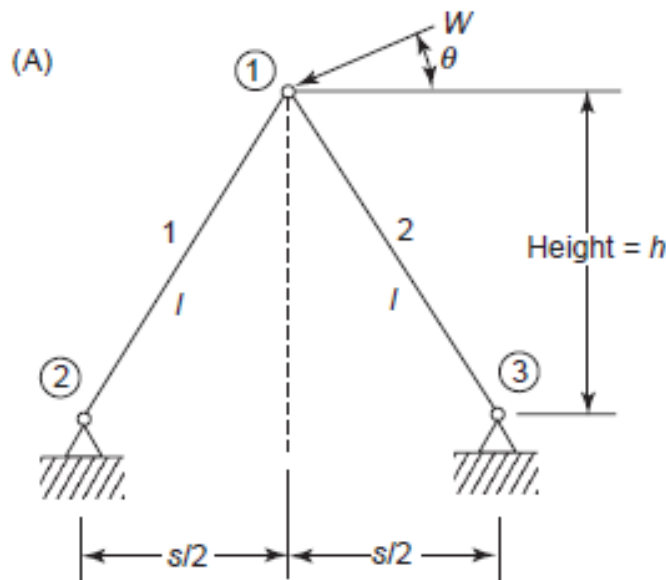
1. $x_1 + x_2 \leq 240$: $0 + 0 = 0 \leq 240$ (Satisfied)
2. $x_1 + x_3 \leq 300$: $0 + 300 = 300 \leq 300$ (Satisfied)
3. $x_1 \leq 200$: $0 \leq 200$ (Satisfied)
4. $x_2 + x_4 \leq 200$: $0 + 0 = 0 \leq 200$ (Satisfied)
5. $x_1 + x_2 + x_3 + x_4 \geq 300$: $0 + 0 + 300 + 0 = 300 \geq 300$ (Satisfied)

All constraints are satisfied by this solution. The optimization algorithm determined that the most cost-effective way to meet all constraints is to produce 300 units of product 3 (x_3) and none of the other products.

This solution makes sense because product 3 has the lowest cost coefficient (17.2) among all products, allowing for the minimization of the total cost while meeting the production requirement of at least 300 units total.

Two-Bar Structure (1)

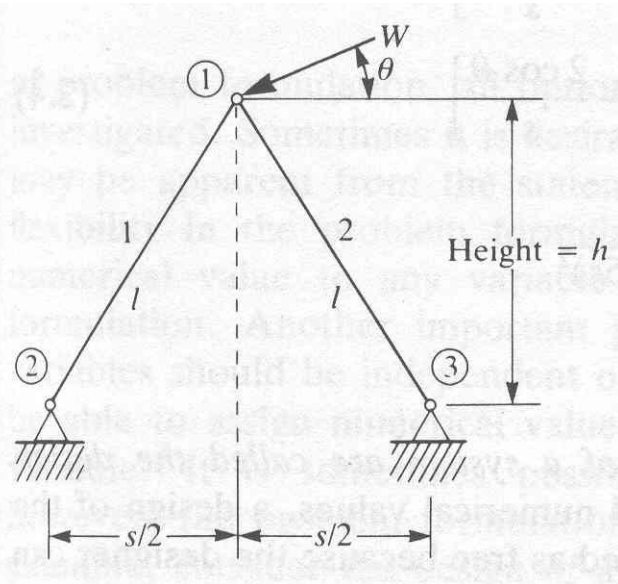
- Step 1: Problem Statement
 - Design a two-bar bracket to support a force W without failure
 - Cost directly related to the size of the two bars
 - To minimize the total mass of the bracket while satisfying performance, fabrication, and space limitations



$$\begin{cases} \sum F_x = -F_1 \sin \alpha + F_2 \sin \alpha - W \cos \theta = 0 \\ \sum F_y = -F_1 \cos \alpha - F_2 \cos \alpha - W \sin \theta = 0 \end{cases}$$

$$\sin \alpha = \frac{s}{2l}, \quad \cos \alpha = \frac{h}{l}, \quad l = \sqrt{h^2 + \left(\frac{s}{2}\right)^2}$$

Two-Bar Structure (2)



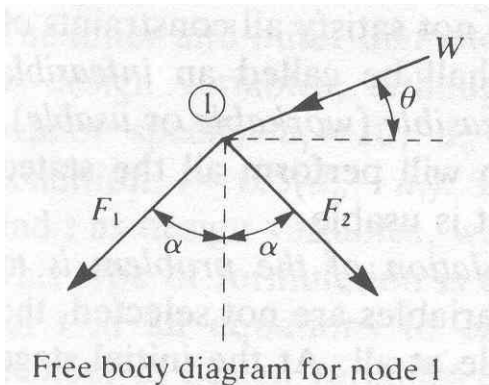
$$\begin{cases} \sum F_x = -F_1 \sin \alpha + F_2 \sin \alpha - W \cos \theta = 0 \\ \sum F_y = -F_1 \cos \alpha - F_2 \cos \alpha - W \sin \theta = 0 \end{cases}$$

$$\sin \alpha = \frac{s}{2l}, \quad \cos \alpha = \frac{h}{l}, \quad l = \sqrt{h^2 + \left(\frac{s}{2}\right)^2}$$

$$\rightarrow F_1 = \frac{-W}{2 \sin \alpha \cos \alpha} (\cos \alpha \cos \theta + \sin \alpha \sin \theta)$$

$$= -0.5W \frac{2l^2}{sh} \left(\frac{h}{l} \cos \theta + \frac{s}{2l} \sin \theta \right) = -0.5Wl \left(\frac{2 \cos \theta}{s} + \frac{\sin \theta}{h} \right) : \text{compression}$$

$$\rightarrow \sigma_1 = \frac{F_1}{A_1} = \frac{2W \sqrt{x_1^2 + (0.5x_2)^2}}{\pi(x_3^2 - x_4^2)} \left(\frac{2 \cos \theta}{x_2} + \frac{\sin \theta}{x_1} \right) \leq \sigma_a$$



$$F_2 = -0.5Wl \left(-\frac{2 \cos \theta}{s} + \frac{\sin \theta}{h} \right) \rightarrow \begin{cases} \frac{\sin \theta}{h} > \frac{2 \cos \theta}{s} : \text{compression} \\ \rightarrow \sigma_2 = \frac{F_2}{A_2} = \frac{2W \sqrt{x_1^2 + (0.5x_2)^2}}{\pi(x_5^2 - x_6^2)} \left(-\frac{2 \cos \theta}{x_2} + \frac{\sin \theta}{x_1} \right) \leq \sigma_a \\ \frac{\sin \theta}{h} < \frac{2 \cos \theta}{s} : \text{tension} \\ \rightarrow \sigma_2 = \frac{F_2}{A_2} = -\frac{2W \sqrt{x_1^2 + (0.5x_2)^2}}{\pi(x_5^2 - x_6^2)} \left(-\frac{2 \cos \theta}{x_2} + \frac{\sin \theta}{x_1} \right) \leq \sigma_a \end{cases}$$

solve F1 and F2 in the following equations: -
 $F1\sin(\alpha)+F2\sin(\alpha)-W\cos(\theta)=0$, -
 $F1\cos(\alpha)-F2\cos(\alpha)-W\sin(\theta)=0$

Pro Search

- ✓ Rearrange the given equations to isolate F1 and F2
- ✓ Solve the rearranged equations for F1 and F2 given specific values for alpha, W, and theta
- ✓ Summarize the results of the solutions for F1 and F2

Sources

$-W\sin(\theta)/(2\cos(\alpha)) - W\cos(\theta)/(2\sin(\alpha))$ -
Wolfram|Alpha
wolframalpha · 1

$-W\sin(\theta)/(2\cos(\alpha)) + W\cos(\theta)/(2\sin(\alpha))$ -
Wolfram|Alpha
wolframalpha · 2

Answer

To solve for F1 and F2 in the given equations, I'll use the results from the symbolic solution provided in the search results. The equations we're solving are:

- $-F1\sin(\alpha) + F2\sin(\alpha) - W\cos(\theta) = 0$
- $-F1\cos(\alpha) - F2\cos(\alpha) - W\sin(\theta) = 0$

The solution for F1 and F2 is:

$$F1 = -W\sin(\theta)/(2\cos(\alpha)) - W\cos(\theta)/(2\sin(\alpha))$$
$$F2 = -W\sin(\theta)/(2\cos(\alpha)) + W\cos(\theta)/(2\sin(\alpha))$$

Let's break down these solutions:

F1 Solution

$$F1 = -W\sin(\theta)/(2\cos(\alpha)) - W\cos(\theta)/(2\sin(\alpha))$$

This can be simplified to:

$$F1 = -1/2 W (\sec(\alpha) * \sin(\theta) + \csc(\alpha) * \cos(\theta))$$

Or, using trigonometric identities:

$$F1 = -W * \csc(2\alpha) * \cos(\alpha - \theta)$$

F2 Solution

$$F2 = -W\sin(\theta)/(2\cos(\alpha)) + W\cos(\theta)/(2\sin(\alpha))$$

This can be simplified to:

$$F2 = 1/2 W (\csc(\alpha) * \cos(\theta) - \sec(\alpha) * \sin(\theta))$$

Or, using trigonometric identities:

$$F2 = W * \csc(2\alpha) * \cos(\alpha + \theta)$$

These solutions represent the forces F1 and F2 in terms of the weight W, the angles alpha and theta. The solutions satisfy both original equations and provide a complete description of how F1 and F2 depend on the other variables in the system.

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Two-Bar Structure (3)

- Step 3: Design variables (hollow circular tubes)

- x_1 : height of the truss, x_2 : span of the truss
- x_3, x_4 : outer/inner diameters of member 1
- x_5, x_6 : outer/inner diameters of member 2

$$A_1 = \frac{\pi}{4}(x_3^2 - x_4^2), A_2 = \frac{\pi}{4}(x_5^2 - x_6^2)$$

$(d_0, r) \text{ where } r = \frac{d_i}{d_0}$
 (d_0, d_i)
 $(d_0, d_i, r)?$

- Step 4: Cost function

- Minimize the mass: $m = \rho[l(A_1 + A_2)] = \rho\sqrt{x_1^2 + (0.5x_2)^2} \frac{\pi}{4}(x_3^2 - x_4^2 + x_5^2 - x_6^2)$

- Step 5: Constraints

- stress in each member \leq material allowable stress
- Side constraints

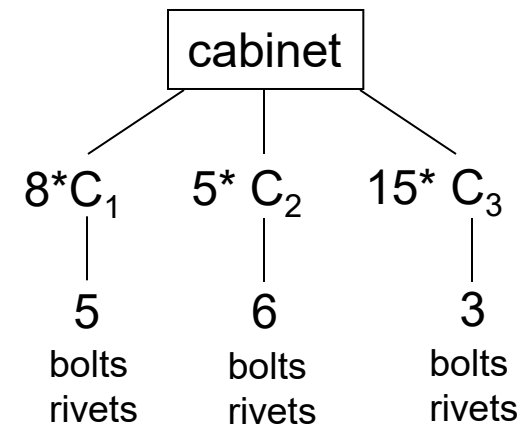
$$\left| \sigma_i = \frac{F_i}{A_i} \right| \leq \sigma_a \quad (i=1,2)$$

$$x_{il} \leq x_i \leq x_{iu} \quad (i=1,\dots,6)$$

Design of a Cabinet

- Determine the number of components to be bolted and riveted to minimize the cost
 - Each cabinet requires $8 \cdot C_1$, $5 \cdot C_2$, $15 \cdot C_3$ components
 - Assembly of C_1 needs either 5 bolts or 5 rivets; C_2 6 bolts or 6 rivets ; C_3 3 bolts or 3 rivets
 - A total of 100 cabinets must be assembly daily
 - Bolting and riveting capacities per day are 6000 and 8000, respectively

Cost (\$)	C_1	C_2	C_3
bolt	0.7	1.0	0.6
rivet	0.6	0.8	1.0



Formulation 1 (component level)

- Design variables (for 100 cabinets)
 - $x_1/ x_3/ x_5$ = number of $C_1/ C_2/ C_3$ to be bolted
 - $x_2/ x_4/ x_6$ = number of $C_1/ C_2/ C_3$ to be riveted
- Cost function (bolts/rivets)
- Constraints
 - Number of components
 - Capacity of bolts/rivets

Formulation 2 (bolt/rivet level)

- Design variables
 - $x_1/ x_2/ x_3$ = total number of bolts required for all $C_1/ C_2/ C_3$
 - $x_4/ x_5/ x_6$ = total number of rivets required for all $C_1/ C_2/ C_3$
- Cost function (bolts/rivets)
- Constraints
 - Number of bolts/rivets in a component
 - Capacity of bolts/rivets

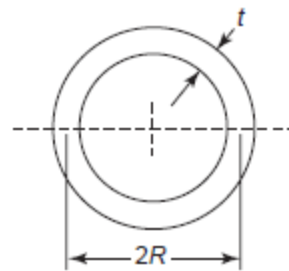
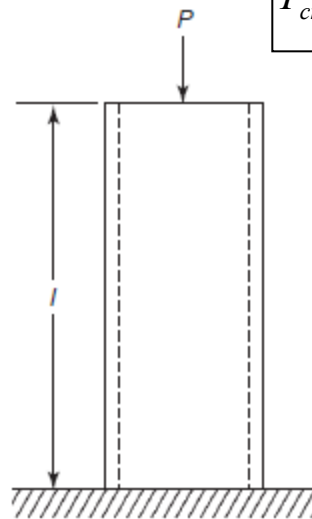
Formulation 3 (←Formulation 1)

- Design variables (for one cabinet)
 - $x_1/ x_3/ x_5$ = number of $C_1/ C_3/ C_5$ to be bolted on one cabinet
 - $x_2/ x_4/ x_6$ = number of $C_2/ C_4/ C_6$ to be riveted on one cabinet
- Cost function
- Constraints
 - Number of components
 - Capacity of bolts/rivets

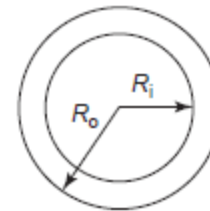
Minimum Weight Tubular Column Design

- Step 1: Problem Statement
 - Straight columns: structural elements (street light pole, traffic light post, water tower support)
 - Design a minimum mass tubular column of length l supporting a load P w/o buckling or overstressing
- Step 2: Data and Information Collection
 - Buckling load

$$P_{cr} = \frac{\pi^2 EI}{4l^2}$$

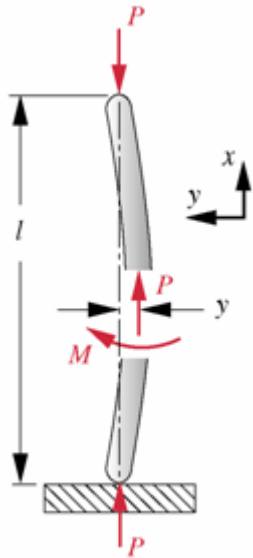


Formulation 1



Formulation 2

Buckling of an Euler Column



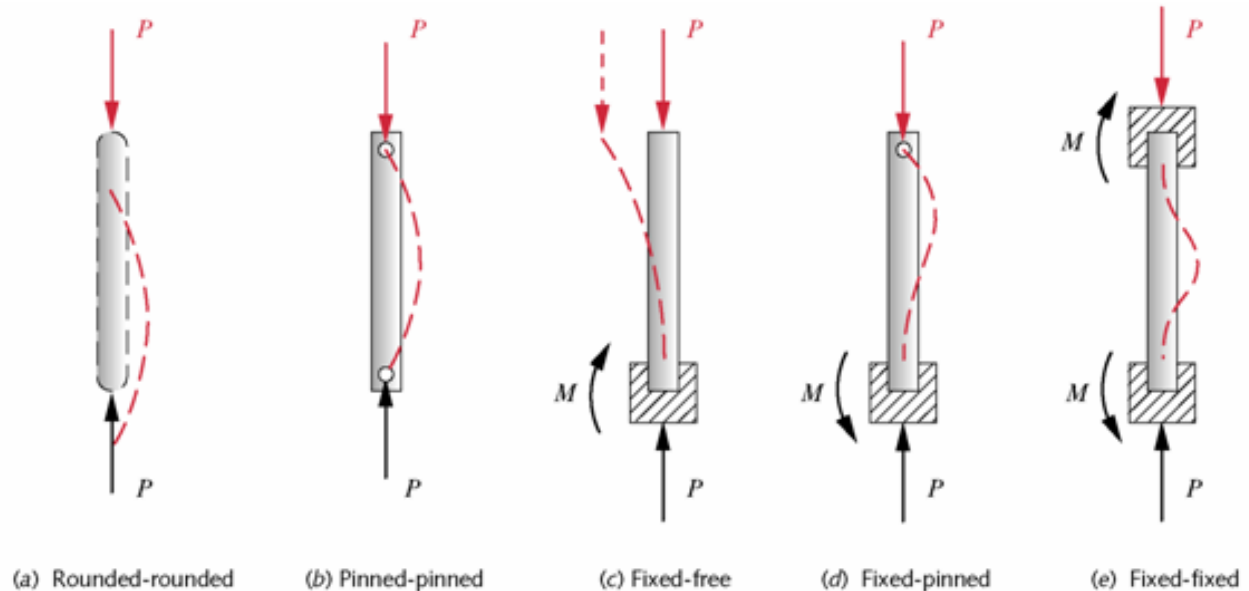
$$\frac{d^2 y}{dx^2} = \frac{M}{EI} \rightarrow EI \frac{d^2 y}{dx^2} = M = -Py$$

$$\frac{d^2 y}{dx^2} + \left(\frac{P}{EI} \right) y = 0$$

$$y = c_1 \sin \left(\sqrt{\frac{P}{EI}} x \right) + c_2 \cos \left(\sqrt{\frac{P}{EI}} x \right)$$

+ boundary conditions

$$\rightarrow P_{cr} = \frac{\pi^2 EI}{l_{eff}^2}$$



End Conditions	Theoretical Value	AISC* Recommends	Conservative Value
Rounded-Rounded	$l_{eff} = l$	$l_{eff} = l$	$l_{eff} = l$
Pinned-Pinned	$l_{eff} = l$	$l_{eff} = l$	$l_{eff} = l$
Fixed-Free	$l_{eff} = 2l$	$l_{eff} = 2.1l$	$l_{eff} = 2.4l$
Fixed-Pinned	$l_{eff} = 0.707l$	$l_{eff} = 0.80l$	$l_{eff} = l$
Fixed-Fixed	$l_{eff} = 0.5l$	$l_{eff} = 0.65l$	$l_{eff} = l$

Formulation 1

- Step 3: Design variables
 - R (mean radius of column) / t (wall thickness)
- Step 4: Cost function

$$\text{mass} = \rho(lA) = 2\rho l\pi R t$$
$$\left[\text{assuming thin wall } (R \gg t) \rightarrow A = 2\pi R t; I = \pi R^3 t \right]$$

- Step 5: Constraints

$$\begin{cases} \sigma = \frac{P}{A} = \frac{P}{2\pi R t} \leq \sigma_a \\ P \leq \frac{\pi^2 EI}{4l^2} = \frac{\pi^3 ER^3 t}{4l^2} \end{cases}$$
$$R_{\min} \leq R \leq R_{\max}; \quad t_{\min} \leq t \leq t_{\max}$$

Formulation 2

- Step 3: Design variables
 - R_o (outer radius of column) / R_i (inner radius of column)

- Step 4: Cost function

$$\text{mass} = \rho(lA) = \pi\rho l(R_o^2 - R_i^2) \quad \left[A = \pi(R_o^2 - R_i^2); \quad I = \frac{\pi}{4}(R_o^4 - R_i^4) \right]$$

- Step 5: Constraints

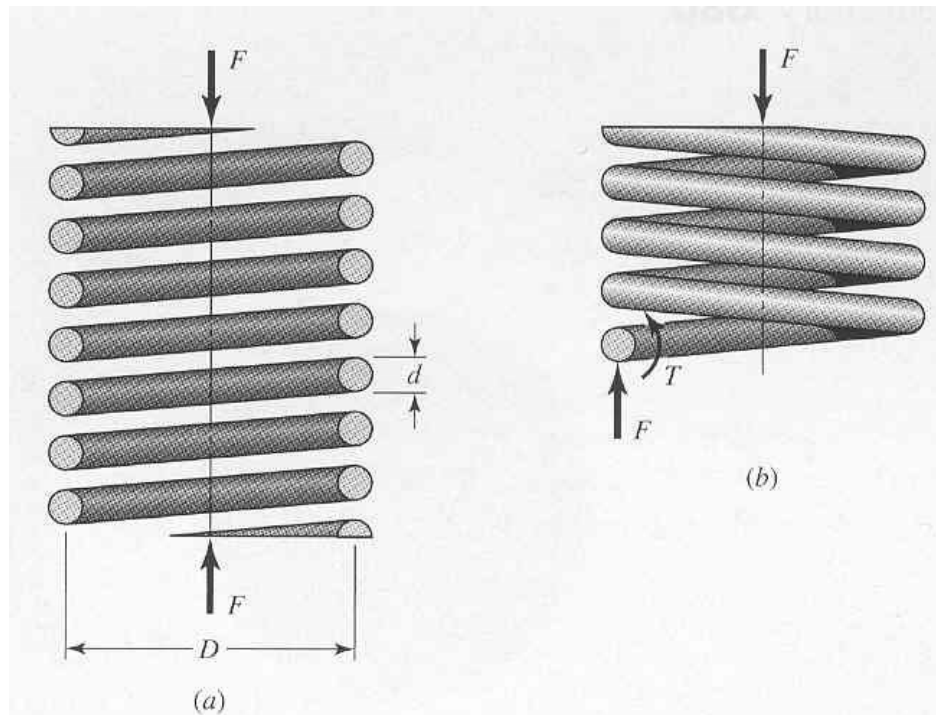
$$\begin{cases} \sigma = \frac{P}{A} = \frac{P}{\pi(R_o^2 - R_i^2)} \leq \sigma_a \\ P \leq \frac{\pi^2 EI}{4l^2} = \frac{\pi^3 E}{16l^2}(R_o^4 - R_i^4) \end{cases}$$

$$(R_o)_{\min} \leq R_o \leq (R_o)_{\max}; \quad (R_i)_{\min} \leq R_i \leq (R_i)_{\max}$$

$$R_o > R_i; \quad \underbrace{\frac{R}{t} = \frac{R_o + R_i}{2(R_o - R_i)}}_{\text{to avoid local buckling}} \leq k \quad (\text{thin-walled: } R \gg t, k \geq 20)$$

Design of Coil Spring

- Step 1: Problem Statement
 - To design a minimum mass spring to carry a given axial load without material failure and while satisfying two performance requirement: the spring must deflect by at least Δ (in), and the frequency of surge waves must not be less than ω_0 (Hz)



Step 2: Data and Information Collection (1)

- Deflection along the axis of the spring: δ (in)
- Mean coil diameter: D (in)
- Wire diameter: d (in)
- Number of active coils: N
- Gravitational constant: $g = 386$ (in/s²)
- Frequency of surge waves: ω (Hz)

Step 2: Data and Information Collection (2)

- Material property
 - Weight density: $\gamma = 0.285 \text{ (lb/in}^3 \text{)}$
 - Shear modulus: $G = 1.15\text{E}7 \text{ (lb/in}^2 \text{)}$
 - Mass density: $\rho = 7.38342\text{E-}4 \text{ (lb-s}^2\text{/in}^4 \text{)}$
 - Allowable shear stress: $\tau_a = 80000 \text{ (lb/in}^2 \text{)}$
- Other data
 - Number of inactive coils: $Q = 2$
 - Applied load: $P = 10 \text{ (lbs)}$
 - Minimum spring deflection: $\Delta = 0.5 \text{ (in)}$
 - Lower limit on surge wave frequency: $\omega_0 = 100 \text{ (Hz)}$
 - Limit on outer diameter of the coil: $D_0 = 1.5 \text{ (in)}$

Design equations for the spring (1)

- Load-deflection

$$U = \underbrace{\frac{T^2 L}{2GJ}}_{\text{torsion}} + \underbrace{\frac{F^2 L}{2GA}}_{\text{shear}} = \frac{F^2 (D/2)^2 \pi D (N+Q)}{2G (\pi d^4 / 32)} + \frac{F^2 \pi D (N+Q)}{2G (\pi d^2 / 4)} = \frac{4F^2 D^3 (N+Q)}{d^4 G} + \frac{2F^2 D (N+Q)}{d^2 G}$$

$$\xrightarrow{\text{by Castigliano's theorem}} \delta = \frac{\partial U}{\partial F} = \frac{8FD^3 (N+Q)}{d^4 G} + \frac{4FD (N+Q)}{d^2 G}$$

$$\xrightarrow{C = \frac{D}{d}} \delta = \frac{8FD^3 (N+Q)}{d^4 G} \left(1 + \frac{1}{2C^2} \right) \approx \frac{8FD^3 (N+Q)}{d^4 G}$$

- Shear stress

$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A} = \frac{F(D/2)(d/2)}{(\pi d^4 / 32)} + \frac{F}{\pi d^2 / 4} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

$$= \frac{8FD + 4Fd}{\pi d^3} = \left(1 + \frac{d}{2D} \right) \frac{8FD}{\pi d^3} = K_s \frac{8FD}{\pi d^3}$$

$$\begin{cases} K_s = 1 + 0.5 \frac{d}{D} \\ K_w = \frac{4D-1}{4(D-d)} + \frac{0.615d}{D} \end{cases}$$

Design equations for the spring (2)

- Frequency of surge waves

$$\frac{\partial^2 u}{\partial y^2} = \frac{W}{kgl^2} \frac{\partial^2 u}{\partial t^2}, \quad \text{B.C. } u(0,t) = 0 \text{ and } u(l,t) = 0$$

$$W = AL\gamma = \left(\frac{\pi d^2}{4} \right) (\pi DN) \gamma = \frac{\pi^2 d^2 DN \gamma}{4}$$

$$\omega_m = m\pi \sqrt{\frac{kg}{W}}, \quad \text{fundamental frequency } (m = 1)$$

$$f_1 = \frac{\omega_1}{2\pi} = \frac{1}{2} \sqrt{\frac{kg}{W}} = \frac{2}{\pi N} \frac{d}{D^2} \sqrt{\frac{Gg}{32\gamma}} = \frac{d}{2\pi ND^2} \sqrt{\frac{G}{2\rho}}$$

Problem Formulation

- Step 3: Identification of design variables
 - Wire diameter: d
 - Mean coil diameter: D
 - Number of active coils: N
- Step 4: Identification of an objective function
 - Mass
$$m = \rho AL = \rho \left(\frac{\pi d^2}{4} \right) \pi D (N + Q) = \frac{\pi^2 \rho d^2 D (N + Q)}{4}$$
- Step 5: Identification of constraints
 - Deflection: $\delta \geq \Delta$
 - Shear stress: $\tau \leq \tau_a$
 - Frequency of surge waves: $\omega \geq \omega_0$
 - Diameter: $D + d \leq D_0$
 - Side constraints: $d_{\min} \leq d \leq d_{\max}, D_{\min} \leq D \leq D_{\max}, N_{\min} \leq N \leq N_{\max}$

Mathematical Formulation

$$\text{Minimize}_{d,D,N} m = \frac{\pi^2 \rho d^2 D (N + Q)}{4}$$

$$\text{subject to } \frac{8FD^3(N+Q)}{d^4G} \geq \Delta \rightarrow 1 - \frac{8FD^3(N+Q)}{d^4G\Delta} \leq 0$$

$$\left[\frac{4D-d}{4(D-d)} + \frac{0.615d}{D} \right] \frac{8FD}{\pi d^3} \leq \tau_a \rightarrow \left[\frac{4D-d}{4(D-d)} + \frac{0.615d}{D} \right] \frac{8FD}{\pi d^3 \tau_a} - 1 \leq 0$$

$$\frac{d}{2\pi ND^2} \sqrt{\frac{G}{2\rho}} \geq \omega_0 \rightarrow 1 - \frac{d}{2\pi ND^2 \omega_0} \sqrt{\frac{G}{2\rho}} \leq 0$$

$$D + d \leq D_0 \rightarrow \frac{D+d}{D_0} - 1 \leq 0$$

$$d_{\min} \leq d \leq d_{\max}$$

$$D_{\min} \leq D \leq D_{\max}$$

$$N_{\min} \leq N \leq N_{\max}$$

Standard Design Optimization Model

Find an n -vector $\mathbf{x} = (x_1, \dots, x_n)$ of design variables
to minimize a cost function

$$f(\mathbf{x}) = f(x_1, \dots, x_n)$$

subject to

$$\left\{ \begin{array}{l} \text{the } p \text{ equality constraints} \\ h_j(\mathbf{x}) = h_j(x_1, \dots, x_n) = 0; \quad j = 1, \dots, p \\ \text{and the } m \text{ inequality constraints} \\ g_i(\mathbf{x}) = g_i(x_1, \dots, x_n) \leq 0; \quad i = 1, \dots, m \end{array} \right.$$

bounds on design variables:

$$x_i \geq 0 \quad \text{or} \quad x_{il} \leq x_i \leq x_{iu}; \quad i = 1, \dots, n$$

Observations (1)

- Functions must depend on design variables.
- Number of independent equality constraints: $p \leq n$
 - $p > n$: overdetermined system of equations
 - redundant equality constraints
 - Inconsistent formulation
 - $p = n$: no optimization is necessary
- Inequality constraints written as “ ≤ 0 ”
 - No restriction on the number of inequality constraints
- Scaling effect
 - optimum design does not change. optimum cost function value, however, changes.
 - cost function by a positive constant
 - Inequality constraints by a positive constant
 - equality constraints by any constants

Observations (2)

- Maximization problem treatment

$$f(\mathbf{x}) = -F(\mathbf{x})$$

- “ \geq type” constraints

$$G_j(\mathbf{x}) \geq 0 \rightarrow g_j(\mathbf{x}) = -G_j(\mathbf{x}) \leq 0$$

- Discrete and Integer design variables

- Approach 1

- Solve the problem assuming continuous DVs
- Assign nearest discrete/integer values
- Check feasibility \leftarrow numerous combinations

- Approach 2 (adaptive numerical optimization)

- Obtain optimum solution with continuous DVs
- Assign only DVs close to their discrete/integer values
- Optimize the problem until all DVs have proper values

Observations (3)

- Feasible set: collection of all feasible designs

$$S = \{\mathbf{x} | h_j(\mathbf{x}) = 0; j = 1, \dots, p; \quad g_i(\mathbf{x}) \leq 0; i = 1, \dots, m\}$$

- Inequality constraint:

$$g_i(\mathbf{x}) \leq 0 \rightarrow \begin{cases} \text{active/tight/binding} : g_i(\mathbf{x}^*) = 0 \\ \text{inactive} : g_i(\mathbf{x}^*) < 0 \\ \text{violated} : g_i(\mathbf{x}^*) > 0 \end{cases}$$