Classification of Optimization Problems (1)

- Variables
  - Continuous, discrete or mixed
- Objective
  - Function of a single variable
  - Function of many variables
  - Linear
  - Sum of squares
  - Nonlinear
  - Smooth / non-smooth
  - Convex / non-convex
  - 1st derivatives are available
  - 2nd derivatives are available
- Constraints
  - None
  - Simple bounds
  - Linear
  - Non-linear
  - Equality / inequality
  - Smooth / non-smooth
  - 1st derivatives are available
  - 2nd derivatives are available
- Optimum
  - Local
  - Global

Classification of Optimization Problems (2)

- The choice of solution method is very dependent on
  - the class of the problem
  - the size of the problem
  - the structure of the problem
  - the cost of function and gradient evaluation
  - etc.

Classification of Optimization Problems (3)

- Some more jargon
  - Gradient
  - Hessian
  - Sensitivity analysis
  - Scaling
  - Normalization
  - Mathematical Programming
  - LP
  - NLP

- Optimization software
  - In
    - EXCEL GRG2
    - MATLAB OPTIM toolbox
    - NAG
    - NETLIB
  - Specialized packages
    - NPSOL
    - IDESIGN
    - LANCELOT
  - Plus hundreds of others

Structural Optimization (1)

- Rational establishment of a structural design that is the best of all possible designs within a prescribed objective and a given set of geometrical and/or behavioral limitations
- Mathematics and mechanics with engineering
- Broad multidisciplinary field
  - Aeronautical, civil, mechanical, nuclear, off-shore engineering, space technology
- Motivation
  - Limited energy resources, shortage of economic and some material resources, strong technological competition, environmental problem

Structural Optimization (2)

- Minimum cost or weight of the structure for given performance / Maximum performance for a bound on cost
  - Decreasing the weight of space, aero, or land-borne structures
  - Cost reduction of load-carrying structures for given capacity, strength, and/or stiffness requirements
  - Increasing the efficiency of fibers in composite materials by optimizing their distribution and orientation
  - Minimizing dynamic response of rotating machinery or structures subjected to external excitation
- Research in optimal structural design
  - Fundamental aspects of structural optimization
  - Development of effective numerical solution procedures for optimization of complex practical structures

Analysis Problem

- Completely specified in deterministic problems / Given in terms of probabilities in probabilistic problems
  - Structural design, properties of materials, support/loading conditions
- Determine the structural response
  - Equilibrium (or state) / constitutive equations, compatibility / boundary conditions
  - Stress, strain, deflection, natural vibration frequencies, load factors for elastic instability

Redesign (or Sensitivity Analysis)

- Design, material, or support parameters are changed (or varied) and the corresponding changes (or variations) of the structural response are determined via repeated (or special) analysis
- Conventional design procedure
  - A series of repeated changes of the structural parameters followed by analysis
  - A series of redesign analyses until a structure fulfills the behavioral requirements and is reasonable in cost
  - Changes decided by guesswork based on information obtained from the previous analysis

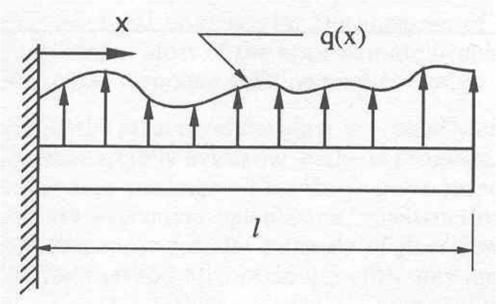
Optimization of Structures

- Set of structural parameters is subdivided into preassigned parameters and design variables
- Problem consists in determining optimal values of the design variables such that they maximize or minimize a specific function termed the objective (or criterion, or cost) function while satisfying a set of geometrical and/or behavioral requirements, which are specified prior to design, and are called constraints.

Beam Design

- Structural analysis

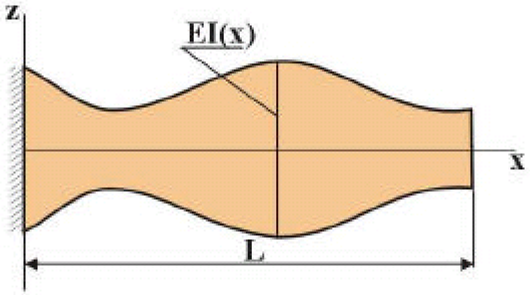
$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right)=q\left(x\right)$$

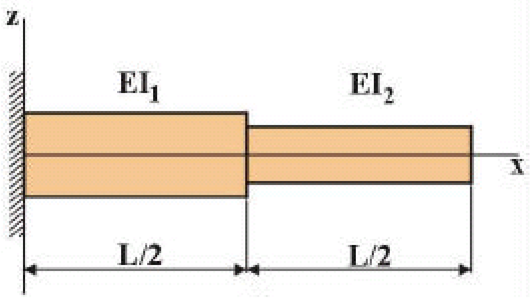


- Structural designer
  - Optimal distribution of the moment of inertia *I(x)* of the beam along its length
  - Objective function : mass $m=c\int_0^l I^p\left(x\right)dx$
  - Constraints : displacement $w_{\max}=\max_{0\le x\le l} w\left(x\right)\le w_0$
  - Optimality condition : in the form of a differential equation in *I(x)* and *w(x)* → “calculus of variations”

Function vs. Parameter Optimization

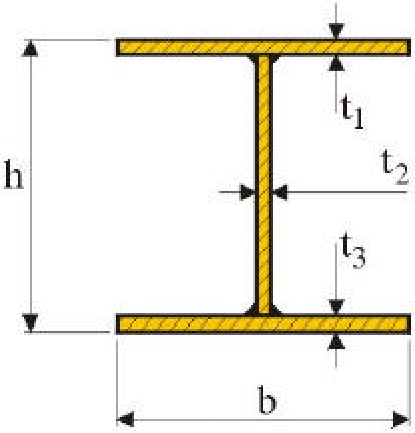
	Before 60s	After 60s
analysis solution	analytic solutions (e.g., by using infinite series)	computer implementation (e.g., finite element method)
unknown	function	discrete value
equation	differential	algebraic
discipline	calculus of variations	mathematical programming





Elements of Problem Formulation (1)

- Design variables: $\mathbf{x} = \left(x_1, x_2, \ldots, x_n \right)$
  - Parameters controlling the geometry of the structure
    - Cross-sectional dimensions
    - Member sizes
  - Material properties
  - Continuous
    - Range of variation
  - Discrete
    - Isolated values
    - Manufacturing considerations
  - Critical to the success of the optimization process



Elements of Problem Formulation (2)

- Objective function : $\mathbf{f}(\mathbf{x}) = \left[f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_p(\mathbf{x}) \right]$
  - Measure of effectiveness of the design
    - Weight, displacements, stresses, vibration frequencies, buckling loads, cost
  - Multicriteria(multiobjective) optimization
    - Generate a composite objective function
    - Select the most important as the only objective function and impose limits on the others
    - Edgeworth-Pareto optimization
- Constraints
  - limits on the design variables : side constraints
  - Impose upper or lower limits on quantities : inequality constraints
  - Equality constraints $\rightarrow$ inequality constraints (some solution strategies)

Design Variables (1)

- Cross-sectional DVs: Properties of structural elements
  - Cross-sectional areas (of a bar, rod or beam)
  - Second area moment (of a beam, column or arch)
  - Thickness (of a plate)
  - Continuous (function of the spatial coordinates) / Discrete (distinct, standardized sizes)
- DVs describing the layout of a structure
  - Topological DVs: number, spatial sequence, and mutual connectivity of members and joints (integer)
  - Configurational (or geometrical) DVs: coordinates of joints, centerlines or midsurfaces of structural members (bar, beam, arch, shell)

Design Variables (2)

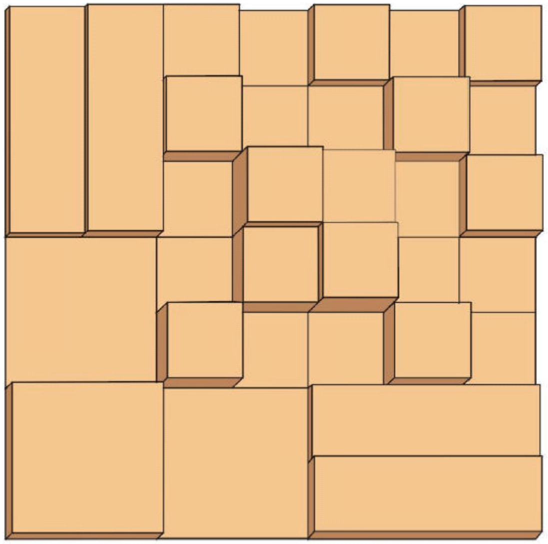
- Shape DVs
  - Shape of external boundaries or interface of a structure
  - Cross-sectional shape of a rod, column, or beam
  - Boundary shape of a disk, plate, or shell
- Material DVs
  - Material properties (discrete)
  - Fiber composite materials: concentration and direction of the fibers (continuous)
- Support or loading DVs
  - Support (or boundary) conditions or the distribution of loading on a structure
  - Location, number, and type of support or the external forces

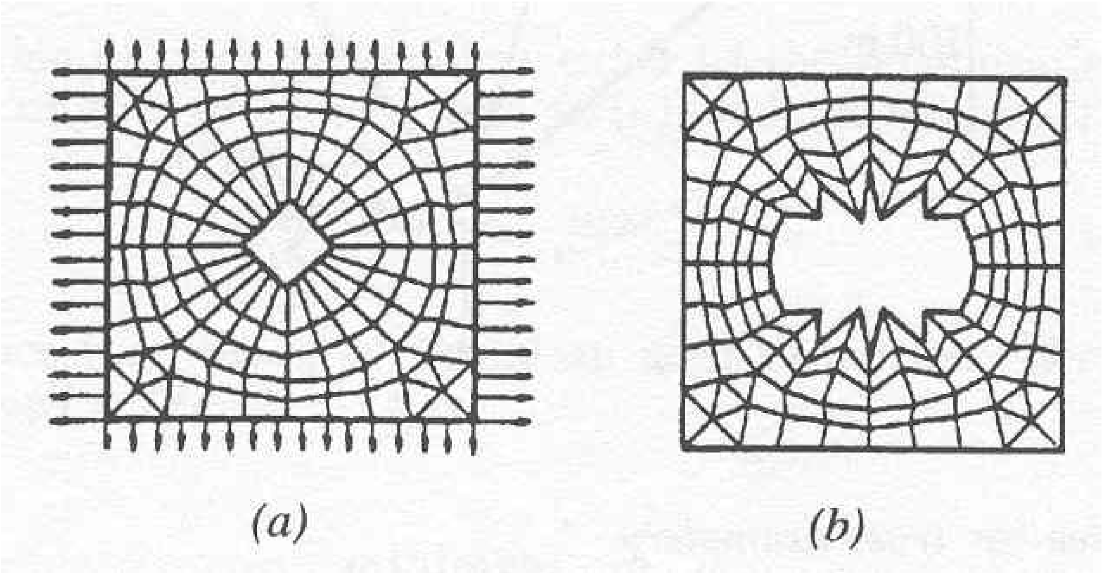
Continuous vs. Discrete

- Continuous (or distributed parameter) optimization problem
  - DVs are considered to vary continuously over the length or domain of the element
  - Rod, beam, arch, plate
- Discrete (or parameter) optimization problem
  - Inherently discrete structure
  - Truss, frame, complex practical structures
- The governing equations of both types of problem (as well as mixed types) can be derived by variational analysis

Design Variables

- Finite element model
  - Distribution of DVs should be much coarser
  - Optimal thickness distribution of a plate
    - Thickness of the FE model, 7X7?
  - Optimized shape of a hole in a plate
    - Coordinates of nodes of the FE model





Objective Function

  - Cost or criterion function
  - Function whose value is to be minimized or maximized by the optimal set of values of DVs within the feasible design space
- Structural weight or cost
- Local or global measure of the structural performance
  - Stress, displacement, stress intensity factor, stiffness, plastic collapse load, fatigue life, buckling load, natural vibration frequency, aeroelastic divergence, flutter speed, etc.
- Single-criterion / Multicriteria

Problems with Multiple Objectives (1)

$$\left.\begin{array}{l} Min\ \ f_1(\boldsymbol{x}) \\ \qquad\vdots \\ Min\ \ f_M(\boldsymbol{x}) \end{array}\right\} \rightarrow F(\boldsymbol{x}) = f\left[f_1(\boldsymbol{x}),\ldots,f_M(\boldsymbol{x})\right]$$

- Individual objectives are usually in contradiction with one another, hence
- If x_1^*, …, x_M^* are the solutions to individual objectives, then $x_1^* \neq \ldots \neq x_M^*$
- If the individual objectives are controlled by different sets of variables, then the optimum of *f* can be obtained by optimizing the individual f_i's.

$$F(\boldsymbol{x}) = f_1(\boldsymbol{x}_1) + \cdots + f_M(\boldsymbol{x}_M) = \sum_{i=1}^{M} f_i(\boldsymbol{x}_i) \quad where \quad \boldsymbol{x} = (\boldsymbol{x}_1,\ldots,\boldsymbol{x}_M)$$

Problems with Multiple Objectives (2)

- All objectives are controlled by the same set of variables:
  - Composite objective function

$$F(\boldsymbol{x}) = \alpha_1 f_1(\boldsymbol{x}) + \cdots + \alpha_M f_M(\boldsymbol{x}) = \sum_{i=1}^{M} \alpha_i f_i(\boldsymbol{x}_i)$$

  - Choose the most important to Max(Min), and put limits on the others.

$$Min(Max) \quad f(\boldsymbol{x}) = f_2(\boldsymbol{x}) \ such\ that \quad f_1(\boldsymbol{x}) \geq A_1 \quad \cdots \quad f_M(\boldsymbol{x}) \geq A_M$$

  - Optimize each of the objectives w.r.t. ***x*** individually to find f_i^* and the corresponding $\boldsymbol{x}_i^*$.

$$Min \quad \underset{i=1,\ldots,M}{Max}[d_i(\boldsymbol{x})] \quad or \quad Min \sum_{i=1}^{M} d_i^{\,2}(\boldsymbol{x}) \quad where \quad d_i(\boldsymbol{x}) = \frac{f_i(\boldsymbol{x}) - f_i^*}{f_i^*}$$

Constraints

  - Directly or indirectly impose limits on the range of variations of DVs
  - Design space / hypersurfaces → feasible or admissible designs
- Geometrical (or side) constraints
  - Explicit restrictions on DVs
  - Manufacturing limitations, physical practicability, aesthetics
  - Typically inequality constraints: lower and upper bounds
- Behavioral constraints
  - Generally nonlinear and implicit
  - Equality: state and compatibility equations governing the structural response associated with the loading conditions
  - Inequality: restrictions on those quantities that characterize the response of the structure
    - Local (stress, deflection) / global (compliance, natural vibration frequencies)

Solution Process

- Selection of the active constraint set
- Calculation of a search direction
  - Based on the objective function and the active constraint set
- Determination of a travel distance
  - One dimensional line search
- Termination criteria
  - No improvement of the objective function w/o violating constraints
  - Check for optimality (Kuhn-Tucker conditions)

Numerical Search Techniques (1)

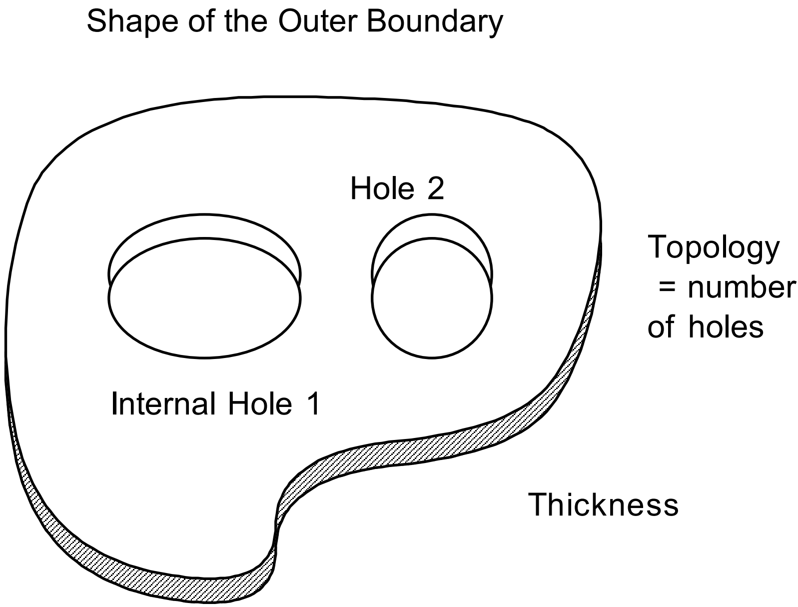
- Procedure
  - Selection of an initial design in the *n*-dimensional space
  - Evaluation of the function (objective and constraints) at a given point in the design space
  - Comparison of the current design with all of the preceding designs
  - A rational way to select a new design and repeat the process
- Questions
  - How is the initial design selected and what effect will it have on the outcome of the search?
  - What is a rational way to select the new designs and how does it affect the final outcome?
  - Where to stop the search?

Numerical Search Techniques (2)

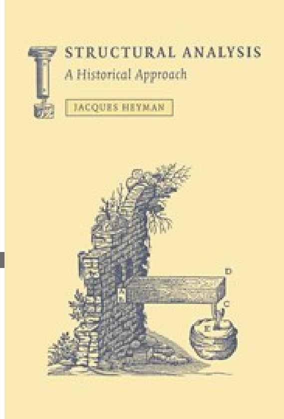
- 50s: simplex method and its variations
  - LP: transportation, scheduling, chemical processes, etc.
- 60s: gradient projection, feasible directions, penalty function methods
- 70s: implementation and serious applications
- 80s: refinement of the algorithms proposed in the 60s and 70s
  - (U.S.) Vanderplaat’s implementation of the feasible directions
  - CONMIN, ADS, MICRODOT
  - (Europe) Fleury’s CONLIN, Schittkowski’s implementation of SQP (NLPL) → IMSL library

Three Major Design Problems

- Sizing Optimization (1960)
  - How thick it is?
  - Thickness
  - cross sectional properties
  - Finite element model is fixed
- Shape Optimization (1973)
  - What are the boundaries?
  - Location and/or radii of holes/arcs
  - Control points of splines
  - Element shapes change during optimization
- Topology Optimization (1988)
  - Where are the holes?
  - Number of holes
  - Shape of holes
  - Finite element topology possible not defined



History (1)


STRUCTURAL ANALYSIS

A Historical Approach

JACQUES HEYMAN

- Galileo's problem
  - Strongest cantilever beam in bending and constant shear for minimum weight under a uniform stress constraint
- Introduction of calculus by Newton and/or Leibniz
  - Development of mathematical optimization
  - Min-max conditions: necessary conditions for optimal solutions
  - Only the unconstrained optimization problems
- Augmented Lagrangian function
  - Extension of simple min-max conditions to constrained optimization problems
  - Lagrangian multipliers: dual variables
    - Weighting factors in establishing the importance of the various constraints at different regions of design space
    - Link between the objective and the constraint functions

History (2)

- Calculus of variation (attributed to Bernoulli, Euler, Lagrange)
  - Brachistochrone problem
  - Generalization of the elementary theory of minima and maxima
  - Dealing with extremum of a function of functions
  - Solution? One or more functions represented by differential equations
  - Solution of D.E. → optimal path, or all the optimal points
  - Euler-Lagrangian equations → most of field equations of mechanics
  - Principles of least action: originally derived by Euler
    - Hamilton's principle → most of dynamic system equations based on Newton's Laws
    - Lagrange's equation → basis for an elegant description of Newtonian dynamics
  - Numerical difficulties in practical applications

History (3)

- The Euler-Lagrange equations: extreme conditions
  - Yield one or more nonlinear differential equations for solution
  - Variational approach: difficult to solve, restricted continuity and differentiability
  - Numerical approach: approximation of derivatives by differences and of integrals by sum
    - Differential equation → algebraic equation
    - Reliable? accuracy, time steps, convergence

History (4)

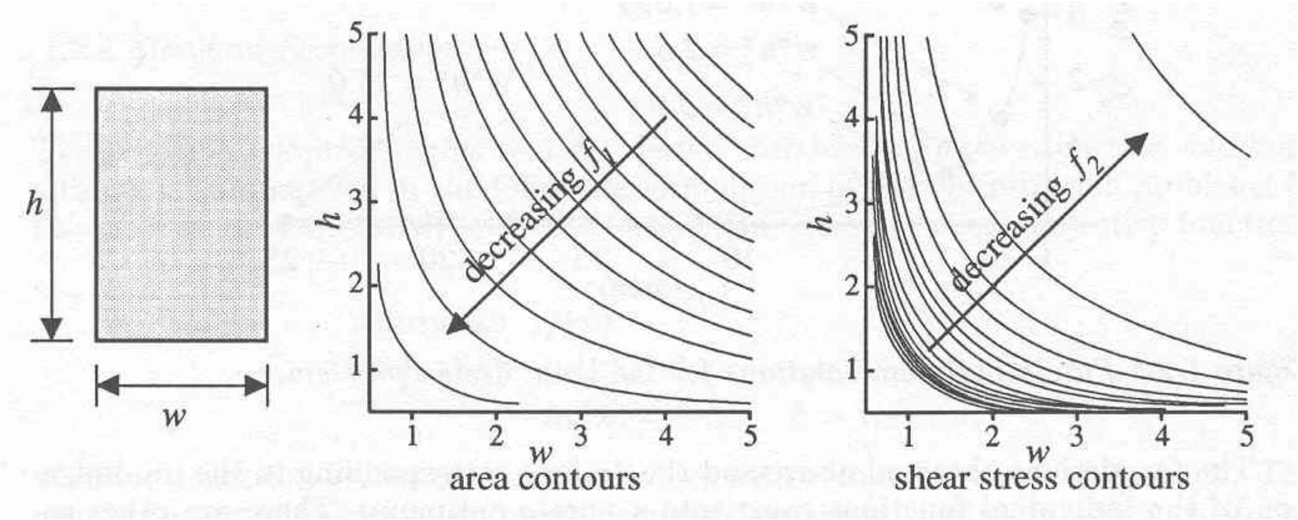
- Separation of the analysis and design as different problems
  - Analysis: determination of the state of the system as a function of time and spatial coordinates
  - Differential equations of analysis are obtained by minimization or maximization of one or more functions
    - e.g., in solid mechanics, potential energy in the system
    - Dependable variable: state variables → define the state of the system
    - Independent variable: spatial coordinates and time
  - Design: minimization or maximization of a predefined performance function subject to a set of constraint conditions
    - Variables: physical parameters that define the configuration of the system, sizes and/or geometrical quantities of the structural elements

Well-Established Areas

- Single-criterion optimization problems
- Optimal plastic design
  - Design against plastic collapse (limit load)
  - Uniform energy dissipation
- Elastic optimal design under static loading
  - Elastic design under strength, stiffness, or stability requirement
- Optimal layout of trusses
- Optimal design under dynamic loading
  - Natural frequency / forced steady state / transient response requirements

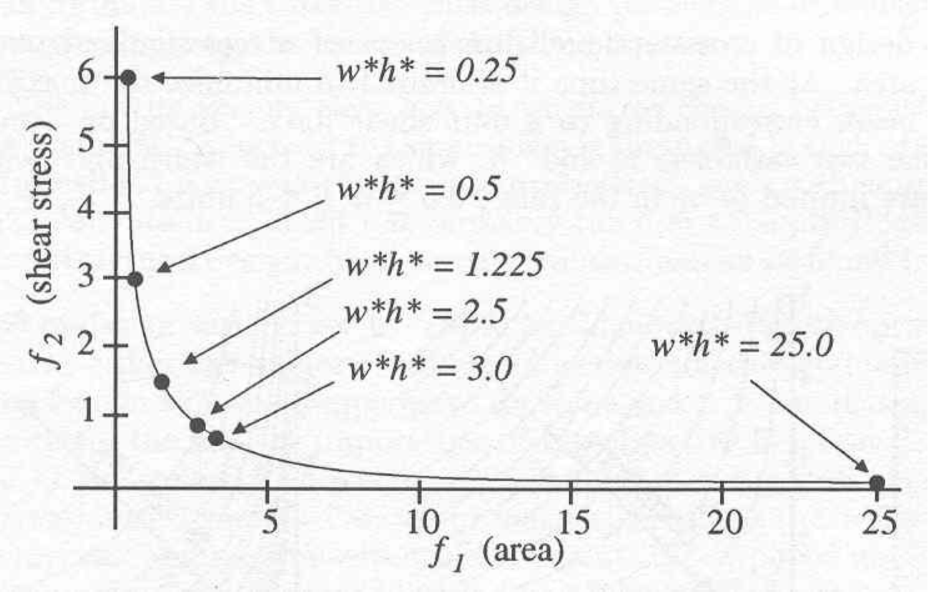
Rectangular Beam (1)

- Design variables
  - Width and height of the cross-section
- Objective functions
  - Minimize the area: $f_1 = A = wh$
  - Minimize the maximum shear stress : $f_2 = \tau_{\max} = 1.5\frac{V}{A} = \frac{3}{2wh}$
- Constraints: $0.5 \le w, h \le 5$



Rectangular Beam (2)

- Weighted sum
- Euclidean norm of the distance from the individual minima

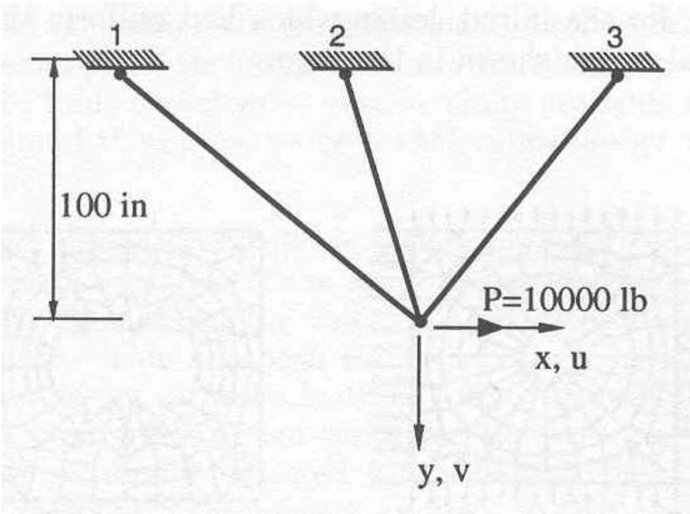


Three-bar Truss: Chapter 2.10

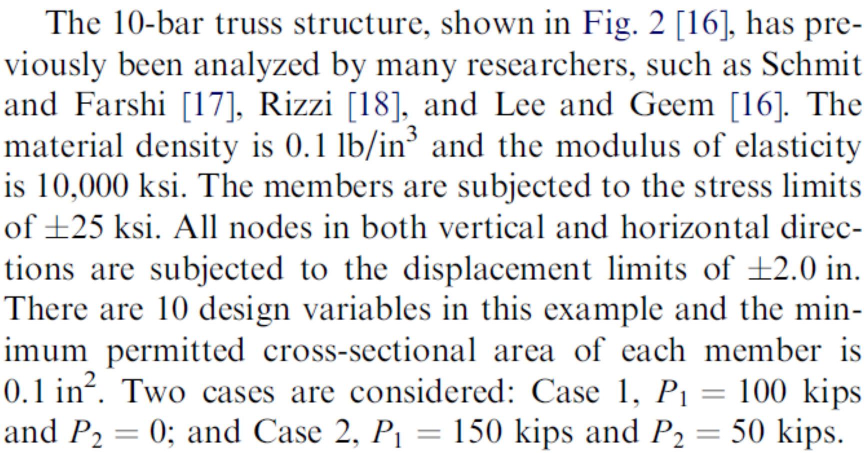
- Design variables
  - Cross-sectional areas: A_1, A_2, A_3
  - Horizontal coordinates: x_1, x_2, x_3
- Objective function
  - Minimize the mass: $m = \rho \sum_{i=1}^{3} A_i \sqrt{x_i^2 + 100^2}$ $\quad \left(\rho = 2.9\, lb / in^3\right)$
- Constraints
  - Allowable stress in tension and compression:

    $$\left|\sigma_i\right| \le 30{,}000\, psi$$
  - Minimum area of any member:

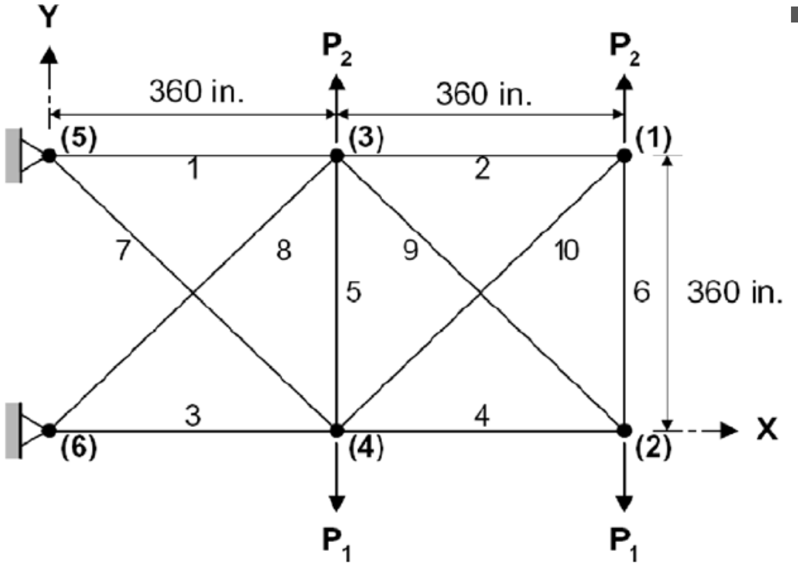
    $$A_i \ge 0.1\, in^2$$

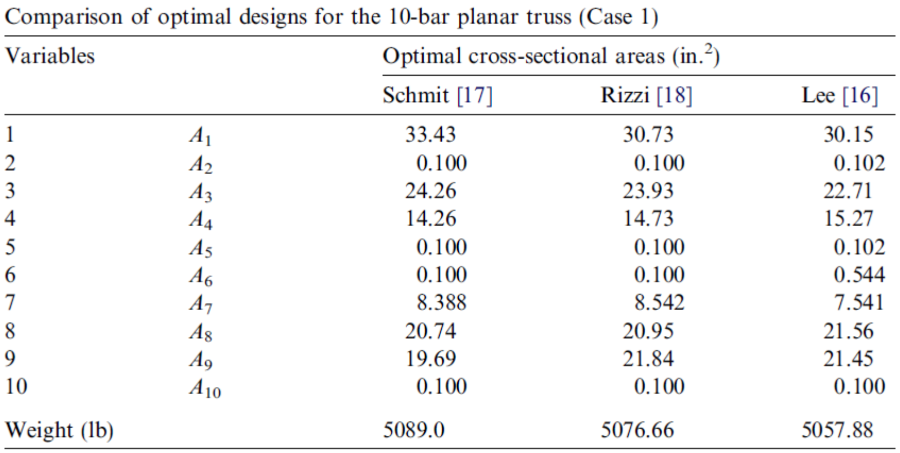


BMT: 10-bar planar truss structure



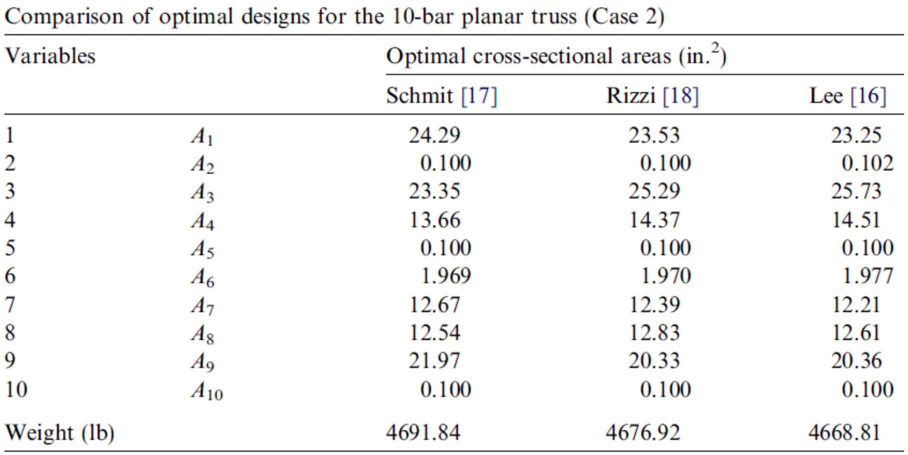
The 10-bar truss structure, shown in Fig. 2 [16], has previously been analyzed by many researchers, such as Schmit and Farshi [17], Rizzi [18], and Lee and Geem [16]. The material density is 0.1 lb/in^3 and the modulus of elasticity is 10,000 ksi. The members are subjected to the stress limits of ±25 ksi. All nodes in both vertical and horizontal directions are subjected to the displacement limits of ±2.0 in. There are 10 design variables in this example and the minimum permitted cross-sectional area of each member is 0.1 in^2. Two cases are considered: Case 1, $P_1 = 100$ kips and $P_2 = 0$; and Case 2, $P_1 = 150$ kips and $P_2 = 50$ kips.





Comparison of optimal designs for the 10-bar planar truss (Case 1)

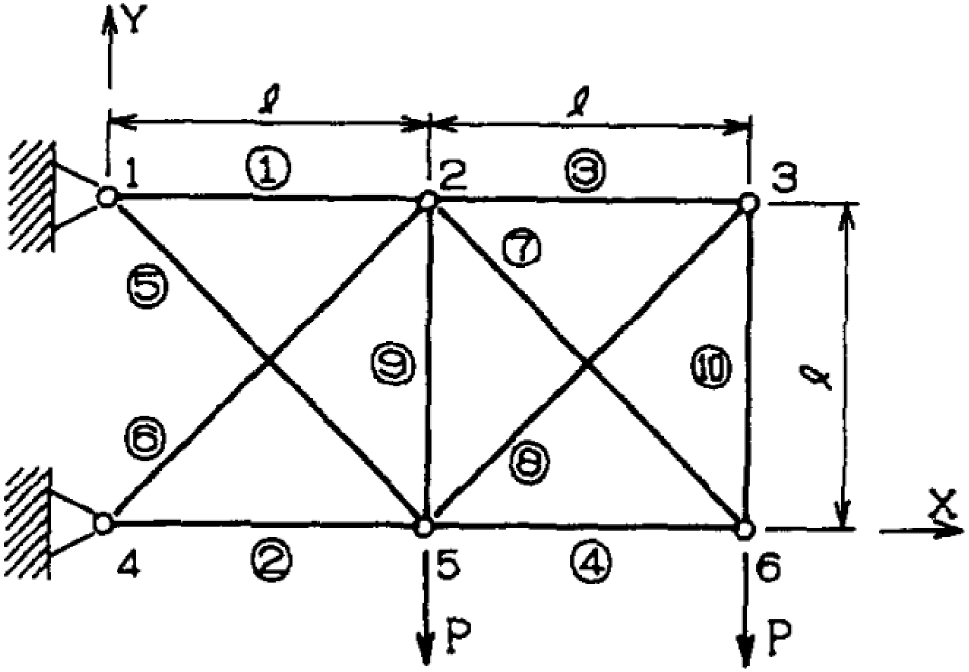
Variables		Optimal cross-sectional areas (in.^{2})		
		Schmit [17]	Rizzi [18]	Lee [16]
1	A_1	33.43	30.73	30.15
2	A_2	0.100	0.100	0.102
3	A_3	24.26	23.93	22.71
4	A_4	14.26	14.73	15.27
5	A_5	0.100	0.100	0.102
6	A_6	0.100	0.100	0.544
7	A_7	8.388	8.542	7.541
8	A_8	20.74	20.95	21.56
9	A_9	19.69	21.84	21.45
10	A_{10}	0.100	0.100	0.100
Weight (lb)		5089.0	5076.66	5057.88



Comparison of optimal designs for the 10-bar planar truss (Case 2)

Variables		Optimal cross-sectional areas (in.^{2})		
		Schmit [17]	Rizzi [18]	Lee [16]
1	A_1	24.29	23.53	23.25
2	A_2	0.100	0.100	0.102
3	A_3	23.35	25.29	25.73
4	A_4	13.66	14.37	14.51
5	A_5	0.100	0.100	0.100
6	A_6	1.969	1.970	1.977
7	A_7	12.67	12.39	12.21
8	A_8	12.54	12.83	12.61
9	A_9	21.97	20.33	20.36
10	A_{10}	0.100	0.100	0.100
Weight (lb)		4691.84	4676.92	4668.81

Optimal Weight Design Problem: 10 Bar Truss



min $$W(A) = \rho \sum_{i=1}^{10} l_i A_i$$

s. t. $$G_i = \sigma_i \le b_i, (i = 1, 2, .., 10)$$

$$G_k = v_k \le b_k, (k = 2, 3, 5, 6)$$

$$A_i^L \le A_i \le A_i^U, \quad (i = 1, 2, .., 10),$$

$$\sigma_i^L \le \sigma_i \le \sigma_i^U, \quad (i = 1, 2, .., 10),$$

$$v_k^L \le v_k \le v_k^U, \quad (k = 2, 3, 5, 6),$$

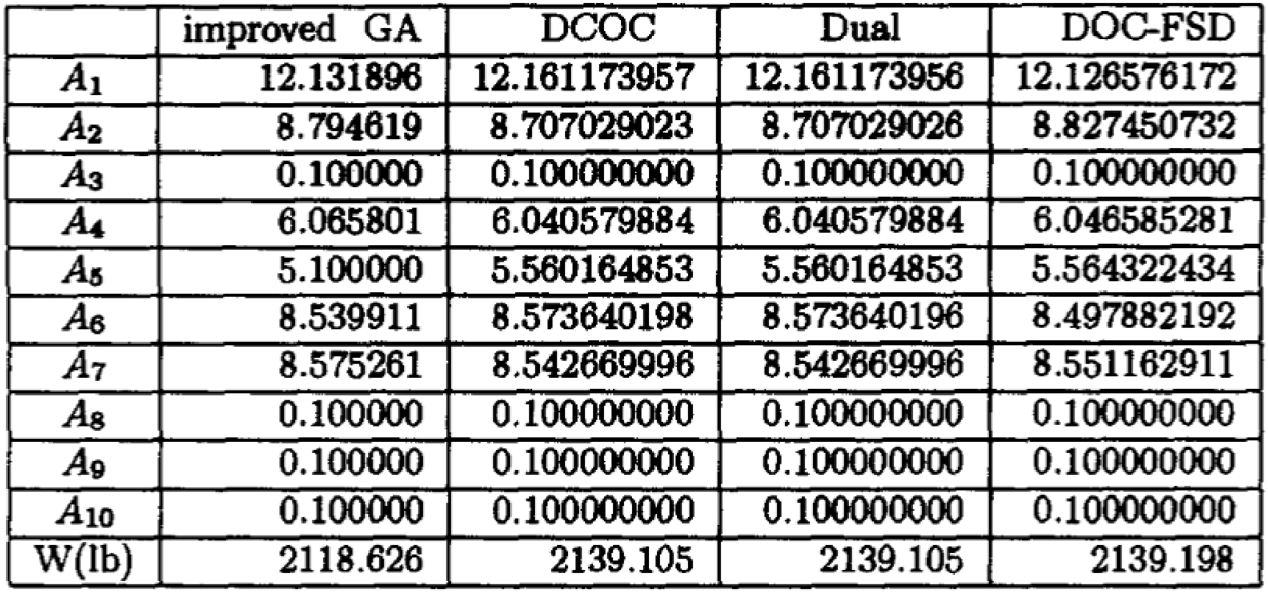
where

$$\sigma_i = \epsilon_i E, (i = 1, 2, .., 10)$$

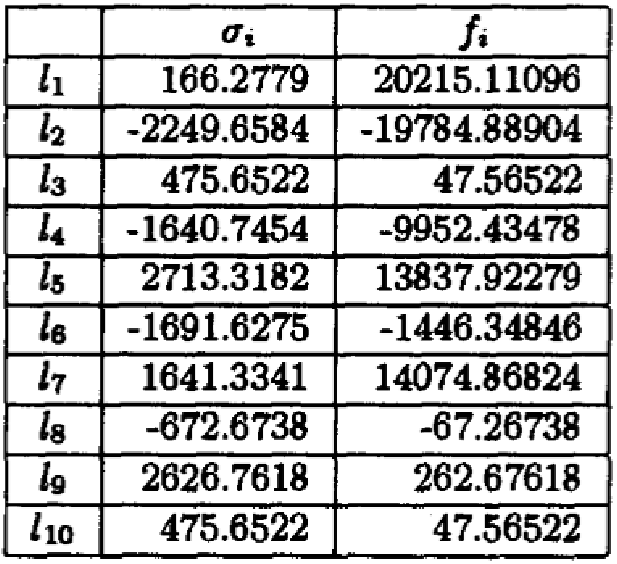
$$\begin{bmatrix} u_k \\ v_k \end{bmatrix} = \{F\} \, [K]^{-1}, (k = 2, 3, 5, 6)$$

$11.5 \le A_1 \le 12.5$	$8.0 \le A_2 \le 9.0$
$0.1 \le A_3 \le 1.0$	$5.5 \le A_4 \le 6.5$
$5.5 \le A_5 \le 6.0$	$8.0 \le A_6 \le 9.0$
$8.0 \le A_7 \le 9.0$	$0.1 \le A_8 \le 1.0$
$0.1 \le A_9 \le 1.0$	$0.1 \le A_{10} \le 1.0$
$E = 10^7$	$\rho = 0.1$
$\lvert\sigma\rvert \le 25000$	$\lvert v_6\rvert \le 5.0$
$l_{1-4,9,10} = 360$	$P = 10^5$
$l_{5-8} = 360\sqrt{2}$	

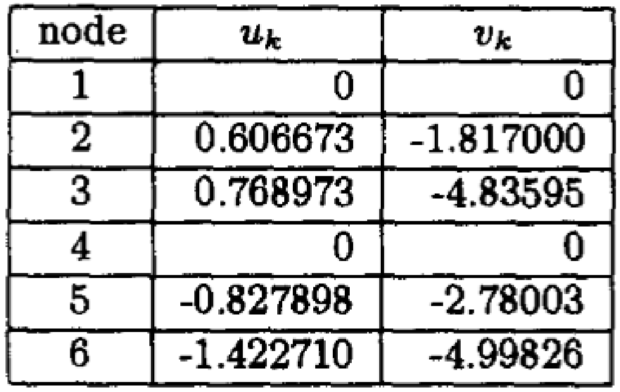
Results: 10 Bar Truss



	improved GA	DCOC	Dual	DOC-FSD
A_1	12.131896	12.161173957	12.161173956	12.126576172
A_2	8.794619	8.707029023	8.707029026	8.827450732
A_3	0.100000	0.100000000	0.100000000	0.100000000
A_4	6.065801	6.040579884	6.040579884	6.046585281
A_5	5.100000	5.560164853	5.560164853	5.564322434
A_6	8.539911	8.573640198	8.573640196	8.497882192
A_7	8.575261	8.542669996	8.542669996	8.551162911
A_8	0.100000	0.100000000	0.100000000	0.100000000
A_9	0.100000	0.100000000	0.100000000	0.100000000
A_{10}	0.100000	0.100000000	0.100000000	0.100000000
W(lb)	2118.626	2139.105	2139.105	2139.198

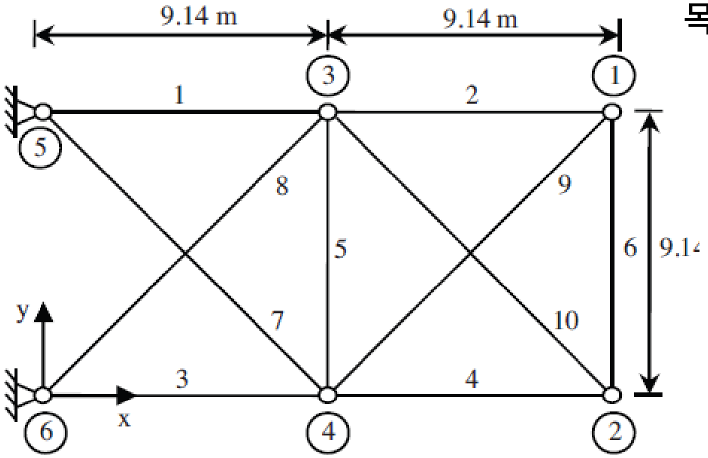


	σ_i	f_i
l_1	166.2779	20215.11096
l_2	-2249.6584	-19784.88904
l_3	475.6522	47.56522
l_4	-1640.7454	-9952.43478
l_5	2713.3182	13837.92279
l_6	-1691.6275	-1446.34846
l_7	1641.3341	14074.86824
l_8	-672.6738	-67.26738
l_9	2626.7618	262.67618
l_{10}	475.6522	47.56522

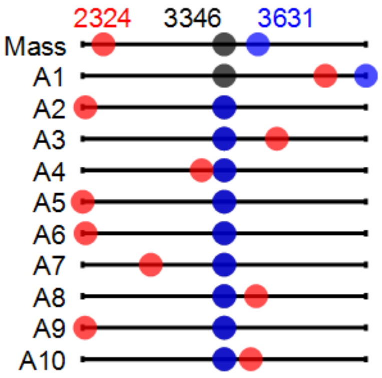


node	u_k	v_k
1	0	0
2	0.606673	-1.817000
3	0.768973	-4.83595
4	0	0
5	-0.827898	-2.78003
6	-1.422710	-4.99826

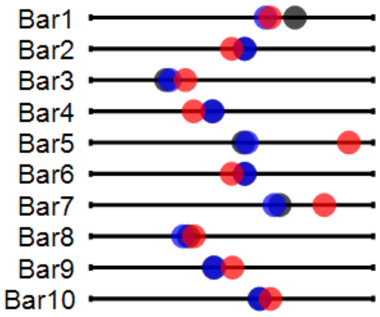
해석 / (적정)설계 / 최적설계



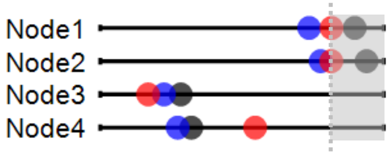
목적함수(질량) / 설계변수(단면적)



응력



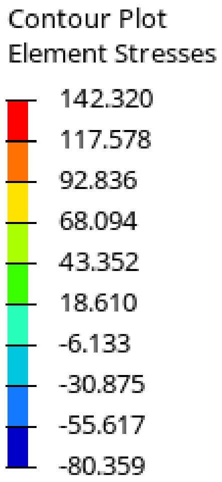
변위



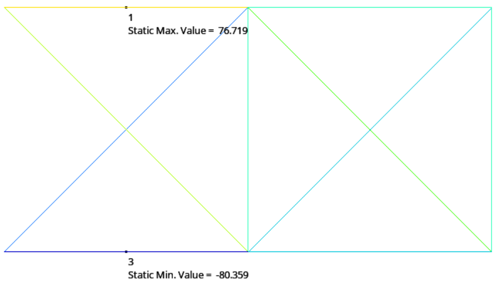
Aluminum 6063-T5: $E = 68.9\text{GPa},\ \rho = 2.8\times10^{-6}\ \text{kg}/\text{mm}^3$

Loading conditions: $P_{2y} = P_{4y} = -4.45\times10^{5}\ \text{N}$

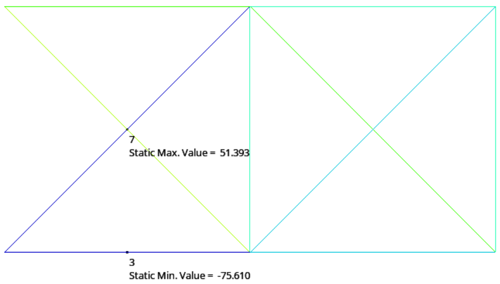
$$64.5\text{mm}^2 \le A_i \le 22,600\text{mm}^2,\ \ |\sigma_a| = 172\ \text{N}/\text{mm}^2,\ \ |\delta_a| = 50.8\text{mm}$$



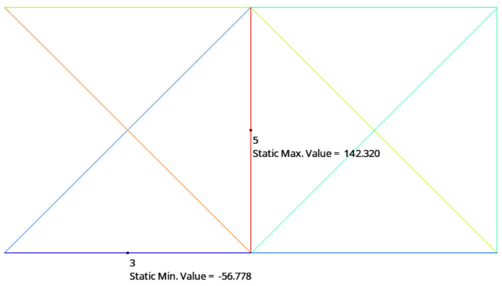
해석: 응력/변위?



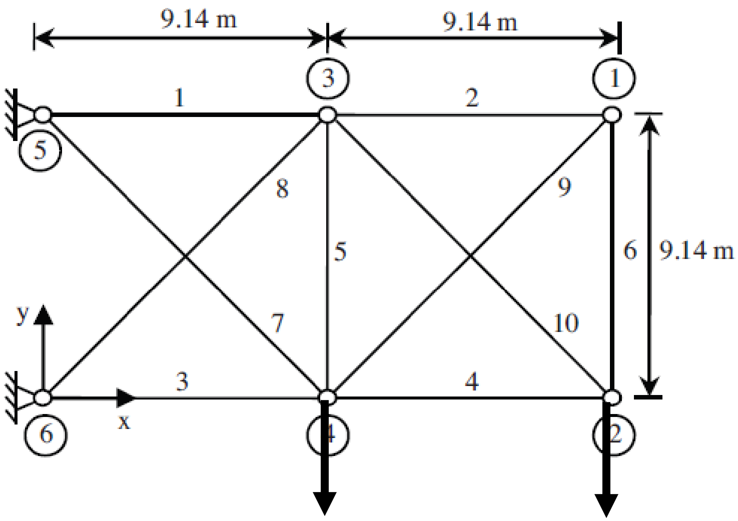
(적정)설계: 허용응력/변위 이내



최적설계: (적정)설계+최소질량



최적설계: 치수 vs. 위상(토폴로지)

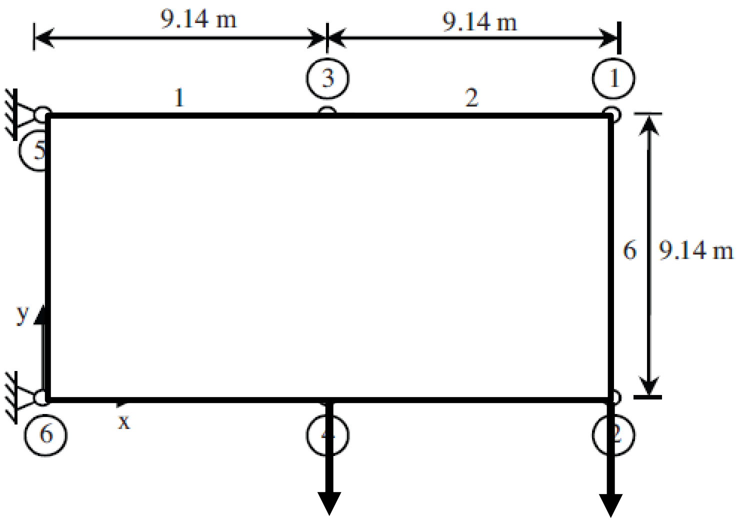


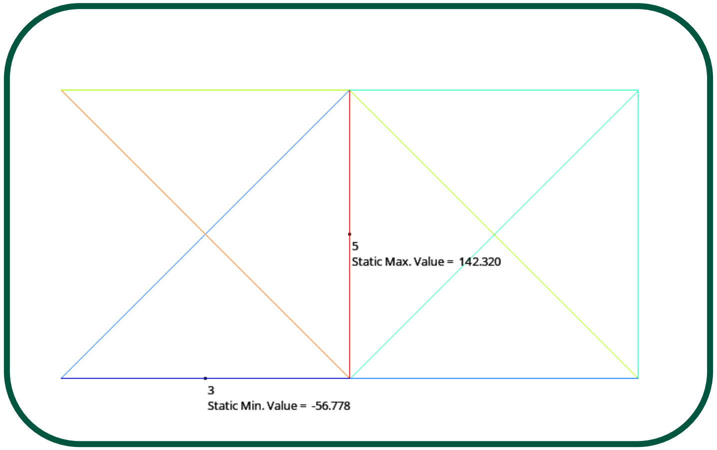
Minimize Mass
Design Variables

subjec to

$|\sigma| \le |\sigma_a| = 172\,\mathrm{N/mm^2}$

$|\delta| \le |\delta_a| = 50.8\mathrm{mm}$

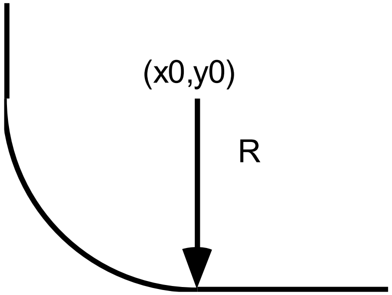


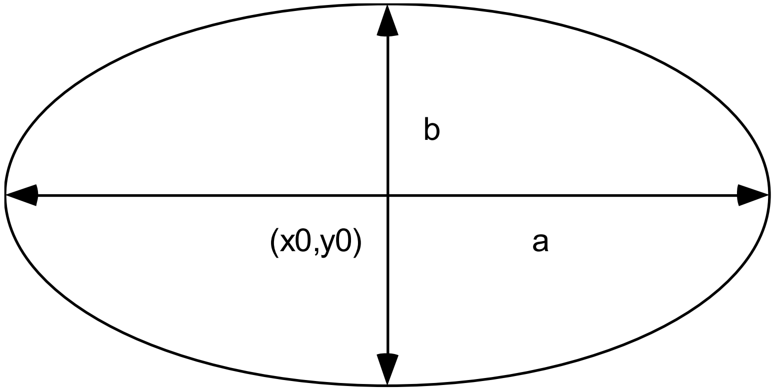


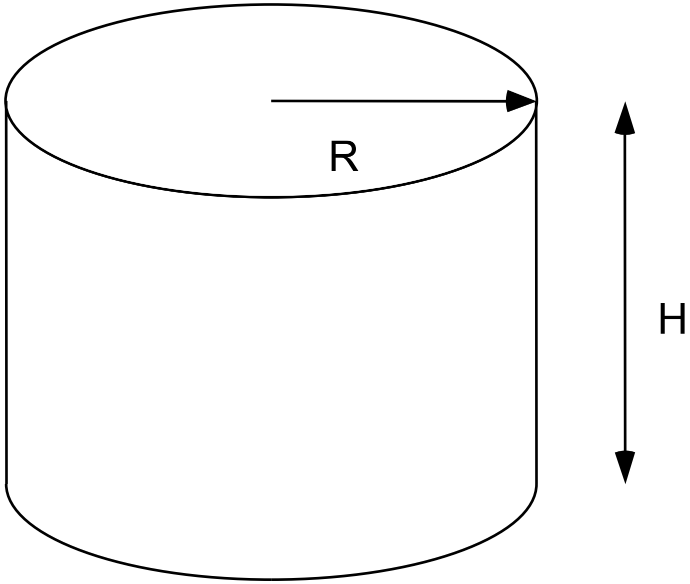
설계변수:
단면적 vs. 밀도(유한요소)

목적함수(질량, kg):
2324 vs. 3417

Size Design in MCAE



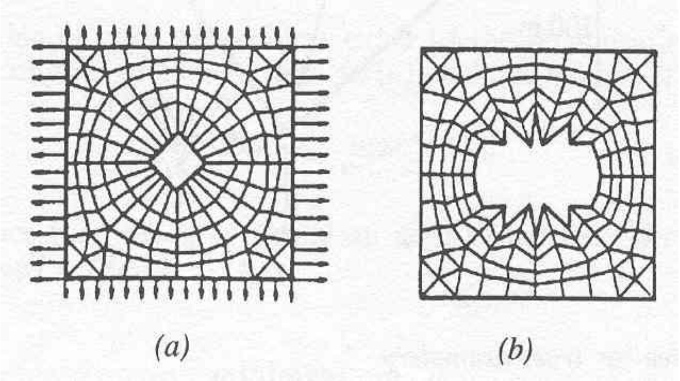




Sizing in Mechanical Design is always related to the shape of a structure !

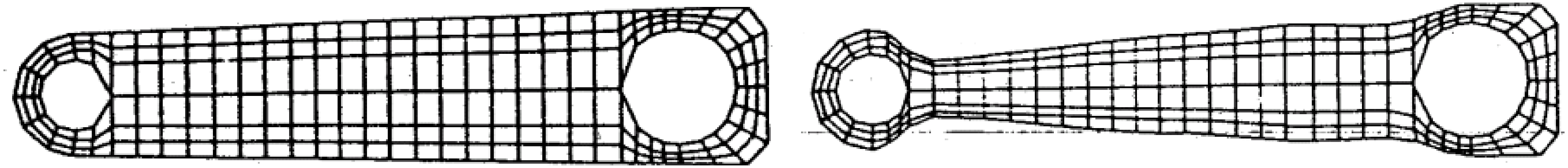
Shape Optimization (1)

- FEM + Design Sensitivity + SLP
  - O. C. Zienkiewicz and J. S. Campbell, Shape Optimization and Sequential Linear Programming, International Symposium on Optimization of Structural Design, University of Wales, Swansea, January 1973
- Adaptation of Nodal Points on the Boundary
- Without using parametric representation, they adapted the nodes of the finite element model → a lot of problem !
  - possibility of non-smoothed optimum shape due to non-smooth stresses on the design boundary
  - possibility of excessive element distortion
  - unclear adaptation schemes



Shape Optimization (2)

- Reducing stresses at a boundary by changing that boundary
- Difficulties in shape optimization
  - Accuracy of the FE analysis? continuously changing FE model
  - Good sensitivity derivatives w.r.t. shape design variables? expensive



R.T. Haftka and R.V. Grandhi, Structural Shape Optimization–A Survey, *Computer Methods in Applied Mechanics and Engineering*, 57, pp.91-10, 1986

Shape: Formulation

Typical Setting of Optimization

$$\min_{\text{design}} \int_{\Omega} \rho \, d\Omega$$

subject to

$$a(\boldsymbol{u},\boldsymbol{v}) = f(\boldsymbol{v}) \forall \boldsymbol{v}$$

$$\bar{\sigma} \le \sigma_{\max}$$

$$|\boldsymbol{u}| \le u_{\max}$$

Ω: variable unknown domain

Finite Element Representation

$$\min_{\text{design}} \int_{\Omega} \rho \, d\Omega = \sum_{e=1}^{nel} \rho_e \Omega_e$$

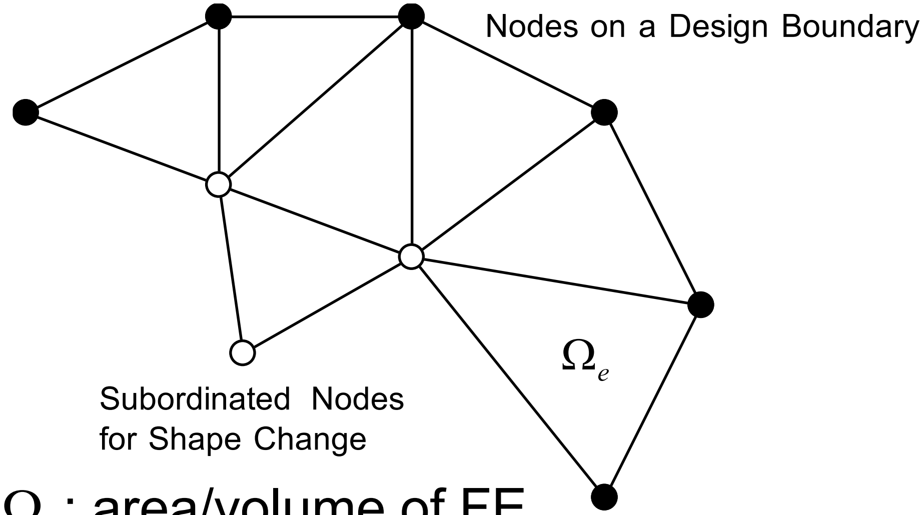
subject to

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}$$

$$\bar{\sigma}_e \le \sigma_{\max}, e=1,\ldots,nel$$

$$|\boldsymbol{u}_i| \le u_{\max}, i=1,\ldots,node$$

Varying in Shape Design



Ω_e: area/volume of FE

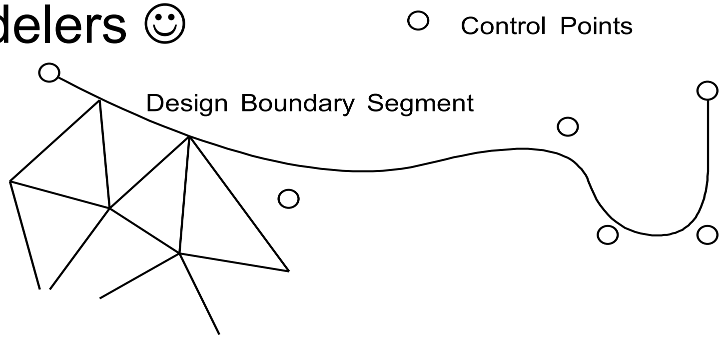
Practical Approach

- Difficulties
  - Every FEA code does have their own special finite elements, and then design sensitivity must be performed in such a FEA code
  - Geometric representation of the control points and the FE nodes must be related, and then this requires full link with CAD representation and mesh generation scheme
  - Full integration of
    - CAD like representation of Design Segments
    - Control Point Adaptation
    - Adaptive Finite Element Method
    - Full Automatic Mesh Generation Method

    is not realistic in practice.
- What is a possible alternate ?

Shape Design Parameters (1)

- Geometry-based mesh parameterization
  - Higher-level geometry data: surface control points, fillet radii
  - Mapped / free meshes
  - Integration with parametric solid modelers ☺
  - Mesh generator must be included ☹



- Reduced basis approach
  - Base configuration with a distinct mesh topology that remains fixed during the optimization
  - How to generate the design velocities (design base shapes for complex FE meshes)?

$$\{X(b)\} = \underset{\text{original nodal coordinates}}{\{X_0\}} + \sum_{k=1}^{N_s} b_k \{V_k\}$$

$\{V_k\} = \{X_k\} - \{X_0\}$: k-th design velocity vector

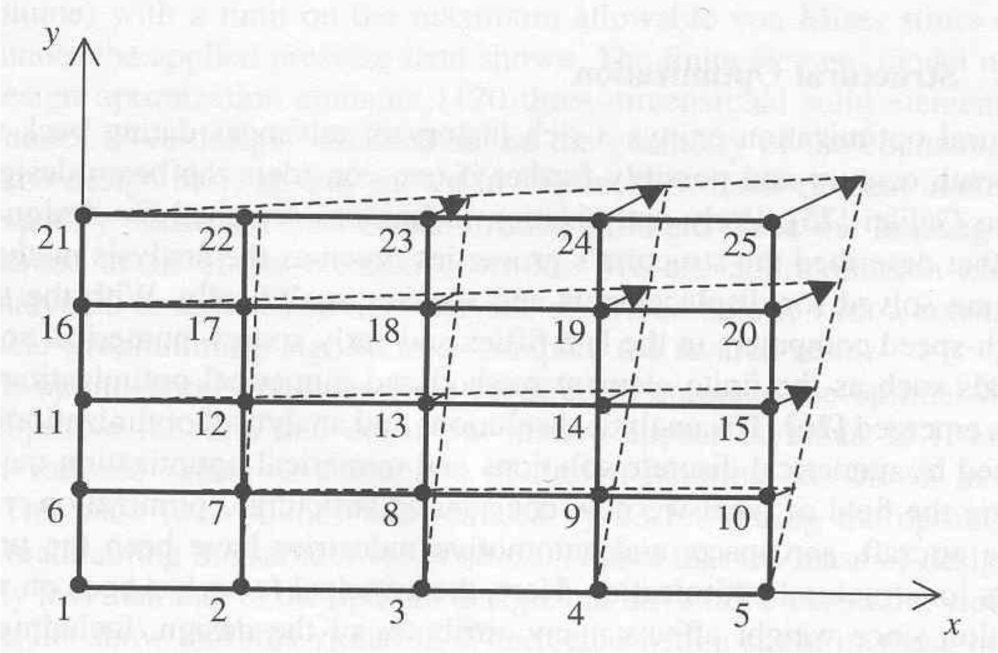
b_k : shape parameters

Shape Design Parameters (2)

- Design velocities giving shape changes as a function of shape design parameters

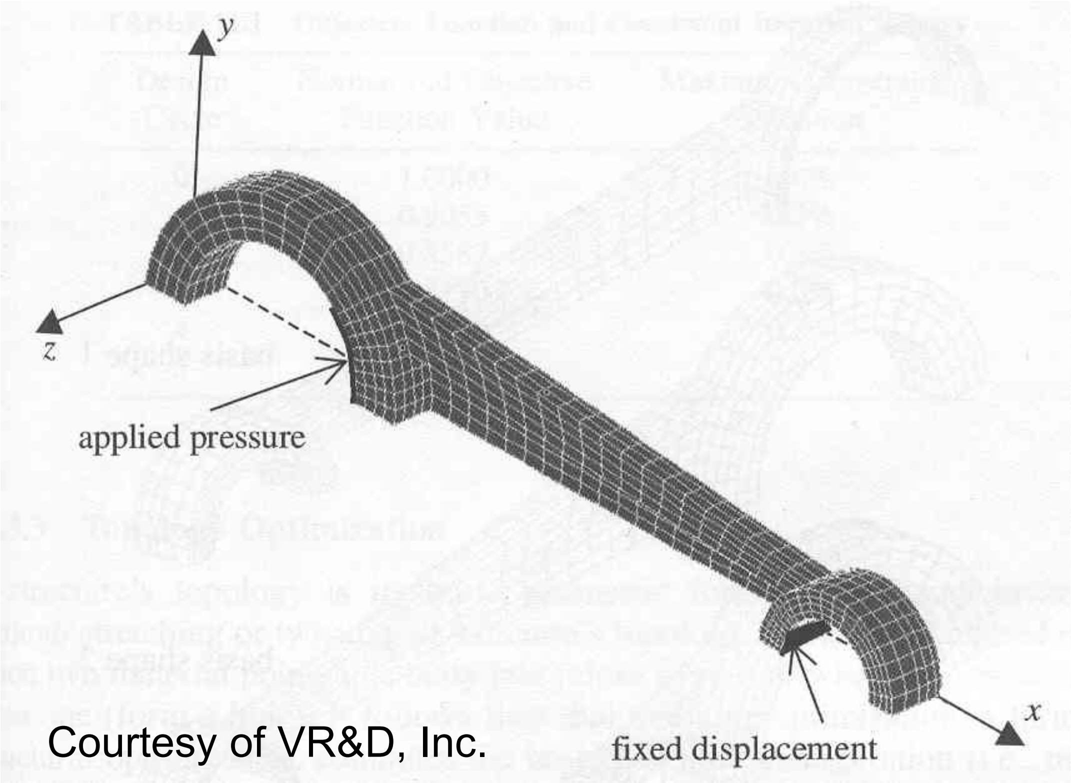
$$\left(\hat{x}_j, \hat{y}_j\right) \xrightarrow{\Delta x_j = b_1 \hat{x}_j \hat{y}_j, \Delta y_j = \frac{b_1}{2} \hat{x}_j \hat{y}_j} \left(x_j, y_j\right)$$

$$\left.\begin{aligned} x_j &= \hat{x}_j + b_1 \hat{x}_j \hat{y}_j \\ y_j &= \hat{y}_j + \frac{b_1}{2} \hat{x}_j \hat{y}_j \end{aligned}\right\} \rightarrow \left(\frac{\partial x_j}{\partial b_1}, \frac{\partial y_j}{\partial b_1}\right) = \left(\hat{x}_j \hat{y}_j, \frac{1}{2} \hat{x}_j \hat{y}_j\right)$$



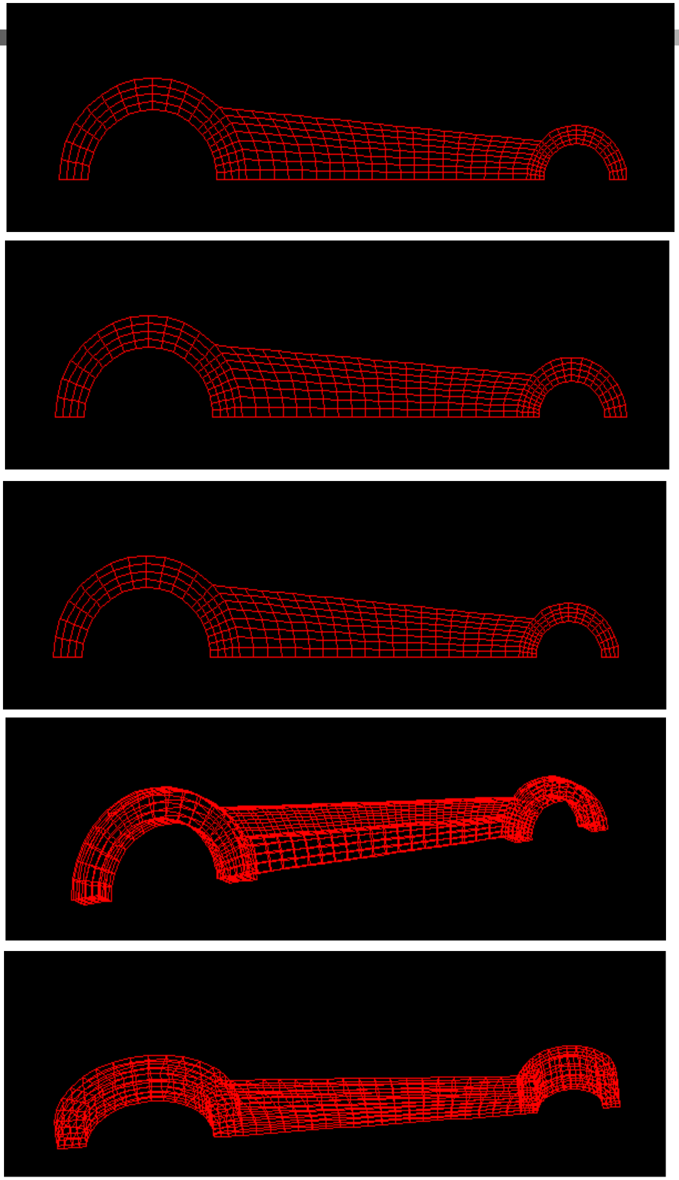
Engine Connecting Rod: Problem Description

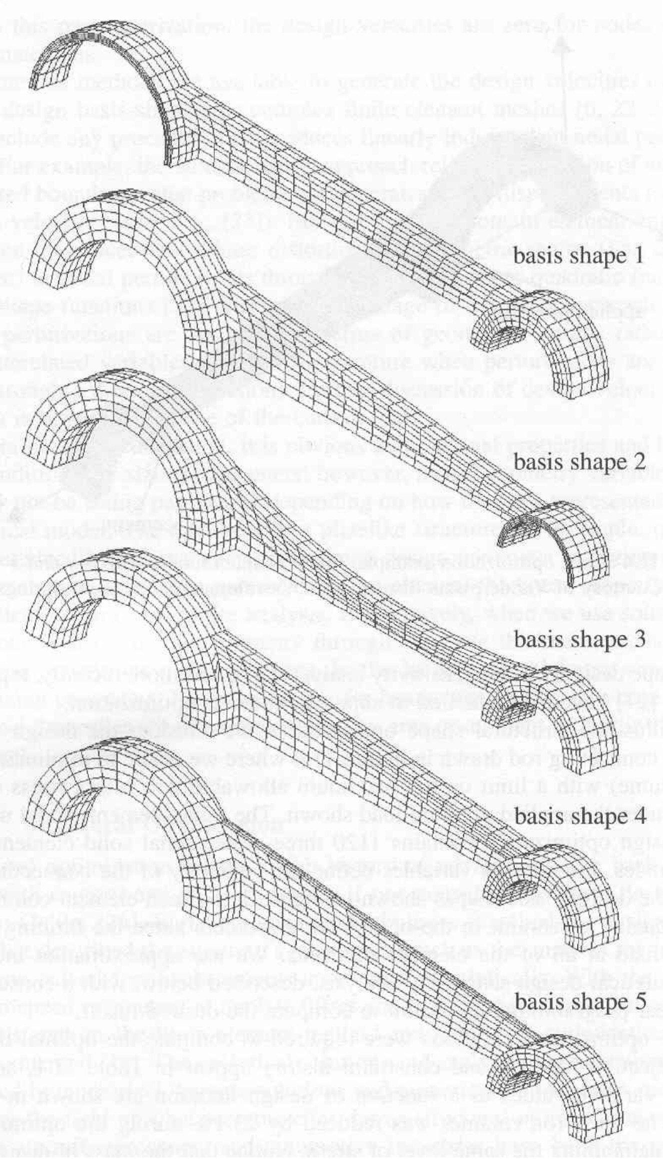
- Minimize mass with a limit on the maximum allowable von Mises stress developed under the applied pressure load
- 1120 3D solid elements
- Design variables
  - Outer radius at crank
  - Outer radius at piston
  - Rod body curvature
  - Flange thickness
  - Flange width



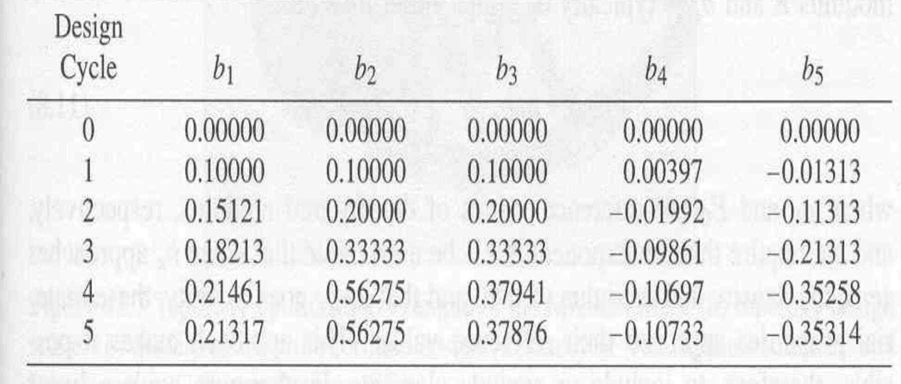
Courtesy of VR&D, Inc.

Engine Connecting Rod: Basis Vectors

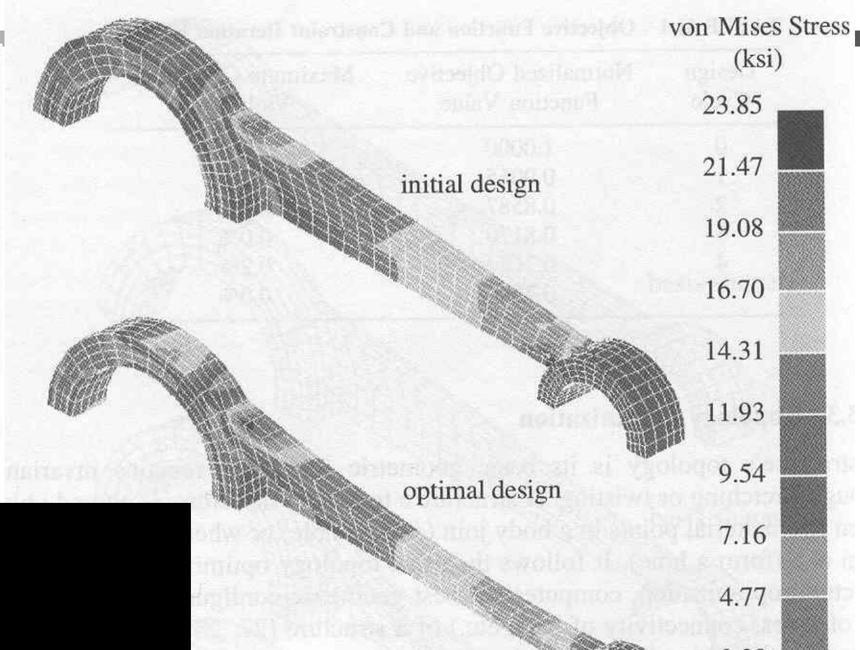


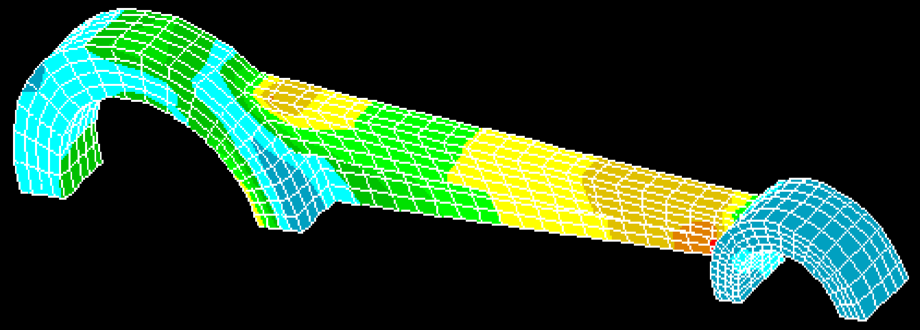


Engine Connecting Rod: Results



Design Cycle	b_1	b_2	b_3	b_4	b_5
0	0.00000	0.00000	0.00000	0.00000	0.00000
1	0.10000	0.10000	0.10000	0.00397	−0.01313
2	0.15121	0.20000	0.20000	−0.01992	−0.11313
3	0.18213	0.33333	0.33333	−0.09861	−0.21313
4	0.21461	0.56275	0.37941	−0.10697	−0.35258
5	0.21317	0.56275	0.37876	−0.10733	−0.35314





weight reduction (23.1%)