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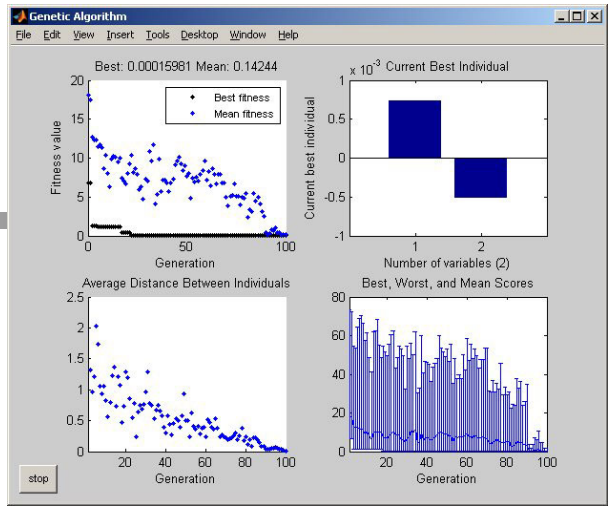

Genetic Algorithm and Direct Search Toolbox

V2.3 (R2008a)

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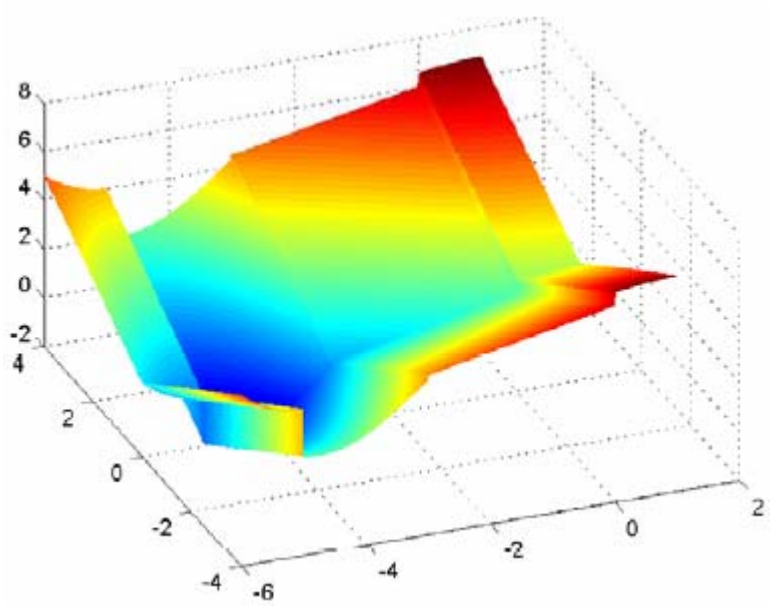
Direct Search

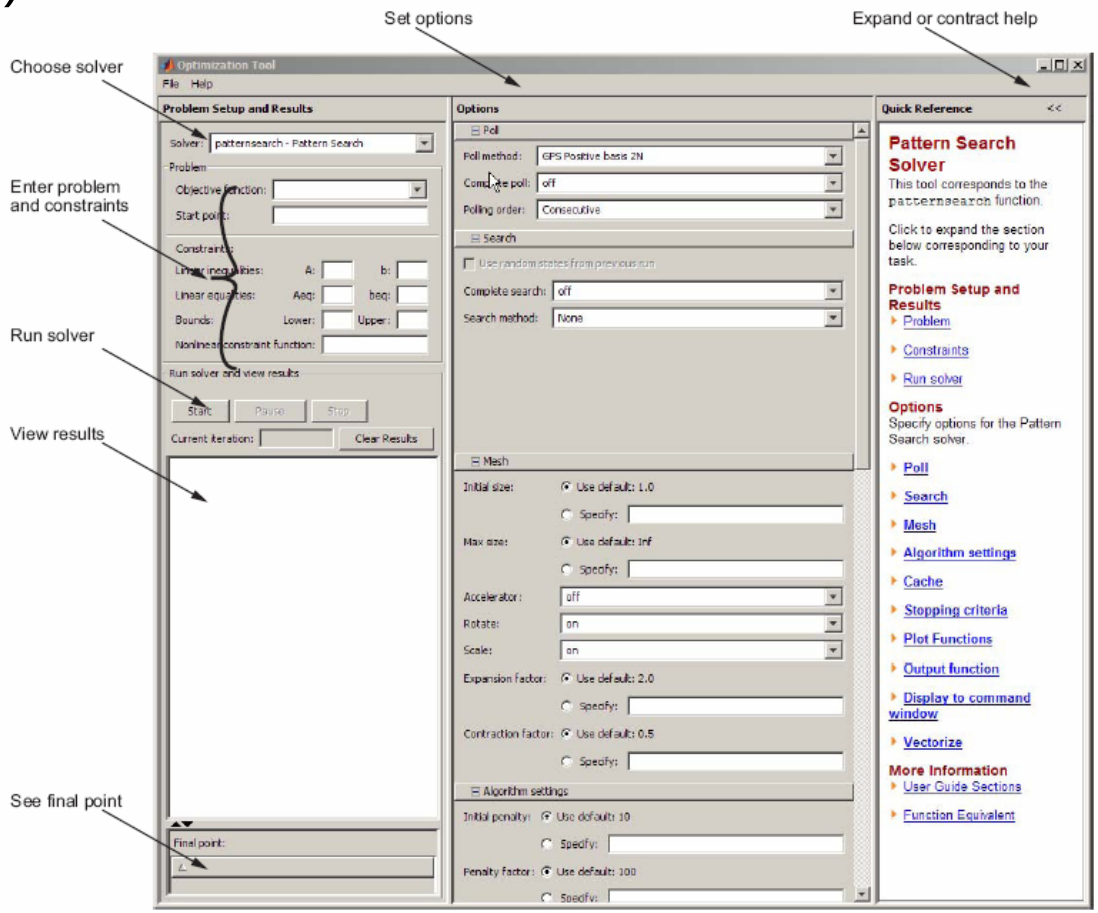
- method for solving optimization problems that does not require any information about the gradient of the objective function
- searches a set of points around the current point, looking for one where the value of the objective function is lower than the value at the current point
- solves problems for which the objective function is not differentiable, stochastic, or even continuous

- generalized pattern search (GPS) algorithm
- mesh adaptive search (MADS) algorithm

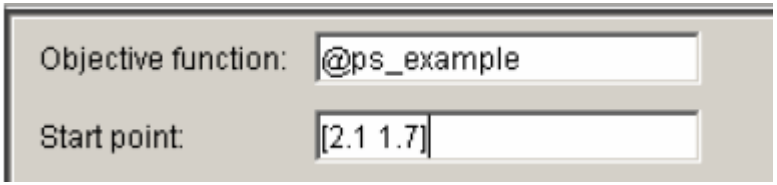
DS: Function Call

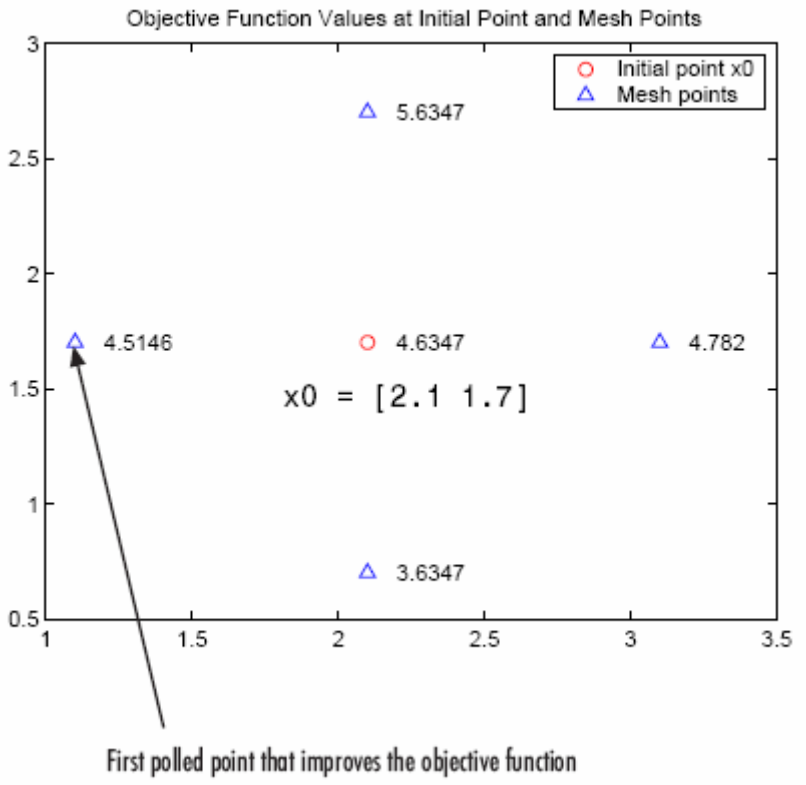
- [x fval exitflag output] =patternsearch(@objfun, x0, A, b, Aeq, beq, lb, ub, nonlcon, options)
- optimtool('patternsearch')





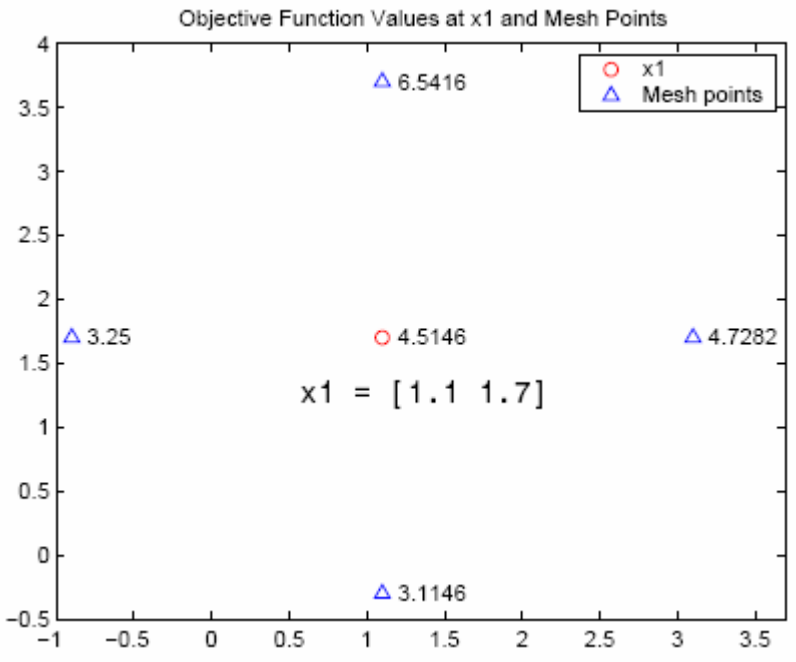
How Pattern Search Works






```
[1 0] + x0 = [3.1 1.7]
[0 1] + x0 = [2.1 2.7]
[-1 0] + x0 = [1.1 1.7]
[0 -1] + x0 = [2.1 0.7]
```

(GPS Positive basis 2N)

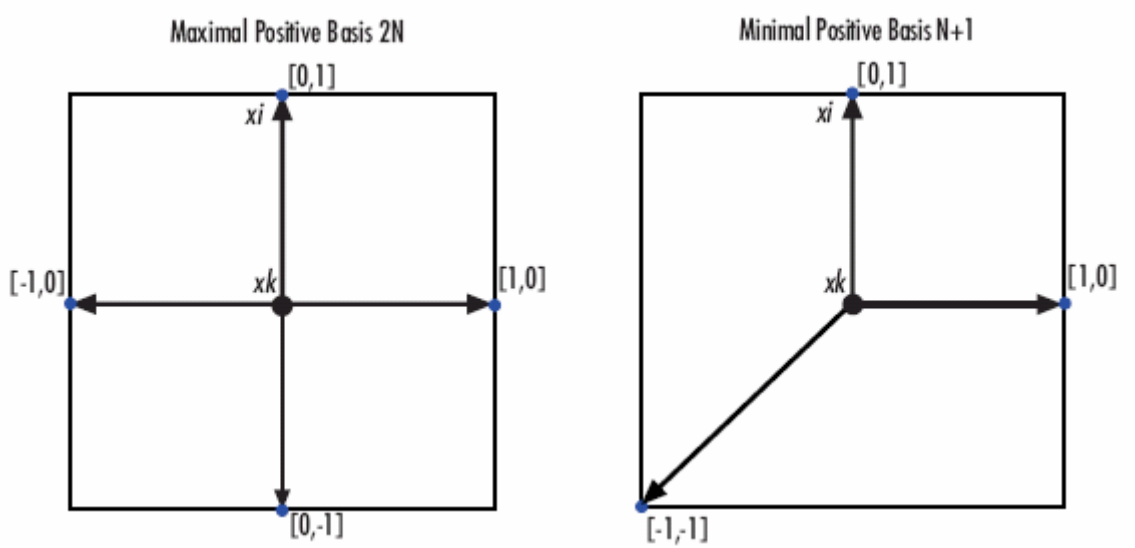



```
2*[1 0] + x1 = [3.1 1.7]
2*[0 1] + x1 = [1.1 3.7]
2*[-1 0] + x1 = [-0.9 1.7]
2*[0 -1] + x1 = [1.1 -0.3]
```

x2 = [-0.9 1.7]

DS: Terminology

- Pattern: direction



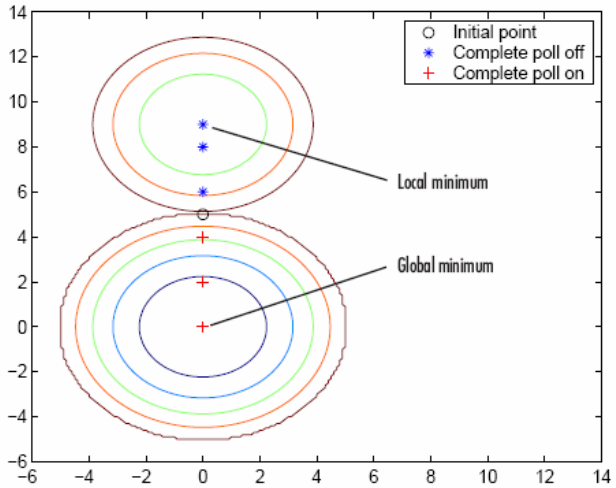
- Mesh: step size (expansion/contraction factor)
- Polling: finds a point whose objective function value is less than that of the current point

DS: Options (1)

– options = psoptimset(@patternsearch)

```
options =
               TolMesh: 1.0000e-006
                TolCon: 1.0000e-006
                  TolX: 1.0000e-006
                TolFun: 1.0000e-006
               TolBind: 1.0000e-003
               MaxIter: '100*numberofvariables'
           MaxFunEvals: '2000*numberofvariables'
             TimeLimit: Inf
       MeshContraction: 0.5000
         MeshExpansion: 2
       MeshAccelerator: 'off'
            MeshRotate: 'on'
       InitialMeshSize: 1
             ScaleMesh: 'on'
           MaxMeshSize: Inf
        InitialPenalty: 10
         PenaltyFactor: 100
            PollMethod: 'gpspositivebasis2n'
          CompletePoll: 'off'
          PollingOrder: 'consecutive'
          SearchMethod: []
        CompleteSearch: 'off'
               Display: 'final'
           OutputFcns: []
             PlotFcns: []
          PlotInterval: 1
                 Cache: 'off'
             CacheSize: 10000
              CacheTol: 2.2204e-016
            Vectorized: 'off'
           UseParallel: 'never'
```

DS: Options (2)



- Poll Method
  - GPS/MADS + Positive basis 2N/Np1
- Complete Poll: Off/On
- Using a Search Method
- Mesh Expansion and Contraction
- Mesh Accelerator
  - When the mesh size is below a certain value
- Using Cache
  - Store a history, eliminate redundant computations
- Setting Tolerances for the Solver
  - Mesh, X, function, nonlinear constraint, bind
- Constrained Minimization Using pattern search

Genetic Algorithm

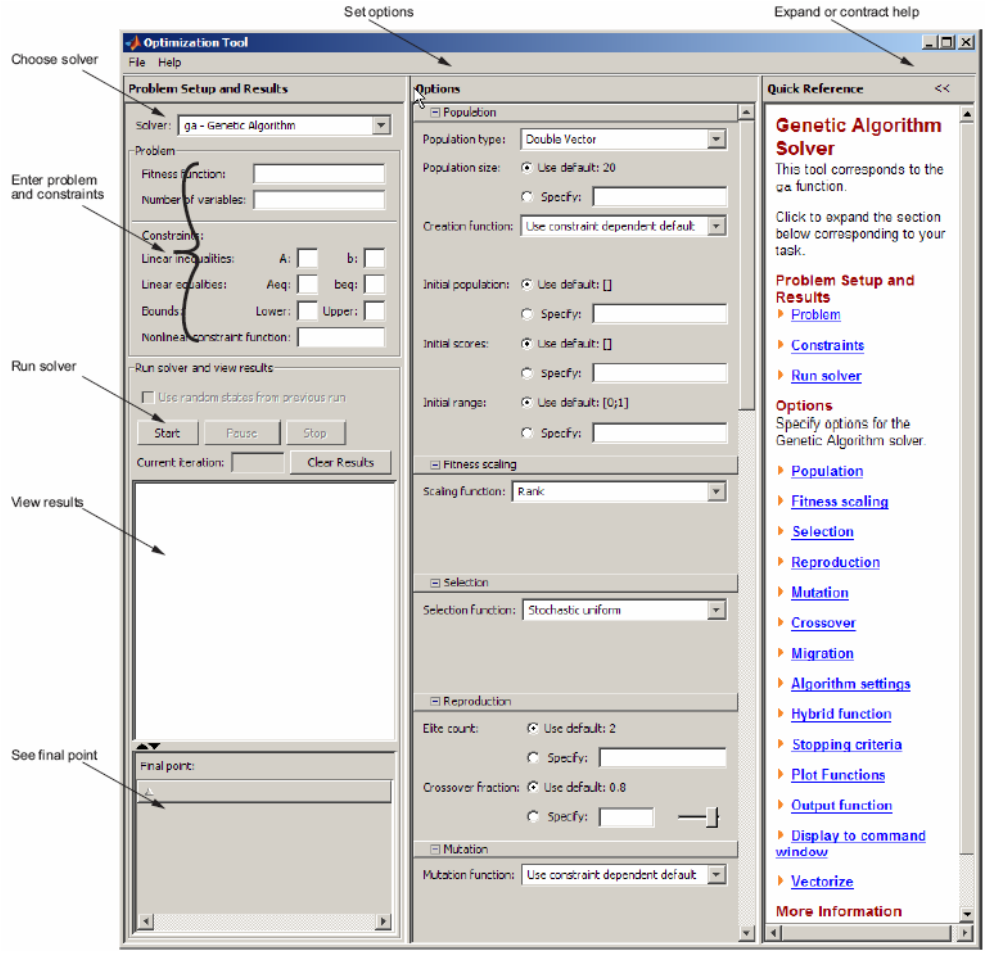
- method for solving both constrained and unconstrained optimization problems that is based on natural selection, the process that drives biological evolution
- solves a variety of optimization problems that are not well suited for standard optimization algorithms, including problems in which the objective function is discontinuous, nondifferentiable, stochastic, or highly nonlinear

Classical Algorithm	**Genetic Algorithm**
Generates a single point at each iteration. The sequence of points approaches an optimal solution.	Generates a population of points at each iteration. The best point in the population approaches an optimal solution.
Selects the next point in the sequence by a deterministic computation.	Selects the next population by computation which uses random number generators.

GA: Function Call

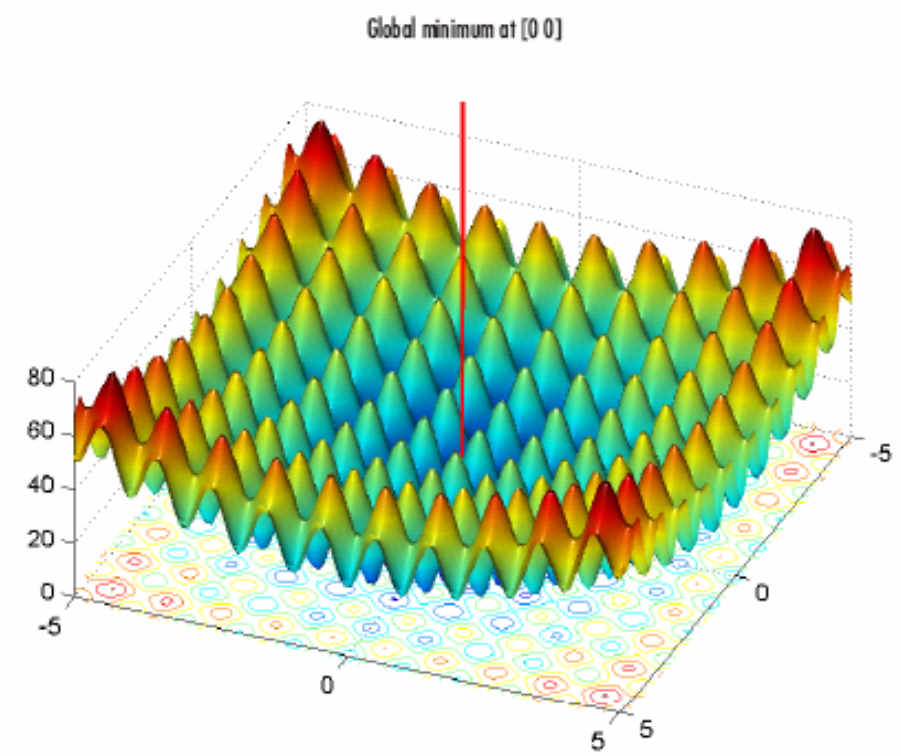
– [x fval exitflag output population scores] = ga(@fitnessfun, nvars, A, b, Aeq, beq, lb, ub, nonlcon, options)

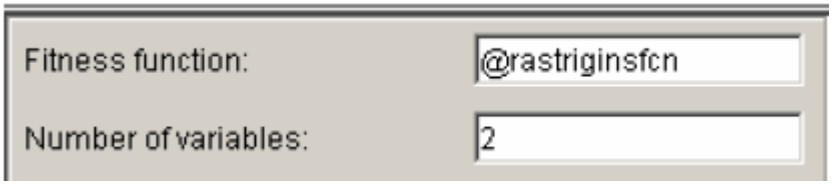
– optimtool('ga')

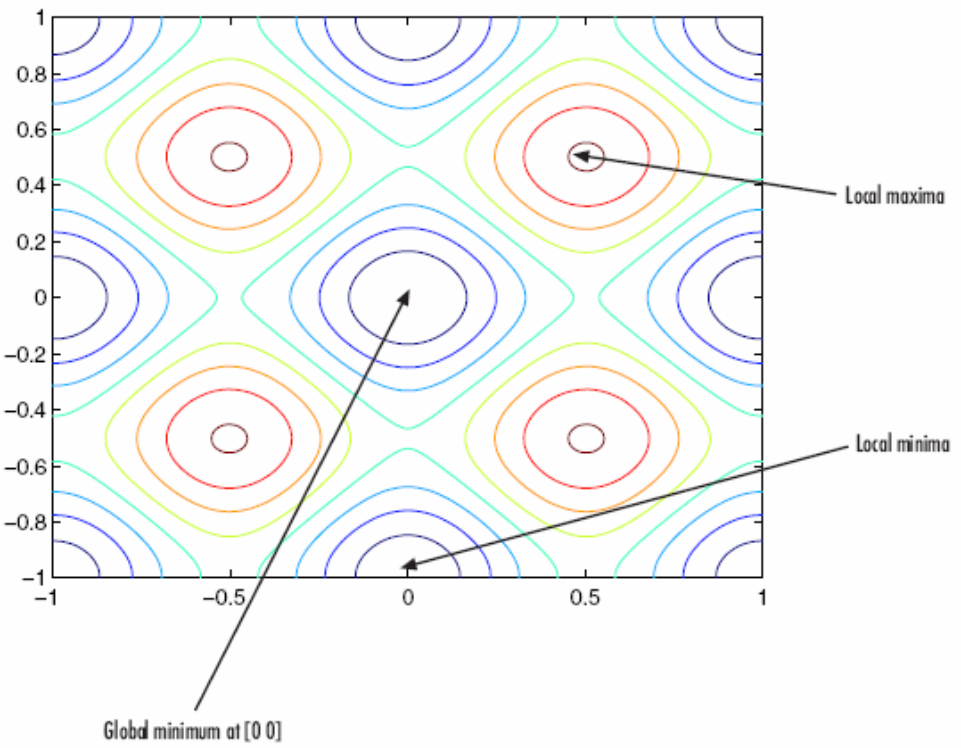


Rastrigin's Function

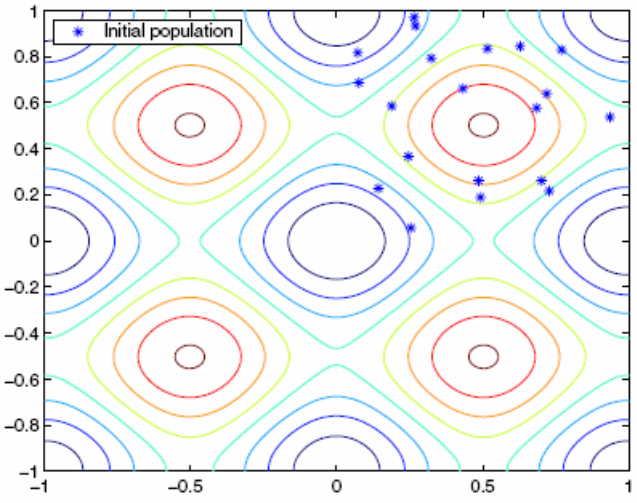
$$Ras(\boldsymbol{x}) = 20 + x_1^{\,2} + x_2^{\,2} - 10\left(\cos 2\pi x_1 + \cos 2\pi x_2\right)$$

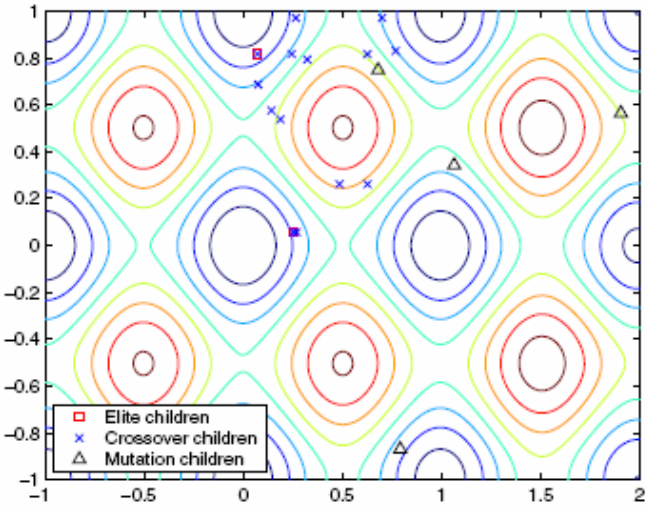


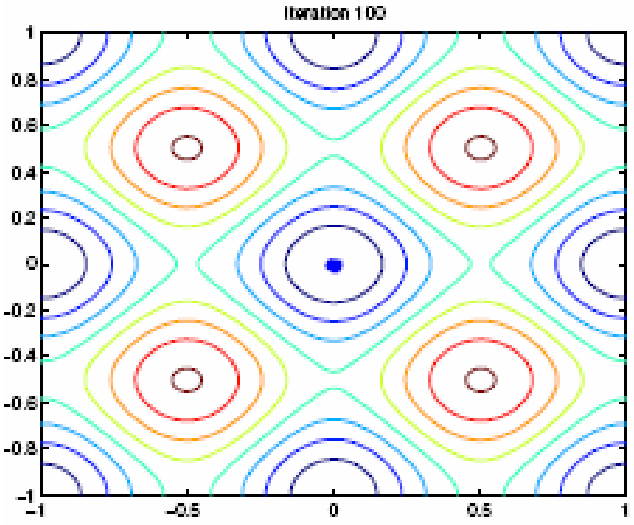


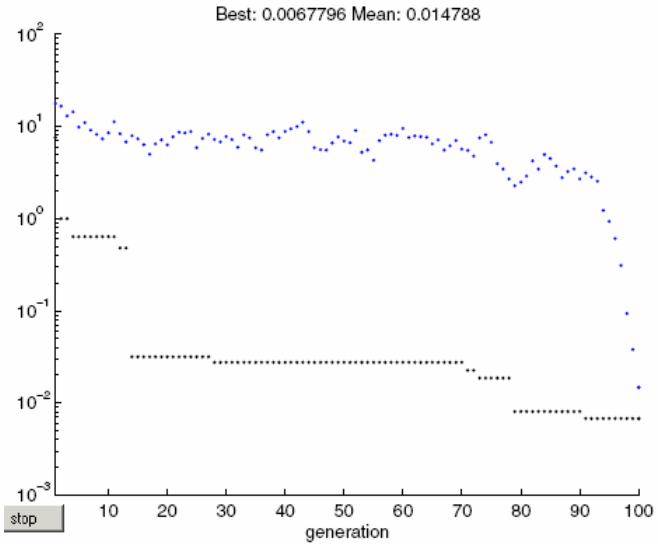


Iteration History



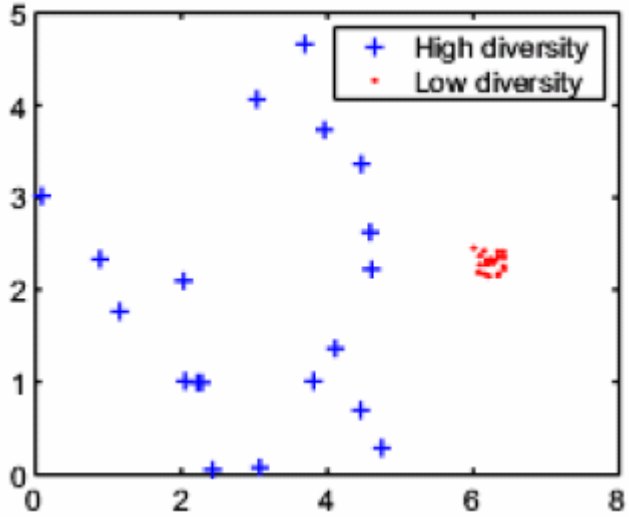






GA: Terminology

- Fitness function: objective function
- Individual: genome (genes)
  - any point to which you can apply the fitness function
- Population: an array of individuals
- Generation: Each successive population
- Diversity
  - average distance between individuals in a population
- Fitness value and best fitness
- Parent and children



GA: Options (1)

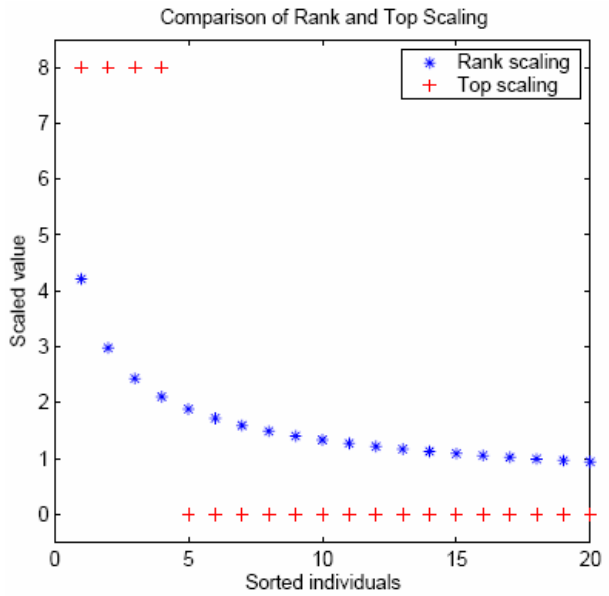
– options = gaoptimset(@ga)

```
options =
        PopulationType: 'doubleVector'
          PopInitRange: [2x1 double]
        PopulationSize: 20
           EliteCount: 2
      CrossoverFraction: 0.8000
         ParetoFraction: []
     MigrationDirection: 'forward'
      MigrationInterval: 20
      MigrationFraction: 0.2000
            Generations: 100
              TimeLimit: Inf
           FitnessLimit: -Inf
          StallGenLimit: 50
          StallTimeLimit: Inf
                 TolFun: 1.0000e-006
                 TolCon: 1.0000e-006
```

```
  InitialPopulation: []
      InitialScores: []
     InitialPenalty: 10
      PenaltyFactor: 100
       PlotInterval: 1
        CreationFcn: @gacreationuniform
  FitnessScalingFcn: @fitscalingrank
       SelectionFcn: @selectionstochunif
       CrossoverFcn: @crossoverscattered
        MutationFcn: {[1x1 function_handle]  [1]  [1]}
 DistanceMeasureFcn: []
          HybridFcn: []
            Display: 'final'
           PlotFcns: []
         OutputFcns: []
         Vectorized: 'off'
        UseParallel: 'never'
```

GA: Options (2)

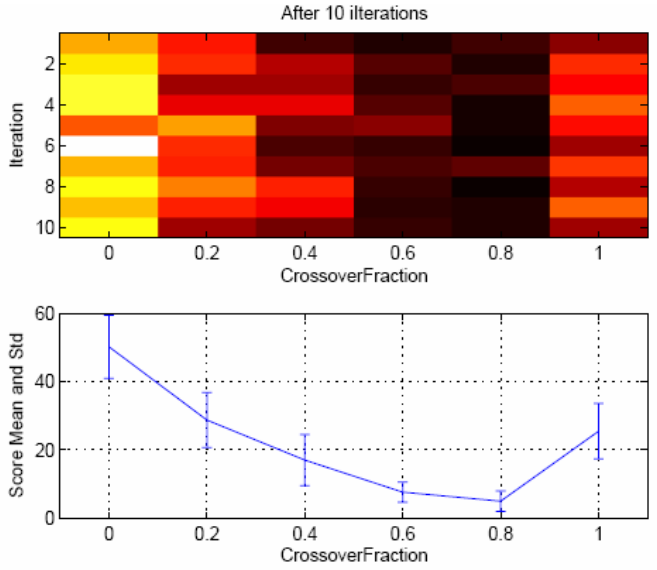
- Population Diversity
  - Setting the initial range / population size
- Fitness Scaling
  - Rank, Proportional, Top, Shift linear
- Selection
  - Stochastic uniform, Remainder, Uniform, Roulette, Tournament



GA: Options (3)

- Reproduction Options
  - Elite count, Crossover fraction
  - >>deterministicstudy
- Crossover
  - Scattered, Single point, Two point, Intermediate, Heuristic, Arithmetic

Population size	20	Children
Elite count	2	2
Crossover fraction	0.8	0.8*18→14
Mutation		4



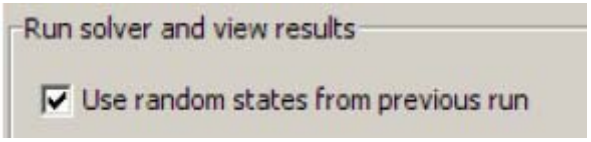
GA: Options (4)

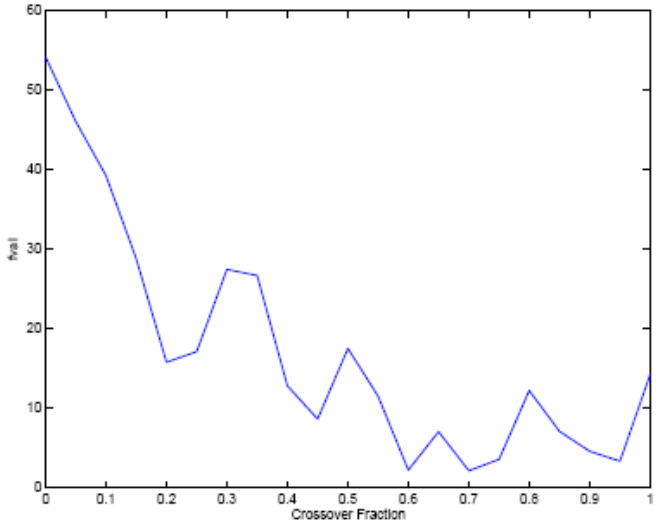
- Mutation
  - Mutation function: Gaussian, Uniform, Adaptive feasible
  - Scale: controls the standard deviation of the mutation
  - Shrink: controls the rate at which the average amount of mutation decreases
- Global vs. Local minima
  - Increase the initial range
- Using a Hybrid Function
  - optimization function that runs after the genetic algorithm terminates in order to improve the value of the fitness function
  - uses the final point from the genetic algorithm as its initial point
- Setting the Maximum Number of Generations
- Vectorizing the Fitness Function
  - 'Vectorized' On (compute the fitness for all individuals at once)
  - tic; ga(@fitnessfun, nvars); toc

Reproducing Results

– run the genetic algorithm with different settings for **Crossover fraction** to see which one gives the best results

```
options = gaoptimset('Generations',300);
rand('twister', 71); % These two commands are only included to
randn('state', 59); % make the results reproducible.
record=[];
for n=0:.05:1
 options = gaoptimset(options,'CrossoverFraction', n);
 [x fval]=ga(@rastriginsfcn, 10,[],[],[],[],[],[],[],options);
 record = [record; fval];
end
plot(0:.05:1, record);
xlabel('Crossover Fraction');
ylabel('fval')
```





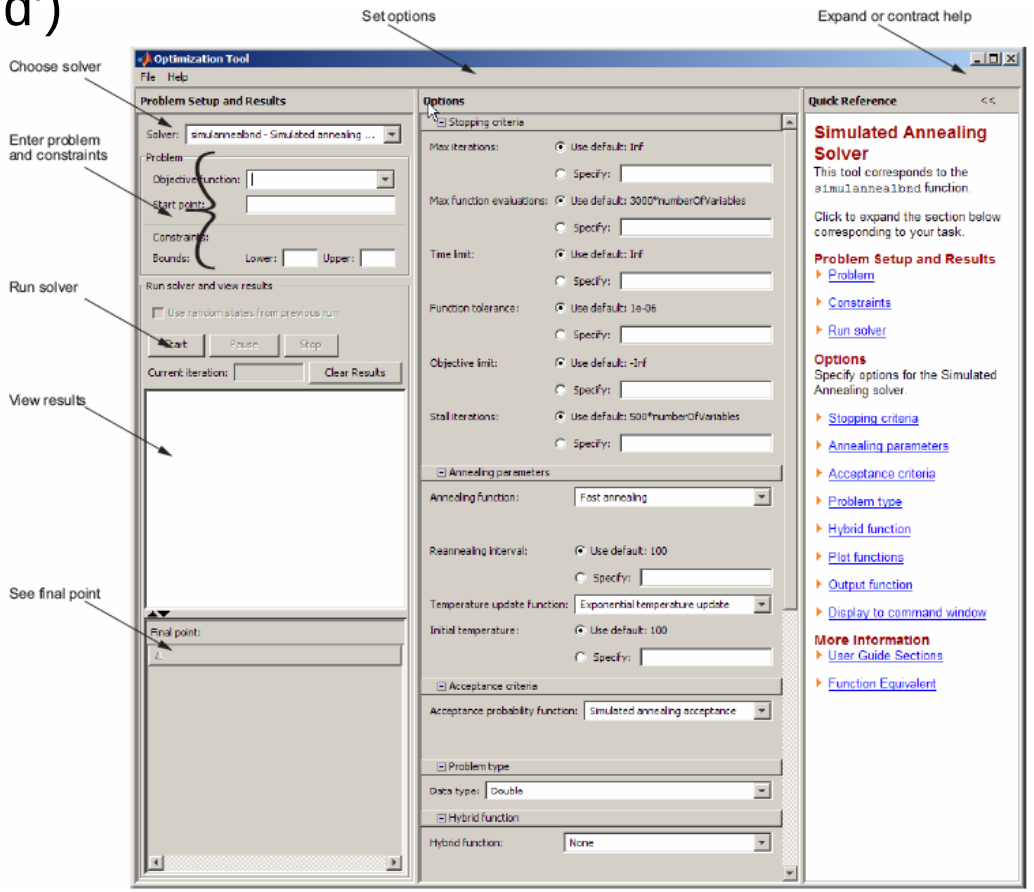
Simulated Annealing and Threshold Acceptance

- method for solving unconstrained and bound-constrained optimization problems
- The method models the physical process of heating a material and then slowly lowering the temperature to decrease defects, thus minimizing the system energy

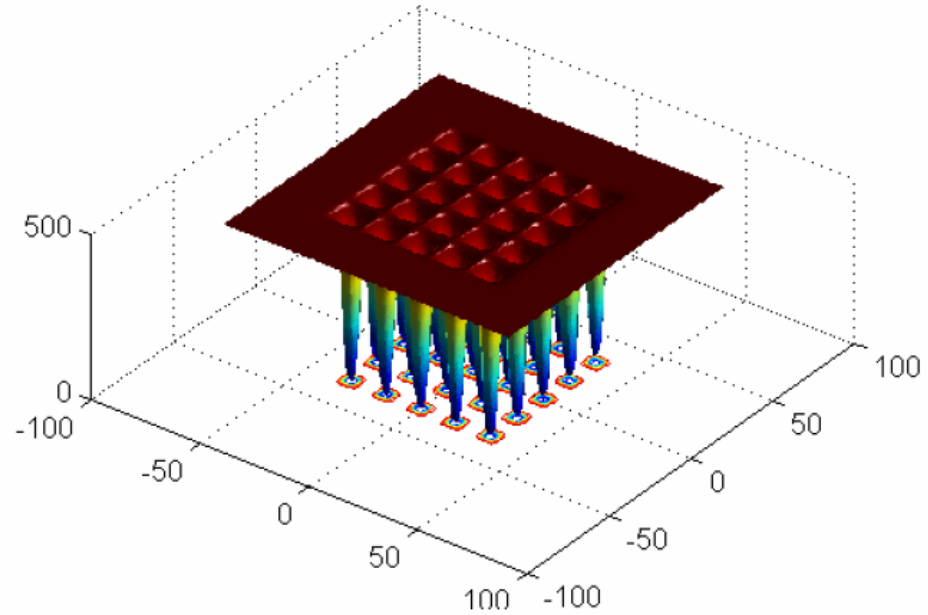
- Threshold acceptance
  - Instead of accepting new points that raise the objective with a certain probability, it accepts all new points below a fixed threshold (faster than SA)
  - The threshold is then systematically lowered, just as the temperature is lowered in an annealing schedule

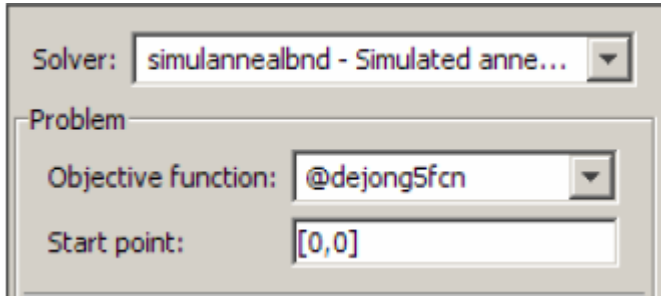
Function Call

- [x fval exitflag output] = simulannealbnd(@objfun, x0, lb, ub, options)
- [x, fval exitflag output] = threshacceptbnd(@objfun, x0, lb, ub, options)
- optimtool('simulannealbnd')



De Jong's Fifth Function






```
fun = @dejong5fcn;
[x fval] = simulannealbnd(fun, [0 0])
```

```
x =
   -31.9779  -31.9595
fval =
    0.9980
```

Terminology

- Objective function
- Temperature: `InitialTemperature,TemperatureFcn`
  - It determines the probability of accepting a worse solution at any step and is used to limit the extent of the search in a given dimension
- Annealing schedule: `TemperatureFcn`
  - The rate by which the temperature is decreased
  - The slower the rate of decrease, the better the chances are of finding an optimal solution, but the longer the run time
- Reannealing: `ReannealInterval`
  - Raises the temperature after a certain number of new points have been accepted, and starts the search again at the higher temperature
- Threshold acceptance
  - accepts a worse point if the objective function is raised by less than a fixed threshold

SA: Options

- options=saoptimset('simulannealbnd')
- options=saoptimset('threshacceptbnd')

```
options =
          AnnealingFcn: @annealingfast
       TemperatureFcn: @temperatureexp
        AcceptanceFcn: @acceptancesa
              TolFun: 1.0000e-006
       StallIterLimit: '500*numberofvariables'
          MaxFunEvals: '3000*numberofvariables'
            TimeLimit: Inf
              MaxIter: Inf
       ObjectiveLimit: -Inf
              Display: 'final'
      DisplayInterval: 10
            HybridFcn: []
       HybridInterval: 'end'
             PlotFcns: []
         PlotInterval: 1
           OutputFcns: []
    InitialTemperature: 100
      ReannealInterval: 100
              DataType: 'double'
```

Nonlinear Constraint Solver

- Augmented Lagrangian Pattern Search (ALPS) algorithm
- Augmented Lagrangian Genetic Algorithm (ALGA)

$$\begin{aligned} &\underset{x}{\text{Minimize}} \quad f(x) \\ &\text{such that} \\ &\quad C_i(x) \le 0, i = 1 \ldots m \\ &\quad C_i(x) = 0,\ i = m+1 \ldots mt \\ &\quad Ax \le b \\ &\quad A_{eq}x = beq \\ &\quad LB \le x \le UB \end{aligned}$$

$$\Theta(x,\lambda,s,\rho) = f(x) - \sum_{i=1}^{m} \lambda_i s_i \log(s_i - c_i(x)) + \sum_{i=m+1}^{mt} \lambda_i c_i(x) + \frac{\rho}{2} \sum_{i=m+1}^{mt} c_i(x)^2$$