Bicycle Frame

Project 1.

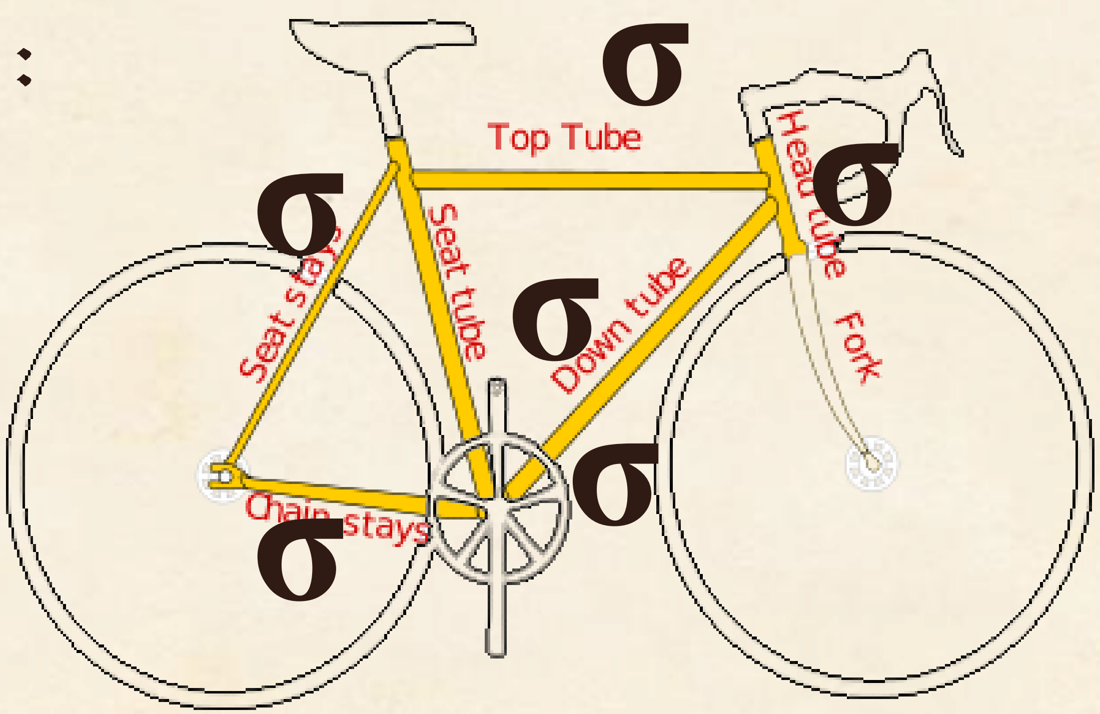
Design Optimization

Archimedes

:이신성, 김승현

Problem Definition

- Objective : Bicycle frame lightening.
- Requirement :



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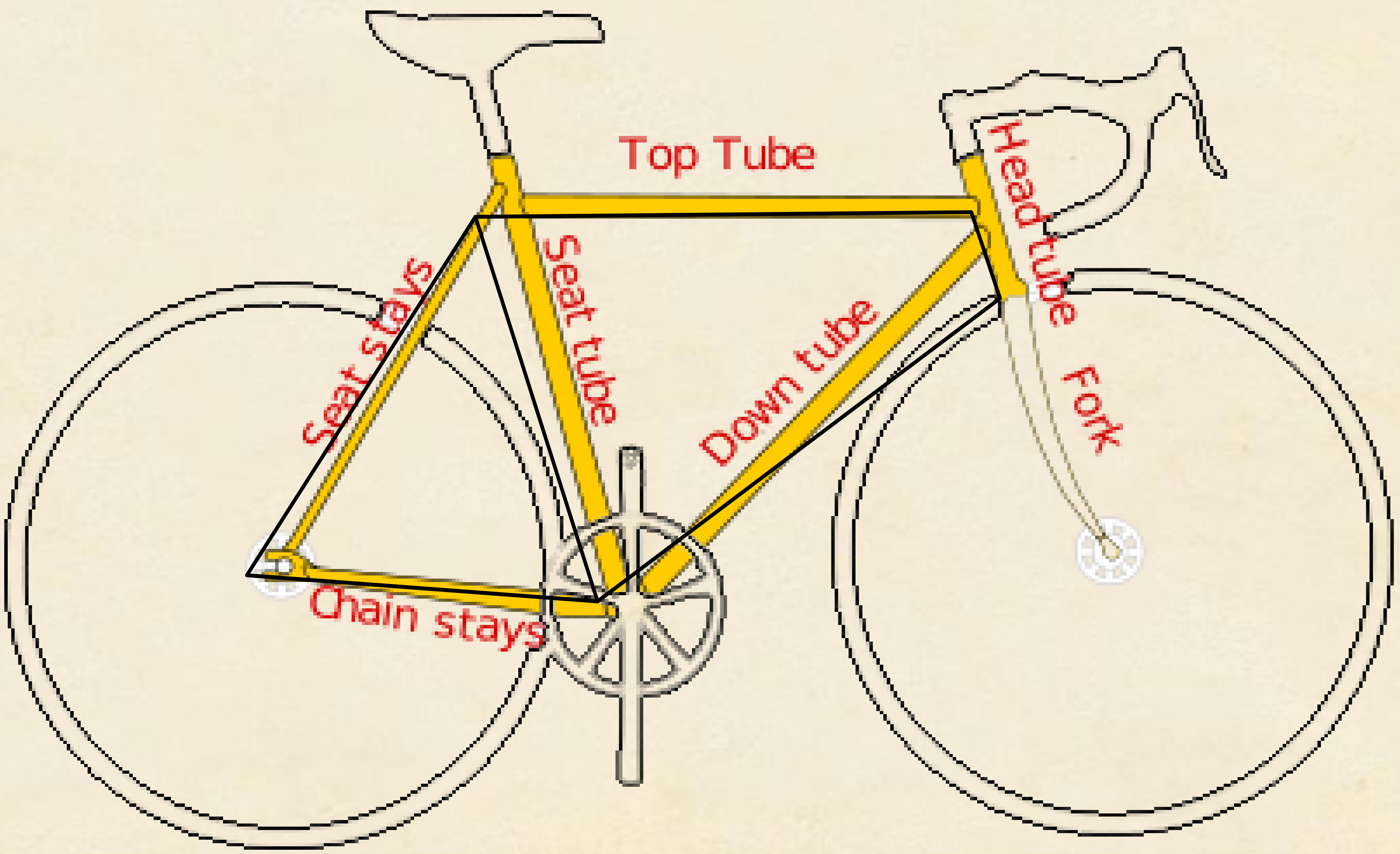
Specified Data



Specified Data

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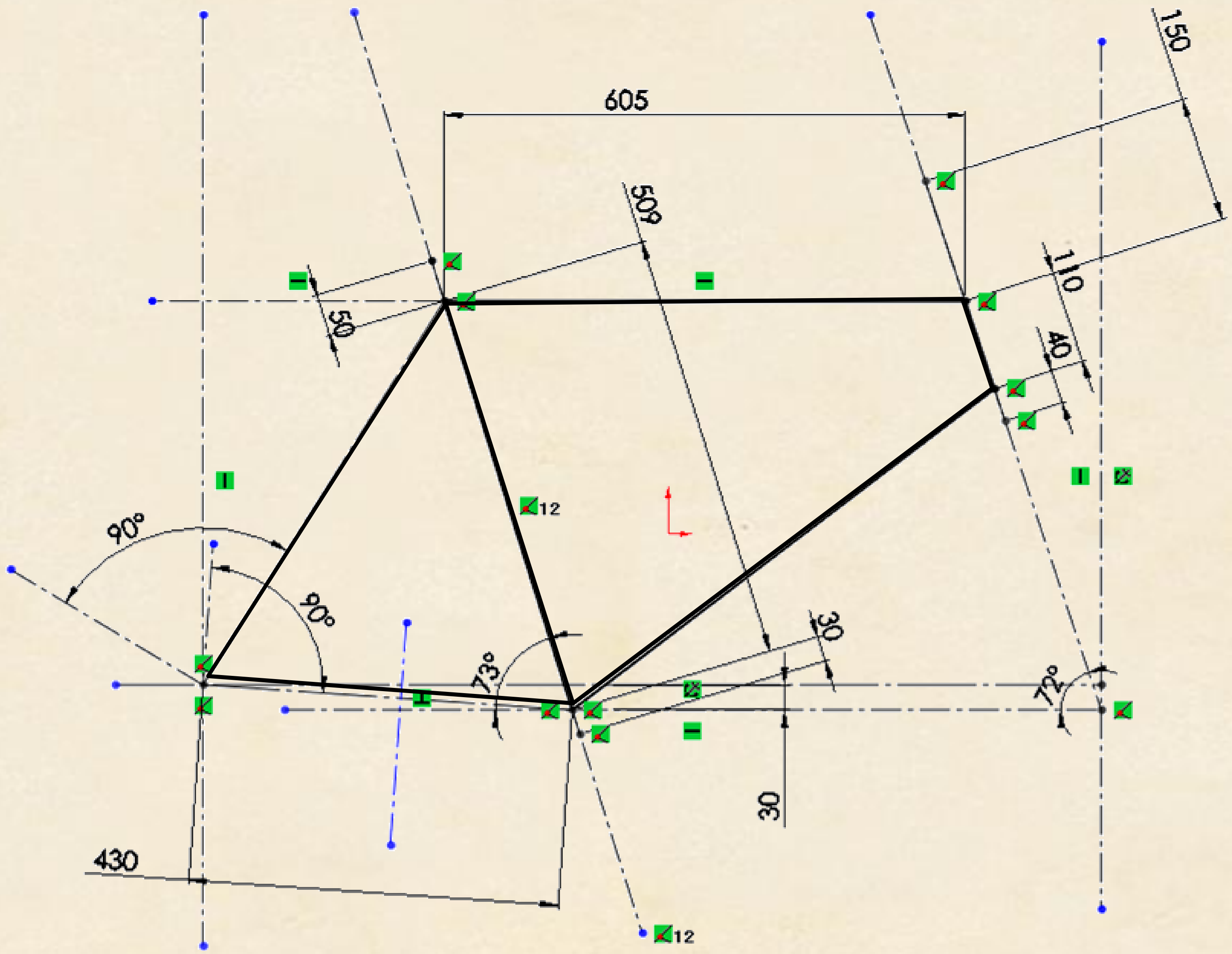
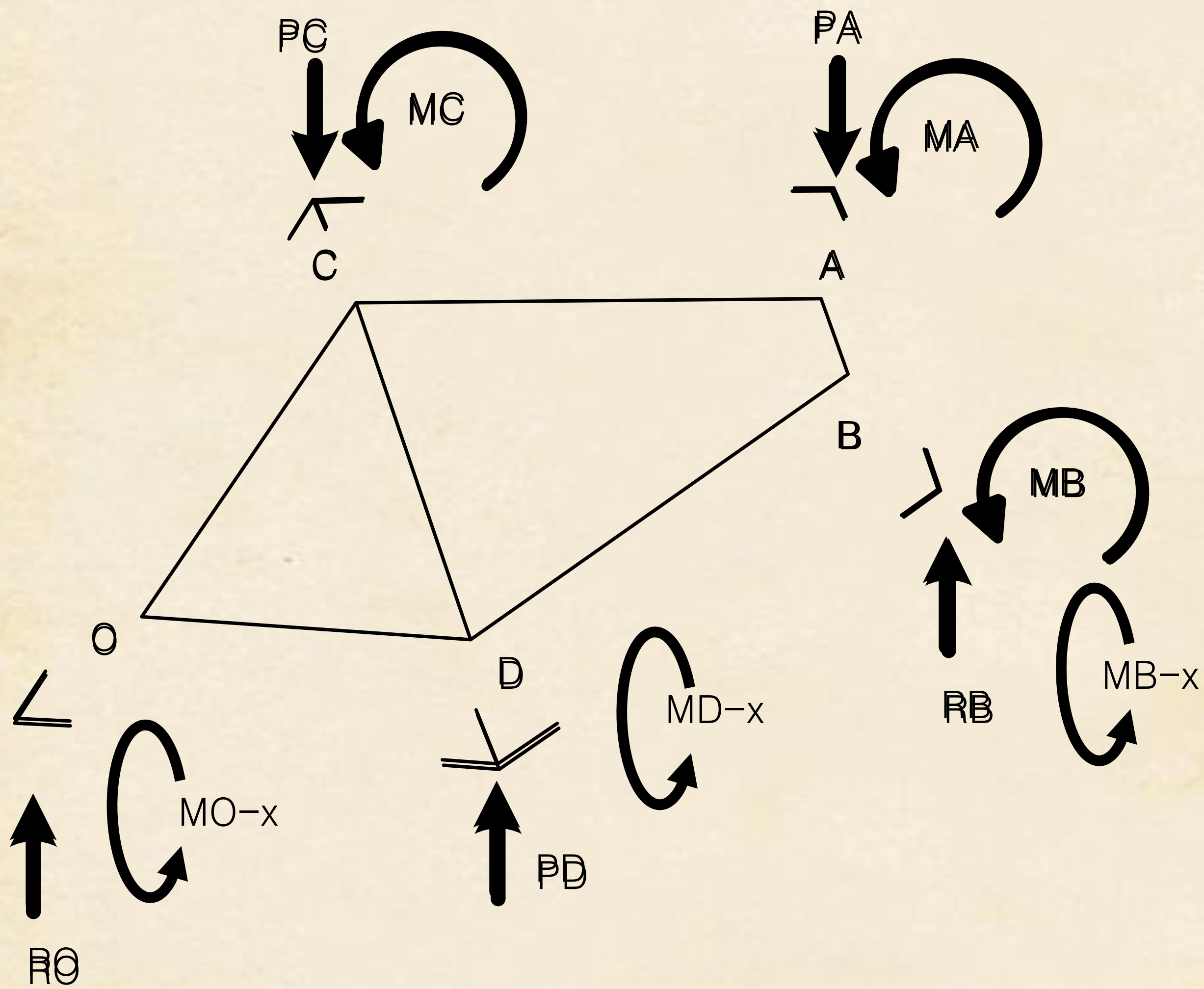


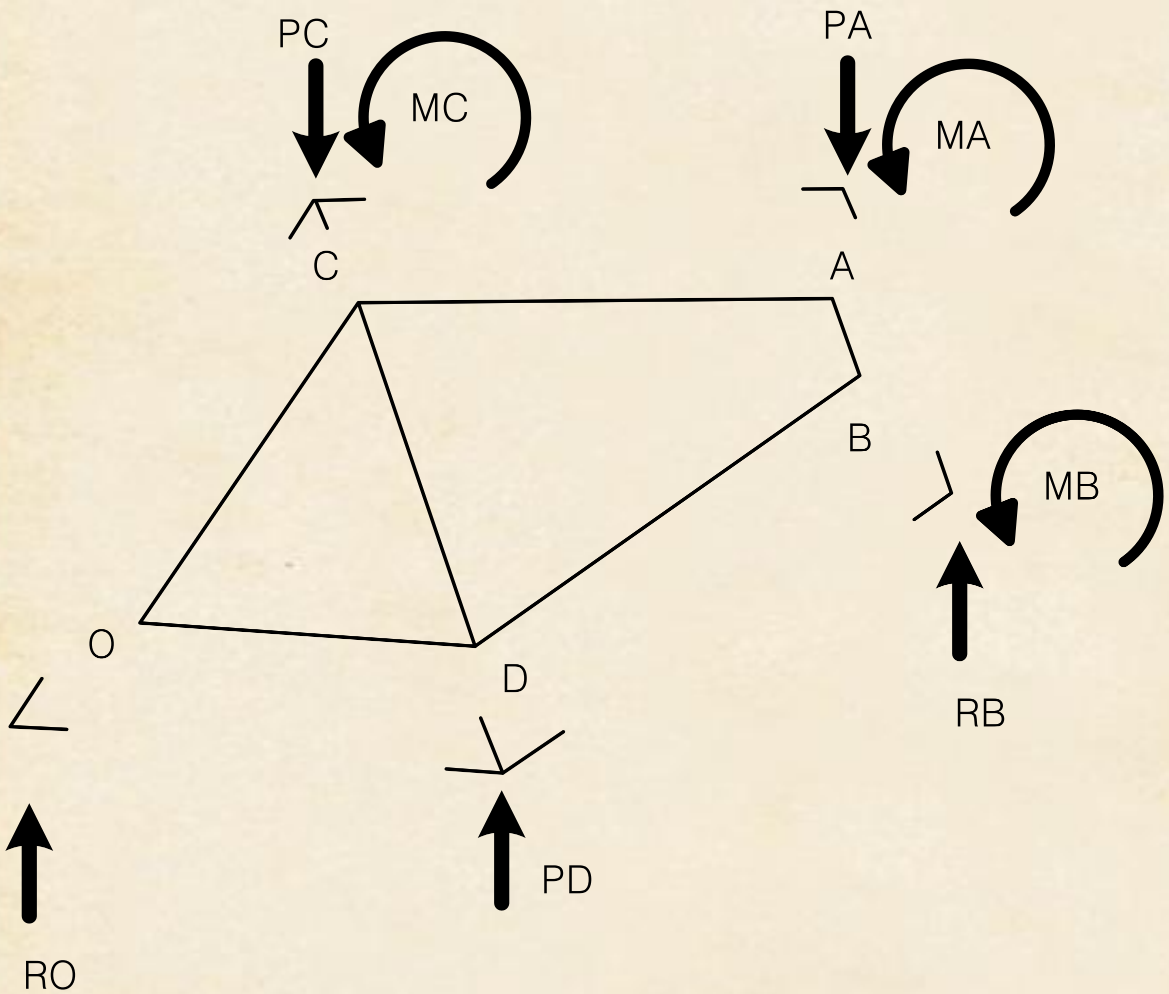
Condition Define



Complicate!

→ Neglect Torsion.

Condition Define



unknown : RO,RB,MB

Indeterminate!

→ Singularity function

```
a=0.28014; b=0.42895; c=0.88514; d=0.91913;
syms Oy C1
eq1='Oy/2*(d)^2-300*(d-a)^2+120*(d-a).....
     +100*(d-b)^2-100*(d-c)^2+40*(d-c)+C1=0';
eq2='Oy/6*(d)^3-100*(d-a)^3+60*(d-a)^2.....
     +100/3*(d-b)^3-100/3*(d-c)^3+20*(d-c)^2+C1*d=0';
[Oy, C1]=solve(eq1,eq2,Oy,C1)
```

```
a=0.28014;b=0.42895;c=0.88514; d=0.91913;
C1=82.0490;Oy=-14.1155;
syms Mb Rb
eq3='Oy-600+200-200+Rb=0';
eq4='120+40+Mb-600*a+200*b-200*c+Rb*d=0';
[Mb, Rb]=solve(eq3,eq4,Mb,Rb)
```

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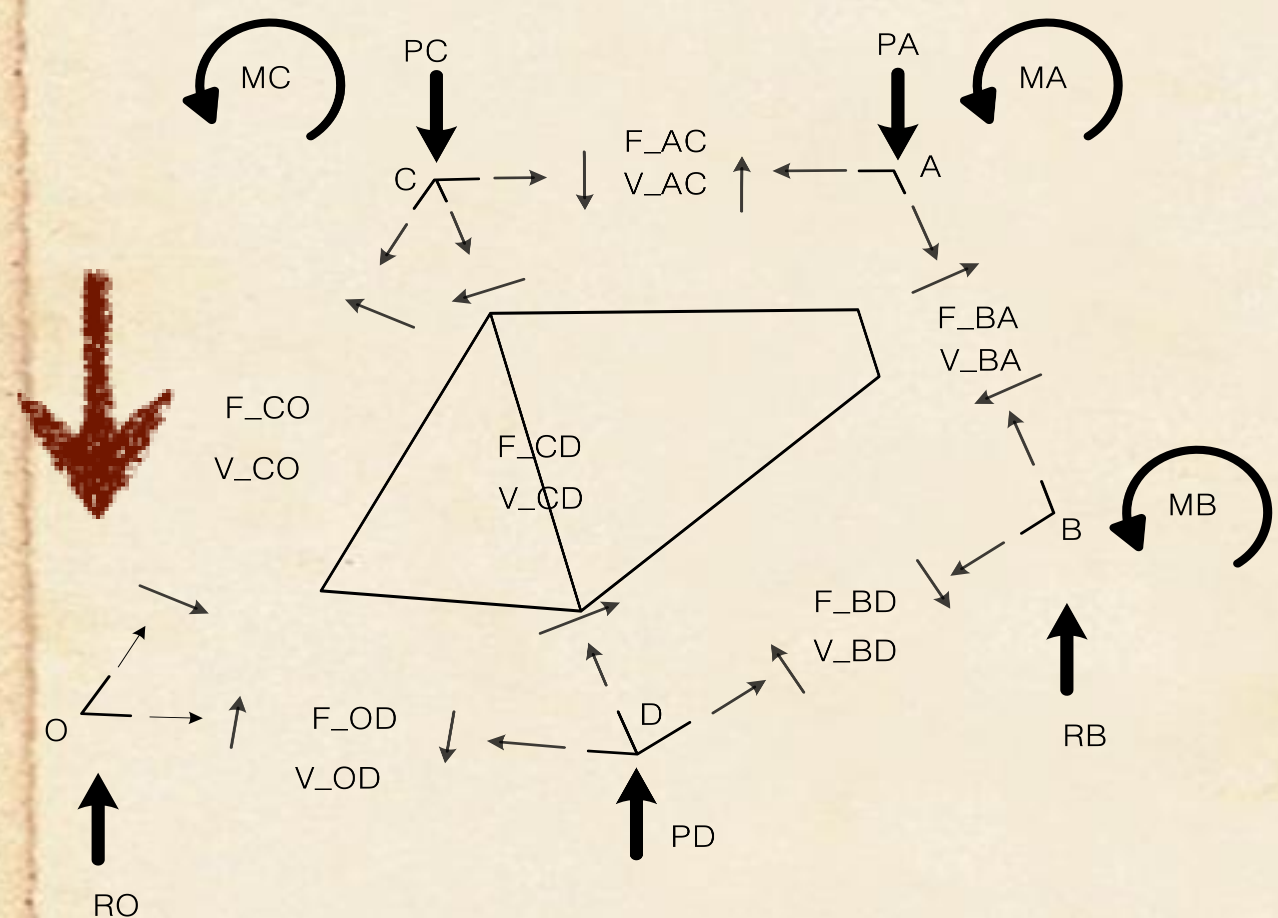
Data Analysis V,F



Indeterminate!!!!!

Take unknown load
+ ~~Castigliano's Theorem~~

Can't solve!!!!!!!!

Assume load
to be divide!

Data Analysis V,F

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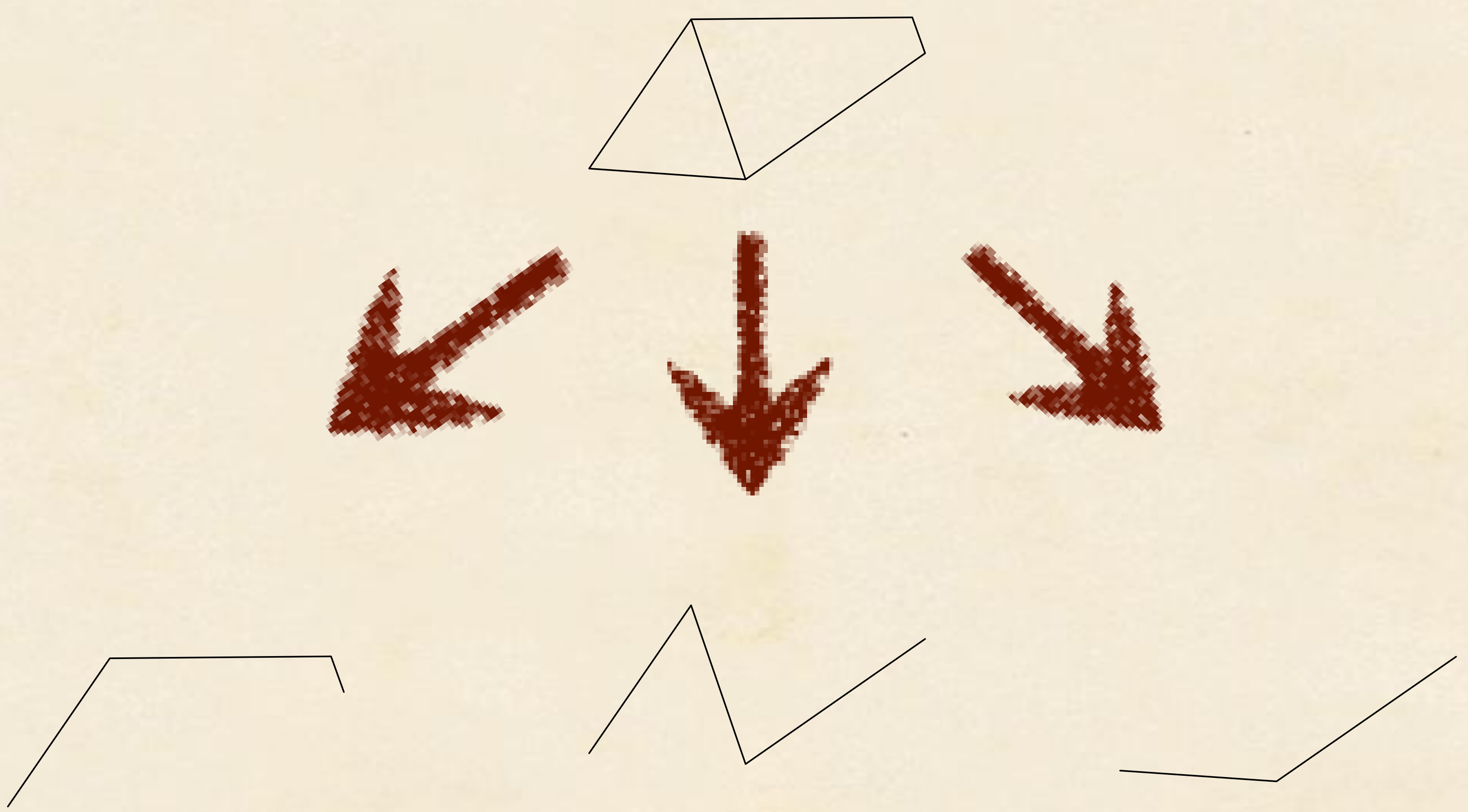

Dime chaser

Data Analysis V,F

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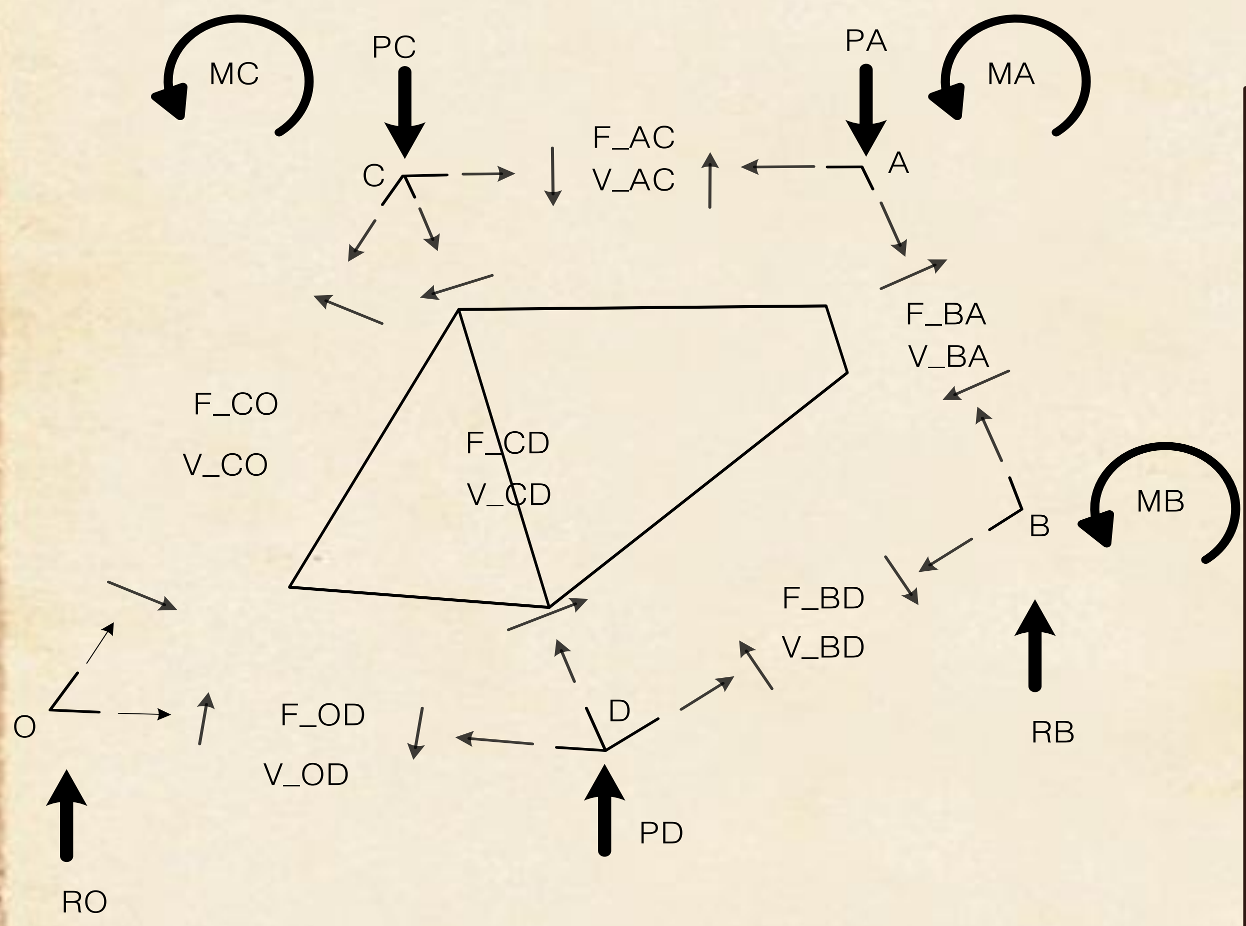
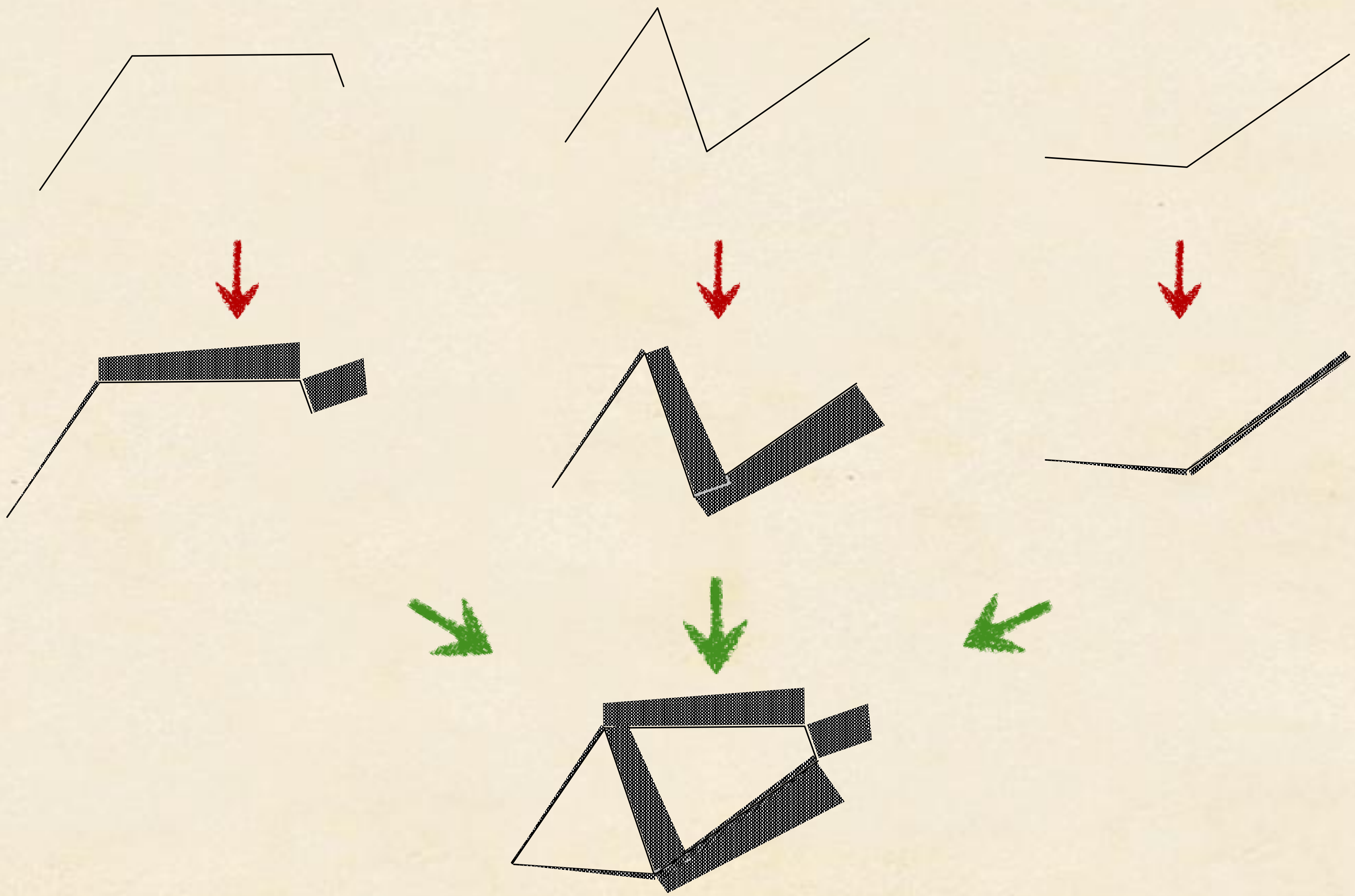
In MATLAB!




```
syms F1 F2 F3 F4 F5 F6 V1 V2 V3 V4 V5 V6 ;
Oy=491.92;By=108.08;
eq1='V6*cos(1.50098)+F6*cos(0.06981)=0';
eq2='V6*sin(1.50098)-F6*sin(0.06981)+245.96=0';
[F6,V6]=solve(eq1,eq2)
eq3='F5*cos(1.0206)-V5*cos(0.5501)=0';
eq4='F5*sin(1.0206)+V5*sin(0.5501)+245.96=0';
[F5,V5]=solve(eq3, eq4)
eq5='-F3*cos(0.6621)+V3*cos(0.9086)=0';
eq6='-F3*sin(0.6621)-V3*sin(0.9086)+54.04=0';
[F3,V3]=solve(eq5, eq6)
eq7='-F1*cos(1.2566)-V1*cos(0.3141)=0';
eq8='F1*sin(1.2566)-V1*sin(0.3141)+54.04=0';
[F1,V1]=solve(eq7, eq8)
F1=-51.396;V1=16.7;
eq9='-51.396*cos(1.2566)+16.7*cos(0.3141)-F2=0';
eq10='51.396*sin(1.2566)+16.7*sin(0.3141)-V2-200=0';
[F2,V2]=solve(eq9, eq10)
```

Data Analysis M



Verified 93% at **Mb**

Dime chaser

Design optimization

Minimize$_{a,b}$ $f = \rho L A$

$$\leftarrow A = \pi\{(a+t)(b+t) - ab\}$$

a : Major axis, b : Minor axis

Subeject to

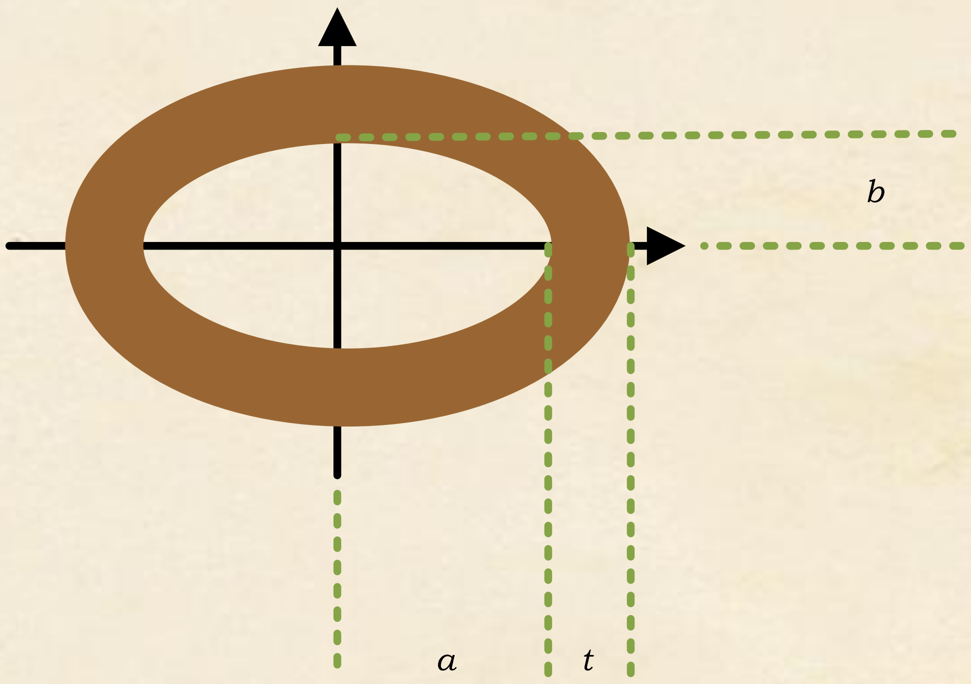
$$\begin{cases} g_1 = \dfrac{4Mb}{\pi[(a+t)(b^3+3b^2t+3bt^2+t^3)-ab^3]} + \dfrac{F}{A} - \sigma_a \le 0 \\ g_2 = \dfrac{4}{3}\cdot\dfrac{V}{A} - \tau_a \le 0 \\ g_3 = -a < 0 \\ g_4 = -b < 0 \end{cases}$$

$\sigma_a = 410\,\mathrm{Mpa}$

$\tau_a = 220\,\mathrm{Mpa}$

$t = 0.005\,\mathrm{m}$

F : Inner force
V : Shear force
M : $\mathrm{Moment}_{\max}$



Karush-Kuhn-Tucker METHOD

$$\mathscr{L} = f + \sum_{j=1}^{4} u_j - \{g_j(x) + s_j{}^2\}$$

$$= \rho\pi L\{(a+t)(b+t) - ab\} + u_1(g_1 + s_1{}^2) + u_2(g_2 + s_2{}^2) + u_3(g_3 + s_3{}^2) + u_4(g_4 + s_4{}^2)$$

- ◇ $$\frac{\partial L}{\partial a} = \rho\pi L t + u_1\left\{\frac{-4Mb(3b^2t + 3bt^2 + t^3)}{\pi[(a+b)(b+t)^3 - ab^3]^2} - \frac{F}{\pi}\cdot\frac{t}{(t^2 + at + bt)^2}\right\} + u_2\left\{\frac{V}{3\pi}\cdot\frac{t}{(t^2 + at + bt)^2}\right\} - u_3 = 0$$

- ◇ $$\frac{\partial L}{\partial b} = \rho\pi L t + u_1\left\{\frac{4M\left\{(a+b)(b+t)^3 - ab^3\right\} - 6tab^2 - 3abt^2 - 3tb^3 - 6t^2b^2 - 3t^3b}{\pi\left\{(a+b)(b+t)^3 - ab^3\right\}^2} - \frac{F}{\pi}\cdot\frac{t}{(t^2 + at + bt)^2}\right\}$$
  $$- u_2\left\{\frac{V}{3\pi}\cdot\frac{t}{(t^2 + at + bt)^2}\right\} - u_4 = 0$$

- ◇ $$\frac{\partial L}{\partial u_1} = \frac{4Mb}{\pi\left\{(a+b)(b+t)^3 - ab^3\right\}} - \frac{F}{\pi\{(a+t)(b+t) - ab\}} - \sigma_a + s_1{}^2 = 0$$

- ◇ $$\frac{\partial L}{\partial u_2} = \frac{4V}{3\pi\{(a+t)(b+t) - ab\}} - \tau_a + s_2{}^2 = 0$$

- ◇ $$\frac{\partial L}{\partial u_3} = -a + s_3{}^2 = 0$$

- ◇ $$\frac{\partial L}{\partial u_4} = -b + s_4{}^2 = 0$$

- ◇ $$\sum_{i=1}^{4}\frac{\partial L}{\partial s_i} = \sum_{i=1}^{4} u_i s_i = 0$$

- ◇ $u_i \geq 0$

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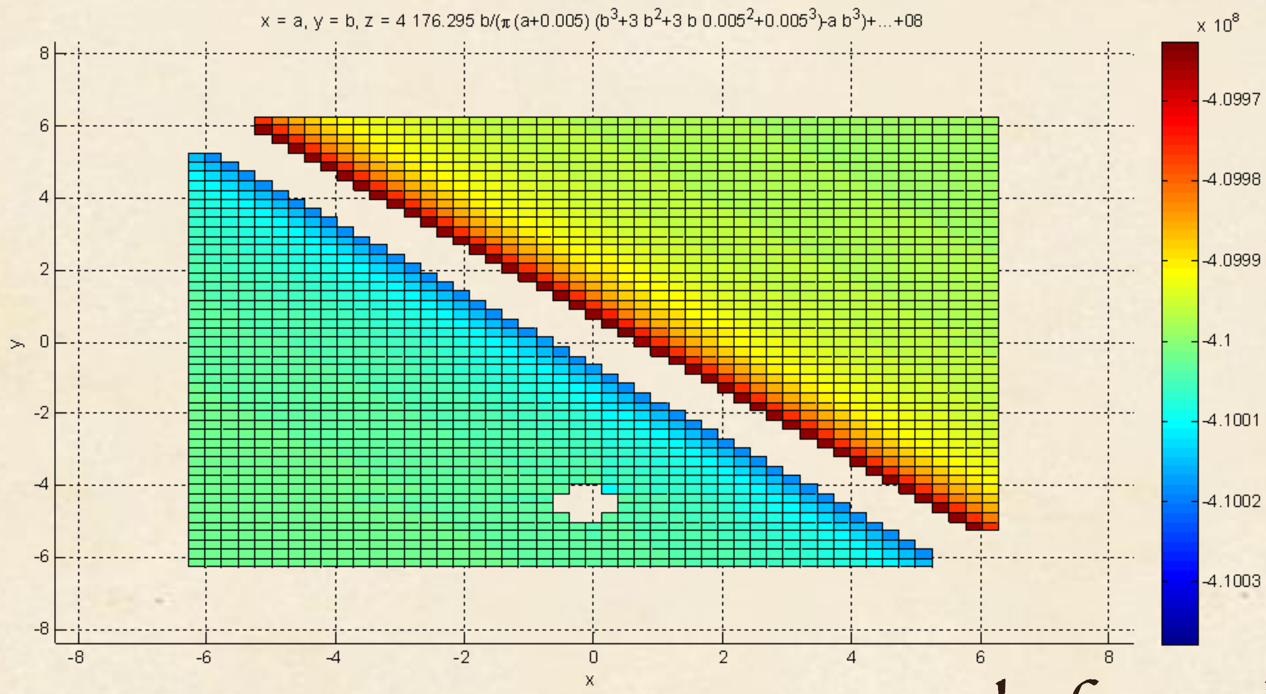
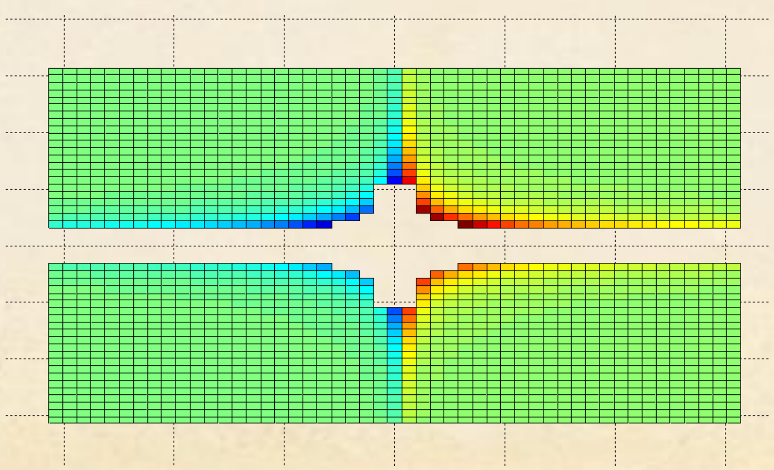
Karush-Kuhn-Tucker METHOD



Can't solve!!!!!!!!!

Infeasible problem..



Re-design optimization

Minimize $_{r,t}$ $\quad f = \rho A L$

$$\leftarrow A = 2\pi r t, \rho = 2710\,\text{kg/m}^3$$

Subeject to

$$\begin{cases} g_1 = \dfrac{Mr}{I} + \dfrac{F}{A} - \sigma_a \le 0 \\ g_2 = \dfrac{4}{3}\cdot\dfrac{V}{A} - \tau_a \le 0 \\ g_3 = -t + 0.003 \le 0 \end{cases}$$

$\sigma_a = 110\,\text{Mpa}$

$\tau_a = 68.9\,\text{Mpa}$

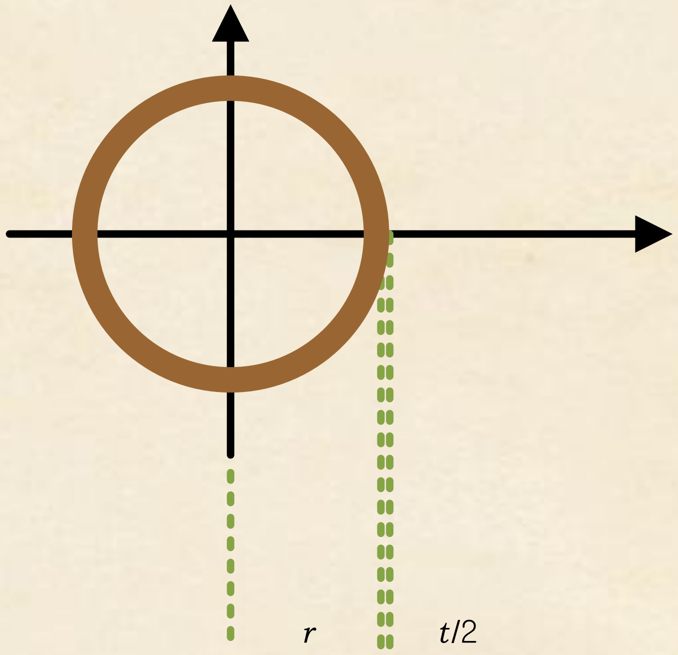
$I = \pi r^3 t$

F : Inner force

V : Shear force

M : $\text{Moment}_{\max}$

$\boxed{Safety\ factor = 10}$



Karush-Kuhn-Tucker

METHOD 2nd

$$\mathscr{L} = f + \sum_{j=1}^{4} u_j - \{g_j(x) + s_j^{\,2}\}$$

$$= 2\rho\pi Lrt + u_1(g_1 + s_1^{\,2}) + u_2(g_2 + s_2^{\,2}) + u_3(g_3 + s_3^{\,2})$$

- $\dfrac{\partial \mathscr{L}}{\partial r} = \rho\pi Lt + u_1\left(\dfrac{M}{\pi t}\cdot\dfrac{-2r}{r^4} - \dfrac{F}{2\pi t}\cdot\dfrac{1}{r^2}\right) - u_2\left(\dfrac{12V}{3\pi t}\cdot\dfrac{1}{r^2}\right)$
- $\dfrac{\partial \mathscr{L}}{\partial t} = \rho\pi Lt + u_1\left(\dfrac{M}{\pi r^2}\cdot\dfrac{-2r}{t^2} - \dfrac{F}{2\pi r}\cdot\dfrac{1}{t^2}\right) - u_2\left(\dfrac{2V}{3\pi r}\cdot\dfrac{1}{t^2}\right) - u_3$
- $\dfrac{\partial \mathscr{L}}{\partial u_1} = \dfrac{M}{\pi r^2 t} + \dfrac{F}{2\pi rt} - \sigma_a + s_1^{\,2} = 0$
- $\dfrac{\partial \mathscr{L}}{\partial u_2} = \dfrac{2V}{3\pi rt} - \tau_a + s_2^{\,2} = 0$
- $\dfrac{\partial \mathscr{L}}{\partial u_3} = -t + 0.005 + s_3^{\,2} = 0$

Can't solve!!!!!!!!!

We don't know WHY!!

- $\displaystyle\sum_{i=1}^{4}\frac{\partial L}{\partial s_i} = \sum_{i=1}^{4} u_i s_i = 0$
- $u_i \geq 0$

Optimization Result from graphical METHOD

Discrimination with EXCEL SOLVER'

t =	0.0030	m	all the same.
BEAM 1r=	0.0295	m	
BEAM 2r=	0.0246	m	
BEAM 3r=	0.0372	m	
BEAM 4r=	0.0261	m	
BEAM 5r=	0.0022	m	
BEAM 6r=	0.0027	m	

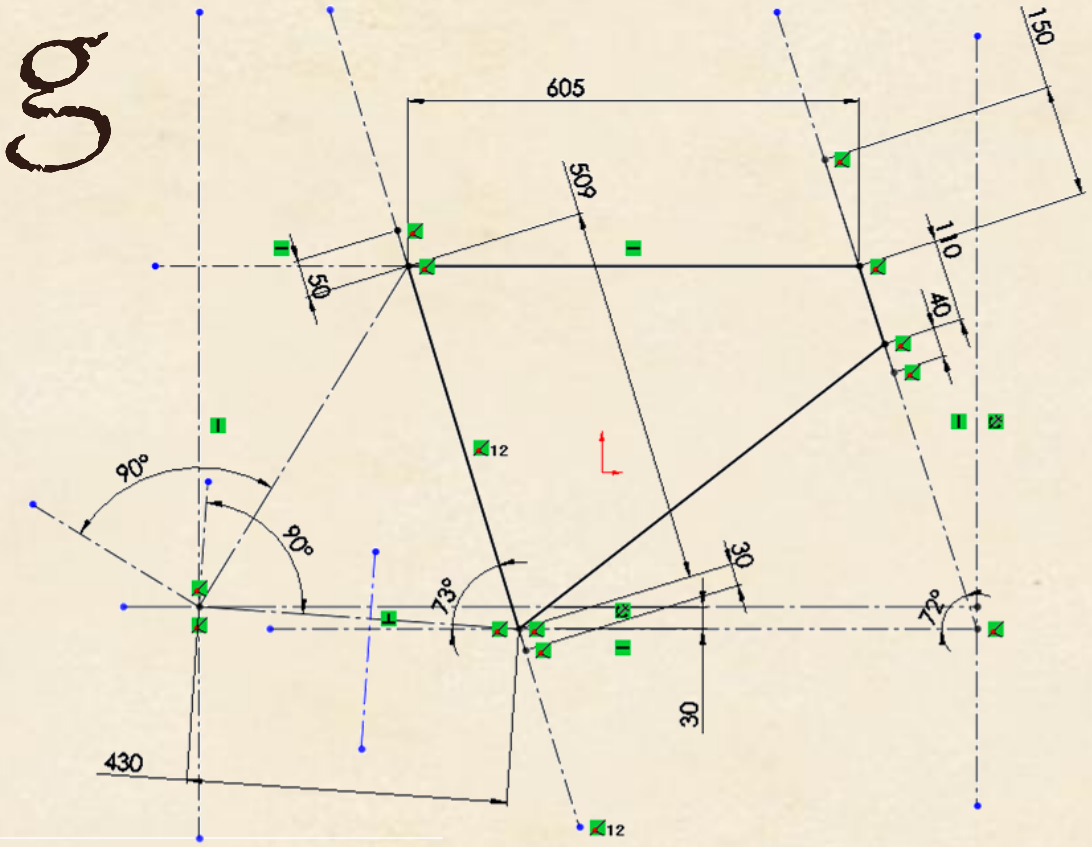
Conclusion

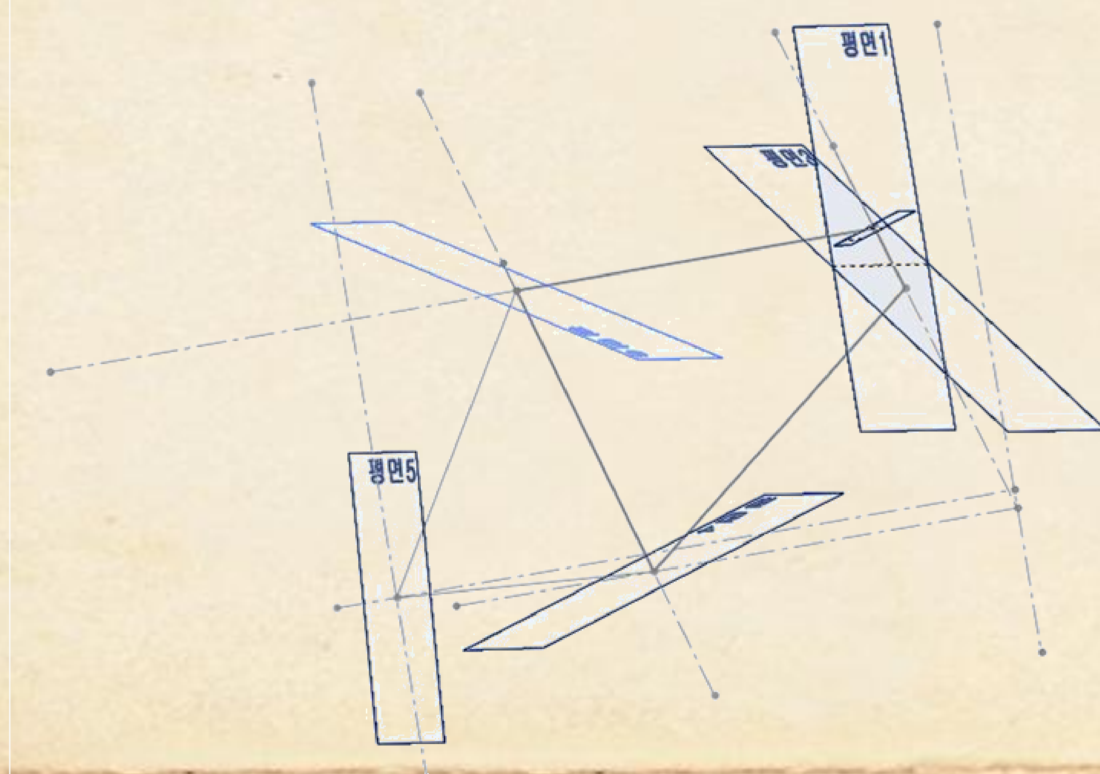
It's too hard to design

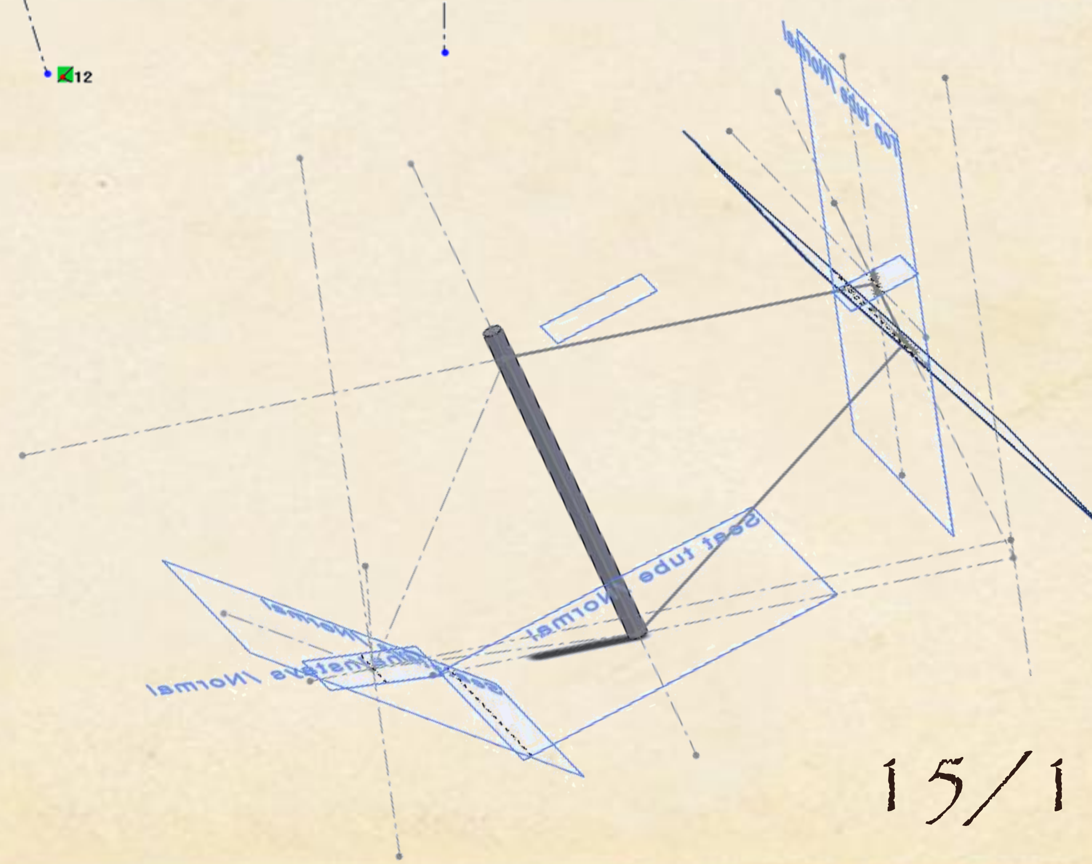
'BICYCLE FRAME'

※ Final Weight =3.19 kg

Modeling


605
509
150
110
40
50
90°
90°
73°
72°
30
30
430


평면1
평면5


Top tube Normal
Seat tube Normal

Rendering

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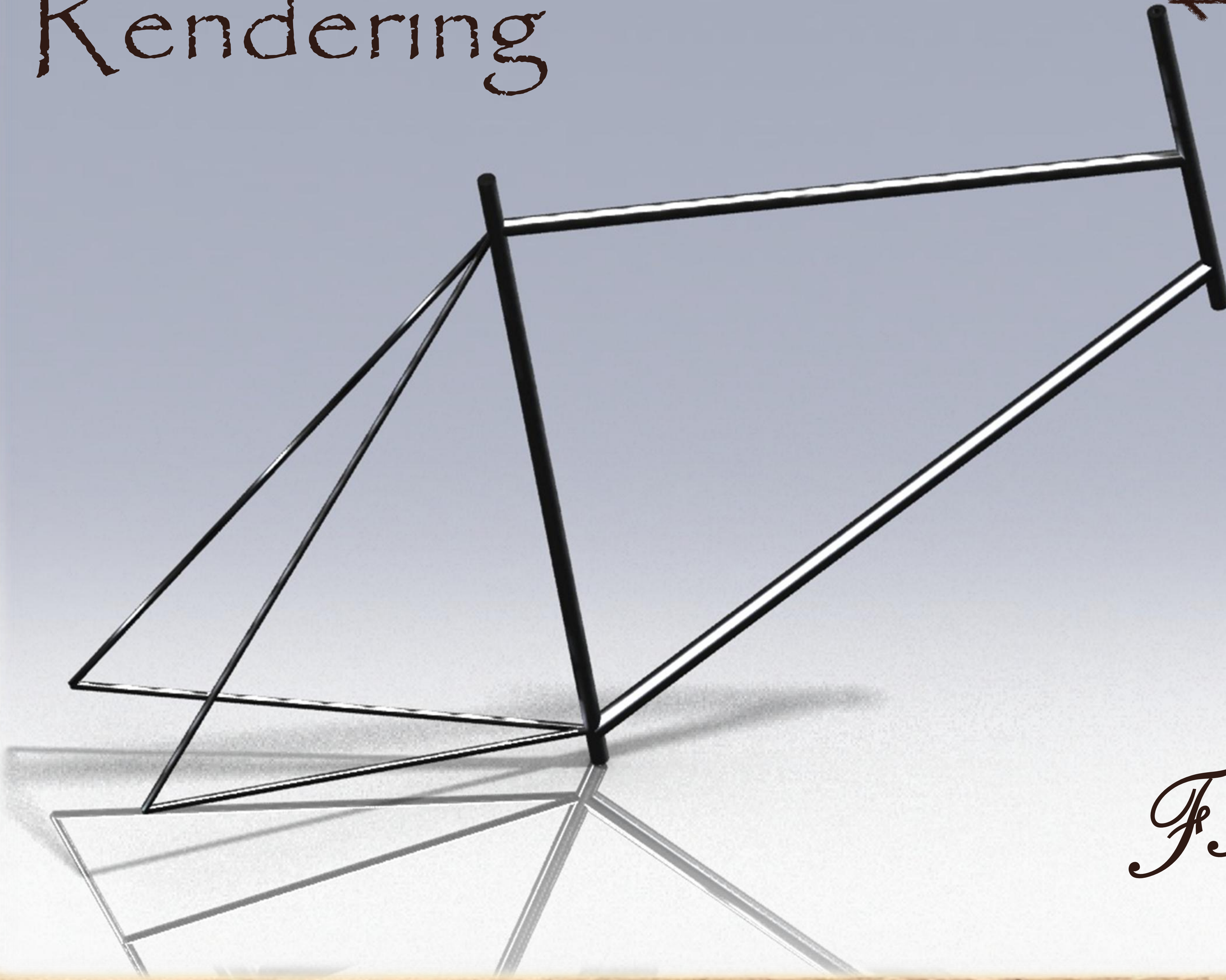
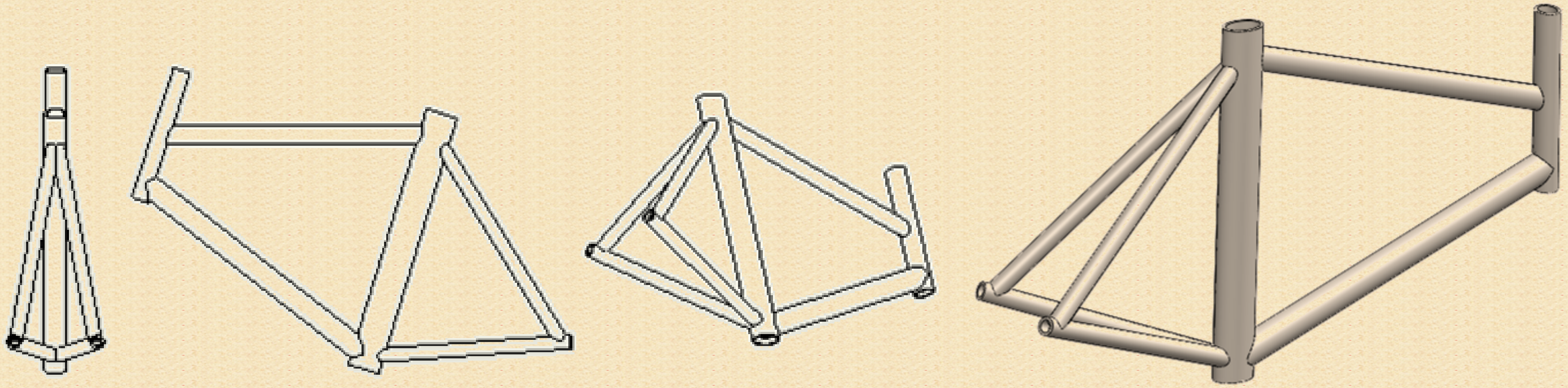
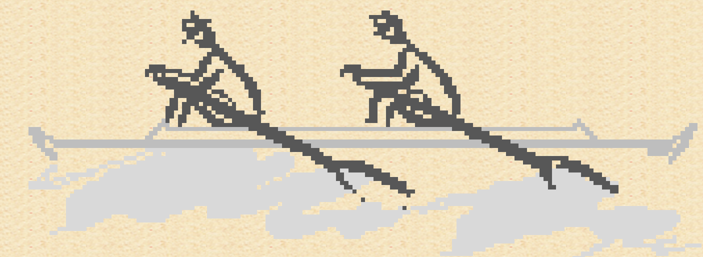


FIN

16/16





QUESTIONS?

Angle (rad)		
Angle (rad)	COx	1.02
	rOC-x	0.55
	DOx	0.07
	rDOx	1.50
	ABx	1.88
	CDB	1.27
	rCDB	0.30
	BAx	1.26
	rBAx	0.31
	DBx	2.48
	DB-x	0.66
	DBy	0.91
	OC-x	1.02
	OC-y	0.55
	DCA	1.27

Load (N)	P1_A	200.00
	P2_C	600.00
	P3_D	200.00
	Mz_A	40.00
	Mz_C	120.00

Point location from O (m)-x dir	A	0.88514
	B	0.91913
	C	0.28014
	D	0.42895

Length (m)	A-B	0.116
	A-C	0.605
	B-D	0.622
	C-D	0.509
	C-O	0.536
	D-O	0.430

	Output
Ry_B	614.12
Mz_B	-465.13
Ry_O	-14.12

σ_a=	4.10E+08	Pa
τ_a=	2.20E+08	Pa
ρ=	2.80E+03	kg/m³

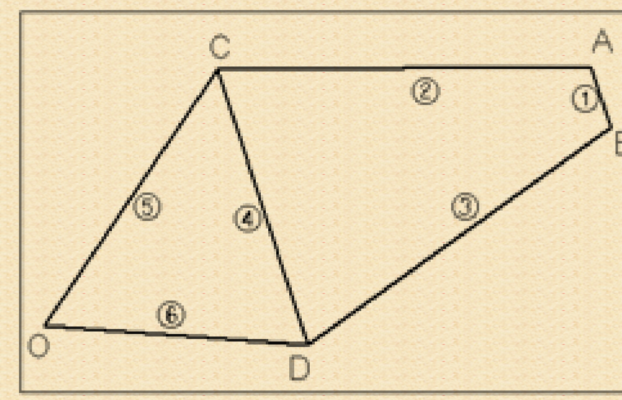
(t=0.005 m)

section	item	value(m)	A(m²)	Ixx(m⁴)	Mass(kg)
A-B	a	0.020	0.0012566	1.8113E-07	2.13
①	b	0.020			
A-C	a	0.020	0.0018850	4.1773E-07	0.61
②	b	0.030			
B-D	a	0.020	0.0025133	7.8393E-07	4.37
③	b	0.040			
C-D	a	0.020	0.0018850	4.1773E-07	2.83
④	b	0.030			
C-O	a	0.020	0.0012566	1.8113E-07	1.51
⑤	b	0.020			
D-O	a	0.014	0.0006158	7.2183E-08	0.88
⑥	b	0.014			

: Solved by 'find solution'

	Shear Force	Equation	Target:0
V_1	94.89	ΣF_x|BA	0.00
V_2	107.06	ΣF_y|A	
V_3	242.16	ΣF_x|BD	0.00
V_4	146.19	ΣF_x|D	0.00
V_5	3.69	ΣF_x|OC	0.00
V_6	7.04	ΣF_x|OD	0.00

	Inner Force	Equation
F_1	-292.05	
F_2	0.00	ΣF_x|A
F_3	188.79	
F_4	-478.15	
F_5	6.02	
F_6	-0.49	



section	item	M_max (N/m)	value(Pa)	Total axial stress
A-B	σ_centric	176.30	2.32E+05	1.97E+07
①	σ_bending		1.95E+07	
A-C	σ_centric	125.26	0.00E+00	9.00E+06
②	σ_bending		9.00E+06	
B-D	σ_centric	283.90	7.51E+04	1.46E+07
③	σ_bending		1.45E+07	
C-D	σ_centric	134.90	2.54E+05	9.94E+06
④	σ_bending		9.69E+06	
C-O	σ_centric	0.99	2.39E+03	1.12E+05
⑤	σ_bending		1.09E+05	
D-O	σ_centric	1.51	4.00E+02	2.94E+05
⑥	σ_bending		2.94E+05	

section	item	Q(m³)	value(Pa)	Total shear stress
A-B	τ_centric	5.0833E-06	7.55E+04	2.09E+05
①	τ_bending		1.33E+05	
A-C	τ_centric	8.4167E-06	5.68E+04	1.29E+05
②	τ_bending		7.19E+04	
B-D	τ_centric	1.2417E-05	9.64E+04	1.92E+05
③	τ_bending		9.59E+04	
C-D	τ_centric	8.4167E-06	7.78E+04	1.76E+05
④	τ_bending		9.82E+04	
C-O	τ_centric	5.0833E-06	1.47E+03	6.65E+03
⑤	τ_bending		5.18E+03	
D-O	τ_centric	2.7433E-06	5.72E+03	2.48E+04
⑥	τ_bending		1.91E+04	

Discrimination	Discrimination
σ<σ_a : OK	τ<τ_a : OK
σ<σ_a : OK	τ<τ_a : OK
σ<σ_a : OK	τ<τ_a : OK
σ<σ_a : OK	τ<τ_a : OK
σ<σ_a : OK	τ<τ_a : OK
σ<σ_a : OK	τ<τ_a : OK

Moment of at each point													
Route	O	C			D			A		B		B-Total	compare with Mb
		C-o	C+a	C+d	D-c	D-o	D+b	A-c	A+b	B-a	B-d		
OCAB	0.00	-0.49	-60.49					-125.26	-165.26	-176.30		-460.19	-4.94
OCDB	0.00	-0.49	-60.49	-134.90	-134.90		-134.90				-210.16		
ODB	0.00					1.51	1.51				-73.74		

95.24