

# Profit Maximization Problem (1)

*Step 1: Project/problem description.* A company manufactures two machines, A and B. Using available resources, either 28 A or 14 B can be manufactured daily. The sales department can sell up to 14 A machines or 24 B machines. The shipping facility can handle no more than 16 machines per day. The company makes a profit of \$400 on each A machine and \$600 on each B machine. How many A and B machines should the company manufacture every day to maximize its profit?

*Step 2: Data and information collection.* Data and information are defined in the project statement. No additional information is needed.

*Step 3: Definition of design variables.* The following two design variables are identified in the problem statement:

$x_1$  = number of A machines manufactured each day

$x_2$  = number of B machines manufactured each day

*Step 4: Optimization criterion.* The objective is to maximize daily profit, which can be expressed in terms of design variables using the data given in step 1 as

$$P = 400x_1 + 600x_2, \$ \quad (a)$$

*Step 5: Formulation of constraints.* Design constraints are placed on manufacturing capacity, on sales personnel, and on the shipping and handling facility. The constraint on the shipping and handling facility is quite straightforward:

$$x_1 + x_2 \leq 16 \text{ (shipping and handling constraint)} \quad (b)$$

# Profit Maximization Problem (2)

Constraints on manufacturing and sales facilities are a bit tricky because they are either “this” or “that” type of requirements. First, consider the manufacturing limitation. It is assumed that if the company is manufacturing  $x_1$  A machines per day, then the remaining resources and equipment can be proportionately used to manufacture  $x_2$  B machines, and vice versa. Therefore, noting that  $x_1/28$  is the fraction of resources used to produce A and  $x_2/14$  is the fraction used to produce B, the constraint is expressed as

$$\frac{x_1}{28} + \frac{x_2}{14} \leq 1 \text{ (manufacturing constraint)} \quad (c)$$

Similarly, the constraint on sales department resources is given as

$$\frac{x_1}{14} + \frac{x_2}{24} \leq 1 \text{ (limitation on sale department)} \quad (d)$$

Finally, the design variables must be nonnegative as

$$x_1, x_2 \geq 0 \quad (e)$$

# Graphical Solutions (1)

## Profit Maximization Problem

$$\text{Maximize } f = 400x_1 + 600x_2$$

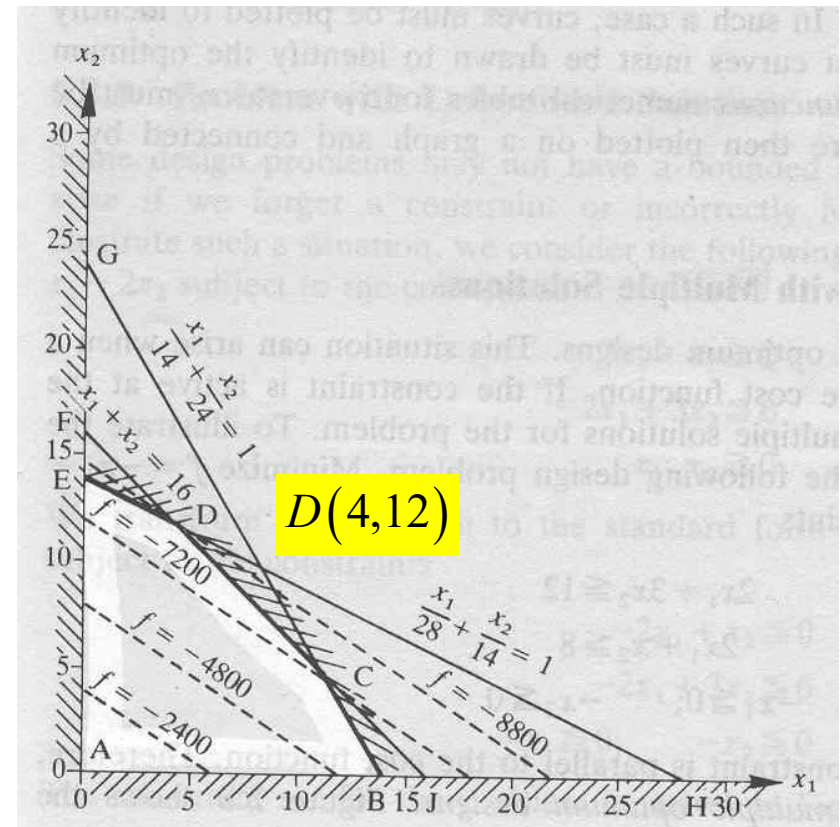
$x_1, x_2$

subject to

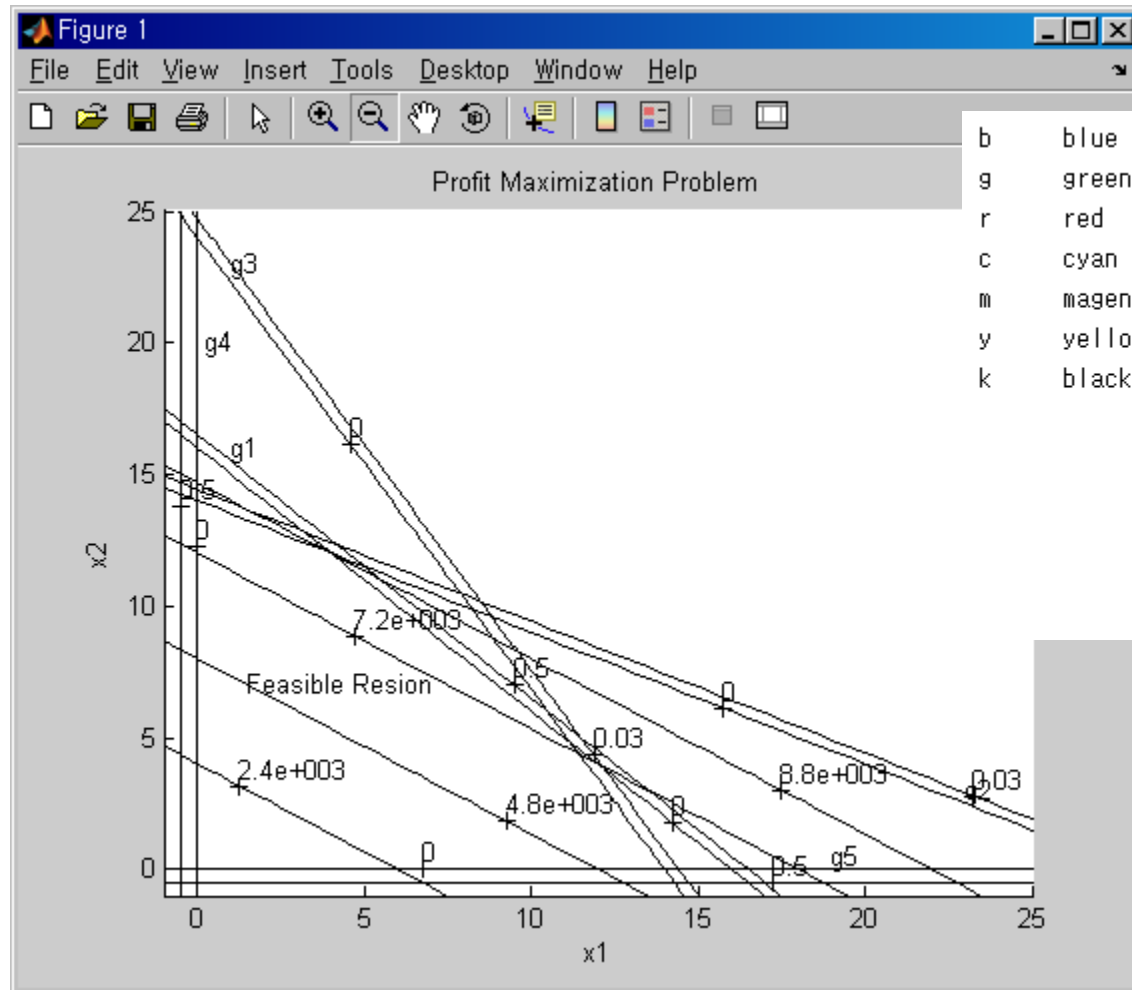
$$\begin{cases} x_1 + x_2 \leq 16 & \text{(shipping and handling)} \\ \frac{x_1}{28} + \frac{x_2}{14} \leq 1 & \text{(manufacturing)} \\ \frac{x_1}{14} + \frac{x_2}{24} \leq 1 & \text{(limitations on sales dept.)} \\ x_1, x_2 \geq 0 \end{cases}$$

$x_1$  = # of A machines manufactured each day

$x_2$  = # of B machines manufactured each day



# Graphical Solutions (2)



b	blue	.	point	-	solid
g	green	o	circle	:	dotted
r	red	x	x-mark	-.	dashdot
c	cyan	+	plus	--	dashed
m	magenta	*	star	(none)	no line
y	yellow	s	square		
k	black	d	diamond		
		v	triangle (down)		
		^	triangle (up)		
		<	triangle (left)		
		>	triangle (right)		
		p	pentagram		
		h	hexagram		

# Minimum Weight Tubular Column Design

Minimize  $f = 2\rho l\pi Rt = (2.4608 \times 10^5) Rt$

subject to

$$\begin{cases} \sigma = \frac{P}{A} = \frac{P}{2\pi Rt} \leq \sigma_a \rightarrow g_1 = \frac{10 \times 10^6}{2\pi Rt} - 248 \times 10^6 \leq 0 \\ P \leq \frac{\pi^2 EI}{4l^2} = \frac{\pi^3 ER^3 t}{4l^2} \rightarrow g_2 = 10 \times 10^6 - \frac{\pi^3 (207 \times 10^9) R^3 t}{4(5)^2} \leq 0 \\ R, t \geq 0 \rightarrow \begin{cases} g_3 = -R \leq 0 \\ g_4 = -t \leq 0 \end{cases} \end{cases}$$

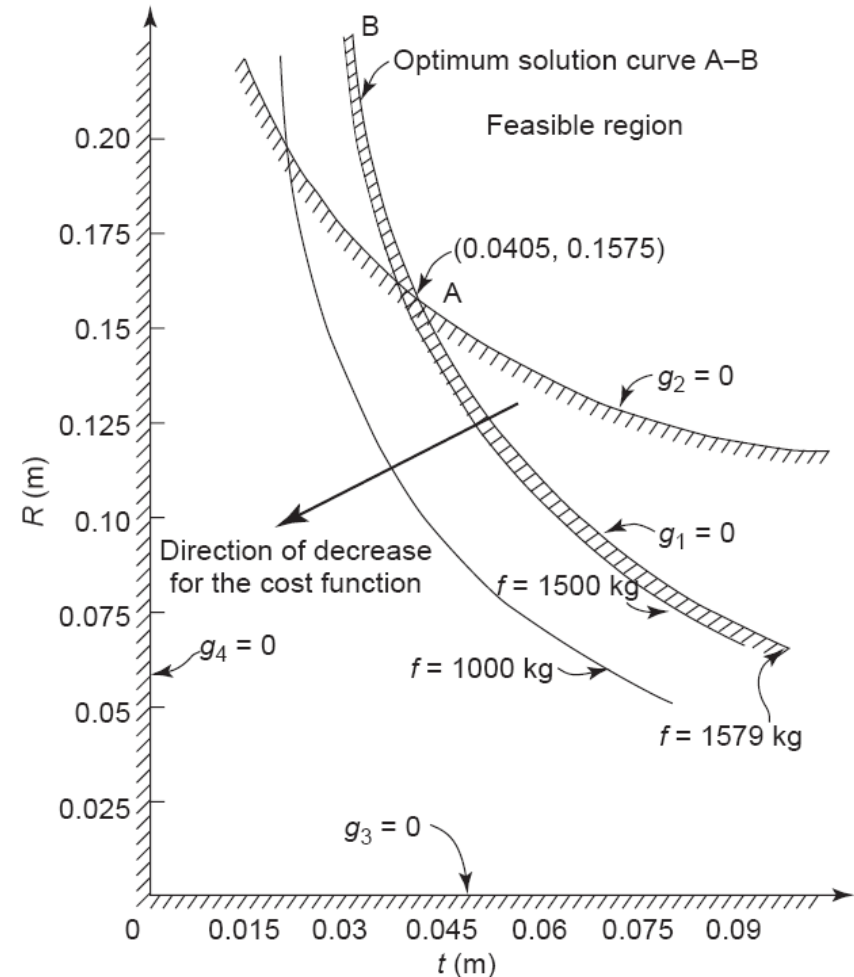
$P = 10\text{MN}$

$E = 207\text{GPa}$

$\rho = 7833\text{kg/m}^3$

$l = 5.0\text{m}$

$\sigma_a = 248\text{MPa}$



\* cost function contours run parallel to the stress constraint  $g_1$

# Beam Design Problem (1)

*Step 1: Project/problem description.* A beam of rectangular cross-section is subjected to a bending moment  $M$  (N·m) and a maximum shear force  $V$  (N). The bending stress in the beam is calculated as  $\sigma = 6M/bd^2$  (Pa), and average shear stress is calculated as  $\tau = 3V/2bd$  (Pa), where  $b$  is the width and  $d$  is the depth of the beam. The allowable stresses in bending and shear are 10 and 2 MPa, respectively. It is also desirable that the depth of the beam does not exceed twice its width and that the cross-sectional area of the beam is minimized. In this section, we formulate and solve the problem using the graphical method.

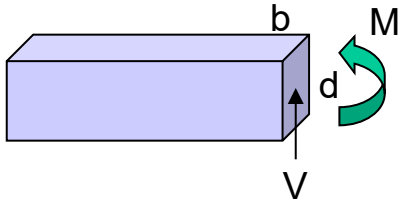
*Step 2: Data and information collection.* Let bending moment  $M = 40$  kN·m and the shear force  $V = 150$  kN. All other data and necessary equations are given in the project statement. We shall formulate the problem using a consistent set of units, N and mm.

*Step 3: Definition of design variables.* The two design variables are

$d$  = depth of beam, mm

$b$  = width of beam, mm

# Beam Design Problem (2)



Minimize  $f = bd$   
 $\substack{b,d}$

subject to

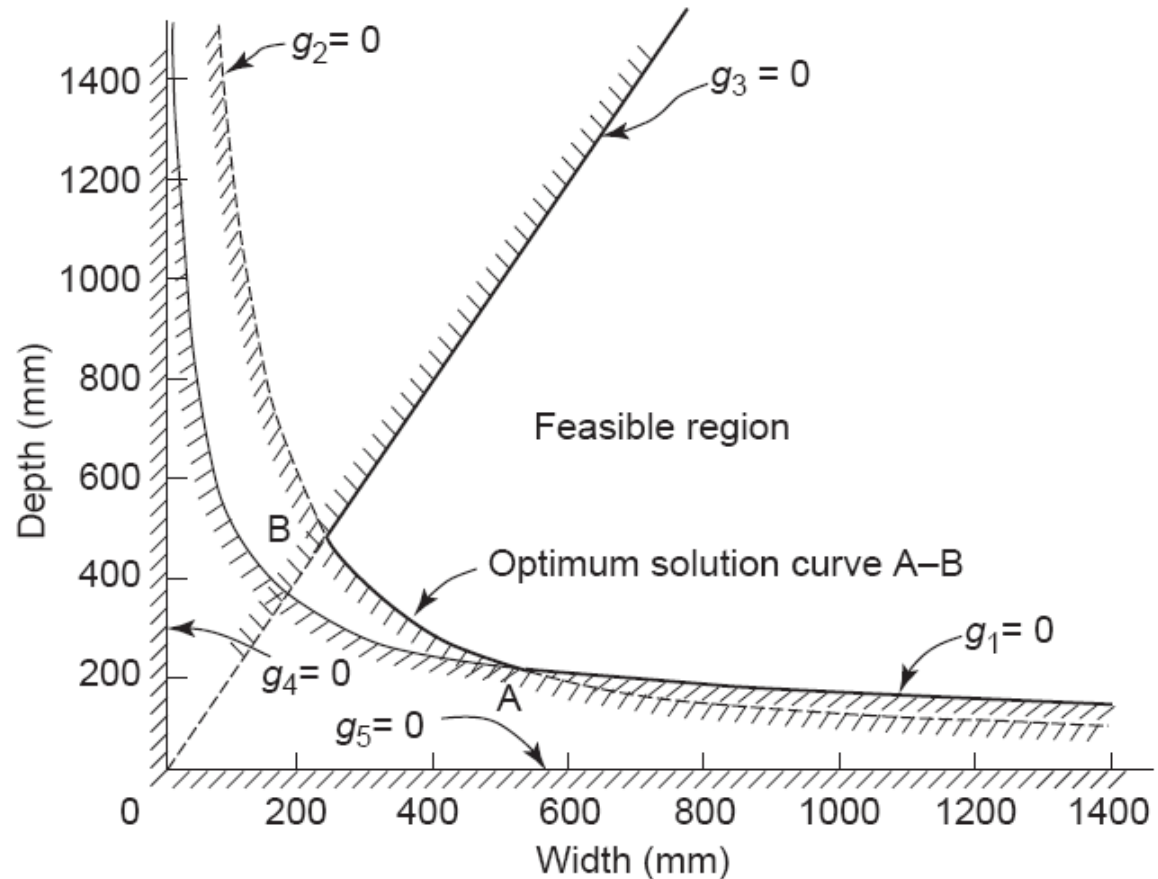
$$\begin{cases} \sigma = \frac{6M}{bd^2} \leq (\sigma_a)_{bending} \\ \tau = \frac{3V}{2bd} \leq (\tau_a)_{shear} \\ d \leq 2b \\ b, d \geq 0 \end{cases}$$

$$M = 40 \text{ kN} \cdot \text{m}$$

$$V = 150 \text{ kN}$$

$$(\sigma_a)_{bending} = 10 \text{ MPa}$$

$$(\tau_a)_{shear} = 2 \text{ MPa}$$



\* cost function is parallel to the stress constraint  $g_2$

$$\left. \begin{array}{l} b^* = 237 \text{ mm}, d^* = 474 \text{ mm @ point B} \\ b^* = 527.3 \text{ mm}, d^* = 213.3 \text{ mm @ point A} \end{array} \right\} \rightarrow f^* = 115,000 \text{ mm}^2$$

find b and d to minimize  $f=b*d$  subject to  $\sigma=6*M/(b*d^2)\leq\sigma_a$ ,  $\tau=3*V/(2*b*d)\leq\tau_a$ ,  $d\leq 2*b$ ,  $b\geq 0$ ,  $d\geq 0$  where  $M=40\text{kNm}$ ,  $V=150\text{kN}$ ,  $\sigma_a=10\text{MPa}$ ,  $\tau_a=2\text{MPa}$

Pro Search

Calculate the constraints based on the given values of M, V, sigma\_a, and tau\_a

Set up and solve the optimization problem to find the values of b and d that minimize f=b\*d

Perplexity

Based on the optimization results provided, I can interpret the solution for you:

The optimal values for b and d that minimize the objective function  $f = b*d$  while satisfying all the given constraints are:

$b \approx 0.39148676 \text{ m}$   
 $d \approx 0.78297353 \text{ m}$

Let's verify that this solution satisfies all the constraints:

- $\sigma = 6M/(bd^2) \leq \sigma_a$   
 $\sigma = 6 * 40000 / (0.39148676 * 0.78297353^2) \approx 10 \text{ MPa} = \sigma_a$  (Satisfied, equality holds)
- $\tau = 3V/(2b*d) \leq \tau_a$   
 $\tau = 3 * 150000 / (2 * 0.39148676 * 0.78297353) \approx 2 \text{ MPa} = \tau_a$  (Satisfied, equality holds)
- $d \leq 2*b$   
 $0.78297353 \leq 2 * 0.39148676 = 0.78297352$  (Satisfied, equality holds)
- $b \geq 0$   
 $0.39148676 \geq 0$  (Satisfied)
- $d \geq 0$   
 $0.78297353 \geq 0$  (Satisfied)

The minimum value of the objective function  $f = b*d$  at this solution is:

$f = 0.39148676 * 0.78297353 \approx 0.30652 \text{ m}^2$

This solution represents the optimal cross-sectional dimensions of a beam or structural member subject to the given constraints:

- $M = 40 \text{ kNm}$  (bending moment)
- $V = 150 \text{ kN}$  (shear force)
- $\sigma_a = 10 \text{ MPa}$  (allowable normal stress)
- $\tau_a = 2 \text{ MPa}$  (allowable shear stress)

The optimization has found the smallest possible cross-sectional area ( $b*d$ ) that can withstand the applied loads without exceeding the allowable stresses. The solution shows that:

- Both stress constraints (normal and shear) are active, meaning the member is fully stressed under the given loads.
- The depth ( $d$ ) is exactly twice the width ( $b$ ), which is the most efficient rectangular cross-section for this loading condition.

This solution provides the most economical use of material for the given loading and stress constraints, ensuring both safety and efficiency in the design.



find b and d to minimize  $f=b*d$  subject to  $\sigma=6*M/(b*d^2)\leq\sigma_a$ ,  $\tau=3*V/(2*b*d)\leq\tau_a$ ,  $d\leq 2*b$ ,  $b\geq 0$ ,  $d\geq 0$  where  $M=40kNm$ ,  $V=150kN$ ,  $\sigma_a=10MPa$ ,  $\tau_a=2MPa$

Pro Search

- Set up the optimization problem with the objective function and constraints based on the provided parameters
- Solve the optimization problem to find the values of b and d that minimize the objective function f
- Verify that the solution satisfies all the given constraints and calculate the minimum value of f

Perplexity

Based on the optimization results provided, I can interpret the solution for you:

The optimal values for b and d that minimize the objective function  $f = b*d$  while satisfying all the given constraints are:

$b \approx 0.3354 \text{ m}$   
 $d \approx 0.3354 \text{ m}$

However, upon checking the constraints, we find that not all constraints are satisfied:

- $\sigma = 6M/(bd^2) \leq \sigma_a$   
 $\sigma \approx 6.36 \text{ MPa} < 10 \text{ MPa}$  (Satisfied)
- $\tau = 3V/(2b*d) \leq \tau_a$   
 $\tau \approx 2.00012 \text{ MPa} > 2 \text{ MPa}$  (Not Satisfied)
- $d \leq 2*b$   
 $0.3354 \leq 2 * 0.3354 = 0.6708$  (Satisfied)
- $b \geq 0$   
 $0.3354 \geq 0$  (Satisfied)
- $d \geq 0$   
 $0.3354 \geq 0$  (Satisfied)

The minimum value of the objective function  $f = b*d$  at this solution is:

$f = 0.3354 * 0.3354 \approx 0.1125 \text{ m}^2$

It's important to note that this solution does not fully satisfy all constraints, particularly the shear stress constraint. The optimization algorithm has found a solution that minimizes the cross-sectional area, but it slightly exceeds the allowable shear stress.

To find a valid solution that satisfies all constraints, we may need to adjust the optimization parameters or use a different optimization method. A valid solution would likely result in slightly larger dimensions to ensure that both normal and shear stress constraints are met.

For a more accurate solution, I recommend:

- Increasing the precision of the optimization algorithm.
- Using a constrained optimization method that enforces strict constraint satisfaction.
- Possibly starting with a different initial guess.

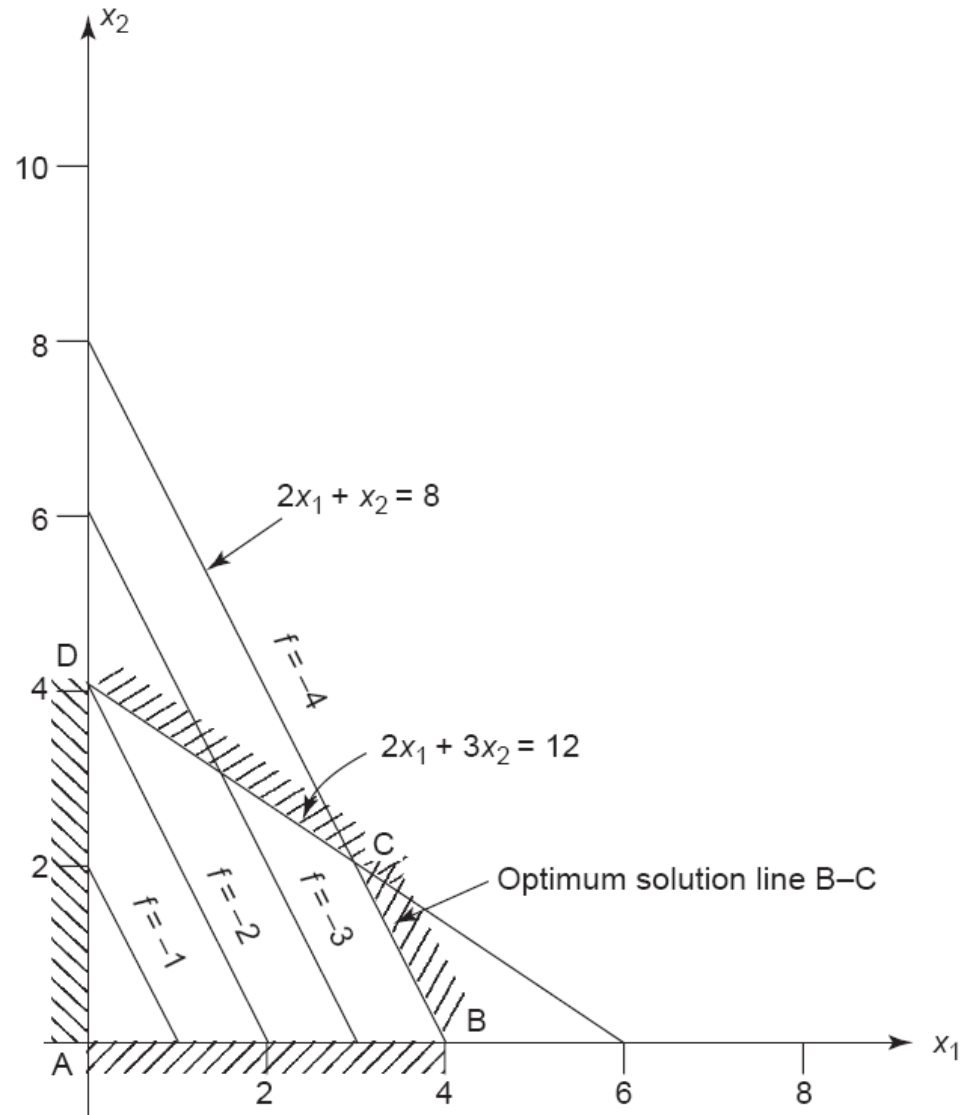
Given the current results, a conservative approach would be to slightly increase both b and d until all constraints are satisfied, even though this would result in a larger cross-sectional area than the theoretical minimum.

# Multiple Solutions

Minimize  $f(\mathbf{x}) = -x_1 - 0.5x_2$

subject to

$$\begin{cases} 2x_1 + 3x_2 \leq 12 \\ 2x_1 + x_2 \leq 8 \\ -x_1 \leq 0 \\ -x_2 \leq 0 \end{cases}$$



# Unbounded Solutions

Maximize  $f(\mathbf{x}) = x_1 - 2x_2$

subject to

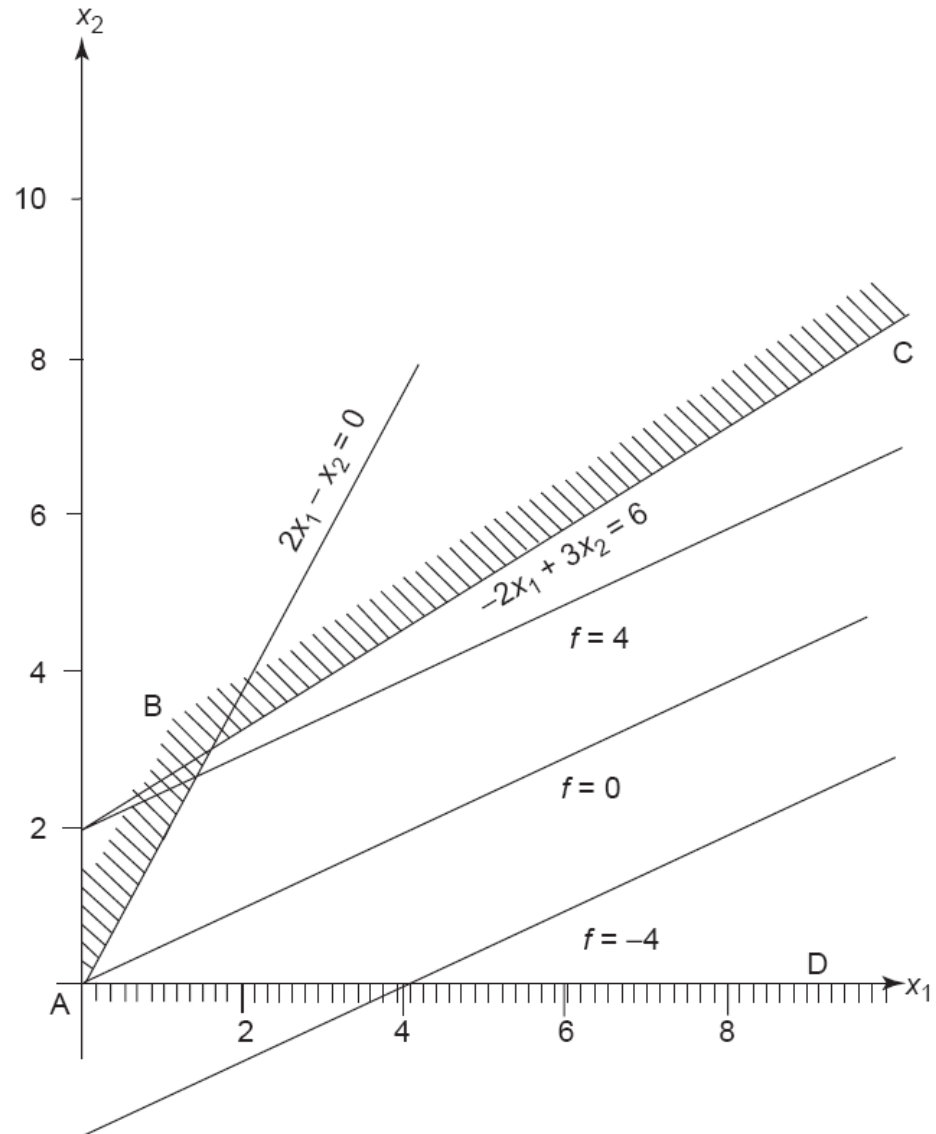
$$\begin{cases} 2x_1 - x_2 \geq 0 \\ -2x_1 + 3x_2 \leq 6 \\ x_1, x_2 \geq 0 \end{cases}$$



Minimize  $f(\mathbf{x}) = -x_1 + 2x_2$

subject to

$$\begin{cases} -2x_1 + x_2 \leq 0 \\ -2x_1 + 3x_2 \leq 6 \\ -x_1 \leq 0 \\ -x_2 \leq 0 \end{cases}$$



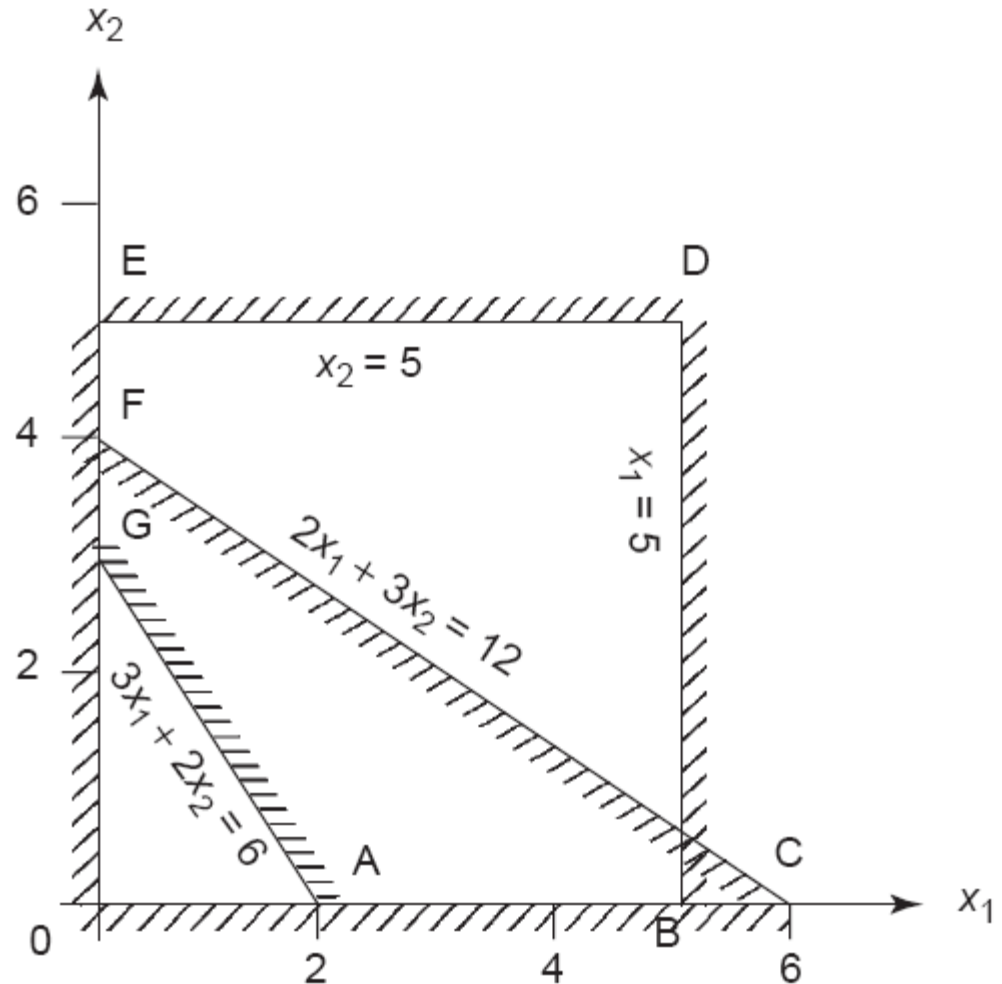
# Infeasible Problem

- Too many constraints

Minimize  $f(\mathbf{x}) = x_1 + 2x_2$

subject to

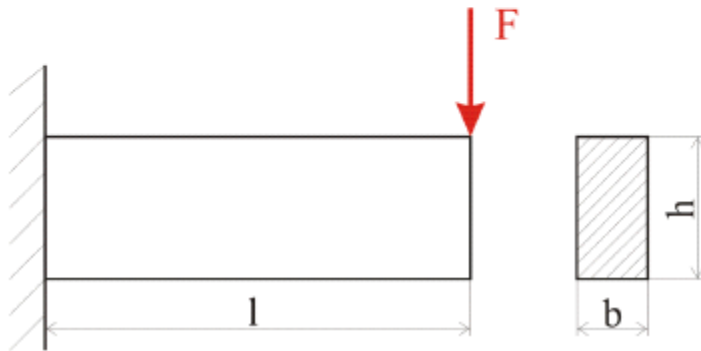
$$\begin{cases} 3x_1 + 2x_2 \leq 6 \\ 2x_1 + 3x_2 \geq 12 \\ x_1 \leq 5 \\ x_2 \leq 5 \\ x_1, x_2 \geq 0 \end{cases}$$



# Example

A **cantilever beam** loaded with force  $F=24000$  N. Where the cross-section parameters: Width  $b_{[20,40]}$  and height  $h_{[30,90]}$  can vary on their range to minimize the beam weight, subject to these constraint:

- 1) Max normal stress can not exceed the  $\sigma_{\max}$  value,
- 2) Max shear stress can not exceed the  $\tau_{\max}$  and
- 3) Height  $h$  should not be larger than twice the width  $b$ .



# Problem Formulation

Mathematically this problem can be stated as:

**Objective:**  $\min \text{Weight}(b,h)$

**Design Variables:**  $b_L < b < b_U, \quad 20 < b < 40$

$h_L < h < h_U, \quad 30 < h < 90$

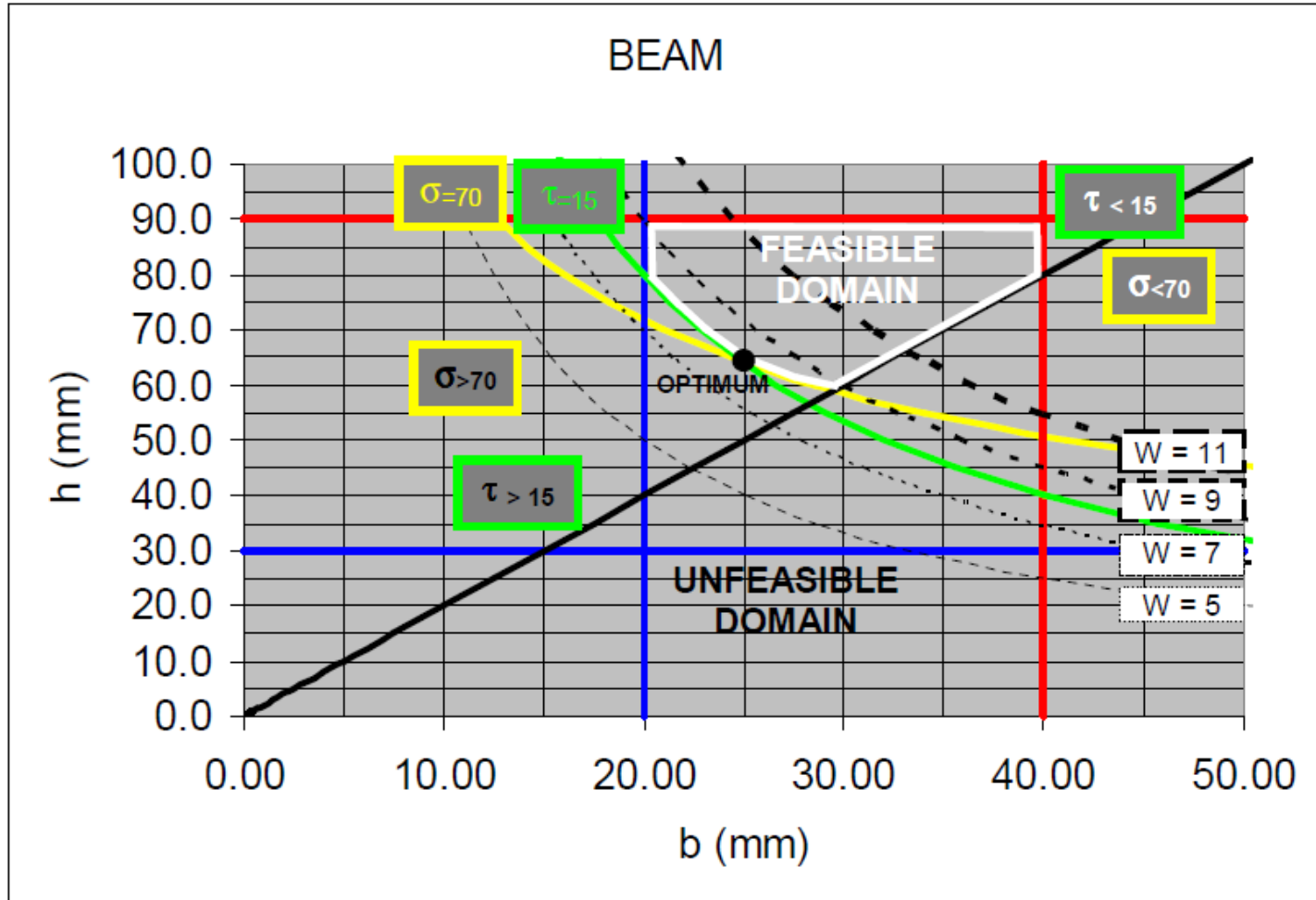
**Design Constraints:**  $\sigma(b,h) = 6F/(bh^2) \leq \sigma_{\max}, \quad \text{with} \quad \sigma_{\max} = 70 \text{ MPa}$

$\tau(b,h) = F/(bh) \leq \tau_{\max}, \quad \text{with} \quad \tau_{\max} = 15 \text{ MPa}$

$h \geq 2*b$

# Graphical Solution: EXCEL

*This problem can be described graphically as showed below:*



Cantilever beam problem (Optimum ( $b=24.9$ ,  $h=64.3$ )  $W = 8$ ).

# MATLAB CODE

- 정식화 내용을 m-file에 아래와 같이 표현

```
[b,h]=meshgrid(0:0.5:50,0:0.5:100);
```

```
f=b.*h;
```

```
g1=(6*24000*50)./(b.*(h.^2))-70;
```

```
g2=24000./(b.*h)-15;
```

```
g3=2*b-h;
```

```
g4=b-40;
```

```
g5=-b+20;
```

```
g6=h-90;
```

```
g7=-h+30;
```



# PLOT: CONTOUR (1)

## – 작성할 그래프를 Contour 함수로 표현

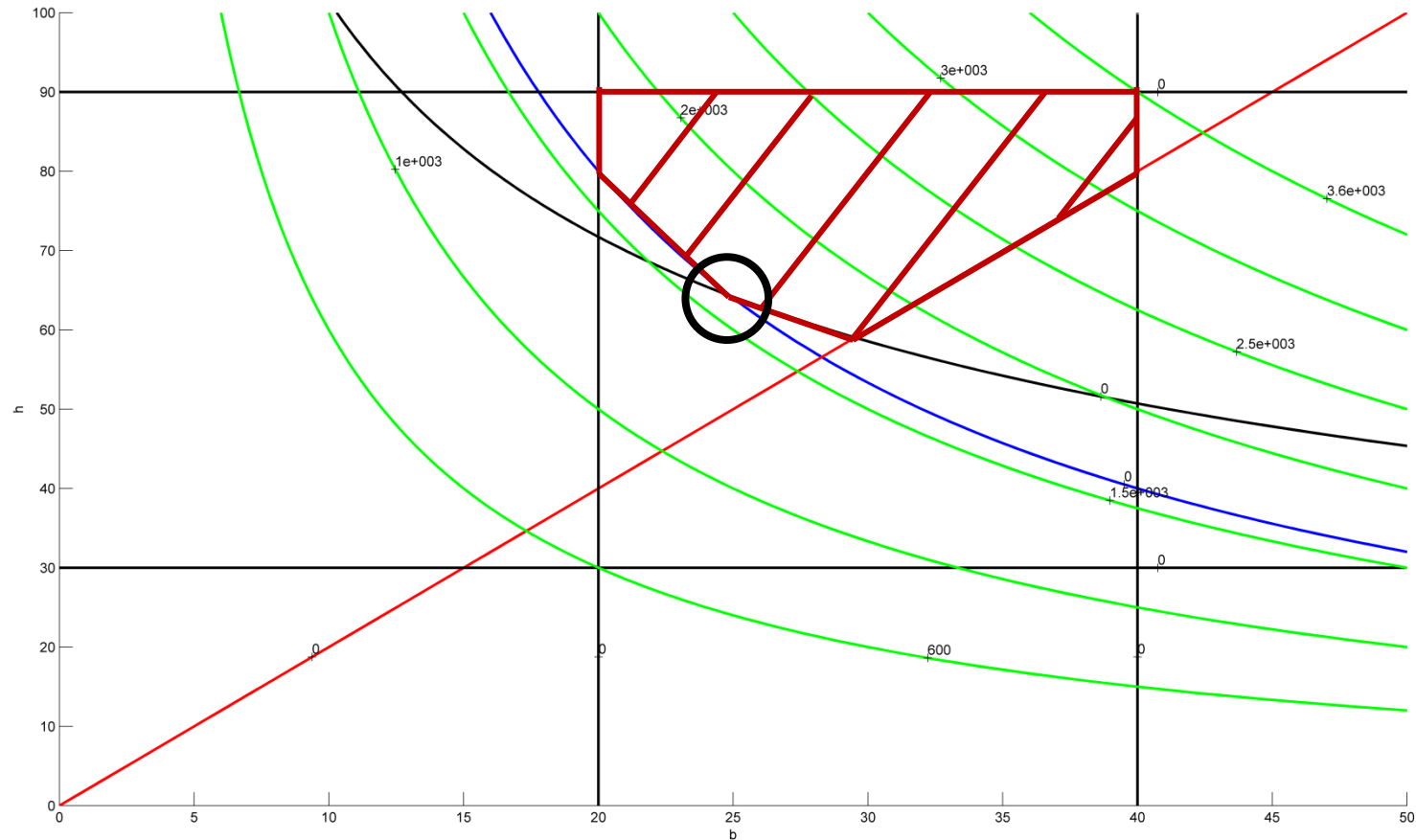
```

cla reset
axis auto
xlabel('b'), ylabel('h')
hold on
cv1=[0 1];
const1=contour(b,h,g1,[0 0],'k','linewidth',2);
clabel(const1)
const2=contour(b,h,g2,[0 0],'b','linewidth',2);
clabel(const2)
const3=contour(b,h,g3,[0 0],'r','linewidth',2);
clabel(const3)
const4=contour(b,h,g4,[0 0],'k','linewidth',2);
clabel(const4)
const5=contour(b,h,g5,[0 0],'k','linewidth',2);
clabel(const5)
const6=contour(b,h,g6,[0 0],'k','linewidth',2);
clabel(const6)
const7=contour(b,h,g7,[0 0],'k','linewidth',2);
clabel(const7)
const8=contour(b,h,f,[600 1000 1500 2000 2500 3000 3600],'g','linewidth',2);
clabel(const8)

```

# PLOT: CONTOUR (2)

– Feasible region



# FSOLVE

- fsolve 함수를 이용하여 교점을 찾음

```
% function을 다음과 같이 먼저 정의
function f=fopt(x)
f=[(6*24000*50)./(x(1).*(x(2).^2))-70;24000./(x(1).*x(2))-15];
```

```
% 초기값은 25,60으로 설정
>>x0=[25,60];
>>x=fsolve('fopt',x0)
```

```
x =
    24.8889    64.2857
```

목적함수  
1600