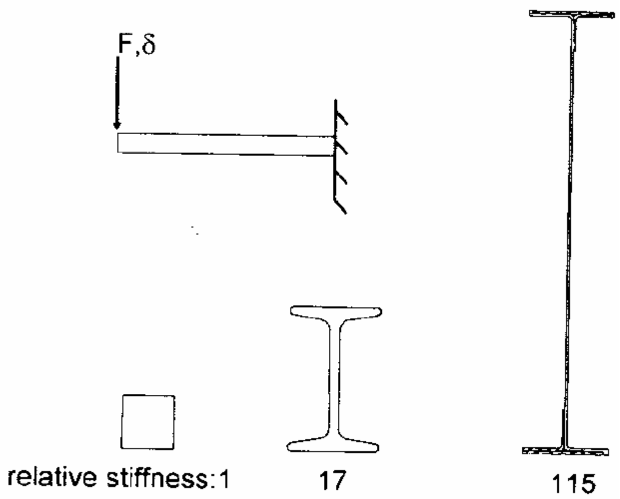
Project #3: Thin Wall Beam Section Design in Automobiles

Why are automotive sections so often thin walled? We will use a simple thought experiment to answer this question. Consider a steel cantilever beam with a tip load. In this experiment the cross-section area is fixed (and therefore the beam mass is fixed), and we are free to choose the cross-section shape to maximize strength and stiffness. From basic theory we know for a cantilever beam that

$$k = \frac{3EI}{L^3} \quad \text{and} \quad F_{\max} = \frac{I\sigma_{\text{design}}}{Lc} \quad \text{where } I = \int z^2 dA \tag{1}$$

where L is the beam length, c the distance from the neutral axis to the outer fiber, I the moment of inertia, z the coordinate for distance from the neutral axis, dA a differential area on the section, and σ_{design} the allowable design stress. In this experiment we will take the material yield stress as the design stress, $\sigma_{\text{design}} = \sigma_Y$.

First let us imagine a square cross section of unit area as our base for stiffness and strength performance, Fig. 1. Now observe from equation (1) that both stiffness and strength increase with moment of inertia, and as section material is moved away from the neutral axis the moment inertia is increased. This observation leads to an I beam shape with increased moment of inertia for constant cross section area yielding improved strength and stiffness. For example, Fig. 1(b) shows an I beam of the same mass as the base square resulting in 17 times the base stiffness. With no apparent difficulties with this design approach, we can move the material further from the neutral axis to result in the I beam of Fig. 1(c) and a performance of 115 times the base stiffness. Given our assumptions, this approach could be continued until a very high I beam of paper thickness resulted in presumably a very high stiffness and strength.



(a) (b) (c)

Fig. 1 Relative bending stiffness for equal cross-section area

If we move from a strictly thought experiment to testing two of the designed sections; the thick walled I beam of Fig. 1(b), and the thin walled I beam of Fig. 1(c) we could find that while very stiff, the thin walled section fails at an unexpectedly low load. The difference between expected performance and actual performance for the thinner walled section is the existence of a new failure mode – elastic plate buckling of the compressive elements of the section.

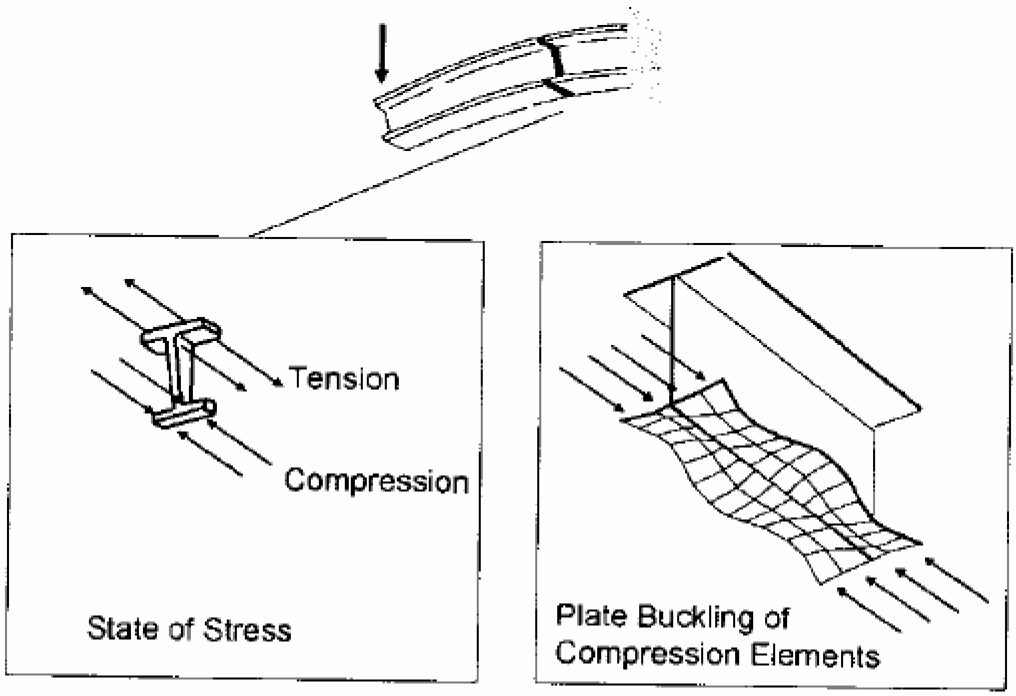


Fig. 2 Deflection behavior of thin-walled members

The general behavior of a compressively loaded plate is to react loads by direct stress. However, if the plate is sufficiently thin it bifurcates into the buckled shape, Fig. 2. The compressive stress at which this occurs depends on the plate width to thickness ratio, b/t. The design stress for a thin plate under compressive loading is then the lower of yield stress or plate buckling stress, Fig. 3.

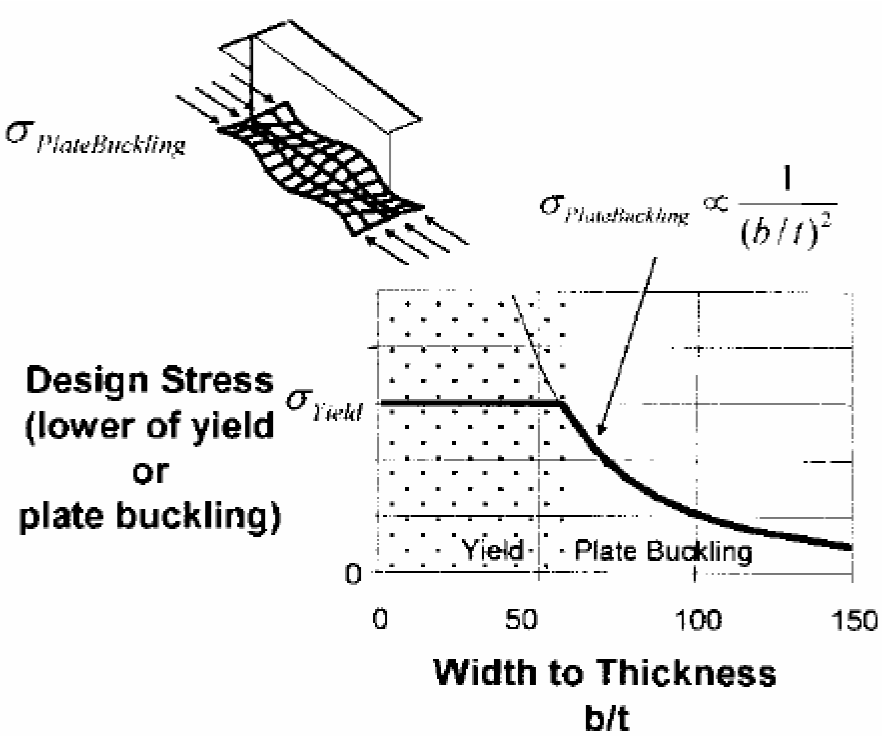


Fig. 3 Plate buckling stress

Thus in this project, we deal with a trade-off: thick walled sections with higher strength but lower stiffness performance, or thin walled sections with higher stiffness but lower strength performance due to plate buckling. Selection of the best section proportion than depend on the relationship of strength requirement to stiffness requirement for the section.

Consider a highly idealized convertible, Fig. 4. The bending performance of this vehicle is provided by the two rocker sections which act as center loaded simply supported beams. The specified bending stiffness requirement for this vehicle is 3335 N/mm and strength requirement is 3335 N. These requirements may be visualized graphically, Fig. 5, with required performance shown as the shaded acceptable region. We wish to determine the mild steel rocker section width, b, and thickness, t, which meets both strength and stiffness requirements at a minimum mass. For this project we arbitrarily hold the section height at 1.5 times the width.

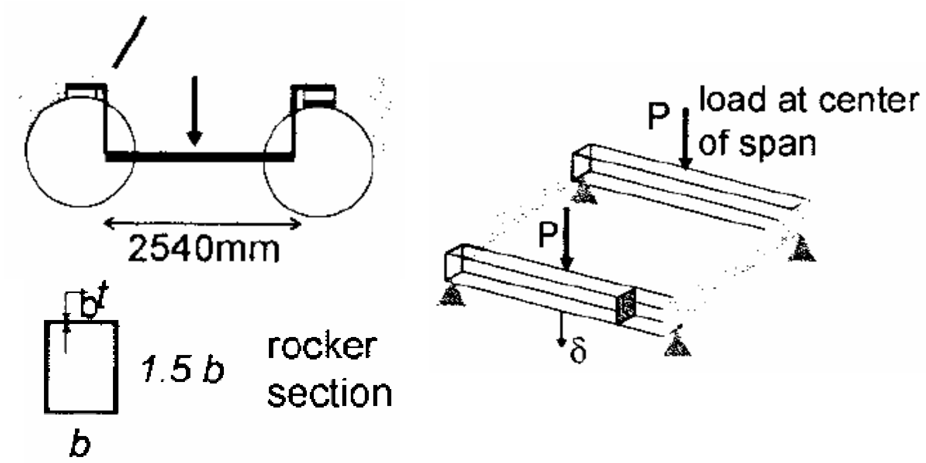


Fig. 4 Convertible rocker sizing for bending

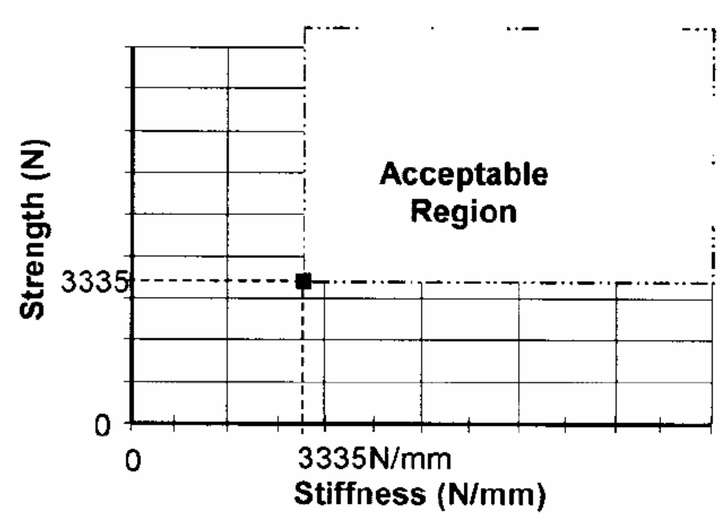


Fig. 5 Convertible bending requirements