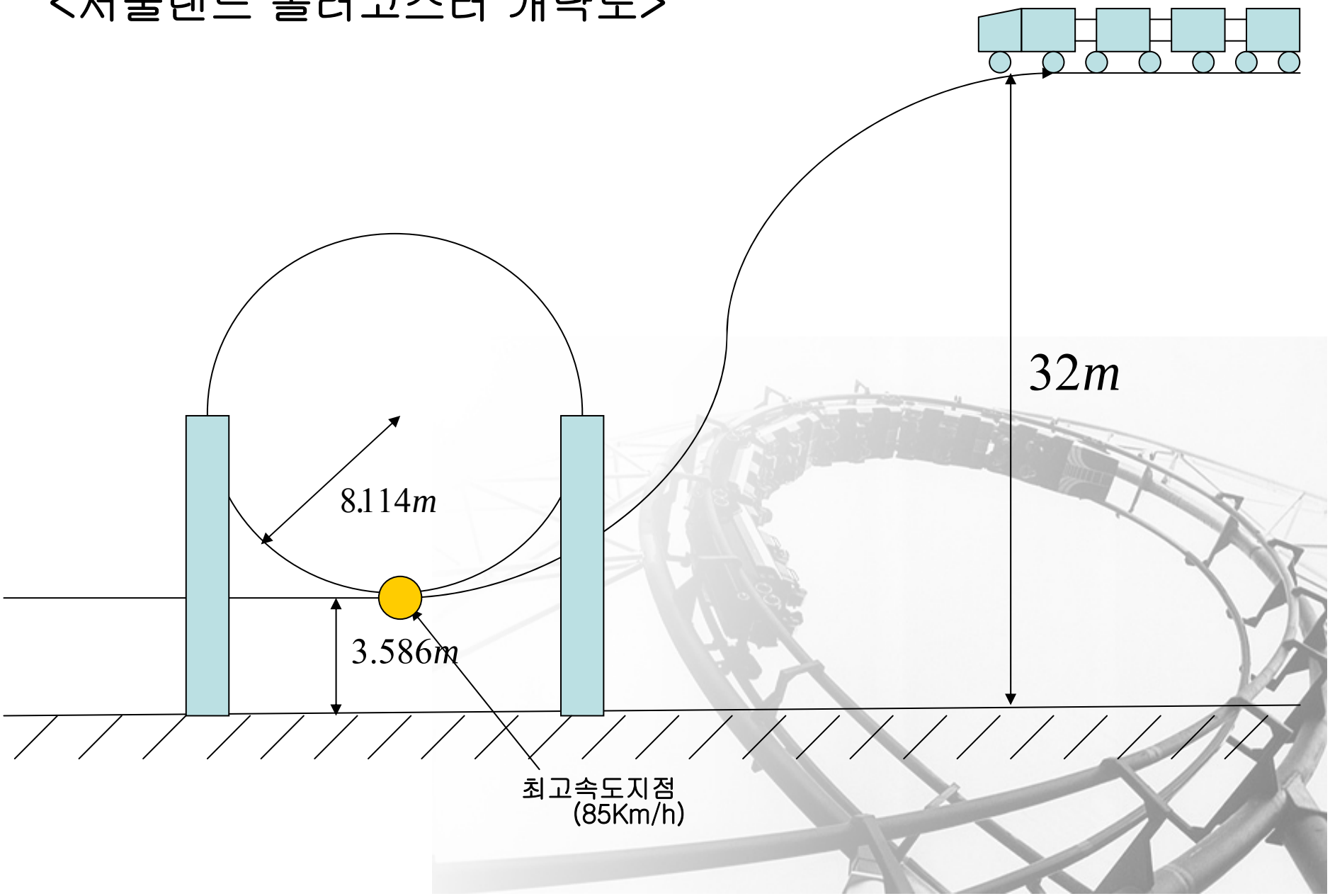
# 롤러코스터 지지대 최적설계

## Flying Support

(진승수,김준호,최범석)

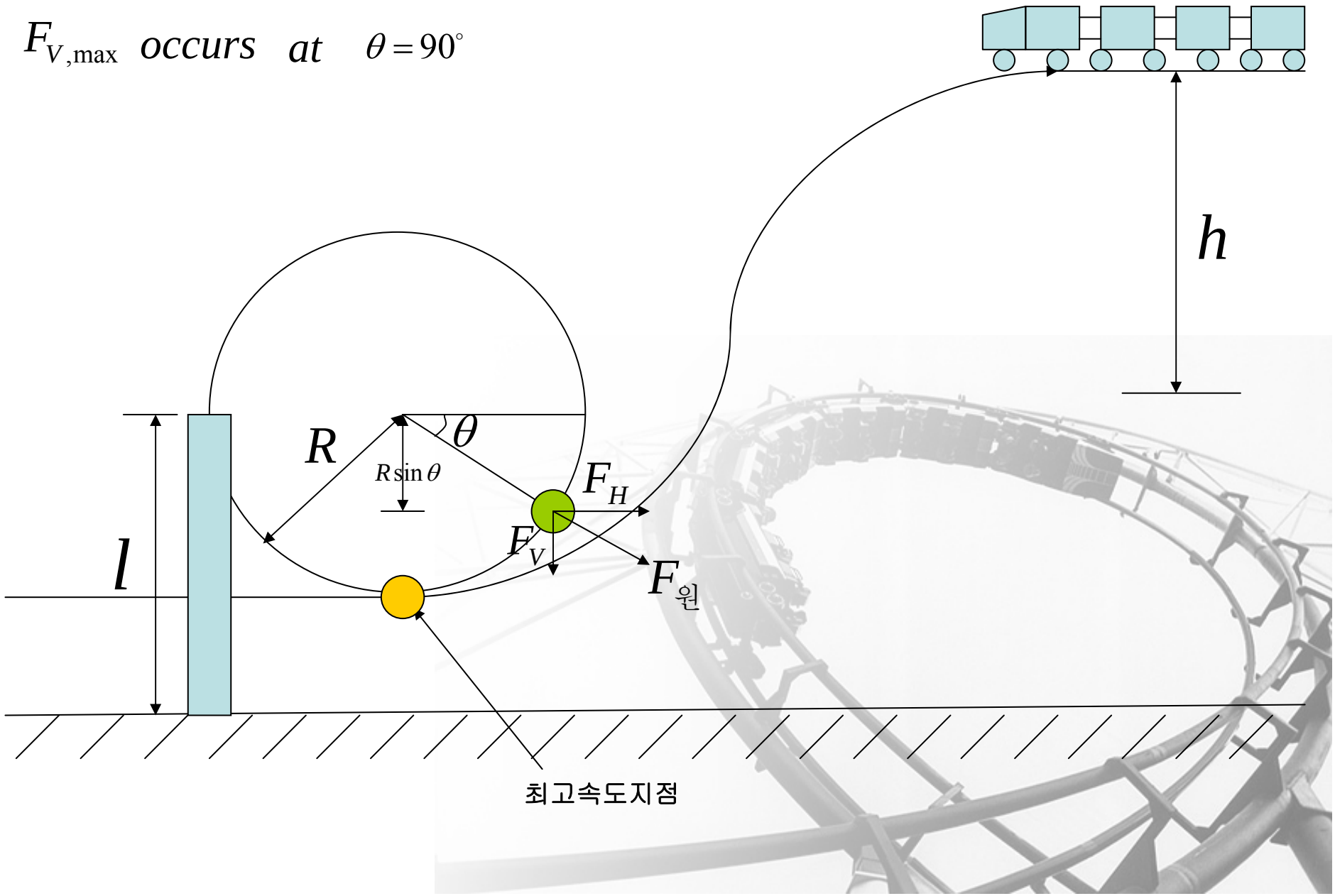
# 이동식 롤러코스터를 위한 지지대의 무게 최적설계

# <서울랜드 롤러코스터 개략도>



$F_{H,\max}$ occurs at $\theta = 17.647^\circ$

$F_{V,\max}$ occurs at $\theta = 90^\circ$



- 레일의 질량 : $M = 30.5 \times 2\pi \times 8.114kg$
- 롤러 코스터의 질량 : $m = 10,880kg$

수직력이 가장 클 때 : 원형레일의 최하지점에서 수직력이 가장 큼.(롤러코스터의 속도가 최하지점에서 최대이기 때문)

$$F_V = (M+m)g + \frac{2mg(h+R\sin\theta)}{r} = 900KN$$

수평력이 가장 클 때 : 수평력은 원심력의 크기에 따라 달라짐.

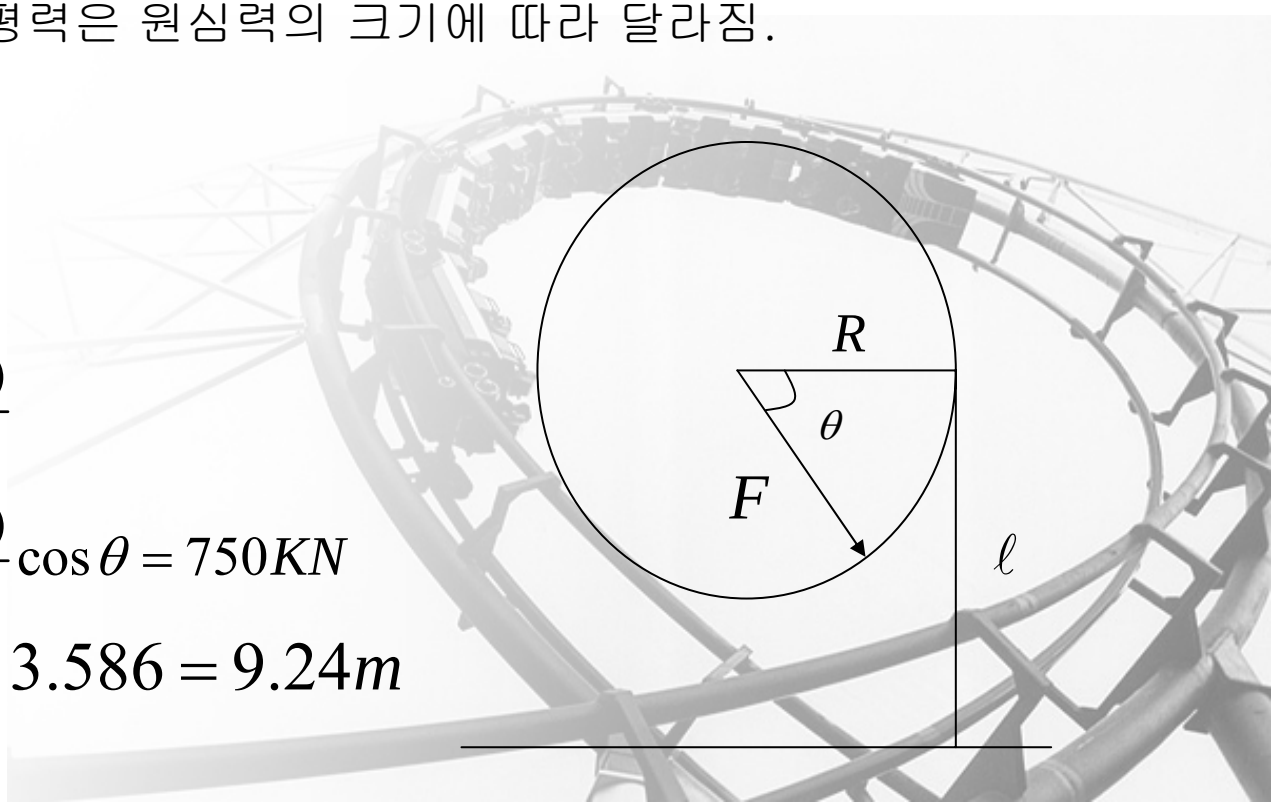
$$v = \sqrt{2g(h+R\sin\theta)}$$

$$F_{원} = \frac{mv^2}{r}$$

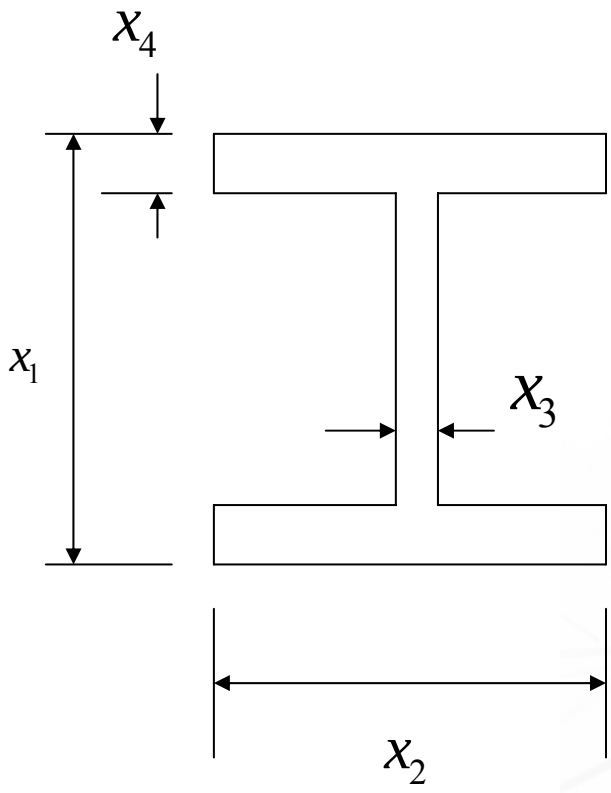
$$F_{원} = \frac{2mg(h+R\sin\theta)}{r}$$

$$F_H = \frac{2mg(h+R\sin\theta)}{r}\cos\theta = 750KN$$

$$\ell(\theta) = R - R\sin\theta + 3.586 = 9.24m$$



# Variables




$$A = x_1 \cdot x_2 - (x_1 - 2x_4)(x_2 - x_3)$$
$$= x_2 \cdot x_3 + 2x_2 \cdot x_4 - 2x_3 \cdot x_4$$

$$I = \frac{1}{12} x_2 \cdot x_1^3 - \frac{1}{12}(x_2 - x_3)(x_1 - 2x_4)^3$$

$$Length \quad l = R(1 - \sin x_5) + 3.586$$

# Assumption

Steel structural(ASTM-A36) 사용

-Max normal stress : 250MPa

-Max shear stress : 145MPa

-Modulus of elasticity: 200GPa

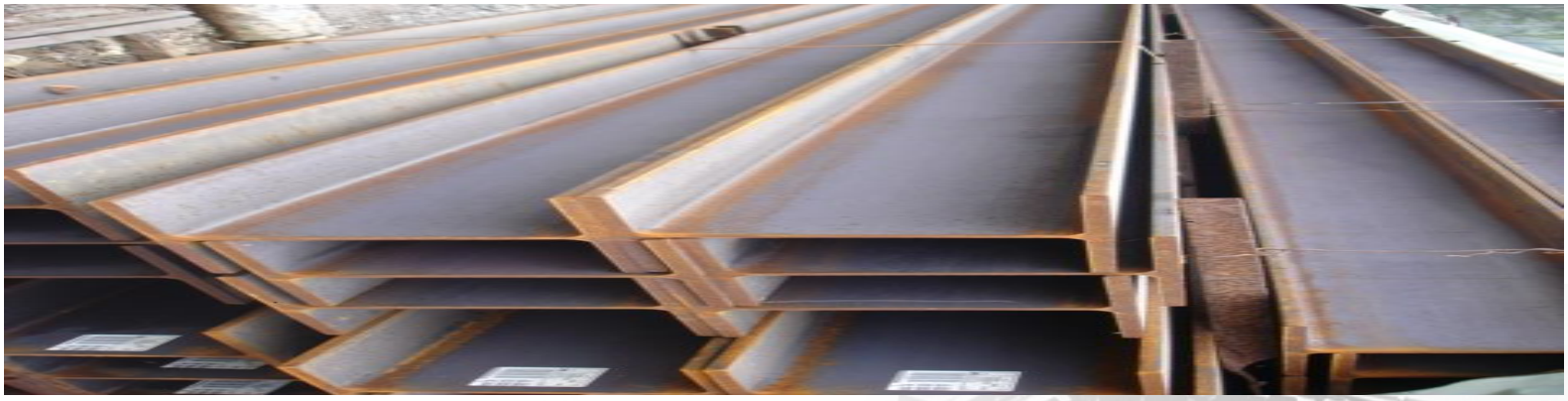
-density : 7850 $kg/m^3$

- 좌우대칭으로 가정하고 우측 지지대만 고려

- 목적함수(최소무게)

$$f(A, l) = \rho Al$$

# 기존 설계



$W360\times551$사용 $\rightarrow A=0.701m^2$

기존설계시 지지대의 총 무게

-->$W=\rho A\ell=664.5342kg$

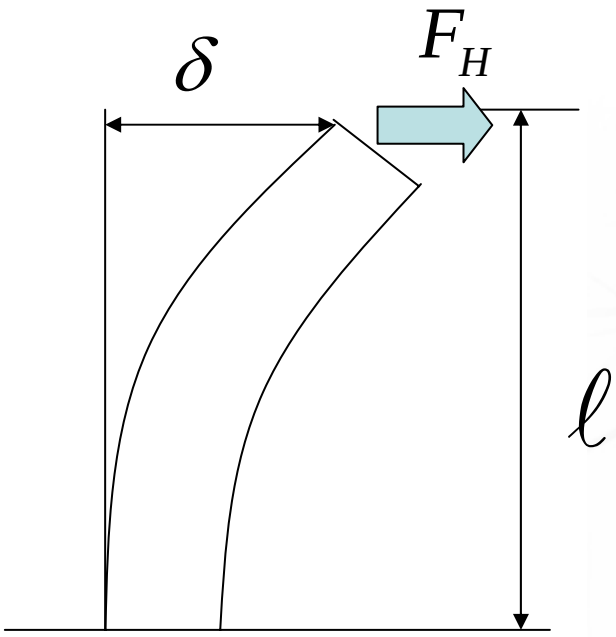
## 좌 굴 (Buckling)


– 압축력을 받을때만 발생하며 한계 압축력에 도달 했을때

횡방향으로 변형을 일으키며 불안정한 상태가 되는 것

$$P_{cr} = \frac{\pi^2 EI}{Le^2}$$

$$P \leq Pcr$$

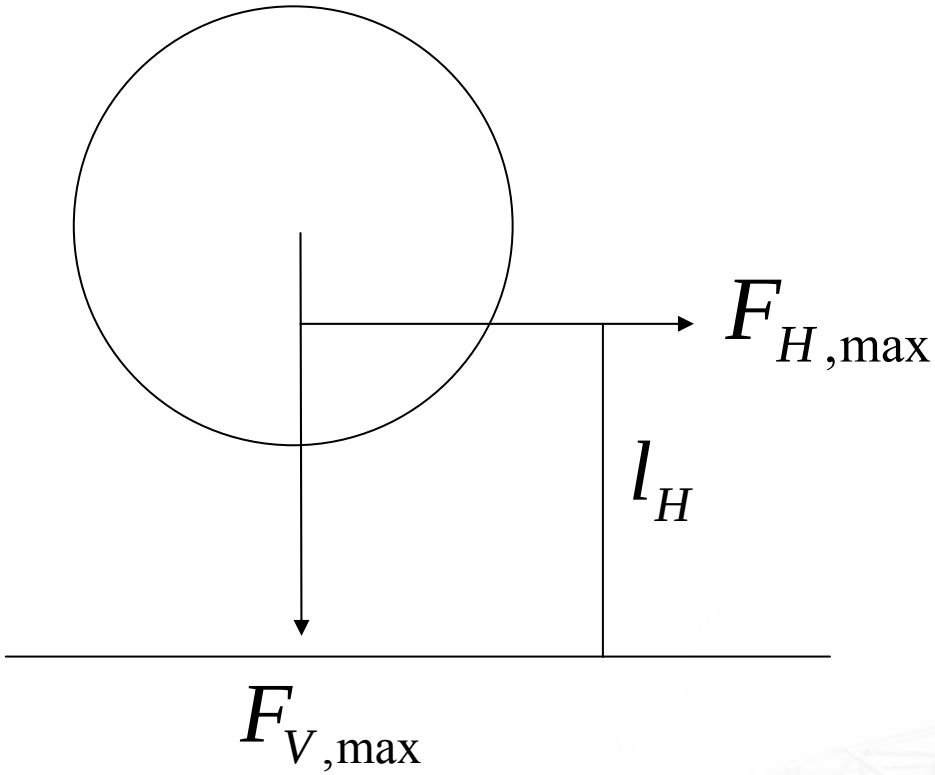



$$\delta \leq \frac{\ell}{1000}$$

$$F = k\delta = (\frac{3EI}{l^3})\delta$$

$$\frac{3EI}{l^3}\frac{l}{1000} \geq F_H$$

$$\tau \geq \frac{F_H}{A}$$




$$\sigma = \frac{My}{I} + \frac{F}{A}$$

$$= \frac{F_{V,\max} \times (R\cos\theta)^2 - F_{H,\max} \times \{9.24 - (R - R\sin\theta + 3.586)\}^2}{I} + \frac{F_{V,\max}}{A}$$

$$\leq \frac{\sigma_Y}{2}$$

# Constraints

1. Allowable normal stress
2. Allowable shear stress
3. Bucking load (critical load)
4. Displacement
5. Length of support
6. radius of support

$$\sum \sigma \le \sigma_{allow}$$

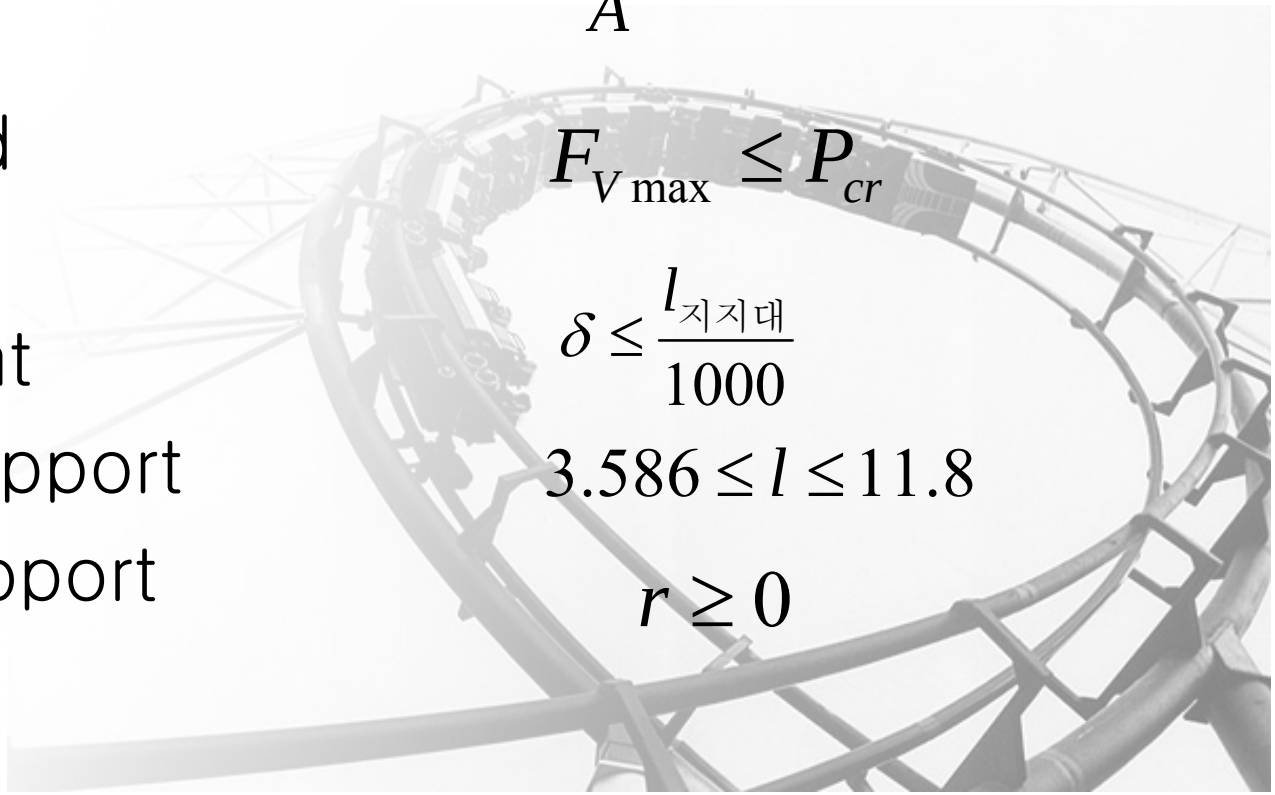
$$\frac{F_{H\ \max}}{A} \le \tau_{allow}$$

$$F_{V\max} \le P_{cr}$$

$$\delta \le \frac{l_{지지대}}{1000}$$

$$3.586 \le l \le 11.8$$

$$r \ge 0$$



# Equation

- The minimize the mass function

$$f(A,l) = \rho Al = \rho A(R(1-\sin\theta)+3.586)$$

- Subject

$$g_1(A,\theta) = \frac{F_{V,\max}\times(R\cos\theta)^2 - F_{H,\max}\times\{9.24-(R-R\sin\theta+3.586)\}^2}{I} + \frac{F_{V,\max}}{A} - \sigma_{allow} \le 0$$

$$g_2(A,\theta) = \frac{F_{H,\max}}{A} - \tau_{allow} \le 0$$

$$g_3(A,\theta) = F_{V,\max} - \frac{\pi^2 EI}{4(R-R\sin\theta+3.586)^2} \le 0$$

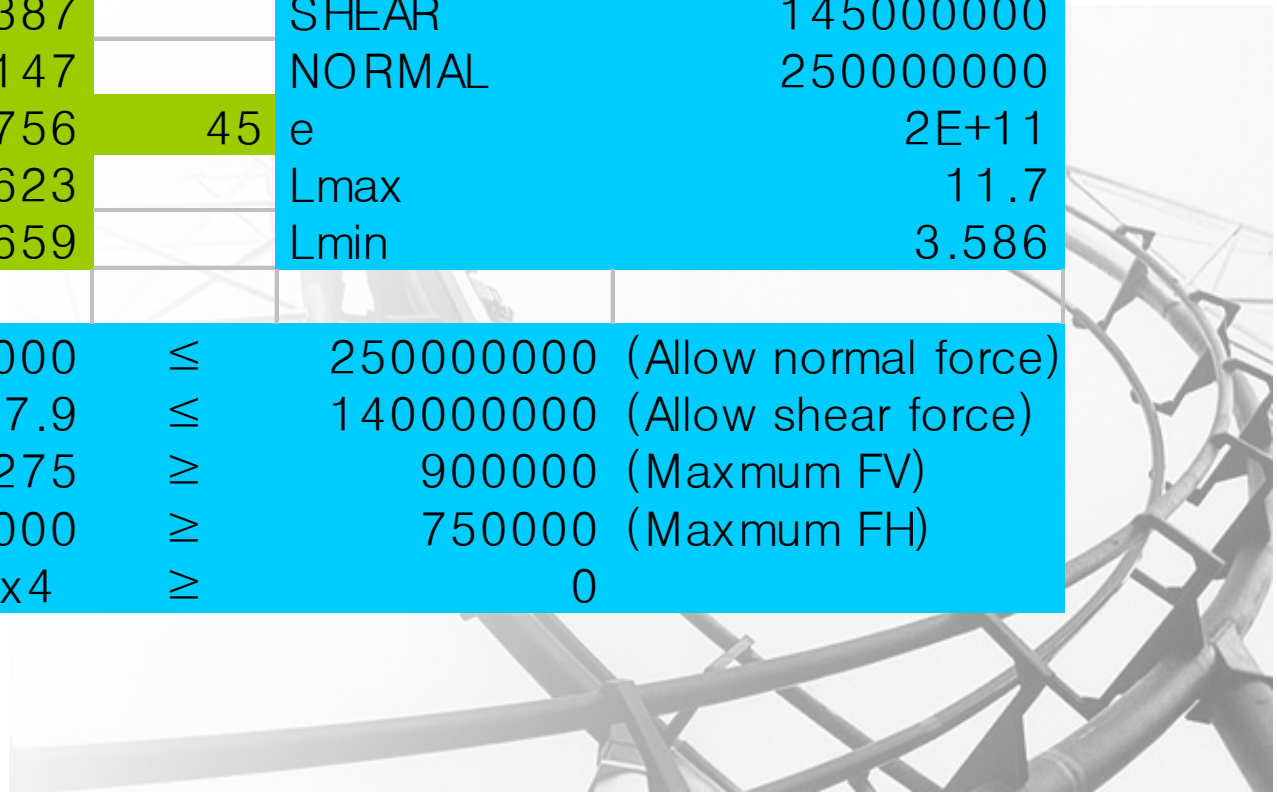
$$g_4(A,\theta) = \frac{R-R\sin\theta+3.586}{1000} \le 0$$

$$g_5(A,\theta) \quad ----> \quad 3.586 \le R-R\sin\theta+3.586 \le 11.7$$

$$g_6(A,\theta) \quad ----> \quad x_1, x_2, x_3, x_4 > 0$$

# Using Excel

| | | | | |
|---|---|---|---|---|
| object | 0.53610068 | | | |
| | | | | |
| x1 | 1.3109279 | | R | 8.116 |
| x2 | 1.87010573 | | FV | 900000 |
| x3 | 4.15597303 | | FH | 750000 |
| x4 | 1.17610387 | | SHEAR | 145000000 |
| l | 7.51407147 | | NORMAL | 250000000 |
| angle | 2.59940756 | 45 | e | 2E+11 |
| A | 0.07134623 | | Lmax | 11.7 |
| i | 0.07057659 | | Lmin | 3.586 |
| | | | | |
| CONST1 | 250000000 | ≤ | 250000000 | (Allow normal force) |
| CONST2 | 10512117.9 | ≤ | 140000000 | (Allow shear force) |
| CONST3 | 616850275 | ≥ | 900000 | (Maxmum FV) |
| CONST4 | 750000 | ≥ | 750000 | (Maxmum FH) |
| CONST5 | x1,x2,x3,x4 | ≥ | 0 | |



# Using Excel

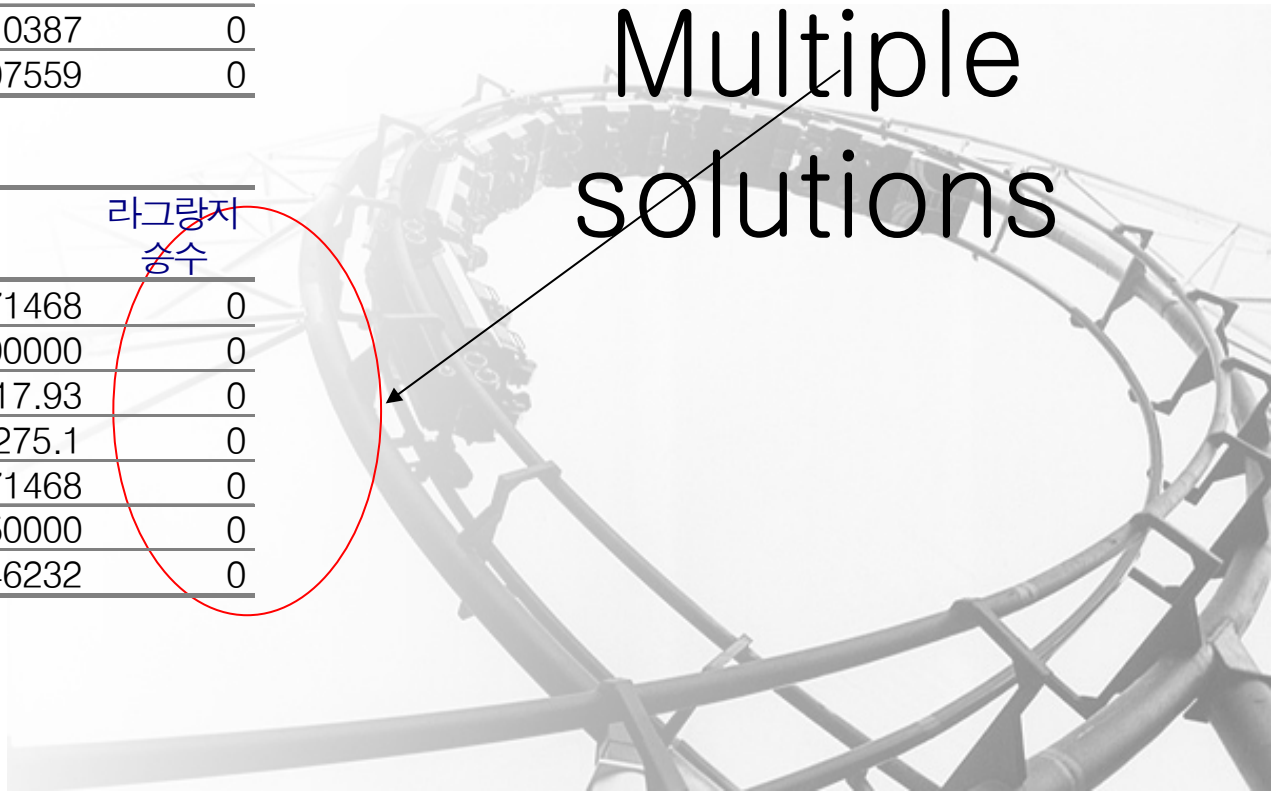
값을 바꿀 셀

| 셀 | 이름 | 계산 값 | 한계 기울기 |
|---|---|---|---|
| $B$4 | x1 | 1.3109279 | 0 |
| $B$5 | x2 | 1.870105727 | 0 |
| $B$6 | x3 | 4.155973032 | 0 |
| $B$7 | x4 | 1.17610387 | 0 |
| $B$9 | angle | 2.599407559 | 0 |

제한 조건

| 셀 | 이름 | 계산 값 | 라그랑지 승수 |
|---|---|---|---|
| $B$8 | I | 7.514071468 | 0 |
| $B$13 | CONST1 | 250000000 | 0 |
| $B$14 | CONST2 | 10512117.93 | 0 |
| $B$15 | CONST3 | 616850275.1 | 0 |
| $B$8 | I | 7.514071468 | 0 |
| $B$16 | CONST4 | 750000 | 0 |
| $B$10 | A | 0.071346232 | 0 |

Multiple solutions



# Using Matlab (fmincon사용)

•options = optimset('LargeScale','off','Display','iter','maxiter',2);

- 빔의 형태와 관련된 4개의변수로 실행
- 1이면 초기값 자신을 1회 계산하여 다시 초기값을 해로 나타내므로 반복횟수를 2로 하였음
- Multiple solution으로 초기값을 바꾸어가며 20번 실행
- 초기값(1.2 0.5 0.5 1)

| | | |
|---|---|---|
| object | 0.2800044 | |
| x1 | 1.967 | |
| x2 | 0.026 | |
| x3 | 0.014 | |
| x4 | 0.982 | 1.964 |
| l | 11.702 | |
| angle | 0 | 0 |
| A | 0.023932 | |
| i | 0.01648941 | |

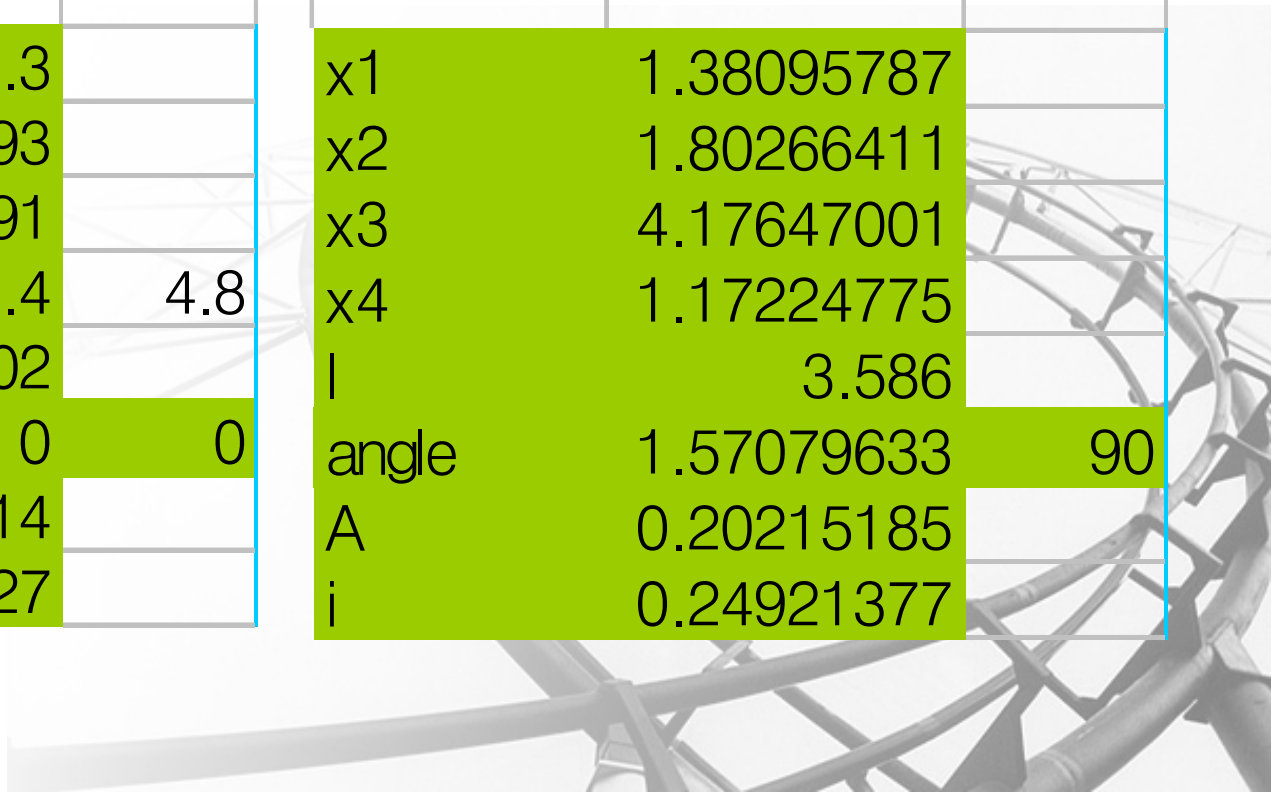
# Using Excel

초기값(1,1,1,1,0)

| | | |
|---|---|---|
| object | 0.06267857 | |
| | | |
| x1 | 5.3 | |
| x2 | 0.07159793 | |
| x3 | 0.0715491 | |
| x4 | 2.4 | 4.8 |
| l | 11.702 | |
| angle | 0 | 0 |
| A | 0.00535714 | |
| i | 0.88827327 | |

초기값(1,1,1,1,90)

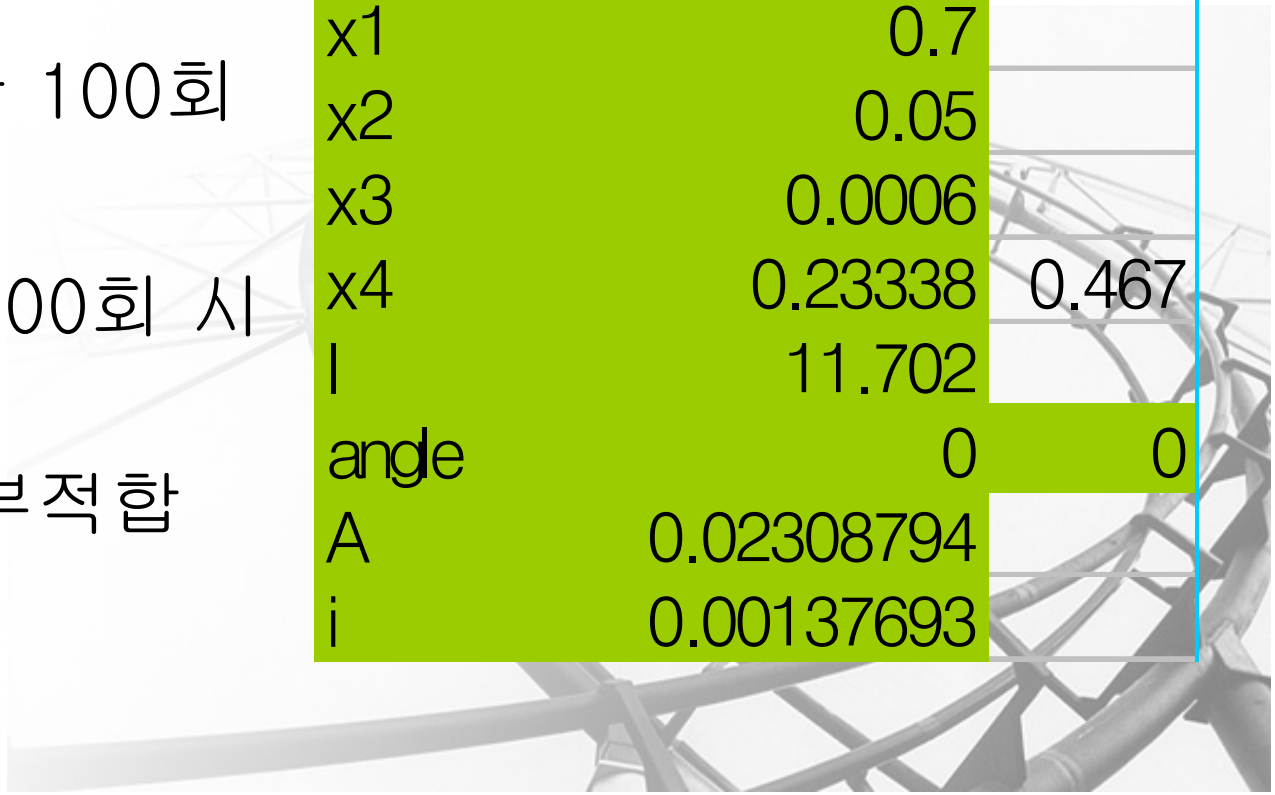
| | | |
|---|---|---|
| object | 0.72491655 | |
| | | |
| x1 | 1.38095787 | |
| x2 | 1.80266411 | |
| x3 | 4.17647001 | |
| x4 | 1.17224775 | |
| l | 3.586 | |
| angle | 1.57079633 | 90 |
| A | 0.20215185 | |
| i | 0.24921377 | |



# Using Matlab (GA)

- 초기값 설정이 어려워 GA사용
- 계산횟수 제한 100회
- 20번 시행
- 계산 대부분 100회 시행
- 빔의 형상에 부적합

| | | |
|---|---|---|
| object | 0.27012894 | |
| | | |
| x1 | 0.7 | |
| x2 | 0.05 | |
| x3 | 0.0006 | |
| x4 | 0.23338 | 0.467 |
| l | 11.702 | |
| angle | 0 | 0 |
| A | 0.02308794 | |
| i | 0.00137693 | |



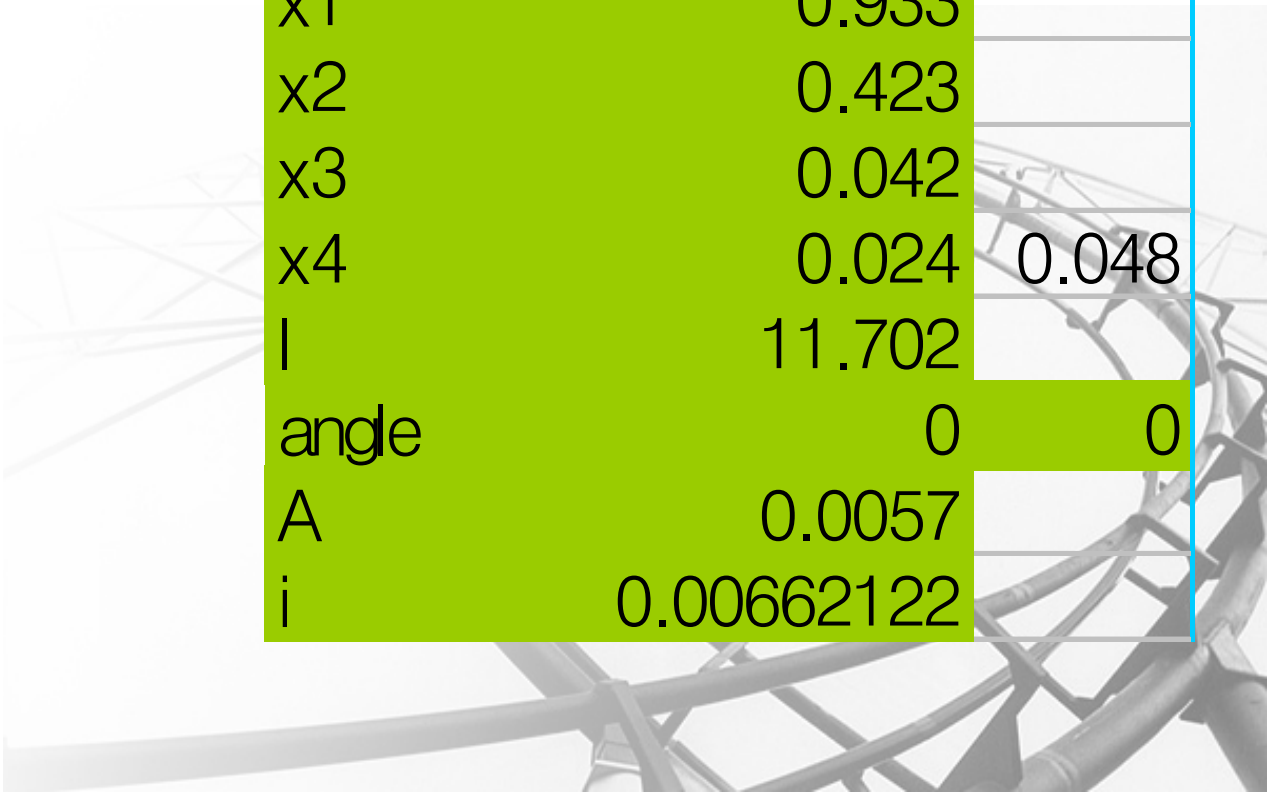
# Solution(Using Excel)

| | | | | |
|---|---|---|---|---|
| object | 0.06267857 | | | |
| | | | | |
| x1 | 3.05395627 | | R | 8.116 |
| x2 | 0.07254674 | | FV | 900000 |
| x3 | 0.07249791 | | FH | 750000 |
| x4 | 0.99999121 | 2 | SHEAR | 145000000 |
| l | 11.2957617 | | NORMAL | 250000000 |
| angle | 0.05007493 | 2.869 | e | 2E+11 |
| A | 0.00535714 | | Lmax | 11.7 |
| i | 0.17219205 | | Lmin | 3.586 |
| | | | | |
| CONST1 | -6.657E+09 | ≤ | 250000000 | (Allow normal force) |
| CONST2 | 140000000 | ≤ | 140000000 | (Allow shear force) |
| CONST3 | 665965610 | ≥ | 900000 | (Maxmum FV) |
| CONST4 | 809717.086 | ≥ | 750000 | (Maxmum FH) |
| CONST5 | x1,x2,x3,x4 | ≥ | 0 | |

# 대안 설계

- W Shape(W920X446)
- X1=0.933
- X2=0.423
- X3=0.042
- X4=0.024
- 길이 :11.7m

| | | |
|---|---|---|
| object | 0.06669 | |
| | | |
| x1 | 0.933 | |
| x2 | 0.423 | |
| x3 | 0.042 | |
| x4 | 0.024 | 0.048 |
| l | 11.702 | |
| angle | 0 | 0 |
| A | 0.0057 | |
| i | 0.00662122 | |



# Conclusion

- 기존 설계(W360x551)
  →664.534Kg

- Excel을 이용한 설계
  →493.513Kg (25%감소)

- 대안 설계(W920x446)
  →523.534Kg (21%감소)

# Thank you!!
# Any Question?