

목 차

- 구조최적설계
- 위상최적설계
 - 정식화
 - 사례: 자동차 분야

구조최적설계: 개념

- 설계변수 (d)
 - 부재크기 (두께, 단면적, 길이), 경계 (절점/조절점 좌표)

- 상태변수 (U)
 - 중량, 응력, 변위, 온도, 고유진동수, 좌굴하중
 - 목적함수, 제약조건

상태방정식
지배방정식



- 결정사항
 - 해석 종류: 상/편 미분방정식
 - 설계 공간 (설계변수)
 - 목적함수 및 제약함수 (구조 거동, 구조 기하)

구조최적설계: 설계변수

■ 치수 (Size)

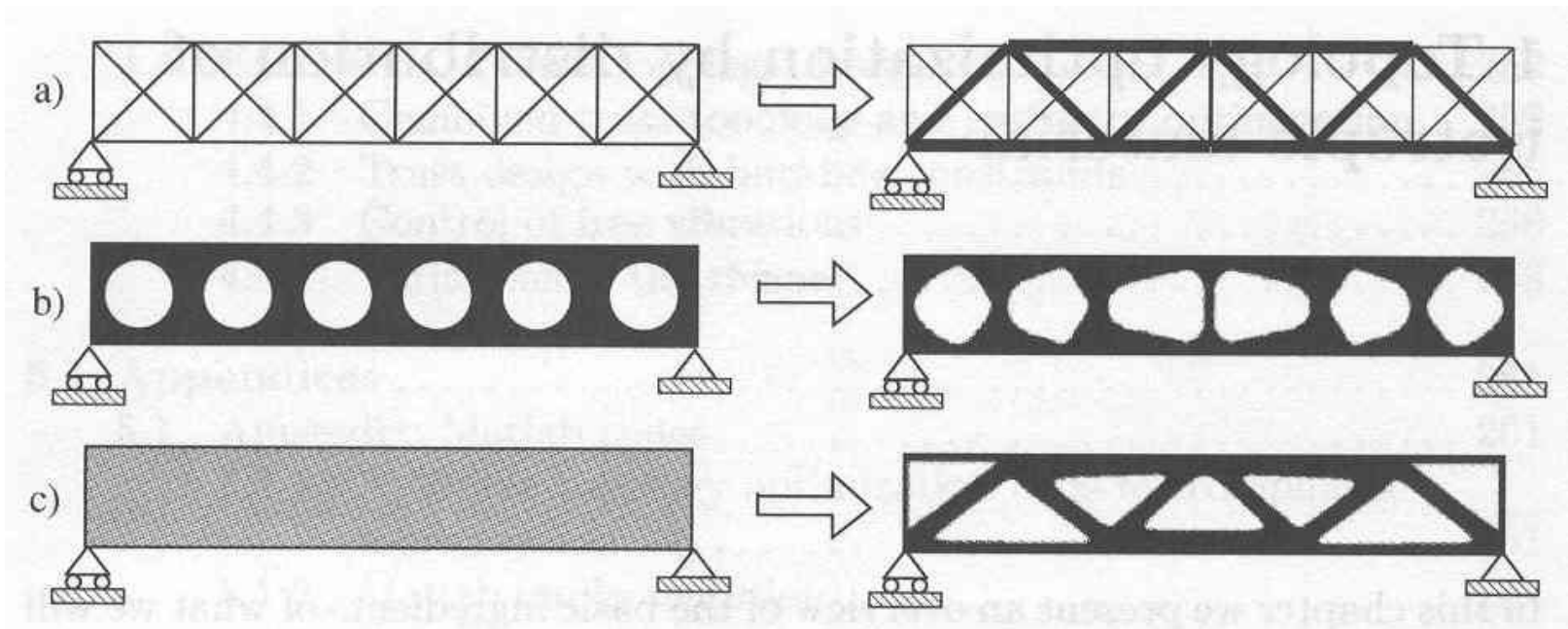
- 부재크기
- 두께, 단면적, 길이

■ 형상 (Shape)

- 경계
- 절점/조절점 좌표

■ 위상 (Topology)

- 재료 유/무
- 요소밀도



M.P.Bendsøe and O. Sigmund, Topology Optimization: Theory, Methods and Applications, Springer, 2003

구조최적설계: 정식화

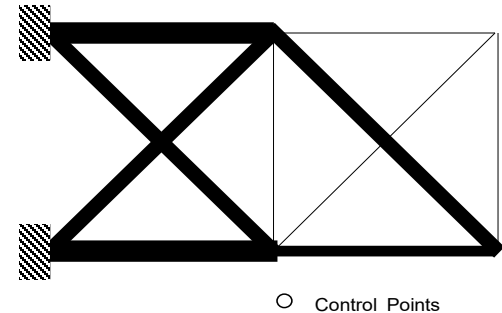
- 설계변수 :
- 목적함수 : 부피(중량) 최소화
- 제약조건 : 상태방정식(해석), 최대응력, 최대변위

$$\min_d \int_{\Omega} \rho d\Omega$$

$$\text{subject to } \begin{cases} a(\mathbf{u}, \mathbf{v}) = f(\mathbf{v}) \forall \mathbf{v} \\ \bar{\sigma} \leq \sigma_{\max} \\ |\mathbf{u}| \leq u_{\max} \end{cases}$$

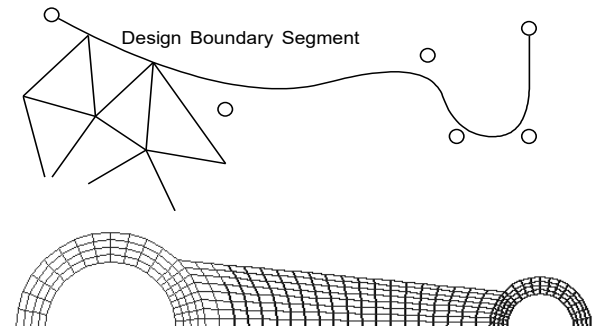
■ 치수최적설계

- 항공/토목 구조물: 트러스, 빔, 프레임
- 기계구조물: 형상과 관련



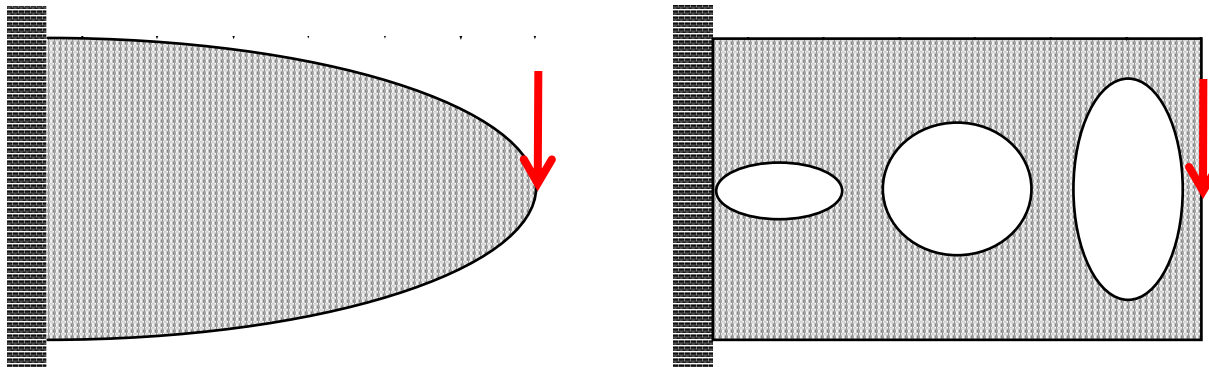
■ 형상최적설계

- 파라메트릭 기하 표현: 자동 요소망 생성 필요
- 기저형상 활용



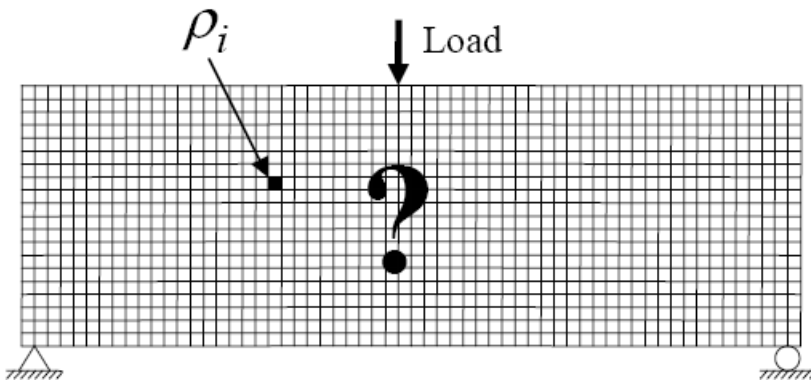
구조물 경량화

- 치수(size) 및 형상(shape) 최적설계
 - 초기설계에 크게 의존
 - 최적설계에 의한 경량화 효과 적음 (우수한 초기설계인 경우)
- 효과적인 구조물 경량화: 구조물 내 구멍 생성
 - 구멍의 위치/크기/형상: 패러다임 변화 필요
 - 위상(topology)최적설계 제안



새로운 패러다임: 위상최적설계

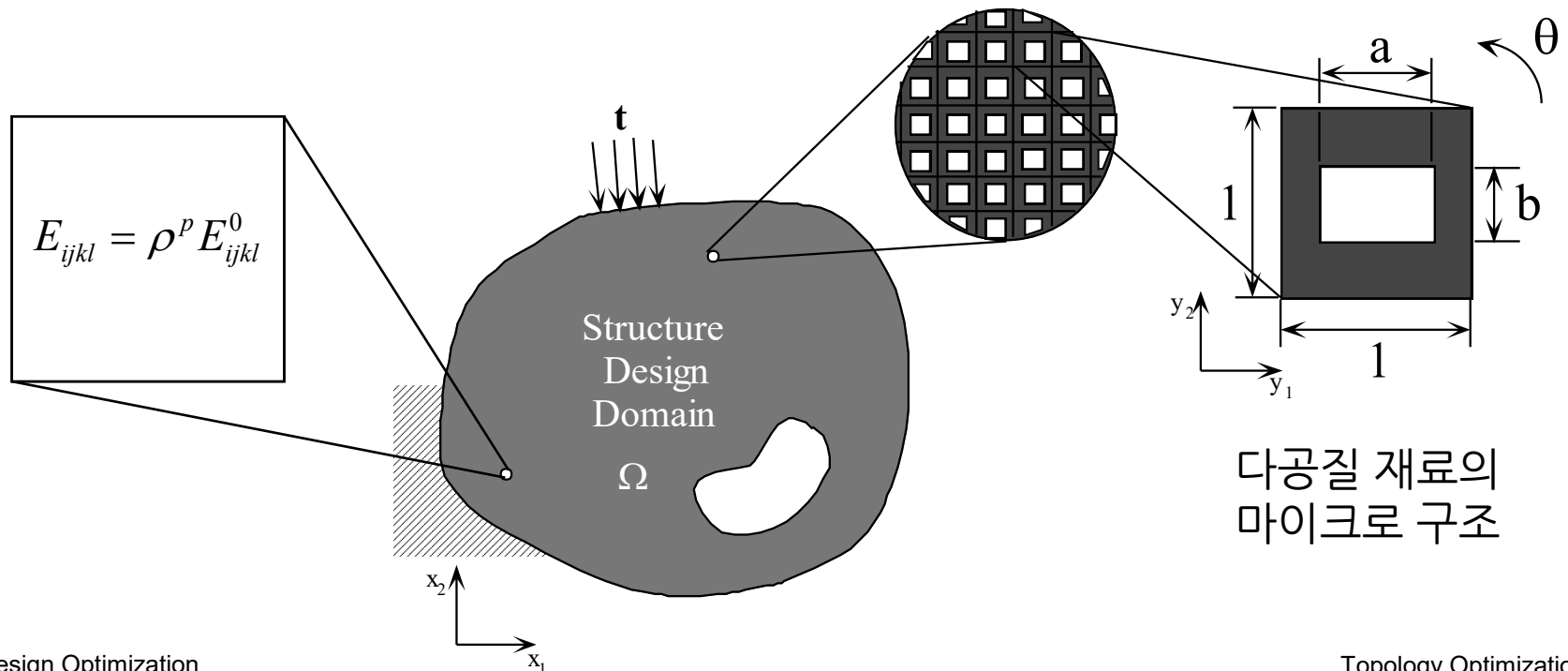
- 형상최적설계의 문제점
 - 형상변화에 따른 해석모델 재생성
 - 설계변화계산의 어려움, 제한적 형상변화
- 아이디어?
 - 해석모델 고정: 구조최적설계방법이 유한요소생성과 별개
 - 형상을 유한요소의 밀도로 표현(pixel, voxel 개념)



위상최적설계: 개념

- 설계변수

- 구조물의 재료분포(탄성계수)를 표현하는 값
- 균질화설계법: 다공질구조로 모델링, 복합재역학이론으로 균질화된 물성 계산
- 밀도법: 유한요소의 밀도로 물성계산



위상최적설계: 정적 문제

- 설계변수 (ρ)
 - 이산화한 각 유한요소의 밀도
- 문제 정식화
 - 부피 최소화, 응력/변위 제약조건: 주어진 강도/강성을 만족하는 경량화 설계
 - 응력: 트러스, 빔, 프레임 구조물에서는 유한값, 연속체에서는 국부적 물리량
 - 전역적 물리량 (평균컴플라이언스) 도입: 강성 표현

$$\min_{\rho} \int_{\Omega} \rho d\Omega \rightarrow \sum_{e=1}^N v_e \rho_e$$

subject to $\left\{ \begin{array}{l} a(\mathbf{u}, \mathbf{v}) = f(\mathbf{v}) \forall \mathbf{v} \rightarrow \mathbf{K}(\rho) \mathbf{U} = \mathbf{F} \\ \bar{\sigma} \leq \sigma_{\max} \\ |\mathbf{u}| \leq u_{\max} \end{array} \right\} \rightarrow \mathbf{U}^T \mathbf{F} \leq l_{\max}$

➡

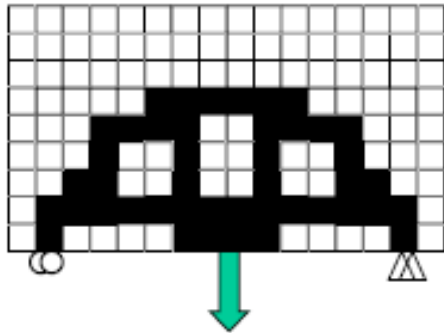
$$\min_{\rho} \mathbf{U}^T \mathbf{F}$$

subject to $\left\{ \begin{array}{l} \mathbf{K}(\rho) \mathbf{U} = \mathbf{F} \\ \sum_{e=1}^N v_e \rho_e \leq V^* \\ 0 < \rho_{\min} \leq \rho_e \leq 1 \end{array} \right.$

위상최적설계: 밀도법

■ 정수(0/1) 문제

- 모든 조합? Ill-conditioned



$$\min_{\rho} U^T F$$

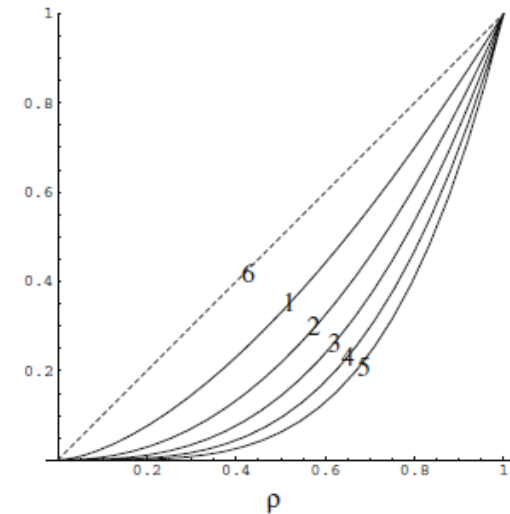
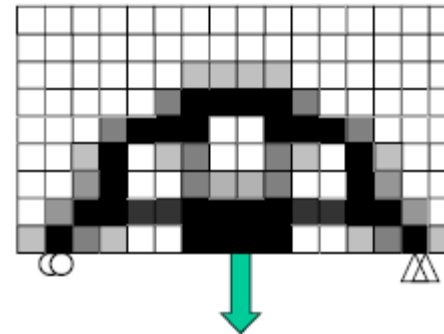
$$\text{subject to } \begin{cases} K(\rho)U = F \\ \sum_{e=1}^N v_e \rho_e \leq V^* \\ 0 < \rho_{\min} \leq \rho_e \leq 1 \end{cases}$$

$$E(\rho_e) = \rho_e^p E_0$$

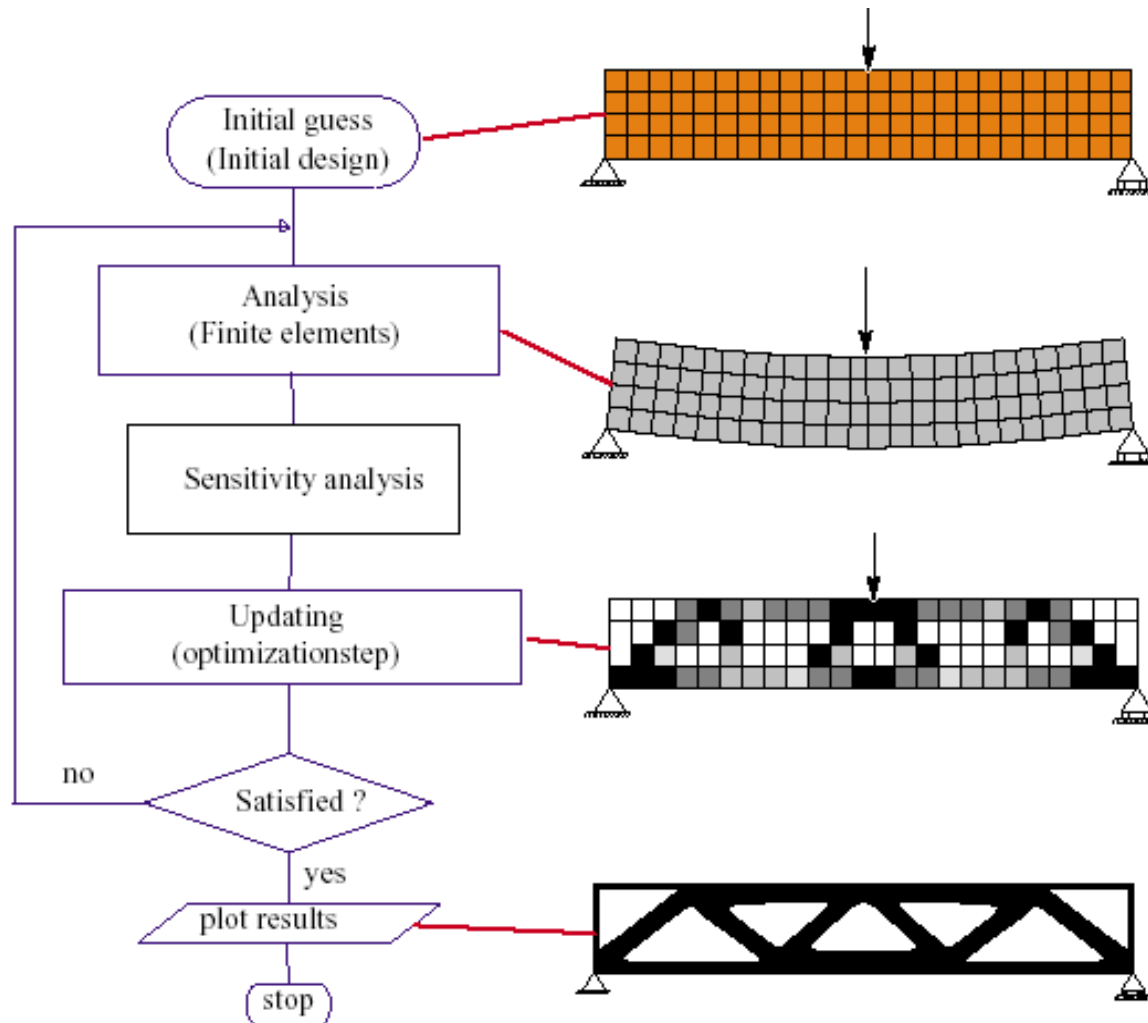
$$p > 1$$

■ 실수(0~1) 문제

- 완화, 중간밀도?



위상최적설계: 흐름도



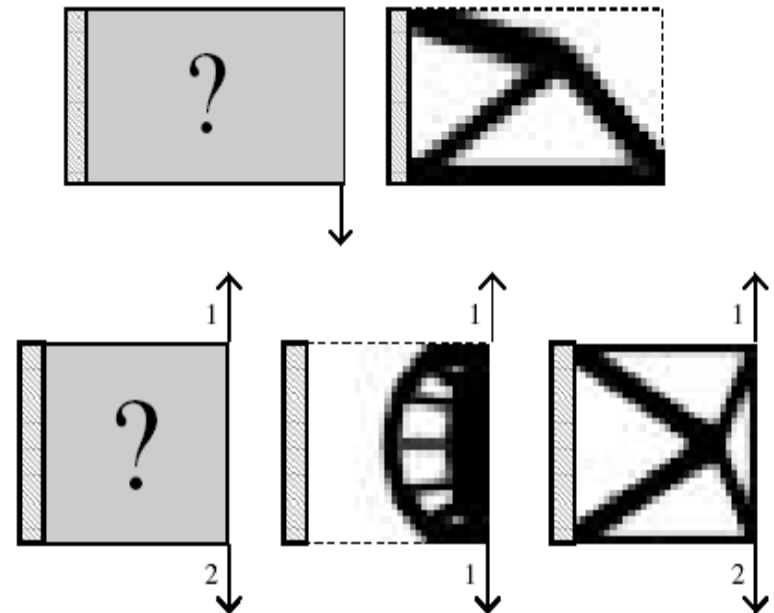
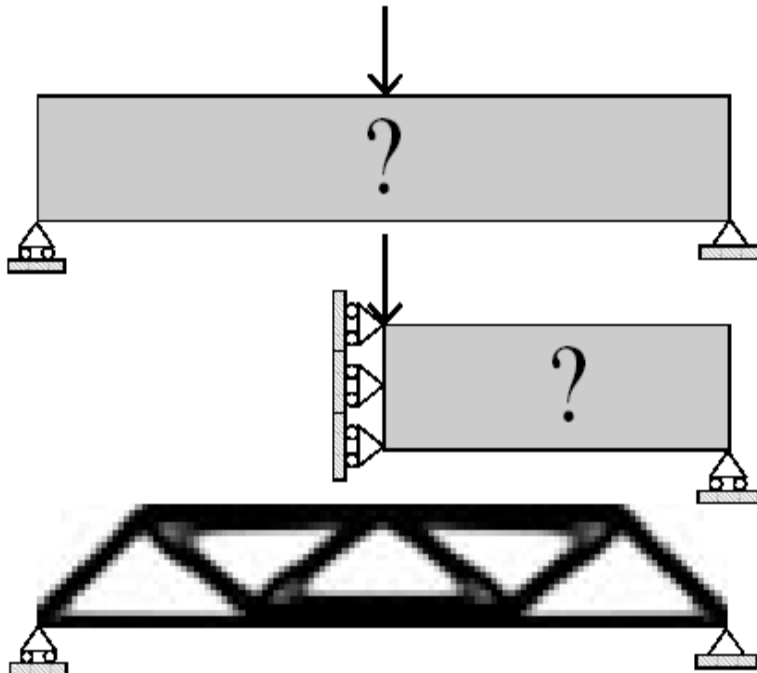
Topology Optimization: Formulation

$$\begin{aligned}
 & \left. \begin{aligned}
 & \min_{DV} \int_{\Omega} \rho d\Omega \rightarrow \sum_{e=1}^N v_e \rho_e \\
 & \text{subject to } a(\mathbf{u}, \mathbf{v}) = f(\mathbf{v}) \forall \mathbf{v} \rightarrow \mathbf{K}(\rho) \mathbf{U} = \mathbf{F} \\
 & \left. \begin{aligned}
 & \bar{\sigma} \leq \sigma_{\max} \\
 & |\mathbf{u}| \leq u_{\max}
 \end{aligned} \right\} \rightarrow \mathbf{u}^T \mathbf{f} \rightarrow \mathbf{U}^T \mathbf{F}
 \end{aligned} \right\} \xrightarrow[E(\rho_e) = \rho_e^p E_0]{\text{dual problem}} \left\{ \begin{aligned}
 & \min_{\rho} \mathbf{U}^T \mathbf{F} = \phi(\mathbf{U}(\rho)) \\
 & \text{subject to } \left\{ \begin{aligned}
 & \mathbf{K}(\rho) \mathbf{U} = \mathbf{F} \\
 & \sum_{e=1}^N v_e \rho_e \leq V^* \\
 & 0 < \rho_e \leq 1
 \end{aligned} \right.
 \end{aligned} \right. \\
 \\
 & L(\rho, \mathbf{U}, \lambda, \Lambda, \lambda_e^-, \lambda_e^+) = \phi + \lambda^T (\mathbf{K} \mathbf{U} - \mathbf{F}) + \Lambda \left(\sum_{e=1}^N v_e \rho_e - V^* \right) + \sum_{e=1}^N \lambda_e^- (-\rho_e) + \sum_{e=1}^N \lambda_e^+ (\rho_e - 1) \\
 \\
 & \left. \begin{aligned}
 & \frac{\partial L}{\partial \rho} = \underbrace{\frac{d\phi}{d\rho} + \frac{\partial \phi}{\partial \mathbf{U}} \frac{\partial \mathbf{U}}{\partial \rho}}_{\frac{d\phi}{d\rho} + \left(\frac{\partial \phi}{\partial \mathbf{U}} + \lambda^T \mathbf{K} \right) \frac{\partial \mathbf{U}}{\partial \rho} + \lambda^T \frac{\partial \mathbf{K}}{\partial \rho} \mathbf{U} - \frac{\partial \mathbf{F}}{\partial \rho}} + \lambda^T \left(\frac{\partial \mathbf{K}}{\partial \rho} \mathbf{U} + \mathbf{K} \frac{\partial \mathbf{U}}{\partial \rho} - \frac{\partial \mathbf{F}}{\partial \rho} \right) + \Lambda v_e - \lambda_e^- + \lambda_e^+ = 0 \\
 & \frac{\partial L}{\partial \mathbf{U}} = \frac{\partial \phi}{\partial \mathbf{U}} + \lambda^T \mathbf{K} = 0 \rightarrow \mathbf{F} + \mathbf{K}^T \lambda = 0 \rightarrow \lambda = -\mathbf{U} \\
 & -\mathbf{U} \frac{\partial \mathbf{K}}{\partial \rho} \mathbf{U} \rightarrow -\mathbf{U}_e^T \frac{\partial \left(\sum_e \mathbf{K}_e \right)}{\partial \rho_e} \mathbf{U}_e = -\mathbf{U}_e^T \left(p \rho_e^{p-1} \mathbf{K}_e^0 \right) \mathbf{U}_e
 \end{aligned} \right\} \rightarrow \left\{ \begin{aligned}
 & -\mathbf{U}_e^T \left(p \rho_e^{p-1} \mathbf{K}_e^0 \right) \mathbf{U}_e + \Lambda v_e = 0 \\
 & 1 = \frac{\mathbf{U}_e^T \left(p \rho_e^{p-1} \mathbf{K}_e^0 \right) \mathbf{U}_e}{\Lambda v_e} = B_e \\
 & \rho_e^{new} = \rho_e (B_e)^\eta
 \end{aligned} \right.
 \end{aligned}$$

Educational Design Tool (1)

– TOPOPT (www.topopt.dtu.dk)

- A 99 line topology optimization code written in Matlab, Struct Multidisc Optim 21, pp.120-127, 2001
- A web-based topology optimization program, Struct Multidisc Optim 22, pp.179-187, 2001
- Interactive topology optimization on hand-held devices, Struct Multidisc Optim 47, pp.1-6, 2013



Educational Design Tool (2)

The screenshot displays the 'Educational Design Tool' interface within a Netscape browser window. The main window shows a design problem titled 'Try your own design!' with instructions to 'Just construct your own design problem and press 'Submit' button.' The design area features a rectangular plate with a central square void, supported by two pin supports at the bottom corners. Two upward-pointing arrows indicate single loading at the top corners. A toolbar on the left provides various design tools, including a 'Reset' button. Three optimization results are shown as separate windows:

- Result 9:** Shows a single loading configuration with an objective value of 28.21. The density scale ranges from 10^{-1} to 1.
- Result 10:** Shows a multiple loading configuration with an objective value of 39.81. The density scale ranges from 10^{-2} to 1.
- Result 12:** Shows a multiple loading configuration with a void area, with an objective value of 56.74. The density scale ranges from 10^{-3} to 1.

The results windows also display the 'last ready frame' and the status of the optimization process (Stopped or Finished).

Single loading

Multiple loading

Multiple loading with void area

99 Lines of Matlab Code (2001)

Educational article

Struct Multidisc Optim 21, 120–127 © Springer-Verlag 2001

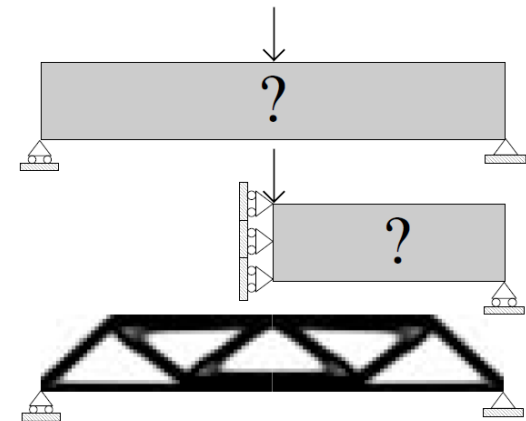
A 99 line topology optimization code written in Matlab

O. Sigmund

```
top(nelx,nely,volfrac,penal,rmin)
```

- nelx and nely: number of elements in the horizontal and vertical directions
- volfrac: volume fraction
- penal: penalization power (=3)
- rmin: filter size(divided by element size)

```
>>top(60,20,0.5,3.0,1.5)
```



Alternative '88 line Matlab code' (2011)

New 99 line Matlab code (2020)

Struct Multidisc Optim (2011) 43:1–16
DOI 10.1007/s00158-010-0594-7

EDUCATIONAL ARTICLE

Efficient topology optimization in MATLAB using 88 lines of code

Erik Andreassen · Anders Clausen · Mattias Schevenels ·
Boyan S. Lazarov · Ole Sigmund

```
top88(nelx,nely,volfrac,penal,rmin,ft)
```

Structural and Multidisciplinary Optimization
<https://doi.org/10.1007/s00158-020-02629-w>

EDUCATIONAL PAPER

A new generation 99 line Matlab code for compliance topology optimization and its extension to 3D

Federico Ferrari¹  · Ole Sigmund¹

Received: 18 February 2020 / Revised: 2 May 2020 / Accepted: 10 May 2020
© Springer-Verlag GmbH Germany, part of Springer Nature 2020

```
top99neo(nelx,nely,volfrac,penal,rmin,ft,ftBC,eta,beta,move,maxit)
```

```
top3D125(nelx,nely,nelz,volfrac,penal,rmin,ft,ftBC,eta,beta,move,maxit)
```

topopt.mek.dtu.dk

- conv → imfilter: image process toolbox

The 2D MBB beam example:

```
>> top99neo(300,100,0.5,3,8.75,3,'N',0.5,2,0.2,500);
```

The 3D cantilever beam example

```
>> top3D125(24,12,12,0.12,3,sqrt(3),1,'N',0.5,1,0.2,100);
```

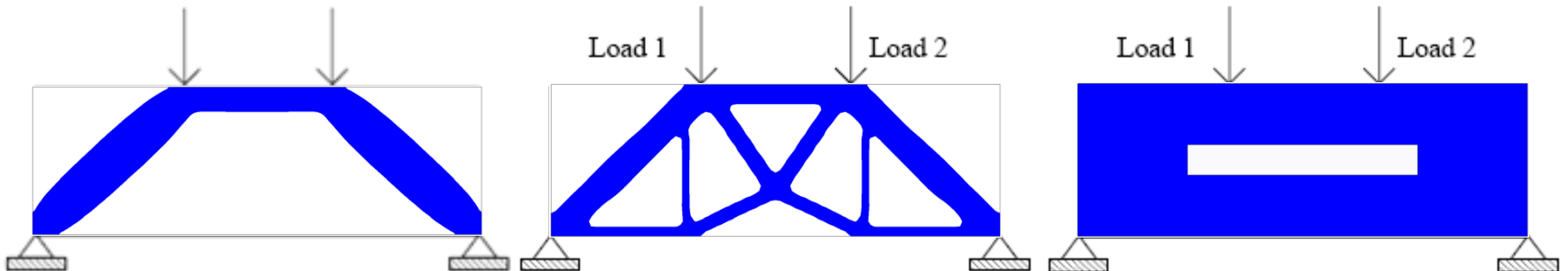
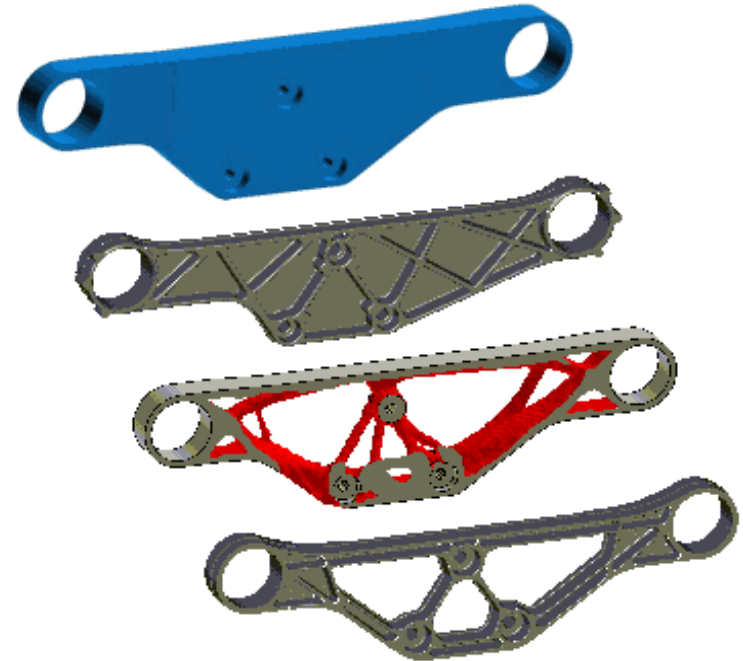
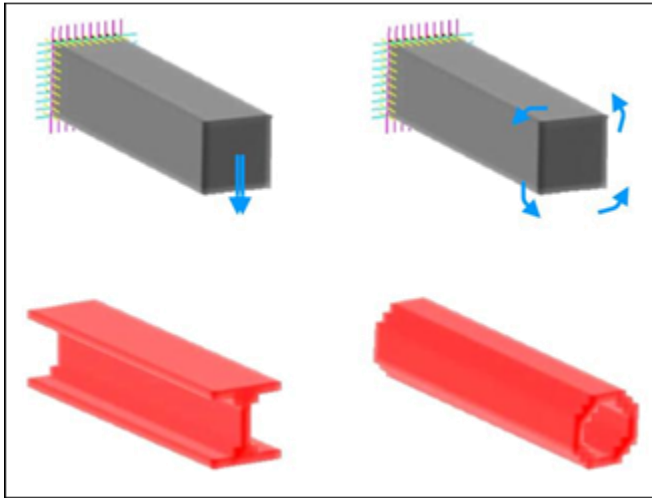
NOTE: the codes contain the external function "fsparse" that is part of the "Fast" package by Stefan Engblom. The package can be downloaded here:

<https://github.com/stefanengblom/stenglib>

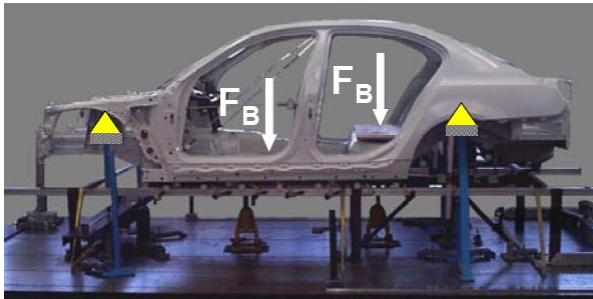
and installed in Matlab following the instructions in the README file.

정적 문제: 예제

■ 고체역학 검증: 하중 경로

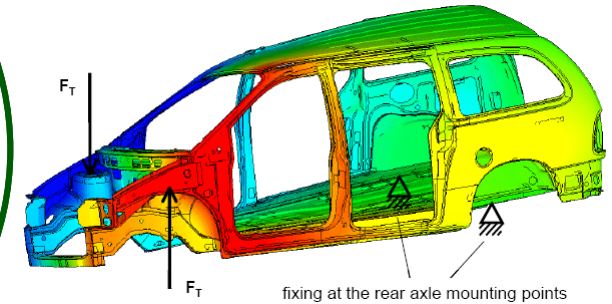
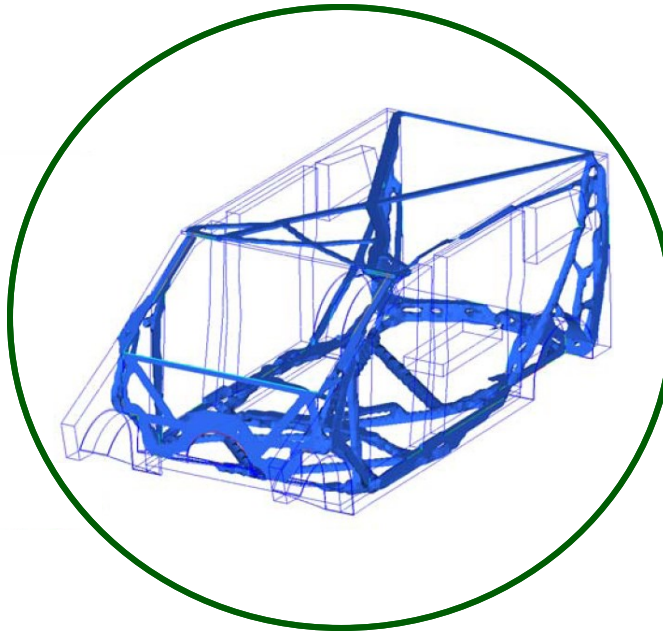
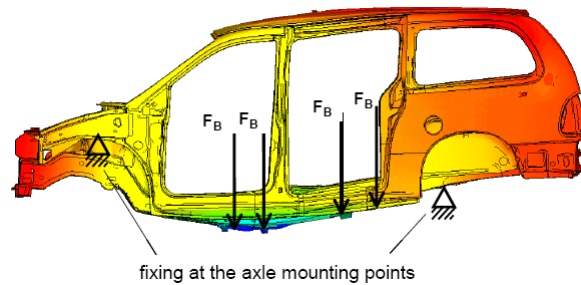


위상최적설계: 개념 설계에 활용

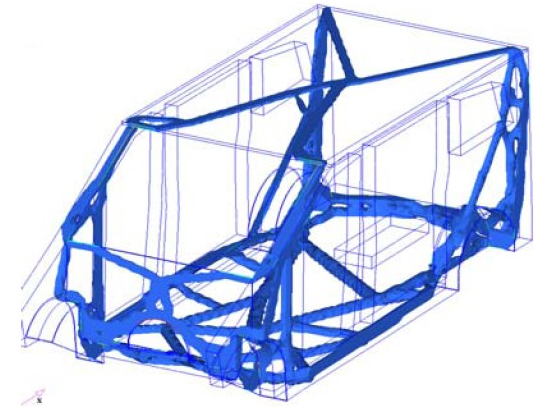
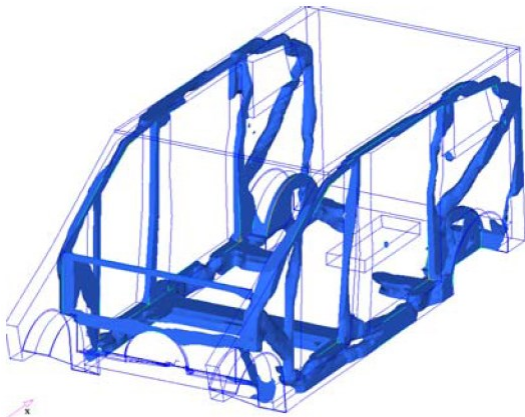


굽힘

비틀림



굽힘 + 비틀림



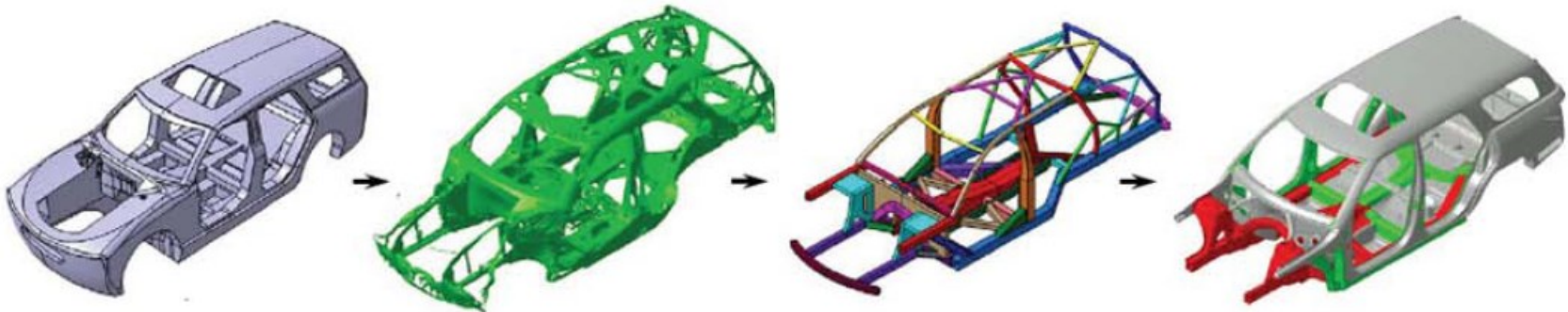
위상최적설계: 차체구조설계 사례



Light Weight Body Structural Optimization Process

2008 GDIS

Topology Optimization used to define Major Load Paths



Packaging Volume

Holistic Drivers

- Safety
- NVH Refinement
- Durability

Topology Results

Spatial load image

Beam Model

87 paths

BIW Design

202 components

Chassis: Ferrari F458 Italia

- 설계 목적

경량화

- 설계 변수 (cascade)

Topology optimization: 밀도

Topometry optimization : 절점 높이

Size optimization : 쉘 두께

- 성능 요구조건

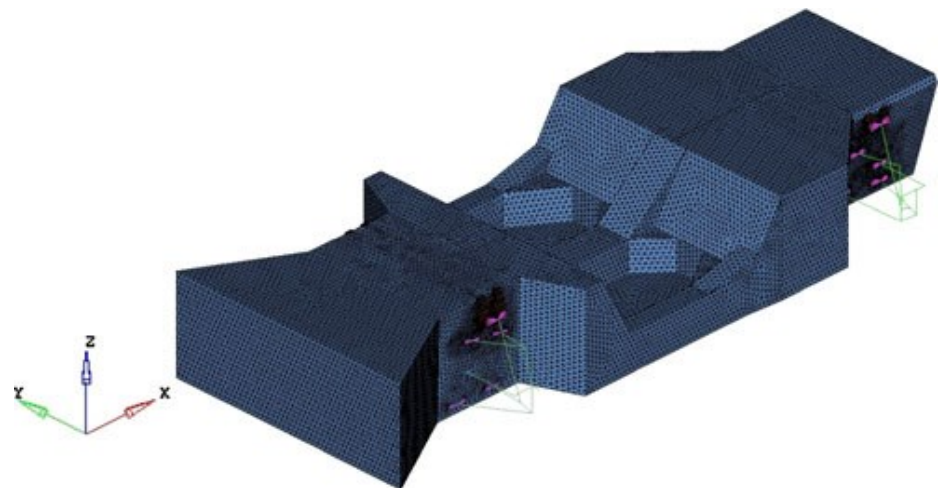
global bending stiffness
global torsional stiffness
Local stiffness of the suspensions, engine and gearbox
Modal response
Crash linearization

Struct Multidisc Optim (2011) 44:45–56
DOI 10.1007/s00158-010-0578-7

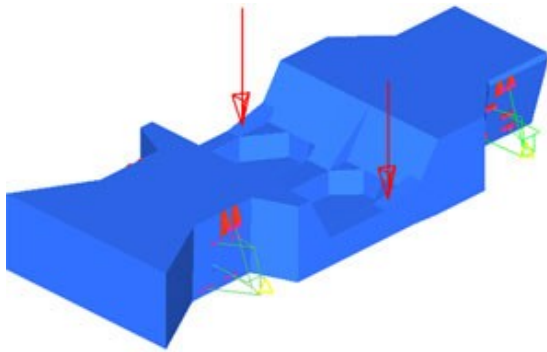
INDUSTRIAL APPLICATION

High performance automotive chassis design: a topology optimization based approach

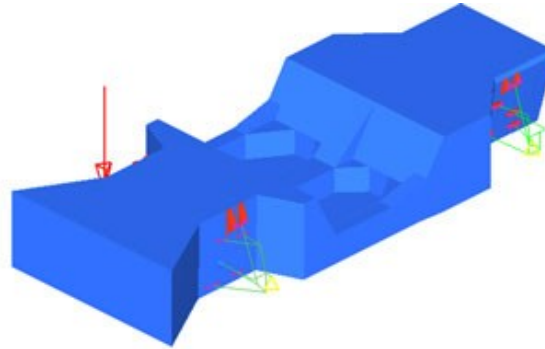
Marco Cavazzuti · Andrea Baldini · Enrico Bertocchi ·
Dario Costi · Enrico Torricelli · Patrizio Moruzzi



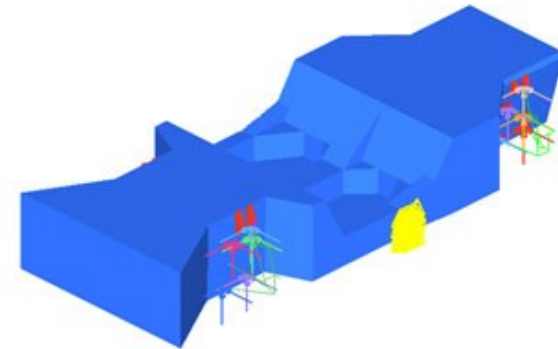
Chassis: 하중조건



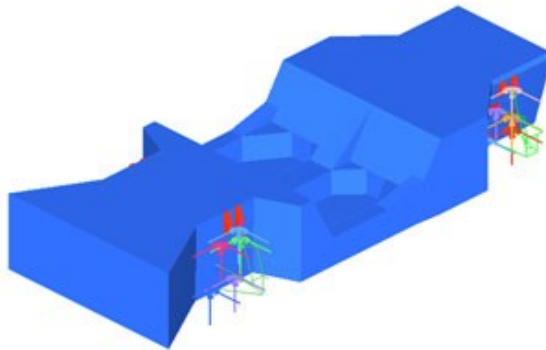
bending stiffness (1 FEA)



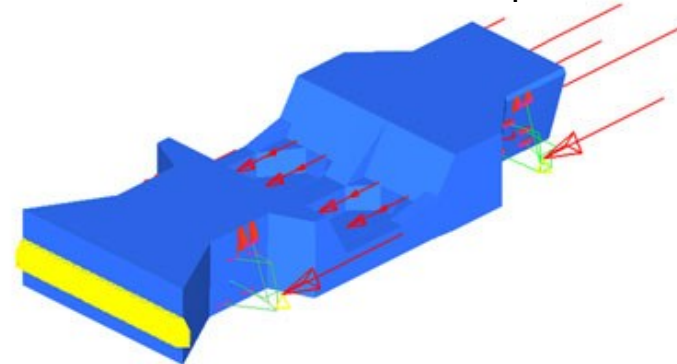
torsional stiffness (1 FEA)



local stiffness with
clamped sills (32 FEA)

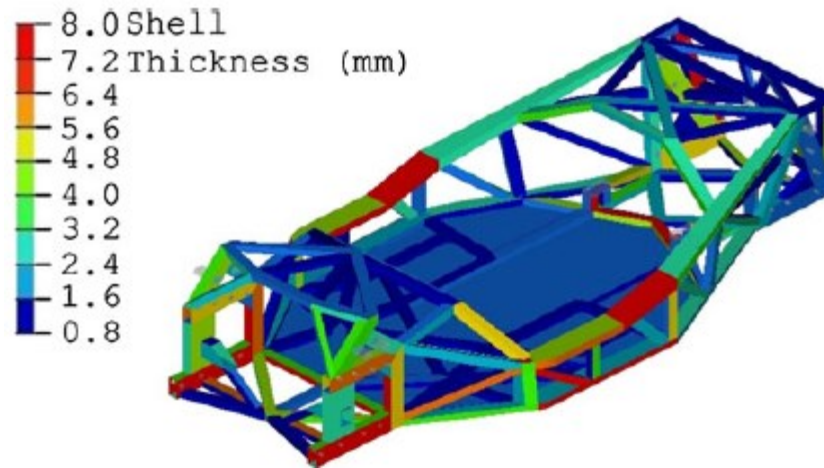
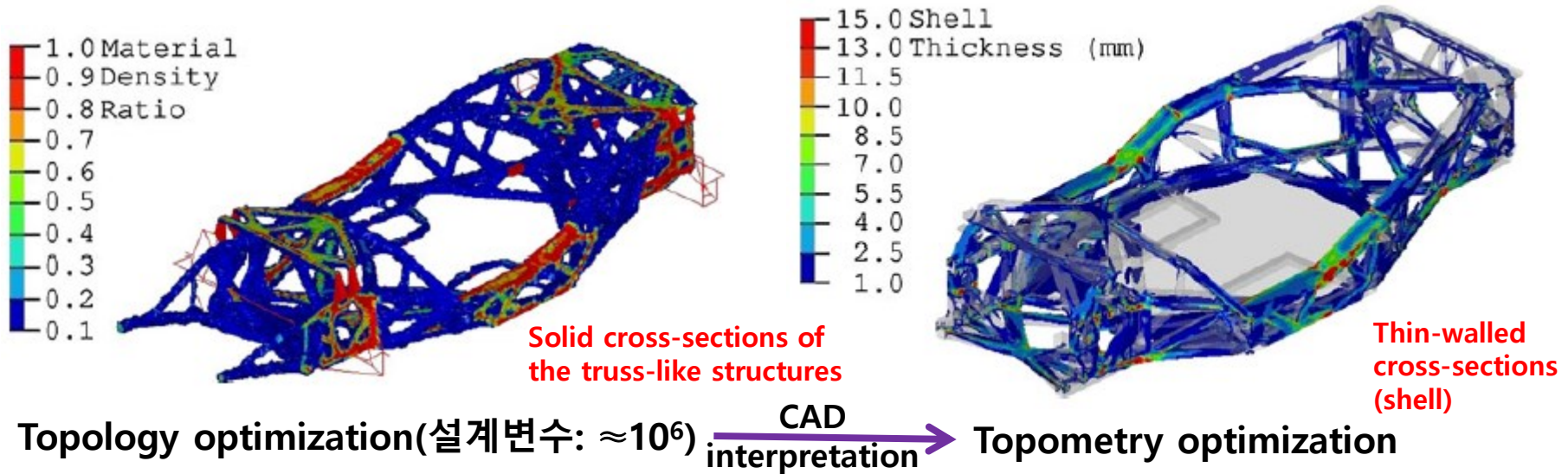


local stiffness with
inertia relief (32 FEA)



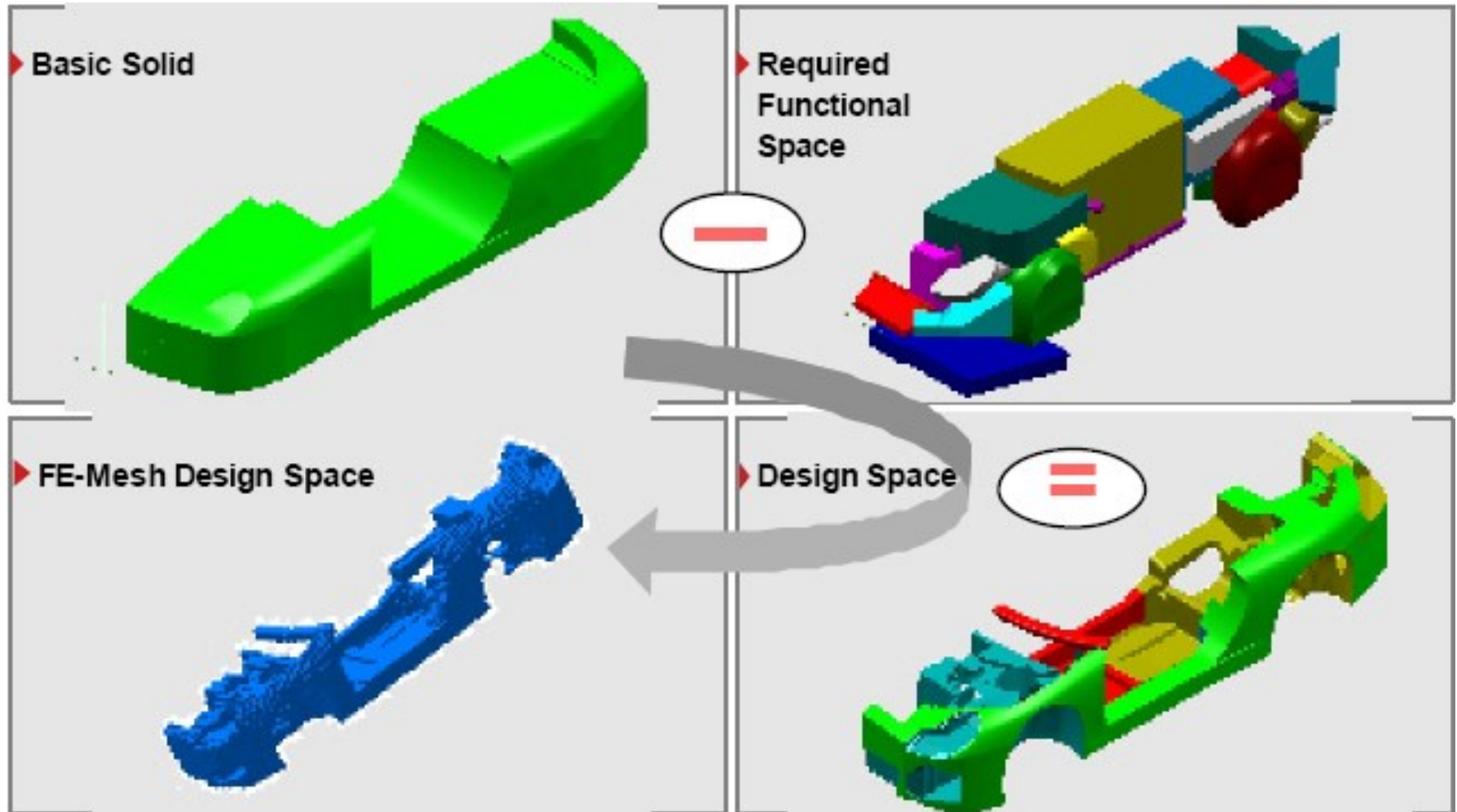
crash linearization (1 FEA)

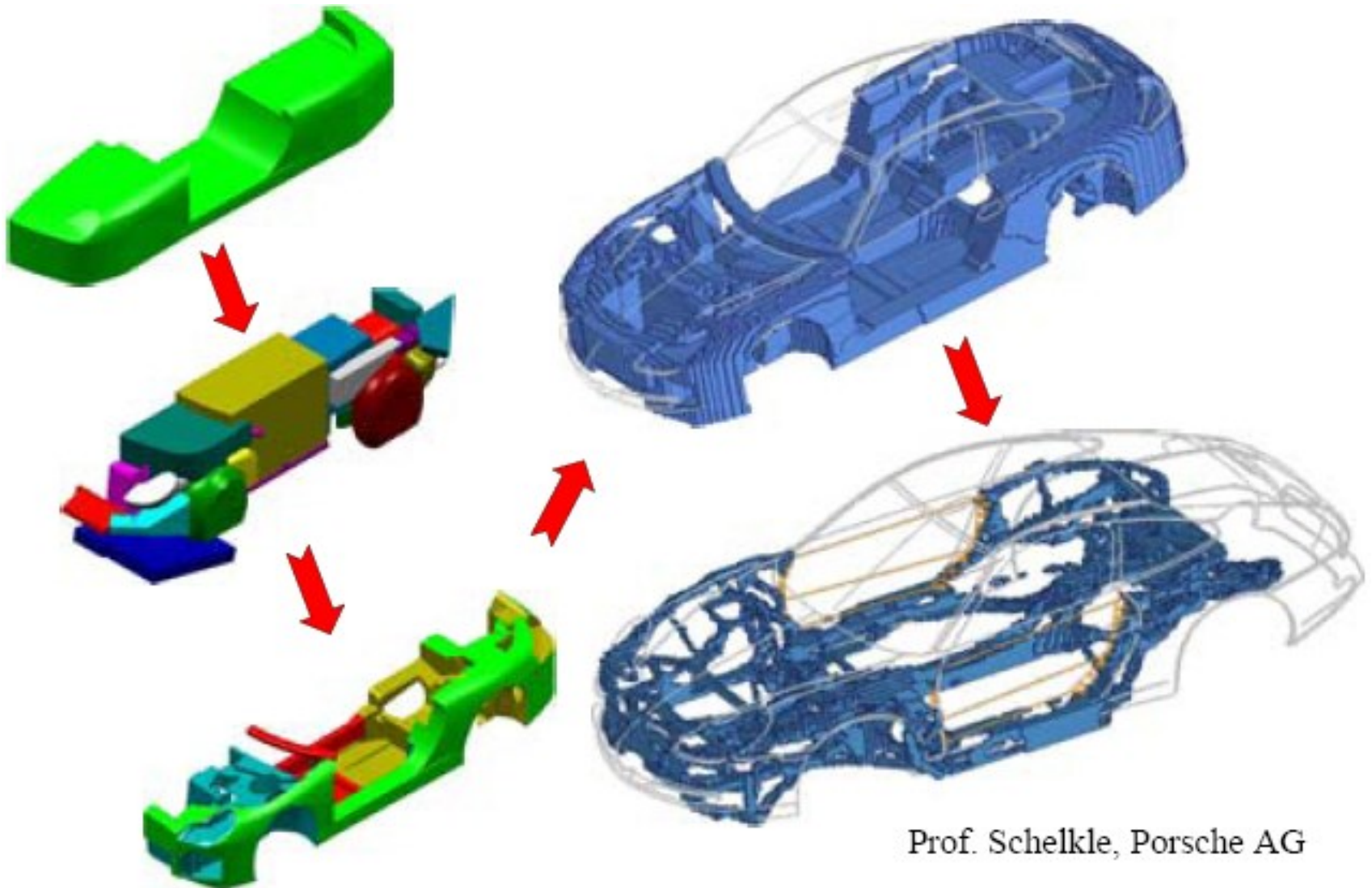
Chassis: 최적설계



Size optimization(설계변수: ≈ 330)

Load Path Analysis Sequence

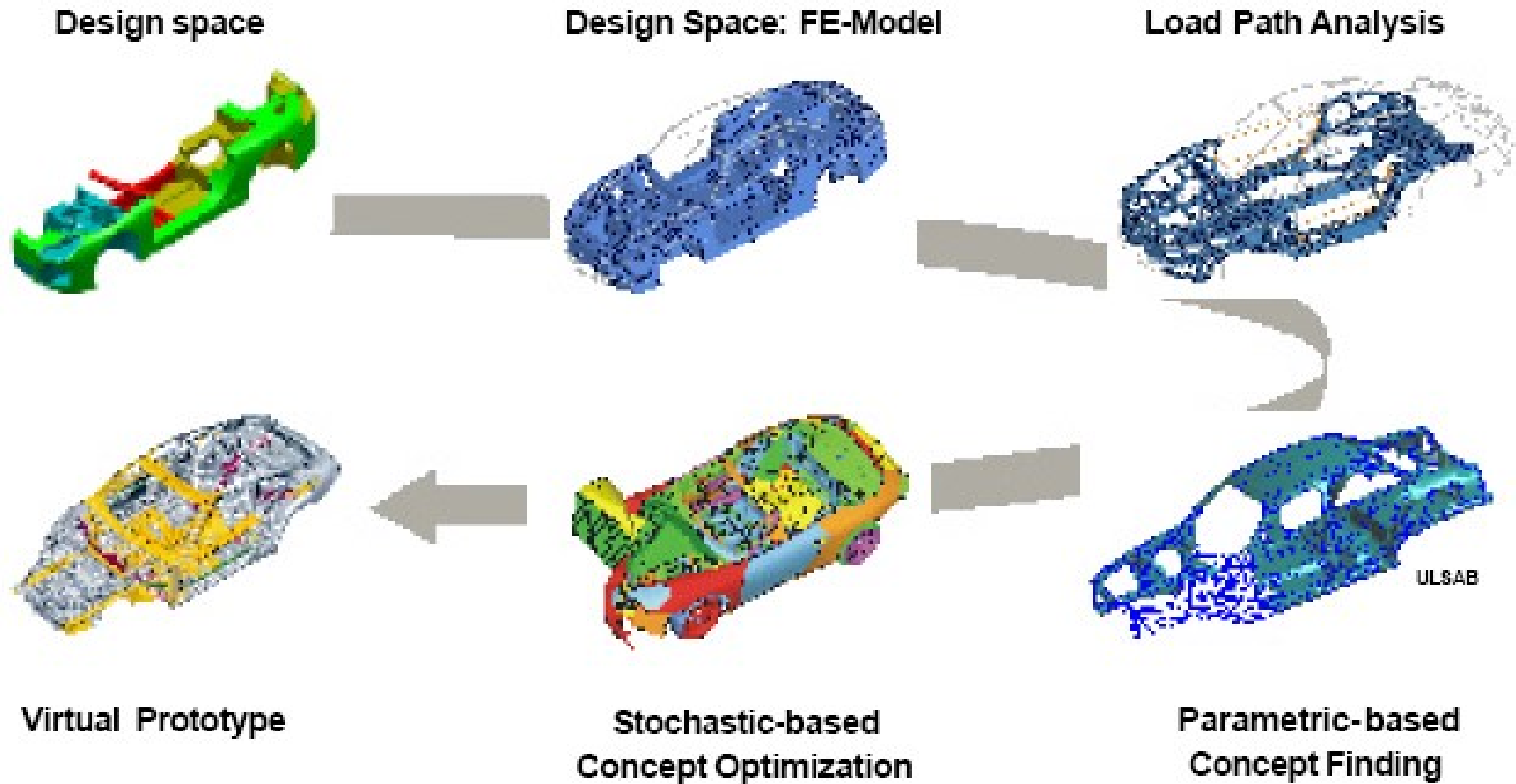




Prof. Schelkle, Porsche AG



Concept Stage of Vehicle Development



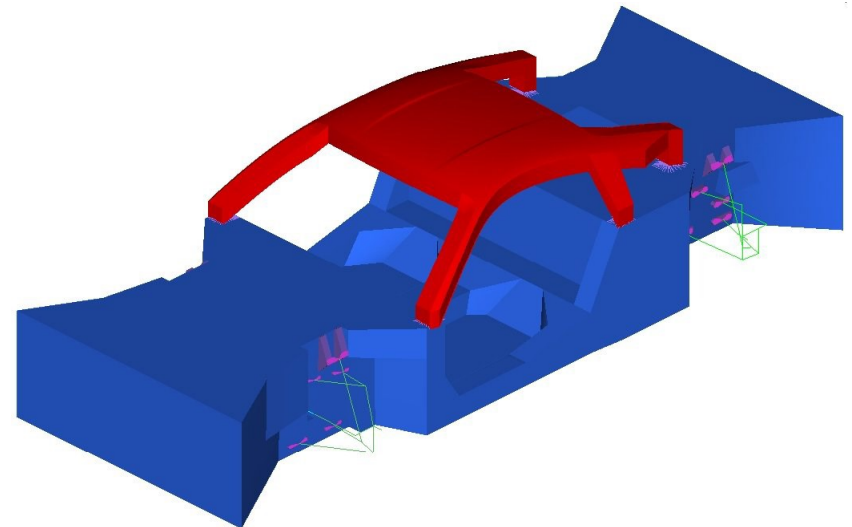
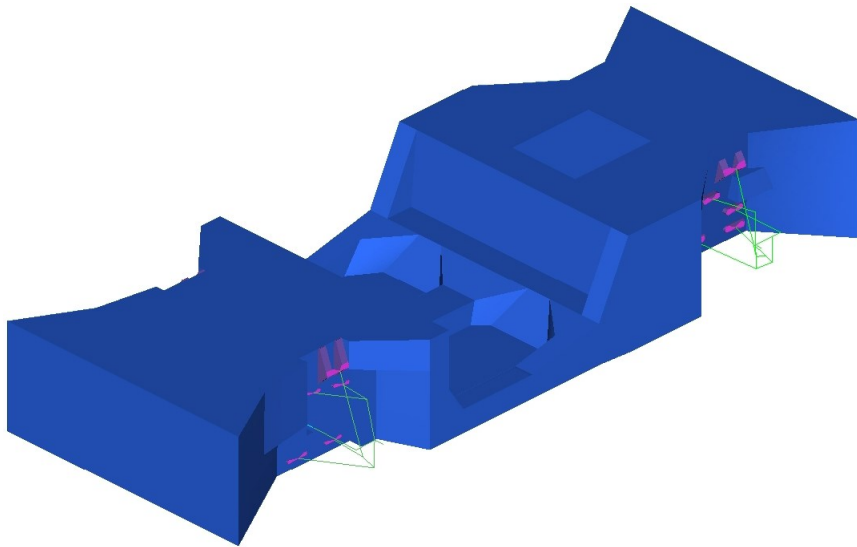
Automotive Chassis

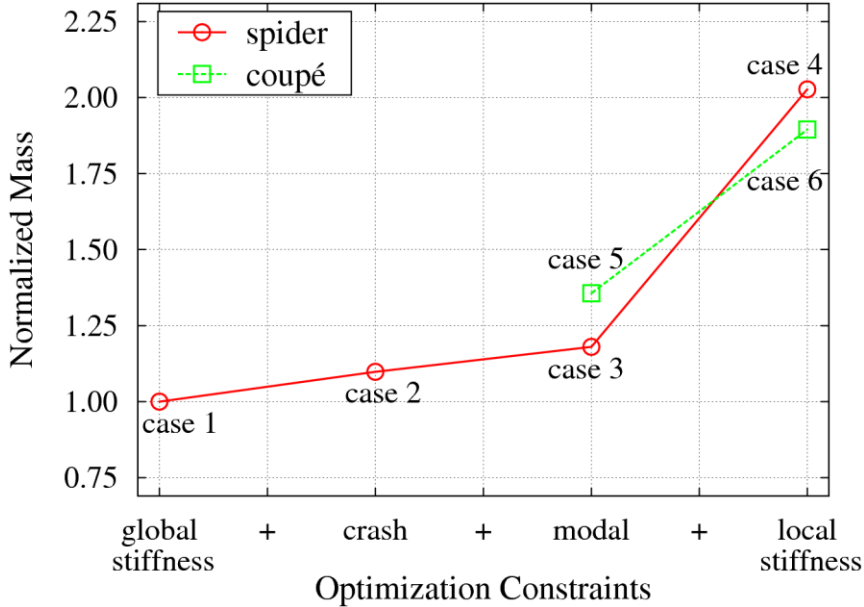
Proceedings of the World Congress on Engineering 2011 Vol III
WCE 2011, July 6 - 8, 2011, London, U.K.

Automotive Chassis Topology Optimization: a Comparison Between Spider and Coupé Designs

Marco Cavazzuti, Dario Costi, Andrea Baldini, Patrizio Moruzzi

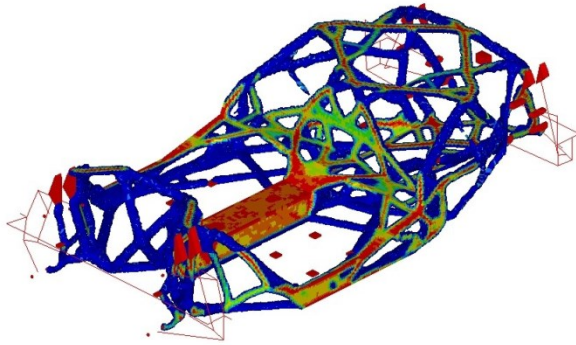
- Comparison Between Spider and Coupe Designs



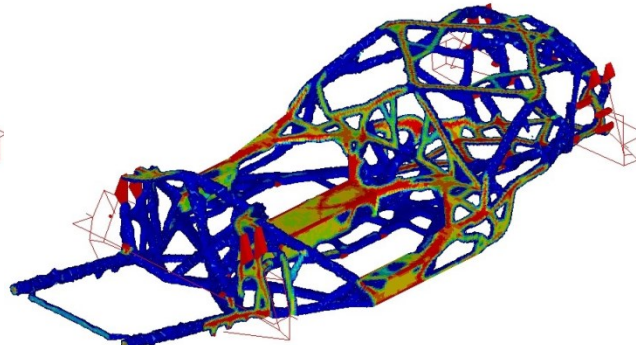


Active Optimization Constraints	Cases					
	Spider				Coupé	
	1	2	3	4	5	6
Global bending stiffness	✓	✗	✓	✓	✓	✓
Global torsion stiffness	✓	✓	✓	✓	✓	✗
Crash seat joints displacement	–	✗	✗	✗	✓	✗
Crash engine joints displacement	–	✓	✓	✗	✓	✓
Crash A-pillar displacement	–	✓	✓	✓	✓	✓
Crash pedal displacement	–	✗	✓	✗	✗	✗
Crash flame shield displacement	–	✓	✓	✗	✗	✗
Crash dashboard joints displacement	–	✗	✗	✗	✗	✓
Crash compliance	–	✓	✓	✓	✓	✓
First natural mode	–	–	✓	✓	✓	✓
Local front wheel stiffness along x	–	–	–	✓	–	✓
Local front wheel stiffness along y	–	–	–	✓	–	✓
Local front wheel stiffness along z	–	–	–	✓	–	✓
Local rear wheel stiffness along x	–	–	–	✓	–	✓
Local rear wheel stiffness along y	–	–	–	✓	–	✓
Local rear wheel stiffness along z	–	–	–	✗	–	✗
Local engine joint stiffness along z	–	–	–	✓	–	✓
Local gearbox joint stiffness along z	–	–	–	✓	–	✗
Total	$\frac{2}{2}$	$\frac{5}{9}$	$\frac{8}{10}$	$\frac{12}{18}$	$\frac{7}{10}$	$\frac{12}{18}$

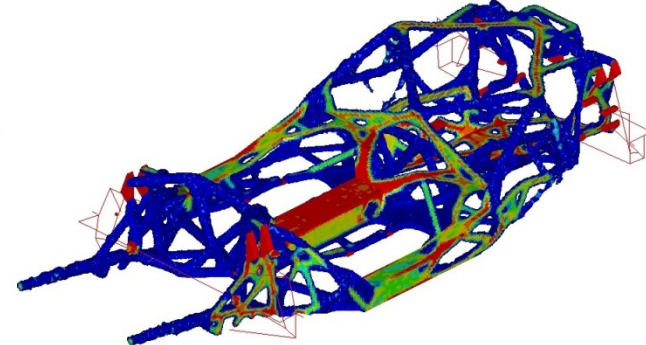
Results (1)



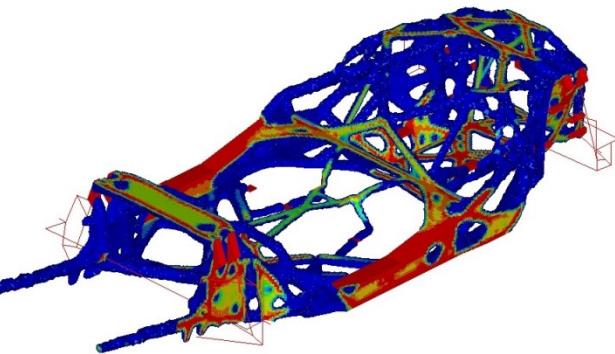
Case 1



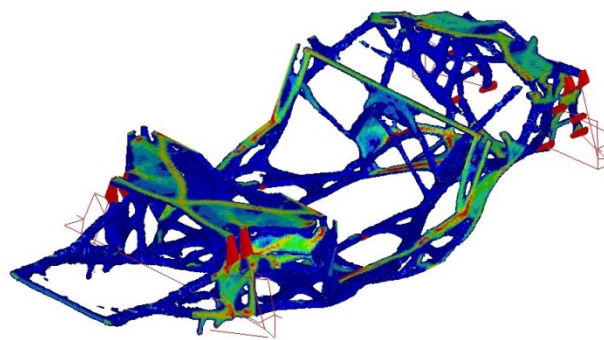
Case 2



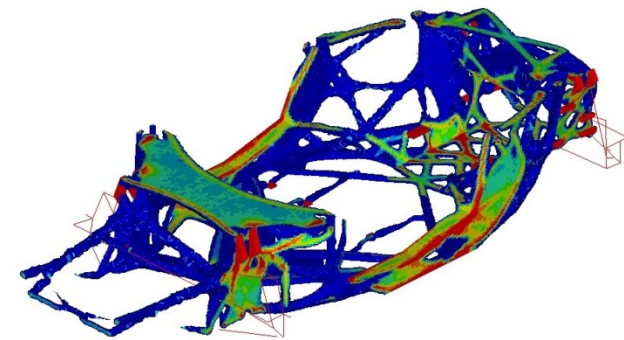
Case 3



Case 4

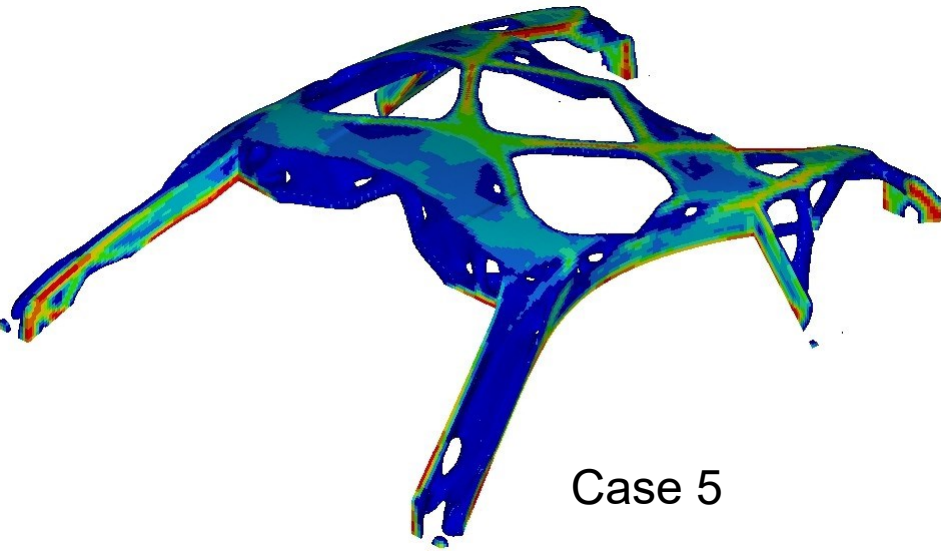


Case 5

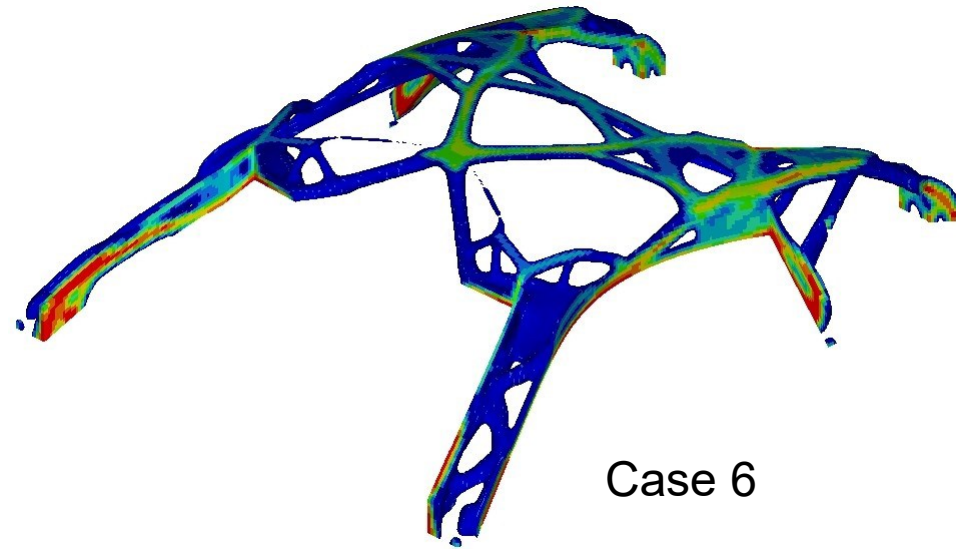


Case 6

Results (2)

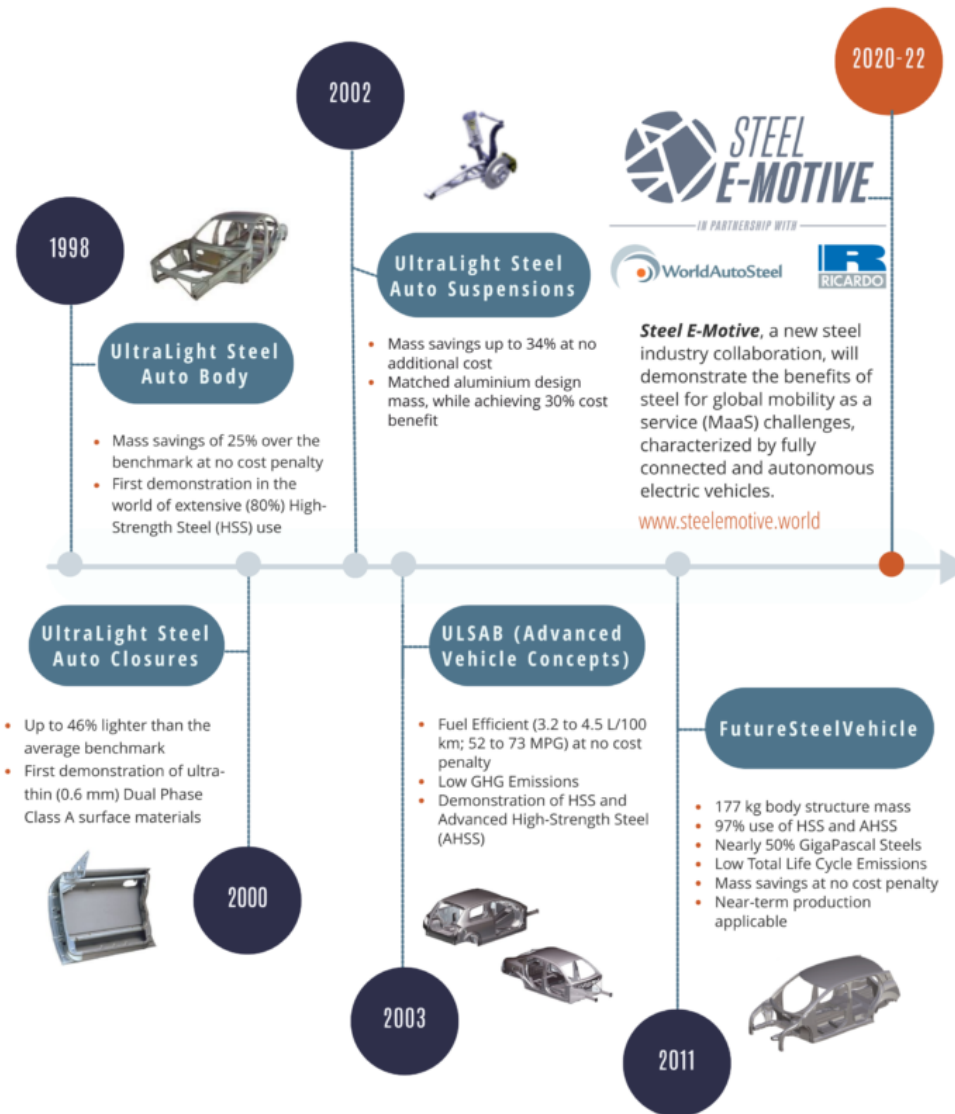


Case 5



Case 6

History of Steel Industry Innovation



설계프로세스에 적용

FutureSteelVehicle



ULSAB
UltraLight Steel Auto Body



ULSAC
UltraLight Steel Auto Closures



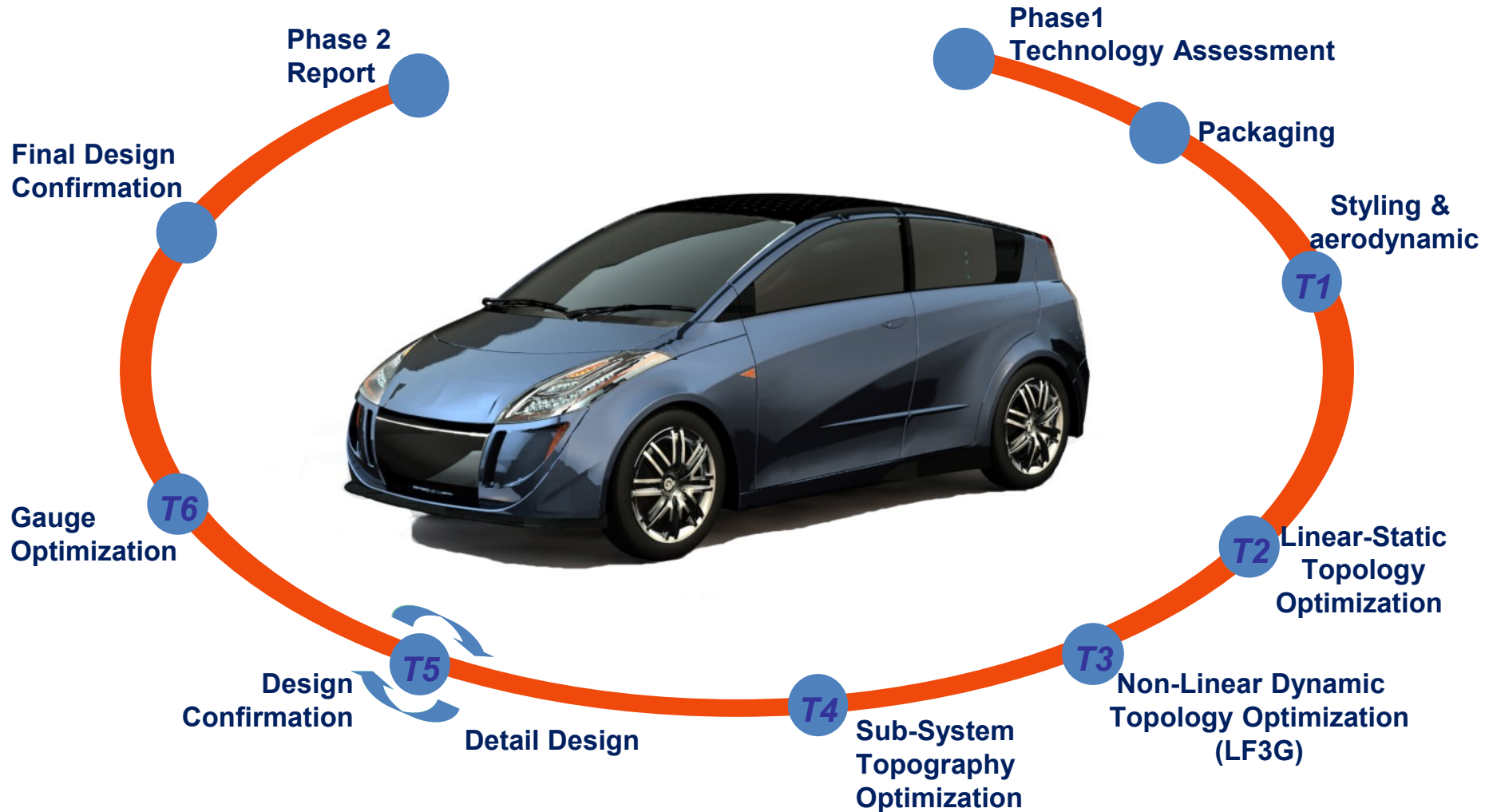
ULSAS
UltraLight Steel Auto Suspensions

*Nature's Way to Mobility*

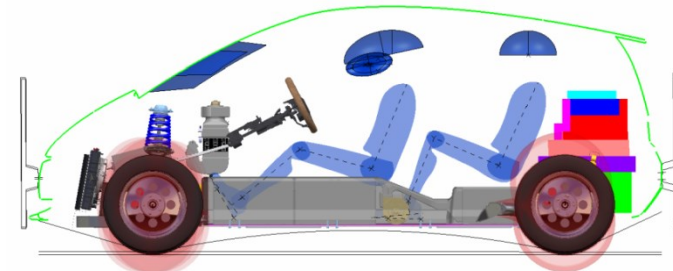
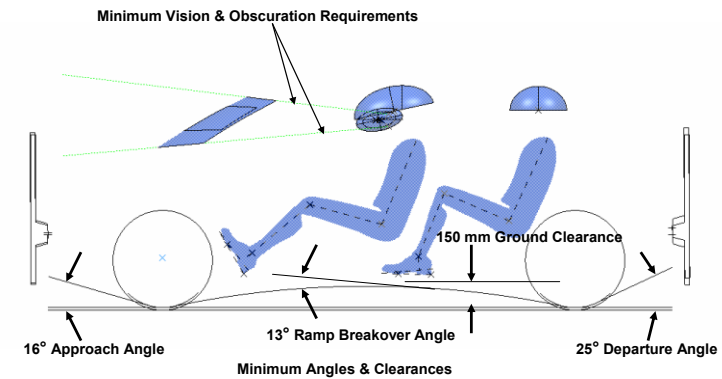
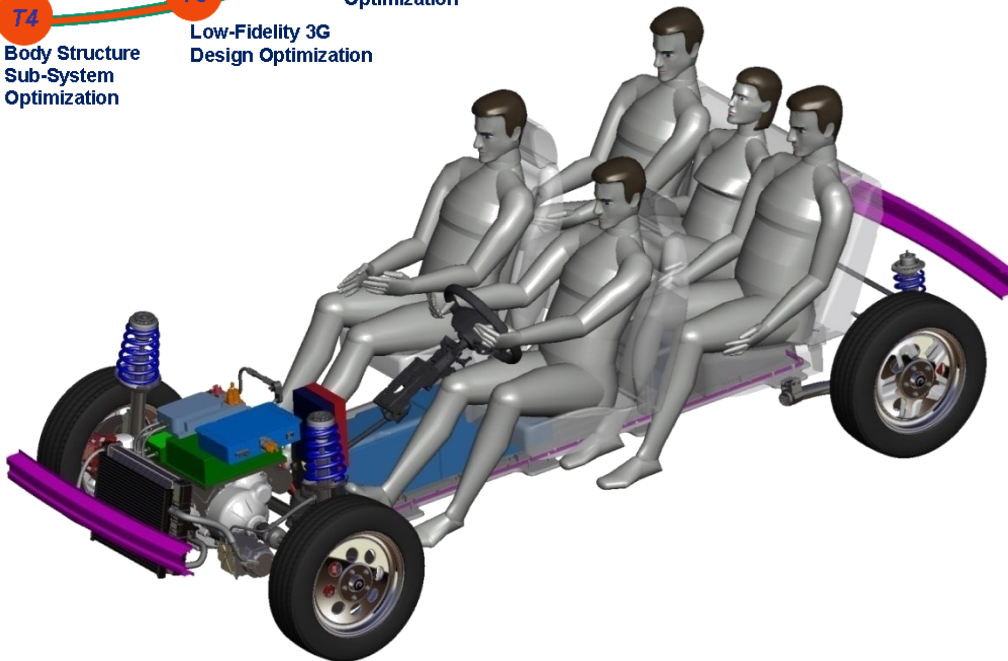
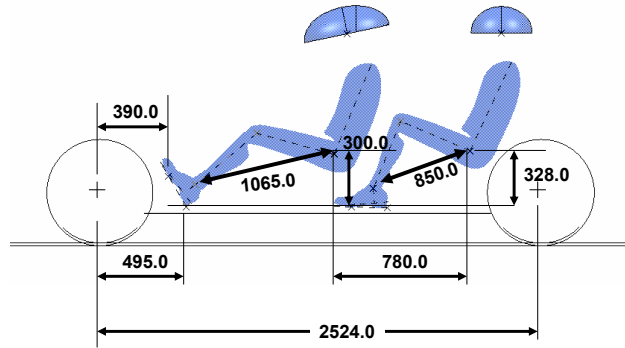
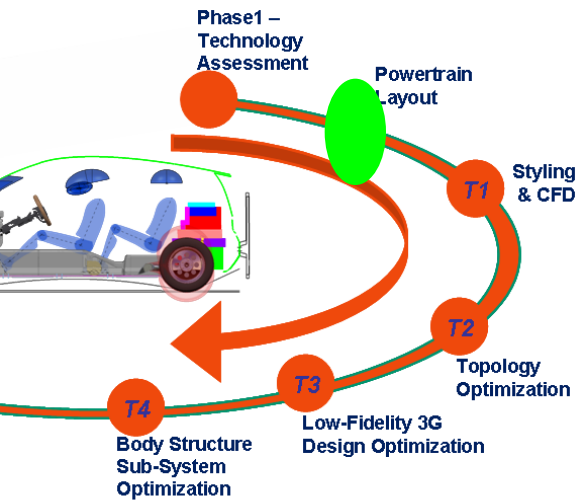
ULSAB-AVC
Advanced Vehicle Concepts



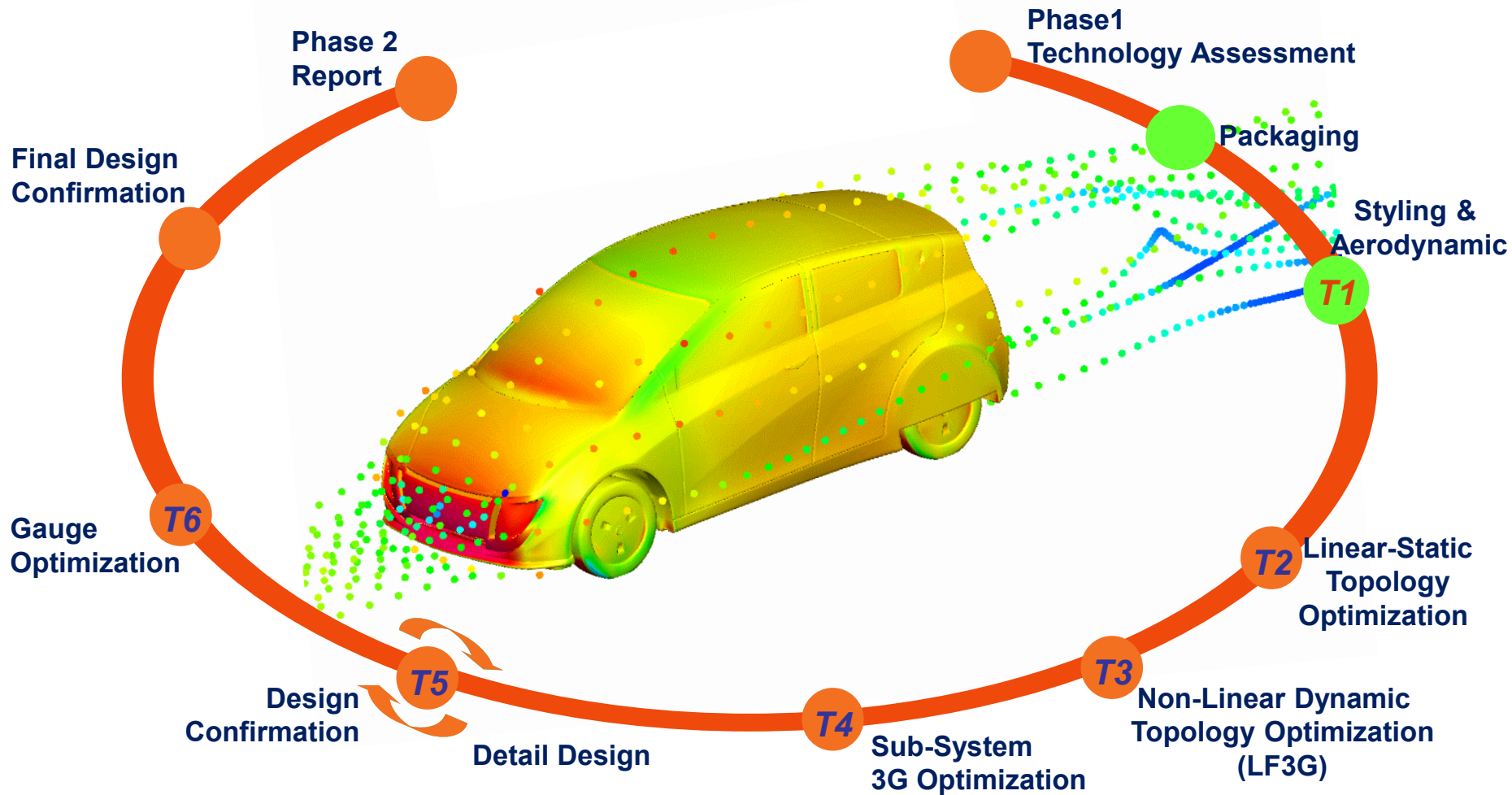
FSV Design Methodology



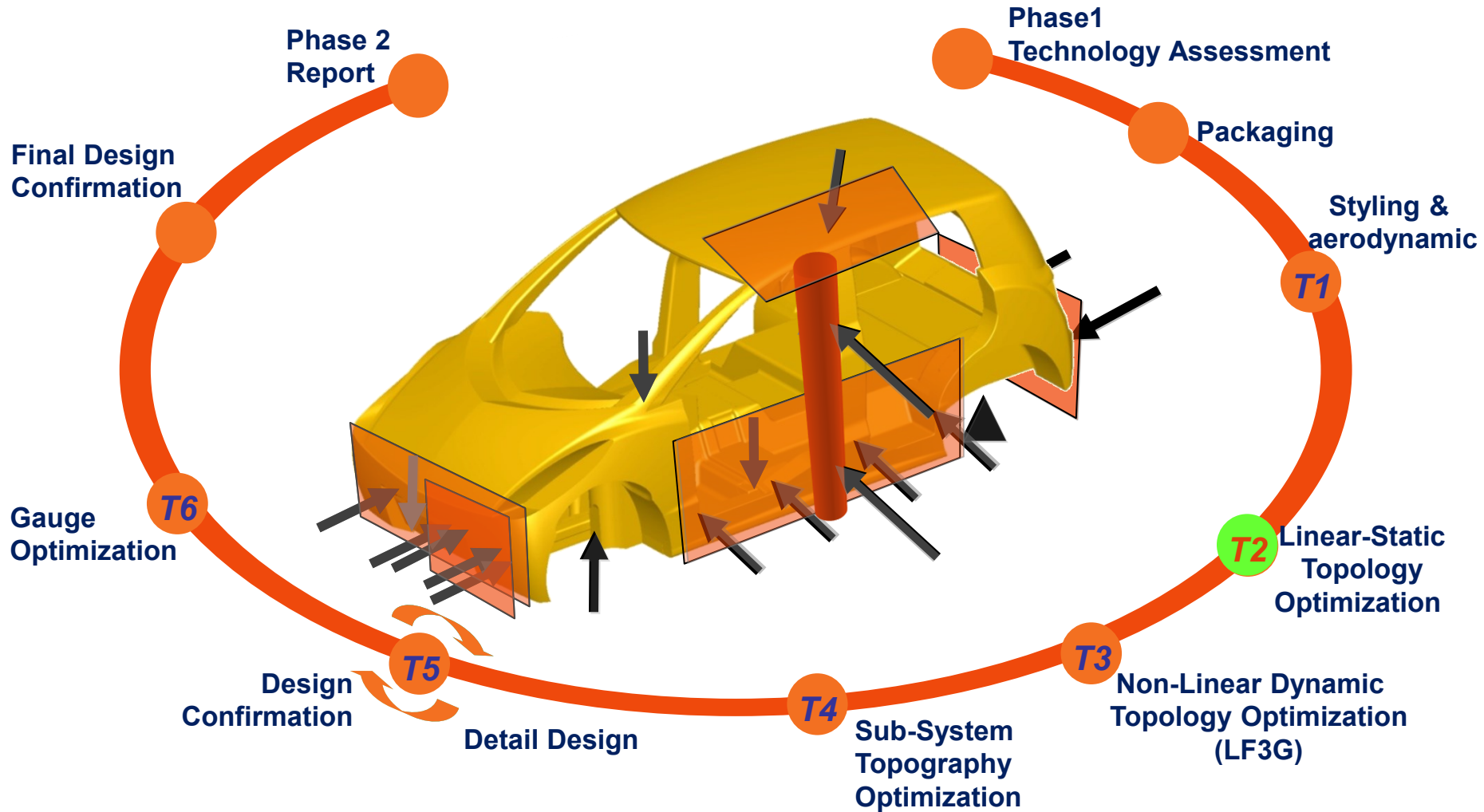
FSV BEV Packaging



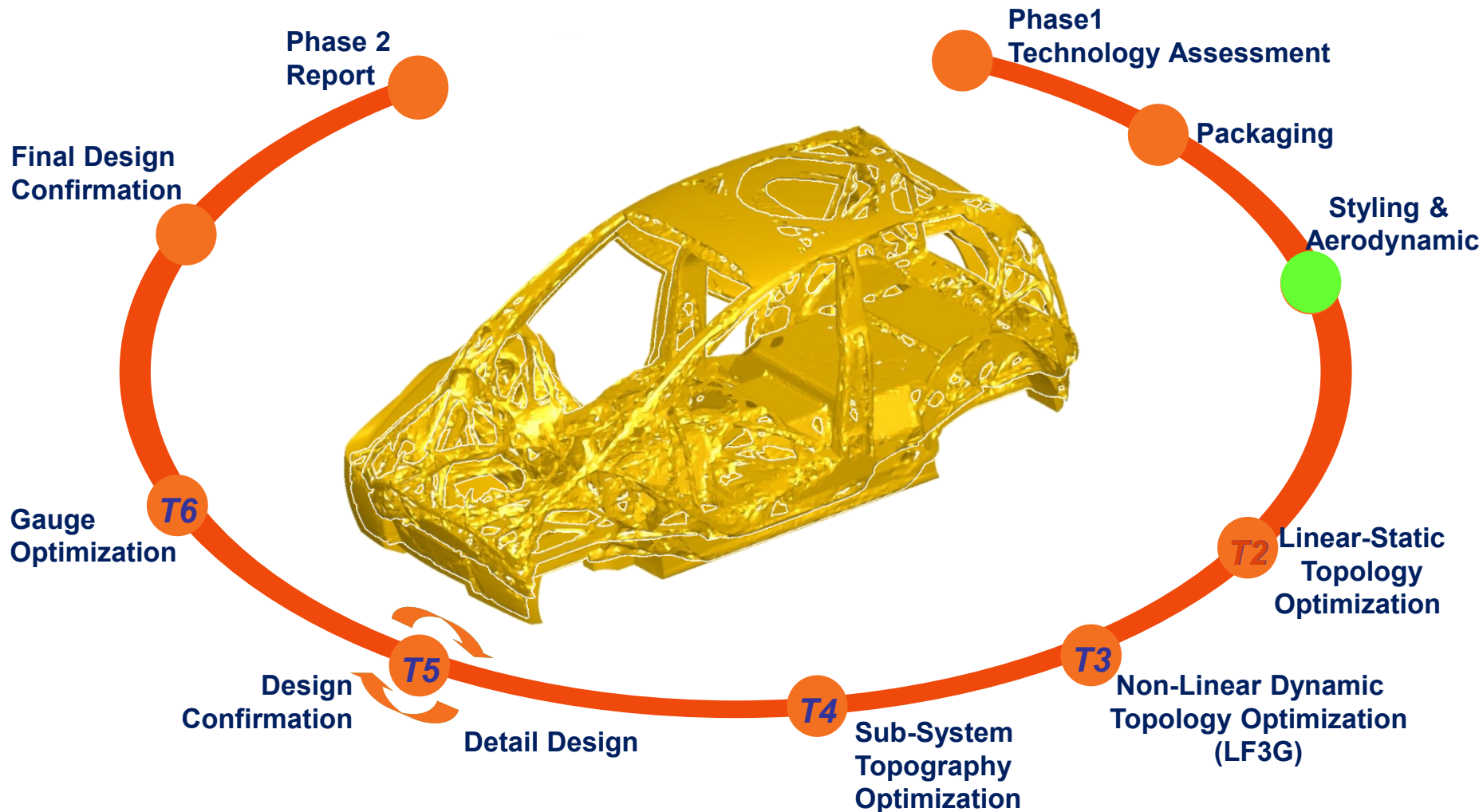
Aerodynamics & Styling



Topology Optimization Load Cases

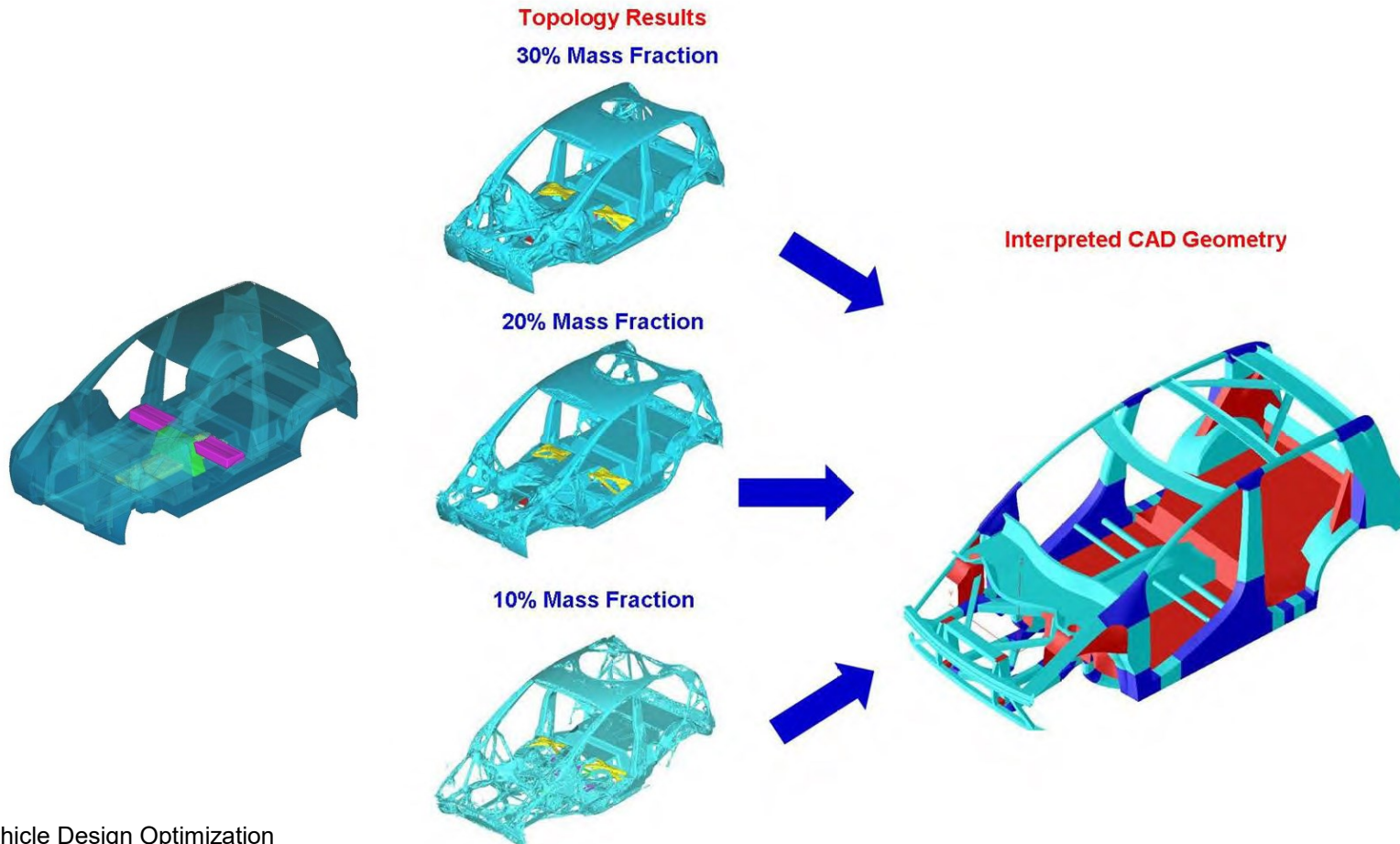


Linear-Static Topology Optimization

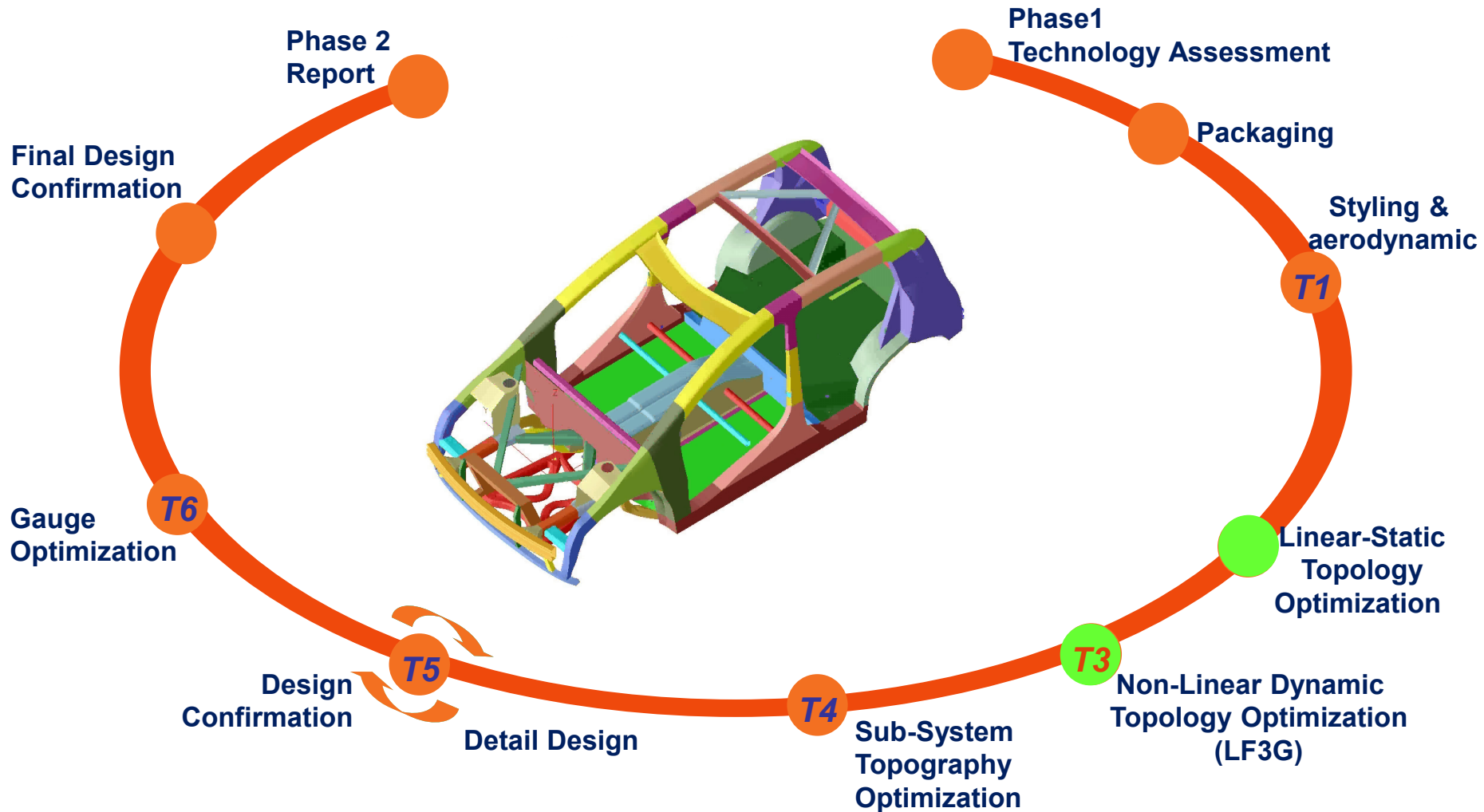


T2: Topology Optimization

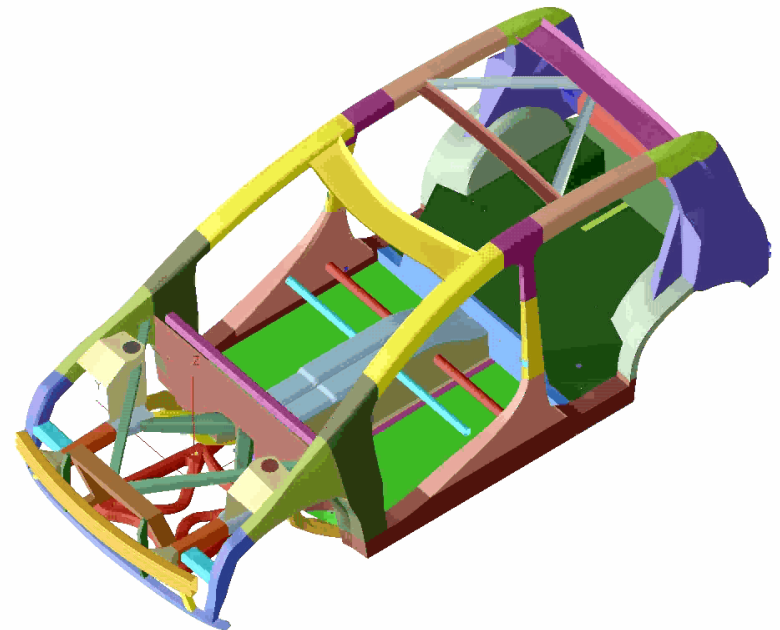
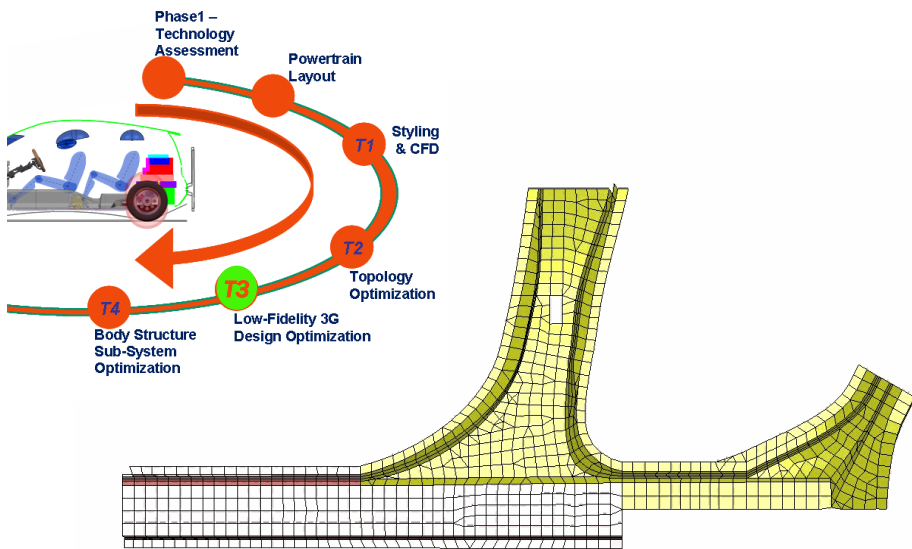
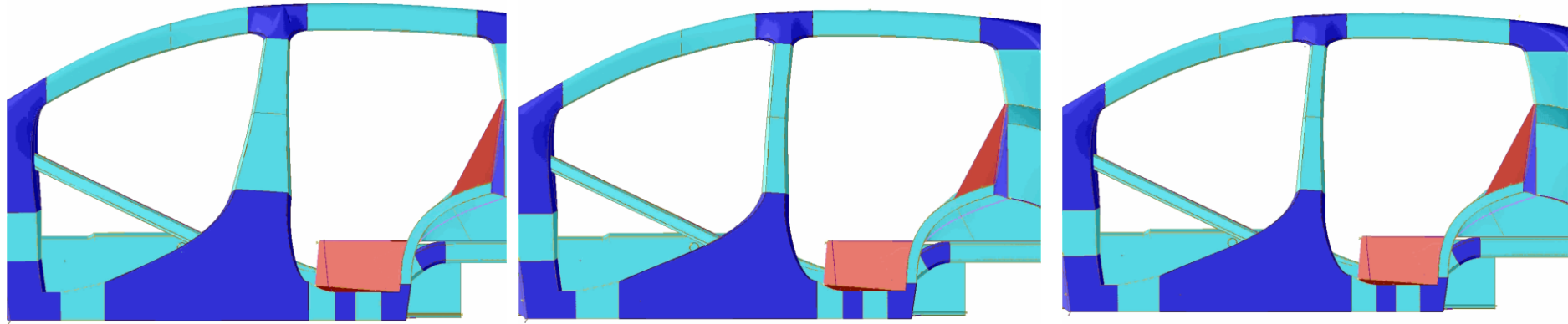
- geometry developed by the topology optimization was manually interpreted into a CAD (Computer Aided Design) model using engineering judgment



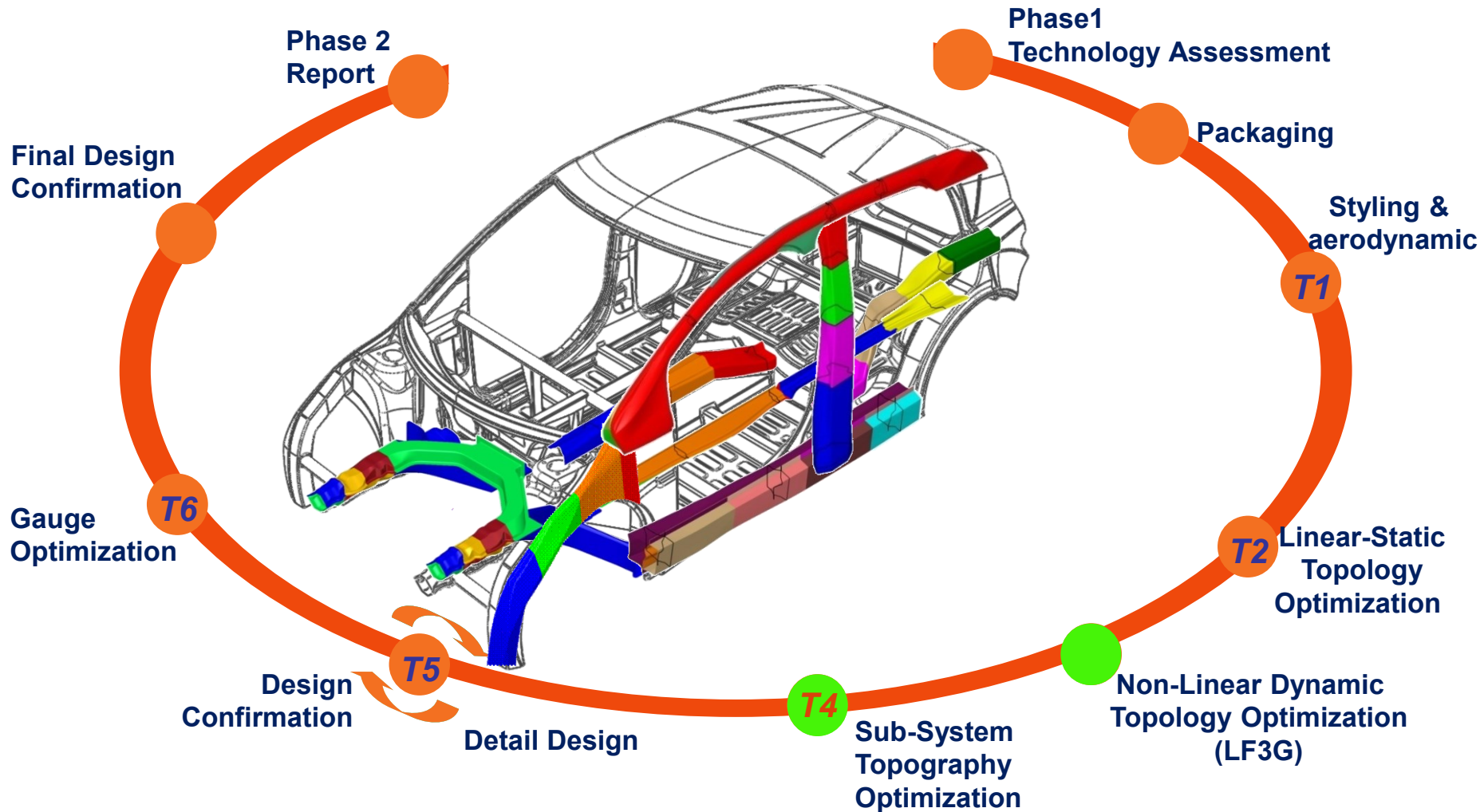
LF3G Load Path and 3G Optimization



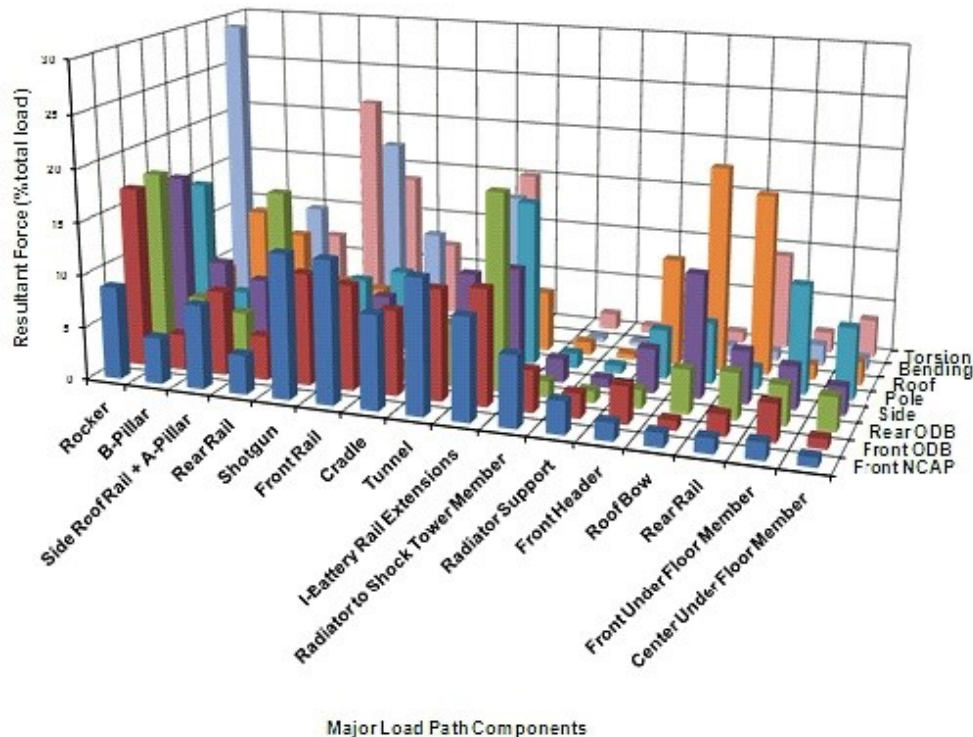
Low Fidelity 3G (Geometry, Gauge & Grade) Optimization



Sub-Systems 3G Optimization

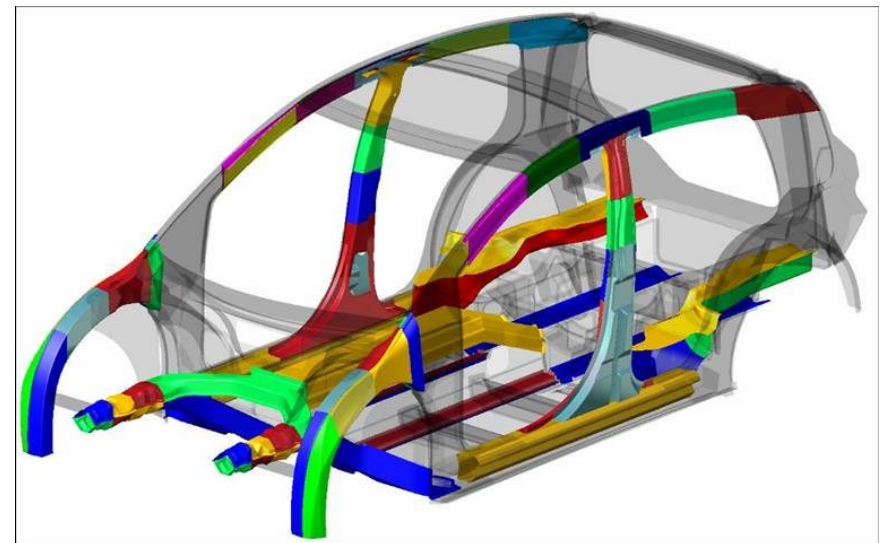


T4 Load Path Mapping

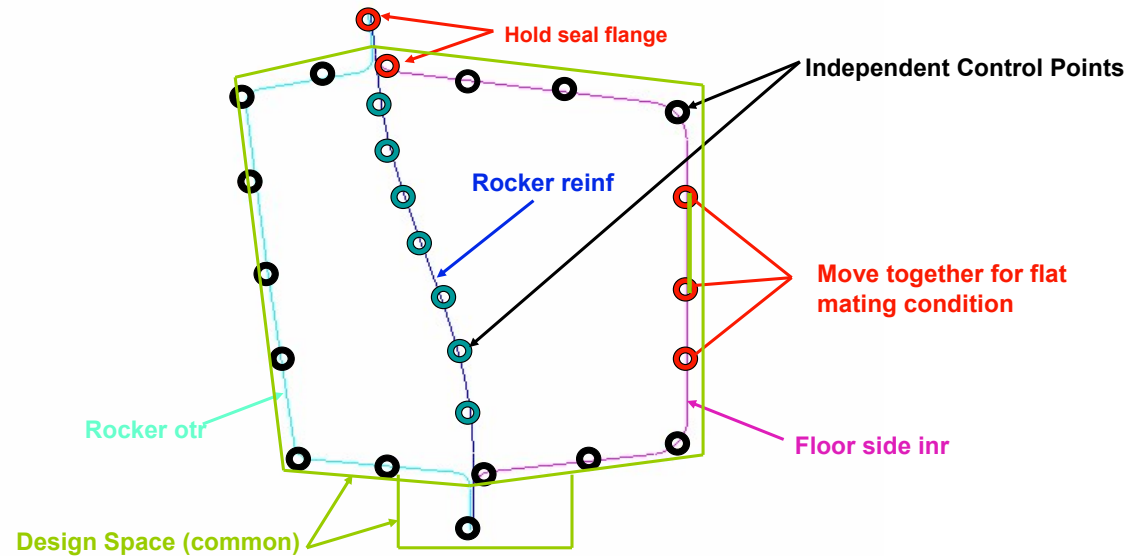
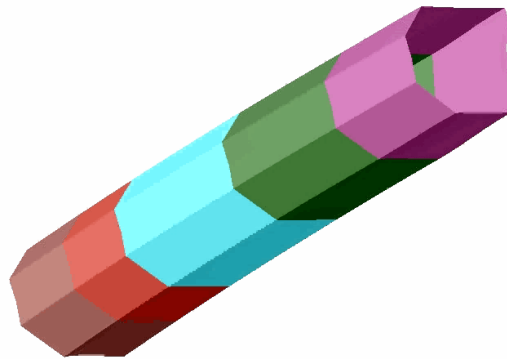
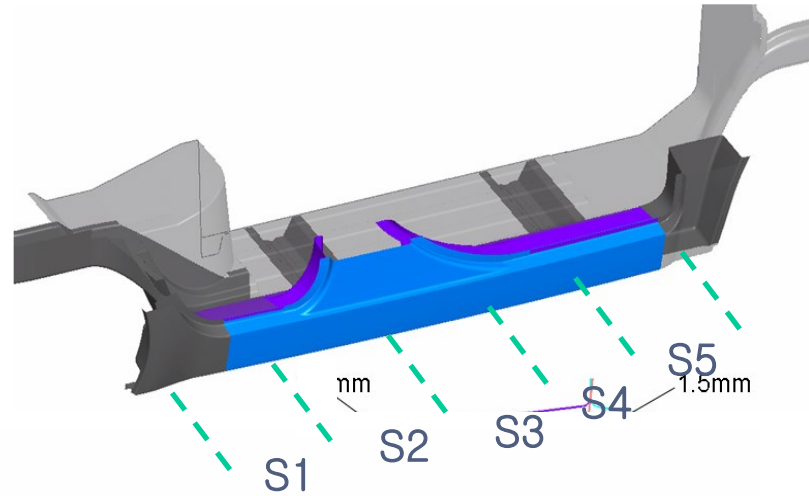
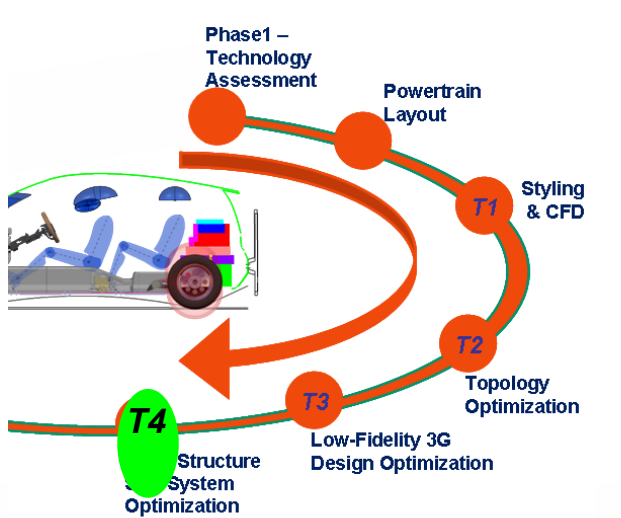


Selected Sub-Systems

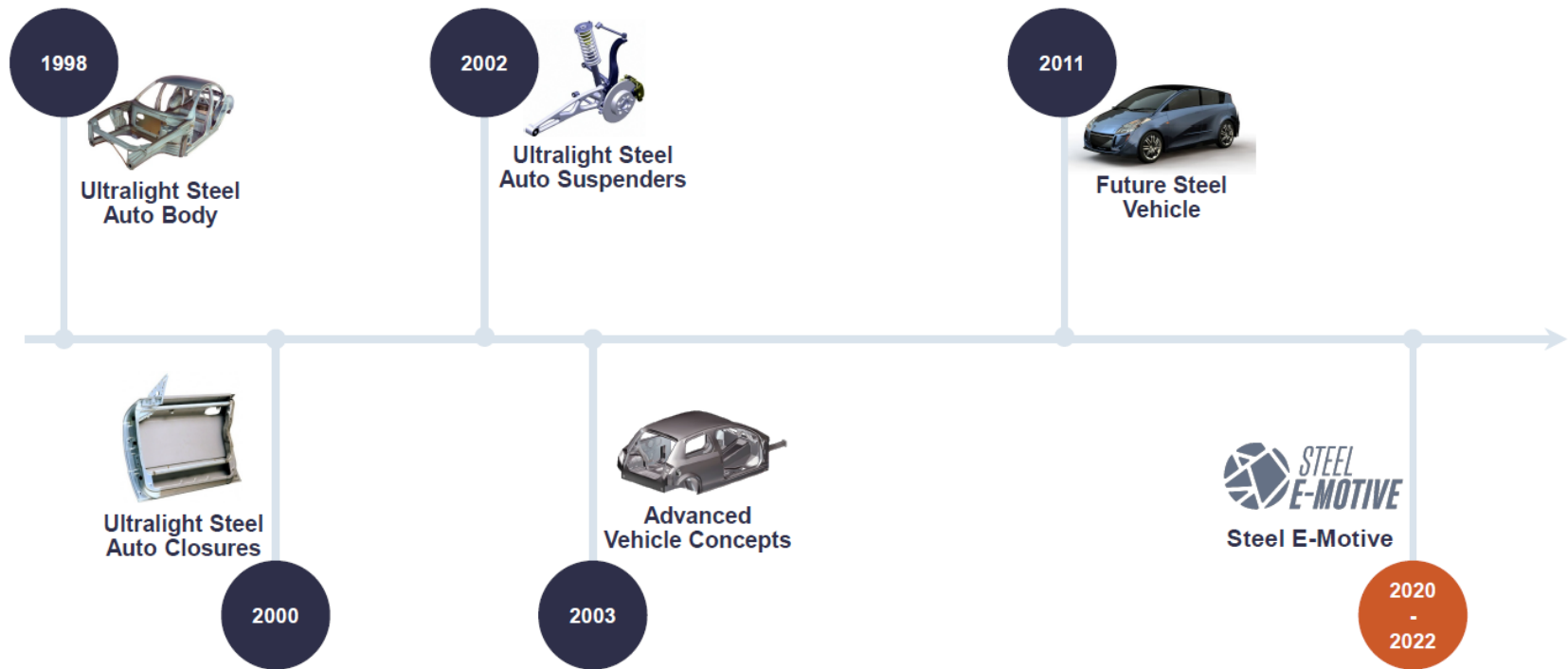
- Front Rail
- Shot Gun
- Rocker
- B-Pillar
- Rear Rail
- Roof Rail
- Tunnel Reinforcement



Body Structure – Sub-System 3G Optimization

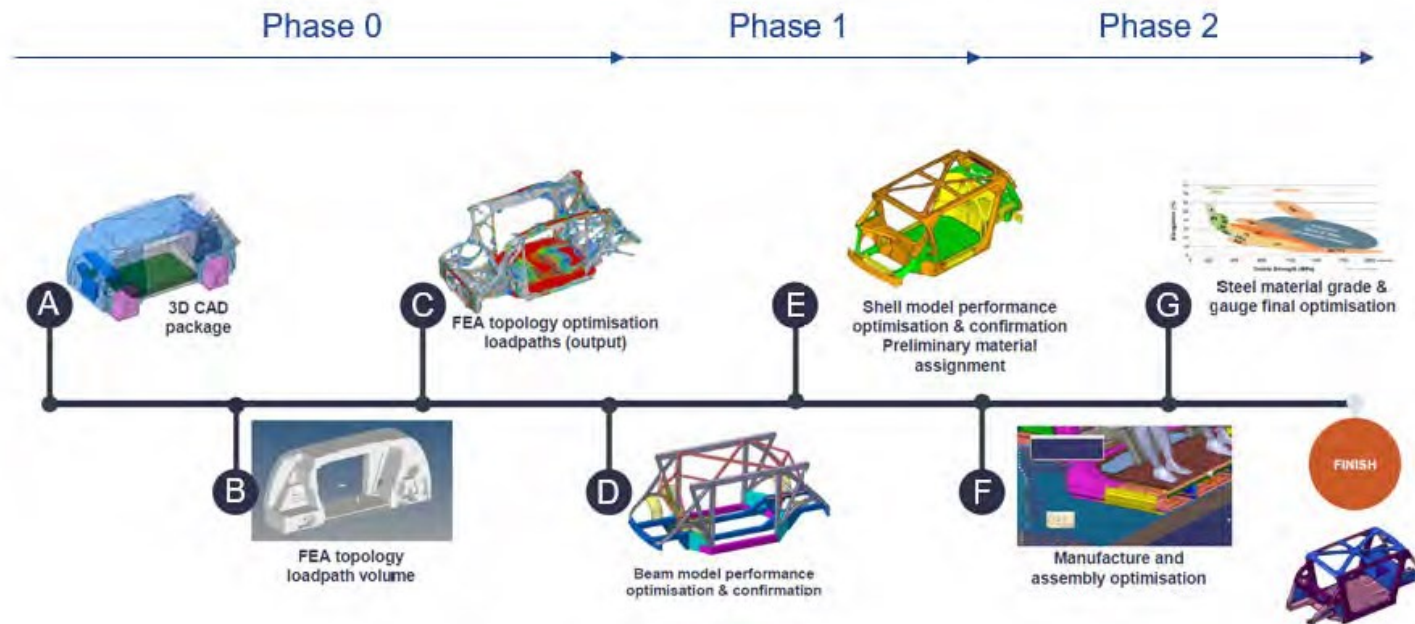


History of WorldAutoSteel Projects



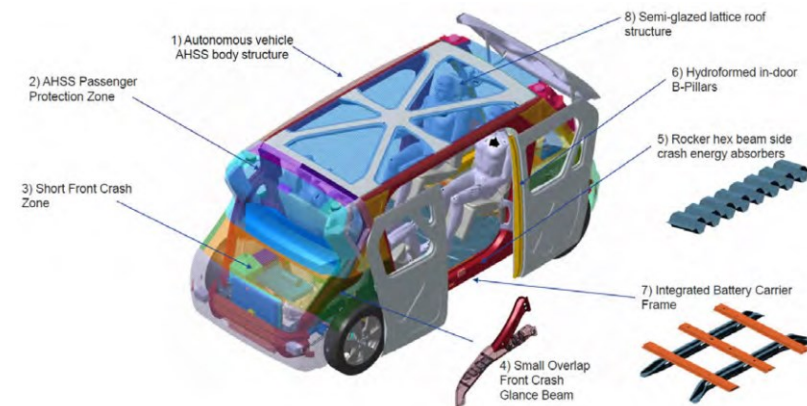
Engineering Approach

- advanced CAD and CAE tools, coupled with the latest Advanced High-Strength Steel grades to produce an innovative vehicle architecture that meets the future Mobility as a Service needs



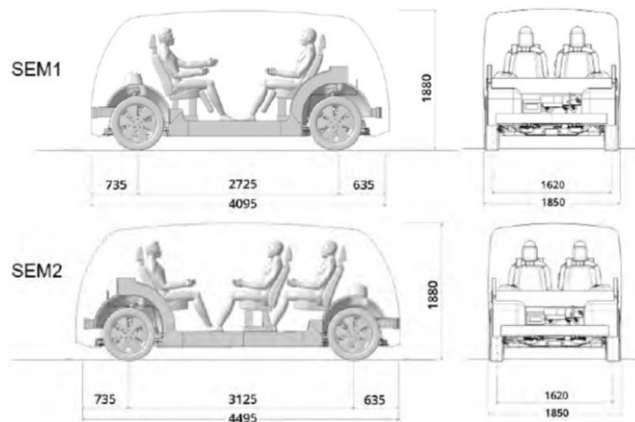
Key Innovations

- Autonomous vehicle AHSS body structure
 - front occupants are positioned rear facing towards the front of the vehicle
 - B pillars are housed within the door structure
 - passenger side doors have a “scissor” type motion
 - seating position and door configuration enable a more open, spacious feeling interior
 - competitive weight and stiffness performance
- AHSS Passenger Protection Zone
- Short Front Crash Zone structure
- Small Overlap Front Crash Glance Beam
- (Rocker) Hex beam energy absorbers
- Hydroformed in-door B-Pillar
- Integrated Battery Carrier Frame
- Semi-glazed lattice roof structure



Technical Approach, Phase 0

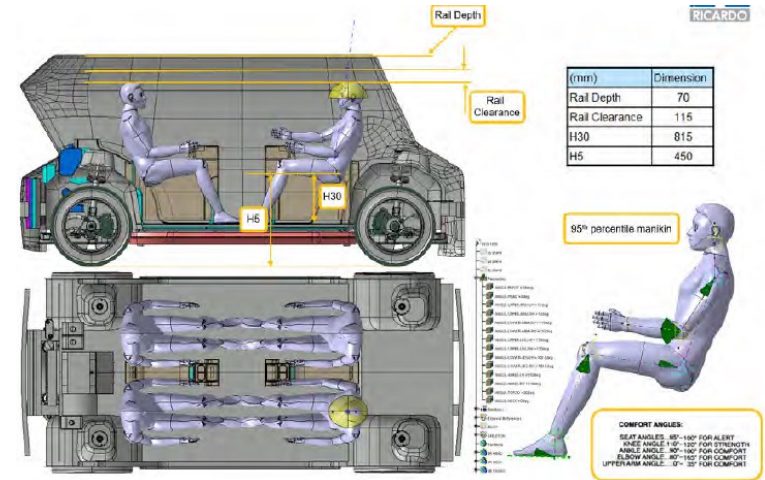
- Powertrain sizing and specification study
- Chassis system selection and sizing
- Vehicle package study
- Topology Loadpath Optimisation Simulation
- Competitor Vehicle Benchmarking
- Exterior Styling Study
- Vehicle curb weight verification and system weight budget allocation



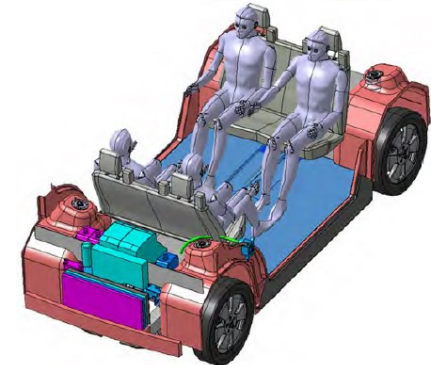
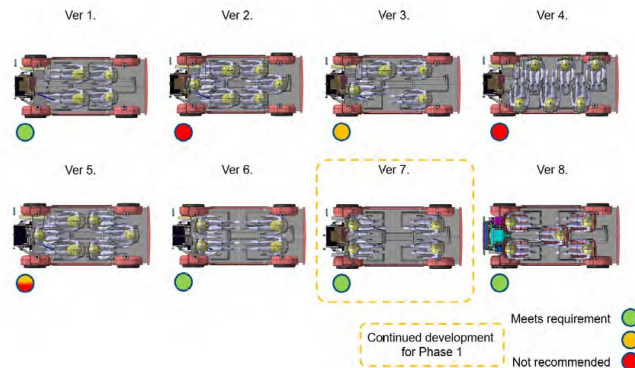
	SEM1	SEM2
Number of passengers	4	6
Battery (kWh)	75	96
Motor	front	front/rear
Curb weight (kg)	1512	1873
Gross weight (kg)	2012	2548

Occupant seating position/configuration

		Ver. 1	Ver. 2	Ver. 3	Ver. 4	Ver. 5	Ver. 6	Ver. 7	Ver. 8
M S C W	Meets requirement								
	Not recommended								
M	Comfort								
M	Safety								
S	Ingress/Egress								
C	Socializing space								
C	Work space								
C	Level 4 compatible								
M	Passengers number	4	6	4	6	6	4	4	6
Conclusion for Steel E-Motive application									

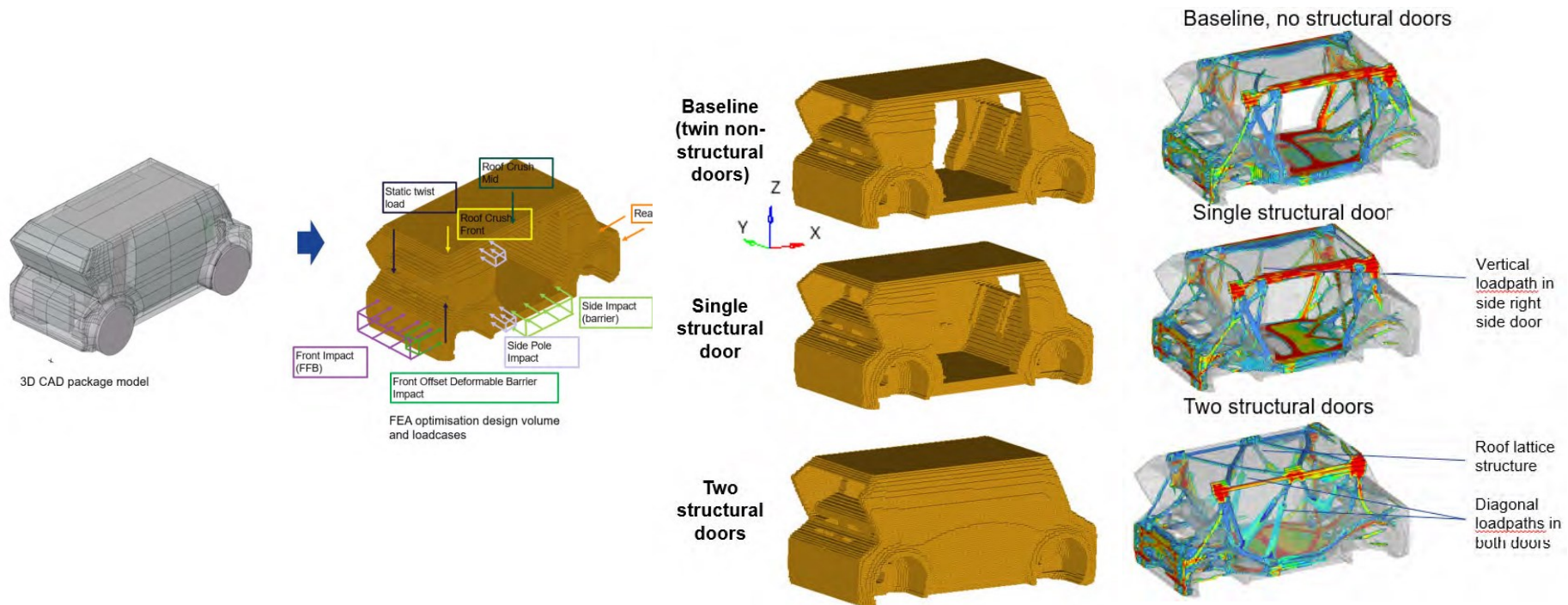


M S C W	M	Must have – Non negotiable need or requirement
	S	Should have – Important, not vital but add significant value
	C	Could have – Nice to have initiatives that have a small impact if left out
	W	Will not have – Not a priority at this point in time

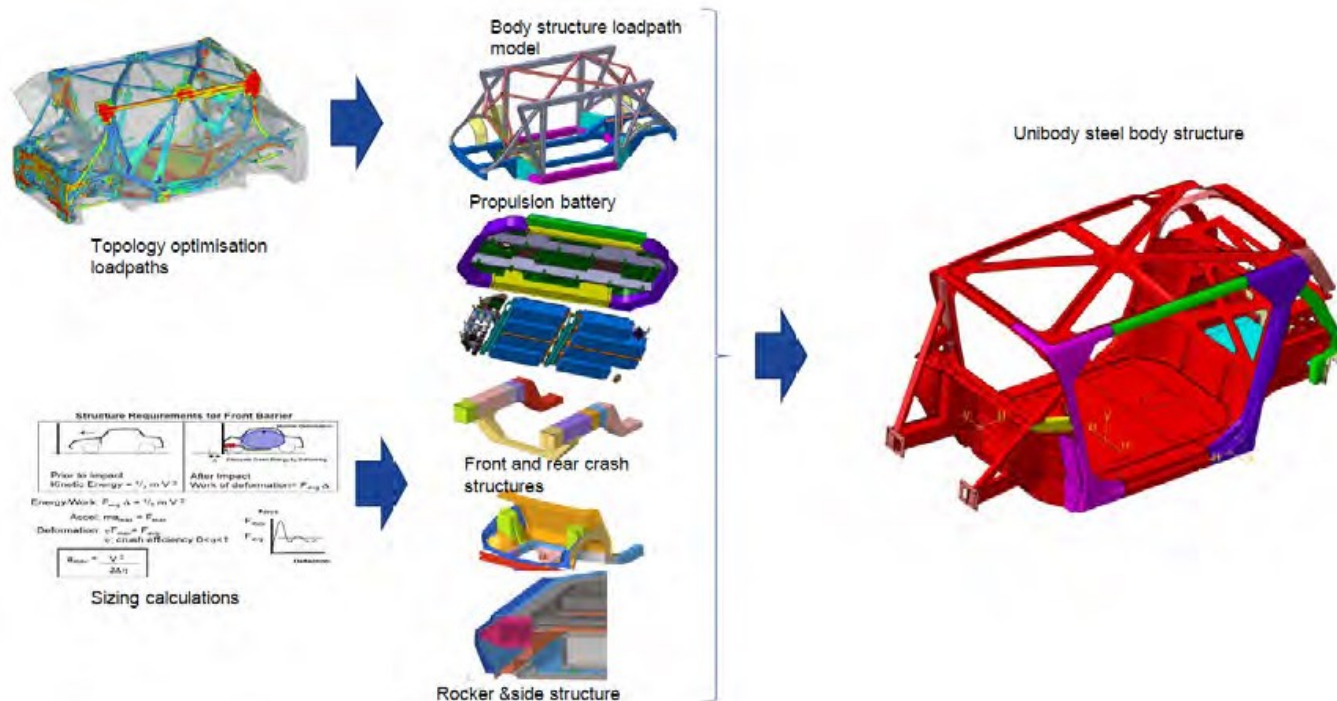
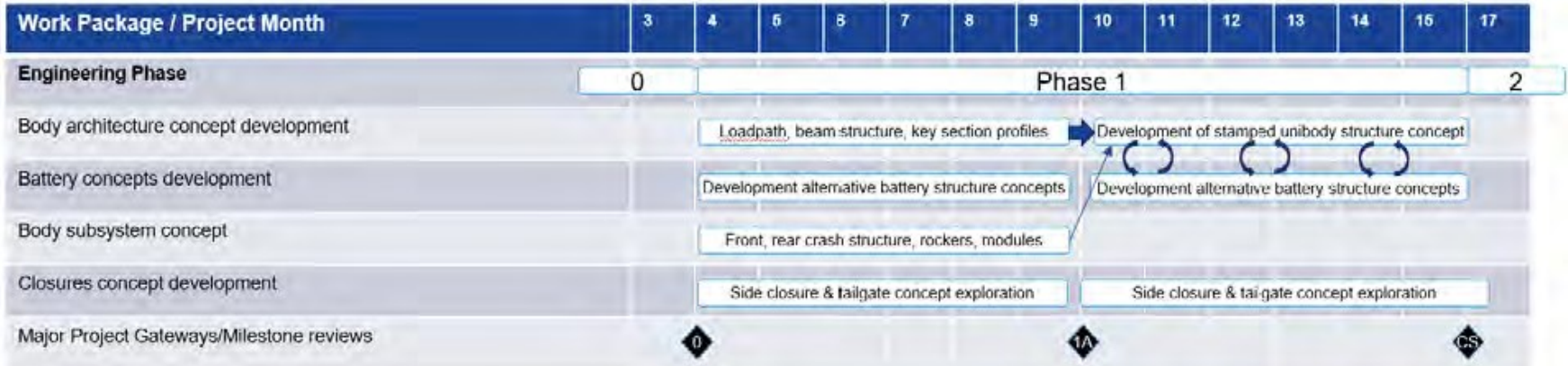


Topology Loadpath Optimisation Simulation

- define the major structural loadpaths in the body given a set of loads that represent the typical conditions that the vehicle may experience during its lifetime, including crashworthiness loads

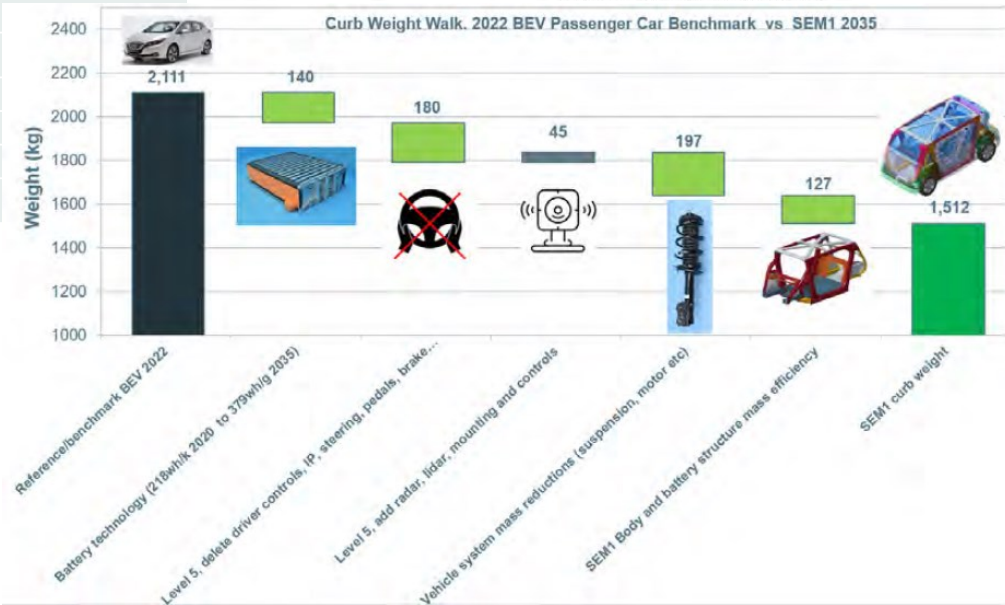
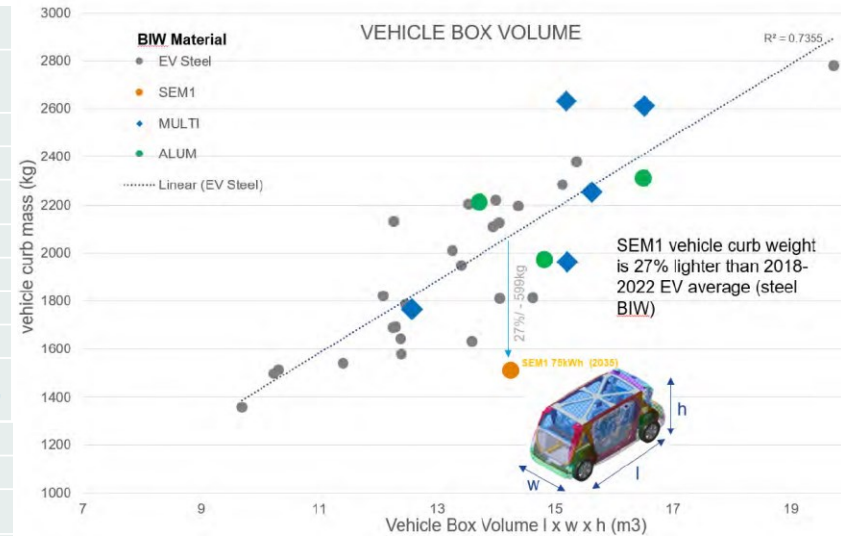


Technical Approach, Phase 1



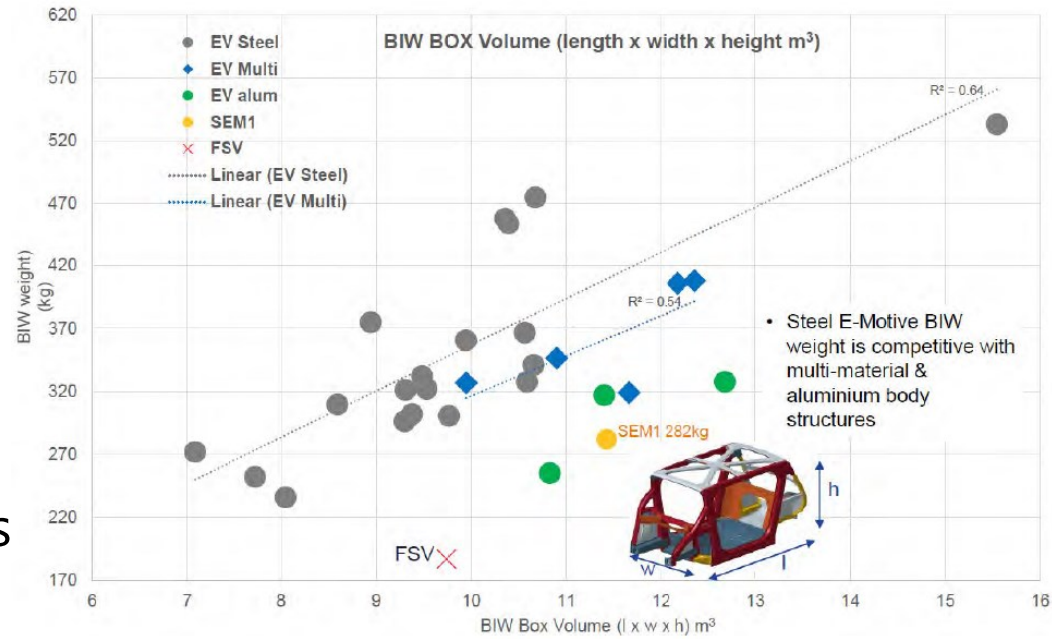
Vehicle Weight

Subsystem	SEM 1 Status CD	Source/estimation
Body non-structure	177.2	Estimated from a2mac1 statistical analysis. Includes seating, trim panels, glazing, front end clip, exterior panels
Body Structure	282.0	<CD> design value from 3D CAD
Front sub-frame	16.5	<CD> design value from 3D CAD
Battery case (structure)	59.0	<CD> design value from 3D CAD
Rear sub-frame	10.2	<CD> design value from 3D CAD
Front suspension	114.0	<CD> design value from 3D CAD
Rear suspension	114.0	<CD> design value from 3D CAD
Braking	59.7	Estimated from a2mac1 statistical analysis
Steering	26.0	Estimated from a2mac1 statistical analysis (steering rack and tie rods only, no wheel column, EPAS etc)
Tires and wheels	84.0	Estimated from a2mac1 statistical analysis
Motor Trans Front	63.0	Estimated from a2mac1 statistical analysis and Fiat 500e
Battery system less case	245.6	<CD> Ref BOM including 5kg of fluids, busbars, cooling plate, modules and PDU
Propulsion controls	60.0	Estimated from a2mac1 statistical analysis
Electrical-non propulsion	35.7	Estimated from a2mac1 statistical analysis
Cooling and heating	30.0	Estimated from a2mac1 statistical analysis
Closures	109.3	<CD> design value from 3D CAD
Bumpers	26.0	Estimated from a2mac1 statistical analysis
Vehicle curb weight	1512.2	kg



Features for Weight, Stiffness and NVH performance

- BIW weight
- Body Structure Stiffness
- Gauge thickness for weight, stiffness and strength
- BIW Joining Methods
- Lightweight design coefficient
- Static vertical bending stiffness
- NVH performance



Loadcase/requirement	Target value	SEM1 performance
Body in white weight	<309.3kg	282kg
Static torsional stiffness	>25,000 Nm/deg	63,285 Nm/deg
Static vertical bending stiffness	>9,000 N/mm	13,438 N/mm
Trimmed BIW first mode	>28Hz	32Hz
Trimmed BIW first battery mode	>35Hz	35Hz
Trimmed BIW local dynamic attachment stiffness	>5 x equivalent dynamic bushing stiffness	5 x dynamic bushing stiffness achieved

정리: 위상최적설계

- 구조물 설계에 있어서 CAE 주도 설계: 위상최적설계가 최적격이자 필수
- 초기설계 없이도 구조물 최적설계 가능
 - 개념설계단계에서 효과적
 - 선행(Up-Front) CAE에 적합
 - 구조물 경량화에 큰 공헌: 혁신설계
- 설계자에게 사용 가능한 도구로 발전하기 위한 주안점

