

터널 단면 최적 설계



팀명 : 두더지

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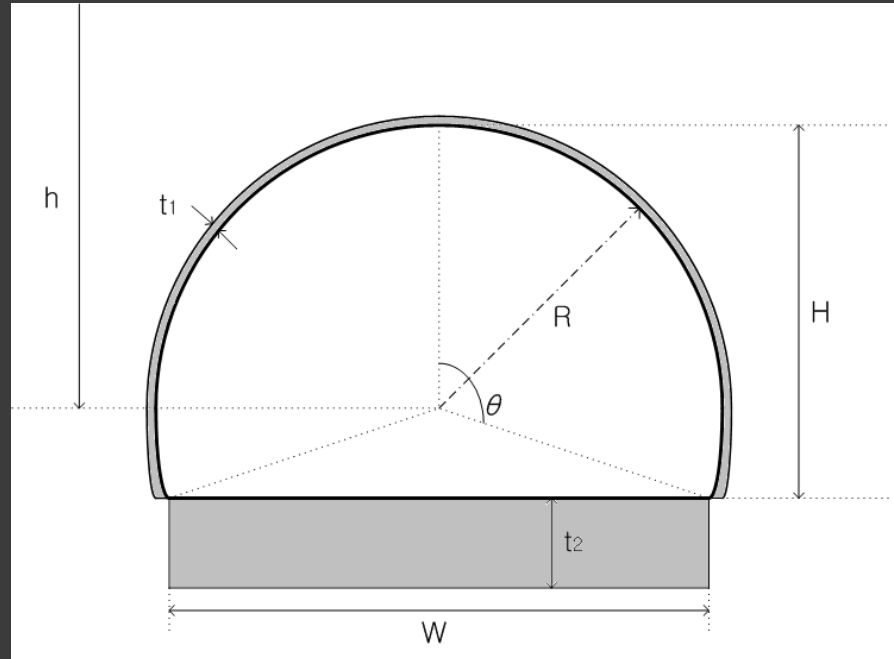
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Step1 : Problem Statement

- 터널 단면의 최적 설계
- 라이닝 콘크리트는 허용응력을 넘지 않도록 한다.
(라이닝 : 터널 벽면을 이루는 부재)
- 도로 폭은 최소 8m, 높이는 5m, 편평율 0.7~1.0
- 라이닝 두께의 최소값은 0.1m
- 겉 표면 법선의 응력만을 고려하여 라이닝 두께 설계.
- 하단부 응력 고려 하여 지반 두께 설계

Step2 : Data and Information

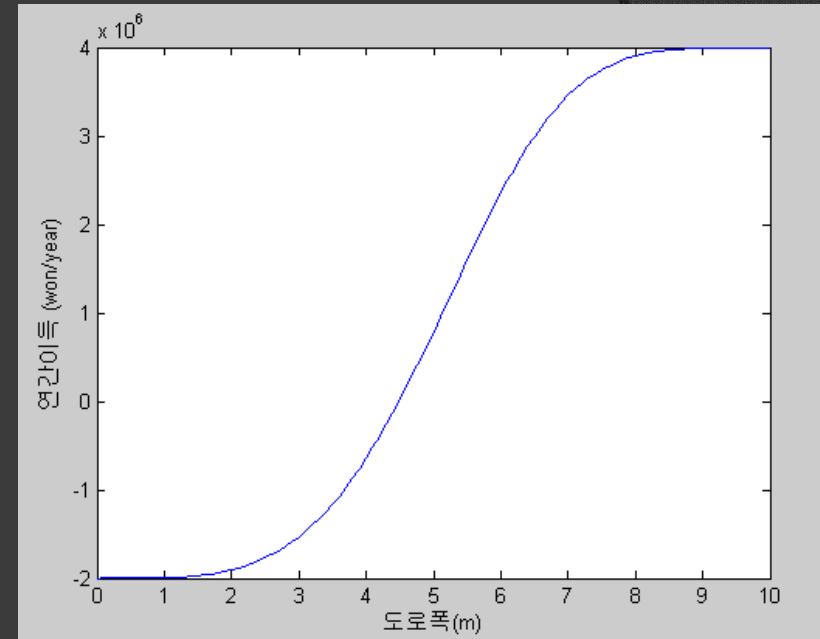


- $h(\text{깊이}) = 22.6 \text{ m}$
- $\sigma_y = 23.5 \text{ Mpa}$
- S.F.(안전계수) = 5
- $\sigma_{al} = \sigma_y / \text{S.F.} = 4.708 \text{ Mpa}$

Step2 : Data and Information

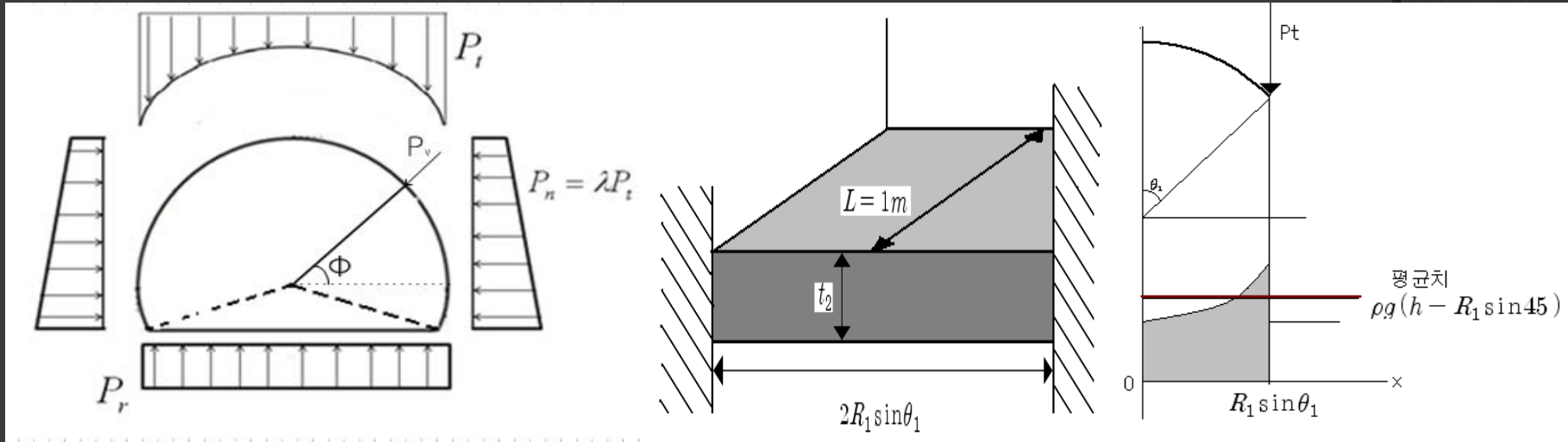
- 편평율 = H/W (0.7~1.0)
- λ (측압계수) = 0.5
- 흙의 비중 = 1.23
- 콘크리트 비중 = 2.32
- 콘크리트 m^2 당 재료비+인건비
= 30000 won/ m^2
- 철근 비용 = 70000 won/ m^2
- 레미콘 비 = 48000 won/ m^2
- 도로 폭당 이익비율(won/year)
= $4 \times 10^6 (1 - 1.5 \times \exp(-0.001 W^4))$

(ref. 남산 터널 조사 결과 하루 4만 여대
한대당 이득 500원 365일 1500m 로 계산)



도로 폭당 이익비율

Step2 : Data and Information



$$P_t = \rho g(h - R_1 \sin \phi)$$

$$P_n = \lambda P_t$$

$$P_v = P_t(\lambda \cos \phi + \sin \phi)$$

$$\sigma_1 = \frac{2P_v(R + t_1)^2}{(R + t_1)^2 - R^2} \leq \sigma_{al}$$

$$x = R_1 \cos \phi, \sin \phi = \sqrt{1 - (x / R_1)^2}$$

$$P_r = 2 \int_0^{R_1 \sin \theta} \rho g(h - R_1 \sin \phi) dx$$

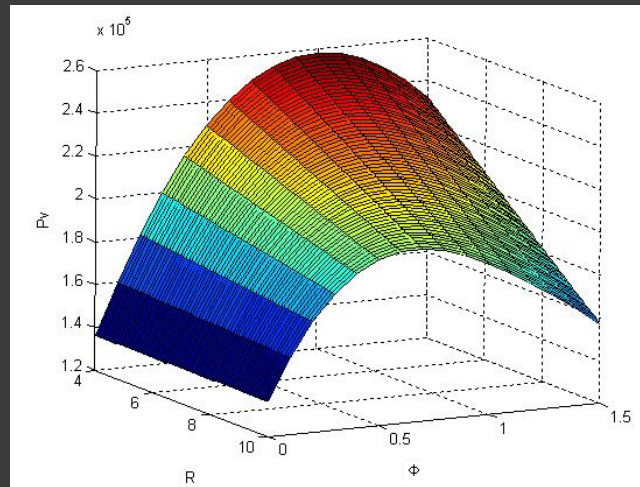
$$F_{dn} = 2 \rho g L R \sin \theta (h - R / \sqrt{2})$$

$$M_{\max} = \frac{2 \times 3 F_{dn} R \sin \theta}{8}$$

$$\sigma_2 = \frac{6 M_{\max}}{t_2^2} \leq \sigma_{al}$$

Step2 : Data and Information

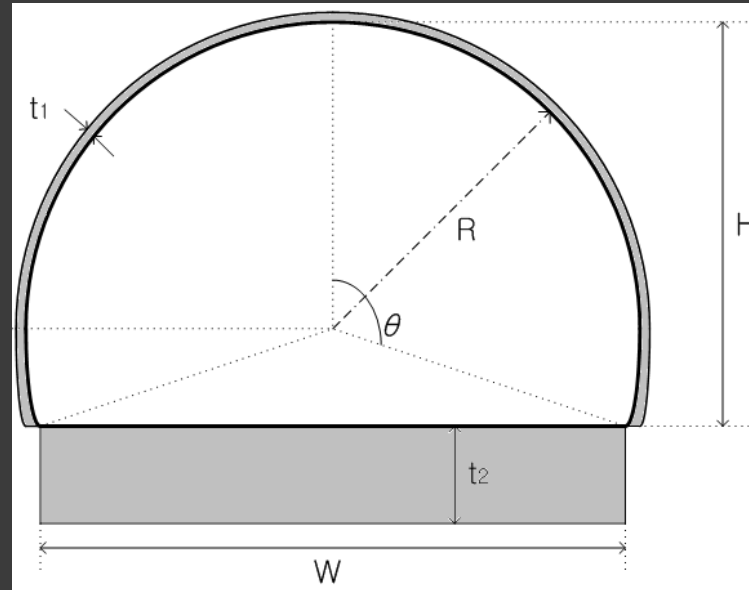
- 최고 응력 각도 산출



$$P_v = \gamma (h - 1 \sin \phi) [\lambda \cos \phi + \sin \phi]$$

⇒ 모든 경우에 대하여 최고응력 각도를 산출 하기 어려워 $\phi = 45^\circ$ 로 가정.

Step3 : Design Variables



- R : 터널의 반지름
- θ : 중심으로 부터 각도
- t_1 : 터널 라이닝 두께
- t_2 : 터널 지반 두께

Step4 : Objective Function

- $f(R, \theta, t_1, t_2) = [\text{Cost}] - [10\text{yrs Benefit}]$

$$f(R, \theta, t_1, t_2) = \left(2t_2 R \sin \theta + \theta \left\{ (R + t_1)^2 - R^2 \right\} \right) \times (30000 + 0.07 \times 10^6 + 0.6 \times 80000) \\ - 10 \times 400000 \left(1 - 1.5e^{-0.001(R_1 \sin \theta)^4} \right)$$

- 목적함수 $f(R, \theta, t_1, t_2)$ 의 최소화.
- 초기값은 Project 1에서의 결과 값 사용
→ $[R, \theta, t_1, t_2] = [7, 90^\circ, 0.363, 1]$

Step5 : Constraints

$$\sigma_{1,2} \leq \sigma_{al}, H \geq 5, W \geq 8, 0.7 \leq \frac{H}{W} \leq 1.0$$

$$g1 = 2\rho g \left\{ \left(h - \frac{R}{\sqrt{2}} \right) \times \frac{(0.5+1)}{\sqrt{2}} \right\} \times \frac{(R+t_1)^2}{(R+t_1)^2 - R^2} - 4.708 \times 10^6 \leq 0$$

$$g2 = 6 \left[3 \left\{ 2\rho g R \sin \theta \times \left(h - \frac{R}{\sqrt{2}} \right) \right\} \times \frac{R \sin \theta}{4} \right] / t_2^2 - 4.708 \times 10^6 \leq 0$$

$$g3 = 5 - R(1 - \cos \theta) \leq 0$$

$$g4 = 8 - 2R \sin \theta \leq 0$$

$$g5 = 0.7 - \frac{R(1 - \cos \theta)}{2R \sin \theta} \leq 0$$

$$g6 = \frac{R(1 - \cos \theta)}{2R \sin \theta} - 1 \leq 0$$

Solution (Excel)

1. Design Variables names	Lower limit	symbol	value	upper limit	units
R1	4	R	7.90423	10	m
theta1	0.785398163	θ	1.901094	3.141592654	rad
t1	0.1	t1	0.39299		m
t2	0.1	t2	4.683593		m
2. Parameter name	symbol	value			
중력가속도	g	9.81			
밀도	ρ	1230			
허용응력	σ_{al}	4708000			
3. Objective function					
시공비용-이득	f	-26086109.95			
4. Constraint	value	부등호	조건		
t1에 관한 허용응력	4708000	$\leq$	4708000	active	
t2에 관한 허용응력	4708000	$\leq$	4708000	active	
높이 조건	10.467764	$\geq$	5	inactive	
폭 조건	14.95394858	$\geq$	8	inactive	
편평율(H/W)	0.7	$\leq$	0.7	$\geq$	1

라그랑지
승수

-0.419286517

-1.100857265

0

0

1843960.603

0

Solution (fmincon)

Problem Setup and Results

Solver:

Algorithm:

Problem

Objective function:

Derivatives:

Start point:

Constraints:

Linear inequalities: A: b:

Linear equalities: Aeq: beq:

Bounds: Lower: Upper:

Nonlinear constraint function:

Derivatives:

Run solver and view results

Current iteration:

Optimization running.
Optimization terminated.
Objective function value: -2.6086109952420816E7
Optimization terminated: magnitude of search
direction less than 2*options.TolX
and maximum constraint violation is less than
options.TolCon.

Final point:

1	2	3	4
7.904	1.901	0.393	4.684

$$R = 7.904 \text{ m}$$

$$\theta = 1.901 \text{ rad} = 108.92^\circ$$

$$t1 = 0.393 \text{ m}, t2 = 4.684 \text{ m}$$

$$f = -26,085,000 \text{ won}$$

$$H = 10.47 \text{ m}, W = 14.95 \text{ m}$$

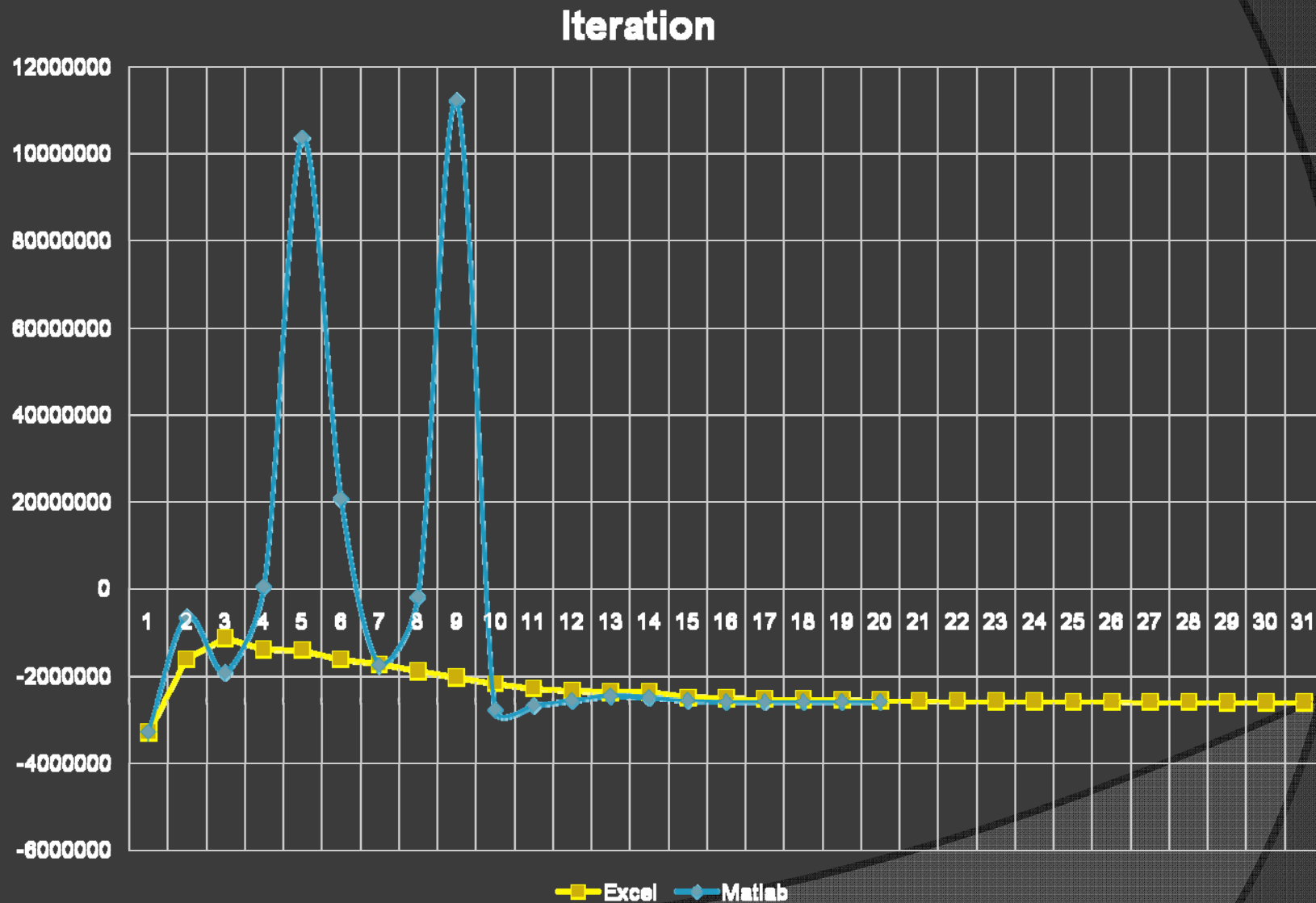
라그랑지 승수 :

$$g1 : 0.4193, g2 : 1.101$$

$$g3 : 0, g4 : 0$$

$$g5 : 1.844 \times 10^7, g6 : 0$$

Solution (Iteration)



Solution (GA)

Problem Setup and Results

Solver:

Problem

Fitness function:

Number of variables:

Constraints:

Linear inequalities: A: b:

Linear equalities: Aeq: beq:

Bounds: Lower: Upper:

Nonlinear constraint function:

Run solver and view results

☐ Use random states from previous run

Current iteration:

Optimization terminated: no feasible point found.

Optimization running.
Optimization terminated.
Objective function value: -70161.74423913985
Optimization terminated: no feasible point found.

Optimization running.
Optimization terminated.
Objective function value: -2.6064467457962863E7
Optimization terminated: maximum number of generations exceeded.

Final point:

1	2	3	4
7.946	1.901	0.394	4.713

- 시행마다 값이 다름
→ 5개 값 평균

[R θ t₁ t₂]

06 : 7.90 1.98 0.393 4.54

07 : 7.90 1.98 0.393 4.54

23 : 7.95 1.90 0.395 4.71

35 : 7.54 1.90 0.381 4.50

42 : 7.95 1.90 0.394 4.71

- 평균 : [7.85 , 1.93, 0.391, 4.60]

- f = -26032000 won

- H = 10.61 m, W= 14.70 m

Solution (patternsearch)

Problem Setup and Results

Solver: patternsearch - Pattern Search

Problem

Objective function: @projectfun

Start point: [7 pi/2 0.363 1]

Constraints:

Linear inequalities: A: b:

Linear equalities: Aeq: beq:

Bounds: Lower: [4 pi/4 0.1 0.1] Upper: [22.6 pi 22.6 22.6]

Nonlinear constraint function: @projectconfun

Run solver and view results

☐ Use random states from previous run

Start Pause Stop

Current iteration: 6 Clear Results

Optimization running.
Optimization terminated.
Objective function value: -3.3090822932095E7
Optimization terminated: no feasible point found.

Optimization running.
Optimization terminated.
Objective function value: -2.151623366275053E7
Maximum number of function evaluations exceeded:
increase options.MaxFunEvals.

Final point:

1	2	3	4
7.654	1.901	0.385	6.637

$R = 7.654 \text{ m}$

$\theta = 1.901 \text{ rad} = 108.92^\circ$

$t_1 = 0.385 \text{ m}$

$t_2 = 6.637 \text{ m}$

$f = -21,516,000 \text{ won}$

$H = 10.48 \text{ m}$

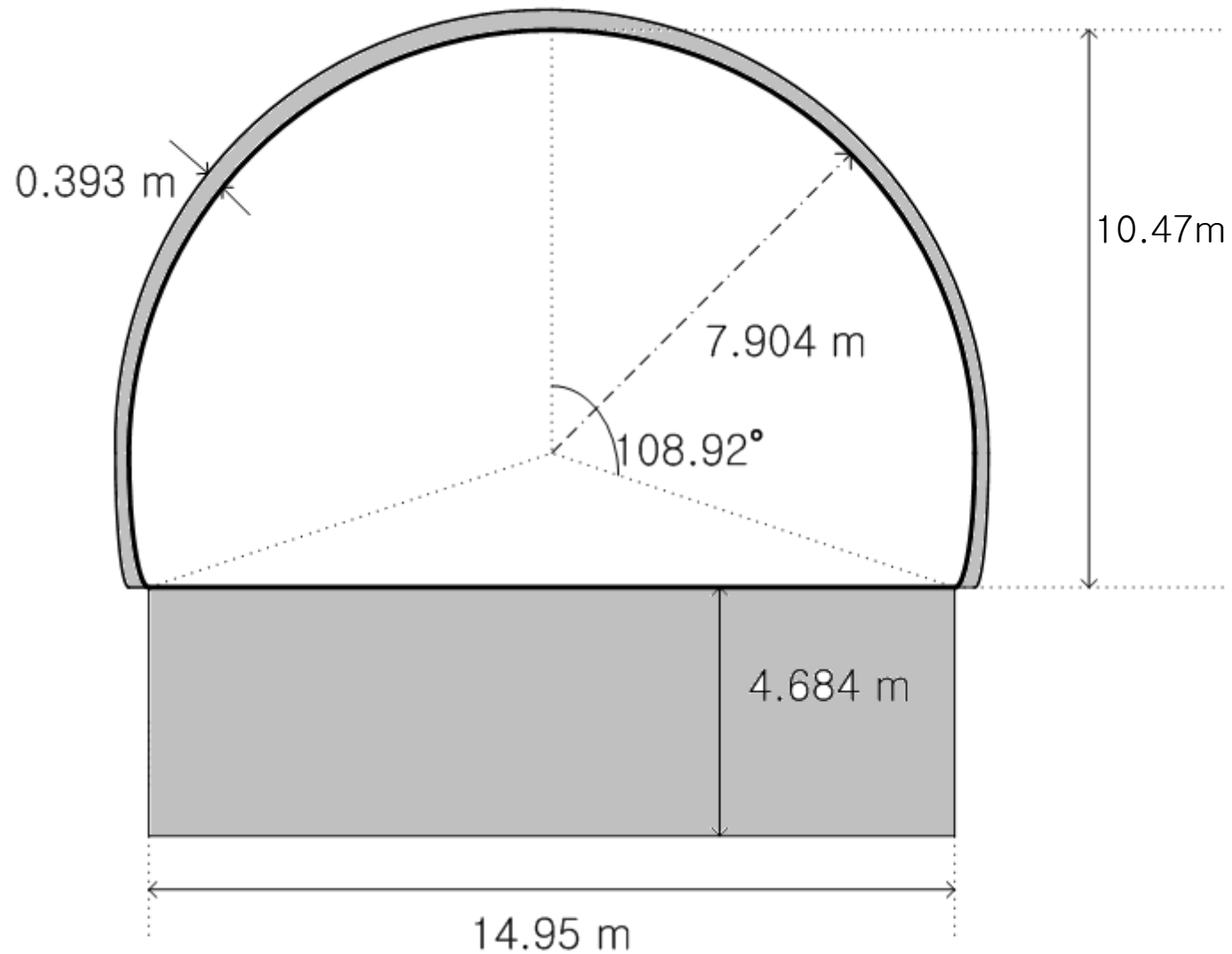
$W = 14.48 \text{ m}$

Solution (결과 값 비교)

	Excel	fmincon	GA	patternsearch
R(m)	7.904	7.904	7.85	7.654
$\theta (^{\circ})$	108.92	108.92	108.92	108.92
t1(m)	0.393	0.393	0.391	0.385
t2(m)	4.684	4.684	4.60	6.637
H(m)	10.47	10.47	10.61	10.48
W(m)	14.95	14.95	14.70	14.48
f(won)	-2609만	-2609만	-2603만	-2152만

- GA 나 patternsearch 의 경우 수렴하지 않는 경우가 많아 Excel 과 fmincon의 결과 값을 최종 설계값으로 결정.
- 라이닝의 두께 : 0.393 m 로 일반적인 터널에서 사용되는 두께인 0.3~0.6 m 와 유사한 결과.

최종 설계 모형



감사 합니다.