

타이어의 최적설계

(Optimum Design of Tire)

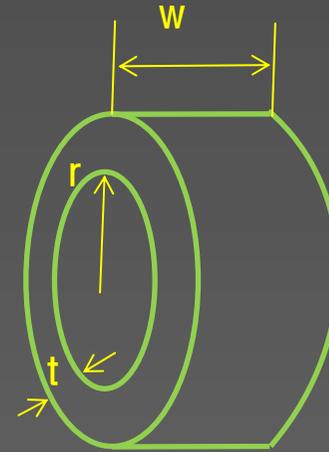
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● Project / Problem Statement



- Ferrari F50과 같은 슈퍼카 계열의 차량은 고출력 엔진을 장착하고 있으며, 그 성능을 최고로 이끌어낼 수 있는 방향으로 설계되고 제작된다. 차량과 노면이 직접 접촉하는 부분인 타이어의 사이징은 고성능 구현과 역동적인 드라이빙을 위한 중요한 설계 요소이다. 실제로는 복잡한 형상과 구조를 가진 타이어를 그림과 같은 solid cylinder로 단순화 시켜 모델링하였다. F50에 사용되는 타이어의 재료는 실리카 컴파운드($\rho = 1000 \text{ kg/m}^3$, $\mu = 0.8$)를 사용하며, 편평도 30 이상의 것을 선택한다. 탑승 인원을 포함한 차량의 총 무게 1640 kg, Final drive ratio(주행 기어비) 3.70:1 에서, 타이어는 엔진의 최대 토크 63.513 N·m, 차량의 최고 속도 325.1km/h 를 견딜 수 있어야 하며 (엔진과 차축, 차축과 휠, 휠과 타이어 간의 동력 전달에 마찰 손실이 없다고 가정한다), wheel housing 및 차량의 전체적 구조와 offset (브레이크 디스크와 휠의 차축 연결부 간의 거리)조건 등에 의해 타이어의 중량은 9kg 이상이 되어야 한다. 타이어의 부피가 최대가 되는 r 과 w , t 의 값은 얼마인가? 계산 결과를 F50 순정 타이어의 제원과 비교하여 Modeling과 formulation의 실효성 여부를 판단하라.

● Data and Information Collection

- Mass moment of inertia : $I = \frac{1}{2} \rho \pi \{ (r+t)^2 - r^2 \}^2 w$
- Density of tire : $\rho = 1000 \text{ kg/m}^3$
- Thickness of sidewall : t
- Friction coefficient between road and tire : $\mu = 0.8$
- Maximum torque : $T_{\max} = 63.513 \text{ N} \cdot \text{m}$
- Maximum speed : $v_{\max} = 90.305 \text{ m/s}$
- Angular velocity : $\omega = 240 \text{ rad/s}$
- Angular acceleration : $\alpha = \frac{a}{r+t}$
- Acceleration : $a = 0.513 \text{ m/s}^2$
- Final drive ratio : 3.70:1
- Mass of car : $m_{\text{car}} = 1640 \text{ kg}$
- Minimum Mass of tire : $m_{\text{tire min}} = 9 \text{ kg}$
- Minimum of oblateness : $O_{\min} = 30$

- **Definition of Design Variables**

- Inner radius of tire (radius of wheel) : r (m)
- Width of tire : w (m)
- **Thickness of sidewall** : t (m)

- **Definition of Cost Function**

- Maximize the volume of tire

$$V = \pi \{ (r + t)^2 - r^2 \} w \text{ (m}^3\text{)}$$

● Identification of Constraints

– Torque : $g_1 = I\alpha - \frac{\mu m_{car} g}{4} - T_{max} \leq 0$

– Oblateness : $g_2 = \frac{t}{w} \times 100 - O_{min} \geq 0$

– Velocity : $g_3 = \omega(r+t) - v_{max} \leq 0$

– Mass : $g_4 = \rho V - m_{tire_{min}} \geq 0$

– Dimension : $g_5 = -r < 0$

$$g_6 = -w < 0$$

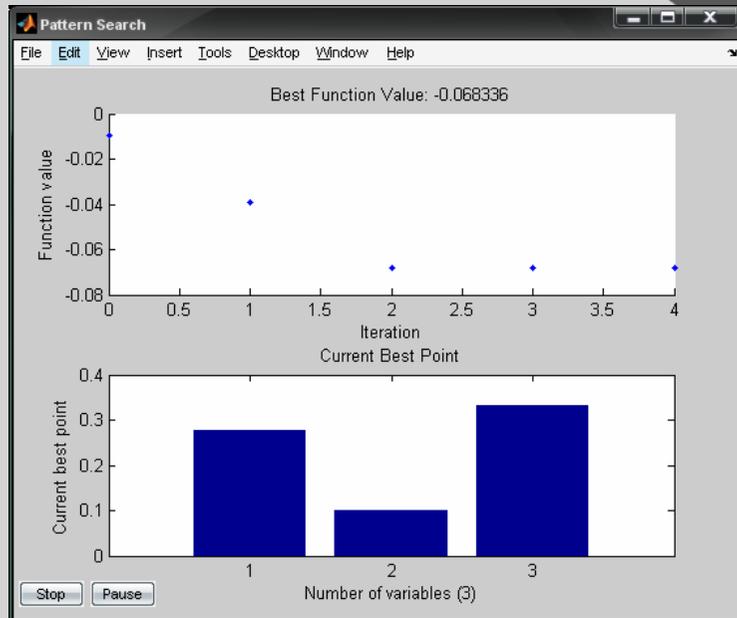
$$g_7 = -t < 0$$

● Solution by MATLAB (pattern search)

초기값 [0.1;0.1;0.1]

→ Optimum : $r = 0.276$ m, $t = 0.1$ m, $w = 0.333$ m *😊*

maximum $V = 0.068$ m³



The screenshot shows the MATLAB Pattern Search Tool interface. The 'Problem Setup and Results' section includes the following fields:

- Objective function: @fun
- Start point: [0.1;0.1;0.1]
- Constraints: Linear inequalities (A, b), Linear equalities (Aeq, beq), Bounds (Lower, Upper), Nonlinear constraint function: @nonlcon
- Plots: Plot interval: 1, Best function value, Best point, Custom function
- Run solver: Start, Pause, Stop buttons, Current iteration: 4, Clear Status button
- Status and results: Pattern search running, Pattern search terminated, Objective function value: -0.0683363551203886, Optimization terminated: mesh size less than options.TolN and constraint violation is less than options.TolCon.
- Final point table:

Index	Value
1	0.27627
2	0.1
3	0.33334

The 'Options' section includes Poll, Poll method (GPS Positive basis 2N), Complete poll (Off), Polling order (Consecutive), Search, Mesh, Algorithm settings, Cache, Stopping criteria, Output function, Display to command window, and Vectorize. The 'Quick Reference' section provides information about the Pattern Search Solver, Objective function, Start point, Constraints, Plot Functions, Run solver, Options, Poll, Search, Mesh, Algorithm settings, Cache, Stopping criteria, Output function, Display to command window, and Vectorize.

● Solution by MATLAB (pattern search)

초기값 [0.1;0.01;0.1]

→ Optimum : $r = 0.255$ m, $t = 0.120$ m, $w = 0.402$ m *☺*
maximum $V = 0.096$ m³

초기값 [0.01;0.01;0.01]

→ Optimum : $r = 0.367$ m, $t = 0.00885$ m, $w = 0.0295$ m *☹*
maximum $V = 6.098 \times 10^{-4}$ m³

초기값 [3;2;2]

→ Optimum : $r = 0.248$ m, $t = 0.014$ m, $w = 0.0467$ m *☹*
maximum $V = 0.00105$ m³

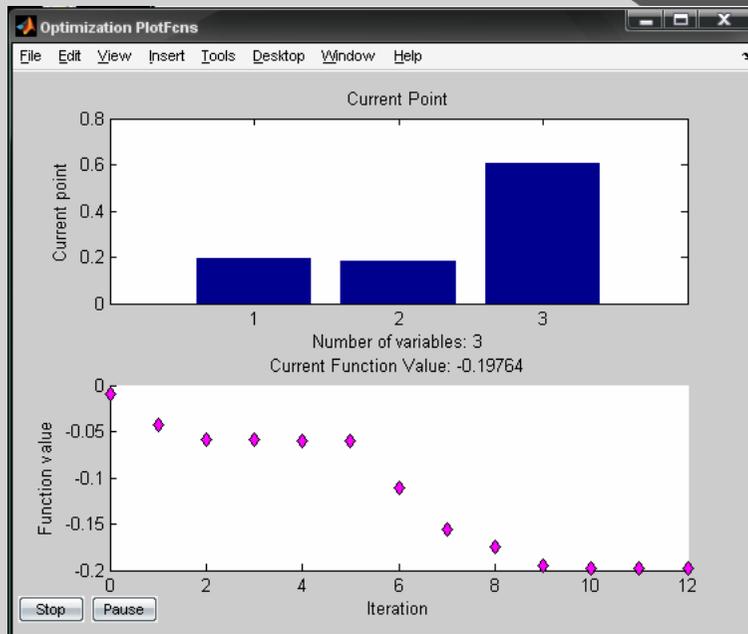
⇒ 원하는 결과값에 가장 근접했으나 초기 값이 최적해에 큰 영향을 미친다.

● Solution by MATLAB (fmincon)

초기값 [0.1;0.1;0.1]

→ Optimum : $r = 0.194$ m, $t = 0.182$ m, $w = 0.606$ m *☺*

maximum $V = 0.197$ m³



The Optimization Tool window displays the following information:

Problem Setup and Results

- Solver: fmincon - Constrained nonlinear minimization
- Algorithm: Medium scale
- Objective function: @fun
- Start point: [0.1;0.1;0.1]
- Derivatives: Approximated by solver
- Constraints: Linear inequalities (A, b), Linear equalities (Aeq, beq), Bounds (Lower, Upper), Nonlinear constraint function: @noncon

Options

- Stopping criteria: Max iterations (Use default: 400), Max function evaluations (Use default: 100*numberOfVariables), Max nodes (Use default), Time limit (seconds) (Use default), X tolerance (Use default: 1.00e-06), Function tolerance (Use default: 1.00e-06), Nonlinear constraint tolerance (Use default: 1.00e-06)

Run solver and view results

- Current iteration: 12
- Optimization running, Optimization terminated, Objective function value: -0.19764173716354028
- Optimization terminated: first-order optimality measure less than options.TolFun and maximum constraint violation is less than options.TolCon.

Final point:

Index	Value
1	0.194
2	0.182
3	0.606

● Solution by MATLAB (fmincon)

초기값 [0.1;0.01;0.1]

→ Optimum : $r = 0.194$ m, $t = 0.182$ m, $w = 0.606$ m *☺*
maximum $V = 0.197$ m³

초기값 [0.01;0.01;0.01]

→ Optimum : $r = 0.37$ m, $t = 0.006$ m, $w = 0.021$ m *☹*
maximum $V = 3.082$ m³

초기값 [2;2;2]

→ Optimum : $r = 0.194$ m, $t = 0.182$ m, $w = 0.606$ m *☺*
maximum $V = 0.197$ m³

⇒ 초기값의 영향은 비교적 적은 편이나 원하는 결과값과 다소 차이를 보임.

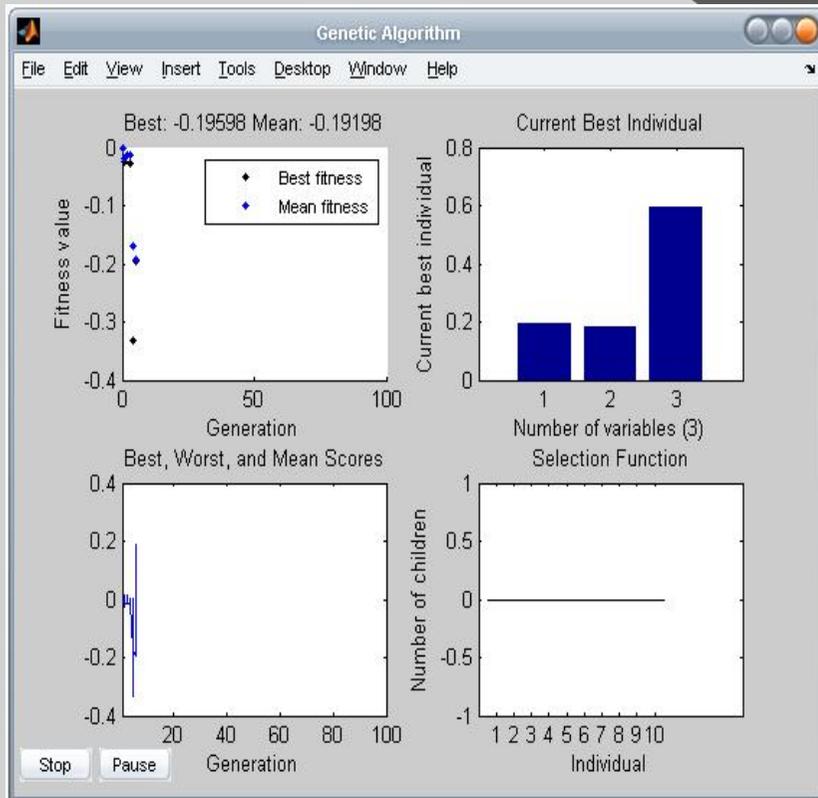
● Solution by MATLAB (genetic algorithm)

변수 3개

→ optimum $r = 0.196$ m, $t = 0.179$ m, $w = 0.594$ m *☺*

maximum $V = 0.192$ m³

→ 실행시마다 얻어진 최적값의
편차가 큼 *☹*



The figure shows the MATLAB Genetic Algorithm tool window with the following sections:

- Problem Setup and Results:**
 - Fitness function: @zfun
 - Number of variables: 3
 - Constraints: Linear inequalities (A, b), Linear equalities (Aeq, beq), Bounds (Lower, Upper), Nonlinear constraint function (@noncon)
 - Plots: Plot interval (1), Best fitness (checked), Best individual (checked), Distance (unchecked), Expectation (unchecked), Genealogy (unchecked), Range (checked), Score diversity (unchecked), Scores (unchecked), Selection (checked), Stopping (unchecked), Max constraint (unchecked), Custom function (unchecked)
 - Run solver: Use random states from previous run (unchecked), Start, Pause, Stop buttons, Current generation: 5, Status and results: GA running, GA terminated, Fitness function value: -0.19200617374460235, Optimization terminated: average change in the fitness value and constraint violation is less than options, TolCon.
 - Final point:

1	2	3
0.19643	0.17953	0.59473
- Options:**
 - Population: Population type (Double Vector), Population size (20), Creation function (Uniform), Initial population ([]), Initial scores ([]), Initial range ([0, 1])
 - Fitness scaling (checked)
 - Selection (checked)
 - Reproduction (checked)
 - Mutation (checked)
 - Crossover (checked)
 - Migration (checked)
 - Algorithm settings (checked)
 - Hybrid function (checked)
 - Stopping criteria (checked)
 - Output function (checked)
 - Display to command window (checked)
 - Vectorize (checked)
- Quick Reference:**
 - Genetic Algorithm Solver:** This tool corresponds to the ga function.
 - Fitness function (required):** is the objective function you want to minimize. You can specify the function as a function handle of the form @objfun, where objfun.m is an M-file that returns a scalar.
 - Number of variables (required):** is the number of independent variables for the fitness function.
 - Links: Constraints, Plot Functions, Run solver, Population, Fitness scaling, Selection, Reproduction, Mutation, Crossover, Migration, Algorithm settings, Hybrid function, Stopping criteria, Output function, Display to command window.

● Solution by Excel solver

Tire design

Design variables	Lower limit	Symbol	Value
Inner radius of tire	0	r	0
Width of tire	0	t	0.376271
Thickness of sidewall	0	w	1.254236

Objective function	Symbol	Value
Maximize volume of tire	V	0.557867

Parameter	Symbol	Value
Density of tire	rho	1000
Friction coefficient	mu	0.8
Maximum torque	T_max	63.513
Maximum speed	V_max	90.305
Angular velocity	omega	240
Angular acceleration	alpha	1.3633797
Acceleration	a	0.513
Mass of car	m_car	1640
Minimum mass of tire	m_tire,min	9
Minimum oblateness	O_min	30
Mass moment of inertia	I	39.491298
Gravity	g	9.81

Constranints	value	inequality
Torque	-3227.351	<=0
Oblateness	6.404E-07	<=0
Velocity	0	<=0
Mass	-548.8665	<=0
Dimension	0	<=0
Dimension	-0.376271	<=0
Dimension	-1.254236	<=0

→ r 값이 항상 lower limit 값이 되는 문제점 발생 *☹*

● Comparison

	Pattern search	Fmincon	Genetic algorithm	Excel solver	Reference
r (m)	0.276	0.194	0.196	0	0.237
t (m)	0.1	0.182	0.179	0.376	0.1005
w (m)	0.333	0.606	0.594	1.254	0.335

⇒ 실제 설계값과 가깝거나 최적값으로 수용할 수 있을 만한 결과를 얻은 경우가 있었으나 최적해의 안정성이 보장되지 않으므로 이 Modeling 과 fomulation 은 신뢰도가 떨어지며, 실제 설계에 사용하기에 부적합하다!!



● Review

● Modeling 자체의 한계

- ⇒ 실제 형상을 지나치게 단순화
- ⇒ 타이어의 실제 거동과 특성을 표현하는데 한계가 있다

● Formulation 방법의 문제점

- ⇒ 최적해를 찾는데 부적합한 방법을 사용했을 가능성
- ⇒ 타이어의 Sliding, 동력 전달 간의 손실 등의 요소 무시

● Constraints 의 문제점

- ⇒ 정확한 물성치, 사이즈 등의 정보 부족 → 제약 조건의 부족
- ⇒ 실제 제약 요소의 표현 방법의 문제점

FEAR CAN HOLD YOU PRISONER
HOPE CAN SET YOU FREE.



끝났 다 아 ~

- 학우 여러분 그리고 교수님 한 학기 동안 수고 많으셨습니다!!!!!!! -