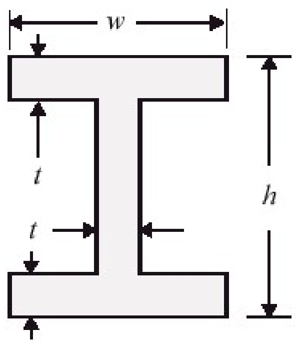
1. Cross-sectional area of an I-beam shown below is to be designed with the objectives of minimizing the cross-sectional area and minimizing the normal stresses resulting from unit bending moment about its horizontal neutral axis. The thickness of the flange and the web of the cross-section are fixed at $t = 0.1$ in. Choosing the design variables to be the width w and height h of the cross-section, determine graphically the designs which minimize the individual objectives. The width and the height are constrained to remain in the range $0.1 \le (w, h) \le 10$ in. (25 pts)



$\sigma = My/I, I = wh^3/12$ (for rectangular beam)

$I_{\text{paralell axis}} = I_{\text{centroidal axis}} + md^2$

m : mass of the body

d : perpendicular distance between the centroidal axis and the parallel axis

2. The equilibrium equations of the stepped bar shown in the figure are given by $\boldsymbol{KY} = \boldsymbol{P}$ with

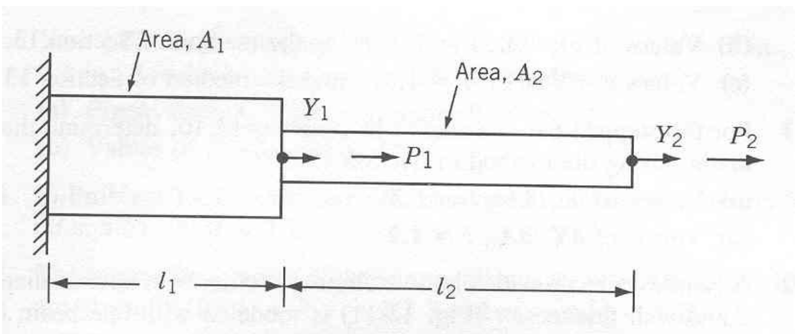
$$\boldsymbol{K} = \begin{bmatrix} \frac{A_1E_1}{l_1} + \frac{A_2E_2}{l_2} & -\frac{A_2E_2}{l_2} \\ -\frac{A_2E_2}{l_2} & \frac{A_2E_2}{l_2} \end{bmatrix}, \quad \boldsymbol{Y} = \begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix}, \quad \boldsymbol{P} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}$$

If $A_1 = 2\,\text{in}^2$, $A_2 = 1\,\text{in}^2$, $E_1 = E_2 = 30 \times 10^6$ psi, $2l_1 = l_2 = 50$ in, $P_1 = 100$ lb, and $P_2 = 200$ lb, determine: (25 pts)

(1) displacements, $\boldsymbol{Y}$

(2) values of $\partial\boldsymbol{Y}/\partial A_1$ and $\partial\boldsymbol{Y}/\partial A_2$

(3) values of $\partial\boldsymbol{\sigma}/\partial A_1$ and $\partial\boldsymbol{\sigma}/\partial A_2$ where $\boldsymbol{\sigma} = \{\sigma_1 \quad \sigma_1\}^T$ denotes the vector of stresses in the bar and $\sigma_1 = E_1Y_1/l_1$ and $\sigma_2 = E_2(Y_2 - Y_1)/l_2$



3. The shortest curve connecting two points is a straight line. Suppose we cannot go in a straight line because of a constraint. When the constraint is $\int u(x)dx = A$, find the shortest curve $u(x)$ between $u(0)=a$ and $u(1)=b$ that has area A below it.

[Euler-Lagrange equation: $\frac{\partial F}{\partial u} - \frac{d}{dx}\left(\frac{\partial F}{\partial u'}\right) = 0$] (25 pts)

4. A topology optimization problem based on the power law approach, where the objective is to minimize compliance can be written as

$$\left.\begin{aligned} &\min_{\boldsymbol{x}} : c(\boldsymbol{x}) = \boldsymbol{U}^T \boldsymbol{K} \boldsymbol{U} = \sum_{e=1}^{N} (x_e)^p u_e^T k_0 u_e \\ &\text{subject to: } \frac{V(\boldsymbol{x})}{V_0} = f \\ &\qquad : \boldsymbol{K}\boldsymbol{U} = \boldsymbol{F} \\ &\qquad : 0 \le x_{\min} \le x \le 1 \end{aligned}\right\}$$

where $\boldsymbol{U}$ and $\boldsymbol{F}$ are the global displacement and force vectors, respectively, $\boldsymbol{K}$ is the global stiffness matrix, u_e and k_e are the element displacement vector and stiffness matrix, respectively, $\boldsymbol{x}$ is the vector of design variables, $x_{\min}$ is a vector of minimum relative densities (non-zero to avoid singularity), N (= nelx×nely) is the number of elements used to discretize the design domain, p is the penalization power (typically p = 3), $V(\boldsymbol{x})$ and V_0 is the material volume and design domain volume, respectively and f (volfrac) is the prescribed volume fraction. Derive the updating scheme for the design variables by solving the optimization problem using a standard Optimality Criteria (OC) methods. (25 pts)