

자전거 프레임 최적 설계

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1. PROBLEM STATEMENT

- ◇ 안장에 가해지는 사람의 하중을 견딜 수 있는 자전거 프레임 설계한다.
- ◇ 자전거의 재료는 알루미늄합금으로 한다.
- ◇ 프레임 질량의 최소화를 목적으로 한다.
- ◇ 뒷바퀴를 고정하는 프레임은 두 개의 대칭으로 되어 있으나, 2차원 분석을 위해 하나로 합친다.
- ◇ 각 부재의 연결은 핀 고정으로 가정하였다.
- ◇ 각 부재는 원통형으로 가정하였다.
- ◇ 정지 기준으로 인장 전단 응력을 분석하였다.
- ◇ 탑 튜브, 다운 튜브, 시트 튜브, 시트 스테이, 체인 스테이 네 가지 부재로 단순화 시켰다.

1. PROBLEM STATEMENT

추가

사람의 하중의 전체가 페달에 걸리는 경우를 고려한다.

지속적인 하중에 의한 피로파괴를 고려하여 설계한다.

변형률이 제한값 이내로 이루어지도록 한다.

2. DATA & INFORMATION COLLECTION



Ti-3AL-2.5V 티타늄 소재
중량 : 1340g(M(17) 모델)

	POSITION	TOP FORCE 3.0			
		14	15	17	18
탑튜브 수평길이 (Top Tube Horizon Length)	A	537	557	577	587
탑튜브 길이 (Top Tube Length)	B	524	539	553	563
시트 튜브길이-상단 (Seat Tube Length)	C	356	381	432	457
시트 튜브길이-중심 (Seat Tube Length)	D	306	331	382	407
체인스테이길이 (Chain Stay Length)	G	424	424	424	424
헤드튜브각도 (Cast Angle))	R1	70	71	71	70
시트 튜브각도 (Seat Tube Angle)	R2	73	73	73	73
할가드롭 (Hanger Drop)	E	35	35	35	35
휠베이스 (Wheel Base)	W	1020	1030	1050	1060
포크오프셋 (Fork Offset)	F	38	38	38	38
헤드 러그 길이 (Head Lug Length)	H	110	110	110	110

2. DATA & INFORMATION COLLECTION



Ti-3AL-2.5V 티타늄 소재

Mechanical Properties

Hardness, Brinell	256
Hardness, Knoop	278
Hardness, Rockwell C	24
Hardness, Vickers	280
Tensile Strength, Ultimate	620 MPa
Tensile Strength, Yield	500 MPa
Elongation at Break	15 %
Modulus of Elasticity	100 GPa
Compressive Yield Strength	690 MPa
Notched Tensile Strength	790 MPa
Poisson's Ratio	0.3
Charpy Impact	86 J
Fatigue Strength	170 MPa
Fatigue Strength	280 MPa
Fatigue Strength	280 MPa
Fatigue Strength	380 MPa
Fracture Toughness	100 MPa-m ^{1/2}
Shear Modulus	44 GPa

2. DATA & INFORMATION COLLECTION

재료	정하중	동하중		충격하중
		반복하중	교번하중	
주철 및 취약 금속	4	6	10	15
연강	3	5	8	12
주강	2	5	8	15
주철 및 취약 금속	4	6	10	15
목재	7	10	15	20
석재	20	30	-	-

산악용 자전거이므로 충격 등을 고려하여 가장 큰 안전률을 고려

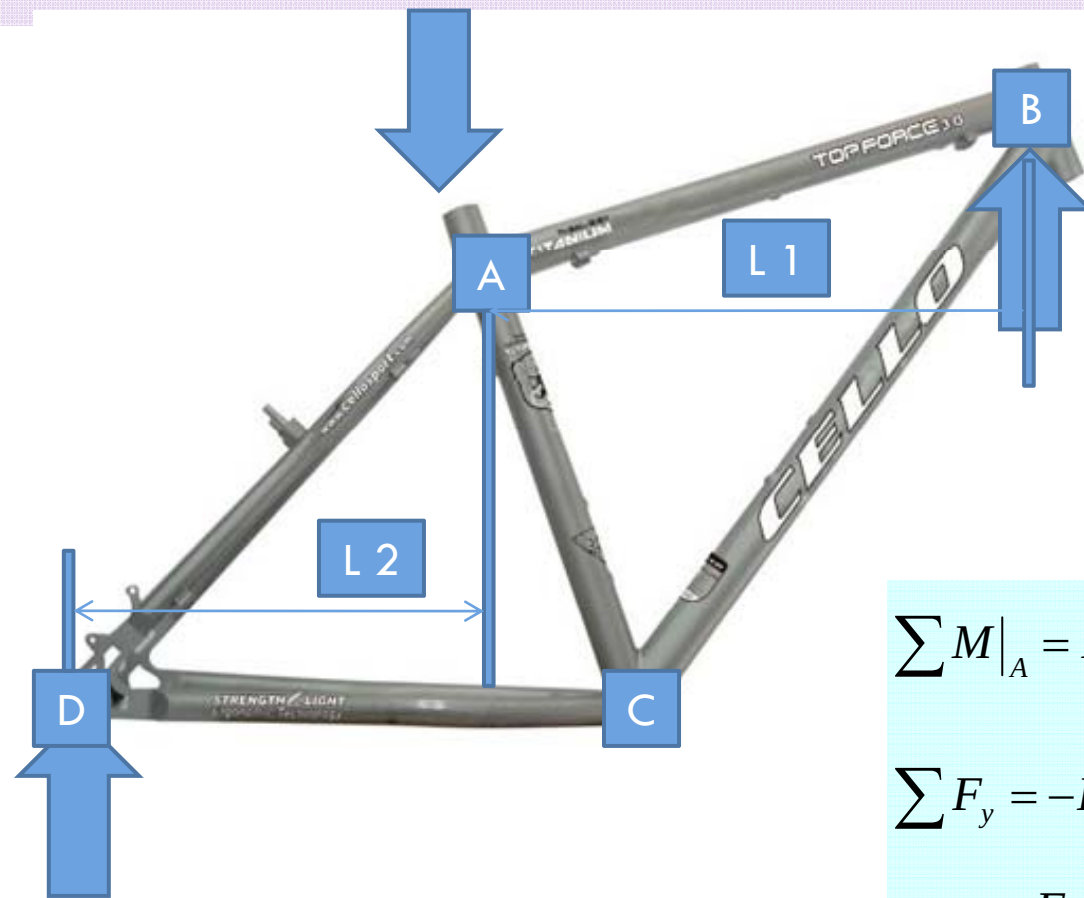
2. DATA & INFORMATION COLLECTION

최대 변형률에 대한 고려

보행자 및 차량하중 등 동하중을 주로 받는 구조물의 최대 허용처짐 단순 또는 연속 경간의 부재는 활하중과 충격으로 인한 처짐이 경간의 $1/800$ 을 초과하지 않아야 한다. 보행자가 지나가는 도시지역의 교량은 $1/1000$ 을 초과하지 않아야 한다.

최대 변형률은 $1/1000$

2. DATA & INFORMATION COLLECTION

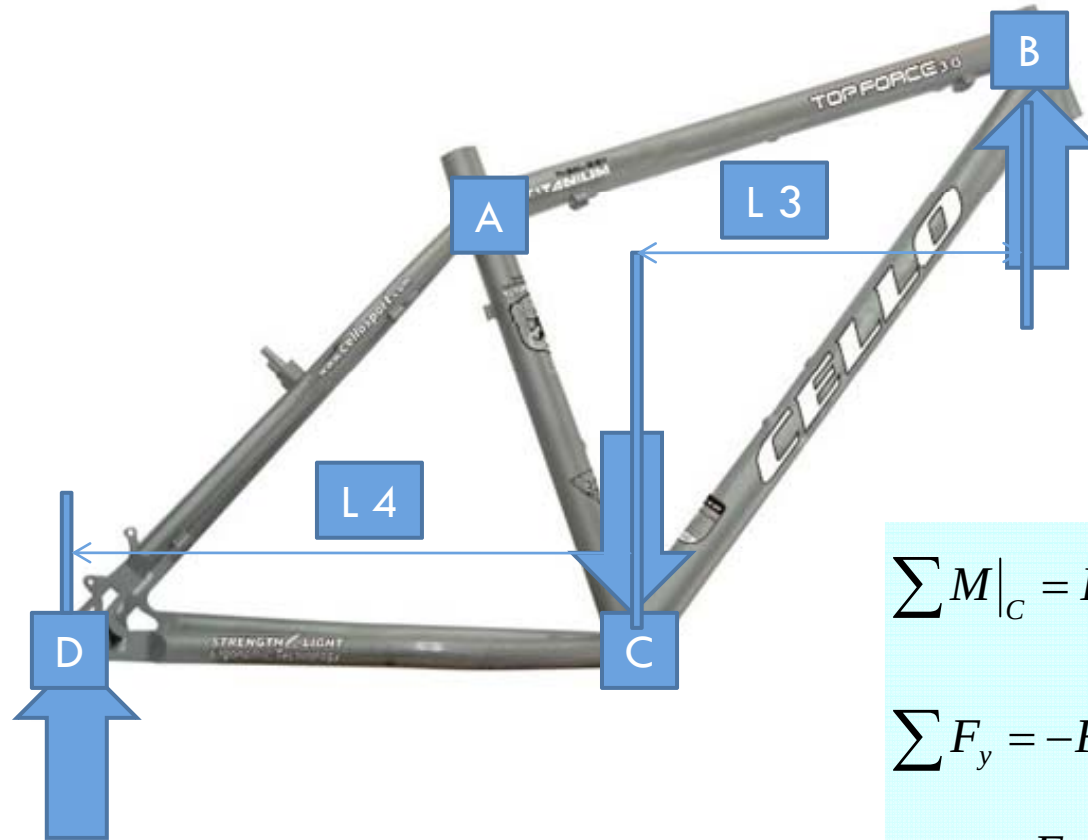


$$\sum M|_A = L_1 F_b - L_2 F_d = 0, \quad F_b = \frac{L_2}{L_1} F_d$$

$$\sum F_y = -F + F_b + F_d = 0, \quad F = \left(1 + \frac{L_2}{L_1}\right) F_d$$

$$F_d = \frac{F}{1 + \frac{L_2}{L_1}}, \quad F_b = \frac{L_2}{L_1} \cdot \frac{F}{1 + \frac{L_2}{L_1}}$$

2. DATA & INFORMATION COLLECTION

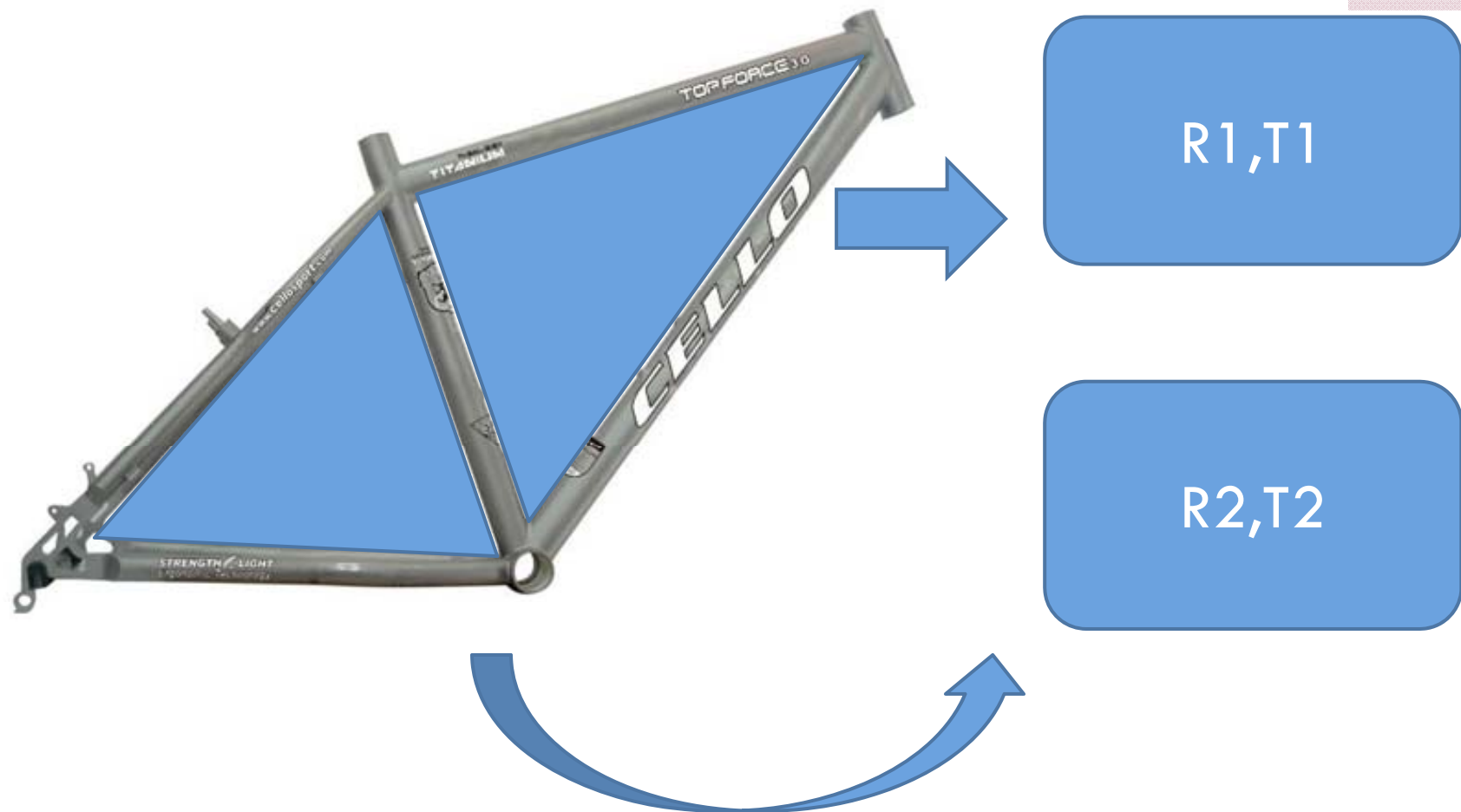


$$\sum M|_C = L_3 F_b - L_4 F_d = 0, \quad F_b = \frac{L_4}{L_3} F_d$$

$$\sum F_y = -F + F_b + F_d = 0, \quad F = \left(1 + \frac{L_4}{L_3}\right) F_d$$

$$F_d = \frac{F}{1 + \frac{L_4}{L_3}}, \quad F_b = \frac{L_4}{L_3} \cdot \frac{F}{1 + \frac{L_4}{L_3}}$$

3. DESIGN VARIABLES



4. IDENTIFICATION OF A CRITERION TO BE OPTIMIZED

$$\text{minimize_} F(R1, T1) = 2\pi R1 T1 \rho L_{total1}$$

$$\text{minimize_} F(R2, T2) = 2\pi R2 T2 \rho L_{total2}$$

$$mass_{total} = F(R1, T1) + F(R2, T2)$$

5. IDENTIFICATION OF CONSTRAINTS



$$\sum F_y = F_{AD} \sin \theta_{AD} - F + F_{AC} \sin \theta_{AC} - F_{AB} \sin \theta_{AB} = 0$$

$$\sum F_x = F_{AD} \cos \theta_{AD} - F_{AC} \cos \theta_{AC} - F_{AB} \cos \theta_{AB} = 0$$

$$F_{AC} = \frac{F_{AD} \cos \theta_{AD} - F_{AB} \cos \theta_{AB}}{\cos \theta_{AC}}$$

$$\sum F_y = F_d - F_{AD} \sin \theta_{AD} = 0, \quad F_{AD} = \frac{F_d}{\sin \theta_{AD}}$$

$$\sum F_x = -F_{CD} - F_{AD} \cos \theta_{AD} = 0, \quad F_{CD} = -F_{AD} \cos \theta_{AD} = -\frac{F_d}{\tan \theta_{AD}}$$

$$\sum F_y = F_b + F_{AB} \sin \theta_{AB} + F_{BC} \sin \theta_{BC} = 0$$

$$\sum F_x = +F_{AB} \cos \theta_{AB} + F_{BC} \cos \theta_{BC} = 0$$

$$F_{AB} = \frac{-F_{BC} \cos \theta_{BC}}{\cos \theta_{AB}}$$

$$F_{BC} = \frac{F_b}{\cos \theta_{BC} \tan \theta_{AB} - \sin \theta_{BC}} \quad F_{AB} = \frac{-\cos \theta_{BC}}{\cos \theta_{AB}} \frac{F_b}{\cos \theta_{BC} \tan \theta_{AB} - \sin \theta_{BC}}$$

5. IDENTIFICATION OF CONSTRAINTS

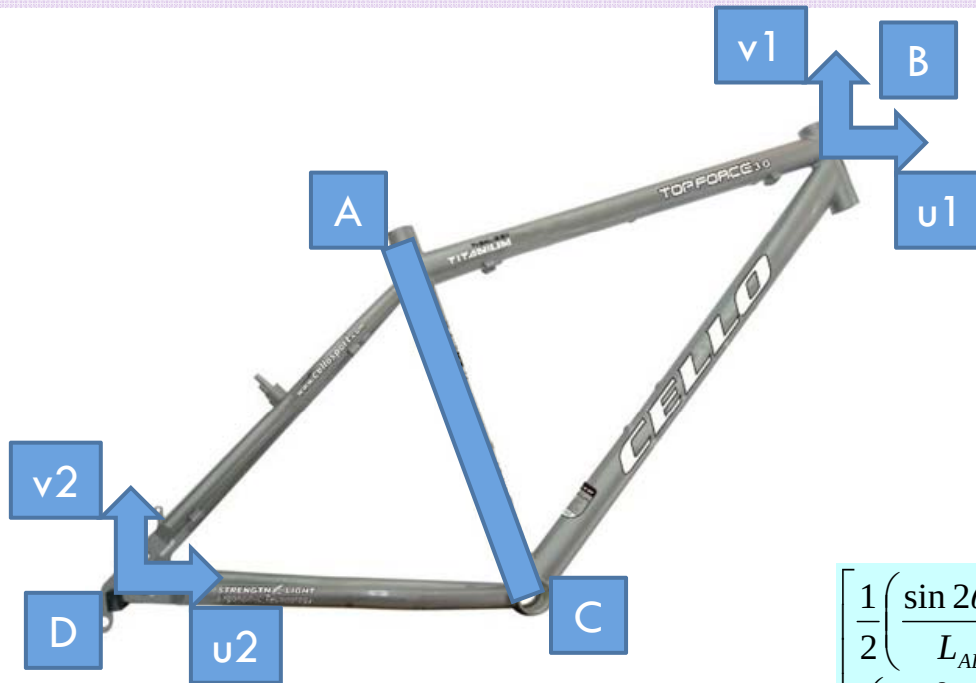


$$\begin{aligned}\sum F_y &= F_{AD} \sin \theta_{AD} + F_{AC} \sin \theta_{AC} - F_{AB} \sin \theta_{AB} = 0 \\ \sum F_x &= F_{AD} \cos \theta_{AD} - F_{AC} \cos \theta_{AC} - F_{AB} \cos \theta_{AB} = 0 \\ F_{AC} &= \frac{F_{AD} \cos \theta_{AD} - F_{AB} \cos \theta_{AB}}{\cos \theta_{AC}}\end{aligned}$$

$$\begin{aligned}\sum F_y &= F_d - F_{AD} \sin \theta_{AD} = 0, \quad F_{AD} = \frac{F_d}{\sin \theta_{AD}} \\ \sum F_x &= -F_{CD} - F_{AD} \cos \theta_{AD} = 0, \quad F_{CD} = -F_{AD} \cos \theta_{AD} = -\frac{F_d}{\tan \theta_{AD}}\end{aligned}$$

$$\begin{aligned}\sum F_y &= F_b + F_{AB} \sin \theta_{AB} + F_{BC} \sin \theta_{BC} = 0 \\ \sum F_x &= +F_{AB} \cos \theta_{AB} + F_{BC} \cos \theta_{BC} = 0 \\ F_{AB} &= \frac{-F_{BC} \cos \theta_{BC}}{\cos \theta_{AB}} \\ F_{BC} &= \frac{F_b}{\cos \theta_{BC} \tan \theta_{AB} - \sin \theta_{BC}} \quad F_{AB} = \frac{-\cos \theta_{BC}}{\cos \theta_{AB}} \frac{F_b}{\cos \theta_{BC} \tan \theta_{AB} - \sin \theta_{BC}}\end{aligned}$$

5. IDENTIFICATION OF CONSTRAINTS



$$F_{AB} = \frac{2\pi R_1 t_1 E}{L_{AB}} (u_1 \cos \theta_{AB} + v_1 \sin \theta_{AB})$$

$$F_{BC} = \frac{2\pi R_1 t_1 E}{L_{BC}} (u_1 \cos \theta_{BC} + v_1 \sin \theta_{BC})$$



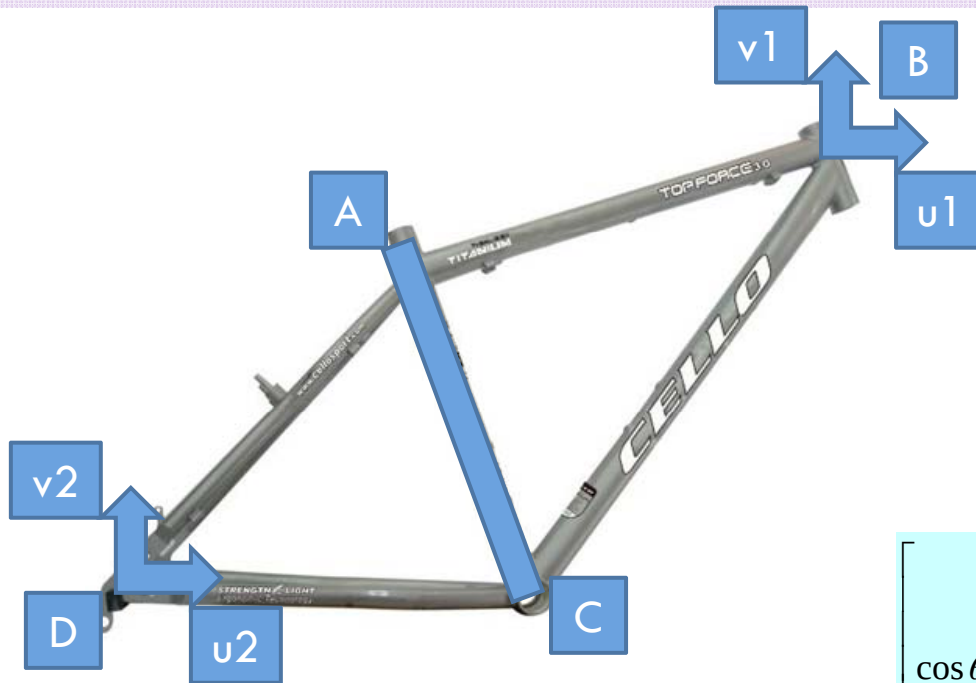
$$\sum F_y = F_b + F_{AB} \sin \theta_{AB} + F_{BC} \sin \theta_{BC} = 0$$

$$\sum F_x = +F_{AB} \cos \theta_{AB} + F_{BC} \cos \theta_{BC} = 0$$

$$\begin{bmatrix} \frac{1}{2} \left(\frac{\sin 2\theta_{AB}}{L_{AB}} + \frac{\sin 2\theta_{BC}}{L_{BC}} \right) & \left(\frac{\sin^2 \theta_{AB}}{L_{AB}} + \frac{\sin^2 \theta_{BC}}{L_{BC}} \right) \\ \left(\frac{\cos^2 \theta_{AB}}{L_{AB}} + \frac{\cos^2 \theta_{BC}}{L_{BC}} \right) & \frac{1}{2} \left(\frac{\sin 2\theta_{AB}}{L_{AB}} + \frac{\sin 2\theta_{BC}}{L_{BC}} \right) \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \frac{F_b}{2\pi E R_1 t_1} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \left(\frac{\sin 2\theta_{AB}}{L_{AB}} + \frac{\sin 2\theta_{BC}}{L_{BC}} \right) & \left(\frac{\sin^2 \theta_{AB}}{L_{AB}} + \frac{\sin^2 \theta_{BC}}{L_{BC}} \right) \\ \left(\frac{\cos^2 \theta_{AB}}{L_{AB}} + \frac{\cos^2 \theta_{BC}}{L_{BC}} \right) & \frac{1}{2} \left(\frac{\sin 2\theta_{AB}}{L_{AB}} + \frac{\sin 2\theta_{BC}}{L_{BC}} \right) \end{bmatrix}^{-1} \begin{bmatrix} \frac{F_b}{2\pi E R_1 t_1} \\ 0 \end{bmatrix}$$

5. IDENTIFICATION OF CONSTRAINTS



$$F_{AD} = \frac{2\pi R_2 t_2 E}{L_{AD}} (u_2 \cos \theta_{AD} + v_2 \sin \theta_{AD})$$

$$F_{CD} = \frac{2\pi R_2 t_2 E}{L_{CD}} (u_2 \cos \theta_{CD} + v_2 \sin \theta_{CD})$$



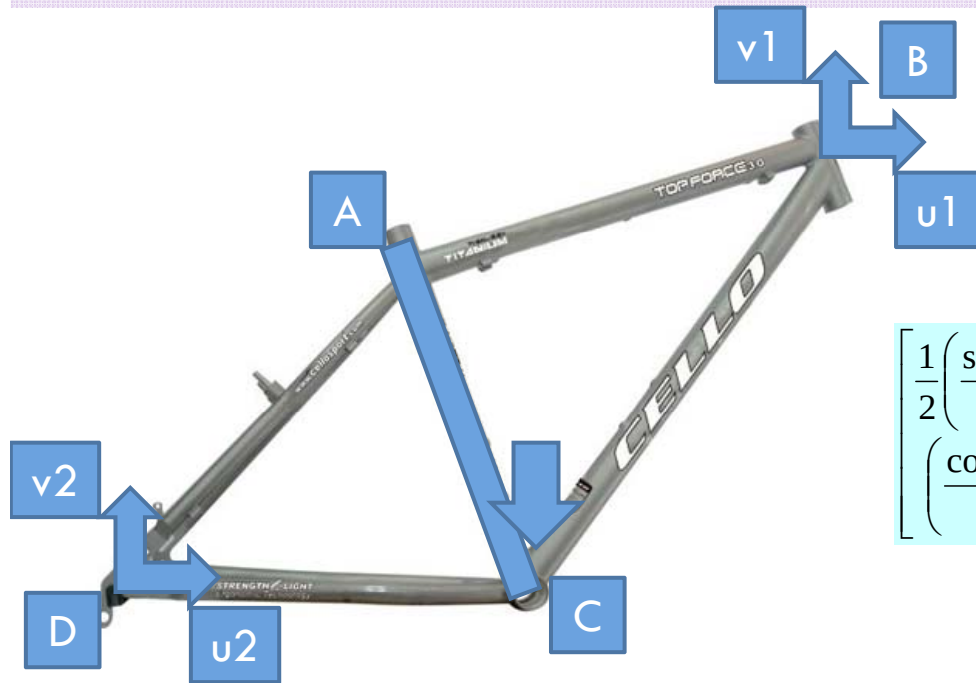
$$\sum F_y = F_d - F_{AD} \sin \theta_{AD} = 0$$

$$\sum F_x = F_{CD} - F_{AD} \cos \theta_{AD} = 0,$$

$$\begin{bmatrix} \frac{1}{2} \sin 2\theta_{AD} & \sin^2 \theta_{AD} \\ \cos \theta_{CD} - \frac{1}{2} \cos^2 \theta_{AD} & \sin \theta_{CD} - \frac{1}{2} \sin^2 \theta_{AD} \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{F_d}{2\pi E R_2 t_2} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} u_2 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \sin 2\theta_{AD} & \sin^2 \theta_{AD} \\ \cos \theta_{CD} - \frac{1}{2} \cos^2 \theta_{AD} & \sin \theta_{CD} - \frac{1}{2} \sin^2 \theta_{AD} \end{bmatrix}^{-1} \begin{bmatrix} \frac{F_d}{2\pi E R_2 t_2} \\ 0 \end{bmatrix}$$

5. IDENTIFICATION OF CONSTRAINTS



$$\begin{bmatrix} \frac{1}{2} \sin 2\theta_{AD} & \sin^2 \theta_{AD} \\ \cos \theta_{CD} - \frac{1}{2} \cos^2 \theta_{AD} & \sin \theta_{CD} - \frac{1}{2} \sin^2 \theta_{AD} \end{bmatrix} \begin{bmatrix} u_4 \\ v_4 \end{bmatrix} = \begin{bmatrix} \frac{F_d}{2\pi E R_2 t_2} \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} \left(\frac{\sin 2\theta_{AB}}{L_{AB}} + \frac{\sin 2\theta_{BC}}{L_{BC}} \right) & \left(\frac{\sin^2 \theta_{AB}}{L_{AB}} + \frac{\sin^2 \theta_{BC}}{L_{BC}} \right) \\ \left(\frac{\cos^2 \theta_{AB}}{L_{AB}} + \frac{\cos^2 \theta_{BC}}{L_{BC}} \right) & \frac{1}{2} \left(\frac{\sin 2\theta_{AB}}{L_{AB}} + \frac{\sin 2\theta_{BC}}{L_{BC}} \right) \end{bmatrix} \begin{bmatrix} u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} \frac{F_b}{2\pi E R_1 t_1} \\ 0 \end{bmatrix}$$

$$P_c = \frac{\pi^2 EI}{4l^2}, \quad \sigma_c = \frac{\pi^2 ER^2}{8l^2}$$

페달에 힘이 주어지는
경우는 F_b 와 F_d 를
달리하여 계산

5. IDENTIFICATION OF CONSTRAINTS

$$\text{minimize } f(R_1, t_1) = 2\pi R_1 t_1 \rho L_{total1}$$

안장에 하중을 주었을 때

$$g1_ \sigma_{AB} - \sigma_{allow} / SF \leq 0$$

$$g2_ \sigma_{BC} - \sigma_{allow} / SF \leq 0$$

$$g3_ \sigma_{AC} - \sigma_{allow} / SF \leq 0$$

$$g4_ \sigma_{AB} - \sigma_{buckling} / SF \leq 0$$

$$g5_ u_1 - L_{AB} / 1000 \leq 0$$

$$g6_ v_1 - L_{AB} / 1000 \leq 0$$

$$g7_ t_1 - R_1 \leq 0$$

$$g8_ R_1 - 0.015 \leq 0$$

페달에 하중을 주었을 때

$$g9_ \sigma_{AB} - \sigma_{allow} / SF \leq 0$$

$$g10_ \sigma_{BC} - \sigma_{allow} / SF \leq 0$$

$$g11_ \sigma_{AC} - \sigma_{allow} / SF \leq 0$$

$$g12_ \sigma_{AB} - \sigma_{buckling} / SF \leq 0$$

$$g13_ u_3 - L_{AB} / 1000 \leq 0$$

$$g14_ v_3 - L_{AB} / 1000 \leq 0$$

$$\text{minimize } f(R_2, t_2) = 2\pi R_2 t_2 \rho L_{total2}$$

안장에 하중을 주었을 때

$$g1_ \sigma_{CD} - \sigma_{allow} / SF \leq 0$$

$$g2_ \sigma_{AD} - \sigma_{allow} / SF \leq 0$$

$$g3_ \sigma_{CD} - \sigma_{buckling} / SF \leq 0$$

$$g4_ u_2 - L_{CD} / 1000 \leq 0$$

$$g5_ v_2 - L_{CD} / 1000 \leq 0$$

$$g6_ t_2 - R_2 \leq 0$$

$$g7_ R_2 - 0.005 \leq 0$$

페달에 하중을 주었을 때

$$g8_ \sigma_{CD} - \sigma_{allow} / SF \leq 0$$

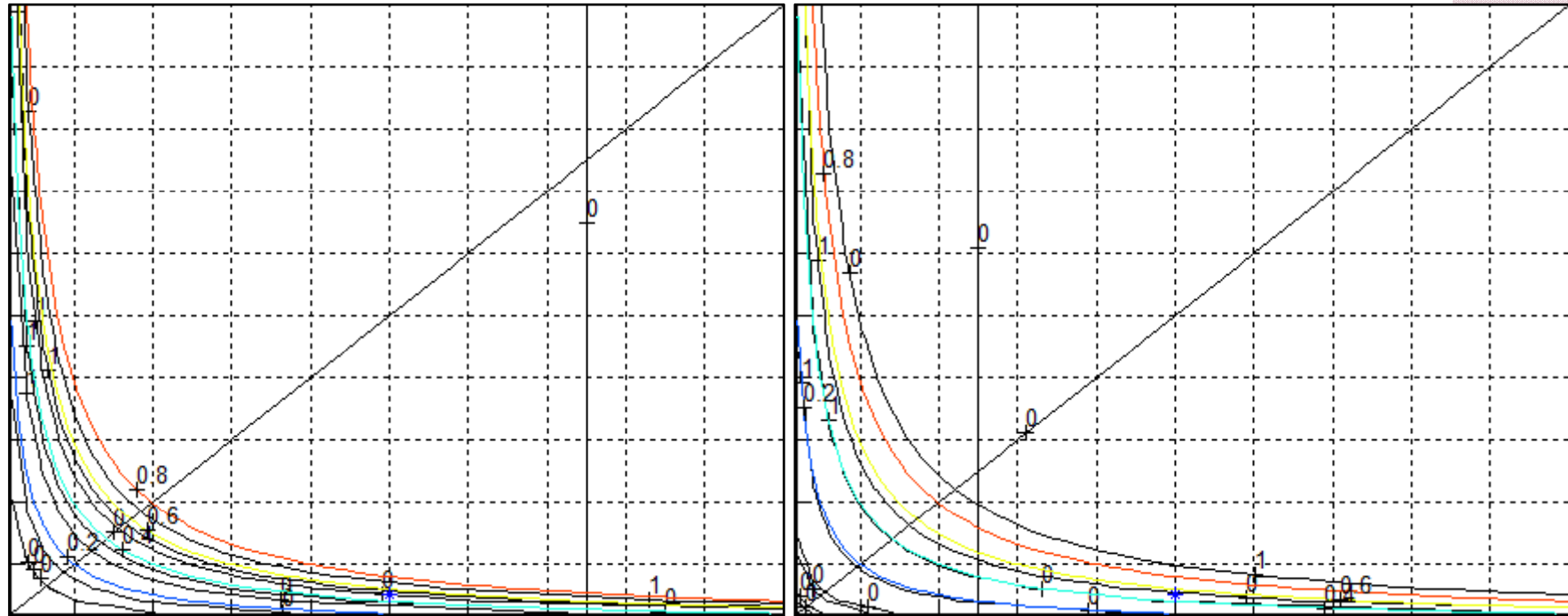
$$g9_ \sigma_{AD} - \sigma_{allow} / SF \leq 0$$

$$g3_ \sigma_{CD} - \sigma_{buckling} / SF \leq 0$$

$$g4_ u_4 - L_{CD} / 1000 \leq 0$$

$$g6_ v_4 - L_{CD} / 1000 \leq 0$$

결과값 비교(excel solver)

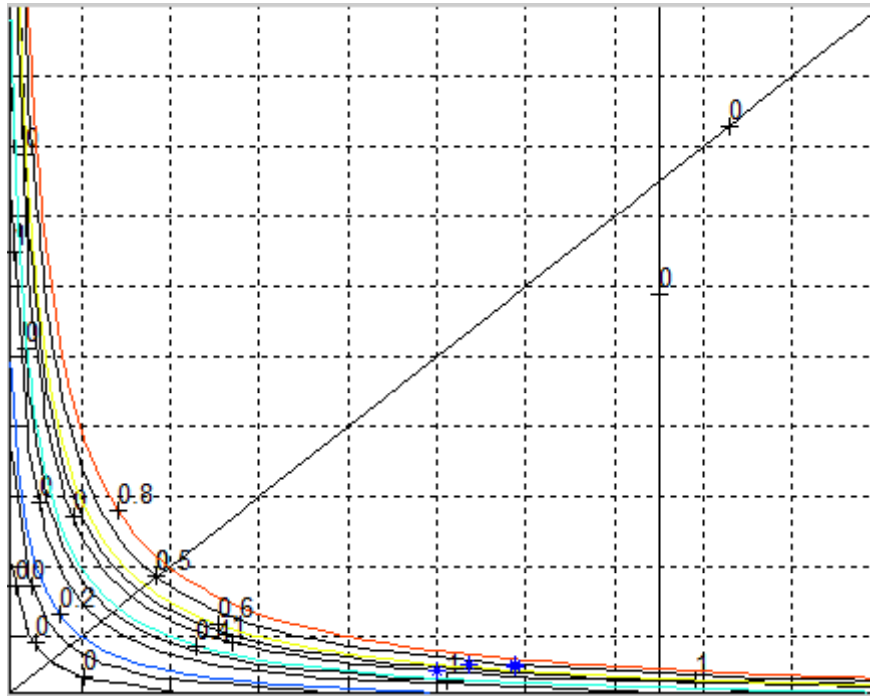


초기값(0.1,0.01)
 $R1=0.015\text{m}$ $t1=0.906\text{mm}$
 $\text{Mass}=0.692\text{kg}$

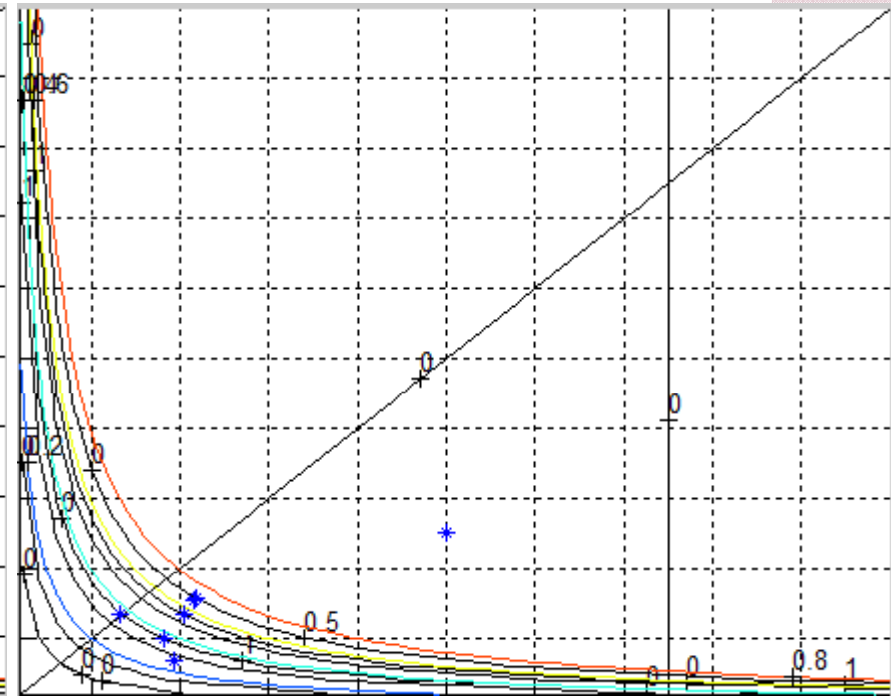
초기값(0.1,0.01)
 $R2=0.00494\text{m}$ $t2=1.96\text{mm}$
 $\text{Mass}=0.635\text{kg}$

Total Mass=1.327kg(feasible region 내의 점을 초기점으로 해도 같은 결과)

결과값 비교(matlab_fmincon)



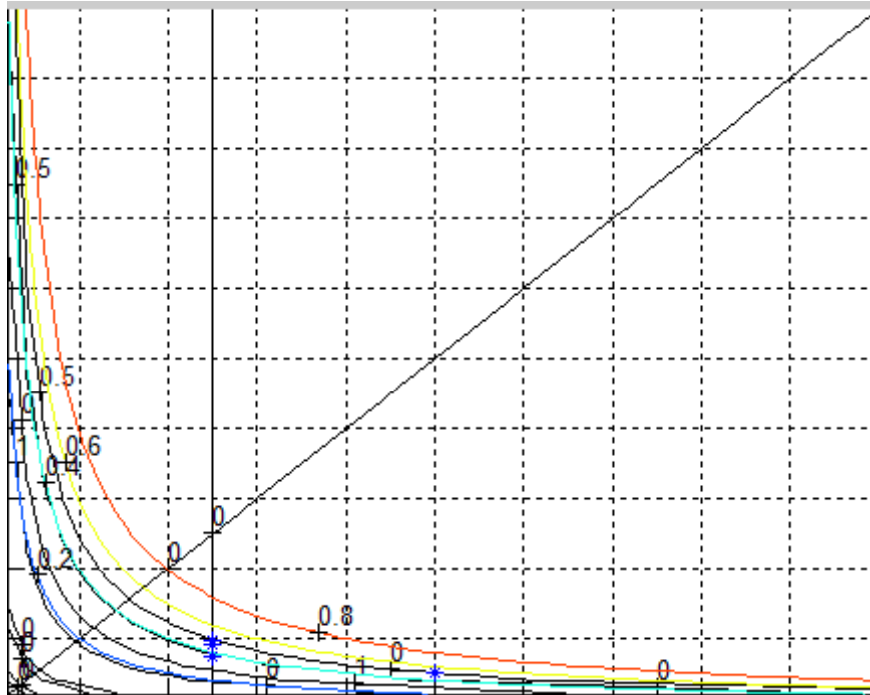
초기값(0.01,0.001)
 $R2=0.0118m$ $t2=1.2mm$
Mass=0.6920kg



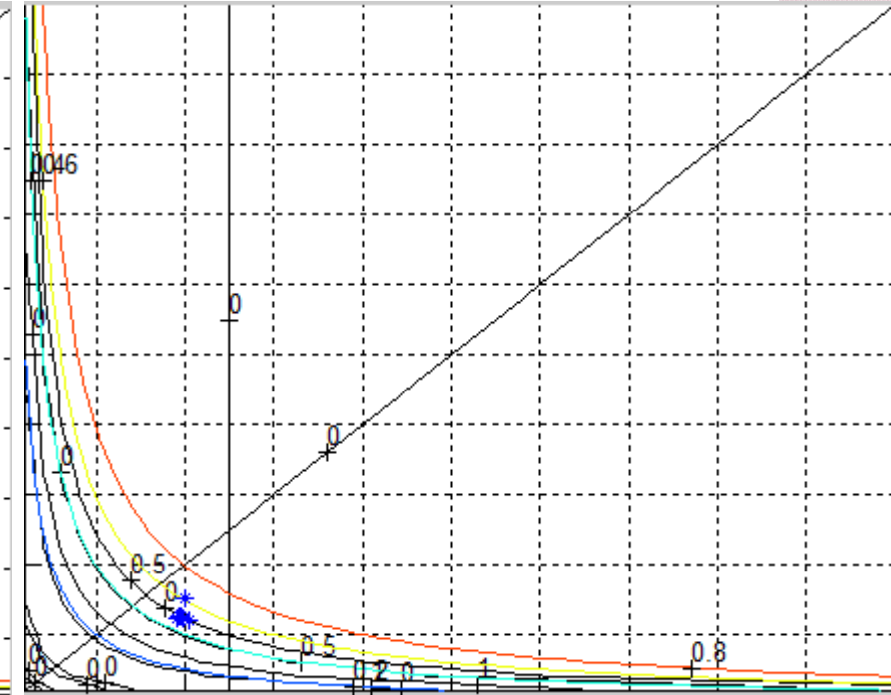
초기값(0.01,0.005)
 $R2=0.0044m$ $t2=3.1mm$
Mass= 0.6920kg

(feasible region 내의 점을 초기 점으로 경우 다른 점을 출력)

결과값 비교(matlab_fmincon)



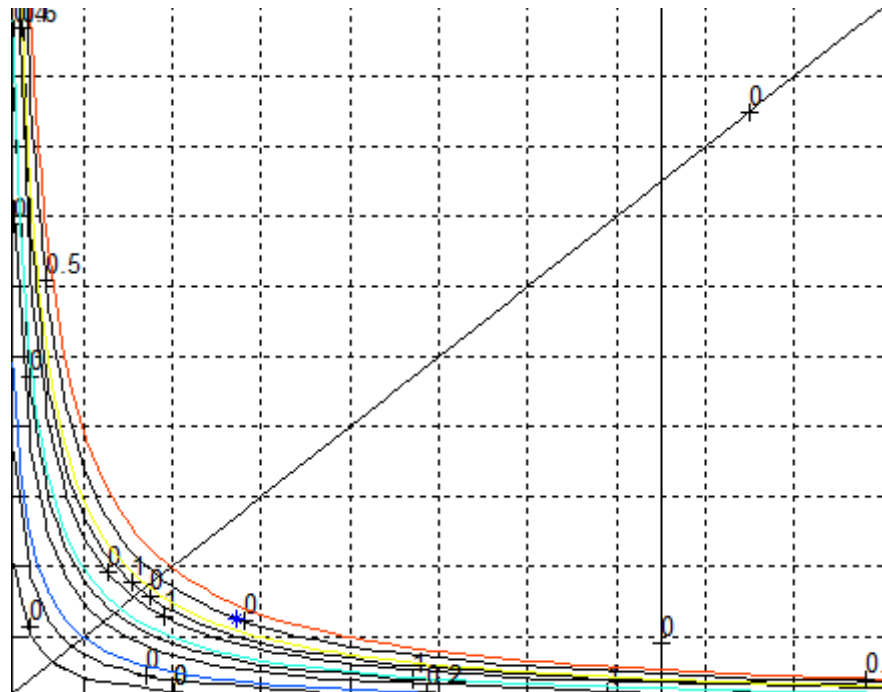
초기값(0.01,0.001)
 $R2=0.005m$ $t2=1.96mm$
Mass=0.635kg



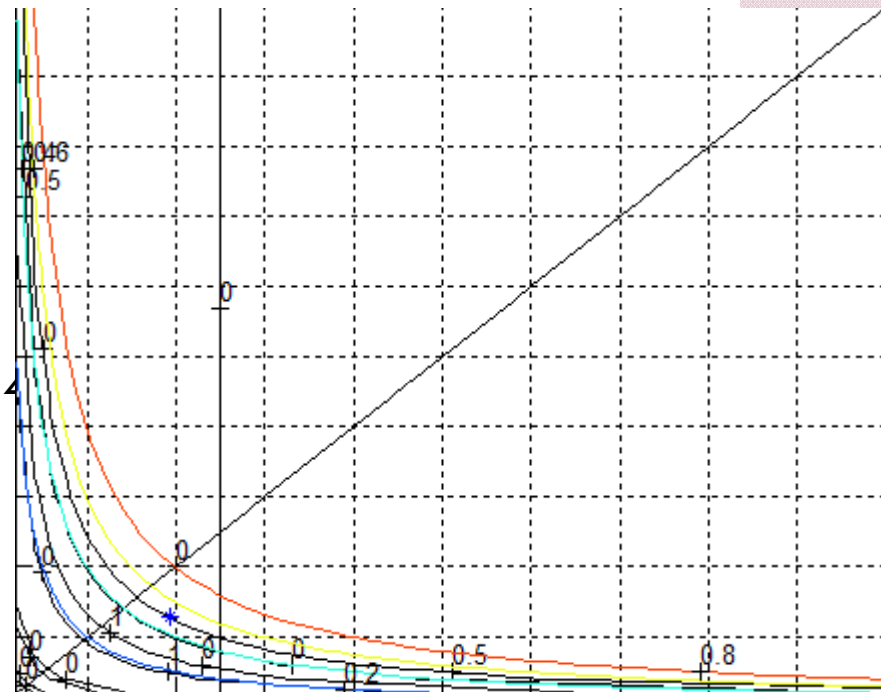
초기값(0.004,0.003)
 $R2=0.0039m$ $t2=2.5mm$
Mass=0.635kg

Total Mass=1.327kg(feasible region 내의 점을 초기 점으로 경우 다른 점을 출력)

결과값 비교(Ga)



초기값(0.01,0.001)
R2=0.01m t2=1mm
Mass=0.5092kg



초기값(0.01,0.001)
R2=0.005m t2=1 mm
Mass=0.3274kg

feasible region 바깥의 값을 출력

결과값 비교

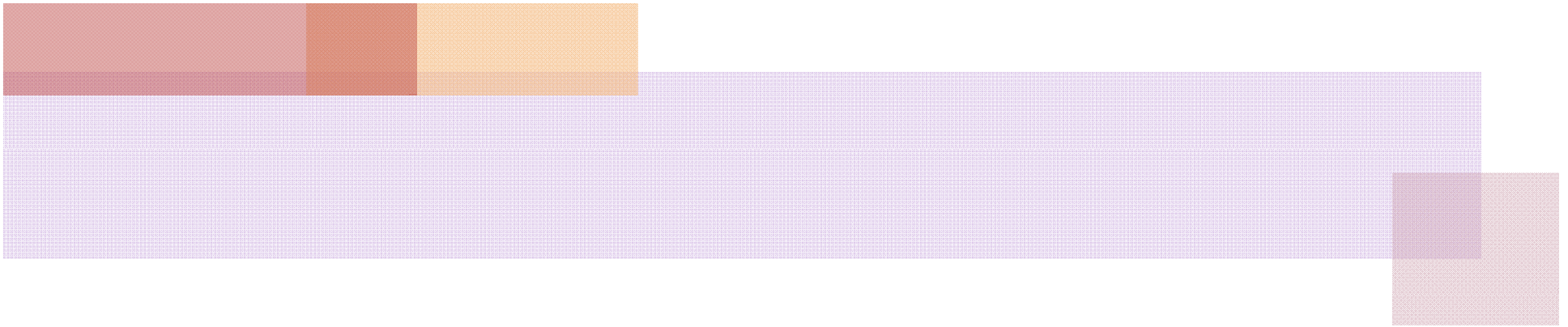
	R1	t1	R2	t2	Total mass
Excel solver	0.015m	0.906mm	0.00494m	1.96mm	1.327kg
fmincon	0.0118m	1.2mm	0.005m	1.96mm	1.327kg
ga	0.01m	1mm	0.005m	1 mm	0.8366kg

검토

Ti-3AL-2.5V 티타늄 소재
중량 : 1340g(M(17) 모델)

R1	t1	R2	t2	Total mass
0.015m	0.906mm	0.00494m	1.96mm	1.327kg
0.0118m	1.2mm	0.005m	1.96mm	1.327kg

최적화
성공



감 사 합 니 다.